



NATIONAL SENIOR CERTIFICATE

IBANGA 12

SEPTEMBA 2020

IMATHEMATIKA P2

AMANQAKU: 150

IXESHA: 3 iiyure

Eli phepha lemibuzo linamaphepha ali 15, kudibene nephepha eli 1 lolwazi,
kunye nencwadi yokuphendulela enamaphepha ayi 25.

IMIYALELO NOLWAZI

Funda imiyalelo ngocoselelo phambi kokuphendula imibuzo.

1. Eli phepha lemibuzo linemibuzo eyi 10.
2. Phendula YONKE imibuzo kwiNCWADI EKHETHIWEYO YOKUPHENDULELA enikiweyo.
3. Bonisa ngokucacileyo ZONKE iikhaltyhuleyshini, iidayagram, iigrafu, njl.ozisebenzisileyo ukubonisa iimpendulo zakho.
4. Iimpendulo kuphela azinyanzelekanga UKUNIKWA amanqaku apheleleyo.
5. Ungayisebenzisa ikhaltyhuleythha esayentifikhi evunyiweyo (engaprogranyangwa nengenagrafikhi), ngaphandle kokuba uxelwelwe ngeny'indlela.
6. Ukuba kunyanzelekile, sondeza iimpendulo kwiindawo EZIMBINI zedesimal, ngaphandle kokukba uxelwelwe ngeny'indlela.
7. Iidayagram AZI zotywanga ngokwesikeyile (scale).
8. Iphepha lolwazi elineefomyula lifakiwe ekugqibeleni kwiphepha lemibuzo.
9. Bhala ngokucocekileyo nangokucacileyo.

UMBUZO 1

Itheybhile elandelayo ibonisa uthelkiso lwamanqaku okugqibela abafundi bebanga le 12 ngonyaka ka2019 kunye namanqaku eSchool Based Assessment (SBA) abafundi onyaka.

ABAUNDI	1	2	3	4	5	6	7	8	9	10
AMANQAKU ESBA	99	93	77	74	63	62	63	63	47	37
AMANQAKU OKUGQIBELA	94	81	73	65	59	58	55	49	43	31

- 1.1 Fumana i-ikhweyzhini yezona zikwere zincinci zomgca werhigreshin (least squares regression line) zedatha. (Sondeza impendulo yakho kwindawo ye 4 echanekileyo yedesimal). (3)
 - 1.2 Fumana ikhorheleyshin khoefishiyenti phakathi kwamanqaku eSBA namanqaku okugqibela. (1)
 - 1.3 Phawula (Comment) ngekhorheleyshin phakathi kwamanqaku eSBA namanqaku okugqibela. (1)
 - 1.4 Umfundu we11 wafumana i51% kwiSBA. Qikelela amanqaku okugqibela anokuwafumana, lungisa use kwiyunithi ekufutshane. (2)
 - 1.5 Unikwe ukuba imin (mean) yamanqaku okugqibela ngu 60,8; khaltyhuleytha ukuba bangaphi abafundi ababekwi diviyeshini enye yemin. (3)
- [10]**

UMBUZO 2

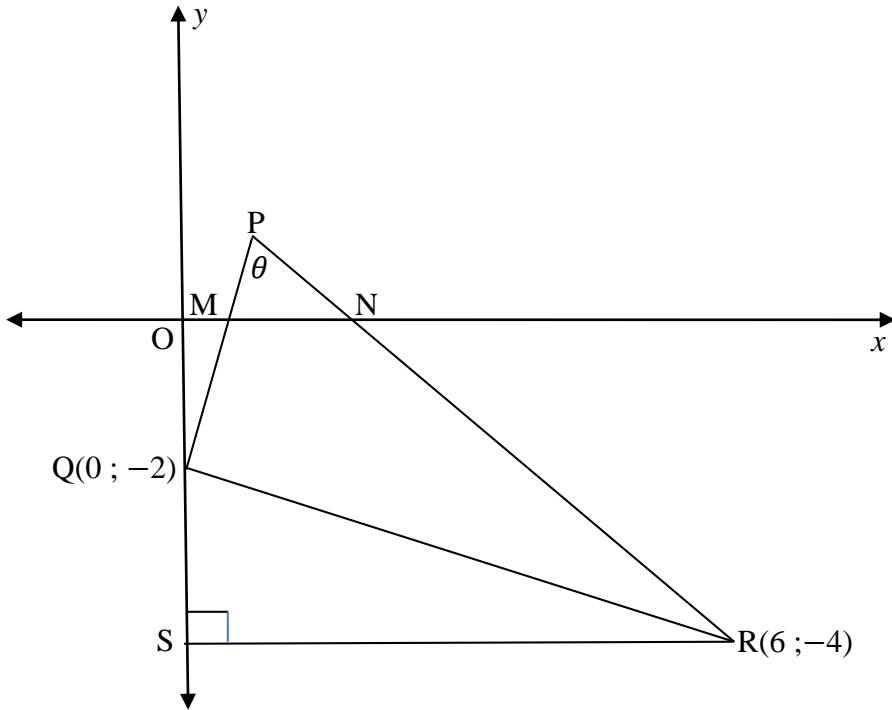
Isantya ngeekhilomitha ngeyure, somkhweli bayisekhile sidlula ipoynti kumda weIronman Race warhekhdishwa watshwankathelwa kwitheybhile engezantsi:

