basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P3

NOVEMBER 2010

MEMORANDUM

MARKS: 100

This memorandum consists of 15 pages.

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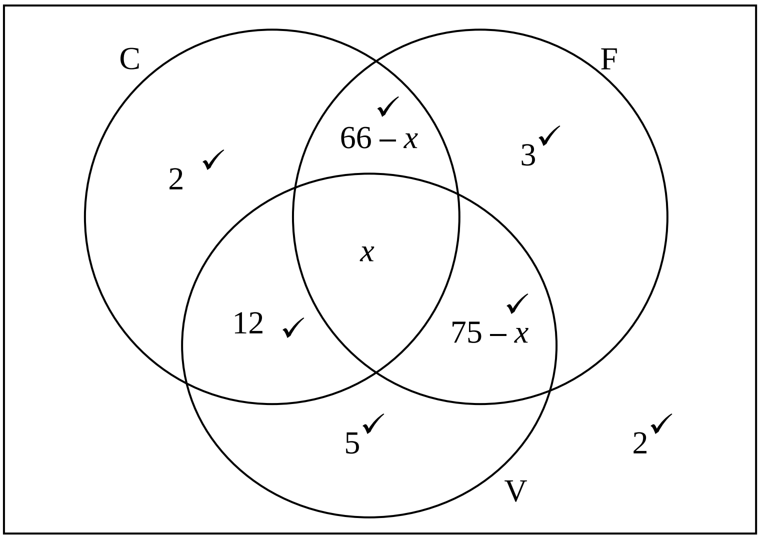
NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent Accuracy applies in **ALL** aspects of the marking memorandum.

QUESTION 1

1.1		(7)
1.2	$5+12+2+x+75-x+66-x+3+2=103$ $x = 62$ **Note:** Although CA applies to the question, the candidate cannot have negative or fraction answers.	✓ equation ✓ answer (2)
1.3.1	P(only eats chicken and fish and no vegetables) $= \frac{4}{103}$	✓ 4 ✓ 103 (2)
1.3.2	P(any two) $= \frac{12+4+13}{103} = \frac{29}{103}$ **Accept** P(any two) $= \frac{91}{103}$ **Note:** Although CA applies to the question, the candidate cannot have negative or value greater than 1.	✓ adding probabilities ✓ $\frac{29}{103}$ (2) ✓ adding probabilities ✓ $\frac{91}{103}$ **[13]**

