Products and Factors - Grade 10 [CAPS]*

Free High School Science Texts Project

Based on *Products and Factors*† by

Rory Adams

Free High School Science Texts Project

Mark Horner

Heather Williams

1 Introduction

In this chapter you will learn how to work with algebraic expressions. You will recap some of the work on factorisation and multiplying out expressions that you learnt in earlier grades. This work will then be extended upon for Grade 10.

2 Recap of Earlier Work

The following should be familiar. Examples are given as reminders.

2.1 Parts of an Expression

Mathematical expressions are just like sentences and their parts have special names. You should be familiar with the following names used to describe the parts of a mathematical expression.

$$\begin{aligned} a\cdot x^k+b\cdot x+c^m=0 \\ d\cdot y^p+e\cdot y+f\le 0 \end{aligned} \quad (1)$$

*Version 1.3: Jun 12, 2011 6:24 pm -0500
†http://cnx.org/content/m31483/1.3/

Name	Examples (separated by commas)
term	$a \cdot x^k$,$b \cdot x$, c^m, $d \cdot y^p$, $e \cdot y$, f
expression	$a \cdot x^k + b \cdot x + c^m$, $d \cdot y^p + e \cdot y + f$
coefficient	a, b, d, e
exponent (or index)	k, p
base	x, y, c
constant	a, b, c, d, e, f
variable	x, y
equation	$a \cdot x^k + b \cdot x + c^m = 0$
inequality	$d \cdot y^p + e \cdot y + f \leq 0$
binomial	expression with two terms
trinomial	expression with three terms

Table 1

2.2 Product of Two Binomials

A *binomial* is a mathematical expression with two terms, e.g. $(ax + b)$ and $(cx + d)$. If these two binomials are multiplied, the following is the result:

$$\begin{aligned} (a \cdot x + b)(c \cdot x + d) &= (ax)(c \cdot x + d) + b(c \cdot x + d) \\ &= (ax)(cx) + (ax)d + b(cx) + b \cdot d \\ &= ax^2 + x(ad + bc) + bd \end{aligned} \quad (2)$$

Exercise 1: Product of two binomials ***(Solution on p. 13.)***

Find the product of $(3x - 2)(5x + 8)$

The product of two identical binomials is known as the *square of the binomial* and is written as:

$$(ax + b)^2 = a^2x^2 + 2abx + b^2 \quad (3)$$

If the two terms are $ax + b$ and $ax - b$ then their product is:

$$(ax + b)(ax - b) = a^2x^2 - b^2 \quad (4)$$

This is known as the *difference of two squares*.

2.3 Factorisation

Factorisation is the opposite of expanding brackets. For example expanding brackets would require $2(x + 1)$ to be written as $2x + 2$. Factorisation would be to start with $2x + 2$ and to end up with $2(x + 1)$. In previous grades, you factorised based on common factors and on difference of squares.

2.3.1 Common Factors

Factorising based on common factors relies on there being common factors between your terms. For example, $2x - 6x^2$ can be factorised as follows:

$$2x - 6x^2 = 2x\left(1 - 3x\right) \quad (5)$$

2.3.1.1 Investigation : Common Factors

Find the highest common factors of the following pairs of terms:

(a) $6y; 18x$	(b) $12mn; 8n$	(c) $3st; 4su$	(d) $18kl; 9kp$	(e) $abc; ac$
(f) $2xy; 4xyz$	(g) $3uv; 6u$	(h) $9xy; 15xz$	(i) $24xyz; 16yz$	(j) $3m; 45n$

Table 2

2.3.2 Difference of Two Squares

We have seen that:

$$\left(ax + b\right)\left(ax - b\right) = a^2x^2 - b^2 \quad (6)$$

Since (6) is an equation, both sides are always equal. This means that an expression of the form:

$$a^2x^2 - b^2 \quad (7)$$

can be factorised to

$$\left(ax + b\right)\left(ax - b\right) \quad (8)$$

Therefore,

$$a^2x^2 - b^2 = \left(ax + b\right)\left(ax - b\right) \quad (9)$$

For example, $x^2 - 16$ can be written as $\left(x^2 - 4^2\right)$ which is a difference of two squares. Therefore, the factors of $x^2 - 16$ are $(x - 4)$ and $(x + 4)$.

