Vectors in One Dimension – Revision

Vector – physical quantity having magnitude and direction.

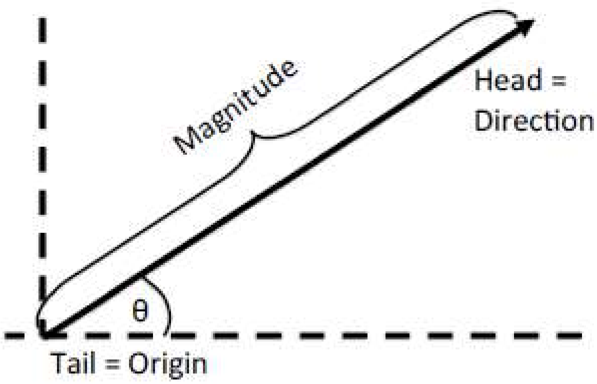
Scalar – physical quantity having magnitude only.

Examples:

Vector	Scalar
Force	Time
	Energy
Weight	Mass
Velocity	Speed
Displacement	Distance
Acceleration	

Graphical Representation of a vector

- Vector is represented by an arrow
- The **length of an arrow** represents the **size (magnitude)** of the vector
- The **arrow-head** represents the **direction of the vector**.



Direction of a horizontal or vertical vector

A positive sign (+) or a negative sign (–) is used to indicate the direction of a vector that are either horizontal or vertical. For each example you must select the sign.

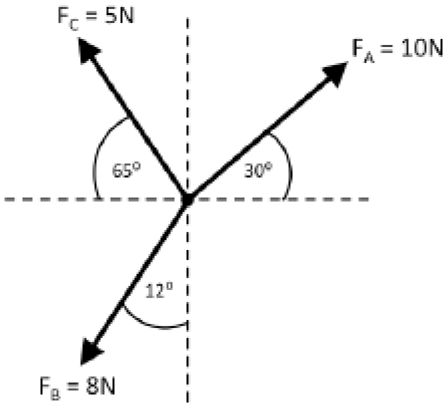
Examples

a) right is +  5 N

b) left is – - 3 N

Three methods to describe the direction of a vector that is not horizontal or vertical

On a graph



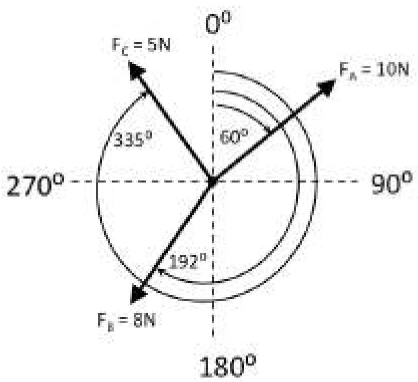
F_A : 10 N at 30° above the positive x- axis (horizontal axis)

F_B : 8 N at 12° left of the negative y- axis (vertical axis)

F_C : 5 N at 65° above the negative x- axis (horizontal axis)

Bearing

- Only for vectors in the horizontal plane i.e parallel to the surface of the Earth
- Use North as 0° and always measure **clockwise**

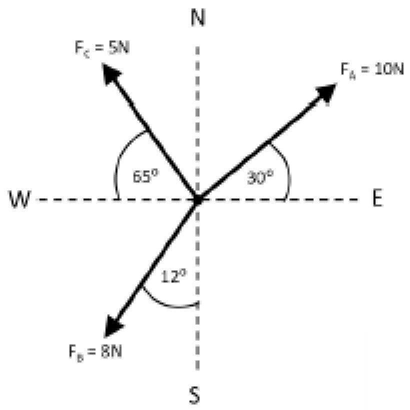


F_A : 10 N on a bearing of 60°

F_B : 8 N on a bearing of 192°

F_C : 5 N on a bearing of 335°

Compass (Cardinal points or directions)



F_A : 10 N at 30° North of East

F_B : 8 N at 12° West of South

F_C : 5 N at 65° North of West

RESULTANT OF VECTORS

Define a resultant as the vector sum of two or more vectors, i.e. a single vector having the same effect as two or more vectors together.

- Resultant vector is greatest when vectors are in the same directions
- Resultant vector is smallest when vectors are in the opposite directions

1. Two vectors acting in the same direction :(one dimension)

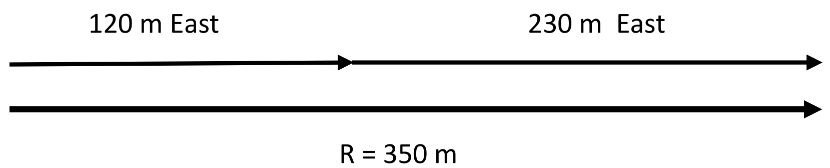
A girl walks 120m due East and then 230m in the same direction. What is her resultant displacement?

