NECT

GRADES 4-9

SUBJECT MATHEMATICS

TERM 3 & 4 2018

TRAINERS GUIDE

CONTENTS

Workshop Objectives

By the end of this training session, participants will:

1. Be aware of the programme for this training session
2. Be informed of the NECT Programme 1 updates
3. Have improved Term 3&4 pedagogical content knowledge.
4. Be fully oriented to the Trainer's Guide that will be used to train teachers on this programme
5. Be motivated to improve their personal facilitation skills
6. Be motivated to improve teaching and learning in their district

Before the Training

1. **Be prepared to model excellence in training and facilitation.**
2. Prepare the venue as best as possible, to ensure that participants are comfortable, that they can all see the presenter, and that the setup is conducive for discussion.
3. Be prepared to show the slide show and videos. Deal with technical issues before the training.
4. Be fully prepared, have all your materials laid out in an orderly fashion.
5. Display the objectives of the workshop and go through these with participants.
6. Display an 'agenda' – a chart listing every activity that will be completed, together with the planned time allocation.
7. At the end of every training day, reflect on the objectives and agenda, and tick off what has been achieved that day.
8. DISPLAY ALL RELEVANT RESOURCE THAT HAVE BEEN PRODUCED BY THE NECT FOR CLASSROOMS, I.E.: POSTERS; RESOURCE PACK ITEMS; ETC. (Make an effort to properly prepare these items to present them in a way that models good practice and will protect resources from wear and tear.)

Tone of the Training

1. Remember that you are training TRAINERS and TEACHERS. Please ensure that you address participants correctly.
2. Be polite, patient and RESPECTFUL at all times. This is possibly the most important aspect of being a trainer.
    - Participants will generally be open to you and to the programme if they are treated with respect.
    - Arrive early and be prepared – for every session!
    - Greet participants by name whenever possible and ensure that names are pronounced correctly.
    - Do not be dismissive of a participant's concern. If you do not have time, or if you know that the issue will be addressed later in the session, create a PARKING LOT. Write down the query and stick it in the parking lot to be addressed later.
    - Do not be dismissive of participants' knowledge, skills and experience. As much as possible, allow participants to contribute to discussions.
3. Remember that humour is always a good strategy – try to add some fun to the training, in a way that does not make anyone uncomfortable.
4. Please remember to use icebreakers and energisers when required – it is important to keep the mood and energy of the training positive.

NECT

GRADES 4-9 MATHEMATICS

TERM 3 & 4 2018 TRAINING PROGRAMME

	TIME	ACTIVITY	TRAINER WORKSHOP	TEACHER WORKSHOP
1	30 minutes	Welcome, housekeeping and updates		
2	30 minutes	Pre-test		
3	30 minutes	Guidelines for facilitators and participants		
4	1 hour	Introductions, reflections and agenda		
5	2 hours 30 minutes	Number sentences (Gr 4 – 6) and Algebraic Equations (Gr 7 – 9)		
6	2 hours	2D shapes (Gr 4 – 7) and Theorem of Pythagoras (Gr 8 – 9)		
7	2 hours	Probability (Gr 4 – 9)		
8a	30 minutes	Selection of topics and preparation for participant presentations		
8b	6 hours 30 minutes	Presentations		
9	2 hours	Orientation to the trainer's guide		
10	30 minutes	Final questions and answers		
11	30 minutes	Post test		
12	30 minutes	Training of teachers: planning session		
13	30 minutes	Closure and evaluation		

What you will need for this training:

ITEM	QUANTITY	CHECK
Flipchart stand and paper	1	
Kokis	10	
Blank A4 paper	100	
Laptop, data-projector and speakers	1	
USB with all materials	1	
Attendance register	1	
Prestik	5	
Evaluation Forms	1 per participant	
White board markers	1 per 2 participants	
Coloured A4 boards	3 per participant	
Dice	1 per participant	
30 cm ruler	1 per participant	
Exercise book (quad)	1 per participant	

1	30 minutes	WELCOME, HOUSEKEEPING AND UPDATES	Facilitator:	What you will need: • Ensure that there is a sign outside your training room

1. Settle the group in plenary.
2. Welcome participants and complete the introductions.
3. Start the day with a short message or prayer if appropriate.
4. Share the relevant housekeeping notes, to ensure that participants are clear about the toilet and catering arrangements.
5. Present any relevant updates or share interesting and successful data or stories.

2	30 minutes	PRE-TEST	Facilitator: MQA	What you will need: • Copies of pre-test

1. Work together to hand out copies of the pre-test to participants.
2. Ask participants to not look at the test yet.
3. Briefly explain the purpose of the pre-test, which is to measure the success of the training, not to measure the scores of individuals.
4. Briefly explain the text conditions, i.e.: to work independently and in silence, for a period of ___ minutes. Ask participants who finish before time to please cover their work and wait quietly for others.
5. As participants complete the pre-test, walk around and offer practical assistance if needed.
6. Once time is up, help to collect and collate pre-tests in an orderly fashion.

3	30 minutes	**GUIDELINES FOR PARTICIPANTS AND FACILITATORS**	**Facilitator:**	**What you will need:** • Flipchart papers • Marker pen

1. Ask participants to close their eyes and take a minute to think about the classrooms they work in, either as a coach, subject advisor, or teacher. Ask them to consider:
   a. Under which conditions does real learning take place?
   b. Should the speaker, whether it is the teacher or a learner, be audible?
   c. Should learners and the teacher be focussed and attentive?
   d. Should there be time for meaningful discussion, and how do you know if this discussion is meaningful?
   e. Should learners and the teacher be asking and responding to questions?
   f. Should the teacher be well organised, and keep the pace of the lesson going?
   g. Should the teacher have some strategies to address uncooperative learners, and what should these strategies be?
   h. Should the teacher model respectful behaviour, in order to encourage the learners to behave respectfully?
   i. What should the teacher do if learners are disrespectful?

2. Ask participants to open their eyes, and to TURN AND TALK to a partner about this.
   a. Then, call participants to attention, and ask them to share any good points they have come up with.
   b. As participants share ideas, document key points on a flipchart, under the heading: **Conditions for real learning.**
   c. Once you have written most points down, thank participants.

3. Next, ask them to please remember this critically important point about teaching and learning: **real learning occurs best when there is effective classroom management.**

4. Finally, ask participants to now think about our workshop situation: how is this different to or the same as a classroom?
5. Look at the list you have written and discuss the difference between a classroom and an adult education event.
6. Have two pieces of flipchart paper ready, with the following headings:
   a. Guidelines for the facilitator
   b. Guidelines for the participants

7. Ask participants to call out guidelines for both the facilitator and the participants and write them on the appropriate sheet.
8. Once completed, read through both lists, and ask participants if they can agree to follow these guidelines, in order for optimum learning to take place.
9. Do your best to follow the guidelines for facilitators.

4	1 hour	**INTRODUCTIONS, REFLECTIONS AND AGENDA**	**Facilitator:**	**What you will need:** • Flipchart papers • Marker pen • Prepared chart of agenda / programme • Blank A4 papers

1. Settle participants so that you have their attention.
2. If there are any new members of the group, or if you are new to the group, briefly do a round of introductions.
3. Next, tell participants that you would like to take some time to get them to reflect on their own experience of the implementation of the training and programme.
4. Make sure each participant has a piece of A4 paper.
5. Ask participants to fold the paper into 4.
6. Next, ask them to do the following:
   a. **In the first square**, **they must write:** their name, position, school or district.
   b. **In the second square**, **they must write:** one thing about the programme that is being successfully implemented in schools. Ask them to please write some details about this, even a short narrative to explain what is happening.
   c. **In the third square**, **they must write:** Something that is still problematic, that the programme has not managed to address. Ask them to write some detail about this, even a short narrative to explain what is happening.
   d. **In the fourth square, they must write:** Anything further that they still want from the NECT. Please point out that this cannot be resources.
7. Draw this diagram on flipchart paper to help participants remember what to do:

Name **Position** **School or District**	**One thing that is working well in schools:**
One thing that is still a problem in schools:	**One thing I think the NECT should do for my subject:**

8. After about 15 minutes, call participants to attention.
9. Ask if anyone would like to share ONE point that they have written down. Listen to as many participants as possible.
10. Thank participants for their input and assure them that you will pass their comments along.
11. Collect all these sheets – you must collate this information for your report.
12. Go through the agenda for this training with participants. If any of their previous needs are being addressed, please point this out.

5	2 hours 30 minutes	NUMBER SENTENCES (Grade 4-6), ALGEBRAIC EQUATIONS (Grade 7-9)	Facilitator:	What you will need: Quad paper exercise book

INTRODUCTION

1. Building and solving number sentences is an important skill in the development of mathematical thinking. The skill is learnt in the Intermediate Phase and extended in the Senior Phase.
2. It is also the basis for understanding algebraic reasoning. The idea that we know some things (the known) but some things are unknown, and we need to solve (the unknown), becomes all important in algebra in later grades.
3. If learners understand and apply the concept of number sentences, they realise that many day-to-day problems can be converted into and solved through the construction of number sentences.
4. What is important, is that teachers emphasize the fact that mathematics – and algebra for that matter – is not separated from learners' real life but is an integral part of making them smart to live their life.

THE PROGRESSION OF THE TOPIC ACROSS GRADES AND PHASES

1. Ask participants to turn to the NECT Grade 4-9 Training Handout Term 3 & 4 2018, page 3, marked ***NUMBER SENTENCES (GR 4-6), ALGEBRAIC EXPRESSIONS & EQUATIONS (GR 7-9): PROGRESSION OF THE TEACHER GUIDELINES AND CLARIFICATION NOTES FOR THE TOPIC ACROSS GRADES AND PHASES***
2. Allow participants some time to browse through the table and observe how this topic is taken forward across grades and phases.
3. Use the ten points below the table to guide a critical group discussion about the build-up towards solving algebraic equations.
4. Remind participants that a pre and post-test for this topic are available in the NECT Mathematics Grade 4-9 Training Handout for use in their teacher training.
5. Tell participants that we will now work through the topics: Number sentences and Algebraic Equations.

NUMBER SENTENCES (Grade 4-6)

TERMINOLOGY

1. Participants note that the terminology we use in this topic is clearly explained in the Content Booklets.
2. Ask participants to discuss the difference between an expression and an equation and in which of these two categories a number sentence belongs.

SUMMARY OF KEY CONCEPTS

1. Remind participants that the section titled: Summary of Key Concepts in the Content Booklets is an overview of all the new concepts that will be taught to learners in this section.
2. Remind participants to use the Planner & Tracker to guide them to teach these in the correct order.
3. Tell participants that the focus today will be on constructing number sentences from real situations and starting to solve number sentences.
4. You will go through the following aspects of Number Sentences:
    - Converting a mathematical problem into a number sentence
    - Expressions and equations
    - Solving by inspection
    - Solving by trial and improvement
    - Checking the solution by substitution
    - Problem types in the Intermediate Phase
    - Setting up a number sentence
    - Solving number sentences and substituting the solution into the number sentence
5. Remind participants that the actual teaching of a new concept should not take more than 10 – 15 minutes.
6. Go through the sub-topics as follows:

Converting a mathematical problem into a number sentence

1. What we know as a mathematical problem is really a life situation that has been converted into a mathematical form to find a solution for the real situation - mathematically.
2. An example is the situation that Thato read 46 pages in 2 hours. The question, the unknown or the problem we want to solve is, how many pages did he read in an hour?
    - The problem converted to a number sentence would be 46 (pp) ÷ 2 (h) = $\square$ (pages in an hour).
    - When we start explaining to learners how to set up a number sentence, we first talk through the problem and get some ideas from the learners.
    - We try to discourage them to give the answer immediately and encourage them to focus on HOW we get to an answer or a solution.
    - The teacher can say the words above (pages, hours and pages in one hour) verbally while writing on the chalk board: $46 \div 2 = \square$.

Expressions and equations

1. An expression contains an unknown but is a mathematical sentence which is also a statement, not a question – it does not contain an equal sign (=) and does not give enough information to make a solution possible.
2. In the example: $\square$ + 4, we do not know what the value of $\square$ is, because we do not have enough information. This expression can therefore not be solved.
3. An equation contains an unknown and is a mathematical sentence which can be seen as a question. It contains an equal sign (=) and it gives enough information to make a solution/answer possible.
4. In the example: $\square$ + 4 = 7, we can find out what the value of $\square$ is, because we have enough information. This equation can therefore be solved. In Grade 4-6 we solve it either by inspection or by trial and improvement, and we check the answer by substitution. In Grade 7 we solve it by any of inspection, trial and improvement, or substitution.

Solving by inspection

1. When learners solve a problem by inspection, they do not really calculate, but they look at it and "see" what would make sense to make the equation true. A way of teaching them to solve by inspection, is to discuss the number sentence with them, saying "what?" in the open space or the block.
2. In the example: 15 + $\square$ = 19, the teacher may say: "Fifteen plus ***what*** is equal to nineteen?"

Solving by trial and improvement

1. When a problem cannot be solved easily by inspection, learners may solve it by trial and improvement, meaning that they take an initial guess which makes it easy to "see" the answer. They see that the answer is either too big or too small, adapt their first guess to come closer to making the equation true and continue until they find the solution.
2. In the example: 84 – $\square$ = 56, learners may try to subtract 30, which gives an easy answer (54); they see that the answer is too low by 2; they improve the answer by subtracting 2 less, that is they subtract 28.

Checking the solution by substitution

1. Through whatever method learners found the solution, teachers bring them into a habit of checking their answers or solutions by substituting them into the unknown space.
2. In the example: $\square$ + 4 = 7, if a learner found 11 as a solution, they substitute the unknown by 11 and say: "11 + 4 = 7". They immediately see that it does not make sense and go back to improve or correct

the solution. If they found 3 as the solution, they substitute 3 into the unknown space and see 3 + 4 = 7, which makes sense and is the right answer.

Problem types in the Intermediate Phase

Learners start writing number sentences with addition and subtraction situations that were given to them in the form of "word problems". They can also get context free number sentences to write. After that, they move to multiplication and division situations. Below is a summary of all those types of number sentences. In the first example there is a "reasoning" part which should be done for all the problems to guide learners as follows:

Note: Teachers may accept alternative ways in which learners put a number sentence, as long as the idea is right, and it works out on the desired answer or solution.

Addition: Number + number = sum

1. <u>Unknown number + known number = known sum</u>

Example: Jim had some money. He received R25 more, now he has R68. How much money did he start with?

Reasoning: "We do not know how much money Jim had to start with, so we put an open block to start with. He received R25 more, which means we must add R25 to the start money. Now he has R68, which means that these two sums together add up to R68."

Solution: $\square$ + R25 = R68 **OR** R25 + $\square$ = R68 **OR** R68 – R25 = $\square$

2. <u>Known number + unknown number = known sum</u>

Example: Jim had R43 and he received some more money, now he has R68. How much money did Jim receive?