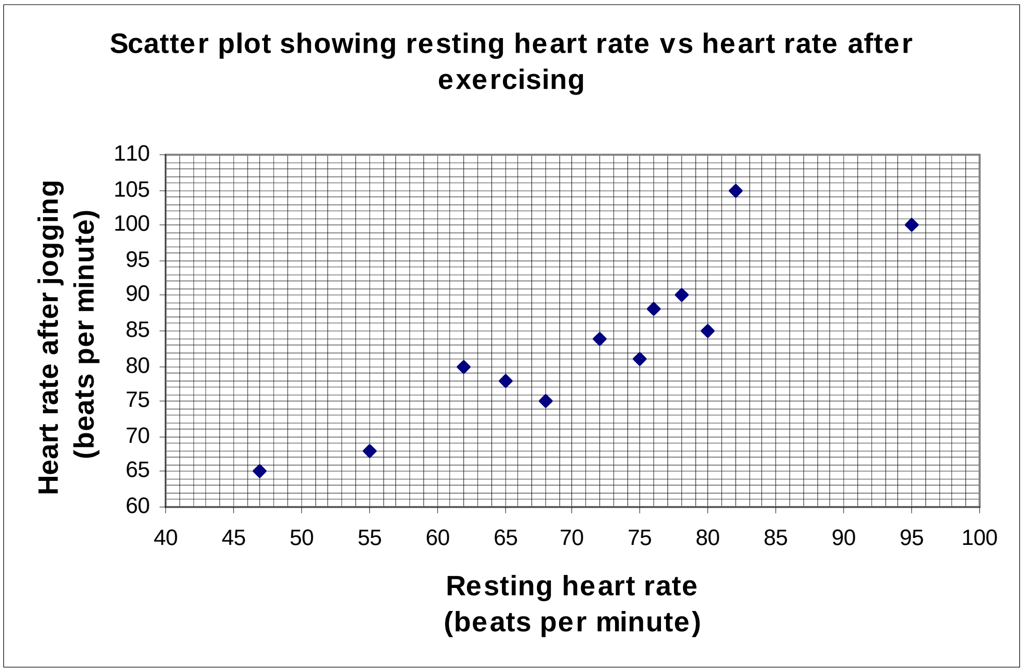
QUESTION 2

2.1	No. They chose a Wednesday morning, when most people are at work. This is not a reliable time to do a survey about customer satisfaction. Most supermarkets are not busy at this time. Only 130 customers of a possible very large sample were interviewed. This is a very small number in comparison to the total number of customers that use a supermarket in a week. **Accept:** Yes, with a reasonable justification related to real life situations for example: very small rural community. **Note:** If the candidate answers YES or NO ONLY, then 0 / 2 marks.	No ✓✓ acceptable reason Yes ✓✓ acceptable reason (2)
2.2	$\frac{22}{100}\times 130$ $= 28{,}6$ **OR** $\frac{78}{100}\times 130$ $= 101{,}4$ $130 - 101{,}4$ $= 28{,}6$ **Accept:** 28 or 29	✓ $\frac{22}{100}$ or 22% ✓ 28 or 29 or 28,6 (2)
2.3	Choose a time when your store is busy, possibly Saturday or Sunday mornings. Interview more people to get a realistic point of view on customer service. Observe customer service over a longer period of time. Make use of questionnaires. **Note:** If yes in 2.1, the reasons must be relevant.	✓✓ any two valid reasons (2) **[6]**

QUESTION 3

3.1	$\frac{68}{100} \times 20000$ = 13 600 **OR** $\frac{66,7}{100} \times 20000$ =13 340 **OR** $\frac{68,3}{100} \times 20000$ =13 660	✓ 68 or 66,7 or 68,3 or $\frac{2}{3}$ ✓ answer (2)
3.2	Lowest weight = 182 – 3(0,454) = 180,638 grams Highest Weight = 182 + 3(0,454) = 183,362 grams Range = 183,362 – 180,638 = 2,724 Answer only: full marks If candidate uses one or two standard deviations: max 2 marks **OR** Range = 6 × 0,454 = 2,724 **Accept:** Range = 8 × 0,454 = 3,632	✓ correct 3 sd ✓ lowest weight ✓ highest weight ✓ difference (4) ✓✓ 6 ✓ 0,454 ✓ answer (4) **[6]**

QUESTION 4

4.1		✓✓✓ all 12 points plotted correctly ✓✓ 7 – 11 points plotted correctly ✓ 2 – 6 points plotted correctly (3)
4.2	$a = 25,23$ (25,22587269…) $b = 0,81$ (0,8143737166…) $\hat{y} = a + bx$ $\hat{y} = 25,23 + 0,81x$ **Note:** If the line of best fit is drawn and its equation then calculated: 0 / 4 marks **If using pen and paper method:** $\bar{x} = 71,25$ $\bar{y} = 83,25$ $a = 25,23$ (25,22587269…) $b = 0,81$ (0,8143737166…) $\hat{y} = a + bx$ $\hat{y} = 25,23 + 0,81x$	✓✓ *a* or *b* ✓ *b* or *a* ✓ $\hat{y} = 25,23 + 0,81x$ (4) ✓ $\bar{x}, \bar{y}$ ✓ *a* ✓ *b* ✓ $\hat{y} = 25,23 + 0,81x$ (4)
4.3	$r = 0,898$ $= 0,90$ (0,8979098935…)	✓✓ answer (2)
4.4	It is a very strong positive relationship.	✓ strong ✓ positive (2)
4.5	$\hat{y} = 25,23 + 0,81x$ $86 = 25,23 + 0,81x$ $x = 75,024...$ Resting heart rate could be 75 beats per minute. If *a* and *b* are not rounded off in the calculation, $x = 74,626$ … $x = 74,63$ If candidate draws in the least square regression line and reads of *x*-value where $y = 86$: full marks	✓ substitute $\hat{y} = 86$ ✓ answer (2) Accept $x = 74,63$ **[13]**