Exercise 2: Factorisation ***(Solution on p. 13.)***
Factorise completely: $b^2y^5 - 3aby^3$

Exercise 3: Factorising binomials with a common bracket ***(Solution on p. 13.)***
Factorise completely: $3a\left(a - 4\right) - 7\left(a - 4\right)$

Exercise 4: Factorising using a switch around in brackets ***(Solution on p. 13.)***
Factorise $5\left(a - 2\right) - b\left(2 - a\right)$

2.3.2.1 Recap

1. Find the products of:

(a) $2y(y+4)$	(b) $(y+5)(y+2)$	(c) $(y+2)(2y+1)$
(d) $(y+8)(y+4)$	(e) $(2y+9)(3y+1)$	(f) $(3y-2)(y+6)$

Table 3

Click here for the solution[1]

2. Factorise:
   a. $2l+2w$
   b. $12x+32y$
   c. $6x^2+2x+10x^3$
   d. $2xy^2+xy^2z+3xy$
   e. $-2ab^2-4a^2b$

   Click here for the solution[2]

3. Factorise completely:

(a) $7a+4$	(b) $20a-10$	(c) $18ab-3bc$
(d) $12kj+18kq$	(e) $16k^2-4k$	(f) $3a^2+6a-18$
(g) $-6a-24$	(h) $-2ab-8a$	(i) $24kj-16k^2j$
(j) $-a^2b-b^2a$	(k) $12k^2j+24k^2j^2$	(l) $72b^2q-18b^3q^2$
(m) $4(y-3)+k(3-y)$	(n) $a(a-1)-5(a-1)$	(o) $bm(b+4)-6m(b+4)$
(p) $a^2(a+7)+a(a+7)$	(q) $3b(b-4)-7(4-b)$	(r) $a^2b^2c^2-1$

Table 4

Click here for the solution[3]

3 More Products

Khan Academy video on products of polynomials.

This media object is a Flash object. Please view or download it at
<http://www.youtube.com/v/fGThIRpWEE4&rel=0>

Figure 9

We have seen how to multiply two binomials in "Product of Two Binomials" (Section 2.2: Product of Two Binomials). In this section, we learn how to multiply a binomial (expression with two terms) by a trinomial

[1]http://www.fhsst.org/lxI
[2]http://www.fhsst.org/lqV
[3]http://www.fhsst.org/lqE

(expression with three terms). We can use the same methods we used to multiply two binomials to multiply a binomial and a trinomial.

For example, multiply $2x+1$ by x^2+2x+1.

$$\begin{array}{rcl} & & (2x+1)\left(x^2+2x+1\right) \\ & = & 2x\left(x^2+2x+1\right)+1\left(x^2+2x+1\right) \quad \text{(apply distributive law)} \\ & = & \left[2x\left(x^2\right)+2x\,(2x)+2x\,(1)\right]+\left[1\left(x^2\right)+1\,(2x)+1\,(1)\right] \\ & = & 2x^3+4x^2+2x+x^2+2x+1 \quad \text{(expand the brackets)} \\ & = & 2x^3+\left(4x^2+x^2\right)+(2x+2x)+1 \quad \text{(group like terms to simplify)} \\ & = & 2x^3+5x^2+4x+1 \quad \text{(simplify to get final answer)} \end{array} \tag{10}$$

TIP: If the binomial is $A+B$ and the trinomial is $C+D+E$, then the very first step is to apply the distributive law:

$$(A+B)(C+D+E)=A(C+D+E)+B(C+D+E) \tag{11}$$

If you remember this, you will never go wrong!

Exercise 5: Multiplication of Binomial with Trinomial *(Solution on p. 13.)*

Multiply $x-1$ with x^2-2x+1.

Exercise 6: Sum of Cubes *(Solution on p. 13.)*

Find the product of $x+y$ and x^2-xy+y^2.

TIP: We have seen that:

$$(x+y)\left(x^2-xy+y^2\right)=x^3+y^3 \tag{12}$$

This is known as a *sum of cubes*.