Solution: R43 + $\square$ = R68 **OR** $\square$ + R43 = R68 **OR** R68 – R43 = $\square$

3. <u>Known number + known number = unknown sum</u>

Example: Jim had R43 and he received R25 more. How much money does Jim have now?

Solution: R25 + R43 = $\square$

Subtraction: Number – number = difference

1. <u>Unknown number – known number = known difference</u>

Example: Thabo's dad had several sheep. He sold 37 sheep and he was left with 56 sheep. How many sheep did he have to start with?

Solution: ☐ – 37 = 56 **OR** 37 + 56 = ☐

2. <u>Known number – unknown number = known difference</u>

Example: Thabo's father had 93 sheep. He sold some of them and was left with 56 sheep. How many sheep did he sell?

Solution: 93 – ☐ = 56 **OR** ☐ + R56 = R93 **OR** R56 + ☐ = R93 **OR** R93 – R56 = ☐

3. <u>Known number – unknown number = known difference</u>

Example: Thabo's father had 93 sheep and he sold 37. How many sheep does he have left?

Solution: 93 – 37 = ☐ **OR** ☐ + R37 = R93 **OR** R37 + ☐ = R93

Multiplication: First number x second number = product

1. <u>First number (unknown) x second number (known) = product (known)</u>

Example: School B has 216 Grade 4 learners, which is 6 times as many as School A. How many Grade 5 learners does School A have?

Number sentence: ☐ x 6 = 216

2. <u>First number (known) x second number (unknown) = product (known)</u>

Example: School A has 36 Grade 5 learners and School B has 216. How many times more learners does School B have than School A?

Number sentence: 36 x ☐ = 216 **OR** ☐ x 36 = 216

3. <u>First number (known) x second number (known) = product (unknown)</u>

Example: School A has 36 Grade 5 learners and School B 6 times more Grade 4 learners. How many Grade 5 learners does school B have?

Number sentence: 36 x 6 = ☐ **OR** ☐ ÷ 36 = 6 **OR** ☐ ÷ 6 = 36 **OR** 6 x 36 = ☐

Division: First number ÷ second number = quotient

1. <u>First number (unknown) ÷ second number (known) = quotient (known)</u>

Example: Susan makes up packets of 7 apples from a box of apples and she makes up exactly 13 packets. How many apples were in the box?

Number sentence: ☐ ÷ 7 = 13 **OR** ☐ ÷ 13 = 7 **OR** 7 x 13 = ☐

2. First number (known) ÷ second number (unknown) = quotient (known)

Example: Su packs 13 bags from a box of 91 apples. How many apples are in each bag?

Number sentence: 91 ÷ ☐ = 13 **OR** 91 ÷ 13 = ☐ **OR** 13 x ☐ = 91

3. First number (known) ÷ second number (known) = quotient (unknown)

Example: From a box with 91 apples, Susan makes up packets of 7 apples each. How many packets of apples does she make up?

Number sentence: 91 ÷ 7 = ☐ **OR** 91 ÷ ☐ = 7 **OR** 7 x ☐ = 91

Context free number sentences

1. There is a number that is 5 more than 12. What is the number?

Solution: ☐ – 12 = 5 **OR** ☐ – 5 = 12 **OR** 12 + 5 = ☐ (the number is 17)

2. There is a number that is 7 less than 23. What is that number?

Solution: ☐ + 7 = 23 **OR** 23 – ☐ = 7 **OR** 23 – 7 = ☐ (the number is 16)

3. 45 is 3 times more than a certain number. What is that number?

Solution: ☐ x 3 = 45 **OR** 45 ÷ ☐ = 3 **OR** 45 ÷ 3 = ☐ (the number is 15)

4. 21 is 4 times less than a certain number. What is that number?

Solution: ☐ ÷ 4 = 21 **OR** 21 x 4 = ☐ **OR** ☐ ÷ 21 = 4 (the number is 84)

5. If I add 12 to a certain number, the sum is three times that number. What is that number?

Solution: ☐ + 12 = 3 x ☐ (the number is 6)

Further examples to consolidate

1. Thili has R350 and she spends R85. How much money does she have left?
2. Luthando collects marbles. He wants to have 200 marbles in the end and he has 144 up to now. How many more marbles must he get to reach his goal number?
3. There are 55 members in the choir of Peme Primary. They perform in the competition together with another primary school who have 23 members in their choir. How many choristers are there altogether?

4. Mamile does not know how many books there were on the teacher's table in the morning. She put 15 extra books on the table and now there are 73 books. How many books were there in the morning?
5. There are 37 test papers on the table and Mary-Jane put more test papers on the pile, now there are 64. How many test papers did Mary-Jane add?

Setting up a number sentence

1. **Setting up number sentences from word problems**: We use the above problem structures to set up the number sentence from a given situation.
2. **Setting up a number sentence from number problems**:
   Learners must also be able to set up number sentences from context free number problems, where only numbers are given, and the problem is not set out in words.

Examples:

1. A certain number is 9 more than 23. What is that number?
2. A number is 15 less than 38. What is that number?
3. There is a number that is 7 times more than 22. What is that number?
4. If I divide a certain number by 4, the answer is 43. What is that number?

Solutions:

1. $23 + 9 = \square$ **OR** $\square - 9 = 23$ **OR** $\square - 23 = 9$
2. $38 - 15 = \square$ **OR** $\square + 15 = 38$
3. $22 \times 7 = \square$ **OR** $\square \div 7 = 22$
4. $\square \div 4 = 43$ **OR** $43 \times 4 = \square$

Solving number sentences and substituting the solution into the number sentence

After setting up the number sentence, learners can solve the problem, or they can solve a number sentence that has already been set up, through inspection or trial and improvement.

After solving the problem, learners can replace the solution into the unknown space of the number sentence to check for correctness of the solution, as has been explained above.

Memoranda for pre- and post-test

MEMORANDUM PRE-TEST

Mark the following Grade 5 number sentences for the situations by giving a ✓ for a correct number sentence or a X for an incorrect number sentence:		
1	There is a number that is two less than 15. What is that number? $\square - 2 = 15$	X
2	Jabu has two times more marbles than Thabo. Jabu has 36 marbles. How many marbles does Thabo have? $2 \text{ X } \square = 36$	✓
3	There are 13 more learners in Grade 6 than in Grade 5. In Grade 6 there are 44 learners. How many learners are in Grade 5? $44 + 13 = \square$	X
4	At the sale Pam paid half price for shoes that were marked R320 before the sale. How much did Pam pay for the shoes? $\square \text{ x } 2 = 320$	✓
5	Mom paid R33 for three-kilogram bananas. What is the price of one-kilogram bananas? $33 \div 3 = \square$	✓

MEMORANDUM POST-TEST

Mark the following Grade 5 number sentences for the situations by giving a ✓ for a correct number sentence or a X for an incorrect number sentence:		
1	There is a number that is five more than 84. What is that number? $\square + 5 = 84$	X
2	Princess has three times more money than Pamela. Princess has R24. How much money does Pamela have? $3 \text{ X } 24 = \square$	X
3	There are 12 fewer learners in Grade 5 than in Grade 6. In Grade 5 there are 43 learners. How many learners are in Grade 6? $43 - 12 = \square$	X
4	At the sale Pam paid half price for a pair of shoes. She paid R125 for the pair of shoes. What was the price of the shoes before the sale? $125 \text{ x } 2 = \square$	✓
5	Uncle Jim paid R60 for 4 litres of petrol. What is the price of one litre petrol? $4 \text{ x } \square = 60$	✓

ALGEBRAIC EQUATIONS (GRADE 7-9)

INTRODUCTION

1. Learning Algebra and Algebraic Equations is achievable for all, you just need to take things one step at a time and learn the basic rules before moving on to more advanced topics.
2. Equations are new to Grade 7.
3. In Grade 8 and 9 they are done early in the year and again in Term 4 (Grade 8) and Term 3 (Grade 9).
4. It is important to note that this section is vital as it forms the foundations for further studies in Mathematical Sciences. Algebra becomes the largest contributing section in Mathematics and helps provide structure for the study of trigonometry, statistics, geometry and many other sections of the FET curriculum.
5. The real purpose of learning to solve equations is to help with problem solving which is the basis of all mathematics.
6. This topic is done with the assumption that the following skills are already in place:
    - collecting like terms
    - using the distributive law
    - understanding and use of the of order of operations.
    - 'undoing' order of operations by using inverse operations.

TERMINOLOGY

1. Participants note that the terminology we use in this topic is clearly explained in the Content Booklets.

SUMMARY OF KEY CONCEPTS

Sequential Table

1. Ask participants to look at the sequential table (from the Grade 8 Content Booklet) in the NECT Grade 4-9 Training Handout Term 3 & 4 2018. This shows all the sub-topics and skills taught in the senior phase.
2. In Grade 7, learners were introduced to algebra through writing number sentences and solving number sentences by inspection or trial and error.
3. In Grade 8, learners start to set up equations to describe problem situations then go on to solve those equations by inspection before using additive and multiplicative inverses to solve them algebraically. Substitution is also included at this stage.
4. In Grade 9, solving quadratic equations by factorising is introduced.

Teaching of new skill / concept

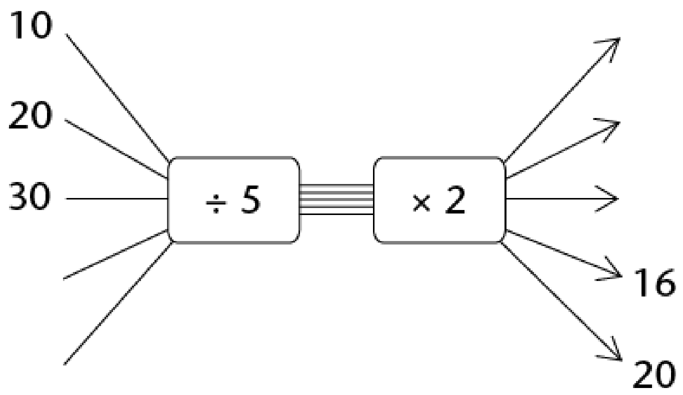
Remind participants that

1. the content booklet has an overview of all the new concepts that will be taught in each section
2. they will use the Planner & Tracker to guide them to teach these in the correct order.

Key Concepts: Grade 7

Writing, analysing and interpreting number sentences to describe problem situations

1. The term, *number sentence* is still used at this stage as it is more familiar to learners.
2. Although learners should have had plenty of opportunities to write and interpret number sentences by now, teachers should still be monitoring each learner's progress to assess their grasp of the concepts and skills required of them.
3. The ability to find input and output values of a flow diagram are important skills in the understanding of equations.
4. For example, consider the following flow diagram (this is available in the NECT Grade 4-9 Training Handout Term 3 & 4 2018):

   Some input numbers, the rule and some output numbers have been given.
5. Working with the input number to find the output number develops the mathematical skill of substitution.
   Working with the output number to find the input number develops the mathematical skill of using inverse operations to work in reverse.
   Both skills are vital for learners to develop a good understanding of solving equations.
6. Complete the exercise of finding the missing values with participants now. The full solutions are shown below:

Input to get output	**Output to get input**
$10 \div 5 \times 2 = 4$	$16 \div 2 = 8$ $8 \times 5 = 40$
$20 \div 5 \times 2 = 8$	$20 \div 2 = 10$ $10 \times 5 = 50$
$30 \div 5 \times 2 = 12$	

7. Discuss the setting out of the last 2. It is important that the following is not done:

$$16 \div 2 = 8 \times 5 = 40$$

This is not acceptable. This implies that $16 \div 2 = 40$!!

Solve and complete number sentences by inspection and trial and improvement

8. Learners can really enjoy *'I think of a number'* puzzles – this should be used to a teacher's advantage. If these are practiced in a game-like situation instead of as an ordinary lesson, learners will feel more relaxed about it.
9. The following kind is always a good starting point. Ask participants to:

- Think of a number. Don't say it aloud.
- Add 3
- Double the result
- Subtract 4
- Halve the result
- Subtract the original number
- Your result is 1!

10. Repeat again, asking participants to choose another number – tell them to be daring and choose a negative integer. (This could be repeated a few times in the classroom, allowing learners to practice some mental arithmetic).
11. Ask participants to work with a partner and show mathematically why this puzzle always works (this probably wouldn't be done at Grade 7 level, but Grade 8's and 9's should certainly be encouraged to find the mathematics in the 'magic').

Let x be the number:

- Add 3: $x + 3$
- Double the result: $2(x + 3) = 2x + 6$
- Subtract 4: $2x + 6 - 4 = 2x + 2$
- Halve the result $\frac{2x+2}{2} = x + 1$
- Subtract the original number:

$$x + 1 - x = 1$$

12. Discuss the process. Clearly the one is essential – ask: *why will there always be the +1 at the end?* Surely, that means the constant before the step of halving needs to be 2. Go back to the step of doubling the number that 3 had been added to – this created the 6. 4 was then chosen to subtract as that would lead back to the 2 which as mentioned was essential.
13. Ask participants to get together in pairs or threes and make up a similar *'I think of a number'* puzzle. After about 5 minutes ask for any volunteers to share their puzzle. When they do, they need to start with: Think of a number, then proceed to issue the instructions that will result in all participants getting the same answer even though they chose different numbers.
14. Changing a word problem into a mathematical statement is one of the most important skills learners will need for mathematics throughout high school as well as in other areas of their lives where they may have to solve problems.
15. In the classroom, it may be a good idea for learners to first be given a list of equations to make up a situation about before having to do it themselves. Ask participants to get together in pairs or threes to make up a problem that is represented in the equations below:
(the variable must be directly related to the actual question)

$5 \times x = 120$	$n - 4 = 10$	$x + 3x = 12$

After a few minutes ask a few volunteers to share their problems.

16. There are many possibilities for each one, here are 3 to use if participants found this a challenge.

$5 \times x = 60$	A child buys 5 packets of sweets and pays R60. How much did each packet cost?
$n - 4 = 10$	There were a certain number of passengers on a bus. 4 people got off at a stop, and 10 passengers remained. How many passengers were on the bus originally?
$x + 3x = 12$	A 12m piece of wire is broken into two parts where the longest piece is three times as long as the shortest piece. Find the length of the shortest piece.

17. In the classroom situation, learners need to spend time solving equations by inspection and by trial and improvement. They have already learned the skill of using inverse operations to work in reverse and this will stand them in good stead for learning how to solve equations algebraically in Grade 8. At this level, however, they must be encouraged to verbalise their process of thinking and how they come to a solution. Groupwork or pair work is ideal for this. Hearing other learner's ideas is an excellent way to consider other ways of solving a problem.
18. Before moving on to variables and constants, ask participants to share examples from their own experience (from their own teaching days or from lessons they have watched) that they think worked well to develop the skills mentioned so far.
19. Thank any participants that volunteered. It is always appreciated to learn from others' good ideas.