By calculation:

Sign of direction: Take to East to be +

R = 120m + 230m = 350m East

By construction:

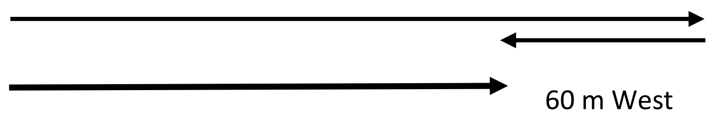


2. Two vectors acting in opposite direction (one dimension)

A boy walks 210m due East. He then turns and walk 60m due West. Determine his resultant displacement.

By calculation: (taking East as positive)

R = 210m + (- 60m) = 150m East



3. Multiple vectors acting in different directions (one dimension)

Determine the resultant(net) force when 8 N force acts to the right, a 10 N force acts to the right, a 25 N force acts to the left and a 12 N force acts to the left

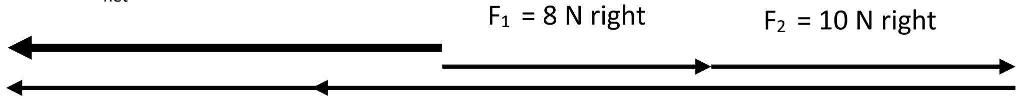
Let to the right be **positive**

$$F_{net} = F_1 + F_2 + F_3 + F_4$$

$$F_{net} = 8 + 10 + (-25) + (-12)$$

$$F_{net} = -19\ N$$

$$F_{net} = 19\ N\ left$$

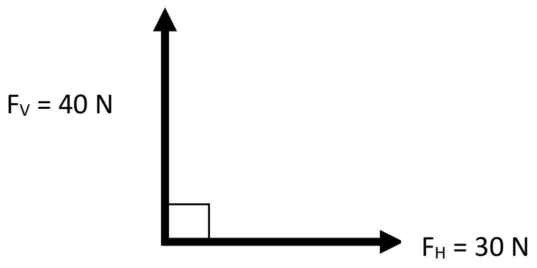


Vectors in Two Dimension

Resultant of perpendicular vectors

- **Perpendicular vectors** are at right angles to each other.
- A horizontal force of 30 N and a vertical force of 40 N that act on an object are an example of two forces that are perpendicular to each other.

Diagram



Adding co-linear vectors

- Vectors that act in one dimension are called co-linear vectors
- The net x-component (R_x) is the sum of the vectors parallel with the x-direction: $R_x = R_{x1} + R_{x2}$
- The net y-component (R_y) is the sum of the vectors perpendicular to the x-direction: $R_y = R_{y1} + R_{y2}$

Worked Example

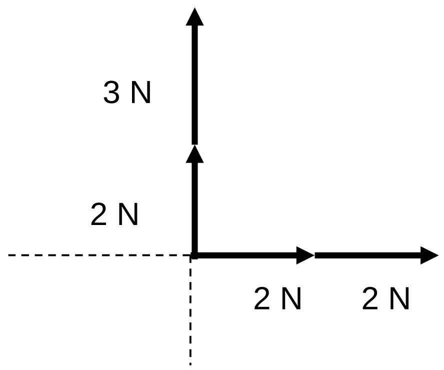
Two forces of 3N and 2N apply an upward force to an object. At the same time two forces each of 2N act horizontally to the right. Find the resultant force acting on the object.

Step 1: Draw a diagram and calculate the net vertical and net horizontal forces

$R_y = R_{y1} + R_{y2}$

$R_y = 2 + 3$

$R_y = 5\ N\ upwards$

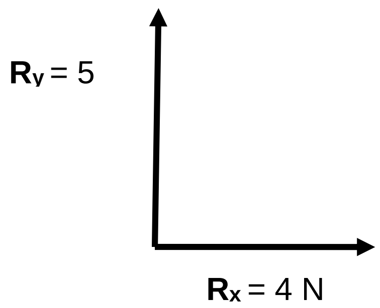


$R_x = R_{x1} + R_{x2}$

$R_x = 2 + 2$

$R_x = 4\ N\ right$

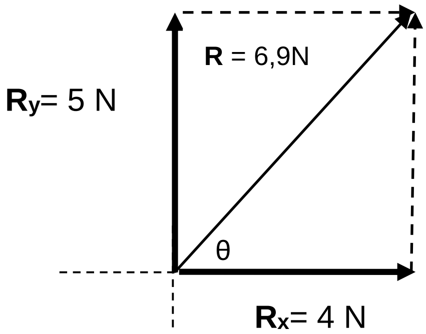
Step 2: Graphical representation of R_x and R_y



