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# GR 12 MATHS

# ANALYTICAL GEOMETRY

## QUESTIONS and ANSWERS

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Work through the Grade 11 Analytical downloads first to ensure your foundation is solid before attempting Grade 12 Analytical Geometry which includes circles.

We wish you the best of luck for your exams.

From  
**The Answer Series team**



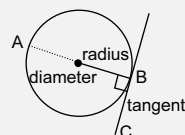
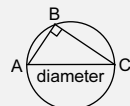
# ANALYTICAL GEOMETRY

## QUESTIONS

### GRADE 12

See why  $\hat{ABC} = 90^\circ$  in these 2 cases ...

- A diameter of a circle makes an angle of  $90^\circ$  at the circumference  $\therefore$  The hypotenuse of a right-angled  $\triangle ABC$  is the diameter of  $\odot ABC$ !
- A tangent to a circle is perpendicular to the radius (or diameter) drawn to the point of contact, i.e.  $BC \perp AB$ .



### CIRCLES – CENTRE AT THE ORIGIN

Equation:  $x^2 + y^2 = r^2$

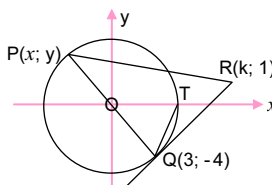
14. The origin O is the centre of the circle in the figure.

P(x; y) and Q(3; -4) are two points on the circle and POQ is a straight line.

R is the point (k; 1) and RQ is a tangent to the circle.

Determine (leave answers in surd form if necessary):

- 14.1 the equation of circle O. (2)
- 14.2 the length of QT. (3)
- 14.3 the length of PQ. (4)
- 14.4 the equation of OQ. (2)
- 14.5 the coordinates of P. (2)
- 14.6 the gradient of QR. (1)
- 14.7 the equation of QR in the form  $y = mx + c$ . (3)
- 14.8 the value of k. (3)
- 14.9 whether the point S(3; 2) lies inside, outside or on the circle. Give a reason for your answer. (2)



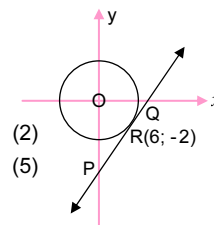
15. PRQ is a tangent to the circle with centre O at the point R(6; -2).

- 15.1 Calculate the equation of
  - 15.1.1 the circle. (2)
  - 15.1.2 the tangent. (5)

- 15.2 Calculate the size of  $\hat{OQR}$ , rounded off to one decimal digit. (3)

- 15.3 Calculate a if (2; a) is a point on the circle  $x^2 + y^2 = 40$ . (3)

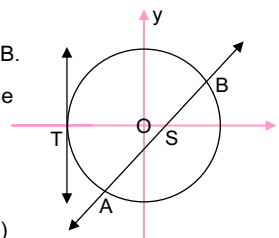
- 15.4 Determine the coordinates of Q. (2)



16. The straight line  $y = x - 1$  cuts the circle  $x^2 + y^2 = 25$  at A and B. AB cuts the x-axis at S. The circle intersects the x-axis at T.

Calculate

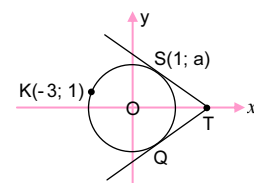
- 16.1 the coordinates of A and B. (6)
- 16.2 the equation of the circle with centre O and radius OS. (3)
- 16.3 the equation of the tangent to the circle at T. (2)



17. In the figure the circle, with centre at the origin O, passes through the point K(-3; 1).

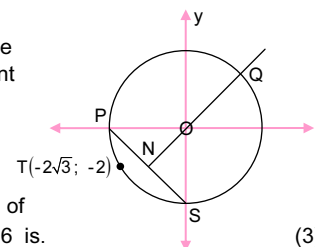
ST is a tangent to the circle at S(1; a).

- 17.1 Determine the radius of the circle. (2)
- 17.2 Write down the equation of the circle. (1)
- 17.3 Show clearly that the value of  $a = 3$ . (2)
- 17.4 Write down the coordinates of a point Q which is symmetrical to the point S with respect to the x-axis. (2)
- 17.5 Write down the gradient of OS. (1)
- 17.6 Hence, show that the equation of the tangent ST is given by  $x + 3y = 10$ . (3)
- 17.7 Write down the coordinates of point T. (2)
- 17.8 Hence, prove that TQ is a tangent to the circle. (3)
- 17.9 Write down the equation of another tangent to the circle (not shown in the figure above) which is also parallel to ST. (2)



18. In the accompanying figure, the circle with the centre at the origin passes through the point T(-2√3; -2) and cuts the x-axis and y-axis at P and S respectively.

