basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

NOVEMBER 2010

MEMORANDUM

MARKS: 150

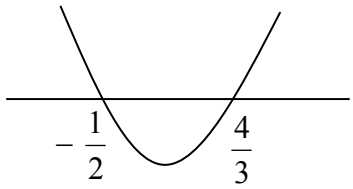
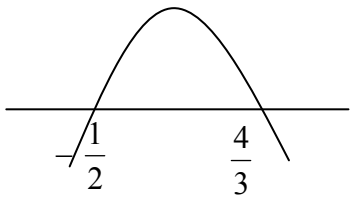
This memorandum consists of 27 pages.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent Accuracy applies in all aspects of the marking memorandum.

QUESTION 1

1.1.1	$(3-x)(5-x)=3$ $15-8x+x^2=3$ $x^2-8x+12=0$ $(x-6)(x-2)=0$ $x=6$ or $x=2$ **OR** $(3-x)(5-x)=3$ $15-8x+x^2=3$ $x^2-8x+12=0$ $(x-4)^2=4$ $x-4=2$ or $x-4=-2$ $x=6$ or $x=2$	**Note:** If answer only : Full Marks If the candidate makes it a linear equation, no marks For only 1 answer: 1 / 3	✓ expansion ✓ factors ✓ answers (3) ✓ expansion ✓ completed square form ✓ answers (3)
1.1.2	$3x^2=2(x+2)$ $3x^2-2x-4=0$ $x=\frac{-(-2)\pm\sqrt{(-2)^2-4(3)(-4)}}{2(3)}$ $=\frac{2\pm\sqrt{52}}{6}$ $x=1{,}54$ or $-0{,}87$ **OR** $3x^2=2(x+2)$ $3x^2-2x-4=0$ $x^2-\frac{2}{3}x=\frac{4}{3}$ $\left(x-\frac{1}{3}\right)^2=\frac{4}{3}+\frac{1}{9}$ $\left(x-\frac{1}{3}\right)^2=\frac{13}{9}$ $x-\frac{1}{3}=\pm\frac{\sqrt{13}}{3}$ $x=\frac{1\pm\sqrt{13}}{3}$ $x=1{,}54$ or $-0{,}87$	**Note:** No penalty for incorrect rounding off of answers. Substitution into incorrect formula, no marks	✓ standard form ✓✓ substitution ✓ answers (4) ✓ expansion ✓ completed square ✓ $\pm\frac{\sqrt{13}}{3}$ ✓ answers (4)

1.1.3	$4+5x>6x^2$ $0>6x^2-5x-4$ $0>(3x-4)(2x+1)$ critical values: $x=\frac{5\pm\sqrt{121}}{12}$ $x=-\frac{1}{2}$ or $\frac{4}{3}$ $+ \quad 0 \quad - \quad 0 \quad +$ $-\frac{1}{2} \qquad \frac{4}{3}$ $-\frac{1}{2}<x<\frac{4}{3}$ **OR** $x\in\left(-\frac{1}{2};\frac{4}{3}\right)$ **OR** $-\frac{1}{2}<x$ and $x<\frac{4}{3}$ **OR** $-6x^2+5x+4>0$ $(-3x+4)(2x+1)>0$ critical values: $-\frac{1}{2}$ and $\frac{4}{3}$ $- \quad 0 \quad + \quad 0 \quad -$ $-\frac{1}{2} \qquad \frac{4}{3}$ $-\frac{1}{2}<x<\frac{4}{3}$ **OR** $x\in\left(-\frac{1}{2};\frac{4}{3}\right)$ **OR** $-\frac{1}{2}<x$ and $x<\frac{4}{3}$	**OR** $-6x^2+5x+4>0$ $6x^2-5x-4<0$ $(3x-4)(2x+1)<0$  
		✓ correct inequality ✓ factors ✓ critical values $-\frac{1}{2}$ and $\frac{4}{3}$ ✓ answer (4) ✓ correct inequality ✓ factors ✓ critical values $-\frac{1}{2}$ and $\frac{4}{3}$ ✓ answer (4)
1.2	$3y=2x$ $y=\frac{2x}{3}$ $x^2-\left(\frac{2x}{3}\right)^2+2x-\left(\frac{2x}{3}\right)=1$ $x^2-\frac{4x^2}{9}+2x-\frac{2x}{3}=1$ $9x^2-4x^2+18x-6x=9$ $5x^2+12x-9=0$ $(5x-3)(x+3)=0$ $x=\frac{3}{5}$ or $x=-3$	**OR** $x^2-\left(\frac{2x}{3}\right)^2+2x-\left(\frac{2x}{3}\right)=1$ $\frac{5x^2}{9}+\frac{4x}{3}-1=0$ $x=\frac{-\frac{4}{3}\pm\sqrt{\left(\frac{4}{3}\right)^2-4\left(\frac{5}{9}\right)(-1)}}{2\left(\frac{5}{9}\right)}$ $=\frac{-\frac{4}{3}\pm\sqrt{\frac{16}{9}+\frac{20}{9}}}{\frac{10}{9}}$ $x=0{,}6$ or $x=-3$
		✓ $y=\frac{2x}{3}$ ✓ substitution ✓ simplification ✓ standard forms ✓ factors or substitution into correct formula ✓ x-answers ✓ y-answers

$y = \frac{2}{5}$ or $y = -2$ $\quad$ $y = 0{,}4$ or $y = -2$ (7)

$(x\,;y) = \left(\frac{3}{5};\frac{2}{5}\right)$ or $(-3\,;-2)$

OR

$3y = 2x$

Note:
If mathematical breakdown
eg. if $y = 2x - 3$ is used, max 3 / 7

$x^2 - y^2 + 2x - y = 1$ ✓ simplification of original quadratic

$4x^2 - 4y^2 + 8x - 4y = 4$

$(2x)^2 - 4y^2 + 8x - 4y = 4$ ✓ substitution $2x = 3y$

$(3y)^2 - 4y^2 + 4(3y) - 4y = 4$ ✓ simplification

$9y^2 - 4y^2 + 8y = 4$ ✓ standard form

$5y^2 + 8y - 4 = 0$

$(5y - 2)(y + 2) = 0$ ✓ factors or substitution into correct formula

$y = \frac{2}{5}$ or $y = -2$ ✓ y-answers

$x = \frac{3}{5}$ or $x = -3$ ✓ x-answers

$(x\,;y) = \left(\frac{3}{5};\frac{2}{5}\right)$ or $(-3\,;-2)$ (7)

OR

$3y = 2x$ ✓ $x = \frac{3y}{2}$

$x = \frac{3y}{2}$

$\left(\frac{3y}{2}\right)^2 - y^2 + 2\left(\frac{3y}{2}\right) - y = 1$ ✓ substitution