3.1 Investigation : Difference of Cubes

Show that the difference of cubes (x^3-y^3) is given by the product of $x-y$ and x^2+xy+y^2.

3.2 Products

1. Find the products of:

(a) $\left(-2y^2-4y+11\right)(5y-12)$	(b) $(-11y+3)\left(-10y^2-7y-9\right)$
(c) $\left(4y^2+12y+10\right)\left(-9y^2+8y+2\right)$	(d) $\left(7y^2-6y-8\right)(-2y+2)$
(e) $\left(10y^5+3\right)\left(-2y^2-11y+2\right)$	(f) $(-12y-3)\left(12y^2-11y+3\right)$
(g) $(-10)\left(2y^2+8y+3\right)$	(h) $\left(2y^6+3y^5\right)(-5y-12)$
(i) $\left(6y^7-8y^2+7\right)(-4y-3)\left(-6y^2-7y-11\right)$	(j) $\left(-9y^2+11y+2\right)\left(8y^2+6y-7\right)$
(k) $\left(8y^5+3y^4+2y^3\right)(5y+10)\left(12y^2+6y+6\right)$	(l) $(-7y+11)(-12y+3)$
(m) $\left(4y^3+5y^2-12y\right)(-12y-2)\left(7y^2-9y+12\right)$	(n) $(7y+3)\left(7y^2+3y+10\right)$
(o) $(9)\left(8y^2-2y+3\right)$	(p) $(-12y+12)\left(4y^2-11y+11\right)$
(q) $\left(-6y^4+11y^2+3y\right)(10y+4)(4y-4)$	(r) $\left(-3y^6-6y^3\right)(11y-6)(10y-10)$
(s) $\left(-11y^5+11y^4+11\right)\left(9y^3-7y^2-4y+6\right)$	(t) $(-3y+8)\left(-4y^3+8y^2-2y+12\right)$

Table 5

Click here for the solution[4]

4 Factorising a Quadratic

Khan Academy video on factorising a quadratic.

This media object is a Flash object. Please view or download it at
<http://www.youtube.com/v/eF6zYNzlZKQ&rel=0>

Figure 12

Factorisation can be seen as the reverse of calculating the product of factors. In order to factorise a quadratic, we need to find the factors which when multiplied together equal the original quadratic.

Let us consider a quadratic that is of the form $ax^2 + bx$. We can see here that x is a common factor of both terms. Therefore, $ax^2 + bx$ factorises to $x(ax + b)$. For example, $8y^2 + 4y$ factorises to $4y(2y + 1)$.

Another type of quadratic is made up of the difference of squares. We know that:

$$(a + b)(a - b) = a^2 - b^2. \tag{13}$$

This is true for any values of a and b, and more importantly since it is an equality, we can also write:

$$a^2 - b^2 = (a + b)(a - b). \tag{14}$$

This means that if we ever come across a quadratic that is made up of a difference of squares, we can immediately write down what the factors are.

Exercise 7: Difference of Squares ***(Solution on p. 14.)***

Find the factors of $9x^2 - 25$.

These types of quadratics are very simple to factorise. However, many quadratics do not fall into these categories and we need a more general method to factorise quadratics like $x^2 - x - 2$?

We can learn about how to factorise quadratics by looking at how two binomials are multiplied to get a quadratic. For example, $(x + 2)(x + 3)$ is multiplied out as:

$$\begin{aligned} (x+2)(x+3) &= x(x+3) + 2(x+3) \\ &= (x)(x) + 3x + 2x + (2)(3) \\ &= x^2 + 5x + 6. \end{aligned} \tag{15}$$

We see that the x^2 term in the quadratic is the product of the x-terms in each bracket. Similarly, the 6 in the quadratic is the product of the 2 and 3 in the brackets. Finally, the middle term is the sum of two terms.

So, how do we use this information to factorise the quadratic?