Isanya (km/h)	Ifrikwensi (f)	Tyhumulethive Frikwensi
$0 < x \leq 10$	10	10
$10 < x \leq 20$		30
$20 < x \leq 30$	45	
$30 < x \leq 40$	72	
$40 < x \leq 50$		170

- 2.1 Gqibeza itheybhile engentla, kwiNCWADI YOKUPHENDULELA oyinikiwego. (2)
 - 2.2 Sebenzisa iekhziz enikiwego kwiNCWADI YOKUPHENDULELA ukuzoba ityhumulethive frikwensi khevü (cumulative frequency curve) yedatha engentla. (3)
 - 2.3 Bonisa ngokucacile kwigrafu apha uqikelelo lwe quartile esezantsi (lower quartile) (Q_1) nemidiyen (M) spidi (median speed) zinokufundeka. Bhala olu qikelelo. (2)
 - 2.4 Zoba ibhokisi nedayagram yewhisker yedatha. Sebenzisa umgca manani kwiNCWADI YOKUPHENDULELA. (2)
 - 2.5 Sebenzisa igrafu yakho ukuqikelela inani labakhweli zibhayisekile abagqitha kwipoynti ngesantya esingaphezulu kune 35 km/h (km ngeyure). (1)
- [10]**

UMBUZO 3

Kwidayagram, P, Q(0 ; -2) no R(6 ; -4) ziivethesis zetrayengli PQR. I-ikhweyzhin ka PQ ka $3x - y - 2 = 0$. I-ikhweyzhin ka PR ngu $y = -x + 2$. U RS uyiphephendityhula ukusuka ku R ukuya kwi y-axis. $\hat{QPR} = \theta$.

