



Province of the
EASTERN CAPE
EDUCATION

**NATIONAL SENIOR
CERTIFICATE/*NASIONALE
SENIOR SERTIFIKAAT***

GRADE/*GRAAD* 11

NOVEMBER 2020

**TECHNICAL MATHEMATICS P2/*TEGNIESE WISKUNDE V2*
MARKING GUIDELINE/*NASIENRIGLYN*
(*EXEMPLAR/EKSEMPLAAR*)**

MARKS/*PUNTE*: 150

This marking guideline consists of 17 pages./
Hierdie nasienriglyn bestaan uit 17 bladsye.

NOTE:

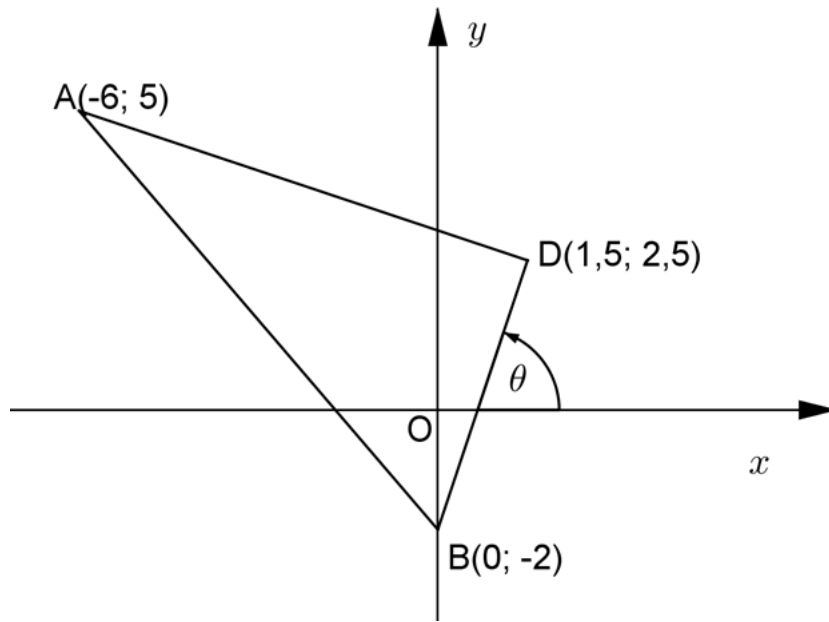
- Continuous accuracy (CA) applies only where indicated in this marking guideline.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

- *Volgehoue akkuraatheid (CA) is slegs van toepassing soos aangedui in hierdie nasienriglyn.*
- *Aanvaarding van waardes/antwoorde om 'n probleem op te los, is onaanvaarbaar.*

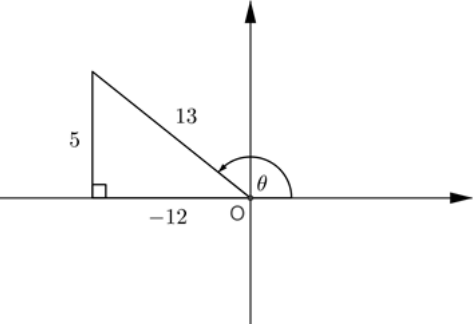
MARKING CODES / NASIENKODES	
M	Method / Metode
A	Accuracy / Akkuraatheid
AO	Answer only / Slegs antwoord
CA	Consistent accuracy / Deurlopende akkuraatheid
F	Formula / Formule
I	Identity / Identiteit
R	Rounding / Afronding
S	Simplification / Vereenvoudiging
ST	Statement / Bewering
RE	Reason / Rede
ST RE	Statement and correct reason / Bewering en korrekte rede
SF	Substitution correctly in correct formula / Korrekte vervanging in die korrekte formule
NPU	No penalty for omitting units / Geen penalisering vir eenhede uitgelaat