- 18.1 Show that the equation of the circle is  $x^2 + y^2 = 16$  is. (3)
- 18.2 Determine the coordinates of P and S. (2)
- 18.3 Determine the equation of the line PS. (3)
- 18.4 If the straight line QN which is perpendicular to PS passes through the origin, write down its equation. (2)
- 18.5 Calculate the coordinates of N if QN meets PS at N. (4)
- 18.6 Find the coordinates of a point E, which is the reflection of point T with respect to the line  $y = -x$ . (2)

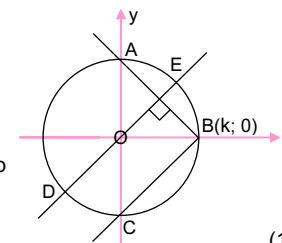


19. A circle with its centre O at the origin passes through the point P(2√3; 2).

- 19.1 Determine the equation of the circle. (2)
- 19.2 Determine the gradient of line OP. (1)
- 19.3 Hence, without using a calculator, determine the size of the angle between OP and the positive x-axis. (2)
- 19.4 Determine the equation of the tangent to the circle at the point P(2√3; 2) in the form  $y = ax + q$ . (4)

20. In the figure, a circle is defined by the equation  $x^2 + y^2 = 9$ , two secants AB and BC meet at a common point B(k; 0), a second line DE passes through the origin and is perpendicular to AB with E a point on the circle.

- 20.1 Write down the value of k. (1)
- 20.2 Determine the length of AB. (2)
- 20.3 Calculate  $\hat{BAC}$ , giving reasons. (2)
- 20.4 Calculate the area of  $\triangle ABC$ . (2)
- 20.5 Determine the equation of line DE. (2)
- 20.6 Hence, without using a calculator, determine the coordinates of D. (4)

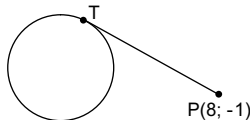


## CIRCLES – ANY CENTRE

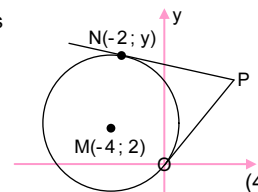
Equation:  $(x - a)^2 + (y - b)^2 = r^2$



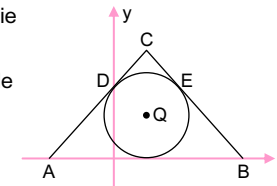
21. Determine the equation of the circle with
- 21.1 centre (2; 3) and radius 7 units (3)
- 21.2 centre (-1; 4) and radius  $\sqrt{5}$  units. (3)
- 22.1 Convert the following *general form* of the equation of a circle to the *standard form*:  $x^2 - 6x + y^2 + 8y - 11 = 0$  (3)
- 22.2 Write down the centre and the radius of the circle. (3)
23. Determine the equation of the tangent that touches the circle defined by:
- 23.1  $(x - 1)^2 + (y + 2)^2 = 25$  at the point (4; 2) (4)
- 23.2  $x^2 - 2x + y^2 + 4y = 5$  at the point (-2; -1) (5)
24. A diameter AB of a circle with points A(-3; -2) and B(1; 4) is given.
- 24.1 Determine the equation of the circle. (4)
- 24.2 Determine the equation of the tangent to the circle at A. (5)
- 25.1 Prove that  $y = x + 7$  is a tangent to the circle  $x^2 + y^2 + 8x + 2y + 9 = 0$ . (6)
- 25.2 Determine the point of contact of the tangent and the circle in 25.1. (2)
26. Find the equation of the circle centre (-2; 5) equal in radius to the circle  $x^2 + y^2 + 8x - 2y - 47 = 0$ . (5)
27. The equation of a circle in the Cartesian plane is  $x^2 + y^2 + 6x - 2y - 15 = 0$ .
- 27.1 Rewrite the equation in the form  $(x - p)^2 + (y - q)^2 = t$ . (4)
- 27.2 Calculate the length of the tangent drawn to the circle from point P(8; -1) outside the circle. (8)
- 27.3 Determine the y-intercepts of the circle. (4)