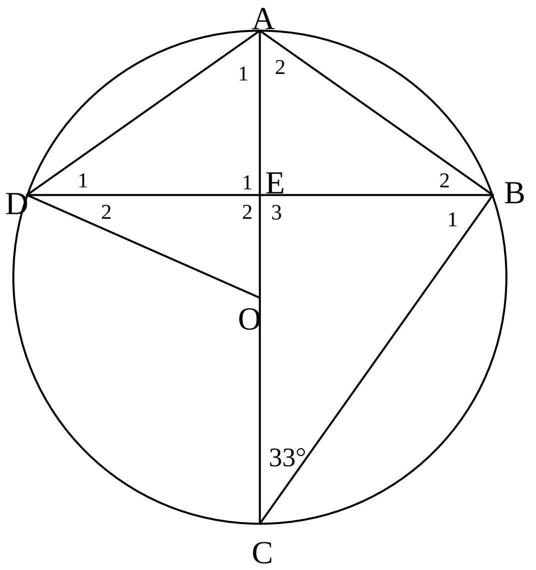
QUESTION 5

5.1	Number licence plates available $= 21\times 21\times 21\times 10\times 10\times 10$ $= 21^3.10^3$ $= 9\,261000$	✓ 21 ✓ 10 ✓ answer (3)
5.2	P(starting with Y) $= \frac{1\times 21\times 21\times 10\times 10\times 10}{21\times 21\times 21\times 10\times 10\times 10}$ $= \frac{441000}{9261000}$ $= \frac{1}{21}$ Answer only: full marks	✓ $21^2\times 10^3$ ✓ denominator (CA with5.1) ✓ answer (3)
5.3	P(contains number 7) $= \frac{21\times 21\times 21\times 1\times 9\times 9 + 21\times 21\times 21\times 9\times 1\times 9 + 21\times 21\times 21\times 9\times 9\times 1}{9261000}$ $= \frac{3(21^3).1.9.9}{9261000}$ $= \frac{243}{1000}$ or 0,243 If did not multiply by 3: max 2 **OR** P(contains number 7) $= \frac{1\times 9\times 9 + 9\times 1\times 9 + 9\times 9\times 1}{1000}$ $= \frac{243}{1000}$ or 0,243	✓ 3 ✓ 1.9.9 ✓ denominator (3) ✓ 3 or $1\times 9\times 9 + 9\times 1\times 9 + 9\times 9\times 1$ ✓ 1.9.9 ✓ denominator
5.4	Number of unique number plates available with no repetition $= 21\times 20\times 19\times 10\times 9\times 8$ $= 5\,745\,600$ **OR** $^{21}P_3 . ^{10}P_3$ $= \frac{21!}{18!}\times\frac{10!}{7!}$ $= 5\,745\,600$	✓ $21\times 20\times 19$ ✓ $10\times 9\times 8$ ✓ answer (3) ✓ $^{21}P_3$ ✓ $^{10}P_3$ ✓ answer (3) **[12]**

QUESTION 6

6.1	$T_1 = 3$ $T_{1+1} = 3 - 4(1) + 5 = 4$ $T_{2+1} = 4 - 4(2) + 5 = 1$ $T_{3+1} = 1 - 4(3) + 5 = -6$ If 3 ; 0 ; – 7 ; – 18 : max 2 marks	✓ T_2 ✓ T_3 ✓ T_4 (3)
6.2	Quadratic sequence. It adds a linear sequence to the preceding term. **OR** 3 4 1 –6 1 – 3 –7 – 4 – 4 Quadratic Sequence Constant second difference of – 4 **OR** Recursive Need the previous term to calculate the next term	✓ quadratic ✓ reason (2) ✓recursive ✓reason (2) **[5]**