QUESTION/VRAAG 1



1.1	$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(-6 - 0)^2 + (5 + 2)^2} = \sqrt{(0 + 6)^2 + (-2 - 5)^2}$ $= \sqrt{85} \text{ or } 9,22$	<p>✓F</p> <p>✓SF A</p> <p>✓CA</p>	(3)
1.2	$m_{AD} = \frac{y_2 - y_1}{x_2 - x_1}$ $= \frac{5 - 2,5}{-6 - 1,5} = \frac{2,5 - 5}{1,5 + 6}$ $= -\frac{1}{3}$	<p>✓F</p> <p>✓SF A</p> <p>✓ gradient / gradiënt CA</p>	(3)
1.3	<p>BD: $y = 3x - 2$</p> <p>∴ $m_{AC} = m_{BD} = 3$ (parallel lines / ewewydige lyne)</p> <p>AC: $y - y_1 = m(x - x_1)$</p> <p>∴ $y - 5 = 3(x + 6)$</p> <p>∴ $y - 5 = 3x + 18$</p> <p>∴ $y = 3x + 23$</p>	<p>✓M BD standard form / standaardvorm</p> <p>✓M gradient / gradiënt</p> <p>✓M substitute point A / vervang punt A</p> <p>✓CA equation / vergelyking</p>	(4)
1.4	<p>$m_{BD} = 3$ (from/vanaf 1.3)</p> <p>$m_{AD} \times m_{BD} = -\frac{1}{3} \times 3$</p> <p>$= -1$</p> <p>∴ $AD \perp BD$ (product of gradients = -1 / produk van gradiënte = -1)</p>	<p>✓M</p> <p>✓R</p>	(2)

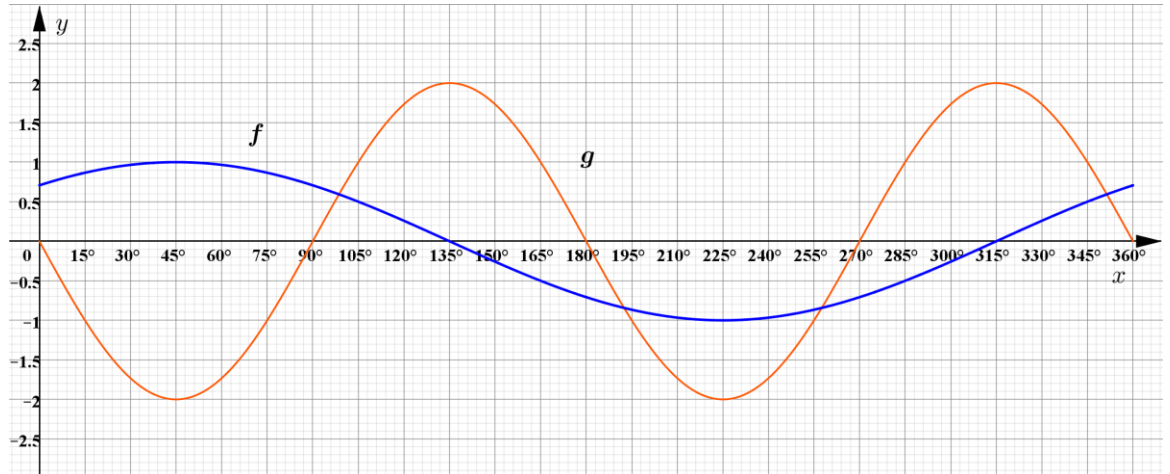
1.5	$M_{AB} = \left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2} \right)$ $= \left(\frac{-6+0}{2}; \frac{5-2}{2} \right)$ $= \left(-3; \frac{3}{2} \right)$	✓F ✓SF A ✓CA	(3)
1.6	$\tan \theta = m_{BD}$ $= 3$ $\theta = \tan^{-1}(3)$ $= 71,565^\circ$	✓M ✓CA value of θ in degree / <i>waarde van θ in grade</i>	(2)
1.7	ACBD is a rectangle (all angles = 90°) <i>ABCD is 'n reghoek (alle hoeke = 90°)</i>	✓A rectangle / <i>reghoek</i> ✓R angles/ <i>hoeke = 90°</i>	(2)
1.8	$CD = \sqrt{85}$ or/of 9,22 (diagonals of rectangle =) <i>(hoeklyne van reghoek =)</i>	✓CA from / vanaf 1.1	(1)
1.9	$\frac{x_C + x_D}{2} = x_{\text{midpt AB}}$ and $\frac{y_C + y_D}{2} = y_{\text{midpt AB}}$ $\frac{x_C + 1,5}{2} = -3$ and $\frac{y_C + 2,5}{2} = 1,5$ $x_C + 1,5 = -6$ and $y_C + 2,5 = 3$ $\therefore x_C = -7,5$ and $y_C = 0,5$	✓M ✓S CA ✓CA x_C ✓CA y_C	(4)
1.10	$AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(-6-1,5)^2 + (5-2,5)^2}$ $= 2,5\sqrt{10}$ $BD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(0-1,5)^2 + (-2-2,5)^2}$ $= 1,5\sqrt{10}$ Area = AD \times BD <i>Oppervlakte = $2,5\sqrt{10} \times 1,5\sqrt{10}$</i> $= 37,5$ sq units / <i>vk eenhede</i>	✓CA length/ <i>lengte</i> AD ✓CA length/ <i>lengte</i> BD ✓M ✓CA area	(4)
			[28]

