

TECHNICAL MATHEMATICS

QUADRIATIC EQUATIONS

GRADE 12



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA



Foreword

In order to improve learning outcomes the Department of Basic Education conducted research to determine the specific areas that learners struggle with in Grade 12 examinations. The research included a trend analysis by subject experts of learner performance over a period of five years as well as learner examination scripts in order to diagnose deficiencies or misconceptions in particular content areas. In addition, expert teachers were interviewed to determine the best practices to ensure mastery of the topic by learners and improve outcomes in terms of quality and quantity.

The results of the research formed the foundation and guiding principles for the development of the booklets. In each identified subject, key content areas were identified for the development of material that will significantly improve learner's conceptual understanding whilst leading to improved performance in the subject.

The booklets are developed as part of a series of booklets, with each booklet focussing only on one specific challenging topic. The selected content is explained in detail and include relevant concepts from Grades 10 - 12 to ensure conceptual understanding.

The main purpose of these booklets is to assist learners to master the content starting from a basic conceptual level of understanding to the more advanced level. The content in each booklet is presented in an easy to understand manner including the use of mind maps, summaries and exercises to support understanding and conceptual progression. These booklets should ideally be used as part of a focussed revision or enrichment program by learners after the topics have been taught in class. The booklets encourage learners to take ownership of their own learning and focus on developing and mastery critical content and skills such as reading and higher order thinking skills.

Teachers are also encouraged to infuse the content into existing lesson preparation to ensure in-depth curriculum coverage of a particular topic. Due to the nature of the booklets covering only one topic, teachers are encouraged to ensure learners access to the booklets in either print or digital form if a particular topic is taught.

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2. How to use this booklet

This booklet is designed to clarify the content prescribed for Technical Mathematics. In addition, it has some tips on how you should tackle real life-problems on a daily basis. Candidates will be expected to have already mastered the content outlined for Grades 8-11.

This booklet must be used to master some mathematical rules that you may not have been aware of. The prescribed textbook must also be used.

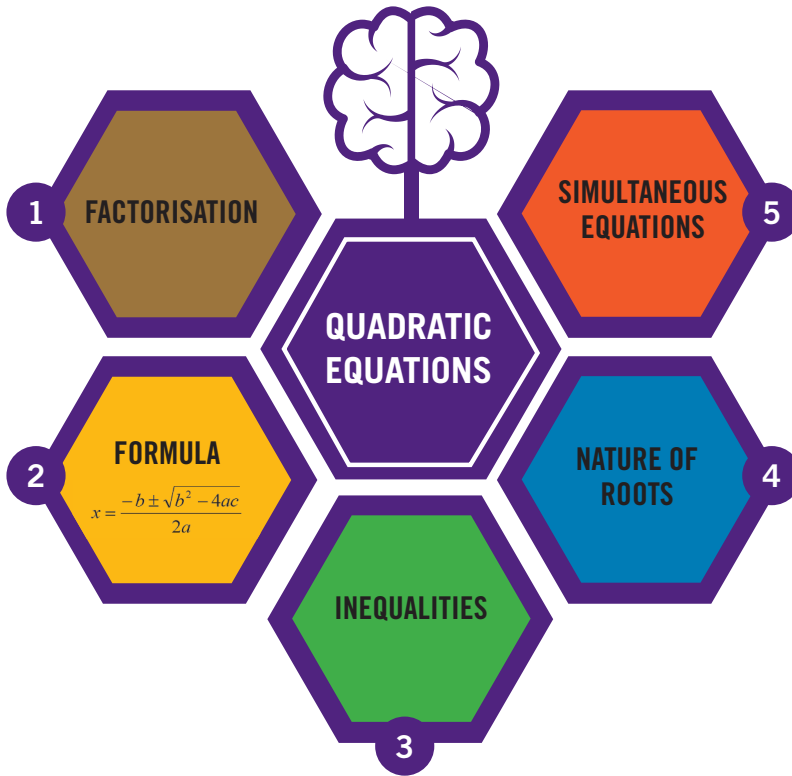
3. Study and examination tips

All learners should be able to acquire sufficient understanding and knowledge to:

- develop fluency in computation skills without relying on the use of a calculator;
- generalise, make conjectures and try to justify or prove them;
- develop problem-solving and cognitive skills;
- make use of the language of Technical Mathematics;
- identify, investigate and solve problems creatively and critically;
- use the properties of shapes and objects to identify, investigate and solve problems creatively and critically;
- encourage appropriate communication by using descriptions in words, graphs, symbols, tables and diagrams;
- practise Technical Mathematics every day.