28. The point M(2; 1) is the midpoint of chord PQ of the circle.  $x^2 + y^2 - x - 2y - 5 = 0$
- 28.1 Determine the coordinates of the centre, A, of the circle. (2)
- 28.2 Determine the radius of the circle. (2)
- 28.3 If chord PQ  $\perp$  AM, determine the equation of chord PQ. (4)
- 28.4 Calculate the coordinates of P and Q. (5)
- 28.5 Determine the equation of the tangent to the circle at the point (2; 3). (4)
29. A(3; -5) and B(1; 3) are two points in a Cartesian plane.
- NB: DRAW A PICTURE!**
- 29.1 Calculate the length of AB and leave the answer in simplified surd form if necessary. (2)
- 29.2 Determine the equation of the circle with AB as diameter in the form:  $(x - a)^2 + (y - b)^2 = r^2$  (4)
- 29.3 Determine the equation of the tangent to the circle at A in the form:  $y = mx + c$ . (5)
30. The equation of a circle with radius  $3\sqrt{2}$  units is  $x^2 + y^2 - 6x + 2y - m = 0$
- 30.1 Determine the coordinates of the centre of the circle. (4)
- 30.2 Determine the value of m. (3)
31. A circle with centre M(-4; 2) has the points O(0; 0) and N(-2; y) on the circumference. The tangents at O and N meet at P.
- Determine:
- 31.1 the equation of the circle. (4)
- 31.2 the value of y. (2)
- 31.3 the equation of OP. (3)
- 31.4 the coordinates of P. (7)
- 31.5 the specific type of figure represented by POMN. (2)

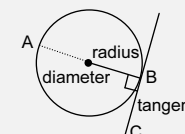


32. The vertices A and B of  $\triangle ABC$  lie on the x-axis.
- The centre of the inscribed circle of  $\triangle ABC$  is Q(4; 5).
- The circle touches AC at the point D(0; 8) and BC at the point E(8; 8).
- Determine:
- 32.1 the equation of the circle. (3)
- 32.2 the equation of BC. (3)
- 32.3 the gradient of AC. (2)
- 32.4 Hence, or otherwise, prove that AC = BC. Give reasons for your answer. (5)
33. A triangle with vertices A(-1; 7), B(8; 4) and C(7; 1) is given.
- 33.1 Show that  $\angle ABC = 90^\circ$ . (4)
- 33.2 Determine the area of the triangle. (4)
- 33.3 Determine the equation of the circle through A, B and C. (7)

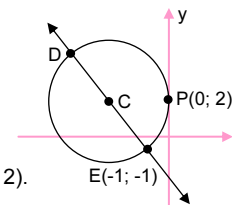
**Remember?**

A tangent to a circle is perpendicular to the radius (or diameter) drawn to the point of contact,

**i.e. tangent  $\perp$  radius.**



34. In the diagram alongside, centre C of the circle lies on the straight line  $3x + 4y + 7 = 0$ .
- The straight line cuts the circle at D and E(-1; -1).
- The circle touches the y-axis at P(0; 2).
- 34.1 Determine the equation of the circle in the form  $(x - n)^2 + (y - q)^2 = r^2$ . (5)
- 34.2 Determine the length of diameter DE. (1)
35. A diameter MN of a circle with points M(-1; 0) and N(3; -2) is given.
- 35.1 Determine the equation of the circle. (4)
- 35.2 Determine the x-intercepts of the circle. (3)
- 35.3 Show that the circle above touches the circle with equation  $(x - 3)^2 + (y - 3)^2 = 5$  (4)



# ANALYTICAL GEOMETRY

## ANSWERS

### CIRCLES – CENTRE AT THE ORIGIN

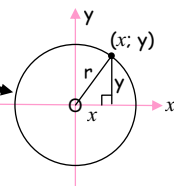
14.1 Equation of  $\odot$  centre O ...

True of any point on the circle is:

$$x^2 + y^2 = r^2 \quad \dots \text{Thm. of Pythagoras}$$

$$\therefore r^2 = x^2 + y^2 = 3^2 + (-4)^2 = 9 + 16 = 25$$

$$\therefore \text{Equation: } x^2 + y^2 = 25 \quad \leftarrow$$



14.2 Length of OT is 5 units  $\Rightarrow$  T(5; 0)1

$$\therefore QT^2 = (5-3)^2 + (0+4)^2 = 4 + 16 = 20$$

$$\therefore QT = \sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \sqrt{5} = 2\sqrt{5} \text{ units} \quad \leftarrow$$

14.3 Diameter, PQ = 10 units  $\dots$  radius = 5

14.4 Gradient,  $m_{OQ} = \frac{y}{x} = \frac{-4}{3}$

$$\therefore \text{Equation of OQ: } y = -\frac{4}{3}x \quad \leftarrow \dots y\text{-intercept, } c = 0$$