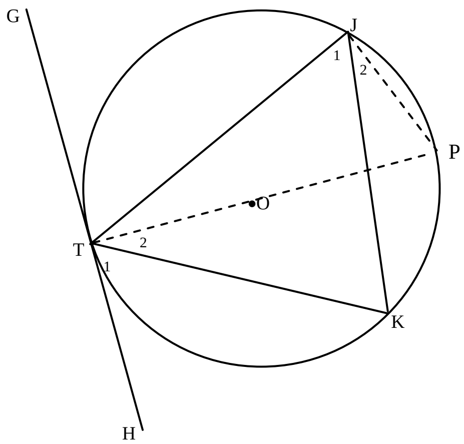
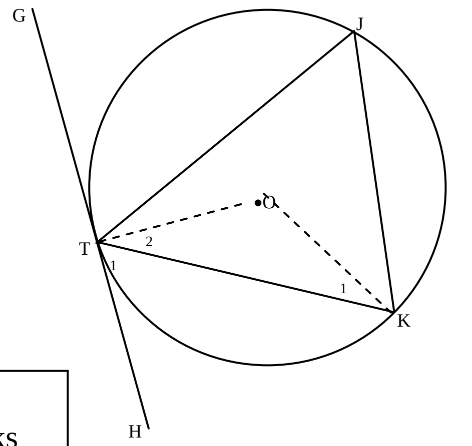
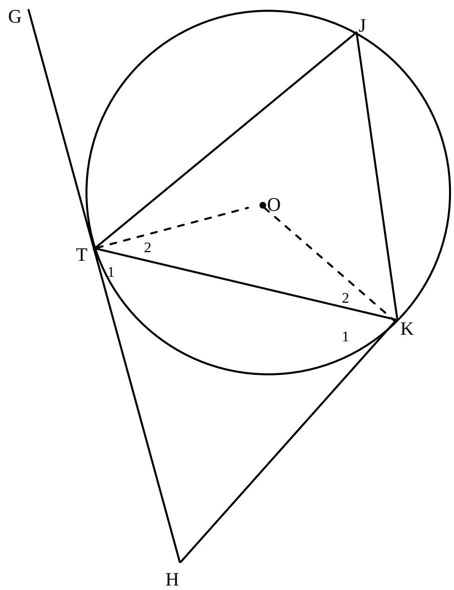
QUESTION 7