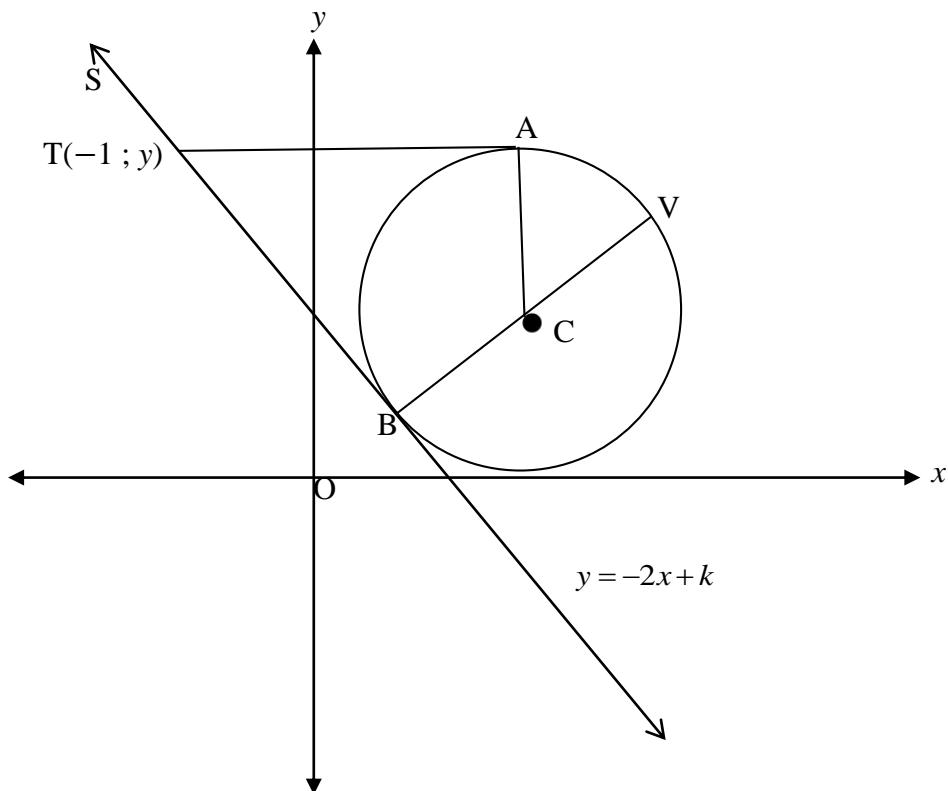


- 3.1 Khaltyhuleytha igradiyent ka QR. (2)
- 3.2 Nqina ukuba $\hat{PQR} = 90^\circ$. (2)
- 3.3 Khaltyhuleytha iikho-odineythi zika P. (3)
- 3.4 Khaltyhuleytha ubude buka QR. Shiya impendulo yakho ikwi surd form. (2)
- 3.5 Fumana i-ikhweyzhin yesekile ugqitha Q, P no R. Nika impendulo yakho ngokwefom: $(x - a)^2 + (y - b)^2 = r^2$. (5)
- 3.6 Khaltyhuleytha isayzi ka engile θ . (5)
- 3.7 Khaltyhuleytha ieriya ka ΔPQR . (3)

[22]

UMBUZO 4

Kwidayagram engezantsi, u C ngumbindi wese kile echazwe ngo $x^2 - 6x + y^2 - 4y + 9 = 0$. T $(-1; y)$ yipo ynti ngaphandle kwesekile. Iithanjenti ezimbini zizotywe ukuya kwise kile ukusuka kuT. U STB yithanjenti ukuya kwise kile ku B kwaye ine ikhwey zhin u $y = -2x + k$ U TA yithanjenti kuse kile A kwaye upharalel kwi x -ekhziz. U BV yidayamitha yesekile.



- 4.1 Fumana iikho-odineythi zika C. (4)
 - 4.2 Fumana i-ikhwey zhin ka BV. (3)
 - 4.3 Fumana i-ikhwey zhin yomgca TA. (1)
 - 4.4 Khaltyhuleytha ubude buka TB. Nika izizathu. (4)
 - 4.5 Khaltyhuleytha ivelyu ka k . (2)
 - 4.6 Khaltyhuleytha isayzi ka \widehat{ACB} . Nika izizathu. (4)
- [18]

UMBUZO 5

5.1 Ukuba $u \cos 22^\circ = p$; fumana iithem ezilandelayozi ka p :

$$5.1.1 \cos 158^\circ \quad (2)$$

$$5.1.2 \sin 112^\circ \quad (2)$$

$$5.1.3 \sin 38^\circ \quad (4)$$

5.2 Fumana zonke iivelyu zikaP kwi intavali ka $[0^\circ; 360^\circ]$ eyonelisa i-ikhweyzhin:

$$\sin P = \sin 2P \quad (4)$$

5.3 Ukuba $u \Delta ABC$ yi scalene triangle, bonisa ukuba: $\cos(A + B) = -\cos C$ (2)

5.4 Nqina olufana lulandelayo:

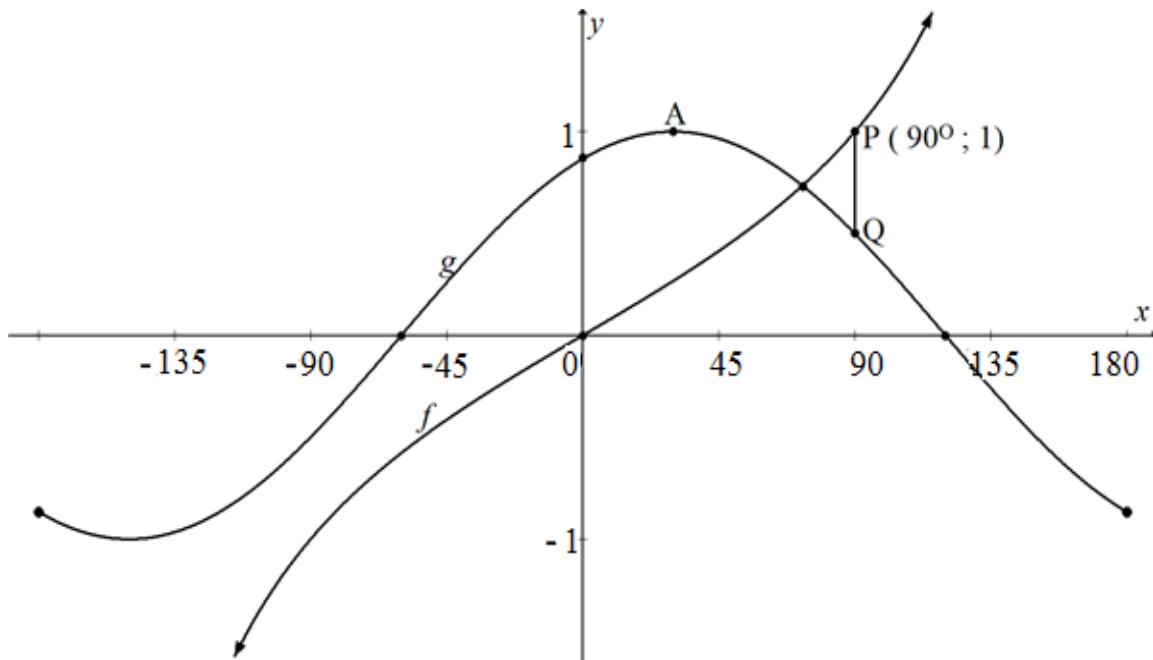
$$\frac{\cos^2 x - \cos x - \sin^2 x}{2 \sin x \cdot \cos x + \sin x} = \frac{1}{\tan x} - \frac{1}{\sin x} \quad (5)$$