14.5 By symmetry/or by rotation  $180^\circ$  about the origin:  
Point P is (-3; 4)  $\leftarrow$

14.6 Note: Radius OQ  $\perp$  tangent QR

$$\therefore m_{QR} = \frac{3}{4} \quad \leftarrow \dots m_{OQ} = -\frac{4}{3}$$

14.7  $m = \frac{3}{4}$  & point Q(3; -4) in:

$$y = mx + c \quad \text{OR} \quad y - y_1 = m(x - x_1)$$

$$\therefore -4 = \left(\frac{3}{4}\right)(3) + c \quad \therefore y + 4 = \frac{3}{4}(x - 3)$$

$$\therefore -4 = 2\frac{1}{4} + c \quad \therefore y + 4 = \frac{3}{4}x - \frac{9}{4}$$

$$\therefore -6\frac{1}{4} = c \quad \therefore y = \frac{3}{4}x - 6\frac{1}{4}$$

$$\therefore \text{Equation: } y = \frac{3}{4}x - 6\frac{1}{4} \quad \leftarrow$$



$$14.8 \quad R(k; 1) \text{ on line QR} \Rightarrow 1 = \frac{3}{4}k - 6\frac{1}{4}$$

$$\therefore 7\frac{1}{4} = \frac{3}{4}k$$

$$\left(\times \frac{4}{3}\right) \quad \therefore \frac{29}{3} = k, \text{ i.e. } k = 9\frac{2}{3} \quad \leftarrow$$

14.9 Inside the circle  $\leftarrow$

$$\text{For } S(3; 2): x^2 + y^2 = 9 + 4 = 13 < 25$$

$\therefore OS < \text{radius } 5 \quad \leftarrow$

OR: Point (3; 4) lies on the circle  $\leftarrow$

by symmetry if Q is reflected in the x-axis.

$\therefore (3; 2)$  lies inside the circle.

$$15.1.1 \quad r^2 = x^2 + y^2 = 6^2 + (-2)^2 = 36 + 4 = 40$$

$$\therefore \text{Equation of } \odot: x^2 + y^2 = 40 \quad \leftarrow$$

$$15.1.2 \quad m_{OR} = \frac{-2}{6} = -\frac{1}{3}$$

$\therefore$  Gradient of tangent = 3

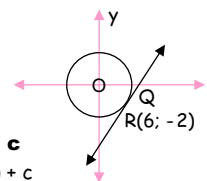
Subst.  $m = 3$  &  $R(6; -2)$  in  $y = mx + c$

$$\therefore -2 = (3)(6) + c$$

$$\therefore -2 = 18 + c$$

$$\therefore -20 = c$$

$$\therefore \text{Equation of tangent: } y = 3x - 20 \quad \leftarrow$$

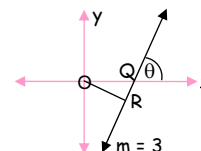


$$15.2 \quad \hat{OQR} = \theta \quad \dots \text{vert. opp. } \angle^s$$

$$\& \tan \theta = 3 \quad \dots m_{QR}$$

$$\therefore \theta = 71,6^\circ$$

$$\therefore \hat{OQR} = 71,6^\circ \quad \leftarrow$$



15.3 Substitute (2; a) in

$$x^2 + y^2 = 40$$

$$\therefore 2^2 + a^2 = 40$$

$$\therefore a^2 = 36$$

$$\therefore a = \pm 6 \quad \leftarrow$$

15.4 At Q,  $y = 0$

$$\therefore 3x - 20 = 0$$

$$\therefore 3x = 20$$

$$\therefore x = \frac{20}{3} = 6\frac{2}{3}$$

$$\therefore Q\left(6\frac{2}{3}; 0\right) \quad \leftarrow$$



$$16.1 \quad \text{At A \& B, } y = x - 1 \quad \dots \textcircled{1}$$

$$\& x^2 + y^2 = 25 \quad \dots \textcircled{2}$$

$\textcircled{1}$  in  $\textcircled{2}$ :

$$\therefore x^2 + (x-1)^2 = 25$$

$$\therefore x^2 + x^2 - 2x + 1 = 25$$

$$\therefore 2x^2 - 2x - 24 = 0$$

$$\therefore x^2 - x - 12 = 0$$

$$\therefore (x-4)(x+3) = 0$$

$$\therefore x = 4 \text{ or } -3$$

$$\textcircled{1}: \therefore y = 3 \text{ or } -4$$

$$\therefore A(-3; -4) \& B(4; 3) \quad \leftarrow$$



$$16.2 \quad \text{At S, } x - 1 = 0 \quad (y = 0)$$

$$\therefore x = 1$$

$$\therefore r = OS = 1$$

$$\therefore \text{Equation of } \odot: x^2 + y^2 = 1 \quad \leftarrow$$

$$16.3 \quad x = -5 \quad \leftarrow$$

$$17.1 \quad r = OK = \sqrt{(-3)^2 + 1^2} = \sqrt{9+1} = \sqrt{10}$$

$$\therefore r = \sqrt{10} \text{ units} \quad \leftarrow$$

$$17.2 \quad \therefore \text{Equation of } \odot: x^2 + y^2 = 10 \quad \leftarrow$$

