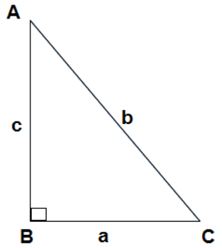
CAMI Mathematics: Grade 10

GRADE 10_CAPS Curriculum
10.9 Trigonometry

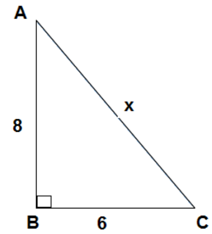
1.1 **Define the trigonometric ratios sinθ, cosθ and tanθ using right-angled triangle.**

(a)



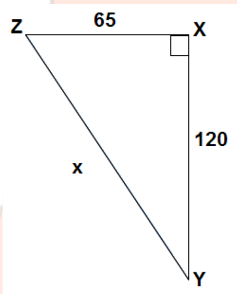
cosA = …
sinC = …
tanA = …

(b)



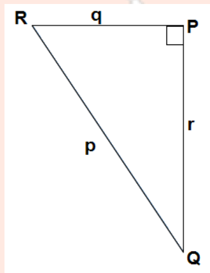
sinA = …
tanC = …
cosC = …

(c)



sinZ = … sinY = …
cosZ = … cosY = …
tanZ = … tanY = …

(d)

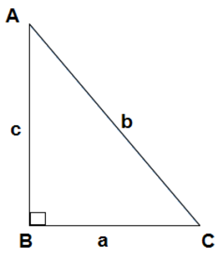


sinQ = …
tanR = …
cosQ = …

1.2 **Extend the definitions of sinθ, cosθ and tanθ for** $0° \leq \theta \leq 360°$.

(a) cos 100° (b) tan 210°
(c) sin 300° (d) tan 135°
(e) sin 315° (f) cos 120°
(g) sin 240° (h) cos 225°
(i) tan 150° (j) sin 135°

1.3 **Define the reciprocals of the trigonometric ratios cosecθ, secθ and cotθ, using right-angled triangles.**



sinA = …	cosec C = …
cosA =	sec C = …
tanA = …	cotC = …
cosecA =…	sin C = …
secA = …	cos C = …
cot A = …	tan C = …

1.4 Derive values of the trigonometric ratios for the special cases (without using a calculator).

(a) $\dfrac{\tan 225°.\sin 135°.\tan 300°}{\cos 315°.\cos 225°.\cos 150°}$

(b) $\dfrac{\tan 120°}{\tan 330°}$

(c) sin60°.cos30°.tan60°

(d) sin30°.tan45°.cos45°

(e) $\dfrac{\tan 120°.\cos 210°}{\sin 240°.\sin 240°}$

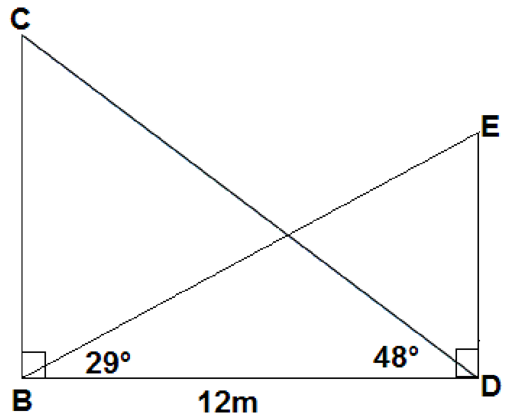
(f) $\dfrac{\cos 330°}{\cos 225°.\cos 315°.\tan 225°}$

1.5 Solve two-dimensional problems involving right-angled triangles.

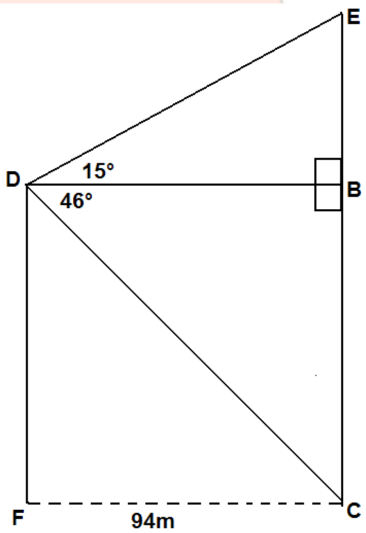
(a) The length of a mast is 8.5m, and the length of the shadow of the mast is 7.25m. Calculate the angle of elevation of the sun at the particular moment.