4 Quadratic Equations

4.1 Mindmap of quadratic equations



4.2 Quadratic equations

A quadratic equation has, at most, two solutions - also referred to as roots. There are some situations, however, in which a quadratic equation has either one solution or no solutions.

4.2.1 Finding the roots of the equation by factorisation

Step 1: The equation should be in the form $ax^2 + bx + c = 0$ (standard form)

Step 2: Factorise the quadratic. _____

- common factors
- difference between two square
- trinomials

Step 3: Write the equation with factors.

Step 4: Solve the equation:

If two brackets are multiplied together and give 0, then one of the brackets must be 0.

*If $A \cdot B = 0$
then $A = 0$ or $B = 0$*

Step 5: Write the final answer.

Examples

- | | | |
|--|--|--|
| <p>1. $3x^2 - 2x = 0$
 $x(3x - 2) = 0$
 $x = 0$ or $(3x - 2) = 0$
 $x = 0$ or $x = \frac{2}{3}$</p> | <p>Take out a common factor
 $fA \cdot B = 0$
 then $A = 0$ or $B = 0$
 Final answer (x – values)</p> | <p>(3)
 \checkmark
 $\checkmark\checkmark$</p> |
| <p>2. $5x^2 = 20$
 $5x^2 - 20 = 0$
 $5(x^2 - 4) = 0$
 $x^2 - 4 = 0$
 $(x - 2)(x + 2) = 0$
 $(x - 2) = 0$ or $(x + 2) = 0$
 $x = 2$ or $x = -2$</p> | <p>Write in standard form

 Divide by 5.
 Factorise the difference between two squares.

 Final answer (x – values)</p> | <p>(4)
 \checkmark

 $\checkmark\checkmark$</p> |
| <p>3. $4x^2 = 9x - 2$
 $4x^2 - 9x + 2 = 0$
 $(4x - 1)(x - 2) = 0$
 $4x - 1 = 0$ or $x - 2 = 0$
 $x = \frac{1}{4}$ or $x = 2$</p> | <p>Standard form
 Factorise

 Final answer (x – values)</p> | <p>(4)
 \checkmark
 \checkmark

 $\checkmark\checkmark$</p> |
| <p>4. $x - 6 - 3x(x - 6) = 0$
 $x - 6 - 3x^2 + 18x = 0$
 $-3x^2 + 19x - 6 = 0$
 $-3x^2 + 19x - 6 = 0$
 $(3x - 1)(x - 6) = 0$
 $(3x - 1) = 0$ or $(x - 6) = 0$
 $x = \frac{1}{3}$ or $x = 2$</p> | <p>Remove brackets
 Standard form
 Make a > 0 by multiplying by (-1)
 Factorise

 Final answer (x – values)</p> | <p>(5)
 \checkmark
 \checkmark

 \checkmark

 $\checkmark\checkmark$</p> |

For you to do:

Solve for x:	Marks	Answers
1.1.1 $(x - 2)(4 + x) = 0$	(2)	$x = 2$ or $x = -4$
1.1.2 $(x + 3)(3x - 1) = 0$	(2)	$x = -3$ or $x = \frac{1}{3}$
1.1.3 $(x - 3)(5x + 2) = 0$	(2)	$x = -3$ or $x = \frac{2}{5}$
1.1.4 $(x^2 - 4)(x - 2) = 0$	(2)	$x = 2$ or $x = -2$
1.1.5 $x^2 + 5x - 6 = 0$	(3)	$x = 6$ or $x = -1$
1.1.6 $x^2 - x - 20 = 0$	(3)	$x = 5$ or $x = -4$
1.1.7 $x^2 + 2x = 0$	(3)	$x = 0$ or $x = -2$
1.1.8 $3x^2 - 7x = 0$	(3)	$x = -0$ or $x = \frac{7}{3}$
1.1.9 $(x - 1)(x + 2) = 4$	(4)	$x = 2$ or $x = -3$
1.1.10 $(x - 1)(x + 8) = 10$	(4)	$x = 2$ or $x = -9$
1.1.11 $2x^2 + x = 3$	(4)	$x = \frac{3}{2}$ or $x = 1$
1.1.12 $x - 18 - 6x(x - 5) = 0$	(5)	$x = \frac{2}{3}$ or $x = \frac{9}{2}$

4.2.2. Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

- Important aspects of using the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ on quadratic equations given with the form: $ax^2 + bx + c = 0$, where "a" is the coefficient of x^2 ; "b" is the coefficient of x ; "c" is the constant term.
- It can be used to solve for x in equations given in any form, i.e. those that can be factorised and those that cannot be factorised.