Let us start with factorising $x^2 + 5x + 6$ and see if we can decide upon some general rules. Firstly, write down two brackets with an x in each bracket and space for the remaining terms.

$$(\quad x \quad\quad)(\quad x \quad\quad) \tag{16}$$

Next, decide upon the factors of 6. Since the 6 is positive, these are:

[4]http://www.fhsst.org/llz

Factors of 6	
1	6
2	3
-1	-6
-2	-3

Table 6

Therefore, we have four possibilities:

Option 1	Option 2	Option 3	Option 4
$(x+1)(x+6)$	$(x-1)(x-6)$	$(x+2)(x+3)$	$(x-2)(x-3)$

Table 7

Next, we expand each set of brackets to see which option gives us the correct middle term.

Option 1	Option 2	Option 3	Option 4
$(x+1)(x+6)$	$(x-1)(x-6)$	$(x+2)(x+3)$	$(x-2)(x-3)$
x^2+7x+6	x^2-7x+6	$\underline{x^2+5x+6}$	x^2-5x+6

Table 8

We see that Option 3 (x+2)(x+3) is the correct solution. As you have seen that the process of factorising a quadratic is mostly trial and error, there is some information that can be used to simplify the process.

4.1 Method: Factorising a Quadratic

1. First, divide the entire equation by any common factor of the coefficients so as to obtain an equation of the form $ax^2+bx+c=0$ where a, b and c have no common factors and a is positive.
2. Write down two brackets with an x in each bracket and space for the remaining terms.

$$(\quad x \qquad)(\quad x \qquad) \tag{17}$$

3. Write down a set of factors for a and c.
4. Write down a set of options for the possible factors for the quadratic using the factors of a and c.
5. Expand all options to see which one gives you the correct answer.

There are some tips that you can keep in mind:

- If c is positive, then the factors of c must be either both positive or both negative. The factors are both negative if b is negative, and are both positive if b is positive. If c is negative, it means only one of the factors of c is negative, the other one being positive.
- Once you get an answer, multiply out your brackets again just to make sure it really works.

Exercise 8: Factorising a Quadratic ***(Solution on p. 14.)***

Find the factors of $3x^2+2x-1$.

4.1.1 Factorising a Trinomial

1. Factorise the following:

(a) $x^2+8x+15$	(b) $x^2+10x+24$	(c) x^2+9x+8
(d) $x^2+9x+14$	(e) $x^2+15x+36$	(f) $x^2+12x+36$

Table 9

Click here for the solution[5]

2. Factorise the following:
   a. $x^2-2x-15$
   b. x^2+2x-3
   c. x^2+2x-8
   d. x^2+x-20
   e. x^2-x-20

   Click here for the solution[6]
3. Find the factors for the following trinomial expressions:
   a. $2x^2+11x+5$
   b. $3x^2+19x+6$
   c. $6x^2+7x+2$
   d. $12x^2+8x+1$
   e. $8x^2+6x+1$

   Click here for the solution[7]
4. Find the factors for the following trinomials:
   a. $3x^2+17x-6$
   b. $7x^2-6x-1$
   c. $8x^2-6x+1$
   d. $2x^2-5x-3$

   Click here for the solution[8]

5 Factorisation by Grouping

One other method of factorisation involves the use of common factors. We know that the factors of $3x+3$ are 3 and $(x+1)$. Similarly, the factors of $2x^2+2x$ are $2x$ and $(x+1)$. Therefore, if we have an expression:

$$2x^2+2x+3x+3 \tag{18}$$

then we can factorise as:

$$2x\left(x+1\right)+3\left(x+1\right). \tag{19}$$

[5] http://www.fhsst.org/liY
[6] http://www.fhsst.org/lir
[7] http://www.fhsst.org/li1
[8] http://www.fhsst.org/liC

You can see that there is another common factor: $x+1$. Therefore, we can now write:

$$(x+1)(2x+3). \tag{20}$$

We get this by taking out the $x+1$ and seeing what is left over. We have a $+2x$ from the first term and a $+3$ from the second term. This is called *factorisation by grouping*.

Exercise 9: Factorisation by Grouping *(Solution on p. 15.)*

Find the factors of $7x+14y+bx+2by$ by grouping

Khan Academy video on factorising a trinomial by grouping.