Step 3: To find resultant (R) of the above vectors, one can using **tail-to-tail** drawing of vectors

Tail to tail method or Parallelogram:

Note: When vectors are drawn tail-to-tail, a parallelogram must be completed in order to determine their resultant.



- Phythagoras theorem is used to calculate the magnitude of the resultant.
- Considering the vector diagram above we can use Pythagoras theorem as follows:

$$R^2 = {R_x}^2 + {R_y}^2$$

$$R^2 = 4^2 + 5^2$$

$$R = \sqrt{4^2 + 5^2}$$

$$R = 6.40\ N$$

- Use trigonometry to find the direction of the resultant as follows:

$$\tan\theta = \frac{R_y}{R_x} = \frac{5}{4}$$

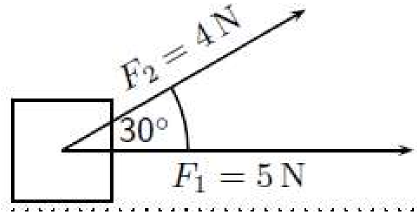
$$\therefore\ \theta = 51{,}34°$$

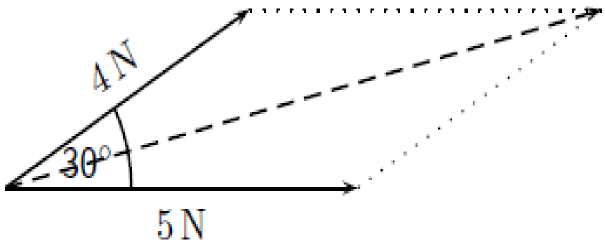
Worked Example:

A force of F_1 = 5N is applied to a block in a horizontal direction. A second force F_2 = 4N is applied to the object at an angle of 30º above the horizontal. Determine the resultant of the two forces**, by accurate scale drawing.**

Step 1: Draw rough sketches of the vector diagrams:

Note: Forces are NOT perpendicular





Step 2: Choose the suitable scale. e.g 1cm : 1N

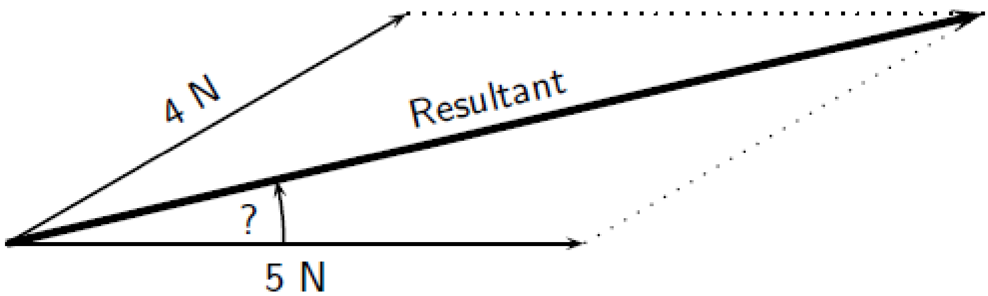
Step 3: Draw the first vector (F_1) on the horizontal, according to the scale.

Step 4: Draw the second scaled vector (F_2) 30º above the horizontal.

Step 5: Complete the parallelogram and draw the diagonal (which is the resultant)

Step 6: Use the protractor to measure the angle between the horizontal and the resultant.

Step 7: Apply scale and convert the measured length to the actual magnitude.



The resultant is 8,7N, 13,3º above the horizontal.

GRAPHICAL DETERMINATION OF THE RESULTANT VECTOR

Tail-to-head method is used to find the resultant of two or more consecutive vectors (vectors that are successive)

Steps to be followed:

- Choose the suitable scale e.g. 10mm : 10N
- Accurately draw the first vector as an arrow according to chosen scale and in the correct direction
- Draw the second vector by placing the tail of the second vector at the tip of the first vector { tail – to – head method}
- Complete the diagram by drawing the resultant from the tail of the first vector to the head of the last vector.
- Make sure that you measure the angles correctly with a protractor.
- Always add arrow heads to vectors to indicate the direction.
- Measure the length and direction of the resultant vector.