Steps to follow when using the quadratic formula:

Step 1:

Identify the values of: "a" , "b" & "c" from the given standard equation $ax^2 + bx + c = 0$

Step 2:

Write the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Step 3:

Substitute the numerical values of "a" , "b" & "c" into the quadratic formula.

Step 4:

Use a calculator to solve for x values.

Examples

Solve for x by using the **quadratic formula**. Correct to two decimal places where necessary, if:

1. $x^2 + 2x - 3 = 0$

Solution:

$$a = 1, b = 2 \text{ \& } c = -3$$

Identify the values of a , b & c

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

✓ Write the quadratic formula.

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(-3)}}{2(1)}$$

✓ Substitute the values of $a = 1$, $b = 2$ & $c = -3$

$$x = \frac{4 \pm \sqrt{16}}{2}$$

Simplify from the previous step.

$$x = -3 \text{ or } x = 1$$

✓✓ Calculate the final values of x

2. $-5x^2 - 4x = 0$

Solution:

$$a = -5, b = -4 \text{ \& } c = 0$$

Identify the values of a , b & c

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

✓ Write the quadratic formula.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(-5)(0)}}{2(-5)}$$

✓ Substitute the values of $a = -5$, $b = -4$ & $c = 0$

$$x = \frac{4 \pm \sqrt{16}}{-10}$$

Simplify from the previous step.

$$x = -\frac{4}{5} \text{ or } x = 0$$

✓✓ Calculate the final values of x

3. $-2x^2 - x - 4 = 0$

Solution:

$a = -2, b = -1 \text{ \& } c = -4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(-2)(-4)}}{2(-2)}$$

$$x = \frac{-(-1) \pm \sqrt{-31}}{-4}$$

No solution

Identify the values of $a, b \text{ \& } c$

✓ Write the quadratic formula.

✓ Substitute the values of $a = -2, b = -1 \text{ \& } c = -4$

✓ Simplify $\sqrt{\quad}$

✓ If $\sqrt{-}$, then the equation will not yield any answer; therefore, there are no real solutions.

4. $-2x^2 - x + 4 = 0$

Solution:

$a = -2, b = -1 \text{ \& } c = 4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(-2)(4)}}{2(-2)}$$

$$x = \frac{-(-1) \pm \sqrt{33}}{-4}$$

$x = -1,69 \text{ or } x = 1,19$

Identify the values of $a, b \text{ \& } c$

✓ Write the quadratic formula.

✓ Substitute the values of $a = -2, b = -1 \text{ \& } c = 4$

Simplify from the previous step.

✓✓ Calculate the final values of x

Exercises 1.2

Solve for x by using the **quadratic formula**. Correct to two decimal places where necessary, if:

No.	Activity:	Marks	Answers
1.2.1	$x^2 - 4x + 4 = 0$	(4)	$x = 2$ or $x = -4$
1.2.2	$x^2 - 2x - 3 = 0$	(4)	$x = -1$ $x = 3$
1.2.3	$x^2 - 2x + 2 = 0$	(4)	No solution.
1.2.4	$-18 + 31x = 6x^2$	(4)	$x = 4,50$ $x = 0,67$
1.2.5	$3x^2 + 5x = 7$	(4)	$x = -2,57$ $x = 0,91$
1.2.6	$x^2 - 2x - 2 = 0$	(4)	$x = -0,73$ $x = 2,73$
1.2.7	$-x^2 - 2 = -5x$	(4)	$x = 0,44$ $x = 4,56$

4.2.3 Quadratic inequalities

The following pre-knowledge is important for you to be able to solve quadratic inequalities: knowledge of inequalities, knowledge of solving linear inequalities, and factorisation.

Inequalities

Symbol	Words	Example
$>$	Greater than	$x > 3$ All real numbers greater than 3.
$<$	Less than	$y < 9$ All real numbers less than 9.
\geq	Greater or equal to	$y \geq 7$ All real numbers equal to or greater than 7, i.e. including 7.
\leq	Less than or equal to	$x \leq 67$ All real numbers equal to or less than 67, i.e. including 67.

Solving inequalities is very much like solving equations. The difference is: when you solve the equation, you get the point; when you solve inequalities, you get an interval.