$\frac{9y^2}{4} - y^2 + 3y - y = 1$ ✓ simplification

$9y^2 - 4y^2 + 8y = 4$ ✓ standard forms

$5y^2 + 8y - 4 = 0$

$(5y - 2)(y + 2) = 0$ ✓ factors or substitution into correct formula

$y = \frac{2}{5}$ or $y = -2$ ✓ y-answers

$x = \frac{3}{5}$ or $x = -3$ ✓ x-answers

$(x\,;y) = \left(\frac{3}{5};\frac{2}{5}\right)$ or $(-3\,;-2)$ (7)

1.3

$$\frac{5^{2007}+5^{2010}}{5^{2008}+5^{2009}}$$
$$=\frac{5^{2007}+5^{2007}.5^3}{5^{2008}+5^{2008}.5}$$
$$=\frac{5^{2007}(1+5^3)}{5^{2008}(1+5)}$$
$$=\frac{126}{5\times 6}$$
$$=\frac{126}{30}$$
$$=\frac{21}{5}$$
$$=4\frac{1}{5}$$
$$\approx 4$$

Note: If the candidate leaves the answer as 4,2 **max 2 / 3 marks**

Answer only of 4,2 **0 / 3 marks**

✓ $\frac{5^{2007}+5^{2007}.5^3}{5^{2008}.5+5^{2008}.5^2}$

✓ simplification to $\frac{1+5^3}{5+5^2}$ or $\frac{126}{30}$ or $\frac{21}{5}$

✓ answer = 4

(3)

OR

$$\frac{5^{2007}+5^{2010}}{5^{2008}+5^{2009}} \quad \text{(divide each term by } 5^{2007}\text{)}$$
$$=\frac{5^{2007}+5^{2007}.5^3}{5^{2007}.5+5^{2007}.5^2}$$
$$=\frac{1+5^3}{5+5^2}$$
$$=\frac{126}{30}$$
$$\approx 4$$

✓ $\frac{5^{2007}+5^{2007}.5^3}{5^{2007}.5+5^{2007}.5^2}$

✓ simplification to $\frac{1+5^3}{5+5^2}$ or $\frac{126}{30}$ or $\frac{21}{5}$

✓ answer = 4

OR

let $x = 2009$

$$\frac{5^{x-2}+5^{x+1}}{5^{x-1}+5^x}$$
$$=\frac{5^x(5^{-2}+5)}{5^x(5^{-1}+1)}$$
$$=\frac{\frac{1}{25}+5}{\frac{1}{5}+1}$$
$$=\frac{21}{5}$$
$$=4\frac{1}{5}$$
$$\approx 4$$

OR

let $x = 2007$

$$\frac{5^x+5^{x+3}}{5^{x+1}+5^{x+2}}$$
$$=\frac{5^x(1+5^3)}{5^x(5+5^2)}$$
$$=\frac{1+125}{5+25}$$
$$=\frac{126}{30}$$
$$=\frac{21}{5}$$
$$=4\frac{1}{5}$$
$$\approx 4$$

OR

let $x = 2010$

$$\frac{5^{x-3}+5^x}{5^{x-2}+5^{x-1}}$$
$$=\frac{5^x(5^{-3}+1)}{5^x(5^{-2}+5^{-1})}$$
$$=\frac{\frac{1}{125}+1}{\frac{1}{25}+\frac{1}{5}}$$
$$=\frac{126}{30}$$
$$=\frac{21}{5}$$
$$=4\frac{1}{5}$$
$$\approx 4$$

✓ $\frac{5^{x-2}+5^{x+1}}{5^{x-1}+5^x}$ or $\frac{5^x+5^{x+3}}{5^{x+1}+5^{x+2}}$ or $\frac{5^{x-3}+5^x}{5^{x-2}+5^{x-1}}$

✓ simplification to $\frac{\frac{1}{25}+5}{\frac{1}{5}+1}$ or $\frac{1+125}{5+25}$ or $\frac{\frac{1}{125}+1}{\frac{1}{25}+\frac{1}{5}}$

✓ answer = 4

(3)

[21]

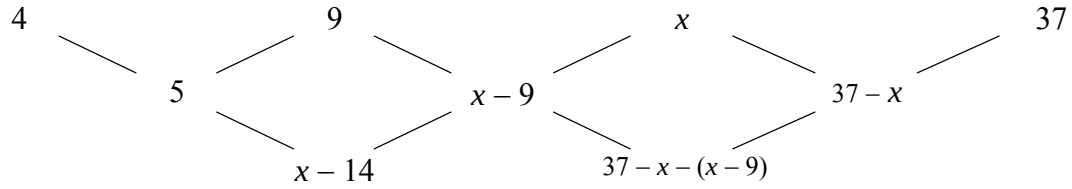
QUESTION 2

2.1	$\sum_{n=1}^{20} 3^{n-2}$ $= \frac{1}{3} + 1 + 3 + \ldots$ to 20 terms $= \frac{\frac{1}{3}\left(3^{20}-1\right)}{3-1}$; $r = 3$; $n = 20$ $= \frac{3^{20}-1}{6}$ $= 581130733{,}33$ OR $581130733\frac{1}{3}$ OR $581130733{,}3$ **Note:** The mark for $n = 20$ can be implied in the substitution to the formula **Note:** If leave only as $\frac{1}{3} + 1 + 3 + 9 + 27 + 81 + 243 + 729 + 2187 + 6561 + 19683 + 59049 + 177147 + 531441 + 1594323 + 4782969 + 14348907 + 43046721 + 129140163 + 387420489$ **only**, then 2 / 4 **Note**: The 20th term is **387 420 489** **Answer only:** 3 / 4 marks	✓ $a = \frac{1}{3}$ ✓ $r = 3$ ✓ $n = 20$ ✓ answer (4)
2.2.1	$5x$; x^2 ; $\frac{x^3}{5}$; … $r = \frac{x}{5}$ $-1 < \frac{x}{5} < 1$ $-5 < x < 5$ **Answer can be written as** $x \in (-5\,;\,5)$ **Note:** If $-1 < x < 1$ 1 mark **Note:** If answer is $-5 \le x \le 5$ then 2 / 3	✓ $r = \frac{x}{5}$ or $\frac{x^2}{5x}$ ✓ $-1 < r < 1$ ✓ answer (3)
2.2.2	$r = \frac{2}{5}$ and $a = 10$ $S_\infty = \frac{10}{1-\frac{2}{5}}$ $= \frac{50}{3}$ or 16,67	✓ $a = 10$ ✓ answer (2)