QUESTION/VRAAG 2			
2.1.1	$\tan 3B + \frac{1}{3} \cos \frac{A}{3}$ $= \tan 3(130,5^\circ) + \frac{1}{3} \cos \frac{310^\circ}{3}$ $= 0,6128 + (-0,07687\dots)$ $\approx 0,5$	<p>✓SF A</p> <p>✓CA</p> <p>AO: Full marks / volpunte</p>	(2)
2.1.2	$-\sec\left(\frac{A}{4} - 2B\right)$ $= -\sec\left(\frac{310^\circ}{4} - 2 \times 130,5^\circ\right)$ $= -\frac{1}{\cos(-183,5^\circ)}$ ≈ 1	<p>✓M $\frac{1}{\cos()}$</p> <p>✓CA</p> <p>AO: Full marks / volpunte</p>	(2)
2.2	 $r = \sqrt{(-12)^2 + 5^2}$ $= \sqrt{144 + 25}$ $= \sqrt{169}$ $= 13$ $20 \operatorname{cosec} \theta - 12 \sec \theta$ $= 20 \times \frac{13}{5} - 12 \times \frac{13}{-12}$ $= 52 + 13$ $= 65$	<p>✓A correct quadrant / korrekte kwadrant</p> <p>✓CA Hypotenuse / skuinssy</p> <p>✓CA $\operatorname{cosec} \theta = \frac{13}{5}$</p> <p>✓CA $\sec \theta = \frac{13}{-12}$</p> <p>✓CA</p>	(5)

2.3	$\frac{\sin(360^\circ - x)\sec(180^\circ + x)}{\tan(180^\circ - x)\operatorname{cosec}(360^\circ + x)}$ $= \frac{(-\sin x)(-\sec x)}{(-\tan x)(\operatorname{cosec} x)}$ $= \frac{(\sin x)(\sin x)}{(\tan x)(-\cos x)}$ $= \frac{(\sin x)(-\tan x)}{\tan x}$ $= -\sin x$	<p>✓A $-\sin x$ ✓A $-\sec x$ ✓A $-\tan x$ ✓A $\operatorname{cosec} x$</p> <p>✓A $\sec x = \frac{1}{\cos x}$ ✓A $\operatorname{cosec} x = \frac{1}{\sin x}$ ✓A $\tan x = \frac{\sin x}{\cos x}$</p> <p>✓CA</p>	(8)
2.4	$\text{LHS/LK} = \frac{1}{1 + \cot^2 x} + \frac{1}{1 + \tan^2 x}$ $= \frac{1}{\operatorname{cosec}^2 x} + \frac{1}{\sec^2 x}$ $= \sin^2 x + \cos^2 x$ $= 1$ $= \text{RHS / RK}$	<p>✓A $\operatorname{cosec}^2 x = 1 + \cot^2 x$ ✓A $\sec^2 x = 1 + \tan^2 x$</p> <p>✓A $\sin x = \frac{1}{\operatorname{cosec} x}$ ✓A $\cos x = \frac{1}{\sec x}$</p> <p>✓A $\sin^2 x + \cos^2 x = 1$</p>	(5)
2.5	$-\frac{2}{3}\sin x + 0,524 = 0$ $-\frac{2}{3}\sin x = -0,524$ $\sin x = 0,786$ <p>Reference / Verwysings $\angle = 51,8^\circ$ $x = 51,8^\circ$ or/of $180^\circ - 51,8^\circ$ $x = 51,8^\circ$ or/of $128,2^\circ$</p>	<p>✓S</p> <p>✓CA Ref / Verw \angle ✓CA $x = 51,8^\circ$ ✓CA $x = 128,2^\circ$</p>	(4)
			[26]