5.5 Fumana ijeneral sholushini ka: $4 + 7 \cos \theta + \cos 2\theta = 0$. (6)

[25]

UMBUZO 6

Kwidayagram engezantsi, iigrafu zika $f(x) = \tan bx$ no $g(x) = \cos(x - 30^\circ)$ zizotywe kwiseti enye ye ekhziz zika $-180^\circ \leq x \leq 180^\circ$. Iipoynti $P(90^\circ; 1)$ no Q ilale ku f no g ngokulandeelana. Sebenzisa idayagram ukuphendula imibuzo elandelayo.



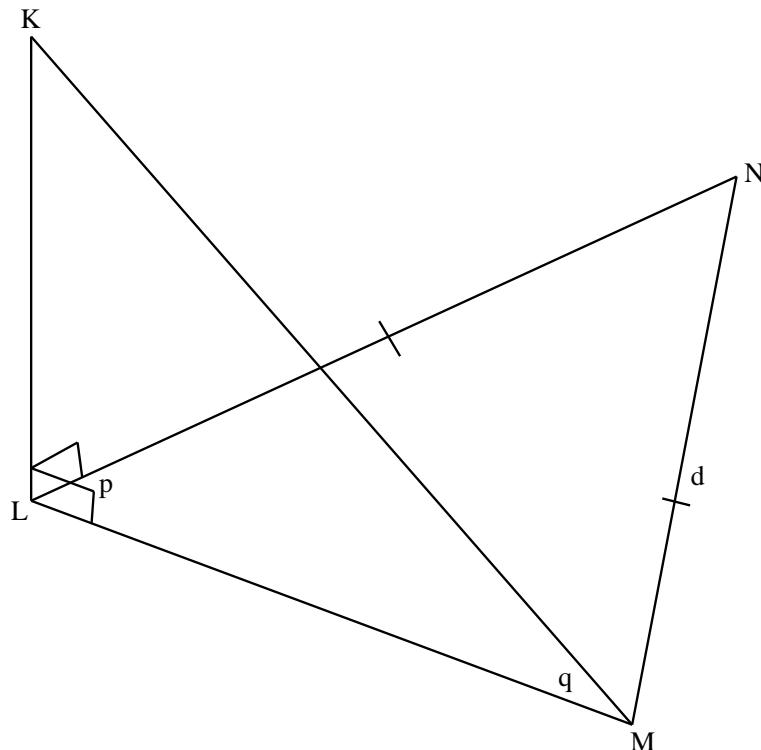
- 6.1 Fumana ivelyu ka b . (1)
- 6.2 Bhala iikho-odineythi zika A, ipoynti ejikayo (turning point) ka g . (2)
- 6.3 Ukuba uPQ upharalel kwi y -ekhziz, fumana iikho-odineythi zika Q. (2)
- 6.4 Bhala i-ikhweyzhini ye asymptote(s) ka $y = \tan b(x + 20^\circ)$ ka $x \in [-180^\circ; 180^\circ]$. (1)
- 6.5 Fumana irheynji ka h ukuba $h(x) = 2g(x) + 1$. (2)
[8]

QUESTION 7

Upoynti L, M no N bakwi pleyini ehorizontali. U KL yivethikhala thawa (vertical tower).

Iengile ye eleveyshin (angle of elevation) ka K ukusuka ku M ngu q° . $\hat{NLM} = p^\circ$;

$NL = NM = d$ no $KL = h$.