Examples

Solve for x

(a) $x + 3 = 7$

(b) $x + 3 \geq 7$

(c) $-x + 3 = 7$

(d) $-x + 3 \geq 7$

Solutions

(a) $x + 3 = 7$

$$x = 7 - 3$$

Isolate x

$$x = 4$$

(b) $x + 3 \geq 7$

$$x \geq 7 - 3$$

Isolate x

$$x \geq 4$$

(c) $-x + 3 = 7$

$$-x = 7 - 3$$

Isolate x

$$-x = 4$$

$$x = -4$$

(d) $-x + 3 \geq 7$

$$-x \geq 7 - 3$$

Isolate x

$$-x \geq 4$$

$$x \leq -4$$

Multiplying by a negative changes the inequality

Solving quadratic inequality

Quadratic equations are equations in the form $ax^2 + bx + c = 0, a \neq 0$

Quadratic inequality are inequalities in the form $ax^2 + bx + c \geq 0, a \neq 0$

or $ax^2 + bx + c \leq 0, a \neq 0$ or $ax^2 + bx + c > 0, a \neq 0$ or

$ax^2 + bx + c < 0, a \neq 0$

To solve quadratic inequality, you have to determine the critical values. In order to determine the critical values, change the inequality to an equal sign and solve quadratic equations.

Note: When the product of the two factors is equal to or less than or zero, solutions are on the left of the smaller critical value, or on the right of the larger critical value.

Examples

Solve for x and represent your answer **graphically**.

(a) $x^2 + x - 2 \geq 0$

Solutions

$$x^2 + x - 2 \geq 0$$

$$(x + 2)(x - 1) \geq 0 \quad \text{Factorise}$$

critical values are $x = -2$ or $x = 1$

Solutions

$$x \leq -2 \text{ or } x \geq 1$$

Graphical representation



(b) $3x^2 - 2x + 1 \leq 0$

$$3x^2 - 2x + 1 \leq 0$$

$$(3x - 1)(x + 1) \leq 0 \quad \text{Factorise}$$

critical values are $x = -1$ or $x = \frac{1}{3}$ The value of x

Solutions

$$-1 \leq x \leq \frac{1}{3}$$

Graphical representation



(c) $x^2 - 5x - 6 \leq 0$

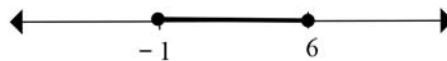
$$(x - 6)(x + 1) \leq 0$$

critical values are $x = -1$ or $x = 6$

Solutions

$$-1 \leq x \leq 6$$

Graphical representation



(d) $-x^2 > -3x$

$$-x^2 + 3x > 0$$

$$x^2 - 3x < 0$$

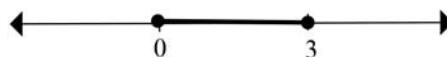
$$x(x - 3) < 0 \quad \text{Factorise}$$

critical values are $x = 0$ or $x = 3$

Solutions

$$0 \leq x \leq 3$$

Graphical representation



(e) $2x^2 - 6 < 0$

$x^2 - 3 < 0$

Divide by

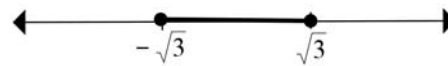
$x^2 < 3$

critical values: $x = -\sqrt{3}$ or $x = \sqrt{3}$

Solutions

$-\sqrt{3} \leq x \leq \sqrt{3}$

Graphical representation



Activities

Solve for x in each of the following and represent the solution graphically.

(a) $2x^2 + 3x + 1 < 0$ (4)

(b) $x^2 - 3x > 2$ (5)

(c) $x^2 < 9$ (5)

(d) $2x^2 - 8 \leq 0$ (4)

(e) $-3x^2 + 5x < 0$ (4)

Solutions

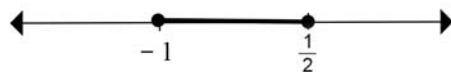
(a) $2x^2 + 3x + 1 < 0$

$(2x + 1)(x + 1) < 0$

critical values are $x = -1$ or $x = -\frac{1}{2}$

Solutions $-1 < x < -\frac{1}{2}$

Graphical representation



✓ Factors

✓ Critical values

✓ $-1 < x < -\frac{1}{2}$

✓ Graphical representation

(4)