(b) The angle of elevation of a glider according to a woman on the ground is 43°. If the glider is 2340m from the woman, calculate the altitude of the glider.

(c) Two towers are 12m apart. From B the angle of elevation to DE is 29° and from D the angle of elevation to BC is 48°. Calculate the difference in the heights of the towers.



(d) A building (DF) and a tower (CE) are 94m apart. From the roof of the building the angle of elevation to the top of the tower is 15° and the angle of depression to the bottom of the tower is 46°. Calculate the height of the tower.



1.6 Solve simple trigonometric equations for angles between 0° and 90°.

(a) $\sin 51° = \cos\beta$, β an acute angle.

(b) $\cos 33° = \sin\alpha$

(c) $\sin 75° = \cos 3\theta$
(d) $\cos 4\alpha = \sin 5\alpha$
(e) $\cos(\beta - 43°) = \sin 65°$
(f) $\sin(\theta + 54°) = \cos(\theta - 8°)$

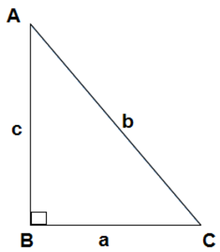
1.7 Use diagrams to determine the numerical values of ratios for angles from 0° and 360°.

(a) If $17\sin A = 15$, $0° \leq A \leq 90°$, determine $\tan A$.
(b) If $9\tan\beta = 40$ and β is an acute angle, determine $\sin\beta$.
(c) If $6\sin\alpha - 5 = 0$ and $\alpha \in [90°;180°]$, determine $\cos\alpha$.
(d) If $-5\cos\beta - 4 = 0$ and $\beta \in [180°;270°]$, determine $\sin\beta$.
(e) If $5\sin\theta - 4 = 0$ and $90° \leq \theta \leq 180°$, determine $\cos\theta$.

MEMO

1.1 Define the trigonometric ratios sinθ, cosθ and tanθ using right-angled triangle. [7.2.1.1]

(a)

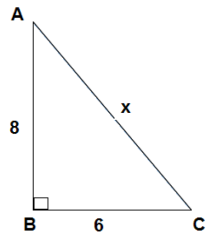


cos A = $\frac{c}{b}$

sin C = $\frac{c}{b}$

tan A = $\frac{a}{c}$

(b)

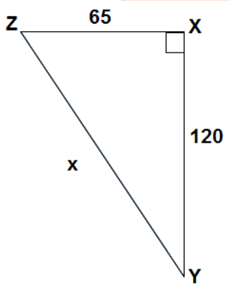


sin A = $\frac{6}{x}$

tan C = $\frac{8}{6}$

cos C = $\frac{6}{x}$

(c)

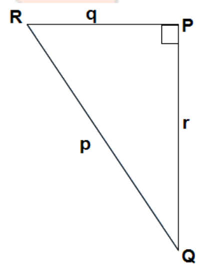


sinZ = $\frac{120}{x}$; sinY = $\frac{65}{x}$

cosZ = $\frac{65}{x}$; cosY = $\frac{120}{x}$

tanZ = $\frac{120}{65}$; tanY = $\frac{65}{120}$

(d)



sinQ = $\frac{q}{p}$

tanR = $\frac{r}{q}$

cosQ = $\frac{r}{p}$

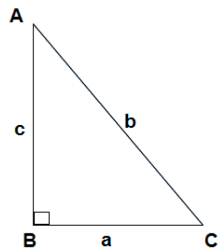
1.2 Extend the definitions of sinθ, cosθ and tanθ for $0° \leq \theta \leq 360°$. [7.4.2.2; 7.4.2.3]

(a) cos 100° = -cos 80°

(b) tan 210° = tan 30°

(c) sin 300° = -sin 60° (d) tan 135° = -tan 45°
(e) sin 315° = -sin 45° (f) cos 120° = - cos 60°
(g) sin 240° = -sin 60° (h) cos 225° = -cos 45°
(i) tan 150° = -tan 30° (j) sin 135° = sin 45°