Identifying variables and constants in formulae or equations

20. The same examples from above can be used to discuss which parts of an equation are variables and which part are constants. Ask participants for a definition of these two terms in an equation.
    (A variable is a symbol for a number that we don't know yet – it represents the unknown. A constant is a number on its own – it will remain constant no matter what the variable is equal to).
21. In the above examples, the variable was the unknown in the question (the cost of the sweets, the original number of passengers and the length of the piece of wire).

Determining numerical values of an expression by substitution

22. Substitution can be practiced by completing a table and making use of a familiar formula.
23. For example (this is available in the NECT Grade 4-9 Training Handout Term 3 & 4 2018):

length	breadth	Area $l \times b$	Perimeter $2l + 2b$
5cm	2cm		
8m	6m		
4cm	7cm		
2m	10m		
9km	3km		

24. Complete the table with participants, discussing how variables can be recapped as well as reminding learners of a topic previously covered. This is an important idea for teachers to try and bring in to their teaching. Learners need to learn early on that topics in mathematics are rarely stand-alone topics.
25. A discussion regarding what happens when *cm* are multiplied by *cm* and what happens when cm are added to cm would also be valuable as this links directly to algebra too.
26. Solutions:

$10cm^2$	$14cm$
$48m^2$	$28m$
$28cm^2$	$22cm$
$20m^2$	$24m$
$27km^2$	$24km$

27. Point out that as substitution is covered in Grade 8 as well, we will only do this example now. Substitution must be done with formulae and equations at this stage.
28. Ask if anyone has any questions or comments before we move onto Grade 8 work.

<u>Additional Key Concepts: Grade 8</u>

Solving equations using additive and multiplicative inverses and the laws of exponents

1. Tell participants that although learners will have already done equations earlier in the year, revision of earlier work needs to be done when they are covered again in Term 4.
2. This includes:
   - solving equations using additive and multiplicative inverses
   - solving equations by using the laws of exponents
   - setting up equations to describe given situations
   - analysing and interpreting equations.
3. Essentially, there is no new concept introduced in Term 4. According to the curriculum, the focus should be on using substitution to generate ordered pairs.
4. Ask participants what the steps are to solving an equation. Write these on the board as they are given.
   - Use inverse operations to get the variables (unknown) on one side (usually the left-hand side) and the constants on the other side
   - Simplify each side of the equation by collecting like terms if possible
   - Use inverse operations to get the unknown variable by itself and hence solve for the unknown in the given equation

5. Complete the following fully worked examples with participants now: (The teachers they will be training need to be encouraged to ask their learners the same questions over and over. Learners should eventually be 'hearing' the questions themselves when they are solving an equation).

Example	Points to discuss as an example is done
$y-5=12$ $y-5+5=12+5$ $y=17$	Ask: *What is in the way of the variable being on its own?* (-5) *How do we 'undo' this operation?* (add 5) Remember: *Whatever we do to one side of an equation, we need to do to the other side – it must stay balanced.*
$5a=20$ $\frac{5a}{5}=\frac{20}{5}$ $a=4$	Ask: *What is in the way of the variable being on its own?* (x 5) *How do we 'undo' this operation?* (divide 5) Remember: *Whatever we do to one side of an equation, we need to do to the other side – it must stay balanced.*
$\frac{x}{2}=12$ $2\times\frac{x}{2}=12\times2$ $x=24$	Ask: *What is in the way of the variable being on its own?* (÷ 2) *How do we 'undo' this operation?* (x 2) Remember: *Whatever we do to one side of an equation, we need to do to the other side – it must stay balanced.*
$4x+2=22$ $4x+2-2=22-2$ $4x=20$ $\frac{4x}{4}=\frac{20}{4}$ $x=5$	Ask: *What is in the way of the variable being on its own?* (+ 2 and x 4) *What are the inverse operations of addition and multiplication?* (subtraction and division) *Which one must we work with first?* (+2) Remember: *Whatever we do to one side of an equation, we need to do to the other side – it must stay balanced.*
$-2a+3=-5$ $-2a+3-3=-5-3$ $-2a=-8$ $\frac{-2a}{-2}=\frac{-8}{-2}$ $a=4$	Ask: *What is in the way of the variable being on its own?* (+ 3 and x -2) *What are the inverse operations of addition and multiplication?* (subtraction and division) *Which one must we work with first?* (+3) Remember: *Whatever we do to one side of an equation, we need to do to the other side – it must stay balanced.*

6. Point out to participants how the mathematical language used became more technically correct as the examples proceeded. The earlier learners use the correct terminology, the better.
7. In Grade 8, learners are also expected to use the laws of exponents while solving equations.

8. Write the following on the flipchart: $3^x = 3^8$

   It should be straightforward to see that $x = 8$ since the bases are the same.

9. Discuss with participants that this – the bases being the same - is the key to solving all exponential equations (at least until Grade 12 when logarithms will be taught and used).

10. Complete the following fully worked examples now:

    (As mentioned before the previous examples, the teachers they will be training need to be encouraged to ask their learners the same questions over and over. Learners should eventually be 'hearing' the questions themselves when they are solving an equation).

Example	Points to discuss with participants
$4^x = 64$ $4^x = 4^3$ $\therefore x = 3$	Ask: *what is essential when solving exponential equations?* (same bases are required) *Therefore, I will need to…?* Rewrite 64 as a base 4
$2.2^x = 16$ $2.2^x = 2^4$ $2^{1+x} = 2^4$ $\therefore 1 + x = 4$ $x = 3$ Option 2: $\frac{2.2^x}{2} = \frac{2^4}{2}$ $2^x = 2^3$ $\therefore x = 3$	Ask: *what is essential when solving exponential equations?* (same bases are required) *Therefore, I will need to?* Rewrite 16 as a base 2 Then, use rules of exponents to simplify Once bases are the same, the exponents are the same. Note: Point out that this question could have been done slightly differently – ask participants for another way.

11. The second example is not the norm in Grade 8 and is more an extension question. However, the mathematics used to solve it, is Grade 8 level. Learners should spend most of their time practicing questions like the first example.

Generate tables of ordered pairs using substitution

12. As mentioned previously, the focus in Term 4 should be on generating ordered pairs. This skill will be useful in the next topic of the term, Graphs, which requires the plotting of points on a Cartesian plane.

13. This brings up an important point: Topics in mathematics are rarely stand-alone topics. A skill required in one part of the curriculum is almost always required in another (or many) parts of the curriculum.

14. Discuss these issues with participants with a focus on how important it is that learners realise this early in their mathematics careers. Teachers need to be encouraged to point out the connections on a regular basis. Learners rarely seem to see the connection between patterns and functions which in turn lead to equations and functions being represented graphically on a Cartesian plane.
15. The following example to be completed now with participants illustrates this:
Ordered pairs can be generated from an equation, using substitution. Learners need to be able to substitute given values (input) to find matching values (output) for the function given.

For example: Given the equation $y = 3x + 2$ complete the following table:
(this is available in the NECT Grade 4-9 Training Handout Term 3 & 4 2018)

x	-2	-1	0	1	2
y					

$y = 3x + 2$	$y = 3x + 2$	$y = 3x + 2$	$y = 3x + 2$	$y = 3x + 2$
$y = 3(-2) + 2$	$y = 3(-1) + 2$	$y = 3(0) + 2$	$y = 3(1) + 2$	$y = 3(2) + 2$
$y = -6 + 2$	$y = -3 + 2$	$y = 0 + 2$	$y = 3 + 2$	$y = 6 + 2$
$y - 4$	$y = -1$	$y = 2$	$y = 5$	$y = 8$

The table can now be completed:

x	-2	-1	0	1	2
y	-4	-1	2	5	8

16. Learners also need to be able to find the input value when given the output value.

For example: Given the equation $y = -2x + 1$ complete the following table:
(this is available in the NECT Grade 4-9 Training Handout Term 3 & 4 2018)

x	-3	0	1		
y				-3	5

The first three calculations are the same as the previous example.

$y = -2(-3) + 1 = 7$ $\quad$ $y = -2(0) + 1 = 1$ $\quad$ $y = -2(1) + 1 = -1$

To find the final two solutions, we need to solve an equation.

$$y = -2x + 1 \qquad\qquad y = -2x + 1$$

$$-3 = -2x + 1 \qquad\qquad 5 = -2x + 1$$

$$2x = 1 + 3 \qquad\qquad 2x = 1 - 5$$

$$2x = 4 \qquad\qquad 2x = -4$$

$$x = 2 \qquad\qquad x = -2$$

The table can now be completed:

x	-3	0	1	2	-2
y	7	1	-1	-3	5

Additional Key Concepts: Grade 9

Solving equations using factorisation (Quadratic Equations)

1. When starting this topic, teachers should spend some time discussing what two numbers can multiply to make zero. Using a few examples (such as 5 x 0 and -2 x 0), learners need to realise that if two numbers multiply to make zero, at least one of those numbers needs to be zero.
2. Secondly, learners need to be able to factorise – this includes, looking for a common factor, factorising a difference of two squares and factorising a trinomial. Although this will have just been completed in the previous topic, it is important that teachers are confident that their learners are able to accomplish this task otherwise solving quadratic equations will be difficult for learners.
3. Teachers should now be ready to start their learners on the concept of quadratic equations. First, a few examples from Term 1 need to be done where the left-hand side of the equation was already factorised to show that two factors were multiplied to equal zero. Examples to be used at this stage:
$$(x - 2)(x + 3) = 0$$
$$2x(x + 5) = 0$$
$$(a + 2)(a - 2) = 0$$
4. Once learners feel confident in the above type they can be introduced to equations not yet factorised.
5. Discuss the following potential problem with participants:
It is surprising how many learners (at Grade 11 level!) get a question similar to those above and proceed to multiply out, collect like terms and factorise. Sadly, the factorising is often incorrect!
6. Examples to be used at this stage:

$$x^2 = 9 \qquad a^2 - 3a = 0 \qquad x^2 + 7x + 10 = 0 \qquad x^2 + x - 12 = 0 \qquad y^2 - 15 = -2y$$

7. Before doing each of these examples, ensure it is clear that the following rules ALWAYS apply to a quadratic equations (an equation where a power of two is with the unknown variable).
    - The '2' or 'square' is already telling the person solving the equation that there will be two possible solutions.
    - Ensure all terms are on one side (usually the left-hand side) of the equal sign and zero on the other.
    - Factorise the expression on the LHS (left hand side).
    - The factorised expression should be made up of two factors.
    - If two factors multiply to make zero, then either one of these factors needs to equal zero (any number multiplied by zero always equals zero).
    - State the two options, making each factor equal to zero and solve each equation.
8. As each example is done, keep referring participants to the steps above.
9. Worked Examples in full:

$$\begin{aligned} x^2 &= 9 \\ x^2 - 9 &= 0 \\ (x+3)(x-3) &= 0 \\ x+3=0 \quad &or \quad x-3=0 \\ x=-3 \quad &or \quad x=3 \end{aligned}$$

$$\begin{aligned} a^2 - 3a &= 0 \\ a(a-3) &= 0 \\ a=0 \quad &or \quad a-3=0 \\ & \qquad a=3 \end{aligned}$$

$$\begin{aligned} x^2 + 7x + 10 &= 0 \\ (x+5)(x+2) &= 0 \\ x+5=0 \quad &or \quad x+2=0 \\ x=-5 \quad & \quad x=-2 \end{aligned}$$

$$\begin{aligned} x^2 + x - 12 &= 0 \\ (x+4)(x-3) &= 0 \\ x+4=0 \quad &or \quad x-3=0 \\ x=-4 \quad & \quad x=3 \end{aligned}$$

$$\begin{aligned} y^2 - 15 &= -2y \\ y^2 + 2y - 15 &= 0 \\ (y+5)(y-3) &= 0 \\ y+5=0 \quad &or \quad y-3=0 \\ y=-5 \quad & \quad y=3 \end{aligned}$$

17. Tell participants that this brings us to the end of our detailed look at the progression of equations in the senior phase, with a focus on Term 3 and 4. Ask if anyone has any questions or points to add before sharing the following with them:

(this is available in the NECT Grade 4-9 Training Handout Term 3 & 4 2018)

This excerpt was taken from Quora digest (an interesting site where anyone can ask any question and someone who considers themselves knowledgeable enough attempts to answer the question):

The question was: **Why do some people find mathematics difficult?**

The answer was:

I once bought a guitar. Just a cheap one, so I could learn some Cat Stevens. Watched YouTube tutorials, diligently practiced my G and E chords. This was going to be fun!

Six months later, I still couldn't play any songs. *Wild World* sounded more like *Awkward Slow World*. My guitar gathered dust. I was frustrated.

Why is guitar hard for most people?

The answer, of course, is that playing guitar is a skill that requires practice. Practice that compounds on itself, but usually too slowly to appreciate the progress.

(Remind you of math yet?)

I had underestimated the sheer number of hours it takes to get good at this new skill. Every time I heard the melodies of a master guitarist - so beautiful, so effortless! - I failed to see the thousands of hours of struggle and pain that had come before.

Practice became tedious and discouraging. I was working hard, but still nowhere near as competent as I thought I was "supposed" to be by now.

I lost steam. Practicing less often, shorter sessions. Why would I want daily reminders that I suck?

Yet without regular practice, my fingers never did develop the muscle memory to play a song smoothly.

And as I sold my guitar to a friend, I sighed and said… *"I guess I'm just not a guitar person."*

Contrary to old beliefs, mathematical ability is not something that some people are born with and others not.

No amount of talent has ever replaced the need for practice. But you can only be great at something if you actually enjoy the struggle.

18. The final statement is very important and teachers should be encouraged to share it with learners – it is difficult to become great at something unless you are willing to practice and enjoy the struggle 😊
19. There is a post-test available in the NECT Grade 4-9 Training Handout Term 3 & 4 2018.

6	2 hours	ACTIVITY 2 Preparatory sub-topics (Grade 4-7), Pythagoras Theorem (Grade 8-9)	Facilitator:	What you will need:

INTRODUCTION

1. For the practical group activity in this topic the emphasis will be on the build-up towards a better understanding of the Pythagoras theorem. The selection of sub-topics for Grade 4-7 was done with this focus in mind and the guiding question was: which knowledge aspects and skills do learners need to understand the Pythagoras theorem in Grade 8 and 9?
2. To fully understand the Pythagoras theorem, learners need to understand at least four basic concepts: perimeter of triangles, right triangles, ratio and similar triangles. The first three concepts are done across the Intermediate Phase, and the last is done in Grade 7 only.
3. Following this activity, the Pythagoras Theorem will be covered in detail as far as the terminology and the key concepts are concerned.