This media object is a Flash object. Please view or download it at
<http://www.youtube.com/v/HXIj16mjfgk&rel=0>

Figure 20

5.1 Factorisation by Grouping

1. Factorise by grouping: $6x+a+2ax+3$
   Click here for the solution[9]
2. Factorise by grouping: $x^2-6x+5x-30$
   Click here for the solution[10]
3. Factorise by grouping: $5x+10y-ax-2ay$
   Click here for the solution[11]
4. Factorise by grouping: $a^2-2a-ax+2x$
   Click here for the solution[12]
5. Factorise by grouping: $5xy-3y+10x-6$
   Click here for the solution[13]

6 Simplification of Fractions

In some cases of simplifying an algebraic expression, the expression will be a fraction. For example,

$$\frac{x^2+3x}{x+3} \tag{21}$$

has a quadratic in the numerator and a binomial in the denominator. You can apply the different factorisation methods to simplify the expression.

$$\begin{array}{rcl} & & \frac{x^2+3x}{x+3} \\ & = & \frac{x(x+3)}{x+3} \\ & = & x \quad \text{provided } x \neq -3 \end{array} \tag{22}$$

[9] http://www.fhsst.org/lih
[10] http://www.fhsst.org/liS
[11] http://www.fhsst.org/liJ
[12] http://www.fhsst.org/liu
[13] http://www.fhsst.org/liz

If x were 3 then the denominator, $x-3$, would be 0 and the fraction undefined.

Exercise 10: Simplification of Fractions *(Solution on p. 15.)*

Simplify: $\frac{2x-b+x-ab}{ax^2-abx}$

Exercise 11: Simplification of Fractions *(Solution on p. 15.)*

Simplify: $\frac{x^2-x-2}{x^2-4} \div \frac{x^2+x}{x^2+2x}$

6.1 Simplification of Fractions

1. Simplify:

(a) $\frac{3a}{15}$	(b) $\frac{2a+10}{4}$
(c) $\frac{5a+20}{a+4}$	(d) $\frac{a^2-4a}{a-4}$
(e) $\frac{3a^2-9a}{2a-6}$	(f) $\frac{9a+27}{9a+18}$
(g) $\frac{6ab+2a}{2b}$	(h) $\frac{16x^2y-8xy}{12x-6}$
(i) $\frac{4xyp-8xp}{12xy}$	(j) $\frac{3a+9}{14} \div \frac{7a+21}{a+3}$
(k) $\frac{a^2-5a}{2a+10} \div \frac{3a+15}{4a}$	(l) $\frac{3xp+4p}{8p} \div \frac{12p^2}{3x+4}$
(m) $\frac{16}{2xp+4x} \div \frac{6x^2+8x}{12}$	(n) $\frac{24a-8}{12} \div \frac{9a-3}{6}$
(o) $\frac{a^2+2a}{5} \div \frac{2a+4}{20}$	(p) $\frac{p^2+pq}{7p} \div \frac{8p+8q}{21q}$
(q) $\frac{5ab-15b}{4a-12} \div \frac{6b^2}{a+b}$	(r) $\frac{f^2a-fa^2}{f-a}$

Table 10

Click here for the solution[14]

2. Simplify: $\frac{x^2-1}{3} \times \frac{1}{x-1} - \frac{1}{2}$

Click here for the solution[15]

7 Adding and subtracting fractions

Using the concepts learnt in simplification of fractions, we can now add and subtract simple fractions. To add or subtract fractions we note that we can only add or subtract fractions that have the same denominator. So we must first make all the denominators the same and then perform the addition or subtraction. This is called finding the lowest common denominator or multiple.

For example, if you wanted to add: $\frac{1}{2}$ and $\frac{3}{5}$ we would note that the lowest common denominator is 10. So we must multiply the first fraction by 5 and the second fraction by 2 to get both of these with the same denominator. Doing so gives: $\frac{5}{10}$ and $\frac{6}{10}$. Now we can add the fractions. Doing so, we get $\frac{11}{10}$.