2.3.1	$T_n = 20 + 3(n-1)$ $101 = 20 + (n-1)3$ $84 = 3n$ $n = 28$ **OR** $T_n = 3n + 17$ $101 = 3n + 17$ $84 = 3n$ $n = 28$	**Note:** If $n = -\frac{17}{3}$ Then 1 / 2 marks **Answer only**: Full marks	✓ $101 = 20 + 3(n-1)$ or $101 = 3n + 17$ ✓ answer (2) ✓ substitution ✓ answer (2)
2.3.2	$23 + 29 + \ldots$ to 14 terms $= \frac{14}{2}[2(23) + (14-1)6]$ OR $\frac{14}{2}[23 + 101]$ $= 868$ **OR** Even numbers = 20 ; 26 ;…; 98 $T_n = 6n + 14$ **OR** $T_n = 20 + (n-1)6$ $98 = 6n + 14$ **OR** $98 = 20 + (n-1)6$ $84 = 6n$ **OR** $84 = 6n$ $14 = n$ **OR** $14 = n$ $S_{remaining} = \frac{28}{2}[2(20) + (27)(3)] - \frac{14}{2}[2(20) + (13)(6)]$ $= 14(121) - 7(118)$ $= 1694 - 826$ $= 868$ **OR** Sequence is ~~20~~; 23; ~~26~~; 29; ~~32~~; 35; ~~38~~; 41; ~~44~~; 47; ~~50~~; 53; ~~56~~; 59; ~~62~~; 65; ~~68~~; 71; ~~74~~; 77; ~~80~~; 83; ~~86~~; 89; ~~92~~; 95; ~~98~~; 101 Sum of odd numbers = 23 + 29 + 35 + 41 + 47 + 53 + 59 + 65 + 71 + 77 + 83 + 89 + 95 + 101 = 868	**Note:** If "to 14 terms" is left out, do not penalise **Note:** If incorrect value for n, max 4 / 6 **Note:** If incorrect formula, max 2 / 6 **Note:** If the candidate only works out the even numbers i.e. 826, then 3 / 6 marks If only 1694 max 1 / 6 marks	✓ $23 + 29 + \ldots$ ✓ $a = 23$ ✓ $n = 14$ ✓ $d = 6$ or $l = 101$ ✓ substitution into correct formula ✓ answer (6) **OR** ✓ $98 = 6n + 14$ or $98 = 20 + (n-1)$ ✓ $14 = n$ ✓substitution into correct formula ✓1694 ✓ 826 ✓ answer (6) Full marks (6) **[17]**

QUESTION 3

3.1



First difference : $5;\ x-9;\ 37-x$

Second difference : $x-14;\ -2x+46$

$$x-14=46-2x$$
$$3x=60$$
$$x=20$$

Note:
Answer only:
Full Marks

✓ first differences $5;\ x-9;\ 37-x$
✓ seconds difference
✓ answer
(3)

OR

$$(x-9)+(x-14)=37-x$$
$$2x-23=37-x$$
$$3x=60$$
$$x=20$$

OR

$$x+(x-9)+(x-14)=37$$
$$3x-23=37$$
$$3x=60$$
$$x=20$$

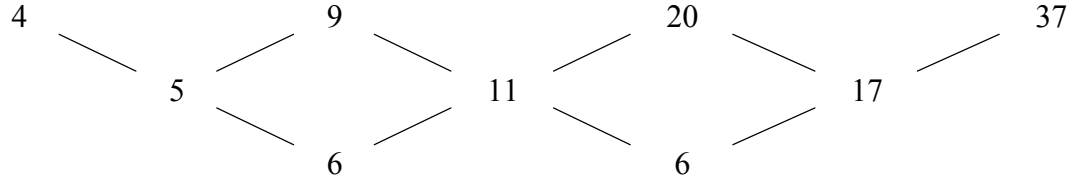
✓ equating
✓ manipulation
✓ answer
(3)

OR

$$(x-9)-5=(37-x)-(x-9)$$
$$x-14=-2x+46$$
$$3x=60$$
$$x=20$$

✓ first differences $5;\ x-9;\ 37-x$
✓ equating
✓ answer
(3)

3.2



Note:
If x is incorrect in 3.1 then
max 2 / 4 marks

$2a=6$
$a=3$
✓ $a=3$

$T_n=3n^2+bn+c$
$3+b+c=4 \quad \ldots T_1$
$b+c=1$
$12+2b+c=9 \quad \ldots T_2$
$2b+c=-3$
$\therefore 9+b=5$
$b=-4$
and
$c=4-(-1)=5$
$\therefore T_n=3n^2-4n+5$

✓ $T_n=3n^2+bn+c$

✓ $b=-4$
✓ $c=5$
(4)

OR

$2a = 6$
$a = 3$
$T_0 = 5$
$c = 5$
$T_n = 3n^2 + bn + 5$
$4 = 3(1)^2 + b + 5$
$b = -4$
$T_n = 3n^2 - 4n + 5$

OR

$a + b + c = 4 \quad \ldots i$
$4a + 2b + c = 9 \quad \ldots ii$
$16a + 4b + c = 37 \quad \ldots iii$
$3a + b = 5$
$12a + 2b = 28$
$6a + b = 14$
$3a = 9$
$a = 3$
$b = -4$
$c = 5$
$T_n = 3n^2 - 4n + 5$

OR

$2a = 6$
$a = 3$
$3a + b = 5$
$b = -4$
$a + b + c = 4$
$3 - 4 + c = 4$
$c = 5$
$T_n = 3n^2 - 4n + 5$

OR

$T_n = 4 + (n-1)5 + \frac{6(n-1)(n-2)}{2}$
$= 4 + 5n - 5 + 3n^2 - 9n + 6$
$= 3n^2 - 4n + 5$

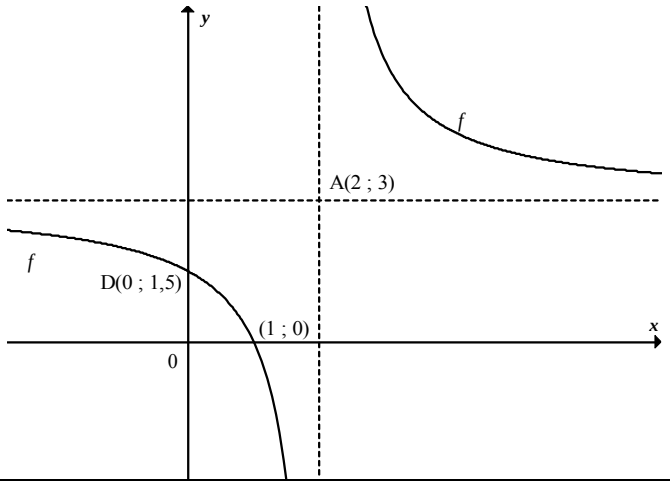
OR

✓ $a = 3$
✓ $c = 5$
✓ method
✓ $b = -4$ (4)

✓ $a = 3$
✓ $c = 5$
✓ method
✓ $b = -4$ (4)

[7]