THE PROGRESSION OF THE TOPIC ACROSS GRADES AND PHASES

1. Ask participants to turn to the page in the NECT Grade 4-9 Training Handout Term 3 & 4 2018, titled 2.1 ***RIGHT TRIANGLES, PERIMETER OF A TRIANGLE, AREA OF A SQUARE AND RATIO (GR 4-6), SIMILAR TRIANGLES (GR 7), PYTHAGORAS THEOREM (GR 8-9): PROGRESSION OF THE TEACHER GUIDELINES AND CLARIFICATION NOTES FOR THE TOPIC(S) ACROSS GRADES AND PHASES.***
2. Lead a critical group discussion (15 min) where participants give their own views of how, in a case like the Pythagoras Theorem, there are skills and knowledge built into the curriculum before the actual topic is covered, to prepare learners to understand the complex or advanced topic.
3. Take note of more interface areas that participants may mention and of critique about those selected in the table.

PRACTICAL GROUP ACTIVITY

1. Ask participants to turn to the page in the NECT Grade 4-9 Training Handout Term 3 & 4 2018, titled 2.2 Training Exercise: Preparing the Skills and Knowledge Used in the Pythagoras Theorem.
2. Complete the exercise (30 min) and allow some discussion following the activity.
3. Proceed towards a detailed discussion of the terminology and key concepts of the Pythagoras theorem for Grade 8-9, as follows:

PYTHAGORAS THEOREM (Grade 8-9)

INTRODUCTION

1. The theorem of Pythagoras is used in many areas and very often from Grade 8 through to Grade 12. An understanding of the Theorem of Pythagoras leads to a better understanding of Analytical Geometry and Trigonometry in future grades.
2. Time spent on understanding and using the theorem will be time well spent. Learners should be given the opportunity to become completely confident in what the theorem is used for and in using it themselves.

TERMINOLOGY

1. Participants note that the terminology we use in this topic is clearly explained in the Content Booklets.

SUMMARY OF KEY CONCEPTS

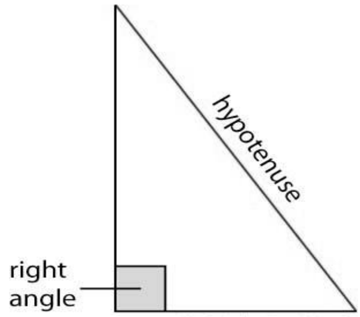
Sequential table

1. Ask participants to look at the sequential table in the Gr 8 Content Booklet.
2. Discuss the importance of understanding what has been done in previous grades as well as where this topic will go in future grades. Having a thorough understanding of the progression in a topic is essential to the teacher.
3. The fact that this is a new concept to Grade 8 makes it very exciting to teach – the learners get to investigate and learn a mathematical concept that they have not ever used before.

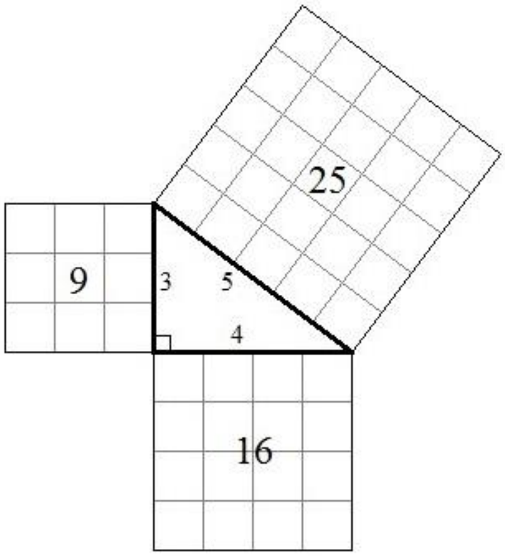
Teaching of new skill / concept

1. Remind participants that the content booklet has an overview of all the new concepts that will be taught to learners in each section and to use the Planner & Tracker to guide them to teach these in the correct order.
2. Tell participants that when they are teaching a new concept, they should not spend more than 10 – 15 minutes on the actual teaching.
3. There is a pre-test available in the NECT Grade 4-9 Training Handout Term 3 & 4 2018.
4. An investigation into the theorem is the best way to introduce it to learners. Talk through the following steps with the participants and ask them to do them with you.
5. Draw a right-angled triangle, with sides of 3cm and 4cm making the right angle.
6. Draw in the side that joins the two sides to form the triangle.

7. Discuss the name of this long side – the hypotenuse (label it)

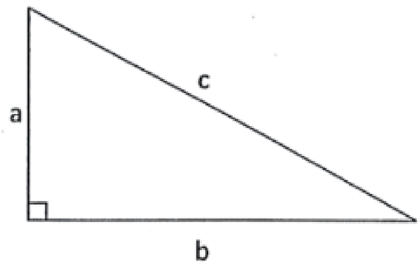


8. Show how the right angle 'points' to the hypotenuse.
9. Teachers should ask learners to measure this side (5cm), then proceed to draw squares on each of the sides and also drawing in the blocks which will help to show the area of the squares.

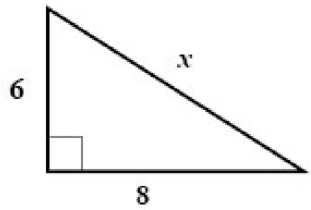


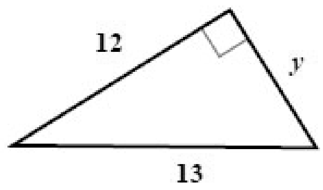
10. At this stage in the lesson, learners would be asked to find a connection between these numbers (the squares of the sides) – as participants clearly know the theorem, this is probably unnecessary today.'
11. Finally, summarise the theorem, in words and with a diagram: In any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

$$c^2 = a^2 + b^2$$



12. Use the theorem to do two basic examples, one to find the hypotenuse and one to find the short side of a right-angled triangle (available in the NECT Grade 4-9 Training Handout Term 3 & 4 2018):



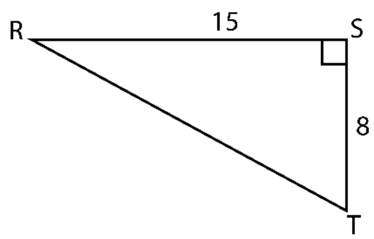
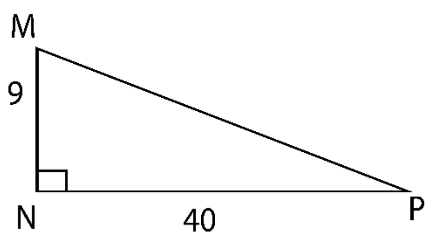


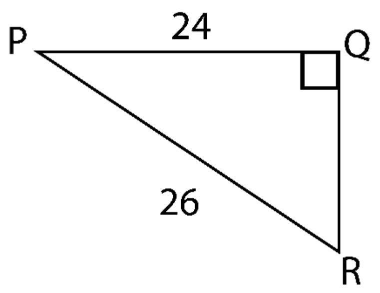
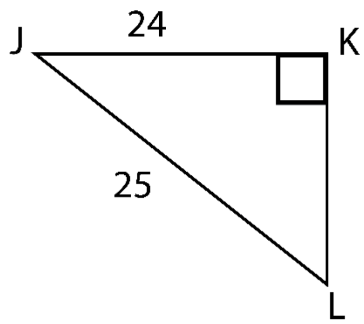
$$6^2 + 8^2 = x^2$$
$$36 + 64 = x^2$$
$$100 = x^2$$
$$\sqrt{100} = \sqrt{x^2}$$
$$\boxed{x = 10}$$

$$12^2 + y^2 = 13^2$$
$$144 + y^2 = 169$$
$$y^2 = 25$$
$$\sqrt{y^2} = \sqrt{25}$$
$$\boxed{y = 5}$$

13. Discuss the importance of the setting out and that whenever the theorem is used, a reason for the first statement should be given. Fill in the reason, *Pythagoras,* next to both opening statements of the two examples now.
14. Remind participants that when teachers are doing this topic with learners, they should allow learners to first practice finding the length of the hypotenuse in several questions before moving on to finding the length of one of the shorter sides.
15. A combination exercise would then follow that in which learners would need to recognise what is required of them.
16. Ensure all participants are happy with the two basic examples before giving them a few of their own examples to try:

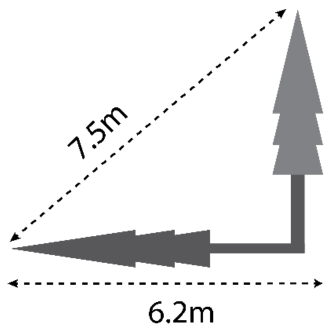
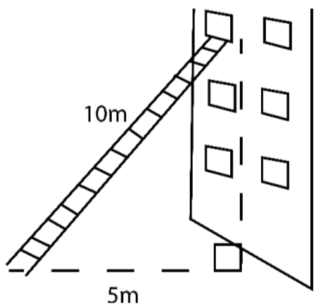
Find the lengths of the unknown sides (these are available in the NECT Grade 4-9 Training Handout Term 3 & 4 2018):

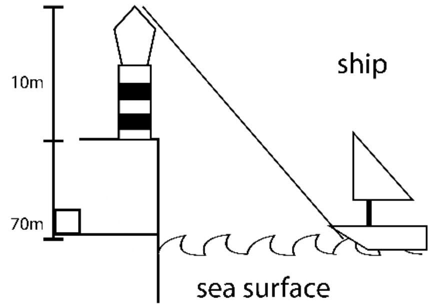
a		b	

c		d	

17. A few practical problems should also be shown and practiced by participants.

    For example (these are available in the <u>NECT Grade 4-9 Training Handout Term 3 & 4 2018</u>):

a	Find the height of the tree using the length of its shadow to help you. 
b	Find the height of the window 
c	A lighthouse is 10m high and is on top of a cliff that is 70m high. The beam of light from the lighthouse shines a beam of light out to a ship which is 192m from the foot of the cliff. How long is the beam of light?



Model Solutions to Examples Given:

Finding lengths of missing sides (8 a-d)

$RS^2 + ST^2 = RT^2$ Pythagoras

$15^2 + 8^2 = RT^2$

$225 + 64 = RT^2$

$289 = RT^2$

RT = 17

$MN^2 + NP^2 = MP^2$ Pythagoras

$9^2 + 40^2 = MP^2$

$81 + 1600 = MP^2$

$1681 = MP^2$

MP = 41

$PQ^2 + QR^2 = PR^2$ Pythagoras

$24^2 + QR^2 = 26^2$

$QR^2 + 576 = 676$

$QR^2 = 676 - 576$

$QR^2 = 100$

QR = 10

$KL^2 + JK^2 = JL^2$ Pythagoras

$KL^2 + 24^2 = 25^2$

$KL^2 + 576 = 625$

$KL^2 = 625 - 576$

$KL^2 = 49$

KL = 7

Practical problems (9 a-c)

The height of the tree:

$T^2 + (6.2)^2 = (7.5)^2$ Pythagoras

$T^2 + 38.44 = 56.25$

$T^2 = 56.25 - 38.44$

$T^2 = 17.81$

∴ T(Tree) = 4.22

Height of the window:

$h^2 + 5^2 = 10^2$ Pythagoras

$h^2 + 25 = 100$

$h^2 = 100 - 25$

$h^2 = 75$ ∴h(height) = 8.66m

Beam of light:

$80^2 + 192^2 = b^2$

$6400 + 36864 = b^2$

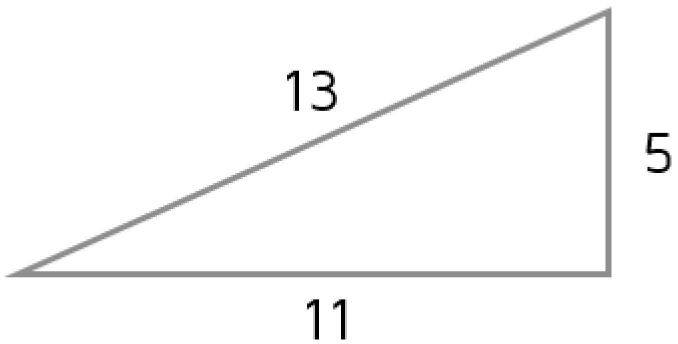
$43264 = b^2$

$\therefore b = \sqrt{43264}$

$\therefore x = 208$

The beam of light is 208m in length.

18. The final aspect in using the theorem of Pythagoras in Grade 8 is to use it to determine whether a triangle is right-angled or not. In this case, the length of all three sides will be given and learners should work out whether the theorem holds for these sides.
For example:
Is the following triangle a right-angled triangle? Show all working.



This example leads to 2 important aspects of geometry:

- Layout of an answer that requires calculating two different answers and comparing them
- The converse of a theorem.

These will be discussed before completing the solution.

19. It is important that learners understand from early on that if they are trying to prove something, they cannot assume that it is already true. In this case, they cannot make the statement:

$$11^2 + 5^2 = 13^2$$

and start doing the calculations.

Teachers must convey this important message to learners: work with the LHS and then the RHS and only after those calculations have been done, can the answers be compared, and the conclusion drawn. This is a VERY important skill for the FET phase in many areas of mathematics.

20. Most theorems have a converse – in other words, the rule can be turned around.

For example, we know that angles on a straight line add up to 180^o, therefore if angles add up to 180^o, they must form a straight line. For the theorem of Pythagoras: In any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Therefore, if the square on one side of a triangle equals the sum of the squares of the other two sides then it must be a right-angled triangle.

21. Solution:

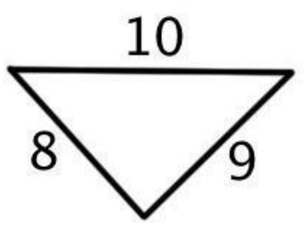
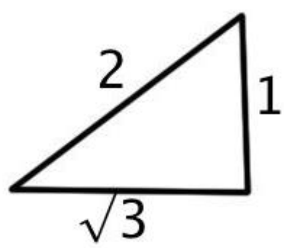
LHS:	RHS:
$11^2 + 5^2$	13^2
$= 121 + 25$	$= 169$
$= 146$	

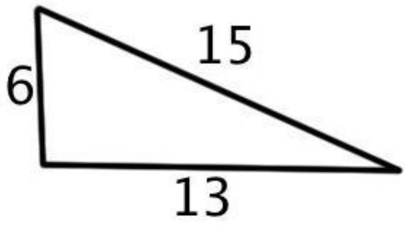
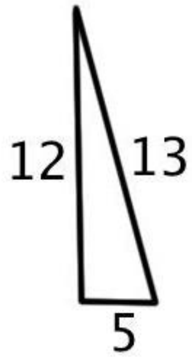
LHS ≠ RHS

∴ the triangle is not a right-angled triangle.

22. Further examples for participants to try. Ensure the setting out is correct. (these are available in the NECT Grade 4-9 Training Handout Term 3 & 4 2018)

23.