$$17.3 \quad OS = \sqrt{10} \text{ too} \Rightarrow 1^2 + a^2 = 10 \quad \dots x^2 + y^2 = r^2$$

$$\text{or, S on } \odot$$

$$\therefore a^2 = 9$$

$$\therefore a = 3 \quad \leftarrow (a > 0)$$

$$17.4 \quad Q(1; -3) \quad \leftarrow$$

$$17.5 \quad m_{OS} = 3 \quad \leftarrow \dots \text{Gradient} = +\frac{3}{1}$$

$$17.6 \quad \text{Gradient of tangent ST} = -\frac{1}{3} \quad \dots \text{tangent } \perp \text{ radius}$$

$$\text{Substitute } m = -\frac{1}{3} \& S(1; 3) \text{ in}$$

$$y = mx + c$$

$$\therefore 3 = \left(-\frac{1}{3}\right)(1) + c$$

$$\therefore 3 = -\frac{1}{3} + c$$

$$\therefore 3\frac{1}{3} = c$$

$$\therefore \text{Eqn. is } y = -\frac{1}{3}x + 3\frac{1}{3}$$

$$(\times 3) \quad \therefore 3y = -x + 10$$

$$\therefore x + 3y = 10 \quad \leftarrow$$

$$\text{OR: in } y - y_1 = m(x - x_1)$$

$$\therefore y - 3 = -\frac{1}{3}(x - 1)$$

$$\therefore y - 3 = -\frac{1}{3}x + \frac{1}{3}$$

$$\therefore y = -\frac{1}{3}x + 3\frac{1}{3}$$

Try and get used to this one!

$$17.7 \quad \text{At T, } y = 0: \quad x = 10$$

$$\therefore T(10; 0) \quad \leftarrow$$



$$24.2 \quad m_{AB} = \frac{4 - (-2)}{1 - (-3)} = \frac{6}{4} = \frac{3}{2}$$

$$\therefore \text{Gradient of tangent} = -\frac{2}{3}$$

$$\therefore \text{Substitute } m = -\frac{2}{3} \text{ \& point } A(-3; -2) \text{ in}$$

$$y - y_1 = m(x - x_1)$$

$$\therefore y + 2 = -\frac{2}{3}(x + 3)$$

$$\therefore y + 2 = -\frac{2}{3}x - 2$$

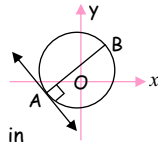
$$\therefore y = -\frac{2}{3}x - 4$$

$$\text{OR: } y = mx + c$$

$$\therefore -2 = \left(-\frac{2}{3}\right)(-3) + c$$

$$\therefore -2 = 2 + c$$

$$\therefore -4 = c, \text{ etc.}$$



25.1 At the point(s) of intersection:

$$x^2 + (x + 7)^2 + 8x + 2(x + 7) + 9 = 0$$

$$\therefore x^2 + x^2 + 14x + 49 + 8x + 2x + 14 + 9 = 0$$

$$\therefore 2x^2 + 24x + 72 = 0$$

$$(\div 2) \quad \therefore x^2 + 12x + 36 = 0$$

$$\text{A perfect square!} \quad \therefore (x + 6)^2 = 0$$

$$\therefore x = -6 \quad \text{only 1 solution!}$$

$\therefore$  The line is a tangent - only 1 point of contact <

$$25.2 \quad y = x + 7 = -6 + 7 = 1$$

$\therefore$  Point of contact is  $(-6; 1)$  <

You can check your answer by making sure that it also satisfies the other equation.

$$26. \quad x^2 + 8x + y^2 - 2y = 47$$

$$\therefore x^2 + 8x + 4^2 + y^2 - 2y + 1^2 = 47 + 16 + 1$$

$$\therefore (x + 4)^2 + (y - 1)^2 = 64$$

The radius,  $r = 8$  units

$$\therefore \text{Equation required: } (x + 2)^2 + (y - 5)^2 = 64 \quad <$$

$$27.1 \quad x^2 + 6x + y^2 - 2y = 15$$

$$\therefore x^2 + 6x + 9 + y^2 - 2y + 1 = 15 + 9 + 1$$

$$\therefore (x + 3)^2 + (y - 1)^2 = 25$$

27.2 Centre,  $M(-3; 1)$  & radius,  $MT = 5$  units  
radius  $MT \perp$  tangent  $PT$

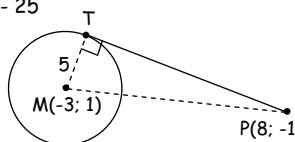
$$\therefore PT^2 = MP^2 - MT^2 \quad \dots \text{Pythagoras}$$