QUESTION 4



4.1	$x = 2$ **OR** x-asymptote = 2 $y = 3$ y-asymptote = 3 If $x = p$; $y = q$ then 1 mark **Note:** If the candidate just writes down the number 2 or 3 or just coordinates (2 ; 3), then no marks	✓ answer ✓ answer (2)
4.2	$f(x) = \frac{a}{x-2} + 3$ $0 = \frac{a}{1-2} + 3$ $0 = -a + 3$ $a = 3$ $f(x) = \frac{3}{x-2} + 3$ **OR** $y = \frac{a}{x-2} + 3$ $y - 3 = \frac{a}{x-2}$ $(x-2)(y-3) = a$ But (1;0) lies on the graph $\therefore (-1)(-3) = a = 3$ $\therefore (x-2)(y-3) = 3$ If the asymptotes are swopped in 4.1, then $f(x) = \frac{a}{x-3} + 2$ $0 = \frac{a}{1-3} + 2$ $a = 4$ $f(x) = \frac{4}{x-3} + 2$	✓subs in of asymptotes ✓ subs in (1 ; 0) ✓ answer (3) ✓ equation ✓ subs in (1 ; 0) ✓ answer (3)
4.3	When $x = 0$, $y = \frac{3}{0-2} + 3$ $= \frac{3}{2}$ $D\left(0 ; \frac{3}{2}\right)$ If asymptotes swopped: $x = 0$ $y = \frac{4}{0-3} + 2$ $y = \frac{2}{3}$ $D\left(0 ; \frac{2}{3}\right)$	✓ $x = 0$ ✓ $y = \frac{3}{2}$ (2)

4.4	$m_{AD} = \frac{3-1,5}{2-0}$ $= \frac{3}{4}$ $y = \frac{3}{4}x + \frac{3}{2}$ **OR** $4y = 3x + 6$ **OR** $y = mx + \frac{3}{2}$ $3 = m(2) + \frac{3}{2}$ $m = \frac{3}{4}$ $y = \frac{3}{4}x + \frac{3}{2}$ If asymptotes swopped: $m_{AD} = \frac{3-\frac{2}{3}}{2-0}$ $= \frac{7}{3} \times \frac{1}{2}$ $= \frac{7}{6}$ $y = \frac{7}{6}x + \frac{2}{3}$	✓ substitution into gradient ✓ $\frac{3}{4}$ ✓ answer (3) ✓ substitution of point (2 ; 3) and $c = \frac{3}{2}$ ✓ $\frac{3}{4}$ ✓ answer (3)
4.5	$\frac{p+0}{2} = 2$ $p = 4$ $\frac{q+\frac{3}{2}}{2} = 3$ $q = 4\frac{1}{2}$ Other point of intersection is $\left(4\ ;\ 4\frac{1}{2}\right)$ **OR** By symmetry the rule to calculate the point of intersection is $(x\ ;\ y) \rightarrow \left(x+2\ ;\ y+\frac{3}{2}\right)$ Other point of intersection is $\left(2+2\ ;\ 3+\frac{3}{2}\right)$ $= \left(4\ ;\ 4\frac{1}{2}\right)$ **Answer only:** Full Marks To help with applying CA the y-coordinate will be $3 + (3 - y)$	✓ $\frac{p+0}{2} = 2$ ✓ $\frac{q+\frac{3}{2}}{2} = 3$ ✓ $x = 4$ ✓ $y = 4\frac{1}{2}$ (4) ✓✓ x-answer ✓✓ y-answer (4)

OR

$$\frac{3}{4}x+\frac{3}{2}=\frac{3}{x-2}+3$$
$$3x(x-2)+6(x-2)=12+12(x-2)$$
$$3x^2-6x+6x-12=12+12x-24$$
$$3x^2-12x=0$$
$$3x(x-4)=0$$
$$x=0 \text{ and } x=4$$

Other point of intersection is $\left(4\,;4\frac{1}{2}\right)$

Note:
If the candidate does not select the *x*-value greater than 2 i.e. a realistic answer, max 3 / 4 marks

If asymptotes swopped:

$$\frac{7}{6}x+\frac{2}{3}=\frac{4}{x-3}+2$$
$$7x(x-3)+4(x-3)=4(6)+2(6)(x-3)$$
$$7x^2-29x=0$$
$$x(7x-29)=0$$
$$x=0 \quad \text{or} \quad x=\frac{29}{7}$$

Other point of intersection is $\left(\frac{29}{7};\frac{11}{2}\right)$

✓ equating
✓ standard form
✓ *x*-values
✓ *y*-value (4)

[14]

QUESTION 5

5.1

$f(x)=4^{-x}-2$

y-intercept: $x=0;\ y=4^0-2=-1\ ;\ (0\,;-1)$

x-intercept:

$$4^{-x}-2=0$$
$$4^{-x}=2$$
$$2^{-2x}=2$$
$$-2x=1$$
$$x=-\frac{1}{2}$$

OR

$$4^{-x}-2=0$$
$$4^{-x}=2$$
$$\log 4^{-x}=\log 2$$
$$-x=\frac{\log 2}{\log 4}$$
$$-x=\frac{\log 2}{2\log 2}$$
$$x=-\frac{1}{2}$$

OR

$$4^{-x}-2=0$$
$$4^{-x}=2$$
$$4^{-x}=4^{\frac{1}{2}}$$
$$-x=\frac{1}{2}$$
$$x=-\frac{1}{2}$$

x-intercept is $\left(-\frac{1}{2}\,;0\right)$

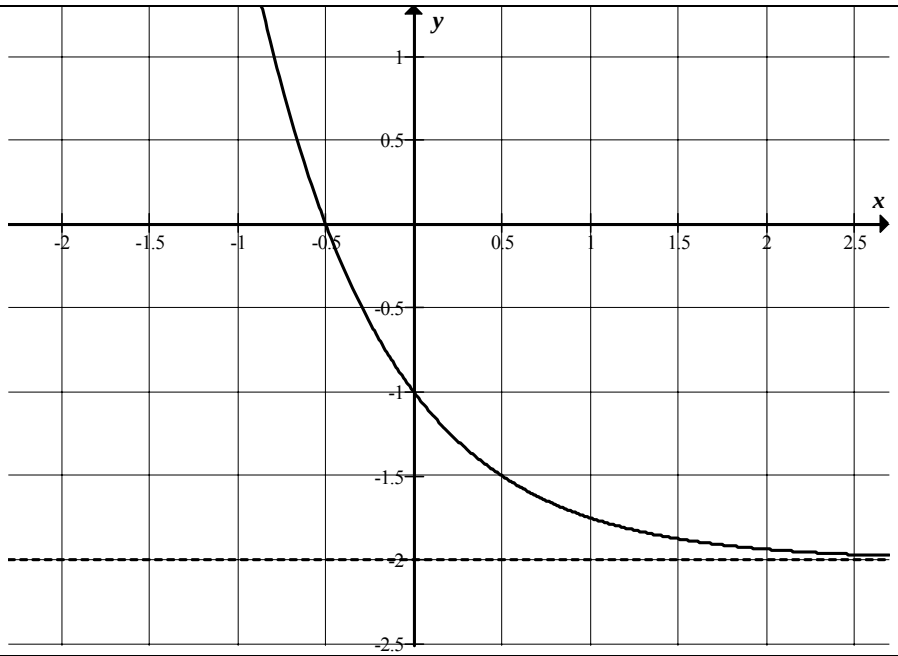
✓✓ *y*-intercept
✓✓ *x*-intercept (4)

Note:
No penalty if the answer is not left as a coordinate.