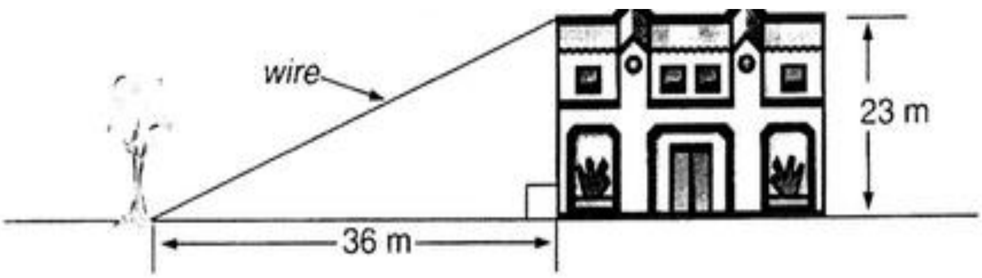
a		b	

c		d	

24. Before moving on to Grade 9 level work, show the following videoclips:
    https://www.youtube.com/watch?v=uaj0XcLtN5c
    https://www.youtube.com/watch?v=CAkMUdeB06o

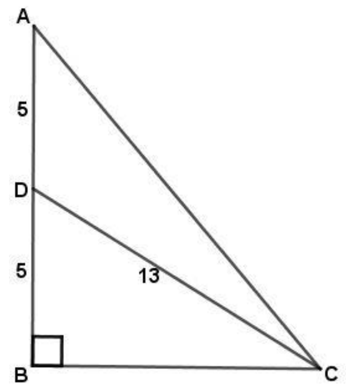
Key Concepts: Grade 9

1. In Grade 9 there are no new concepts of the theorem of Pythagoras taught. Learners need to consolidate what was learned in Grade 8 and use of the theorem to solve more complex problems.
2. Do the following examples with participants. Focus on the logical way forward to solve the problem and on the correct layout. Only do the solution once the problem has been discussed and participants give you what is required and the opening statement etc.
3. Examples (these are available in the NECT Grade 4-9 Training Handout Term 3 & 4 2018):

<table>
<tr><td>a</td><td>How long is the wire that runs from the base of the tree to the roof of the house in the diagram:
</td></tr>
<tr><td colspan="2">Solution:
The wire is the hypotenuse of a right-angled triangle.
$36^2 + 23^2 = w^2$ (Pythagoras)
1296 + 529 = w^2
1825 = w^2
42,72 = w The wire is 42,72m long</td></tr>
</table>

b

Find AC



Discuss:

Note: It is not possible to find AC unless we know BC so that the Theorem of Pythagoras can be used in ΔABC

Solution:

In ΔCBD:

$$BC^2 + BD^2 = CD^2 \qquad \text{Pythagoras}$$

$$BC^2 + 5^2 = 13^2$$

$$BC^2 + 25 = 169$$

$$BC^2 = 169 - 25$$

$$BC^2 = 144$$

$$BC = 12$$

In ΔABC:

$$BC^2 + AB^2 = AC^2 \qquad \text{Pythagoras}$$

$$12^2 + 10^2 = AC^2$$

$$144 + 100 = AC^2$$

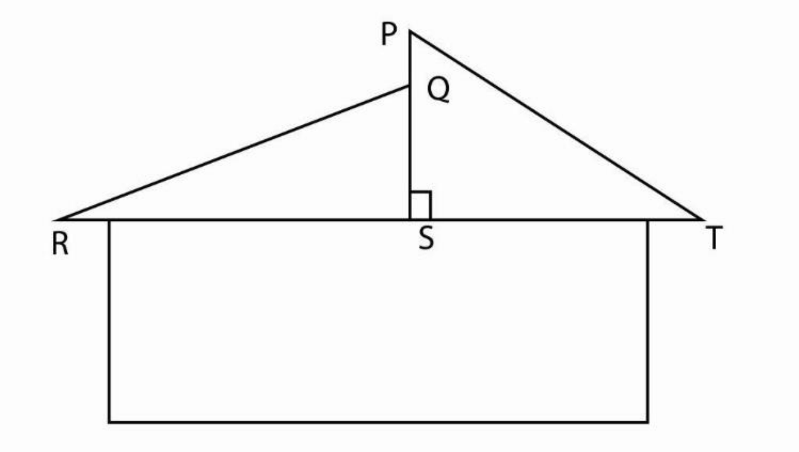
$$244 = AC^2$$

$$15{,}62 = AC$$

c

The triangles form the roof construction of a house.

If $RS = ST = 5m$, $QR = 6{,}5m$ and $PT = 7m$, find the length of PQ.



Discuss:

First fill all the measurements onto the diagram.

To find PQ, we will need to find PS and QS (which are both part of a right-angled triangle).

Once this has been done we can subtract to find the final answer (PQ)

Solution:

In ΔQRS:

$$RS^2 + QS^2 = QR^2 \quad \text{(Pythagoras)}$$

$$5^2 + QS^2 = (6{,}5)^2$$

$$25 + QS^2 = 42{,}25$$

$$QS^2 = 42{,}25 - 25$$

$$QS^2 = 17{,}25$$

$$QS = 4{,}15 \qquad \text{(Fill this in on the diagram)}$$

In ΔPST:

$$PS^2 + ST^2 = PT^2 \quad \text{(Pythagoras)}$$

$$PS^2 + 5^2 = 7^2$$

$$PS^2 + 25 = 49$$

$$PS^2 = 49 - 25$$

$$PS^2 = 24$$

$$\text{PS} = 4{,}9$$

$$\therefore\ PQ = PS - QS$$

$$= 4{,}9 - 4{,}15$$

$$= 0{,}75\ m$$

d. A grade 9 learner did the following work when asked to find the length of BC:

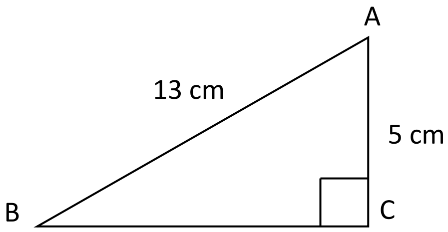
$BC^2 = AB^2 + AC^2$

$= (13)^2 + (5)^2$

$= 169 + 10$

$= 179$

$\therefore BC$ is equal to 179 cm^2.



Discuss three mistakes that were made. Do the full correction at the end.

Solution:

<u>Mistake 1</u>: Making a statement that BC is the hypotenuse (AB is the hypotenuse)

<u>Mistake 2</u>: 5^2 is not equal to 10 (the learner mixed it up with 2 x 5). (It is 25)

<u>Mistake 3:</u> 179 cm^2 cannot be the final answer.

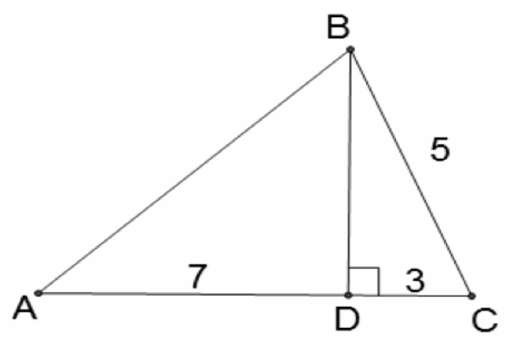
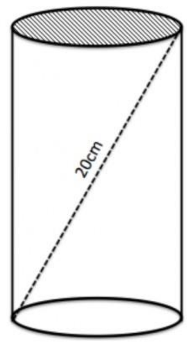
This still needs to be square rooted to find the missing side.

Also, BC is a length, not an area.

Correct solution:

$$AC^2 + BC^2 = AB^2 \qquad \text{(Pythagoras)}$$
$$5^2 + BC^2 = 13^2$$
$$25 + BC^2 = 169$$
$$BC^2 = 169 - 25$$
$$BC^2 = 144$$
$$BC = 12$$

4. Give participants 2 to do on their own. When you mark it together, the focus must be on the layout as well as the solutions. (available in the <u>NECT Grade 4-9 Training Handout Term 3 & 4 2018</u>):

a	Find AB, correct to 2 decimal places. 	b	A cylinder is three times as high as it is wide. Find the radius and height of the cylinder, correct to two decimal places. 

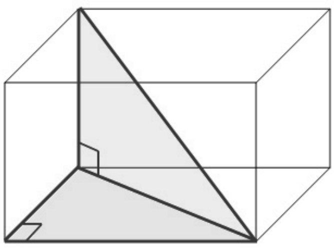
5. Solutions to the above examples:

$BD^2 + 3^2 = 5^2$ (Pythagoras) $BD^2 + 9 = 25$ $BD^2 = 25 - 9$ $BD^2 = 16$ $BD = 4$ $BD^2 + 7^2 = AB^2$ (Pythagoras) $4^2 + 7^2 = AB^2$ $16 + 49 = AB^2$ $65 = AB^2$ $8{,}06 = AB$	Let the width $= d$ $\therefore ht = 3d$ $d^2 + (3d)^2 = 20^2$ (Pythagoras) $d^2 + 9d^2 = 20^2$ $10d^2 = 400$ $d^2 = 40$ $\therefore d = 6{,}32$cm $\therefore r = 3{,}16$cm $\therefore h = 18{,}96$cm

6. Finally, ask participants to work in pairs to solve the following problem (this is available in the <u>NECT Grade 4-9 Training Handout Term 3 & 4 2018</u>):

A box has a base 30cm by 20cm and a height of 25cm. What is the length of the longest stick that can fit in the box? (from a left-hand corner to a right-hand corner)

7. If participants found it a challenge to 'see' the right angles required, show them the following diagram:



Discuss:

Notice that there is a right-angled triangle on the bottom of the box (half of the base) as well as an imaginary right-angled triangle inside the box where the hypotenuse of the triangle on the base of the box becomes one of the short sides in triangle inside the box. The hypotenuse on the 2nd triangle would represent the longest stick you could fit into the box.

It is therefore important that first, the diagonal of the base of the box needs to be found:

$$30^2 + 20^2 = d^2 \qquad \text{(Pythagoras)}$$

$$900 + 400 = d^2$$

$$1300 = d^2$$

$\therefore$ diagonal $= 36{,}06$cm

Next, we need to move onto the imaginary triangle lying inside the box along the diagonal of the base:

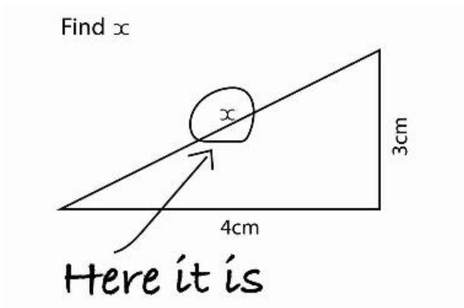
$$(30{,}06)^2 + 25^2 = s^2 \qquad \text{(Pythagoras)}$$

$$1300 + 625 = s^2$$

$$1925 = s^2$$

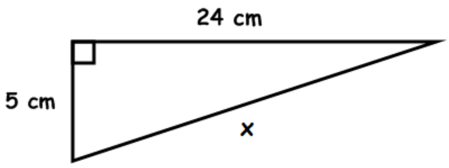
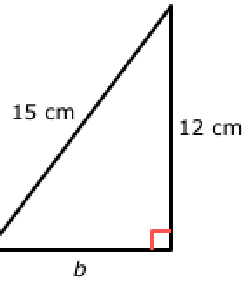
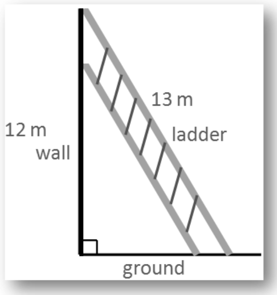
$\therefore$ longest stick to fit in the box = 43,87cm

8. Don't let this happen☺

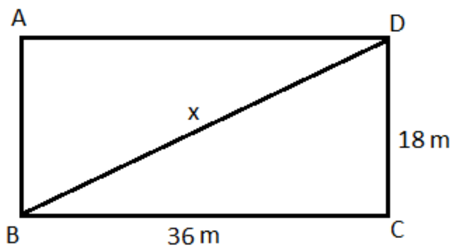
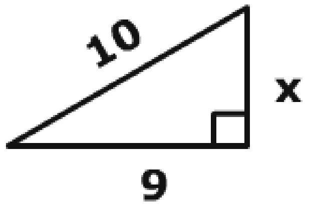
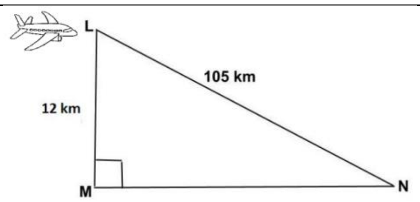


9. Tell participants that we have come to the end of our detailed look at the theorem of Pythagoras. Ask if anyone has any questions or comments before ending. Thank everyone for their participation and attention.
10. There is a post-test available in the NECT Grade 4-9 Training Handout Term 3 & 4 2018.

PRE-TEST MEMORANDUM

1	The theorem of Pythagoras is used in right-angled triangles to find …..	the length of a side ✓
2	Find the length of the missing side: 	$x^2 = 5^2 + 24^2$ ✓ $x^2 = 601$ ✓ $x = 24{,}5$ ✓
3	Find the length of the missing side: 	$b^2 + 12^2 = 15^2$ ✓ $b^2 + 144 = 225$ $b^2 = 225 - 144$ $b^2 = 81$ ✓ $b = 9\text{cm}$ ✓
4	A ladder is 13m long and reaches a window 12m above the ground. How far away from the wall is the foot of the ladder? (hint: draw a triangle)	 $g^2 + 12^2 = 13^2$ ✓ $g^2 + 144 = 169$ $g^2 = 169 - 144$ $g^2 = 25$ ✓ $g = 5m$ ✓

POST-TEST MEMORANDUM

1	The theorem of Pythagoras only works in ….	Right-angled triangles ✓
2	Find the length of the diagonal of the rectangle: 	$x^2 = 36^2 + 18^2$ ✓ $x^2 = 1620$ ✓ $x = 40{,}25m$ ✓
3	Find the length of the missing side: 	$x^2 + 9^2 = 10^2$ ✓ $x^2 + 81 = 100$ $x^2 = 100 - 81$ $x^2 = 19$ ✓ $x = 4{,}36$ ✓
4	An aeroplane is 12km above the ground. The distance of its descent towards the airport is 105km. Calculate the aeroplane's ground distance to the airport. Round off your answer to the nearest whole number. (Hint: draw a triangle)	 $MN^2 + 12^2 = 105^2$ ✓ $MN^2 + 144 = 11025$ $MN^2 = 11025 - 144$ $MN^2 = 10881$ ✓ $MN = 104{,}31$ ✓

7	2 hours	PROBABILITY (Grade 4-9)	Facilitator:	What you will need: Dice

INTRODUCTION

1. Ask participants to please turn to the topic Probability in the Grade 5 and Grade 9 Content Booklets.
2. Ask two participants to read through the introduction in both booklets.
3. Then, explain this further that it is very important to introduce a topic properly. Learners like to know WHY they are studying a topic and what the importance of a topic is.