7.1 Fumana isayizi ka \hat{LMN} ngokweethem zika p . (2)

$$7.2 \text{ Ngqina ukuba } LM = \frac{d \sin 2p}{\sin p}. \quad (2)$$

$$7.3 \text{ Bonisa ukuba } h = 2d \cos p \tan q. \quad (3)$$

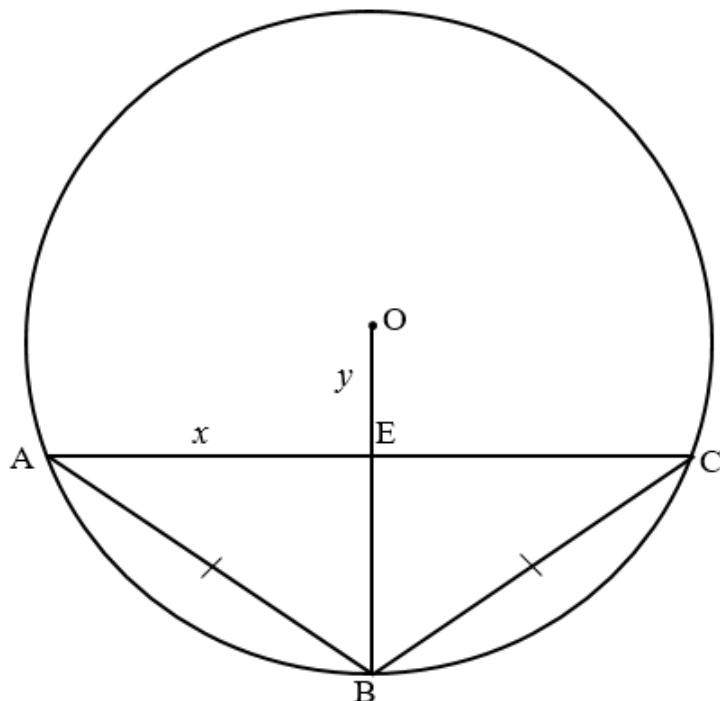
[7]

UMBUZO 8

- 8.1 Gqibezela ingxelo yethiyorem (theorem statement) elandelayo:

*Umgca ozotywe ukusuka embindini wesekile uphephendityula kwikhodi ...
(The line drawn from the centre of a circle perpendicular to a chord ...)* (1)

- 8.2 Kwidayagram engezantsi, isekile ABC enesenta O inikiwe. $OB = 8$ yunithi no $AB = BC = 10$ yunithi. U E yimidi poynti (midpoint) ka AC. U $OE = y$ ze u $AE = x$.



Khaltyhuleytha ngezizathu, ubude buka OE. (5)

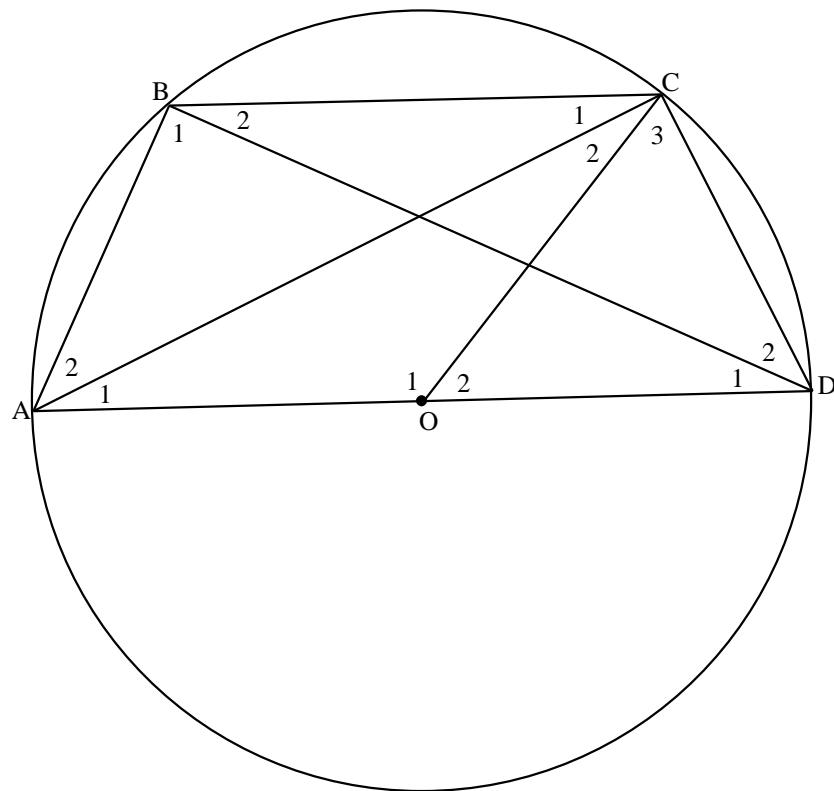
- 8.3 Gqibezela ingxelo yethiyorem (theorem statement) elandelayo:

Iengile ebiyelwe yiakhi kwisenta/embindini wesekile iyi... kwisenta (kwicala elifanayo lekhodi njengesenta).