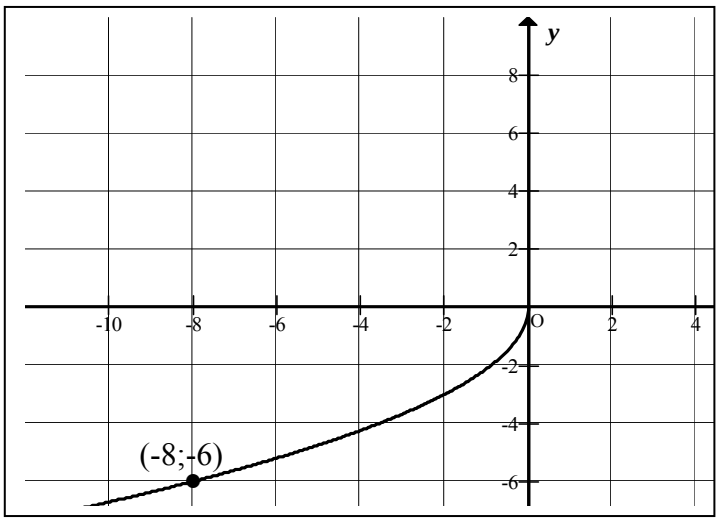
5.2

$y=-2$

✓ equation (1)

5.3		✓ asymptote ✓ *y*-intercept or *x*-intercept ✓ shape (decreasing) (3)
5.4	$g(x) = 4^{-x} - 2 + 2$ $g(x) = 4^{-x}$ **OR** $g(x) = \left(\frac{1}{4}\right)^{x}$ **OR** $g(x) = 2^{-2x}$ **OR** $g(x) = \left(\frac{1}{2}\right)^{2x}$	✓ equation (1)
5.5	$4^{-x} - 2 = 3$ $4^{-x} = 5$ $-x\log 4 = \log 5$ $x = -\frac{\log 5}{\log 4}$ **OR** $x = -\log_4 5$ **OR** $x = \log_{\frac{1}{4}} 5$ **OR** $x = \log_4 \frac{1}{5}$ **OR** $x = -1{,}16$ **OR** $x = \frac{\log 5}{\log \frac{1}{4}}$ **OR** $x = \frac{\log \frac{1}{5}}{\log 4}$	✓ $4^{-x} = 5$ ✓ $-x\log 4 = \log 5$ ✓ answer (3) **[12]**

QUESTION 6

6.1	$f(x) = ax^2$ $-8 = a(-6)^2$ $-8 = 36a$ $a = -\frac{8}{36}$ **OR** $a = -\frac{2}{9}$	✓ substitution ✓ answer (2)
6.2	$f(x): \quad y = -\frac{2}{9}x^2$ $x = -\frac{2}{9}y^2$ $9x = -2y^2$ $-\frac{9x}{2} = y^2$ $y = \pm\sqrt{\frac{-9x}{2}}, \quad$ since $\quad y \le 0$ $y = -\sqrt{-\frac{9x}{2}}$ **OR** $y = -3\sqrt{\frac{-x}{2}}$ **Note:** If candidate does not substitute the value of a the answer is $y = -\sqrt{\frac{x}{a}}$ then 2 / 3 marks	✓ swop x and y ✓ $y^2 = -\frac{9x}{2}$ or $y = \pm\sqrt{\frac{-9x}{2}}$ ✓ $y = -\sqrt{-\frac{9x}{2}}$ (3)
6.3	$y \le 0$ **OR** $y \in (-\infty ; 0]$	✓ answer (1)
6.4		✓ shape (third quadrant) (concave upward) ✓ Any point other than (0 ; 0) that lies on the graph Point corresponding from original graph will be (– 8 ; – 6) (2)

6.5	$y = -f^{-1}(x)$ $= \sqrt{\frac{-9x}{2}}$ **OR** $y = -f^{-1}(x)$ $= 3\sqrt{\frac{-x}{2}}$ **OR** $y = -\frac{2}{9}x^2$ Reflection in $y = x$: $x = -\frac{2}{9}y^2$ $-\frac{9}{2}x = y^2$ $y = -\sqrt{-\frac{9x}{2}}$ Reflection about y-axis: $y = \sqrt{-\frac{9x}{2}}$ **Note:** If candidate has $(x\ ;\ y) \rightarrow (y\ ;\ -x)$ then 2 / 3 marks **Note:** If candidate does not substitute the value of a the answer is $y = \sqrt{\frac{x}{a}}$ then full marks	✓✓ $-f^{-1}(x)$ ✓ answer (3) ✓ $x = -\frac{2}{9}y^2$ ✓ $y = -\sqrt{-\frac{9x}{2}}$ ✓ $y = \sqrt{-\frac{9x}{2}}$ (3) **[11]**

QUESTION 7

7.1	$A = P(1+i)^n$ $2P = P\left(1+\frac{r}{4}\right)^{6\times 4}$ $2 = \left(1+\frac{r}{4}\right)^{24}$ $1+\frac{r}{4} = 2^{\frac{1}{24}}$ $r = 4\left(2^{\frac{1}{24}}-1\right)$ $r = 4\left(2^{\frac{1}{24}}\right)-4$ $r = 0{,}1172\ldots$ rate = 11,72% p.a. compounded quarterly **OR** $A = P(1+i)^n$ $2P = P\left(1+\frac{r}{400}\right)^{6\times 4}$ $2 = \left(1+\frac{r}{400}\right)^{24}$ $1+\frac{r}{400} = 2^{\frac{1}{24}}$ $r = 400\left(2^{\frac{1}{24}}-1\right)$ $r = 400\left(2^{\frac{1}{24}}\right)-400$ $r = 11{,}72\%$ p.a.	**Note:** Penalty 1 for incorrect rounding off. ✓ 2P ✓ $\frac{r}{4}$ and 24 ✓ $1+\frac{r}{4} = 2^{\frac{1}{24}}$ ✓ $r = 4\left(2^{\frac{1}{24}}\right)-4$ ✓ answer (5) ✓ 2P ✓ $\frac{r}{400}$ and 24 ✓ $1+\frac{r}{400} = 2^{\frac{1}{24}}$ ✓ $r = 400\left(2^{\frac{1}{24}}\right)-400$ ✓ answer (5)
7.2.1	$A = 10000\left(1+\frac{0{,}095}{12}\right)^5$ = R 10 402,15	✓ substitution in correct formula ✓ answer (2)

7.2.2

$$10402{,}15 = \frac{450\left[1-\left(1+\frac{0{,}095}{12}\right)^{-n}\right]}{\frac{0{,}095}{12}}$$