1.3 Define the reciprocals of the trigonometric ratios cosecθ, secθ and cotθ, using right-angled triangles. [7.2.1.3; 7.2.1.4; 7.2.1.5; 7.2.1.2]



$\sin A = \frac{a}{b}$ $\quad$ $\text{cosec } C = \frac{b}{c}$

$\cos A = \frac{c}{b}$ $\quad$ $\sec C = \frac{b}{a}$

$\tan A = \frac{a}{c}$ $\quad$ $\cot C = \frac{a}{c}$

$\text{cosec } A = \frac{b}{a}$ $\quad$ $\sin C = \frac{c}{b}$

$\sec A = \frac{b}{c}$ $\quad$ $\cos C = \frac{a}{b}$

$\cot A = \frac{c}{a}$ $\quad$ $\tan C = \frac{c}{a}$

1.4 Derive values of the trigonometric ratios for the special cases (without using a calculator). [7.3.2.1; 7.3.2.3; 7.3.1.5; 7.3.1.1]

(a)

$$\frac{\tan 225°.\sin 135°.\tan 300°}{\cos 315°.\cos 225°.\cos 150°}$$
$$= \frac{\tan 45°.\sin 45°.(-\tan 60°)}{\cos 45°.(-\cos 45°).(-\cos 30°)}$$
$$= \frac{1.\frac{1}{\sqrt{2}}.(-\sqrt{3})}{\frac{1}{\sqrt{2}}.(-\frac{1}{\sqrt{2}}).(-\frac{\sqrt{3}}{2})}$$
$$= -2\sqrt{2}$$

(b)

$$\frac{\tan 120°}{\tan 330°}$$
$$= \frac{-\tan 60°}{-\tan 30°}$$
$$= \frac{\sqrt{3}}{\frac{1}{\sqrt{3}}}$$
$$= 3$$

(c)

$$\sin 60°.\cos 30°.\tan 60°$$
$$= \frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}.\sqrt{3}$$
$$= \frac{3\sqrt{3}}{4}$$

(d)

$$\sin 30°.\tan 45°.\cos 45°$$
$$= \frac{1}{2}.1.\frac{1}{\sqrt{2}}$$
$$= \frac{1}{2\sqrt{2}}$$

(e)

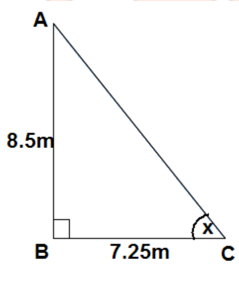
$$\frac{\tan 120°.\cos 210°}{\sin 240°.\sin 240°}$$
$$= \frac{(-\tan 60°).(-\cos 30°)}{(-\sin 60°).(-\sin 60°)}$$
$$= \frac{\sqrt{3}.\frac{\sqrt{3}}{2}}{(\frac{\sqrt{3}}{2})^2}$$
$$= 2$$

(f)

$$\frac{\cos 330°}{\cos 225°.\cos 315°.\tan 225°}$$
$$= \frac{\cos 30°}{(-\cos 45°).\cos 45°.\tan 45°}$$
$$= \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{2}}.1}$$
$$= -\sqrt{3}$$

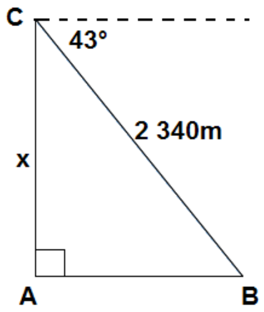
1.5 Solve two-dimensional problems involving right-angled triangles. [7.7.1.1; 7.7.1.2; 7.7.1.3]

(a) The length of a mast is 8.5m, and the length of the shadow of the mast is 7.25m. Calculate the angle of elevation of the sun at the particular moment.



$$\frac{AB}{BC} = \tan x$$
$$\frac{8.5}{7.25} = \tan x$$
$$x = \tan^{-1}(\frac{8.5}{7.25})$$
$$x = 49.5°$$

(b) The angle of elevation of a glider according to a woman on the ground is 43°. If the glider is 2340m from the woman, calculate the altitude of the glider.