(The angle subtended by an arc at the centre of a circle is ... at the circle (on the same side of the chord as the centre)).

(1)

- 8.4 Kwidayagram, u O uysenta yesekile ABCD. U AOD yidayamitha aze u OC abe yirhediya (radius). U AB, BC, CD, AC no BD babe yimgca estreythi (straight lines).



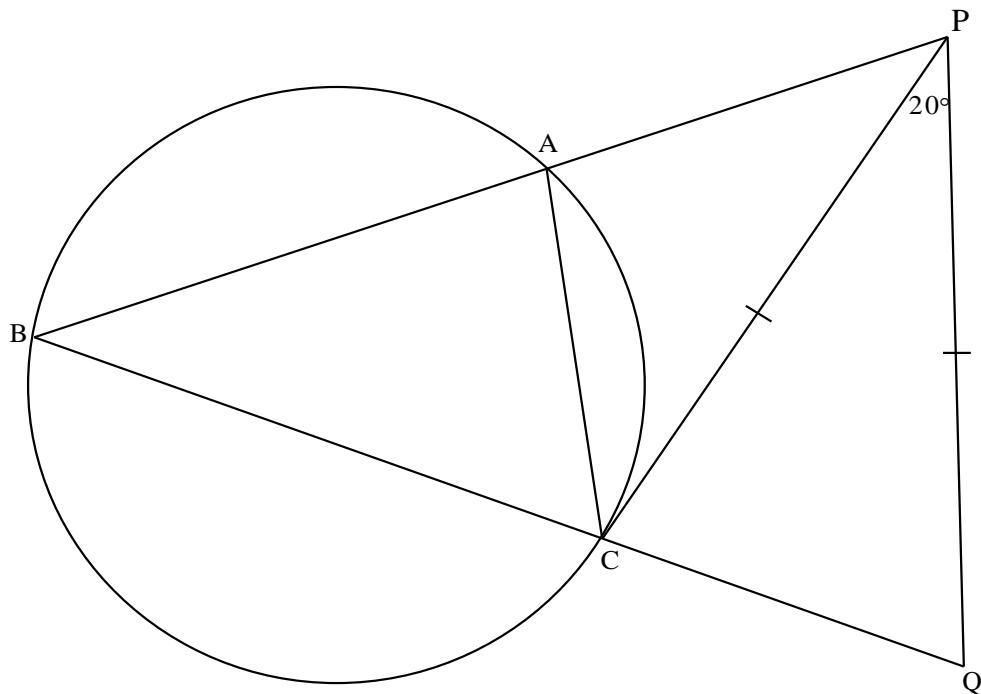
Bhala ngezizathu, i-ikhweyzhin echaza ingxininise uzalwano phakathi kweqela ngalinye lee-engile ezinikiwego.

	IIENGILE	I-IKHWEYZHIN/ UZALANO	ISIZATHU
e.g.	$\hat{M}_3; \hat{P}$	$\hat{M}_3 = 2 \times \hat{P}$	\angle at centre = $2 \times \angle$ at circum.
8.4.1	$\hat{O}_2; \hat{B}_2$		
8.4.2	$\hat{D}_1; \hat{C}_3; \hat{D}_2$		
8.4.3	$\hat{B}_1; \hat{B}_2; \hat{D}_1; \hat{D}_2$		
8.4.4	$\hat{D}_1; \hat{C}_1$		

(8)
[15]

UMBUZO 9

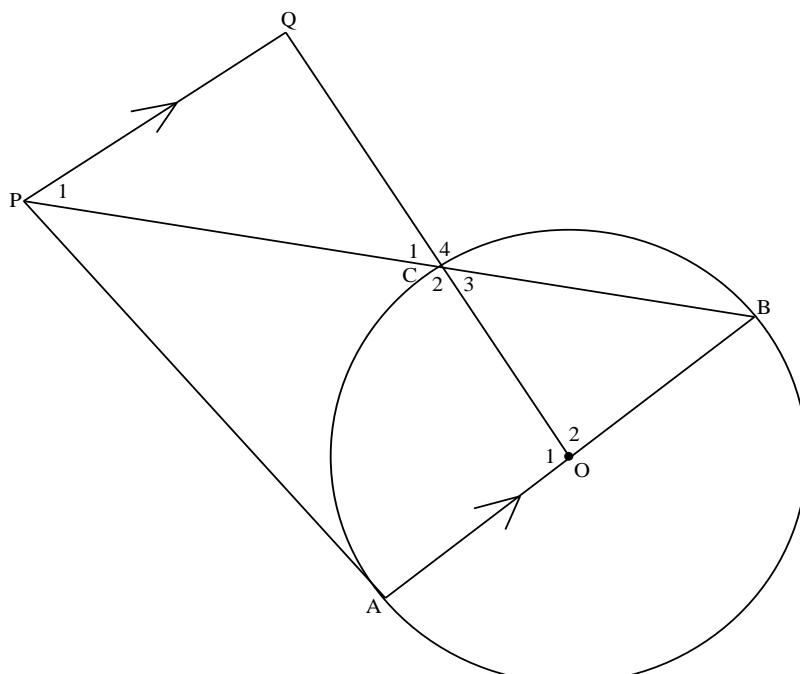
- 9.1 Unikwe ukuba u PC yithanjenti kwisekile ACB; BAP no BCQ bayimigca estreythi.
U $PC = PQ$ and $\hat{CPQ} = 20^\circ$.