PROGRESSION OF THE TOPIC ACROSS GRADES AND PHASES

1. Ask participants to look at the section titled ***3.1 PROBABILITY (GR 4-9): PROGRESSION OF THE TEACHER GUIDELINES AND CLARIFICATION NOTES FOR THE TOPIC ACROSS GRADES AND PHASES*** in the <u>NECT Grade 4-9 Training Handout Term 3 & 4 2018.</u>
2. Invite some critical discussion about the progression of the topic across grades and phases.
3. Explain this further by discussing: Probability is introduced for the first time in Grade 4 in a concrete way and is rapidly progressing towards abstract representations of probability in Grade 9. Care should therefore be taken to introduce the basic concepts properly in the Intermediate Phase where learners experience probability practically and record the outcomes systematically.
   - In Grade 3, learners have not been exposed to probability.
   - In Grade 4, learners do simple practical probability experiments, count and record outcomes, and learn a few crucial words that are added to their mathematics vocabulary.
   - In Grade 5, learners do the same practical experiments and recording as in Grade 4, and they also start doing simple calculations to determine theoretical probability.
   - In Grade 6, learners do more trials and experience how the practical probability moves closer to the theoretical probability through an increased number of trials.
   - In Grade 7, learners continue to perform simple experiments where the possible outcomes are equally likely They also need to list all possible outcomes and determine the probability of each outcome.
   - In Grade 8, learners continue to list outcomes and find the probability of each using the definition of probability. They also compare relative frequency with probability.
   - In Grade 9, learners are introduced to compound events and use tree diagrams and two-way tables to determine the probability of an event.

4. The teaching of this part of data handling requires dice, coins and some paper resources.

PROBABILITY (Grade 4-6)

TERMINOLOGY

1. Remind participants that the terminology needed in this topic is set out in the Content Booklets.
2. Explain this further by saying:
   - Knowing the vocabulary that is used on a regular basis in a topic and subject is a key part of achieving success in a topic.
   - Ensure all learners are exposed to the glossary of terms prior to the teaching of a topic.
   - Displaying it in the classroom is beneficial.
   - Learners should either receive a copy for their own book if possible or write it in themselves.

SUMMARY OF KEY CONCEPTS

1. Remind participants that the section titled: Summary of Key Concepts in the Content Booklets is an overview of all the new concepts that will be taught in this section.
2. Remind participants to use the Planner & Tracker to guide them to teach these in the correct order.
3. Tell participants that the main focus in the Intermediate Phase will be on experiencing probability practically and recording the outcomes of trials systematically.
4. You will go through the following aspects of probability for Grade 4-6:
   - Clarifying concepts
   - Doing an experiment and recording the outcomes
   - Calculating probability
5. The aspects that are applicable to Grade 4 or Grade 5 only, will be marked as such and those applicable to both grades will also be indicated
6. Tell participants that when they are teaching a new concept, they should not spend more than 10 – 15 minutes on the actual teaching.
7. Go through the sub-topics as follows:

Clarifying concepts

Participants go through the following ideas step by step and make a point of using the words:

a. When we flip a coin over and over to see what happens, we do an **experiment**.
b. Each time we flip the coin in the experiment, it is a **trial**.
c. The way the coin lands, is an **outcome** of the trial.

d. Flipping the coin once and having an outcome, make up an **event** of the experiment (from Grade 5).
e. The coin may land heads up or tails up, therefore there are two **possible outcomes**.
f. The chance that an event will occur in an experiment, is the **probability** of that event.
g. The probability that an event will occur is **calculated** out of all the possible outcomes (from Grade 5).
h. The number of times that the coin lands heads up, is the **frequency** of the outcome "heads up". The number of times that the coin lands tails up, is the frequency of the outcome "tails up".
i. When flipping a coin, there are two possible outcomes, therefore the probability that a coin lands heads up is **one out of two**, **or** $\frac{1}{2}$ and the probability to land tails up is one out of two, or $\frac{1}{2}$. Learners like the idea of saying 50/50 chance. They understand it as another way to express equal chance, but as it stands here, mathematically it means 1, rather say: "fifty-fifty".
j. It is **impossible** that the coin will land with nothing up, because there is no face of the coin that has nothing on it.

Doing an experiment and recording the outcomes

1. Ask participants to look at the section titled ***3.2 PROBABILITY (GR 4-6): PRACTICAL ACTIVITY: SPINNER*** in the <u>NECT Grade 4-9 Training Handout Term 3 & 4 2018.</u>
2. In Grade 4 learners only determine probability using coins and dice. From Grade 5 on they also use spinners. This practical activity is designed to put into practice all the concepts that learners need to understand by the end of Grade 6.

Calculating probability

The calculation of probability is best learned by letting learners do experiments of chance, like throwing the die, picking objects out of a bag, dealing cards or flipping a coin.

a) The easiest way of starting to explain the concept of probability is by flipping a coin: the chance or probability that the coin will land heads up, is one out of two, and that it will land tails up, one out of two, or $\frac{1}{2}$, or 50%, because there are only two sides of which it can land either way up.
b) The chance that a die will land with a 2 on top, is 1 out of 6 $\left(\frac{1}{6}\right)$ because there is one two and six numbers altogether on the die.
c) The chance that a die will land with an odd number on top, is 3 out of 6 $\frac{3}{6}$ because there are three odd numbers on a die out of a total of six numbers. (In probability we do not need to insist on the simplification of fractions.)

d) It is all important that learners understand that probability is always in the range 0 to 1, with fractions in between. Another way of putting it, is that the range is from 0% $\left(\frac{1}{100}\right)$ to 100% $\left(\frac{100}{100}\right)$ with any percentage in between, for example a probability of three out of four $\left(\frac{3}{4}\right)$ is a 75% probability.

- If something is totally impossible, or there is no chance for something to happen, like a die falling with the number 7 on top, the probability of this outcome is $\frac{0}{6}$ or zero or 0 or 0%, because there is no 7 on a die.
- If something is sure to happen, like a die falling with a number on top, the probability of such an outcome is $\frac{6}{6}$ or one or 1 or 100%, because the only way it can fall is with a number on top.
- Any probability in between, is a fraction of the total possible outcomes, like the probability of the die falling with an odd number on top, is three out of the six possible outcomes or $\frac{3}{6}$ or $\frac{1}{2}$ or 50%; like the probability of the die falling with the number four on top, is one out of the six possible outcomes or $\frac{1}{6}$.

PROBABILITY (Grade 7-9)

INTRODUCTION

1. Probability is a relatively new topic to many teachers, so it needs to be dealt with in a way that will give each teacher the confidence to teach it and have fun while doing it.
2. This is a topic taught in all grades throughout the senior phase and the FET phase.
3. It is a topic very closely related to real life situations and can be useful for both learners and their family members.
4. Working with and understanding basic probability (as covered in the Senior Phase) is an important skill. Many people wish that they could win the lotto to solve all their financial problems. There are many adults who buy a lotto ticket on a regular basis and believe that winning the lotto is the best way to become financially secure. Unfortunately, you have a better chance of being struck by lightning or being in a plane crash (and there really aren't very many plane crashes) than you have of winning the lotto. The probability of winning is very close to 0. You need a better plan for your financial future than the lottery!

TERMINOLOGY

1. Participants note that the terminology we use in this topic is clearly explained in the Content Booklets.

SUMMARY OF KEY CONCEPTS

Sequential Table

1. Ask participants to look at the sequential table (from Grade 8 Content Booklet) in the NECT Grade 4-9 Training Handout Term 3 & 4 2018. This shows all the sub-topics and skills taught in the senior phase.
2. In Grade 7, learners were already introduced to probability and should have a good idea of what it is all about.
3. Notice that the only extra work to be covered in Grade 8 is that of relative frequency. As there are 4.5 hours allocated to probability it will be possible to cover the work from Grade 7 again to ensure learners leave Grade 8 with a good understanding of this topic.
4. In Grade 9, compound events and the use of two-way tables and tree diagrams are added.

Key Concepts: Grade 7

Teaching of new skill / concept

1. Remind participants that the content booklet has an overview of all the new concepts that will be taught and to use the Planner & Tracker to guide them to teach these in the correct order.
2. Tell participants that when they are teaching a new concept, they should not spend more than 10 – 15 minutes on the actual teaching.
3. There is a pre-test available in the NECT Grade 4-9 Training Handout Term 3 & 4 2018.
4. You will go through the following concepts:
   - the meaning of probability
   - what is an outcome
   - finding the probability of equally likely events

Introducing the topic

To introduce this topic to teachers, start by playing the following video clip. Arthur Benjamin is a professor of mathematics in the United States. He discusses the fact that most topics done in school mathematics generally lead to being able to learn calculus. He, however, believes that statistics and probability are in fact more important and that calculus can always be studied in more detail by students of mathematics who go on to study mathematics at a tertiary level.

https://www.ted.com/talks/arthur_benjamin_s_formula_for_changing_math_education

What is probability?

Probability is the chance that something will happen.

1. It is the likelihood of an event happening, as there are very few things in life that we can predict for certain.
2. ALL probability answers lie from zero (impossible) to one (certain). Most answers are therefore fractions.
3. An example of an event with a probability of 1 is 'Tuesday will follow Monday' or 'picking a red ball from a bag full of red balls'.
4. An example of an event with a probability of zero is 'Monday will follow Tuesday' or 'picking a red ball from a bag full of blue balls'.
5. Probability answers can be written as a common fraction, a decimal fraction or a percentage. For example, $\frac{4}{5} = 0{,}8 = 80\%$. This is a skill covered in Grade 6 and Grade 7 so learners should be able to cope with any conversions.
6. All of these would represent the same probability.

What is an outcome?

1. An outcome is a possible result of a trial or an experiment.
2. Sometimes there can be more than one possible outcome. For example, when throwing a die, there are six possible outcomes. The die can land on the 1, 2, 3, 4, 5 or 6.
3. A few other examples: When a child is born, there are two possible outcomes – a boy or a girl; when a card is chosen from an ordinary pack of playing cards, there can be 52 outcomes. This would depend on the situation though, which will be looked at later in the session.
4. Probability questions often refer to the use of playing cards so it is important that learners and teachers alike are familiar with what makes up a pack of cards.

A regular deck of cards has:

- 52 cards total
- 26 red cards (13 diamonds, 13 hearts) and 26 black cards (13 spades, 13 clubs).
- Each of the 4 groups (suits) has the cards 2-10, Jack, Queen, King, and Ace.
- The J, Q, K are called face- (or picture) cards.
- The Ace and 2 – 10 are called number cards.

Use a pack of cards at this point to demonstrate.

Finding the probability of equally likely outcomes

1. An equally likely outcome means that all the outcomes have the same chance of occurring. This is the only type of probability dealt with in the Senior Phase.
2. To find the probability of any event occurring, the following formula is used:

$$\mathrm{P(event)} = \frac{\text{the number of ways the event could happen}}{\text{the total number of possible equally likely outcomes}}$$

3. Using this formula, it is important to know exactly what the favourable outcome is (what we want to happen) and what all the possible outcomes are.
4. Two worked examples:

   Example 1:

   A die is thrown. What is the probability that it will: (a) land on the 4?

   (b) land on an odd number?

   Solutions: Decide on the total of all the possible outcomes. There are 6. Therefore, 6 will be the denominator in the fraction.

   (a) The favourable outcome is to get the '4'. How many 4's are on the die? There is only one.

   $\therefore$ The probability of getting a 4 is $\frac{1}{6}$

   (There is a one in 6 chance of getting a 4 when rolling a die)

   (b) The favourable outcome is an odd number. How many odd numbers are there on the die? There are 3 (1 or 3 or 5)

   $\therefore$ The probability of getting an odd number is $\frac{3}{6}$

   (There is a three in 6 chance of getting an odd number when rolling a die)

   Note that $\frac{3}{6}$ can be simplified to $\frac{1}{2}$. This should make sense as half of the numbers on the die are odd numbers. Hence, there is a one in two chance of getting an odd number.

   We shouldn't be 'picky' about learners simplifying fractions (although there is no harm done in encouraging it) as the key issue at this stage is to ensure an understanding of what numbers need to be used for the favourable outcomes (numerator) and total possible outcomes (denominator).

Example 2:

A card is chosen from a pack of playing cards. What is the probability of choosing:

(a) an ace

(b) a spade

(c) the 2 of hearts

(d) a king or a queen

Solutions:

Decide on the total of all the possible outcomes. There are 52. Therefore, 52 will be the denominator in the fraction.

(a) The favourable outcome is to get an ace. How many aces are in a pack? There are 4 aces.

$$\therefore \text{The probability of getting an ace is } \frac{4}{52} \left(\frac{1}{13}\right)$$

(There is a 4 in 52 chance or a 1 in 13 chance of getting an ace when choosing any card from a pack of cards)

(b) The favourable outcome is to get a spade. How many spades are in a pack? There are 13 spades.

$$\therefore \text{The probability of getting an ace is } \frac{13}{52}$$

(There is a 13 in 52 chance or a 1 in 4 chance of getting a spade when choosing any card from a pack of cards)

(c) The favourable outcome is to get the 2 of hearts. How many 2 of hearts are in the pack? There is only one.

$$\therefore \text{The probability of getting an ace is } \frac{1}{52}$$

(There is a 1 in 52 chance of getting the 2 of hearts when choosing any card from a pack of cards)

(d) The favourable outcome is to get a king or a queen. Note at this stage that we don't mind whether it is the king or the queen. How many kings and queens are in the pack? There are 4 kings and 4 queens so there are 8 cards in total that we are happy to choose.

$$\therefore \text{The probability of getting a king or a queen is } \frac{8}{52} \left(\frac{2}{13}\right)$$

(There is an 8 in 52 chance or a 2 in 13 chance of getting a king or a queen when choosing any card from a pack of cards)

5. There is a worksheet available for the participants to complete using their pack of cards. It is recommended that they work in pairs, so they can discuss with each other. If participants are familiar with cards, the first of the two worksheets need not be used. Once enough time has been given to complete the worksheet, discuss the solutions with the participants.

Additional Key Concepts: Grade 8

1. You will go through the following concept:
   - the difference between relative frequency and probability

The difference between relative frequency and probability

1. Probability is what is dealt with most often. However, if someone were to perform an experiment and record what outcomes did indeed occur – they would be dealing with relative frequency. To find relative frequency, the following formula is used:

$$\frac{number\ of\ occurences\ of\ a\ certain\ outcome}{number\ of\ trials}$$

   For example:

   A learner rolls a die 20 times and records how many times she gets a 2. After performing the experiment 20 times, she sees that she got the 2 only once. Hence, the relative frequency is $\frac{1}{20}$.

   Note that a different learner could perform the same experiment and find that he gets five 2's.

   This would make the relative frequency $\frac{5}{20}\left(\frac{1}{4}\right)$.

2. Relative frequency can change depending on the actual outcomes of the experiment. Probability will always be the same.
3. Relative frequency is not used a great deal in the FET phase, but an understanding of what makes it different to probability is an important concept for the Senior Phase.