✓ 10 402,15
✓ substitution into present value formula

$$0{,}183000787 = 1-\left(1+\frac{0{,}095}{12}\right)^{-n}$$

Note:
Incorrect Formula
No marks

$$\left(1+\frac{0{,}095}{12}\right)^{-n} = 0{,}816999213$$

$$\log\left(1+\frac{0{,}095}{12}\right)^{-n} = \log 0{,}816999213$$

✓ application of logs

$$-n\log\left(1+\frac{0{,}095}{12}\right) = \log 0{,}816999213...$$

$$n = 25{,}63151282....$$

$$n = 25{,}63 \text{ } months$$

$$n = 26$$

✓ answer

Accept: $n = 31$ (because of first 5 months)

(4)

OR

$$10402{,}15\left(1+\frac{0{,}095}{12}\right)^{n} = \frac{450\left[\left(1+\frac{0{,}095}{12}\right)^{n}-1\right]}{\frac{0{,}095}{12}}$$

✓ 10 402,15
✓ substitution into future value formula

$$10402{,}15\left(1+\frac{0{,}095}{12}\right)^{n} = 56842{,}10526\left[\left(1+\frac{0{,}095}{12}\right)^{n}-1\right]$$

$$56842{,}10526 = 46439{,}95526\left(1+\frac{0{,}095}{12}\right)^{n}$$

$$\log 1{,}223991387 = n\log\left(1+\frac{0{,}095}{12}\right)$$

✓ application of logs

$$n = \frac{\log 1{,}223991387}{\log\left(1+\frac{0{,}095}{12}\right)}$$

$$n = 25{,}63 \quad \text{months}$$

$$n = 26$$

✓ answer

Accept: $n = 31$ (because of first 5 months)

(4)

Note: If the Present value of R 10 000 is used, then $n = 25{,}53$ months is obtained. Max 3 / 4 marks.

7.2.3

Balance outstanding after 25 months

$$= 10402{,}15\left(1+\frac{0{,}095}{12}\right)^{25} - \frac{450\left[\left(1+\frac{0{,}095}{12}\right)^{25}-1\right]}{\frac{0{,}095}{12}}$$

= R 282,36

✓ correct formula
✓ substitution into $\frac{450\left[\left(1+\frac{0{,}095}{12}\right)^{25}-1\right]}{\frac{0{,}095}{12}}$
✓ answer

(3)

OR

Balance Outstanding after 25 months

$$= 10000\left(1+\frac{0{,}095}{12}\right)^{30} - \frac{450\left[\left(1+\frac{0{,}095}{12}\right)^{25}-1\right]}{\frac{0{,}095}{12}}$$

= R 282,36

✓ correct formula
✓ $\frac{450\left[\left(1+\frac{0{,}095}{12}\right)^{25}-1\right]}{\frac{0{,}095}{12}}$
✓ answer

(3)

OR

$n = 25{,}6315128204.... - 25$
$= 0{,}6315128204 ...$

Balance Outstanding after 25 months

$$= \frac{450\left[1-\left(1+\frac{0{,}095}{12}\right)^{-0{,}631512804}\right]}{\frac{0{,}095}{12}}$$

= R 282,36

Note: Accept
If a candidate uses – 0,63, the final answer is R 281,68

✓ correct formula
✓ substitution into $\frac{450\left[1-\left(1+\frac{0{,}095}{12}\right)^{-0{,}631512804}\right]}{\frac{0{,}095}{12}}$
✓ answer

(3)

OR

Present value at beginning of 25 months

$$= 10402{,}15 - \frac{450\left[1-\left(1+\frac{0{,}095}{12}\right)^{-25}\right]}{\frac{0{,}095}{12}}$$

= R 231,84

Balance Outstanding

$$= 231{,}84\left(1+\frac{0{,}095}{12}\right)^{25}$$

= R 282,36

✓ correct formula
✓ substitution into $\frac{450\left[1-\left(1+\frac{0{,}095}{12}\right)^{-25}\right]}{\frac{0{,}095}{12}}$
✓ answer

(3)

[14]

QUESTION 8

8.1

$g(x) = x^2 - 5$

$$g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$$
$$= \lim_{h\to 0} \frac{(x+h)^2 - 5 - (x^2-5)}{h}$$
$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 5 - x^2 + 5}{h}$$
$$= \lim_{h\to 0} \frac{2xh + h^2}{h}$$
$$= \lim_{h\to 0} \frac{h(2x+h)}{h}$$
$$= \lim_{h\to 0} (2x+h)$$
$$= 2x$$

Note:
If the notation is incorrect, penalty 1 mark

If candidate subtracts and gets $x^2 + 2xh + h^2 - 5 - x^2 - 5$ in the numerator and then candidate corrects themselves, max 2 / 5

Answer only: 0 / 5

✓ formula
✓ substitution
✓ expansion
✓$2x + h$
✓ answer
(5)

OR

$g(x) = x^2 - 5$

$$g'(x) = \lim_{h\to 0} \frac{g(x+h)-g(x)}{h}$$
$$= \lim_{h\to 0} \frac{(x+h)^2 - 5 - (x^2-5)}{h}$$
$$= \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$$
$$= \lim_{h\to 0} \frac{(x+h+x)(x+h-x)}{h}$$
$$= \lim_{h\to 0} \frac{h(2x+h)}{h}$$
$$= \lim_{h\to 0} (2x+h)$$
$$= 2x$$

✓ formula
✓ substitution
✓ expansion
✓$2x + h$
✓ answer
(5)

8.2

$$y = \frac{x^6}{2} + 4\sqrt{x}$$
$$y = \frac{1}{2}x^6 + 4x^{\frac{1}{2}}$$
$$\frac{dy}{dx} = 3x^5 + 2x^{-\frac{1}{2}}$$

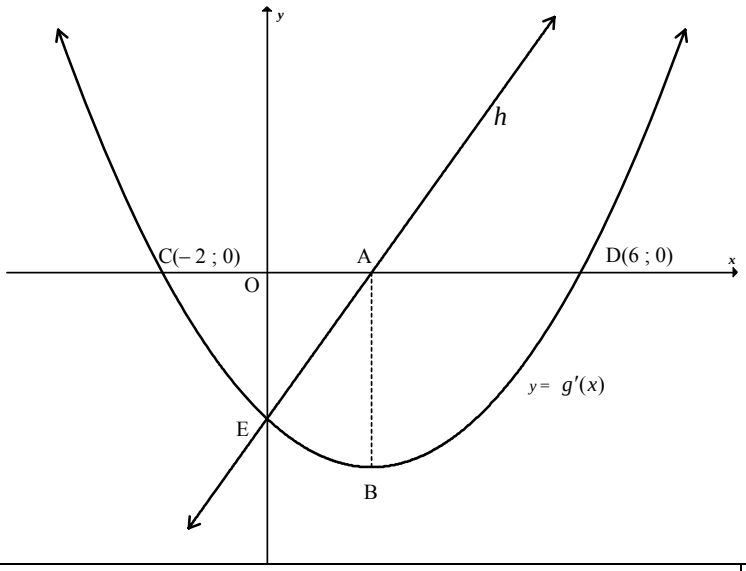
Note:
If $\frac{dy}{dx}$ or y' is left out, penalty 1 mark