Ngqina, nika izizathu, ukuba u BC AKAYIYO idayamitha.

(5)

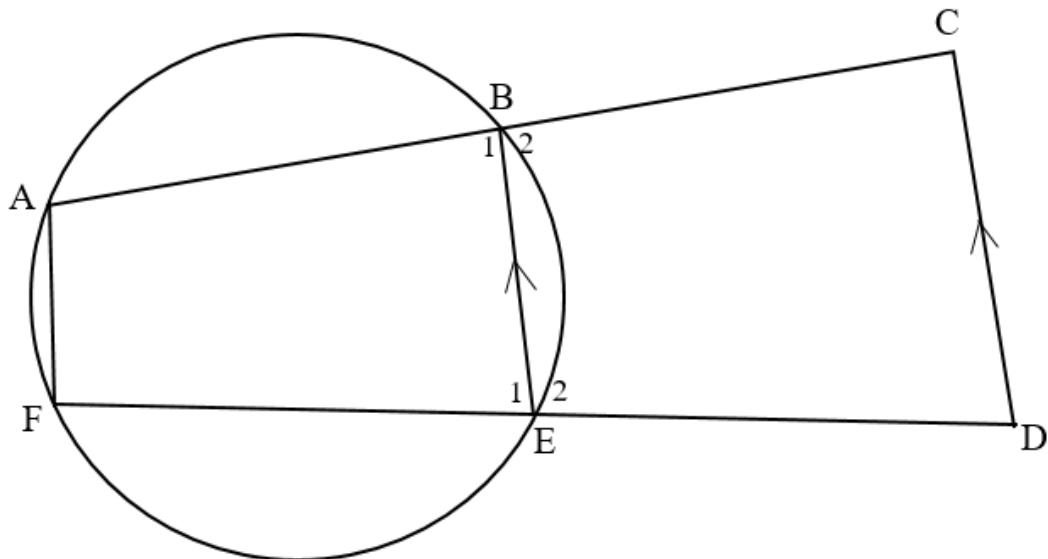
- 9.2 Kwidayagram engezantsi, u O yisenta yesekile ABC. Ithanjenti PA kwisekile ze idayamitha AB zidibane ku A. U OCQ no BCP babeyimigca estreythi. U $PQ \parallel AB$.



Ngqina, unike izizathu, ukuba u $PQ = QC$.

(6)

- 9.3 Kwidayagram engezantsi, iikhodi (chords) AB no FE zezsekile ezinesenta O zitsaliwe ukuya kwiipoyti C no D. U $BE \parallel CD$.



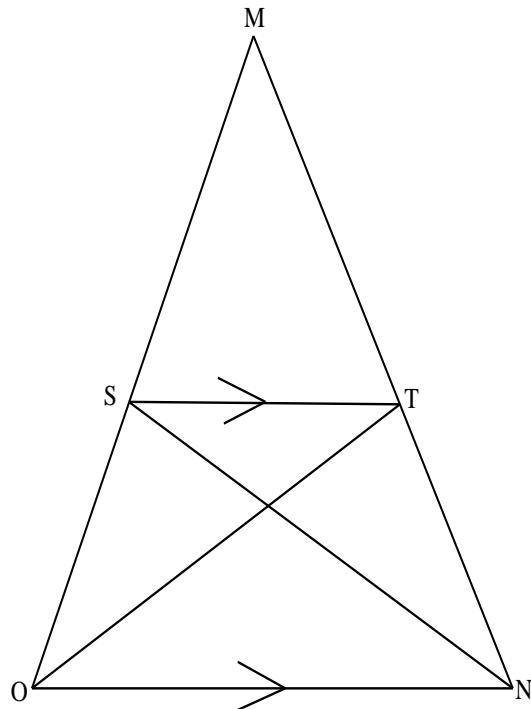
Nggina ukuba u ACDF yi syklikh khwadrilatherali (cyclic quadrilateral).