Additional Key Concepts: Grade 9

1. You will go through the following concepts:
   - probability of compound events using two-way tables
   - probability of compound events using tree diagrams

Compound events

1. A compound event is one in which there is more than *one possible outcome.*
2. The following two events are compound events:
   (a) Tossing 2 coins and getting one head and one tail.
   (b) Tossing a coin and rolling a die and getting heads and a six.

3. It is important to note that what you are hoping to get is not the important feature in the above two examples but rather the fact that more than one thing is happening.
4. When dealing with compound events, there are two methods that help simplify and visualise the situation. These are two-way tables and tree diagrams.

<u>Finding the probability of compound events using two-way tables</u>

1. A two-way table is most useful when a connection (or a relationship) between two variables is part of the situation.
2. For example, the two-way table below has been drawn up after a survey where the learners in a class were asked how they got to school:

	Walk	***Taxi***	***Other***	***Total***
Grade 8	82	30	13	125
Grade 9	70	22	23	115
	152	52	36	240

Probability questions such as the following can now be asked:

What is the probability that:

a) a Grade 8 learner chosen at random walks to school?

b) a learner coming to school by taxi is a Grade 9 learner?

c) a learner does not walk or get a taxi to school?

In answering these questions, it is important to know what the denominator (total) is that is we deal with.

In (a) it will be **125** as the learner chosen is from **Grade 8**

In (b) it will be **52** as the learner chosen is from the group that come by **taxi**

In (c) it will be **240** as the learner is from the **entire group**

Once this has been decided, the focus now needs to move to what is specifically being asked.

In (a) walking (in Grade 8) is the key. The solution is therefore $\frac{82}{125}$

In (b) Grade 9 (by taxi) is the key. The solution is therefore $\frac{22}{52}$

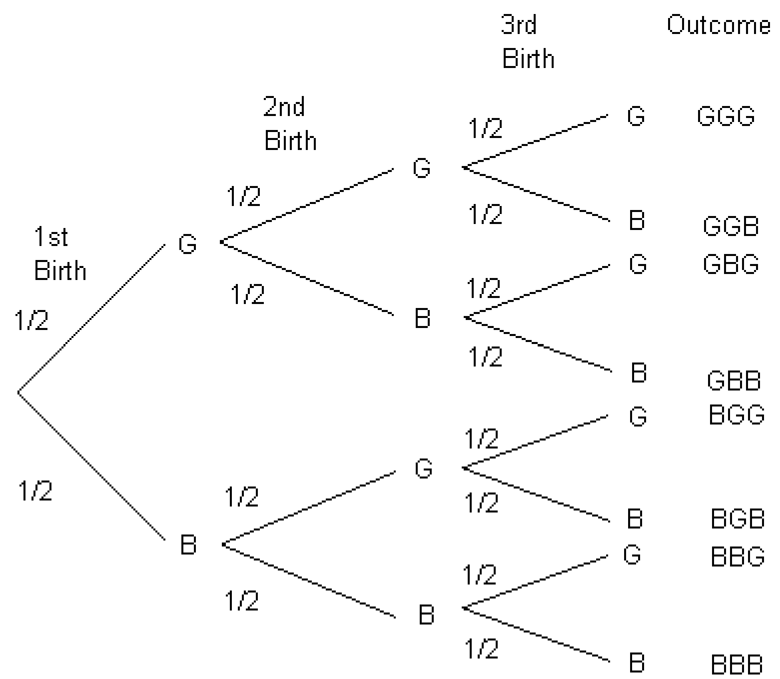
In (c) neither walking or getting a taxi is the key (“other”). The solution is therefore $\frac{36}{240}$

3. Ask participants to make up a few more questions using the table and discuss the solutions.

4. Participants could share this table with teachers to use as part of an assessment if they feel they would have difficulty making up a question.

Finding the probability of compound events using tree diagrams

1. A tree diagram is a visual way of seeing all the possibilities in a given situation. It is a good aid to help calculate the probabilities.
2. For example, if a family have 3 children and the probability of having a boy or a girl is 50% $\left(\frac{1}{2}\right)$, what are the chances the family will have 3 boys? This question could be confusing but with the aid of a tree diagram it can become somewhat easier.
3. A few points regarding the 'rules' of drawing a tree diagram:
    - The outcome (girl or boy in this case) is written at the END of the branch
    - The probability is written ON the branch
    - Ensure there is a space after each outcome and before the next set of branches are drawn, in other words don't join up all the branches
4. The following tree diagram represents the situation up to 3 children:



As you draw this on the flipchart, discuss each birth and each branch in detail. Ask/say:

- *A lady is pregnant – what are the possibilities?* (a boy or a girl – 2 outcomes)
- *What is the chance she has a boy? What is the chance she has a girl?* $\left(\frac{1}{2}\right)$ or 50%
- *Now that she has had a girl, what is the chance she will have a boy next? Or a girl next?*

5. Note the outcomes at the end. The first one is gained by following the first set of branches. Point out that there are 8 possible outcomes in any family that has three children. For example, the first child could be a boy, then the second a girl, then the third a girl again. This possibility is represented 5^{th} in the above tree diagram. There would be a $\frac{1}{8}$ chance of this happening. Reason: There are <u>8</u> possible outcomes altogether (denominator) and there is only <u>1</u> way of having a boy then a girl then a girl again.
6. There is no need to always count the total number of outcomes. To find the probability of getting two girls and one boy *in that order*, follow the branches from the beginning that end at girl, girl, boy. As you go along each branch, note the probability of that branch happening. In this case it is always $\frac{1}{2}$. MULTIPLY each of the probabilities to find the final answer $\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}\right)$
7. However, if the question had been, what is the probability that a family would have two girls and a boy *in any order*: Use the list of outcomes to find all those that have 2 girls and 1 boy. There should be 3 (In the above diagram it is the 2^{nd}, 3^{rd} and 5^{th} outcome). To find this answer mathematically, all the possible outcomes are ADDED. $\left(\frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}\right)$
8. Spend some time on this section with participants. Ensure they all feel comfortable with finding the probability of different situations.
9. There are more tree diagram questions available. These questions deal with events that are not equally likely. If you feel participants need more time on the equally likely events such as throwing a die or tossing a coin rather go through the section in the content booklet with them than move on to more complex skills.
10. In later years, similar situations to the questions below will change to not include replacement. In other words, if a ball is taken out of a bag, at Grade 9 level it will ALWAYS be put back so the chance of getting a certain colour ball will not change each time. In the FET phase, the ball will NOT be replaced causing the chances to change as there would be less balls in the bag.

<u>Further tree diagrams</u>

1. Many people find a tree diagram confusing when it is extended to the second part. Two ideas are represented but only one will happen.
2. Before doing the following examples take the time to do an activity and talk participants through this issue.
3. Have a bag ready with 2 coloured balls/marbles/pieces of card. For example, 3 blue marbles and 5 white marbles. This example will be used here, but whatever is available can be used.
4. Tell participants what you have in the bag and write it on the flipchart.

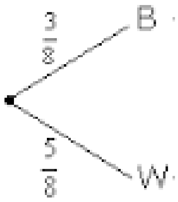
5. Before asking someone to take an item from the bag, tell participants that two people will be drawing from the bag.

   Ask: *what are all the possible outcomes after two draws?* (BB, BW, WB, WW)

   Ask: *what is the probability of drawing a blue marble?* $\left(\frac{3}{8}\right)$

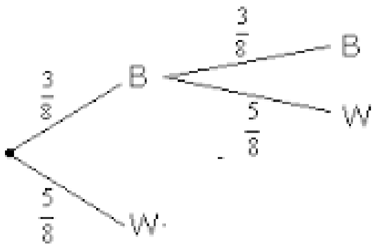
   Ask: *what is the probability of drawing a white marble?* $\left(\frac{5}{8}\right)$

6. Ask for a volunteer to come and draw the first part of the tree diagram on the flipchart. Ensure it has the 2 possible branches, the probability on each branch and the two outcomes at the end of each branch.



7. Ask a participant to draw a marble from the bag. Ask him/her to say what it is. If it is a blue marble circle this on the part of the diagram been drawn and ask the participant to replace it.

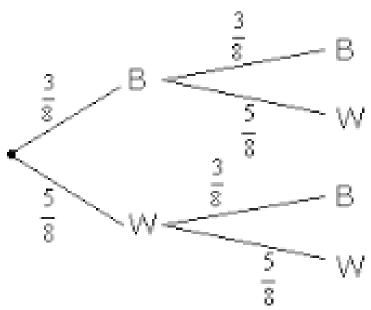
   Say: *now that our focus is on the blue marble being drawn, please can I have a volunteer to draw what could happen in the second draw.*



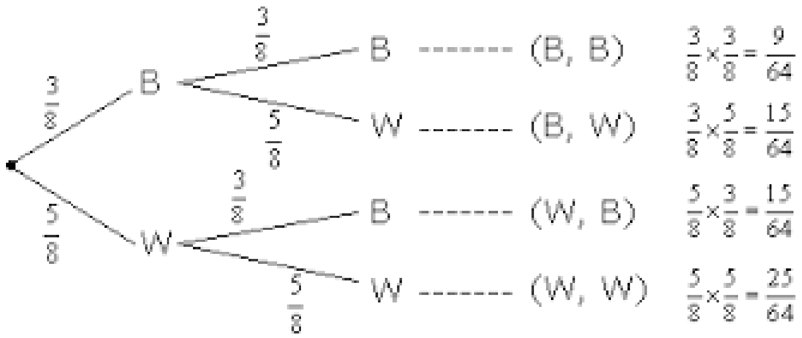
8. Ask a participant to make the second draw. Note what is taken and discuss the outcome (BB or BW)

   Ask: *what was the probability of this outcome happening?* $P(BB) = \frac{3\times3}{8\times8} = \frac{9}{64}$ or $P(BW) = \frac{3\times5}{8\times8} = \frac{15}{64}$

9. Ask: *But what if the first draw had been white? Whether it happened or not, we must consider that it is a possibility and therefore would produce different possible outcomes. Would someone like to draw the final part of the tree diagram in for us?*



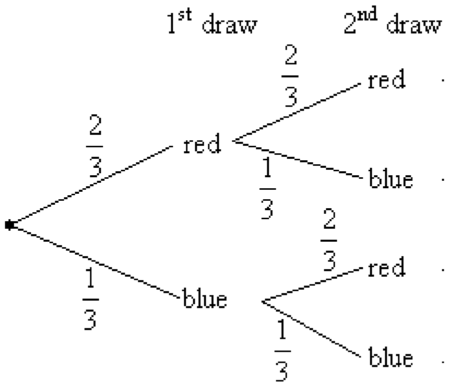
10. Ask a volunteer to come and add all the outcomes as well as the probability of each of the outcomes.



Note that, the procedure and steps would have been a little different if a white marble was drawn first.

11. Ask participants to do the following two questions on their own.
12. There are 3 balls in a bag. Two of them are red and one is blue. One is taken from the bag, and after the colour has been noted it is replaced before a second ball is drawn. Draw a tree diagram of the situation and answer the following questions:
    (a) both balls will be red
    (b) the balls will be different colours
13. You have a pack of cards. Two cards are chosen at random. After the first card is chosen and noted, it is replaced before a second card is chosen. Draw a tree diagram to show the chances of getting a diamond (or not a diamond) then answer the following questions:
    (a) What is the probability of getting no diamonds?
    (b) What is the probability of getting only one diamond?
14. There is a post-test available in the NECT Grade 4-9 Training Handout Term 3 & 4 2018.

Model Solutions to Extra Questions Given:

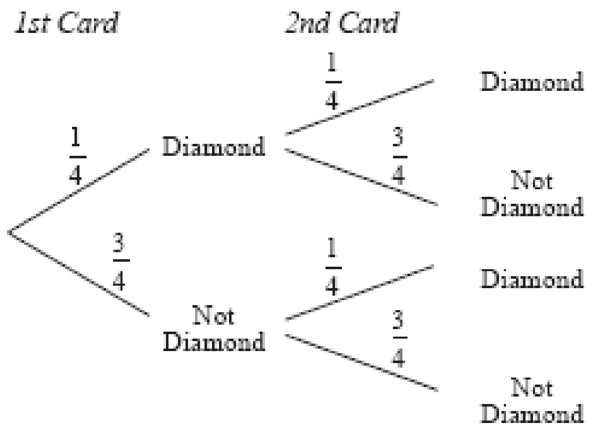


1. a) $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

   b) $\left(\frac{2}{3} \times \frac{1}{3}\right) + \left(\frac{1}{3} \times \frac{2}{3}\right)$ (Red, blue + Blue, red)

$= \frac{2}{9} + \frac{2}{9} = \frac{4}{9}$

2.



a) $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

b) $\left(\frac{1}{4} \times \frac{3}{4}\right) + \left(\frac{3}{4} \times \frac{1}{4}\right)$ (Diamond, Not diamond + Not diamond, Diamond)

$= \frac{3}{16} + \frac{3}{16} = \frac{6}{16} \left(= \frac{3}{8}\right)$

Model Solutions to Card Questions:

How many cards are there in a pack/deck of cards?	**52**
How many different types of cards (ie suits) are there?	**4**
Name the 'suits'	**Spades, Clubs, Diamonds, Hearts**
How many black suits are there?	**2**
How many red suits are there?	**2**
How many cards are there in each suit?	**13**
How many face cards are there in each suit? (An Ace is not considered a face card)	**3**
What is the name of the card given with an 'A' on it?	**Ace**
List the cards in any one suit (No need to write in full – use J for Jack etc)	**Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K**
Name the face cards	**Jack, Queen, King**

How many non-numbered (lettered) cards are in each suit?	**4**
Which card is non-numbered and also not a face card?	**Ace**
How many black cards are there in a fair deck?	**26**
How many red cards are there in a fair deck?	**26**
In a full deck, how many:	
even numbered cards are there?	**20**
odd numbered cards are there?	**16**
lettered cards are there?	**16**
of each kind of card are there? (how many 1's, 2's etc)	**4**

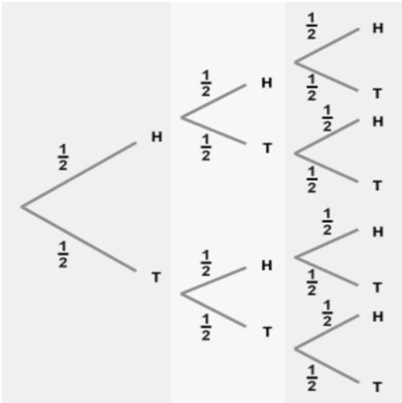
PROBABILITY QUESTIONS FOR A FAIR DECK OF CARDS

is red	$\frac{26}{52} = \frac{1}{2}$	is a black king	$\frac{2}{52} = \frac{1}{26}$
is from the heart suit	$\frac{13}{52} = \frac{1}{4}$	is even numbered	$\frac{20}{52} = \frac{5}{13}$
is a queen	$\frac{4}{52} = \frac{1}{13}$	has a number on it	$\frac{36}{52} = \frac{9}{13}$
is the queen of spades	$\frac{1}{52}$	is a face card	$\frac{12}{52} = \frac{3}{13}$
is a picture card	$\frac{16}{52} = \frac{4}{13}$	is a king or a queen	$\frac{8}{52} = \frac{2}{13}$
is a red jack	$\frac{2}{52} = \frac{1}{26}$	is a spade or diamond	$\frac{26}{52} = \frac{1}{2}$