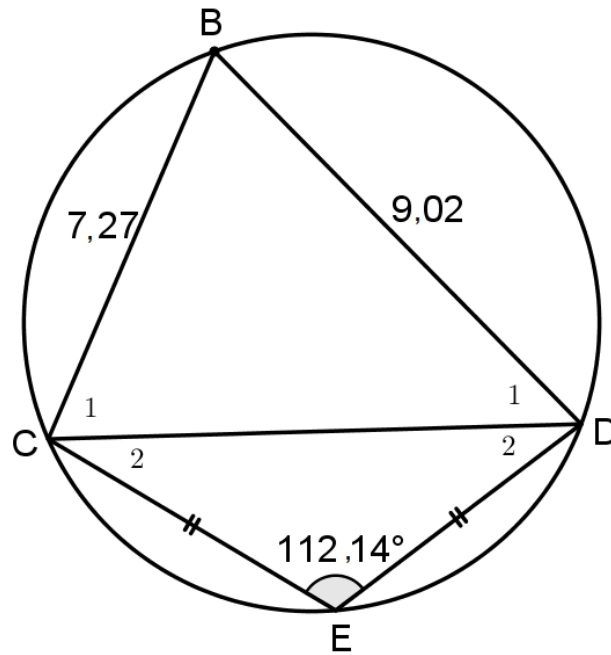
QUESTION/VRAAG 3

3.1



	<p><u>f</u>:</p> <ul style="list-style-type: none"> ✓A endpoints / <i>eindpunte</i> ✓A x-intercepts at / <i>x-afsnitte</i> by 135° & 315° ✓A TP / DP $(45^\circ; 1)$ & $(225^\circ; -1)$ 	<p><u>g</u>:</p> <ul style="list-style-type: none"> ✓A endpoints / <i>eindpunte</i> ✓A x-intercepts at / <i>x-afsnitte</i> by $0^\circ, 90^\circ, 180^\circ, 270^\circ$ & 360° ✓A TP / DP $(45^\circ; -2), (135^\circ; 2), (225^\circ; -2)$ & $(315^\circ; 2)$ 	(6)
3.2	$y \in [-2; 2]$	<ul style="list-style-type: none"> ✓A notation / <i>notasie</i> ✓A end values / <i>eindwaardes</i> 	(2)
3.3	Period = 180°	✓A	(1)
3.4.1	$x = 225^\circ$	✓A	(1)
3.4.2	$x = 45^\circ$	✓A	(1)
3.4.3	$135^\circ \leq x \leq 315^\circ$	<ul style="list-style-type: none"> ✓A notation / <i>notasie</i> ✓A end values / <i>eindwaardes</i> 	(2)
3.4.4	$90^\circ \leq x \leq 135^\circ$	<ul style="list-style-type: none"> ✓A notation / <i>notasie</i> ✓A end values / <i>eindwaardes</i> 	(2)
			[15]