Exercise 12 *(Solution on p. 15.)*

Simplify the following expression: $\frac{x-2}{x^2-4} + \frac{x^2}{x-2} - \frac{x^3+x-4}{x^2-4}$

[14]http://www.fhsst.org/lit
[15]http://www.fhsst.org/lie

8 Two interesting mathematical proofs

We can use the concepts learnt in this chapter to demonstrate two interesting mathematical proofs. The first proof states that n^2+n is even for all $n \in Z$. The second proof states that n^3-n is divisible by 6 for all $n \in Z$. Before we demonstrate that these two laws are true, we first need to note some other mathematical rules.

If we multiply an even number by an odd number, we get an even number. Similarly if we multiply an odd number by an even number we get an even number. Also, an even number multiplied by an even number is even and an odd number multiplied by an odd number is odd. This result is shown in the following table:

	Odd number	**Even number**
Odd number	Odd	Even
Even number	Even	Even

Table 11

If we take three consecutive numbers and multiply them together, the resulting number is always divisible by three. This should be obvious since if we have any three consecutive numbers, one of them will be divisible by 3.

Now we are ready to demonstrate that n^2+n is even for all $n \in Z$. If we factorise this expression we get: $n(n+1)$. If n is even, than $n+1$ is odd. If n is odd, than $n+1$ is even. Since we know that if we multiply an even number with an odd number or an odd number with an even number, we get an even number, we have demonstrated that n^2+n is always even. Try this for a few values of n and you should find that this is true.

To demonstrate that n^3-n is divisible by 6 for all $n \in Z$, we first note that the factors of 6 are 3 and 2. So if we show that n^3-n is divisible by both 3 and 2, then we have shown that it is also divisible by 6! If we factorise this expression we get: $n(n+1)(n-1)$. Now we note that we are multiplying three consecutive numbers together (we are taking n and then adding 1 or subtracting 1. This gives us the two numbers on either side of n.) For example, if $n=4$, then $n+1=5$ and $n-1=3$. But we know that when we multiply three consecutive numbers together, the resulting number is always divisible by 3. So we have demonstrated that n^3-n is always divisible by 3. To demonstrate that it is also divisible by 2, we can also show that it is even. We have shown that n^2+n is always even. So now we recall what we said about multiplying even and odd numbers. Since one number is always even and the other can be either even or odd, the result of multiplying these numbers together is always even. And so we have demonstrated that n^3-n is divisible by 6 for all $n \in Z$.

9 Summary

- A binomial is a mathematical expression with two terms. The product of two identical binomials is known as the square of the binomial. The difference of two squares is when we multiply $(ax+b)(ax-b)$
- Factorising is the opposite of expanding the brackets. You can use common factors or the difference of two squares to help you factorise expressions.
- The distributive law $((A+B)(C+D+E) = A(C+D+E) + B(C+D+E))$ helps us to multiply a binomial and a trinomial.
- The sum of cubes is: $(x+y)\left(x^2-xy+y^2\right) = x^3+y^3$ and the difference of cubes is: $x^3-y^3 = (x-y)\left(x^2+xy+y^2\right)$
- To factorise a quadratic we find the two binomials that were multiplied together to give the quadratic.
- We can also factorise a quadratic by grouping. This is where we find a common factor in the quadratic and take it out and then see what is left over.
- We can simplify fractions by using the methods we have learnt to factorise expressions.
- Fractions can be added or subtracted. To do this the denominators of each fraction must be the same.

10 End of Chapter Exercises

1. Factorise:
   a. $a^2 - 9$
   b. $m^2 - 36$
   c. $9b^2 - 81$
   d. $16b^6 - 25a^2$
   e. $m^2 - (1/9)$
   f. $5 - 5a^2b^6$
   g. $16ba^4 - 81b$
   h. $a^2 - 10a + 25$
   i. $16b^2 + 56b + 49$
   j. $2a^2 - 12ab + 18b^2$
   k. $-4b^2 - 144b^8 + 48b^5$

   Click here for the solution[16]
2. Factorise completely:
   a. $\left(16 - x^4\right)$
   b. $7x^2 - 14x + 7xy - 14y$
   c. $y^2 - 7y - 30$
   d. $1 - x - x^2 + x^3$
   e. $-3\left(1 - p^2\right) + p + 1$