(b) $x^2 - 3x > -2$

$x^2 - 3x + 2 > 0$

critical values are $x = 1$ or $x = 2$

Solutions $x < 1$ or $x > 2$

Graphical representation



✓ Standard form

✓ Factors

✓ Critical values

✓ $x < 1$ or $x > 2$

✓ Graphical representation

(5)

(c) $x < 9$

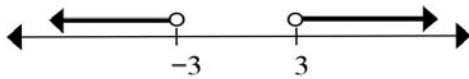
$$x - 3 < 0$$

$$(x + 3)(x - 3) < 0$$

Critical values: $x = -3$ or $x = 3$

Solutions $-3 < x < 3$

Graphical representation



✓ Standard form

✓ Factors

✓ Critical values

✓ $x = -3$ or $x = 3$

✓ Graphical representation

(5)

(d) $2x^2 - 8 \geq 0$

$$x^2 - 4 \geq 0$$

$$(x + 2)(x - 2) \geq 0$$

Critical values: $x = -2$ or $x = 2$

Solutions $x \leq -2$ or $x \geq 2$

Graphical representation



✓ Divide by 2

✓ Factors

✓ Critical values

✓ $x = -2$ or $x = 2$

✓ Graphical representation

(4)

(e) $-3x^2 + 5x < 0$

$$x(-3x + 5) < 0$$

Critical values: $x = 0$ or $x = \frac{5}{3}$

Solutions $x < 0$ or $x > \frac{5}{3}$

Graphical representation



(4)

1.4 Nature of roots

Important aspects when finding the nature of roots:

- Formula: $\Delta = b^2 - 4ac$, where "a" is the coefficient of x^2 , "b" is the coefficient of x and "c" is the constant term.
- The solutions of an equation are the roots of the same equation.
- The following is the nature:

	If:	Then the roots are:
1	$\Delta = 0$	Real, rational and equal.
2	$\Delta > 0$, perfect square	Real, rational and unequal.
3	$\Delta > 0$ not a perfect square	Real, irrational and unequal.
4	$\Delta \geq 0$	Real
5	$\Delta < 0$	Non-real (unreal)

Steps to follow when **determining the nature of roots:**

Step 1:

Identify the values of:"a", "b" and "c" from the given standard equation $ax^2 + bx + c = 0$

Step 2:

Write the Delta (discriminant) formula: $\Delta = b^2 - 4ac$

Step 3:

Substitute the numerical values of "a", "b" and "c"

Step 4:

Use a calculator to find the numerical value.

Step 5:

Use the numerical value found in Step 4 to determine the nature of roots.

Examples

Determine the nature of roots if:

1. $x^2 = 4x - 4$

$$x^2 - 4x + 4 = 0$$

Solution

$$a = 1, b = -4 \text{ \& } c = 4$$

$$\Delta = b^2 - 4ac$$

$$\Delta = (-4)^2 - 4(1)(4)$$

$$\Delta = 0$$

Since $\Delta = 0$ is a perfect square, then the roots are real, rational and equal.

Transpose $4x + 4$ from the right-hand side to the left-hand side.

Identify the values of a , b and c

✓ Write the quadratic formula.

✓ Substitute the values of $a = 1$, $b = -4$ & $c = 4$

✓ Use a calculator to find the numerical value from the previous step.

✓ Conclusion

2. $3x + 2 = 5x^2$

$$-5x^2 + 3x + 2 = 0$$

Solution

$$a = -5, b = 3 \text{ \& } c = 2$$

$$\Delta = b^2 - 4ac$$

$$\Delta = (3)^2 - 4(-5)(2)$$

$$\Delta = 49$$

Since $\Delta = 0$ is a perfect square, the roots are then real, rational and unequal.

Transpose $5x^2$ from the right-hand side to the left-hand side.

Identify the values of a , b and c

✓ Write the quadratic formula.

✓ Substitute the values of $a = -5$, $b = 3$ & $c = 2$

✓ Use a calculator to find the numerical value from the previous step.

✓ Conclusion

3. $x^2 - 4x + 9 = 0$

Solution

$$a = 1, b = -4 \text{ \& } c = 9$$

$$\Delta = b^2 - 4ac$$

$$\Delta = (-4)^2 - 4(1)(9)$$

$$\Delta = -20$$

Since $\Delta < 0$, the roots are then non-real (unreal)

Identify the values of a , b and c

✓ Write the quadratic formula.

✓ Substitute the values of $a = 1$, $b = -4$ & $c = 9$

✓ Use a calculator to find the numerical value from the previous step.