(5)

[16]

UMBUZO 10

- 10.1 Kwidayagram, $\triangle MON$ uzotyiwe. U S yipoynti ku MO no T yipoynti ku MN kangangokuba u $ST \parallel ON$. U SN no OT bazotyiwe.

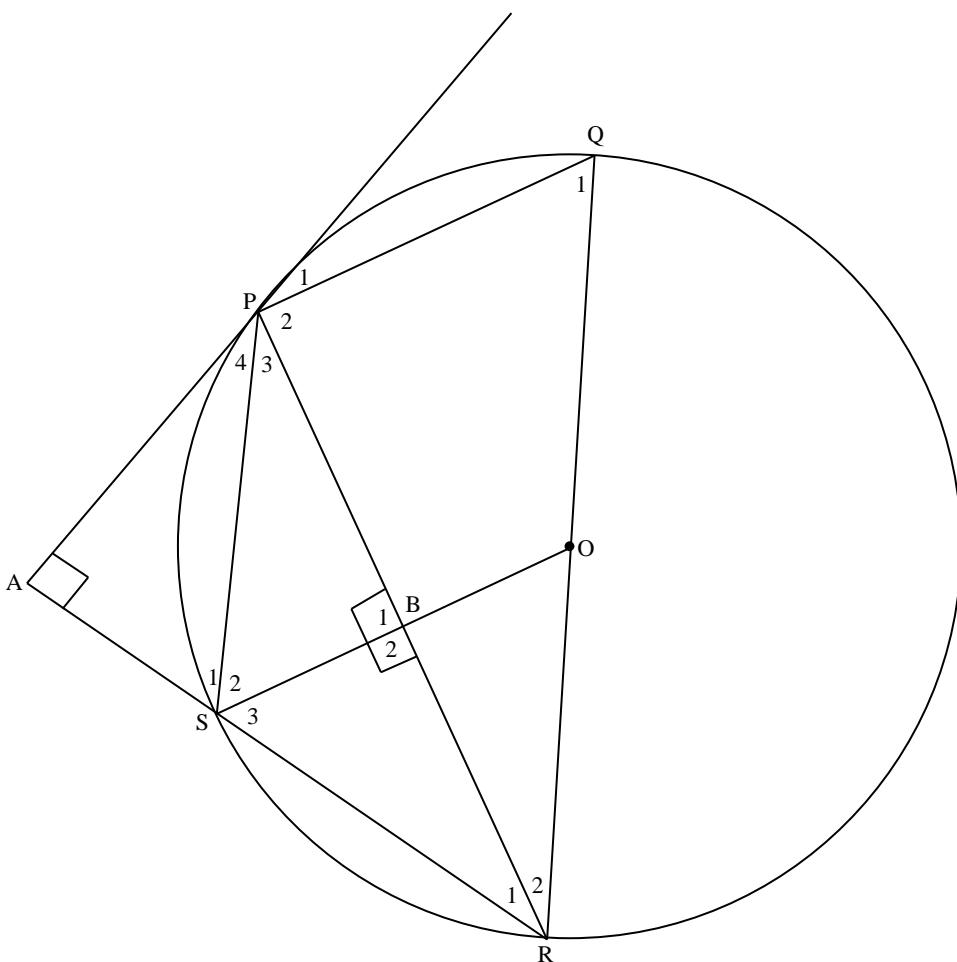


Sebenzisa idayagram kungqina itheorem ethi: a line parallel to one side of a triangle divides the other two sides proportionally. Ngamanye amagama, ngqina

$$\text{ukuba: } \frac{MS}{SO} = \frac{MT}{TN}.$$

(5)

- 10.2 Kwidayagram, u O yisenta yesekile. U PQRS yi sayklikh khwadrilatherali (cyclic quadrilateral). Ithanjenti egqitha kuP i-intasekthe uRS otsalwe ku A. U OB \perp PR no PA \perp AS.



Nggina ukuba:

$$10.2.1 \quad \DeltaAPS \parallel\parallel \DeltaBRS \quad (3)$$

$$10.2.2 \quad AP \cdot RS = BR \cdot PS \quad (1)$$

$$10.2.3 \quad \hat{P}_4 = \hat{R}_2 \quad (4)$$

$$10.2.4 \quad BR \cdot RQ = RS \cdot RP \quad (6) \\ [19]$$

EWONKE: 150

IPHEPHA LOLWAZI: IMATHMATIKA

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \quad A = P(1-ni)$$

$$A = P(1-i)^n$$

$$A = P(1+i)^n$$

$$\sum_{i=1}^n 1 = n \quad \sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r-1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1-r}; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad a^2 = b^2 + c^2 - 2bc \cos A \quad \text{area } \Delta ABC = \frac{1}{2} ab \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$