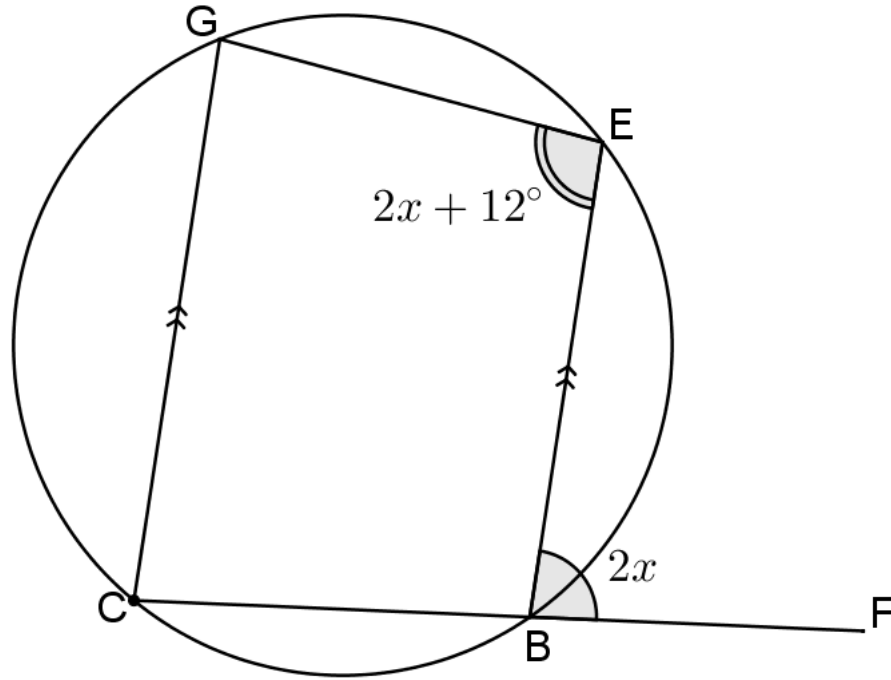
QUESTION/VRAAG 4



4.1	$\hat{B} = 67,86^\circ$ (opp \angle s of cyclic quad) <i>(teenoorst \anglee v kdvk)</i> Area of/van $\triangle BCD = \frac{1}{2} BC \times BD \sin B$ $= \frac{1}{2} \times 7,27 \times 9,02 \sin 67,86^\circ$ $= 30,37$ sq units / vk eenhede	✓ST ✓F ✓SF CA ✓CA	(4)
4.2	$CD^2 = BC^2 + BD^2 - 2BC \times BD \cos B$ $= 7,27^2 + 9,02^2 - 2 \times 7,27 \times 9,02 \cos 67,86^\circ$ $= 84,786\dots$ $CD \approx 9,21$ units / eenhede	✓F ✓SF CA ✓ST ✓CA	(4)

4.3	$\hat{C}_2 = \hat{D}_2 \quad \left(\begin{array}{l} \angle s \text{ opp} = \text{sides} \\ \angle e \text{ teenoor} = \text{sye} \end{array} \right)$ $= \frac{180^\circ - 112,14^\circ}{2} \quad \left(\begin{array}{l} \text{Int } \angle s \text{ of } \Delta \\ \text{Binne } \angle e \text{ van } \Delta \end{array} \right)$ $= 33,93^\circ$ $\frac{CE}{\sin D_2} = \frac{CD}{\sin E}$ $\frac{CE}{\sin 33,93^\circ} = \frac{9,21}{\sin 112,14^\circ}$ $CE = \frac{9,21 \sin 33,93^\circ}{\sin 112,14^\circ}$ $= 5,55 \text{ units / eenhede}$	<p>✓ST</p> <p>✓F</p> <p>✓SF CA</p> <p>✓CA</p>	(4)
			[12]

QUESTION/VRAAG 5			
5.1	Perpendicular bisector / <i>middelloodlyn</i>	✓A	(1)
5.2	$DC = 4,44$ (line from centre \perp to chord <i>loodlyn uit midpt e na koord</i>) $OD^2 = OC^2 - DC^2$ (Pyth) $= 5,63^2 - 4,44^2$ $= 11,9833$ $OD \approx 3,46$ units / <i>eenhede</i>	✓ST ✓RE ✓ST ✓SF CA ✓CA	(5)