✓ Conclusion

4. $3x^2 + 7x - 9 = 0$

Solution

$a = 3, b = -7 \text{ \& } c = -9$

$\Delta = b^2 - 4ac$

$\Delta = (-7)^2 - 4(3)(-9)$

$\Delta = 112$

Since $\Delta > 0$ is not a perfect square, the roots are then real, irrational and unequal.

Identify the values of a, b and c

✓ Write the quadratic formula.

✓ Substitute the values of $a = 3, b = -7 \text{ \& } c = -9$

✓ Use a calculator to find the numerical value from the previous step.

✓ Conclusion

Exercises 1.3A

Determine the nature of roots if:

No.	Activity:	Marks	Answers	Nature of roots
1.3.1	$-2x - 4x + 4 = x^2$	(4)	0	Real, rational and equal
1.3.2	$x^2 - 3 = 2x$	(4)	16	Real, rational and unequal
1.3.3	$x^2 - 2x + 2 = 0$	(4)	-4	Non-real
1.3.4	$-18 + 31x - 6x^2$	(4)	769	Real, irrational and unequal
1.3.5	$3x^2 + 5x = 7$	(4)	109	Real, irrational and unequal
1.3.6	$x^2 = 2x + 2$	(4)	12	Real, irrational and unequal
1.3.7	$-x^2 + 5x - 2 = 0$	(4)	17	Real, irrational and unequal

Important aspects of finding the value of an unknown, if the nature of roots is given:

- Formula: $b^2 - 4ac = \Delta$, where "a" is the coefficient of x^2 , "b" is the coefficient of x and "c" is the constant term.
- The solutions of an equation are the roots of the same equation.
- The following is the nature:

	If the roots are:	Then:
1	Real, rational and equal	$\Delta = 0$
2	Real, rational and unequal	$\Delta > 0$
3	Real, irrational and unequal	$\Delta > 0$
4	Real	$\Delta \geq 0$
5	Non-real (unreal)	$\Delta < 0$

Steps to follow when **finding the unknown if the nature of roots is given:**

Step 1:

Identify the values of: "a", "b" and "c" from the given standard equation $ax^2 + bx + c = 0$

Step 2:

Decide how Delta should be based on the given nature of roots.

Step 3:

Write the Delta (discriminant) formula: $\Delta = b^2 - 4ac$

Step 4:

Solve an equation to find the unknown previous step.

Examples

1. Determine the value of k if the roots of $x^2 = 4x + k = 0$ are real, rational and equal.

$$x^2 = 4x + k = 0$$

Solution:

$$a = 1, b = -4 \text{ \& } c = k$$

$$\Delta = b^2 - 4ac$$

$$\Delta = (-4)^2 - 4(1)(k)$$

$$16 - 4k = 0$$

$$\Delta = 4$$

Identify the values of a , b and c

✓ Write the quadratic formula.

✓ Substitute the values of $a = 1$, $b = -4$ & $c = k$

Then determine the nature of roots.

✓ Solve an equation to find the unknown previous step.

2. Determine the value of k if the roots of $5x^2 + kx + 10 = 0$ are real, rational and equal.

$$5x^2 + kx + 10 = 0$$

Solution:

$$a = 5, b = k \text{ \& } c = 10$$

$$\Delta = b^2 - 4ac$$

$$\Delta = (k)^2 - 4(5)(10)$$

$$k^2 - 100 < 0$$

$$-10 < k < 10$$

Identify the values of a , b and c

✓ Write the quadratic formula.

✓ Substitute the values of $a = 5$, $b = k$ & $c = 10$

✓ Decide how delta should be based on the given nature of roots.

✓ Solve an equation to find the unknown previous step.

3. Determine the value(s) of k if the roots of $x^2 - 4x + k = 0$ are real and unequal.

$$x^2 - 4x + k = 0$$

Solution:

$$a = 1, b = -4 \text{ \& } c = k$$

$$\Delta = b^2 - 4ac$$

$$\Delta = (-4)^2 - 4(1)(k)$$

$$16 - 4k = 0$$

$$k < 4$$

Identify the values of a , b and c

✓ Write the quadratic formula.

✓ Substitute the values of $a = 1$, $b = -4$ & $c = k$

✓ Decide how delta should be based on the given nature of roots.

✓ Solve an equation to find the unknown previous step.