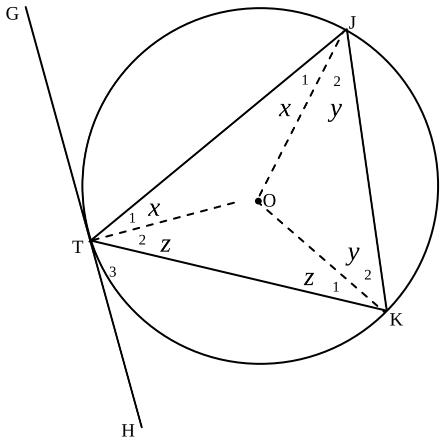


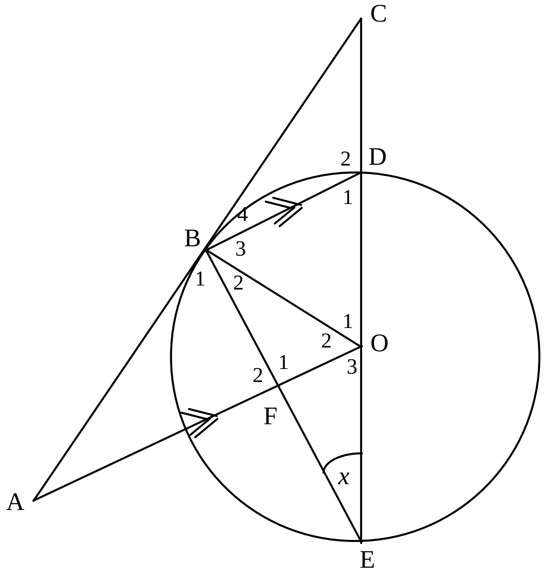
7.1	$\hat{D}_1 = 33°$ (∠ in same segment) $A\hat{E}D = 90°$ (given) $\hat{A}_1 = 57°$ (∠ sum Δ) **OR** $B\hat{E}C = 90°$ (given) $\hat{B}_1 = 57°$ (∠ sum Δ) $\hat{A}_1 = 57°$ (∠ in same segment) **OR** DE = EB (line from circ cent ⊥ ch bis ch) AE is common $A\hat{E}D = \hat{E}_1 = 90°$ (given) $\Delta AED \equiv \Delta AEB$ (SAS) $A\hat{B}C = 90°$ (∠s in semi-circle) $\hat{A}_1 = \hat{A}_2 = 57°$ (∠ sum Δ)	✓ $\hat{D}_1 = 33°$ ✓ ∠ in same segment ✓ $\hat{A}_1 = 57°$ (3) ✓ $\hat{B}_1 = 57°$ ✓ $\hat{A}_1 = 57°$ ✓ ∠ in same segment (3) ✓ DE = EB (S/R) ✓ $\Delta AED \equiv \Delta AEB$ (SAS) ✓ answer (3)
7.2	$\hat{D}_2 + \hat{D}_1 = 57°$ (OD = OA = radii) $\hat{D}_2 = 24°$ **OR** $D\hat{O}C = 114°$ (OD = OA = radii) OR ∠ at the centre theorem $\hat{E}_2 = 90°$ $\hat{D}_2 = 114° - 90°$ $= 24°$	✓ $\hat{D}_2 + \hat{D}_1 = 57°$ ✓ answer (2) ✓ $D\hat{O}C = 114°$ ✓ answer (2)

7.3	$A\hat{B}C = 90°$ ($\angle$ in semi-circle) $\hat{A}_2 = 57°$ ($\angle$ sum Δ) $= \hat{A}_1$ AE bisects $D\hat{A}B$ **OR** DE = EB (line from circ centre bis ch) AE is common $\hat{E}_1 = A\hat{E}B = 90°$ (given) $\Delta ADE \equiv \Delta ABE$ (SAS) $\hat{A}_2 = \hat{A}_1$	✓ $A\hat{B}C = 90°$ ✓ $\angle$ in semi-circle ✓ $\hat{A}_2 = \hat{A}_1$ or AE bisects $D\hat{A}B$ (3) ✓ DE = EB (S/R) ✓ $\Delta AED \equiv \Delta AEB$ (SAS) ✓ $\hat{A}_2 = \hat{A}_1$ or AE bisects $D\hat{A}B$ (3) **[8]**