<p>5.3</p>	$\hat{C} = 2x \quad \left(\begin{array}{l} \text{corrsp } \angle\text{s; GC P BE} \\ \text{ooreenk } \angle\text{e;GC P BE} \end{array} \right)$ $2x + 12^\circ + 2x = 180^\circ \quad \left(\begin{array}{l} \text{opp } \angle\text{s of cyclic quad} \\ \text{teenoor } \angle\text{e van kdvk} \end{array} \right)$ $4x = 168^\circ$ $x = 42^\circ$ $\hat{E} = 2(42^\circ) + 12^\circ$ $= 96^\circ$ <p style="text-align: center;">OR/OF</p> $\hat{G} = 2x \quad \left(\begin{array}{l} \text{Ext } \angle \text{ of cyclic quad} \\ \text{Buite } \angle \text{ van kdvh} \end{array} \right)$ $2x + 12^\circ + 2x = 180^\circ \quad \left(\begin{array}{l} \text{co-int } \angle; \text{ CGPBE} \\ \text{Ko-binne } \angle\text{e;CGPBE} \end{array} \right)$ $4x = 168^\circ$ $x = 42^\circ$ $\hat{E} = 2(42^\circ) + 12^\circ$ $= 96^\circ$	<p>✓ST RE</p> <p>✓ST ✓RE</p> <p>✓ST</p> <p>✓CA</p> <p>✓ST</p> <p>✓CA size / grootte \hat{E}</p> <p style="text-align: center;">OR/ OF</p> <p>✓ST ✓RE</p> <p>✓ST RE</p> <p>✓ST</p> <p>✓CA</p> <p>✓ST</p> <p>✓CA size / grootte \hat{E}</p>	<p>(7)</p> <p>[13]</p>
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QUESTION/VRAAG 6			
6.1	Are equal / is gelyk aan mekaar	✓A	(1)
6.2			
6.2.1	$\hat{F}_1 = \hat{E}$ ($\angle s$ opp = sides $\angle e$ teenoor = sye) $= \frac{180^\circ - 48^\circ}{2}$ ($\text{Int } \angle s$ of Δ $\text{Binne } \angle e$ van Δ) $= 66^\circ$ $\hat{D} = \hat{E}$ ($\angle s$ in same seg $\angle e$ in dies segm) $= 66^\circ$	✓ST RE ✓ST ✓ST ✓RE	(4)
6.2.2	$\hat{F}_3 = \hat{F}_1$ ($\text{vert opp } \angle s$ $\text{regoorst } \angle e$) $= 66^\circ$ $CD = CF$ ($\text{side opp} = \angle s$ $\text{sye teenoor} = \angle e$)	✓ST RE ✓RE	(2)