$$= [(8 + 3)^2 + (-1 - 1)^2] - 25$$

$$= 121 + 4 - 25$$

$$= 100$$

$$\therefore PT = 10 \text{ units} \quad <$$



27.3 On the  $y$ -axis,  $x = 0$  ... so, substitute!

$$\therefore 3^2 + (y - 1)^2 = 25$$

$$\therefore (y - 1)^2 = 16$$

$$\therefore y - 1 = \pm 4$$

$$\therefore y = 1 \pm 4$$

$$\therefore y = 5 \text{ or } -3 \quad <$$



$$28.1 \quad x^2 - x + y^2 - 2y = 5$$

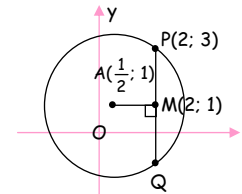
$$\therefore x^2 - x + \left(\frac{1}{2}\right)^2 + y^2 - 2y + 1 = 5 + \frac{1}{4} + 1$$

$$\therefore \left(x - \frac{1}{2}\right)^2 + (y - 1)^2 = 6\frac{1}{4}$$

$$\therefore \text{Centre is } A\left(\frac{1}{2}; 1\right) \quad <$$

$$28.2 \quad r^2 = 6\frac{1}{4} = \frac{25}{4}$$

$$\therefore r = \frac{5}{2} = 2\frac{1}{2} \text{ units} \quad <$$



$$28.3 \quad AM \parallel x\text{-axis} \quad \dots y_A = y_M$$

$$\therefore PQ \parallel y\text{-axis} \quad \dots PQ \perp AM$$

$$\therefore \text{Equation of } PQ: x = 2 \quad <$$

$$28.4 \quad \text{Substitute } x = 2 \text{ in } \left(x - \frac{1}{2}\right)^2 + (y - 1)^2 = 6\frac{1}{4}$$

$$\therefore \frac{9}{4} + (y - 1)^2 = \frac{25}{4}$$

$$\therefore (y - 1)^2 = 4$$

$$\therefore y - 1 = \pm 2$$

$$\therefore y = 1 \pm 2$$

$$\therefore y = 3 \text{ or } -1$$

$$\therefore P(2; 3) \text{ \& } Q(2; -1) \quad <$$

$$28.5 \quad m_{AP} = \frac{3 - 1}{2 - \frac{1}{2}} = \frac{2}{\frac{3}{2}} = \frac{4}{3}$$

$$\therefore \text{Gradient of tangent at } P = -\frac{3}{4}$$

$$\text{Subst. } m = -\frac{3}{4} \text{ \& } (2; 3) \text{ in } y - y_1 = m(x - x_1)$$

$$\therefore y - 3 = -\frac{3}{4}(x - 2)$$

$$\therefore y = -\frac{3}{4}x + 1\frac{1}{2} + 3$$

$$\therefore \text{Equation of tangent: } y = -\frac{3}{4}x + 4\frac{1}{2} \quad <$$

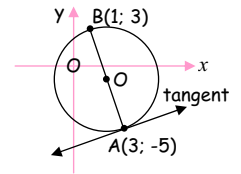


$$29.1 \quad AB^2 = (1 - 3)^2 + (3 + 5)^2$$

$$= 4 + 64$$

$$= 68$$

$$\therefore AB = \sqrt{68} (= \sqrt{4 \times 17} = 2\sqrt{17}) \text{ units}$$



29.2 Centre of  $\odot$  is midpoint of  $AB$ ,  
viz.  $O(2; -1)$  ... by inspection!

$$\& \text{ radius, } r = \frac{1}{2} \text{ diameter} = \sqrt{17} \text{ units}$$

$$\therefore \text{Equation of } \odot: (x - 2)^2 + (y + 1)^2 = 17 \quad <$$

$$29.3 \quad m_{\text{radius}} = \frac{-5 + 1}{3 - 2} = -4$$

$$\therefore m_{\text{tangent}} = \frac{1}{4} \quad \dots \text{tangent} \perp \text{radius}$$

$$\therefore \text{Substitute } m = \frac{1}{4} \text{ \& point } A(3; -5) \text{ in}$$