4. Determine the value(s) of k if the roots of $3x^2 + 7x + k = 0$ are real and unequal.

$$3x^2 + 7x + k = 0$$

Solution:

$$a = 3, b = 7 \text{ \& } c = k$$

$$\Delta = b^2 - 4ac$$

$$\Delta = (7)^2 - 4(3)(k)$$

$$49 - 12k \geq 0$$

$$k \leq \frac{49}{12}$$

Identify the values of a , b and c

✓ Write the quadratic formula.

✓ Substitute the values of $a = 3$, $b = 7$ & $c = k$

✓ Decide how delta should be based on the given nature of roots.

✓ Solve an equation to find the unknown previous step.

Exercises 1.3B

Determine the value(s) of p if the nature of roots is given as follows:

No.	Activity:	If the roots are:	Marks	Answers
1.3.1	$x^2 - 2x + p = 0$	Unreal	4	$p > 1$
1.3.2	$x^2 - 2x + p = 0$	Real, rational and equal	4	$p = 1$
1.3.3	$x^2 - 2x + 2 = 0$	Real, rational and unequal	(4)	$p < 1$
1.3.4	$p + 31x - 6x^2$	Real for all values of p	(4)	$p > \frac{961}{24}$
1.3.5	$3x^2 + 5x = -p$	Non-real	(4)	$p > \frac{25}{12}$
1.3.6	$x^2 - xp + 1 = 0$	Real, rational and unequal	(4)	$p < 2$ or $p > 2$
1.3.7	$-x^2 + 5x + p = 0$	Real, rational and equal	(4)	$p = -\frac{25}{4}$

4.2.3 Simultaneous equations

Solving equations with two unknown values.

In Grade 11, learners are expected to solve linear and quadratic equations.

Revision in Grade 12.

Procedure to solve:

Given the linear and quadratic:

1. Start with the linear equation; make one of the unknowns the subject of the formula.
2. Substitute the variable that was made the subject in the quadratic equation.
3. Apply the rules for solving quadratic equations.
4. Once you get the values, substitute them into the linear equation and then solve the second unknown value.

Example	Explanation	Marks
1. $x + y = -5$ (1)	Name your equations equation 1 and equation 2.	6
$x^2 - 4xy + 15 = 0$ (2)		✓ Subject
Solution		
$x = -5 - y$ (3)	From the linear equation, make x or y the subject of the formula.	✓ Substitution
$x^2 - 4xy + 15 = 0$	Substitute x in (2).	✓ Standard form
$(-5 - y)^2 - 4y(-5 - y) + 15 = 0$	Standard form of quadratic equation	✓ Factors
$25 + 10y + y^2 + 20y + 4y^2 + 15 = 0$	Simplified standard form	✓ Values of y
$5y^2 + 30y + 40 = 0$	Standard form of quadratic equation	
$y^2 + 6y + 8 = 0$	Values of y Substitute y values in (3)	✓ Values of x
$(y + 2)(y + 4) = 0$		
$y = 2$ or $y = -4$		
$x = -5 - y$		
If $y = -2$ or if $y = -4$		
$x = -5 - (-2)$ or $x = -5 - (-4)$		
$= -3$ or $= -1$		
2. $3y = 2x$	Linear is already a subject	(5)
$x^2 - y^2 + 2x - y = 1$	Quadratic is also in terms of y	✓ Equating
3. $y = 8x + 9$		
$y = -x^2 + 4x + 5$		
Solution		
$-x^2 + 4x + 5 = 8x + 9$	In this case, equate the two equations,	
$-x^2 + 4x + 5 - 8x - 9 = 0$	because they are both equal to y .	
$-x^2 - 4x - 4 = 0$		✓ Standard form
$x^2 + 4x + 4 = 0$		✓ Factors
$(x + 2)(x + 2) = 0$		✓ Value of x
$x = -2$		✓ Value of y
If $x = -2$		
$y = 8x + 9$ $y = -x^2 + 4x + 5$	Substitute in either equation.	
$y = 8(-2) + 9$ or		
$y = -(-2)^2 + 4(-2) + 5$		
$y = 3$ or $y = 3$		

Activity	Marks	Answers
1. $x = 2y + 1$ $x^2 - 2y + 3xy = 6$	(6)	$y = \frac{1}{2}$ and $y = -1$ $x = 2$ and $x = -1$
2. $x + 2y = 5$ $2y^2 - xy - 4x^2 = 8$	(6)	$y = \frac{1}{4}$ and $y = 4$ $x = \frac{9}{2}$ and $x = 3$
3. $x + y + 2 = 0$ $x^2 + y^2 = 4$	(5)	$y = 0$ and $y = -2$ $x = -2$ and $x = 0$
4. $2x + y = 3$ $x^2 + y + x = y^2$	(6)	$y = \frac{5}{3}$ and $y = -3$ $x = \frac{2}{3}$ and $x = 3$
5. $y = x - 3$ $y = x^2 - x - 6$	(5)	$y = 0$ and $y = -4$ $x = 3$ and $x = -1$

5. Check your answers

- Indicate answers/responses to the questions and activities in this section.
- Please ensure accurate correlation to the activity in the previous section.
- Indicate mark allocation (use ticks ✓) where required, but more importantly, explain how marks are allocated in the examination.