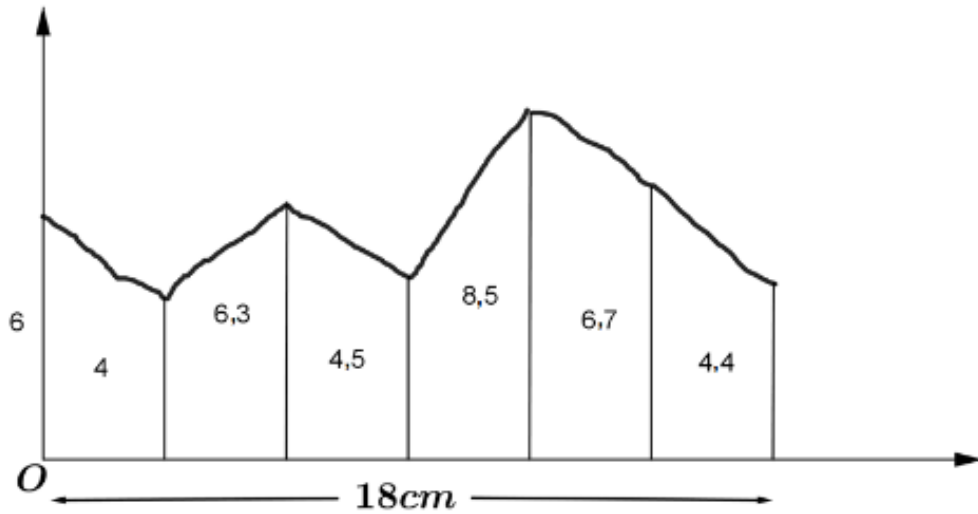
<p>6.2.3</p>	<p>$\hat{CBF} = 19^\circ$ (Ext \angle of Δ Buite \angle van Δ)</p> <p>$\hat{CBE} = 19^\circ + 48^\circ = 67^\circ$</p> <p>$\therefore$ CE is not a diameter (does not subtends \angle of 90° onderspan nie \angle van 90°)</p> <p style="text-align: center;">OR/OF</p> <p>$\hat{CBE} = 67^\circ$ (Int \angles of Δ Binne \anglee van Δ)</p> <p>\therefore CE is not a diameter (does not subtends \angle of 90° onderspan nie \angle van 90°)</p>	<p>✓ST RE</p> <p>✓RE</p> <p>OR/OF</p> <p>✓ST RE</p> <p>✓RE</p>	<p>(2)</p>
<p>6.3</p>			
<p>6.3.1</p>	<p>$\hat{CGA} = 62^\circ$ (\angle at centre = $2 \times \angle$ at circ midpts $\angle = 2 \times$ omtreks \angle)</p>	<p>✓ST ✓RE</p>	<p>(2)</p>
<p>6.3.2</p>	<p>$\hat{DAC} = 62^\circ$ (tan-chord raaklyn – koord)</p>	<p>✓ST ✓RE</p>	<p>(2)</p>
<p>6.3.3</p>	<p>$\hat{ACO} = 28^\circ$ (\angles opp = sides: radii \anglee teenoor = sye: radii)</p>	<p>✓ST ✓RE</p>	<p>(2)</p>

6.3.4	$\hat{G}\hat{C}A = 62^\circ$ $\left(\begin{array}{l} \text{alt } \angle s; CGPDAE \\ \text{verw } \angle e; CGPDAE \end{array} \right)$ $\hat{F} = 118^\circ$ $\left(\begin{array}{l} \text{opp } \angle s \text{ of cyclic quad} \\ \text{teenoor } \angle e \text{ van kdvk} \end{array} \right)$	✓ST RE ✓ST ✓RE	(3)
6.3.5	$\hat{G}\hat{A}E = 62^\circ$ $\left(\begin{array}{l} \text{tan-chord} \\ \text{raaklyn-koord} \end{array} \right)$ $\hat{G}\hat{A}O = 28^\circ$ (Rad \perp tan) OR/OF $\hat{G}\hat{A}E = \hat{C}\hat{G}A = 62^\circ$ $\left(\begin{array}{l} \text{alt } \angle s; CGPDAE \\ \text{verw } \angle e; CGPDAE \end{array} \right)$ $\hat{G}\hat{A}O = 28^\circ$ (Rad \perp tan)	✓ST ✓RE ✓ST ✓RE OR/OF ✓ST ✓RE ✓ST ✓RE	(4)
			[22]