$$y - y_1 = m(x - x_1):$$

$$\therefore y + 5 = \frac{1}{4}(x - 3)$$

$$\therefore y + 5 = \frac{1}{4}x - \frac{3}{4}$$

$$\therefore y = \frac{1}{4}x - 5\frac{3}{4} \quad <$$



$$30.1 \quad x^2 - 6x + y^2 + 2y = m$$

$$\therefore x^2 - 6x + 9 + y^2 + 2y + 1 = m + 9 + 1$$

$$\therefore (x - 3)^2 + (y + 1)^2 = m + 10$$

$$\therefore \text{Centre: } (3; -1) \quad <$$

$$30.2 \quad m + 10 = (3\sqrt{2})^2 (= r^2)$$

$$\therefore m + 10 = 18$$

$$\therefore m = 8 \quad <$$



$$31.1 \quad \text{Radius, } OM^2 = (-4)^2 + 2^2$$

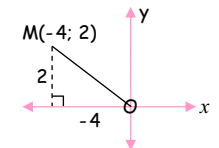
$$= 16 + 4$$

$$= 20$$

& centre is  $(-4; 2)$

$\therefore$  Equation of  $\odot$ :

$$(x + 4)^2 + (y - 2)^2 = 20 \quad <$$



$$31.2 \quad \text{Substitute } N(-2; y): (-2 + 4)^2 + (y - 2)^2 = 20$$

$$\therefore 4 + (y - 2)^2 = 20$$

$$\therefore (y - 2)^2 = 16$$

$$\therefore y - 2 = \pm 4$$

$$\therefore y = 2 \pm 4$$

$$\text{At } N: \therefore y = 6 \quad <$$

$$31.3 \quad m_{OM} = \frac{2}{-4} = -\frac{1}{2}$$

$\therefore m_{OP} = 2 \quad \dots \text{tangent } OP \perp \text{radius } OM$

$\therefore \text{Equation of } OP: y = 2x \quad \leftarrow$

31.4 Equation of NP:

$$m_{MN} = \frac{6-2}{-2-4} = \frac{4}{-6} = -\frac{2}{3}$$

$$\therefore m_{NP} = -\frac{1}{2}$$

&  $(-2; 6)$  in  $y - y_1 = m(x - x_1)$ :

$$\therefore y - 6 = -\frac{1}{2}(x + 2)$$

$$\therefore y - 6 = -\frac{1}{2}x - 1$$

$$\therefore y = -\frac{1}{2}x + 5$$

At P:  $y = 2x$  as well

$$\therefore 2x = -\frac{1}{2}x + 5$$

$$\therefore 2\frac{1}{2}x = 5$$

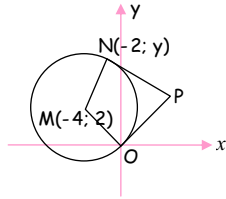
$$\therefore x = 2 \quad \& \quad y = 4$$

$\therefore P(2; 4) \quad \leftarrow$

31.5 A square  $\leftarrow \dots (\hat{MNP} = \hat{MOP} = 90^\circ \quad \& \quad m_{OP} = m_{MN}$

$\Rightarrow \hat{M} = \hat{P} = 90^\circ$  too;

& consec. sides, radii  $MN = MO$ )

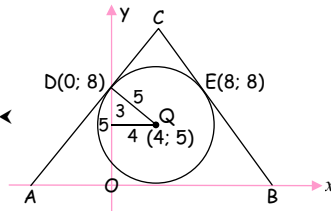


32.1  $r = DQ = 5 \quad \dots \text{look at sketch}$

& centre is  $(4; 5)$

$\therefore \text{Equation of } \odot:$

$$(x - 4)^2 + (y - 5)^2 = 25 \quad \leftarrow$$



$$32.2 \quad m_{QE} = \frac{8-5}{8-4} = \frac{3}{4}$$

$\therefore m_{BC} = -\frac{4}{3} \quad \dots \text{radius } QE \perp \text{tangent } BE$

Substitute  $m_{BC} = -\frac{4}{3}$  & point  $E(8; 8)$  in:

$$y - y_1 = m(x - x_1)$$

$$\therefore y - 8 = -\frac{4}{3}(x - 8)$$

$$\therefore y - 8 = -\frac{4}{3}x + \frac{32}{3}$$

$$\therefore y = -\frac{4}{3}x + 18\frac{2}{3} \quad \leftarrow$$

$$[\text{OR: } 4x + 3y - 56 = 0]$$



$$32.3 \quad m_{DQ} = \frac{8-5}{0-4} = \frac{3}{-4} = -\frac{3}{4}$$

$\therefore m_{AC} = \frac{4}{3} \quad \dots DQ \perp AC - \text{radius} \perp \text{tangent}$

$$32.4 \quad \tan \hat{CAB} = m_{AC} = \frac{4}{3}$$

$$\& \tan \hat{CBA} = -\tan \hat{CBX} = -m_{BC} = -\left(-\frac{4}{3}\right) = \frac{4}{3}$$