QUESTION 8

8.1	Draw diameter TP. Join P to J. $\hat{T}_1 + \hat{T}_2 = 90°$ (tan ⊥ diameter) $\hat{J}_1 + \hat{J}_2 = 90°$ (∠ in semi-circle) $\hat{J}_2 = \hat{T}_2$ (∠ in same seg) $T\hat{J}K = \hat{T}_1$ 	✓ construction ✓ $\hat{T}_1 + \hat{T}_2 = 90°$ ✓ tan ⊥ diameter ✓ S/R ✓ S/R (5)
	OR Draw radii OT and OK Let $\hat{T}_2 = x$ $\hat{K}_1 = x$ (∠ opp = radii) $\hat{T}_1 = 90° - x$ (rad ⊥ tan) $T\hat{O}K = 180° - 2x$ (∠ sum Δ) $T\hat{J}K = 90° - x$ (∠ circ cent) $T\hat{J}K = \hat{T}_1$ (= $90° - x$)  **NOTE:** If there is no construction: 0 / 5 marks If candidate changes lettering and states "Similarly": max full marks	✓ construction ✓ $\hat{T}_1 = 90° - x$ ✓ rad ⊥ tan ✓ S/R ✓ S/R (5)
	 **OR** Draw GT extend to H. Draw tangent KH at K. TH = KH (tan from comm pt) $\hat{K}_1 = \hat{T}_1$ (∠s opp = sides) $T\hat{O}K = 2T\hat{J}K$ (∠ circ cent = 2∠ circumf) $\hat{T}_1 + \hat{T}_2 = 90°$ (tan ⊥ radius) $T\hat{O}K = 180° - (90° - \hat{T}_1 + 90° - \hat{K}_1)$ $= \hat{T}_1 + \hat{K}_1$ $= \hat{T}_1 + \hat{T}_1$ $= 2\hat{T}_1$ $\hat{T}_1 = \frac{1}{2} K\hat{O}T$ $= T\hat{J}K$	✓ construction ✓ S/R ✓ S/R ✓ $\hat{T}_1 + \hat{T}_2 = 90°$ ✓ tan ⊥ radius (5)

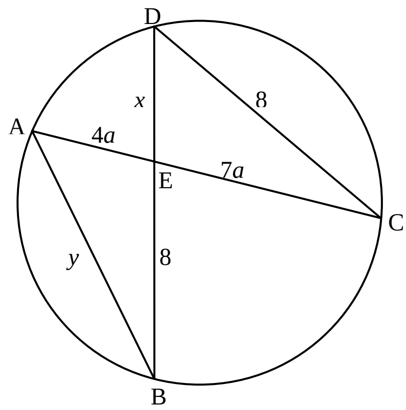
	OR Construct OT, OJ and OK $\hat{T}_1 = \hat{J}_1 = x$ (radii) $\hat{T}_2 = \hat{K}_1 = z$ (radii) $\hat{K}_2 = \hat{J}_2 = y$ (radii) $2x + 2y + 2z = 180°$ ($\angle$ sum Δ) $x + y + z = 90°$ $x + y = 90° - z$ $O\hat{T}H = 90°$ (rad $\perp$ tan)) $\hat{T}_3 = 90° - z$ $= 90° - (90° - (x + y))$ $= 90° - z$ $= T\hat{J}K$ 	✓ construction ✓ S/R ✓ S ✓ $\hat{T}_3 + \hat{T}_2 = 90°$ ✓ rad $\perp$ tan (5)

8.2		
8.2.1	$\hat{B}_4 = x$ (tan chord theorem) $\hat{A} = \hat{B}_4 = x$ (corres ∠; BD ‖ AO) $\hat{B}_2 = x$ (BO = EO = radii) **Note:** If start with $\hat{A} = x$ and do not use tan ch th: max 2 marks	✓ $\hat{B}_4 = x$ ✓ tan chord theorem ✓ $\hat{A} = \hat{B}_4 = x$ with reason ✓ $\hat{B}_2 = x$ (4)
8.2.2	$D\hat{B}E = 90°$ (∠ in semi-circle) $C\hat{B}E = 90° + x$ **OR** $C\hat{B}O = 90°$ (rad ⊥ tan) $C\hat{B}E = 90° + x$ **OR** $\hat{O}_1 = 2x$ (∠ circ cent) $\hat{B}_3 = \hat{D}_1 = 90° - x$ (radii) $C\hat{B}E = x + (90° - x) + x$ $= 90° + x$	✓ $D\hat{B}E = 90°$ ✓ ∠ in semi-circle ✓ $C\hat{B}E = 90° + x$ (3) ✓ $C\hat{B}O = 90°$ ✓ rad ⊥ tan ✓ $C\hat{B}E = 90° + x$ (3) ✓ $\hat{O}_1 = 2x$ ✓ ∠ circ cent ✓ $C\hat{B}E = 90° + x$ (3)
8.2.3	$D\hat{B}E = 90°$ (proved in 8.2.2) $B\hat{F}O = 90°$ (co-int angles supp; BD ‖ AO) BF = FE (line from circ cent ⊥ ch bisect ch) F is the midpoint of EB	✓ $D\hat{B}E = 90°$ ✓ $B\hat{F}O = 90°$ and reason ✓ BF = FE ✓ line from circ cent ⊥ ch bisect ch) (4)