If a candidate shows evidence of how to differentiate from an incorrect function which involves breakdown, then max 1 / 3

✓ $+ 4x^{\frac{1}{2}}$
✓ $3x^5$
✓ $2x^{-\frac{1}{2}}$
(3)

8.3	$g(x) = ax^2 + \frac{b}{x}$ $g(x) = ax^2 + bx^{-1}$ $g'(x) = 2ax - bx^{-2}$ $0 = 2a(4) - \frac{b}{(4)^2}$ $8a = \frac{b}{16}$ $b = 128a$ $96 = a(4)^2 + \frac{b}{4}$ $96 = 16a + \frac{1}{4}(128a)$ $96 = 48a$ $a = 2$ $b = 256$ **OR** $g'(x) = 2ax - \frac{b}{x^2}$ $g'(4) = 8a - \frac{b}{16} = 0$ $g(4) = 16a + \frac{b}{4} = 96$ $32a - \frac{b}{4} = 0$ $48a = 96$ $a = 2$ $b = 256$	**Note:** In the equation $g'(x) = 0$; = 0 must be shown in the equation.	✓ $g'(x) = 2ax - bx^{-2}$ ✓ $0 = g'(x)$ ✓ $2a(4) - \frac{b}{(4)^2}$ ✓ subs (4 ; 96) ✓ $a = 2$ ✓ $b = 256$ (6) ✓ $g'(x) = 2ax - \frac{b}{x^2}$ ✓ $g'(4) = 8a - \frac{b}{16}$ ✓ $g'(x) = 0$ ✓ $g(4) = 16a + \frac{b}{4} = 96$ ✓ $a = 2$ ✓ $b = 256$ (6) **[14]**

QUESTION 9



9.1	The y-intercept of g is E(0 ; – 4) **OR** $x = 0$ and $y = -4$	✓ answer (1)
9.2	$y = a(x+2)(x-6)$ $-4 = a(0+2)(0-6)$ $-4 = -12a$ $a = \frac{1}{3}$ $y = \frac{1}{3}(x+2)(x-6)$ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$ **OR** $g'(0) = -4 = c$ $g'(x) = ax^2 + bx - 4$ $g'(-2) = 0$ $4a - 2b - 4 = 0$ $b = 2a - 2$ $g''(2) = 0$ $2a(2) + b = 0$ $b = -4a$ $2a - 2 = -4a$ $a = \frac{1}{3}$ $b = -\frac{4}{3}$ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$	✓ setting up of equation ✓ subs (0 ; – 4) ✓ $a = \frac{1}{3}$ ✓ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$ (4) ✓ substitution $x = -2$ and $g'(x) = 0$ ✓ $g''(2) = 0$ ✓ $a = \frac{1}{3}$ ✓ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$ (4)

OR

$c = -4$

$4a - 2b - 4 = 0$

$36a + 6b - 4 = 0$

$48a - 16 = 0$

$a = \frac{1}{3}$

$b = -\frac{4}{3}$

$y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$

✓ setting up of equation
✓ simultaneous equation
✓ $a = \frac{1}{3}$
✓ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$

(4)

OR

$y = a(x+2)(x-6)$

$= a(x^2 - 4x - 12)$

$= ax^2 - 4ax - 12a$

$-12a = -4$

$a = \frac{1}{3}$

$y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$

✓ setting up of equation
✓ $ax^2 - 4ax - 12a$
✓ $a = \frac{1}{3}$
✓ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$

(4)

OR

$\frac{dy}{dx} = 2ax + b$

$0 = 2a(2) + b$

$b = -4a$

✓ $b = -4a$

EITHER

subs $(6\ ;\ 0)$

$0 = 36a + 6b - 4$

$4 = 36a + 6b$

$2 = 18a + 3b$

$2 = 18a + 3(-4a)$

$2 = 6a$

$a = \frac{1}{3}$

$b = -\frac{4}{3}$

$y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$

OR

$0 = 4a - 2b - 4$

$0 = 4a - 2(-4a) - 4$

$12a = 4$

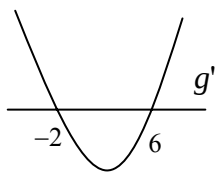
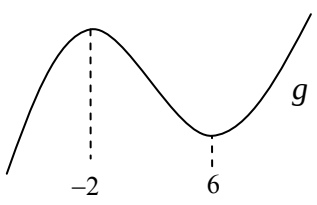
$a = \frac{1}{3}$

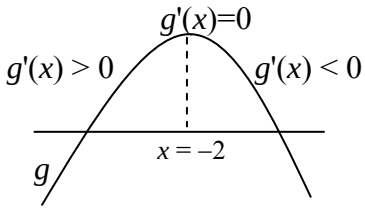
$b = -\frac{4}{3}$

$y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$

✓ simultaneous equation
✓ $a = \frac{1}{3}$
✓ $y = \frac{1}{3}x^2 - \frac{4}{3}x - 4$

(4)

9.3	At turning point $g'(x) = 0$ $x = -2$ and $x = 6$ **Answer only:** Full marks If only 1 value given, max 1 / 2	✓ $g'(x) = 0$ ✓ $x = 6$ and $x = -2$ (2)
9.4	$x = \frac{-2+6}{2}$ $x = 2$ **OR** x-value of point of inflection of g is at A. $g''(x) = 0$ $\frac{2x}{3} - \frac{4}{3} = 0$ $2x - 4 = 0$ $2x = 4$ $x = 2$ **OR** $x = -\frac{b}{2a}$ $x = \frac{\frac{4}{3}}{2\left(\frac{1}{3}\right)}$ $x = 2$ **OR** $g'(x) = \frac{1}{3}(x-2)^2 - \frac{16}{3}$ $x = 2$ **Note:** Answer only Full marks	✓ $x = \frac{-2+6}{2}$ ✓ answer (2) ✓ $2x - 4 = 0$ ✓ answer (2) ✓ $x = \frac{\frac{4}{3}}{2\left(\frac{1}{3}\right)}$ ✓ answer (2) ✓ $g'(x) = \frac{1}{3}(x-2)^2 - \frac{16}{3}$ ✓ answer (2)
9.5	$g'(x) > 0$ for $x < -2$, so g is increasing for $x < -2$. $g'(x) < 0$ for $x > -2$, so g is decreasing for $x > -2$. $\therefore g$ has a local maximum at $x = -2$ because the graph is increasing followed by decreasing **OR**   $\therefore g$ has a local maximum at $x = -2$ **OR**	✓ $g'(x) > 0$ ✓ g is incr for $x < -2$ ✓ g is decr for $x > -2$ (3) ✓ $g'(x) > 0$ for $x < -2$ ✓ $g'(x) < 0$ for $x > -2$ ✓ max at $x = -2$ (3)



✓ $g'(x) > 0$ for $x < -2$
✓ $g'(x) < 0$ for $x > -2$
✓ max at $x = -2$
(3)