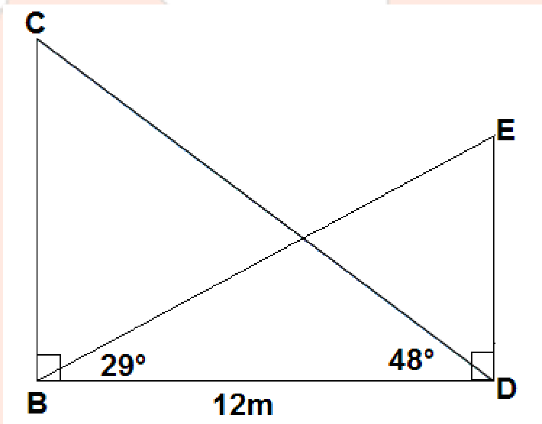
$\frac{AC}{BC}\sin x$

$\frac{x}{2340} = \sin 43°$

$x = 2340 \times \sin 43°$

$x = 1595.9m$

(c) Two towers are 12m apart. From B the angle of elevation to DE is 29° and from D the angle of elevation to BC is 48°. Calculate the difference in the heights of the towers.



ΔBCD:

$\frac{BC}{BD} = \tan 48°$

$\frac{x}{12} = \tan 48°$

$x = 12 \times \tan 48°$

$x = 13.33m$

ΔBDE :

$\frac{DE}{BD} = \tan 29°$

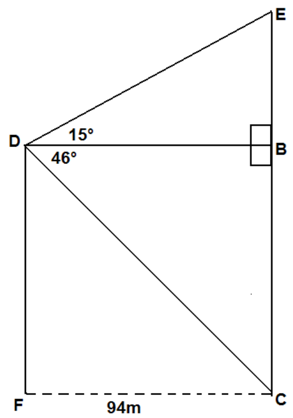
$\frac{y}{12}\tan 29°$

$y = 12 \times \tan 29°$

$y = 6.65m$

$BC - DE = 13.33 - 6.65 = 6.68m$

(d) A building (DF) and a tower (CE) are 94m apart. From the roof of the building the angle of elevation to the top of the tower is 15° and the angle of depression to the bottom of the tower is 46°. Calculate the height of the tower.



$\Delta BCD:$

$$\frac{BC}{DB} = \tan 46^\circ$$
$$\frac{x}{94} = \tan 46^\circ$$
$$x = 94 \times \tan 46^\circ$$
$$x = 97.34m$$

$\Delta BDE:$

$$\frac{BE}{DB} = \tan 15^\circ$$
$$\frac{y}{94} = \tan 15^\circ$$
$$y = 94 \times \tan 15^\circ$$
$$y = 25.19m$$

$$EC = 97.34 + 25.19 = 122.53m$$

1.6 Solve simple trigonometric equations for angles between 0° and 90°. [7.6.2.1; 7.6.2.3; 7.6.2.5]

(a) $\sin 51^\circ = \cos\beta$, β an acute angle.

$$\sin 51^\circ = \cos\beta$$
$$\sin 51^\circ = \sin(90^\circ - \beta)$$
$$\therefore 51^\circ = 90^\circ - \beta$$
$$\therefore \beta = 90^\circ - 51^\circ$$
$$\therefore \beta = 39^\circ$$

(b) $\cos 33^\circ = \sin\alpha$

$\cos 33° = \sin\alpha$
$\cos 33° = \cos(90° - \alpha)$
$\therefore 33° = 90° - \alpha$
$\therefore \alpha = 90° - 33°$
$\therefore \alpha = 57°$

(c) $\sin 75° = \cos 3\theta$

$\sin 75° = \cos 3\theta$
$\sin 75° = \sin(90° - 3\theta)$
$\therefore 75° = 90° - 3\theta$
$\therefore 3\theta = 90° - 75°$
$\therefore 3\theta = 15°$
$\therefore \theta = 5°$

(d) $\cos 4\alpha = \sin 5\alpha$

$\cos 4\alpha = \sin 5\alpha$
$\cos 4\alpha = \cos(90° - 5\alpha)$
$\therefore 4\alpha = 90° - 5\alpha$
$\therefore 9\alpha = 90°$
$\therefore \alpha = 10°$