   Click here for the solution[17]
3. Simplify the following:
   a. $(a-2)^2 - a(a+4)$
   b. $(5a - 4b)\left(25a^2 + 20\text{ab} + 16b^2\right)$
   c. $(2m - 3)\left(4m^2 + 9\right)(2m + 3)$
   d. $(a + 2b - c)(a + 2b + c)$

   Click here for the solution[18]
4. Simplify the following:
   a. $\frac{p^2 - q^2}{p} \div \frac{p+q}{p^2 - \text{pq}}$
   b. $\frac{2}{x} + \frac{x}{2} - \frac{2x}{3}$

   Click here for the solution[19]
5. Show that $(2x-1)^2 - (x-3)^2$ can be simplified to $(x+2)(3x-4)$

   Click here for the solution[20]
6. What must be added to $x^2 - x + 4$ to make it equal to $(x+2)^2$

   Click here for the solution[21]

[16]http://www.fhsst.org/liM
[17]http://www.fhsst.org/lTY
[18]http://www.fhsst.org/lTg
[19]http://www.fhsst.org/lT4
[20]http://www.fhsst.org/lib
[21]http://www.fhsst.org/liT

Solutions to Exercises in this Module

Solution to Exercise (p. 2)

Step 1.

$$\begin{aligned}(3x-2)(5x+8) &= (3x)(5x)+(3x)(8)+(-2)(5x)+(-2)(8)\\ &= 15x^2+24x-10x-16\\ &= 15x^2+14x-16\end{aligned} \quad (23)$$

Solution to Exercise (p. 3)

Step 1.

$$b^2y^5-3aby^3 = by^3\left(by^2-3a\right) \quad (24)$$

Solution to Exercise (p. 3)

Step 1. $(a-4)$ is the common factor

$$3a(a-4)-7(a-4) = (a-4)(3a-7) \quad (25)$$

Solution to Exercise (p. 3)

Step 1.

$$\begin{aligned}5(a-2)-b(2-a) &= 5(a-2)-[-b(a-2)]\\ &= 5(a-2)+b(a-2)\\ &= (a-2)(5+b)\end{aligned} \quad (26)$$

Solution to Exercise (p. 5)

Step 1. We are given two expressions: a binomial, $x-1$, and a trinomial, x^2-2x+1. We need to multiply them together.
Step 2. Apply the distributive law and then simplify the resulting expression.
Step 3.

$$\begin{aligned}& (x-1)\left(x^2-2x+1\right) & \\ =\ & x\left(x^2-2x+1\right)-1\left(x^2-2x+1\right) & \text{(apply distributive law)}\\ =\ & \left[x\left(x^2\right)+x(-2x)+x(1)\right]+\left[-1\left(x^2\right)-1(-2x)-1(1)\right] & \\ =\ & x^3-2x^2+x-x^2+2x-1 & \text{(expand the brackets)}\\ =\ & x^3+\left(-2x^2-x^2\right)+(x+2x)-1 & \text{(group like terms to simplify)}\\ =\ & x^3-3x^2+3x-1 & \text{(simplify to get final answer)}\end{aligned} \quad (27)$$

Step 4. The product of $x-1$ and x^2-2x+1 is x^3-3x^2+3x-1.

Solution to Exercise (p. 5)

Step 1. We are given two expressions: a binomial, $x+y$, and a trinomial, x^2-xy+y^2. We need to multiply them together.

Step 2. Apply the distributive law and then simplify the resulting expression.
Step 3.

$$\begin{array}{rcl} & (x+y)\left(x^2-xy+y^2\right) & \\ = & x\left(x^2-xy+y^2\right)+y\left(x^2-xy+y^2\right) & \text{(apply distributive law)} \\ = & \left[x\left(x^2\right)+x\left(-xy\right)+x\left(y^2\right)\right]+\left[y\left(x^2\right)+y\left(-xy\right)+y\left(y^2\right)\right] & \\ = & x^3-x^2y+xy^2+yx^2-xy^2+y^3 & \text{(expand the brackets)} \\ = & x^3+\left(-x^2y+yx^2\right)+\left(xy^2-xy^2\right)+y^3 & \text{(group like terms to simplify)} \\ = & x^3+y^3 & \text{(simplify to get final answer)} \end{array} \tag{28}$$

Step 4. The product of $x+y$ and x^2-xy+y^2 is x^3+y^3.