OR

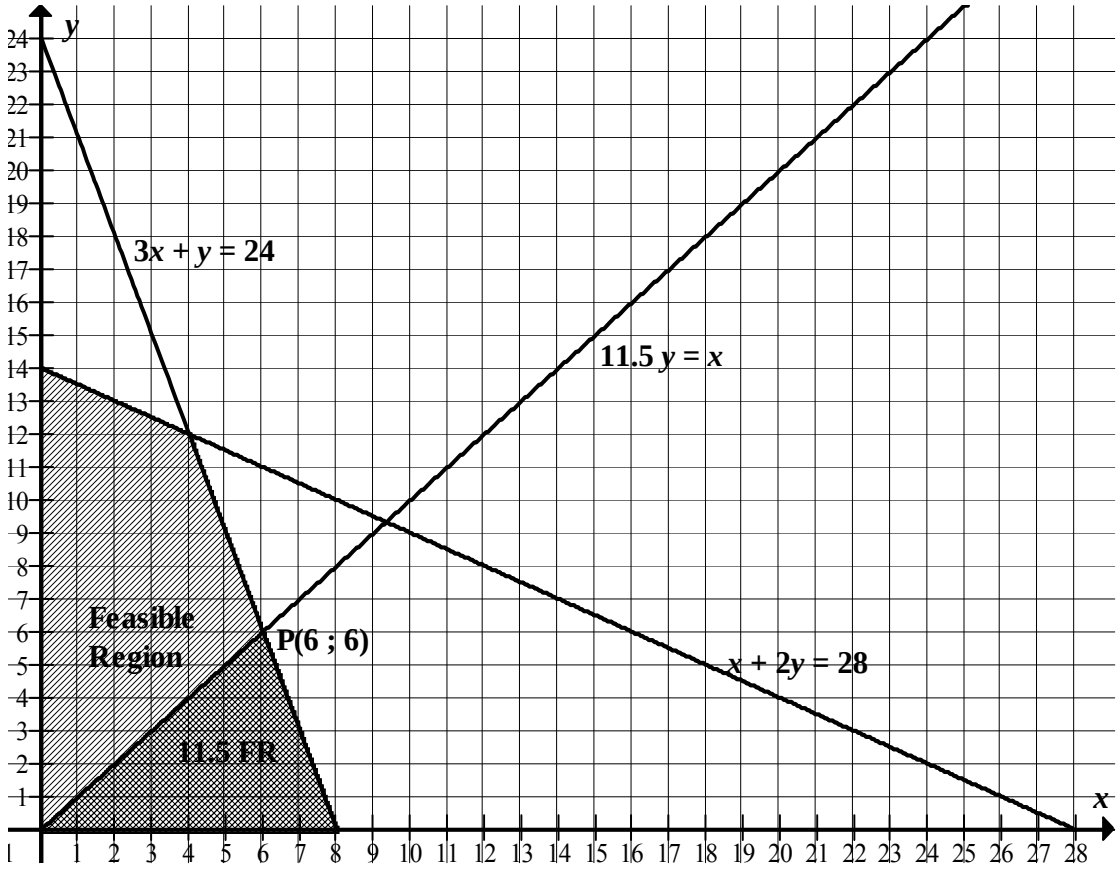
$g'(-2) = 0$
$g''(-2) < 0$ so graph is concave down at $x = -2$, so g has a local maximum

✓ $g'(-2) = 0$
✓ $g''(-2) < 0$
✓ max at $x = -2$
(3)
[12]

QUESTION 10

10.1	$V = \pi r^2 h + 2 \times \frac{1}{2} \times \frac{4}{3}\pi r^3$ $V = \pi r^2 h + \frac{4}{3}\pi r^3$ $\frac{\pi}{6} = \pi r^2 h + \frac{4}{3}\pi r^3$ $\pi r^2 h = \frac{\pi}{6} - \frac{4}{3}\pi r^3$ $h = \frac{\pi}{6\pi r^2} - \frac{4\pi r^3}{3\pi r^2}$ $h = \frac{1}{6r^2} - \frac{4r}{3}$	✓ volume equation ✓ substitution of $\frac{\pi}{6}$ ✓ $h = \frac{\pi}{6\pi r^2} - \frac{4\pi r^3}{\pi r^2}$ (3)
10.2	$S = 2 \times 2\pi r^2 + 2\pi rh$ $S = 4\pi r^2 + 2\pi rh$ $S = 4\pi r^2 + 2\pi r\left(\frac{1}{6r^2} - \frac{4r}{3}\right)$ $S = 4\pi r^2 + \frac{\pi}{3r} - \frac{8\pi r^2}{3}$ $= \frac{4}{3}\pi r^2 + \frac{\pi}{3r}$	✓ surface area equation ✓ substitution of h ✓ simplification (3)
10.3	$S = \frac{4}{3}\pi r^2 + \frac{\pi}{3}r^{-1}$ $\frac{dS}{dr} = \frac{8\pi r}{3} - \frac{\pi}{3r^2} = 0$ $8r = \frac{1}{r^2}$ $8r^3 = 1$ $r = \frac{1}{2}$ Then $S = \frac{4}{3}\pi\left(\frac{1}{2}\right)^2 + \frac{\pi}{3}(2)$ $S = \pi$ square metres $= 3{,}14$ square metres	✓ $\frac{\pi}{3}r^{-1}$ ✓ $\frac{dS}{dr} = \frac{\pi}{3}\left(8r - \frac{1}{r^2}\right)$ or $\frac{dS}{dr} = \frac{\pi}{3}\left(8r - r^{-2}\right)$ ✓ $\frac{dS}{dr} = 0$ ✓ $8r = \frac{1}{r^2}$ ✓ $r = \frac{1}{2}$ ✓ S = π (6) **[12]**

QUESTION 11

11.1	$x, y \in N_0$ $x + 2y \leq 28$ or $y \leq -\frac{x}{2} + 14$ $3x + y \leq 24$ or $y \leq -3x + 24$ **Note:** If inequality signs incorrect or equal signs used: max 3 / 4 marks	✓✓ First inequality ✓✓ Second inequality (4)
11.2		✓graph of $x + 2y \leq 28$ ✓ graph of $3x + y \leq 24$ ✓ feasible region (quadrilateral) (3)
11.3.1	8	✓ answer (1)
11.3.2	14	✓ answer (1)
11.4	Maximise $x + y$ Use search line with gradient -1 4 Type A 12 Type B	✓ 4 Type A ✓ 12 Type B (2)

11.5	$x \geq y$ $y \leq x$ New Feasible region (triangle) in diagram Maximise $x + y$. Maximum at (6 ; 6) Answer: 6 + 6 = 12 braai stands Machine Time = $x + 2y$ $= 6 + 2\times6$ $= 6 + 12$ $= 18$ hours **Note:** Answer only of machine time 18 hours and braai stands 12 Full marks	✓ $y \leq x$ ✓ (6 ; 6) ✓✓ 12 ✓ 18 hours (5) **[16]**

TOTAL: 150