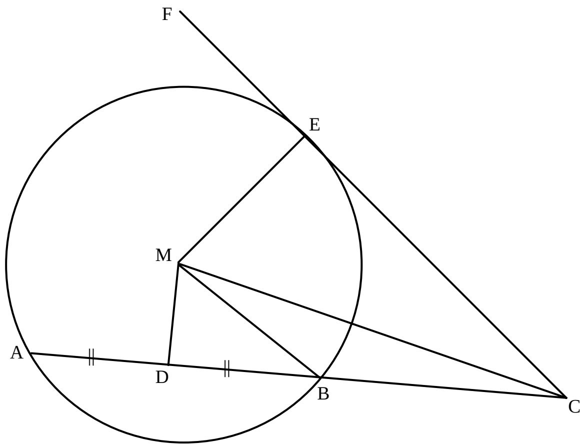
	OR OD = OE (radii) BF = FE (BD || AO) F is the midpoint of EB	✓ OD = OE ✓ radii ✓ BF = FE ✓ BD || AO (4)
	OR $B\hat{F}O = E\hat{F}O = 90°$ (BD || AO) OF is common BO = OE (radii) ΔBOF ≡ ΔEOF (90°HS) BF = FE (≡ Δs)	✓ $B\hat{F}O = E\hat{F}O = 90°$ (BD || AO) ✓ BO = OE ✓ ΔBOF ≡ ΔEOF ✓ BF = FE (4)
	OR $\hat{B}_2 = \hat{A} = x$ (proven) $\hat{O}_2$ is common ΔAOB ||| ΔBOF (AAA) $A\hat{B}O = B\hat{F}O$ $A\hat{B}O = 90°$ (proven) $A\hat{B}O = B\hat{F}O = 90°$ BF = FE (line from circ cent ⊥ ch bisects ch)	✓ ΔAOB ||| ΔBOF ✓ $A\hat{B}O = B\hat{F}O$ ✓ BF = FE ✓ line from circ cent ⊥ ch bisects ch (4)
	OR $D\hat{B}E = 90°$ (∠ in semi-circle) $\hat{B}_3 = 90° - x$ $\hat{O}_2 = 90° - x$ (alt ∠s; BD || FO) $\hat{F}_1 = 90°$ (∠ sum Δ) BF = FE (line from circ cent ⊥ ch bisects ch)	✓ $D\hat{B}E = 90°$ ✓ $\hat{F}_1 = 90°$ ✓ BF = FE ✓ line from circ cent ⊥ ch bisects ch (4)
	OR In ΔOBF and ΔOEF 1. OB = OE (radii) 2. $B\hat{F}O = E\hat{F}O = 90°$ (BD || AO) 3. $\hat{B}_2 = \hat{E}$ (radii) ΔOBF ≡ ΔOEF (AAS) BF = FE	✓ OB = OE ✓ $B\hat{F}O = E\hat{F}O = 90°$ (BD || AO) ✓ ΔOBF ≡ ΔOEF ✓ BF = FE (4)
8.2.4	In ΔCBD and ΔCEB 1. $\hat{E} = \hat{B}_4 = x$ (proven in 8.2.1) 2. $\hat{C}$ is common 3. $\hat{D}_4 = C\hat{B}E = 90° + x$ ΔCBD ||| ΔCEB (AAA)	✓ $\hat{E} = \hat{B}_4 = x$ ✓ $\hat{C}$ is common Or ✓ $\hat{D}_4 = C\hat{B}E = 90° + x$ Any two of the above (2)