6. Message to Grade 12 learners from the writers

Technical Mathematics can be fun, as it requires you to pull together all the pieces you learnt in the lower grades to answer the Grade 12 examination. If you skipped one grade before Grade 12, it will have left a void in the grounding needed to pass this examination.

Please ensure that you know all the axioms and corollaries (all the rules) needed to answer the questions. Answer Technical Mathematics exemplar papers before you sit the final examinations.

Write an exemplar in 3 hours and mark your script on your own, using the memorandum, to gauge if you are ready for the final paper. The memorandum is also available on the DBE website.

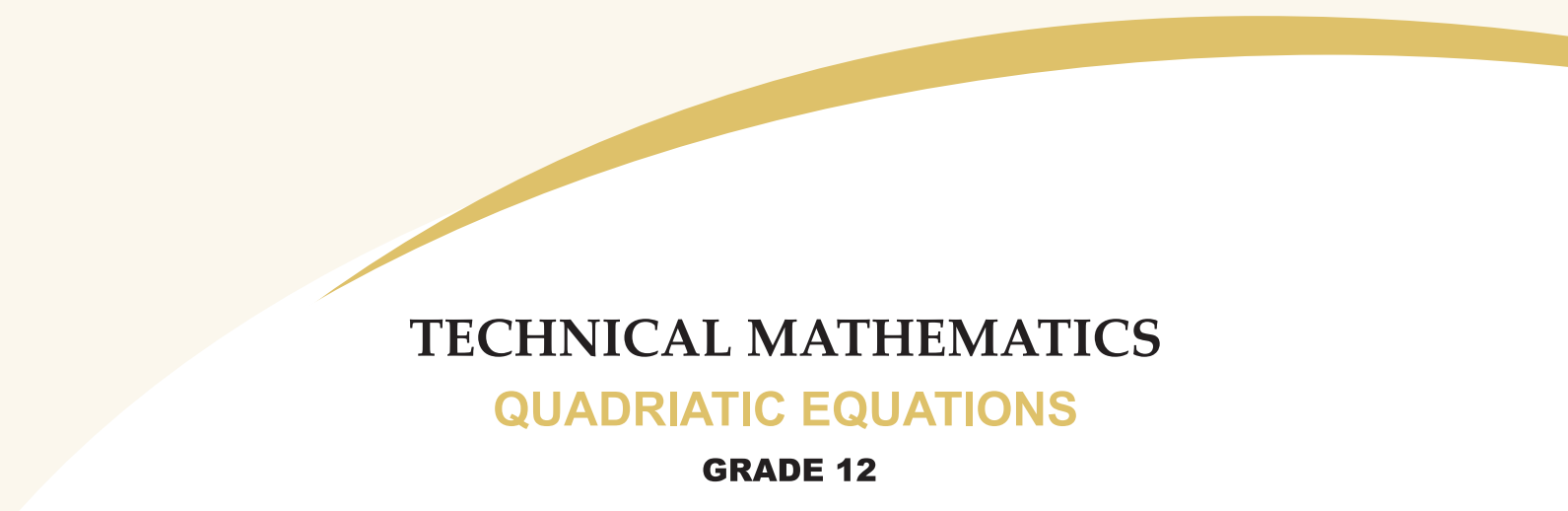
We assure you that this year's final paper will be similar to those of previous years, in both format and style.

Eat only a small food, in order for your cerebrum (brain) to work promptly and effectively. Digestion occupies the functioning of your brain.

Blast the final paper.

7. Thank you and acknowledgements

We hope the explanations provided will be well received as you prepare for the final examinations. Mr Leonard Gumani Mudau, Mrs Nonhlanhla Rachel Mthembu, Mrs Nontobeko Tom and Mr Percy Steven Tebeila all wish you well.



TECHNICAL MATHEMATICS
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GRADE 12

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