QUESTION/VRAAG 7			
7.1	Surface Area of cylinder = $2\pi r^2 + 2\pi rh$ $56\pi = 2\pi\left(\frac{8}{2}\right)^2 + 2\pi\left(\frac{8}{2}\right)h$ $56\pi = 32\pi + 8\pi h$ $24\pi = 8\pi h$ $h = 3 \text{ m}$	✓F A ✓SF A ✓S ✓CA	(4)
7.2	Label = $2\pi rh$ $= 2\pi\left(\frac{7,5}{2}\right)(11)$ $= 82,5\pi \text{ cm}^2$ $= 259,18 \text{ cm}^2 \times \frac{100 \text{ mm}^2}{1 \text{ cm}^2}$ $= 25\,918 \text{ mm}^2$	✓M ✓SF A ✓S CA ✓CA conversion / herleiding	(4)
7.3.1	$V_{\text{cone}} = \frac{1}{3}\pi r^2 h$ $V_{\text{keel}} = \frac{1}{3}\pi (5)^2 (8)$ $= \frac{200}{3}\pi \text{ cm}^3 \text{ or/of } 209,44 \text{ cm}^3$	✓F A ✓SF A ✓CA	(3)
7.3.2	$V_{\text{cone}} = \frac{1}{3}\pi r^2 h$ $V_{\text{keel}} = \frac{1}{3}\pi (10)^2 (8)$ $= \frac{800}{3}\pi \text{ cm}^3 \text{ or/of } 837,76 \text{ cm}^3$	✓A	(1)
7.3.3	$V_{\text{new cone}} : V_{\text{original cone}} = \frac{800}{3}\pi : \frac{200}{3}\pi$ $V_{\text{nuwe keel}} : V_{\text{oorspronklike keel}} = 4 : 1$	✓M ✓CA	(2)
7.4.1	$h^2 = 16,4^2 - \left(\frac{20}{2}\right)^2 \text{ (Pythagoras)}$ $= 168,96$ $h = 12,998\dots$ $\approx 13 \text{ m}$	✓M ✓CA ✓CA	(3)

7.4.2	$V = \frac{1}{3} Bh$ $= \frac{1}{3} (20 \times 20)(13)$ $= 1733,33 \text{ m}^3$	<p>✓F</p> <p>✓SF A</p> <p>✓CA</p>	(3)
7.5.1	<p>Volume of/van sphere/sfeer = Volume of/van cube/kubus</p> $= 60^3 = 216\,000 \text{ mm}^3$	<p>✓M</p> <p>✓A</p>	(2)
7.5.2	$V_{\text{sphere/sfeer}} = \frac{4}{3} \pi r^3 = 216000$ $\pi r^3 = 162000$ $r^3 = 51566,20156$ $r = \sqrt[3]{51566,20156}$ $= 37,22 \text{ mm}$	<p>✓F A</p> <p>✓M equating / gelykstelling</p> <p>✓ST CA</p> <p>✓ST CA</p> <p>✓CA answer / antwoord</p>	(5)
			[27]

QUESTION/VRAAG 8



$$A_T = a \left(\frac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \dots + o_{n-1} \right)$$

$$= 3 \left(\frac{6 + 4,4}{2} + 4 + 6,3 + 4,5 + 8,5 + 6,7 \right)$$

$$= 3 \text{ cm} (35,2 \text{ cm})$$

$$= 30 \text{ mm} \times 352 \text{ mm}$$

$$= 10\,560 \text{ mm}^2$$

OR/OF

$$A_T = a (m_1 + m_2 + m_3 + \dots + m_{n-1})$$

$$A_T = a \left(\frac{6+4}{2} + \frac{4+6,3}{2} + \frac{6,3+4,5}{2} + \frac{4,5+8,5}{2} + \frac{8,5+6,7}{2} + \frac{6,7+4,4}{2} \right)$$

$$= 3(5 + 5,15 + 5,4 + 6,5 + 7,6 + 5,55)$$

$$= 3 \text{ cm} (35,2 \text{ cm})$$

$$= 30 \text{ mm} \times 352 \text{ mm}$$

$$= 10\,560 \text{ mm}^2$$

- ✓F A
- ✓A value of a / waarde van a
- ✓SF CA
- ✓ST
- ✓CA conversion / herleiding
- ✓CA answer / antwoord
- ✓A units / eenhede

OR/OF

- ✓F A
- ✓A value of a / waarde van a
- ✓SF CA
- ✓ST
- ✓CA conversion / herleiding
- ✓CA answer / antwoord
- ✓A units / eenhede

(7)

[7]

TOTAL/TOTAAL: 150