$$\therefore \hat{CAB} = \hat{CBA}$$

$\therefore AC = BC \quad \leftarrow \dots \text{base angles equal}$

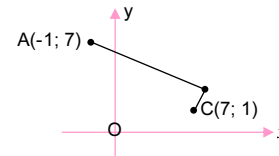
$$33.1 \quad m_{AB} = \frac{4-7}{8-(-1)} = \frac{-3}{9} = -\frac{1}{3}$$

$$\& m_{BC} = \frac{1-4}{7-8} = \frac{-3}{-1} = 3$$

$$\therefore m_{AB} \times m_{BC} = \left(-\frac{1}{3}\right)(3) = -1$$

$\therefore AB \perp BC,$

i.e.  $\hat{ABC} = 90^\circ \quad \leftarrow$



$$33.2 \quad \text{Area of } \triangle ABC = \frac{1}{2} AB \cdot BC$$

$$\& AB^2 = (8+1)^2 + (4-7)^2 = 81+9 = 90$$

$$\& BC^2 = (7-8)^2 + (1-4)^2 = 1+9 = 10$$

$$\therefore \text{Area} = \frac{1}{2} \sqrt{90} \sqrt{10} = \frac{1}{2} \sqrt{900} = \frac{1}{2} (30) = 15 \text{ units}^2 \quad \leftarrow$$

33.3  $\hat{ABC} = 90^\circ \Rightarrow AC$  is the diameter of  $\odot ABC$ !

Centre of  $\odot ABC$  is midpoint of diameter  $AC$ , say  $M$ .

$$\therefore \text{Centre is } \left(\frac{-1+7}{2}; \frac{7+1}{2}\right),$$

$$\therefore M(3; 4)$$

& radius =  $OM = 5$  units  $\dots 3:4:5$  "trip" – Pythagoras

$$\therefore \text{Equation: } (x - 3)^2 + (y - 4)^2 = 25 \quad \leftarrow \dots r^2 = 25$$

34.1 Let centre,  $C$  be  $(n; q)$  – see the question!

The  $\odot$  "touches" the  $y$ -axis  $\Rightarrow y$ -axis is a tangent to the  $\odot$ !

$\therefore CP \perp y$ -axis  $\therefore q = 2$

Also,  $C(n; q)$  lies on line  $3x + 4y + 7 = 0$

$$\therefore 3n + 4(2) + 7 = 0$$

$$\therefore 3n = -15$$

$$\therefore n = -5$$

$\therefore \text{Centre } C(-5; 2)$

$$\& \text{radius}^2, CE^2 = (-1+5)^2 + (-1-2)^2 = 25$$

$$\therefore \text{Equation: } (x + 5)^2 + (y - 2)^2 = 25 \quad \leftarrow$$

34.2  $CE = 5 \Rightarrow \text{diameter } DE = 10 \text{ units} \quad \leftarrow$

35.1 The centre of the circle is the midpoint of  $MN$ :

$$\left(\frac{-1+3}{2}; \frac{0-2}{2}\right) \therefore (1; -1) \quad \& \quad \text{the radius} = \frac{1}{2} MN$$

$$MN^2 = (3+1)^2 + (-2)^2 = 16+4 = 20$$

$$\therefore MN = \sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \sqrt{5} = 2\sqrt{5}$$

$\therefore \text{radius, } r = \sqrt{5}$

$$\therefore \text{Equation of } \odot: (x - 1)^2 + (y + 1)^2 = 5 \quad \leftarrow \dots (= r^2)$$

35.2 On the  $x$ -axis,  $y = 0 \quad \dots \text{so, substitute}$

$$\therefore (x - 1)^2 + 1^2 = 5$$

$$\therefore (x - 1)^2 = 4$$

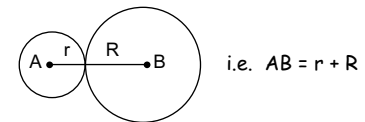
$$\therefore x - 1 = \pm 2$$

$$\therefore x = 1 \pm 2$$

$$\therefore x = 3 \text{ or } -1 \quad \leftarrow$$



35.3 **2  $\odot$ 's touch each other when the distance between their centres equals the sum of their radii**



$$\text{i.e. } AB = r + R$$

Distance between centres  $(1; -1)$  and  $(3; 3)$ :

$$\text{Distance}^2 = (3 - 1)^2 + (3 + 1)^2 = 4 + 16 = 20$$

$$\therefore \text{Distance} = \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$$

& Sum of their radii =  $\sqrt{5} + \sqrt{5} = 2\sqrt{5}$

$\therefore \text{The } \odot\text{'s touch} \quad \leftarrow \dots \text{distance between centres} = \text{sum of radii!}$