MEMORANDUM PRE-TEST

<table>
<tr><td>1</td><td>What is the number value (not the percentage) that we attach to something that is certain to happen?</td><td>1 ✓</td></tr>
<tr><td>2</td><td>A die is thrown, what is the probability it will land on the '3'?</td><td>$\frac{1}{6}$ ✓</td></tr>
<tr><td>3</td><td>A die is thrown, what is the probability it will land on an even number?</td><td>$\frac{3}{6} = \frac{1}{2}$ ✓</td></tr>
<tr><td>4</td><td>There are 4 blue balls and 7 green balls in a bag. What is the probability of getting:
a) a blue ball?
b) a red ball?</td><td>a) $\frac{4}{11}$ ✓
b) 0 ✓</td></tr>
<tr><td>5</td><td>One card is drawn from a pack of playing cards. What is the probability of drawing a:
a) queen
b) red card
c) picture card</td><td>a) $\frac{4}{52} = \frac{1}{13}$ ✓
b) $\frac{26}{52} = \frac{1}{2}$ ✓
c) $\frac{12}{52} = \frac{3}{13}$ ✓</td></tr>
<tr><td>6</td><td>Consider the following table:

Gender compared to handedness
<table>
<tr><td></td><td colspan="2">Handed</td><td></td></tr>
<tr><td></td><td>Left</td><td>Right</td><td></td></tr>
<tr><td>Female</td><td>7</td><td>46</td><td>53</td></tr>
<tr><td>Male</td><td>5</td><td>63</td><td>68</td></tr>
<tr><td></td><td>12</td><td>109</td><td>121</td></tr>
</table>
If a person is chosen at random, what is the probability that the person is:
a) female
b) left handed
c) male and right handed</td><td>a) $\frac{53}{121}$ ✓
b) $\frac{12}{121}$ ✓
c) $\frac{63}{121}$ ✓✓</td></tr>
<tr><td>7</td><td>Consider the following tree diagram that represents tossing a coin 3 times.</td><td>a) $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ ✓✓
b) $1 - \frac{1}{8} = \frac{7}{8}$ ✓✓
(there is only the HHH option that doesn't have at least one T)</td></tr>
</table>

	 a) What is the probability of getting 3 Heads one after the other? b) What is the probability of getting at least one Tails?	**OR:** $\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)+$ $\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)$ $+\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)$ $+\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)$ $=\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8}$ $=\frac{7}{8}$

Post- Assessment Memorandum

1	What is the range in numbers (not percentage) of the probability scale? (from where to where does the probability scale go?)	0 – 1 ✓
2	A die is thrown, what is the probability it will land on the '1'?	$\frac{1}{6}$ ✓
3	A die is thrown, what is the probability it will land on a prime number?	$\frac{3}{6}=\frac{1}{2}$ ✓
4	There are 2 red balls and 5 yellow balls in a bag. What is the probability of getting: a) a yellow ball? b) a black ball?	a) $\frac{5}{7}$ ✓ b) 0 ✓
5	One card is drawn from a pack of playing cards. What is the probability of drawing a: a) 4 b) black card c) king or a Queen	a) $\frac{4}{52}=\frac{1}{13}$ ✓ b) $\frac{26}{52}=\frac{1}{2}$ ✓ c) $\frac{8}{52}=\frac{2}{13}$ ✓

6 Consider the following table:

	Eat Breakfast	Skip Breakfast	Totals
Students: ages 10-13	40	14	54
Students: ages 14-17	12	24	36
Totals	52	38	90

If a person is chosen at random, what is the probability that the person:

a) eats breakfast
b) is 15 years old
c) skips breakfast and is 10 years old

Answers:

a) $\frac{52}{90} = \frac{26}{45}$ ✓

b) $\frac{36}{90} = \frac{2}{5}$ ✓

c) $\frac{14}{90} = \frac{7}{45}$ ✓

7 Consider the tree diagram alongside. It represents picking a ball out of a bag. After the first draw, the ball is replaced. There are 4 Black balls, 2 white balls and 5 red balls.

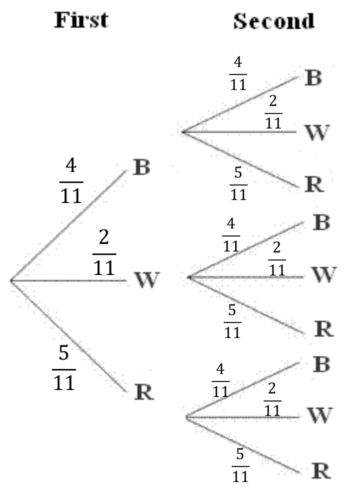
a) Write the probability on each branch using a fraction.

What is the probability that:

b) 2 red balls are drawn one after the other
c) a white and a black ball in any order

Answers:

a)



✓✓✓

b) $\frac{5}{11} \times \frac{5}{11} = \frac{25}{121}$ ✓

c) $\left(\frac{4}{11} \times \frac{2}{11}\right) + \left(\frac{2}{11} \times \frac{4}{11}\right) = \frac{8}{121} + \frac{8}{121}$ ✓ $= \frac{16}{121}$ ✓

8a	30 minutes	Selection of topics for participant presentations	Facilitator:	What you will need: 6 coloured boards per pair Koki pen per participant

1. Settle the participants and explain that they are now going to demonstrate a lesson on one of the topics and concepts that we have covered.
2. Remind participants that they can use the Content Booklet as a guide.
3. Explain that they will have 1 hour to plan the lesson and 15 minutes per pair to give the demonstration with 10 minutes for discussion and feedback from the group.
4. These are the lesson options:
    a. Group 1 = Grade 4: Lesson **Number Sentences** – how to help learners construct a number sentence from a word problem for the first time
    OR Lesson **Measurement** – a concrete way to show how to break up the square of 5 (25) into two other perfect squares
    b. Group 2 = Grade 5: Lesson **Number Sentences** – how to help learners understand "more" does not always mean adding and "less" does not always mean subtraction
    OR Lesson **Probability** – how to explain to learners the concepts of certain /absolutely sure and impossible
    c. Group 3 = Grade 6: Lesson **Number Sentences** – helping learners construct a context free number sentence: "There is a number that is 3 times more than two less than 14"
    OR Lesson **Similar Triangles** – how to understand the proportional relationship between the sides of similar triangles
    d. Group 4 = Grade 7: Lesson **Probability** – an introductory lesson recapping previous knowledge and introducing the concept of listing all possible outcomes
    OR Lesson **Number sentences/Algebraic Equations** – introduce variables instead of place holders into number sentences using word problems
    e. Group 5 = Grade 8: Lesson **Algebraic Equations** – generating ordered pairs
    OR Lesson **Theorem of Pythagoras** – solving word problems
    f. Group 6 = Grade 9: Lesson **Quadratic Equations** – introducing the concept of two solutions to an equation
    OR Lesson **Probability** – introduction to tree diagrams

5. Each group will come to the front and demonstrate the lesson given to them. (Put a timer on your phone to keep track of time).

6. After each group has been given 15 minutes the participants will give constructive feedback. (Always start with a positive observation and then give constructive comments).
7. Thank each group for their effort.

Use this time to ensure that participants do the following:

8. *Clearly and correctly explain the concept*
9. *Use the correct language / terminology*

Also use this time to clear up any misunderstandings or misconceptions that participants may have. Participants must leave with a clear understanding of how to teach these concepts. If a lesson is demonstrated incorrectly, use this time to re-demonstrate the lesson correctly.

8b	6 hours 30 minutes	**Lesson Demonstrations and Feedback**	**Facilitator:**	**What you will need:** • Flipchart Paper • Markers • Improvised resources

1. Tell participants that you are really looking forward to their presentations.
2. Remind participants of these criteria explained in the briefing.
3. Remind participants that their presentations should take 15 minutes.
4. Stop the presentations after the allocated time. You must be strict with the time, otherwise not everybody will have a chance to present.
5. If a group does not manage to do very much within the time, speak to them about time management. Explain that they will not have much more time than this in class to do these presentations. Discuss how the group could speed up.
6. Ask the group to state the grade, topic and subtopic for the lesson that they will present.
7. After each lesson demonstration encourage conversation for critical and constructive feedback.
   Encourage all participants to take part in the feedback session.
   Ensure that all feedback starts with something positive – our approach is to build confidence!

9	2 hours	ORIENTATION TO THE TRAINER'S GUIDE	Facilitator:	What you will need:

Note: If you have any extra time, spend it on this activity, particularly points 4 and 6.

1. Settle participants with all their materials.
2. Give each participant a copy of the **Trainers Guide** and **Training Handout**.
3. Explain to participants that the **Trainers Guide** and **Training Handout** contains all the activities for the Term 3&4 training.

4. Planning the training session:

   a. Tell participants to look carefully at the programme at the front of the trainer's guide.
   b. Go through this programme and tell participants which activities to complete when training other trainers.
   c. Go through this programme and tell participants which activities to complete when training teachers. (They should have 15 hours for this training)

5. Orientation to the guide and handout:

   a. Go through each activity in the trainer's guide, and look at the corresponding resources or section in the training handout.
   b. Work with participants to summarise the key steps and points of each activity.
   c. After you have done this for each activity, revise the order of activities, and the main points for each activity. For example:
      - Start with the **Guidelines for facilitators and participants.**
      - You have 30 minutes for this.
      - You must: tell participants to think about when real learning takes place; get them to discuss this with a partner; write a list of key points; discuss what is the same and different between a classroom and an adult training event; create a list of guidelines for facilitators and participants; ask participants to follow guidelines and commit to following facilitator guidelines.

6. The point of doing this is try and ensure that trainers clearly understand each activity and internalise as much of the workshop as possible.

7. If time allows, allocate different activities to volunteers, and ask them to present a 'dry-run' presentation of the activity. After each presentation, ask the other participants to give feedback based on the following:

a. Was the activity presented correctly?
b. Did the main points of the activity come across clearly?
c. Did the presenter give clear instructions?
d. Was the presenter audible?
e. Did the presenter interact effectively with participants?
f. Did the presenter manage time effectively?

8. Finally, thank participants for their presentations, and hold a closing discussion:

a. Ask: Which activities are you worried about presenting or facilitating? Why?
b. Try to address any concerns that participants may have.
c. Wish participants well for their training.

10	30 minutes	FINAL QUESTIONS AND ANSWERS	Facilitator:	What you will need:

1. Settle participants so that you have their attention.
2. Remind participants that we want them to IMPLEMENT THIS TRAINING IN A MEANINGFUL WAY.
3. Ask participants to think through all the materials, content, skills and information they have engaged with in this workshop. Give them time to look through materials as they do this.
4. Next, ask participants if they have any final questions.
5. Answer each question as clearly as possible. Where appropriate, involve participants in answering.

11	30 minutes	POST TEST	Facilitator:	What you will need: • Copies of post test

1. Work together to hand out copies of the post-test to participants.
2. Remind participants that the purpose of these tests is to measure the success of the training, not to measure the scores of individuals.
3. Remind participants of the test conditions and available time.
4. As participants complete the test, walk around and offer practical assistance if needed.
5. Once time is up, help to collect and collate tests in an orderly fashion.

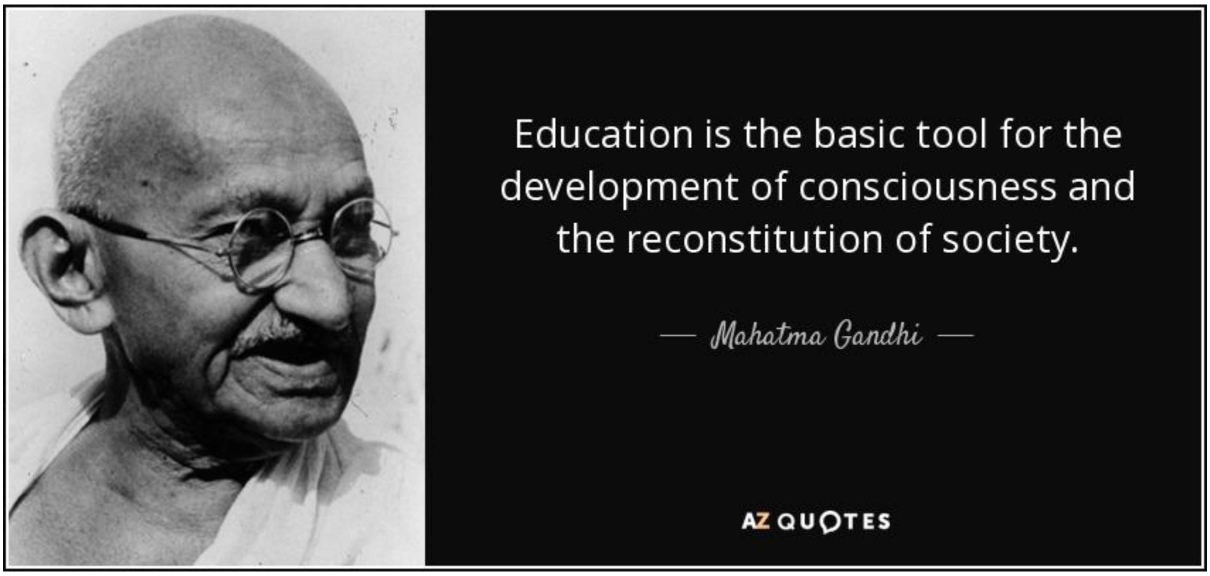
12	30 minutes	TRAINING OF TEACHERS: PLANNING SESSION	Facilitator:	What you will need:

1. Explain that this is an opportunity for Coaches and Subject Advisors to work together to talk about the logistics of the teacher training sessions in their district.
2. Allow participants to sit together in groups and discuss relevant issues.
3. If all the logistics are sorted, then participants should talk about co-facilitation, and who will present which activities.
4. They should also speak about resources in their district, like data-projectors and speakers.

13	30 minutes	CLOSURE AND EVALUATION	Facilitator:	What you will need: Evaluation forms

1. Settle participants so that you have their attention.
2. Give participants an evaluation form, briefly take them through the form, and then ask them to please complete it thoughtfully and carefully.
3. Collect the completed evaluation forms.
4. Call participants to attention and ask them to share some of the positives that they take away from this training. This can be absolutely anything: new content that they have learned or clarified; a new skill; a better understanding of the curriculum; new enthusiasm for their job; a closer working relationship with a colleague; etc.
5. Document what participants say for your report.
6. Thank the participants for their ongoing commitment to education, and to the development of South Africa.

9. Wish participants well for their own training.



Thank you for your ongoing dedication and commitment to this cause.