Solution to Exercise (p. 6)

Step 1. We see that the quadratic is a difference of squares because:

$$(3x)^2 = 9x^2 \tag{29}$$

and

$$5^2 = 25. \tag{30}$$

Step 2.

$$9x^2 - 25 = (3x)^2 - 5^2 \tag{31}$$

Step 3.

$$(3x)^2 - 5^2 = (3x-5)(3x+5) \tag{32}$$

Step 4. The factors of $9x^2-25$ are $(3x-5)(3x+5)$.

Solution to Exercise (p. 7)

Step 1. The quadratic is in the required form.
Step 2.

$$(\quad x \qquad)(\quad x \qquad) \tag{33}$$

Write down a set of factors for a and c. The possible factors for a are: (1,3). The possible factors for c are: (-1,1) or (1,-1).
Write down a set of options for the possible factors of the quadratic using the factors of a and c. Therefore, there are two possible options.

Option 1	Option 2
$(x-1)(3x+1)$	$(x+1)(3x-1)$
$3x^2-2x-1$	$\underline{3x^2+2x-1}$

Table 12

Step 3.

$$\begin{aligned}(x+1)(3x-1) &= x(3x-1)+1(3x-1)\\ &= (x)(3x)+(x)(-1)+(1)(3x)+(1)(-1)\\ &= 3x^2-x+3x-1\\ &= x^2+2x-1.\end{aligned} \quad (34)$$

Step 4. The factors of $3x^2+2x-1$ are $(x+1)$ and $(3x-1)$.

Solution to Exercise (p. 9)

Step 1. There are no factors that are common to all terms.
Step 2. 7 is a common factor of the first two terms and b is a common factor of the second two terms.
Step 3.

$$7x+14y+bx+2by = 7(x+2y)+b(x+2y) \quad (35)$$

Step 4. $x+2y$ is a common factor.
Step 5.

$$7(x+2y)+b(x+2y) = (x+2y)(7+b) \quad (36)$$

Step 6. The factors of $7x+14y+bx+2by$ are $(7+b)$ and $(x+2y)$.

Solution to Exercise (p. 10)

Step 1. Use *grouping* for numerator and *common factor* for denominator in this example.

$$\begin{aligned}&= \frac{(ax-ab)+(x-b)}{ax^2-abx}\\ &= \frac{a(x-b)+(x-b)}{ax(x-b)}\\ &= \frac{(x-b)(a+1)}{ax(x-b)}\end{aligned} \quad (37)$$

Step 2. The simplified answer is:

$$= \frac{a+1}{ax} \quad (38)$$

Solution to Exercise (p. 10)

Step 1.

$$= \frac{(x+1)(x-2)}{(x+2)(x-2)} \div \frac{x(x+1)}{x(x+2)} \quad (39)$$

Step 2.

$$= \frac{(x+1)(x-2)}{(x+2)(x-2)} \times \frac{x(x+2)}{x(x+1)} \quad (40)$$

Step 3. The simplified answer is

$$= 1 \quad (41)$$

Solution to Exercise (p. 10)

Step 1.

$$\frac{x-2}{(x+2)(x-2)} + \frac{x^2}{x-2} - \frac{x^3+x-4}{(x+2)(x-2)} \quad (42)$$

Step 2. We make all the denominators the same so that we can add or subtract the fractions. The lowest common denominator is $(x-2)(x+2)$.

$$\frac{x-2}{(x+2)(x-2)} + \frac{(x^2)(x+2)}{(x+2)(x-2)} - \frac{x^3+x-4}{(x+2)(x-2)} \quad (43)$$

Step 3. Since the fractions all have the same denominator we can write them all as one fraction with the appropriate operator

$$\frac{x-2+\left(x^2\right)(x+2)-x^3+x-4}{(x+2)(x-2)} \tag{44}$$

Step 4.

$$\frac{2x^2+2x-6}{(x+2)(x-2)} \tag{45}$$

Step 5.

$$\frac{2\left(x^2+x-3\right)}{(x+2)(x-2)} \tag{46}$$