8.2.5	$\frac{EB}{BD}=\frac{CE}{CB}$ (sim Δs ∴ sides in proportion) EB.CB = CE.BD but EB = 2EF (F is the midpoint of BE) 2EF.CB = CE.BD	✓ $\frac{EB}{BD}=\frac{CE}{CB}$ ✓ EB.CB = CE.BD ✓ EB = 2EF (3) **[21]**

QUESTION 9

9.	$\hat{A}=\hat{D}$ (∠ in same seg) $\hat{B}=\hat{C}$ (∠ in same seg) $A\hat{E}B=D\hat{E}C$ (vert opp ∠s) ΔDEC ||| ΔAEB (∠∠∠) $\frac{DE}{AE}=\frac{EC}{EB}=\frac{DC}{AB}$ (sides in prop) Let AC = 11a $\frac{x}{4a}=\frac{7a}{8}$ $x=\frac{28a^2}{8}$ $x=3{,}5a^2$ $\frac{y}{8}=\frac{8}{7a}$ (sides in prop) $y=\frac{64}{7a}$  If candidate proves similarity of two triangles: full marks. If candidate does not prove similarity max 3 marks. The triangles have to be in the correct order in order to be given 3 marks.	✓ $\hat{A}=\hat{D}$ S/R ✓ $\hat{B}=\hat{C}$ S/R ✓ $A\hat{E}B=D\hat{E}C$ ✓✓✓ ΔDEC ||| ΔAEB (∠∠∠) **[6]**

QUESTION 10



10.1	$M\hat{E}C = 90°$ (tan ⊥ rad) $M\hat{D}C = 90°$ (line from cent bisects ch) $M\hat{E}C + M\hat{D}C = 180°$ $\therefore$ MDCE a cyclic quad (opp ∠s of quad supplementary) **NOTE:** If the word *cyclic* is used in the last reason: max 2 / 3 marks **OR** $M\hat{E}C = 90°$ (tan ⊥ rad) $M\hat{D}A = 90°$ (line from cent bisects ch) $M\hat{E}C = M\hat{D}A$ $\therefore$ MDCE a cyclic quad (ext ∠ quad = int opp)	✓ $M\hat{E}C = 90°$ (tan ⊥ rad) ✓ $M\hat{D}C = 90°$ ✓ opp ∠s of quad supplementary (3) ✓ $M\hat{E}C = 90°$ (tan ⊥ rad) ✓ $M\hat{D}A = 90°$ ✓ ext ∠ quad = int opp (3)
10.2	$MD^2 = MB^2 - DB^2$ (Pythagoras; ΔMBD) $MC^2 = MD^2 + DC^2$ (Pythagoras; ΔMDC) $= MB^2 - DB^2 + DC^2$	✓ $MD^2 = MB^2 - DB^2$ ✓ Pythagoras ✓ $MC^2 = MD^2 + DC^2$ (3)
10.3	DB = 30 (given) MB = 40 (radii) $MC^2 = (40)^2 + (50)^2 - (30)^2$ $= 3\ 200$ $MC = 40\sqrt{2} = 56,57$ $MC^2 = ME^2 + CE^2$ (Pythagoras) $CE^2 = 3\ 200 - 1\ 600$ $CE^2 = 1\ 600$ CE = 40 mm **OR** $MC^2 = CE^2 + ME^2 - 2CE.ME.\cos M\hat{E}C$ $3200 = CE^2 + (40)^2 - 2CE.(40).\cos 90°$ $= CE^2 + 1600$ $CE^2 = 1600$ $CE = 40$	✓ MB = ME ✓ DB = 30 ✓ $MC^2 = 3200$ or $MC = 40\sqrt{2}$ or MC = 56,57 ✓ answer (4) ✓ cosine rule ✓ ME = 40 ✓ $MC^2 = 3200$ ✓ answer (4) **[10]**

TOTAL: 100