(e) $\cos(\beta - 43°) = \sin 65°$

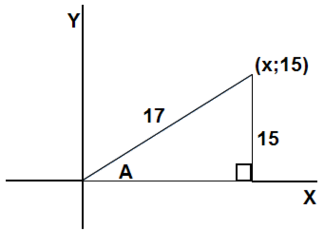
$\cos(\beta - 43°) = \sin 65°$
$\cos(\beta - 43°) = \cos(90° - 65°)$
$\therefore \beta - 43° = 90° - 65°$
$\therefore \beta = 68°$

(f) $\sin(\theta + 54°) = \cos(\theta - 8°)$

$\sin(\theta + 54°) = \cos(\theta - 8°)$
$\sin(\theta + 54°) = \sin(90° - (\theta - 8°)$
$\sin(\theta + 54°) = \sin(90° - \theta + 8°)$
$\therefore \theta + 54° = 90° - \theta + 8°$
$\therefore 2\theta = 90° - 54° + 8°$
$\therefore 2\theta = 44°$
$\therefore \theta = 22°$

1.7 Use diagrams to determine the numerical values of ratios for angles from 0° and 360°. [7.6.3.1; 7.6.3.3; 7.6.3.5; 7.6.5.1]

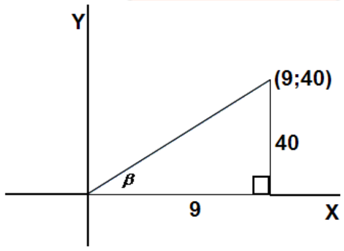
(a) If 17sinA = 15, 0°≤ A ≤ 90°, determine tan A.



$$x^2 + y^2 = r^2$$
$$x^2 + (15)^2 = (17)^2$$
$$x^2 = 64$$
$$x = 8$$

$$\therefore \tan A = \frac{15}{8}$$

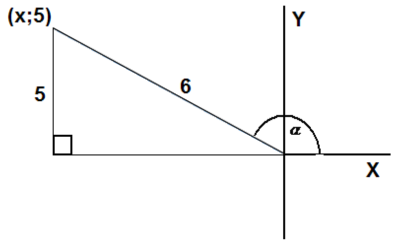
(b) If 9tan β = 40 and β is an acute angle, determine sin β.



$$x^2 + y^2 = r^2$$
$$(9)^2 + (40)^2 = r^2$$
$$81 + 1600 = r^2$$
$$r^2 = 1681$$
$$r = 41$$

$$\therefore \sin \beta = \frac{40}{41}$$

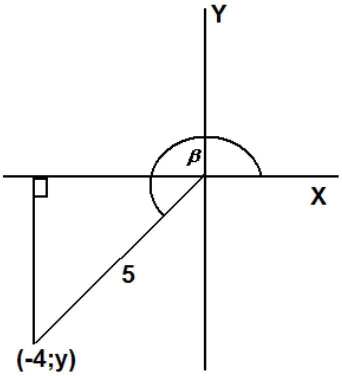
(c) If 6sin α – 5 = 0 and $\alpha \in [90°;180°]$, determine cos α.



$$x^2 + y^2 = r^2$$
$$x^2 + (5)^2 = (6)^2$$
$$x^2 = 36 - 25$$
$$x = -\sqrt{11}$$

$$\therefore \cos\alpha = \frac{-\sqrt{11}}{6}$$

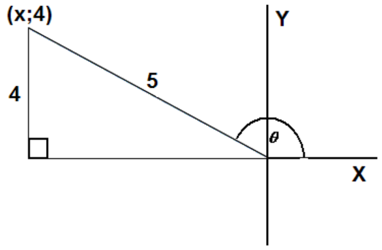
(d) If -5cos β – 4 = 0 and $\beta \in$ [180°;270°], determine sin β.



$$x^2 + y^2 = r^2$$
$$(-4)^2 + y^2 = (5)^2$$
$$16 + y^2 = 25$$
$$y = -3$$

$$\therefore \sin\beta = \frac{-3}{5}$$

(e) If 5sin θ – 4 = 0 and 90° ≤ θ ≤ 180°, determine cos θ.



$x^2 + y^2 = r^2$

$x^2 + (4)^2 = (5)^2$

$x^2 + 16 = 25$

$x = -3$

$\therefore \cos\theta = \dfrac{-3}{5}$