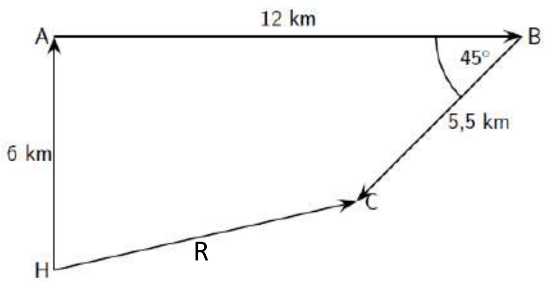
Use the scale to determine the real magnitude of the resultant.

Worked Example 1:

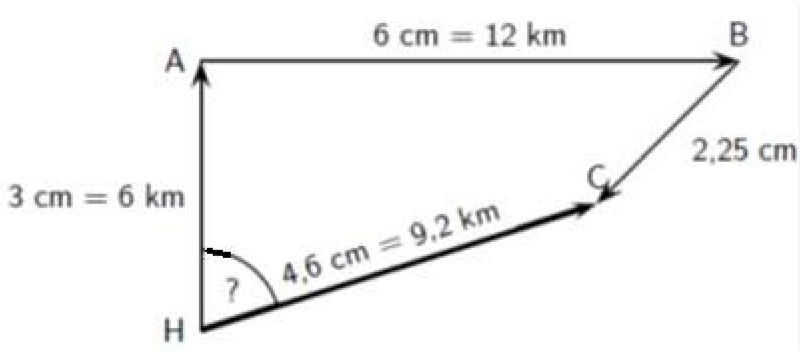
A ship leaves a harbour H and sails 6km north to port A. From here the ship travels 12 km east to port B, before sailing 5,5 km at 45^0 south-west to port C.

Determine the ship's restaurant displacement using the tail-to-head technique.

Rough sketch:



Using a scale 1cm : 2km, the accurate drawing of vectors is:



Measure the angle between the North line and the resultant with a protractor to find that the direction of the resultant displacement:

Resultant displacement of the ship is 9,2 km on a bearing of 72, 3º.

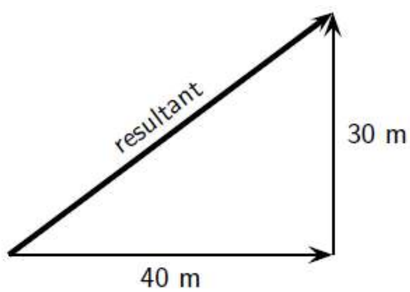
Example 2:

A man walks 40m East, then 30m North. Use a scale of 1 cm: 10 m and answer the following questions:

1. What was the total distance he walked?
2. Determine by construction his resultant displacement?
3. Calculate determine the direction of the resultant.
4. Calculate the magnitude of resultant displacement

Solutions:

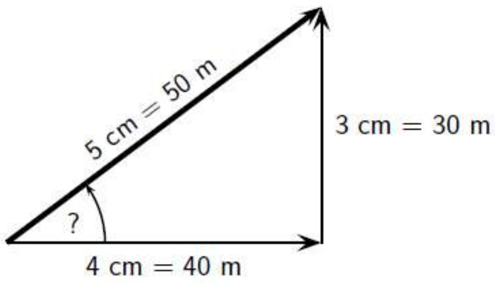
1. Rough sketch



Total distance = 40m + 30m

= 70m

2. Scale: 1cm : 10m



The resultant is 50m, 37º from the horizontal

3. $\tan \Phi = \frac{30}{40}$

   $\Phi = 36,87^\circ$

4. $R^2 = x^2 + y^2$

   $= 40^2 + 30^2$

   $= 2500$

   $R = 50m$

THE TRIANGLE RULE FOR FORCES IN EQUILIBRIUM

Closed vector diagram

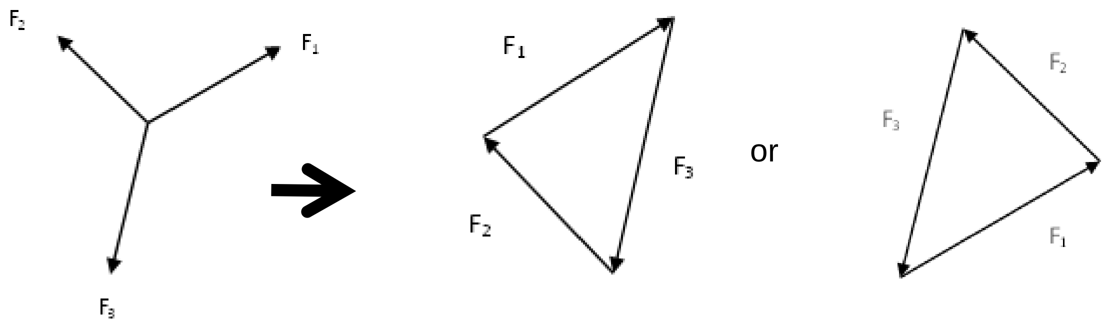
- When drawing force vectors **at equilibrium**, a closed quadrilateral such as triangle (closed vector diagram) will be obtained. In that case, **the resultant is zero and all vectors are drawn from head-to-tail.**
- The forces F_1, $F_2$and F_3 act on the same object and keep it in equilibrium so that the object does not move, or continues moving with the constant velocity. (No change in motion occurs).
- These three forces can be shifted to form a close triangle, where the sides of the triangle still represent the magnitude and direction of the forces.

The triangle rule for forces in equilibrium is as follows:

When three forces acting at the point are in equilibrium, they can be represented in both magnitude and direction by the three sides of a triangle taken in order.

- The triangle is formed because the three forces are in equilibrium.
  $F_1+F_2+F_3=0$

For example: when the forces F_1, F_2 and F_3 are in equilibrium, they can be represented by a closed triangle as:

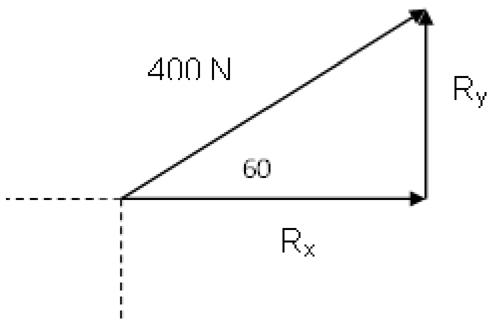


RESOLUTION OF A VECTOR INTO ITS PARALLEL AND PERPENDICULAR COMPONENTS

- The process of breaking down the vector quantity into its components that are at right angles to each other is known as ***resolving a vector*** into its components.

Worked Example

A force of 400N acts at an angle 60^0 to the horizontal.



Horizontal component:

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos 60^o = \frac{R_x}{400\ \text{N}}$$

$$R_x = 400\text{N} \cdot \cos 60^o$$

$$R_x = 200\text{N}$$

Vertical component:

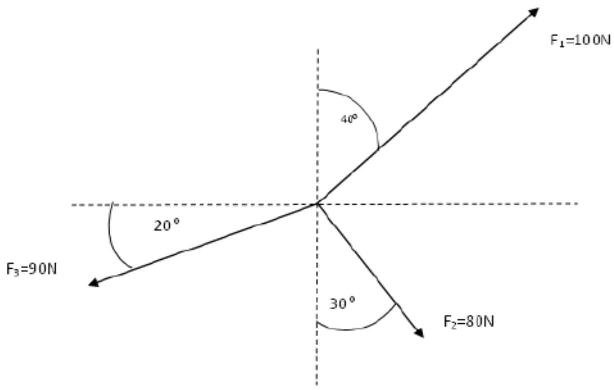
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 60^o = \frac{R_y}{400\ \text{N}}$$

$$R_y = 400\text{N} \cdot \sin 60^o$$

$$R_y = 346{,}41\text{N}$$

Finding resultant of vectors acting at angles (using component method)



Step 1: Find horizontal and vertical components of each force.

Components of F_1:

Horizontal (F_x)

(F_x) = $F_1 \sin\Phi$

= 100 sin40º

= 64,28N (right)

Vertical (F_y)

(F_y) = $F_1 \cos\Phi$

= 100 cos40º

= 76,60N (up)

Components of F_2:

Horizontal (F_x)

(F_x) = $F_2 \sin\Phi$

= 80 sin30º

= 40N (right)

Vertical (F_y)

(F_y) = $F_2 \cos\Phi$

= 80 cos30º

= 69,28N (down)

Components of F_3:

Horizontal (F_x)

(F_x) = $F_3 \cos\Phi$

= 90 cos20º

= 84,57N (left)

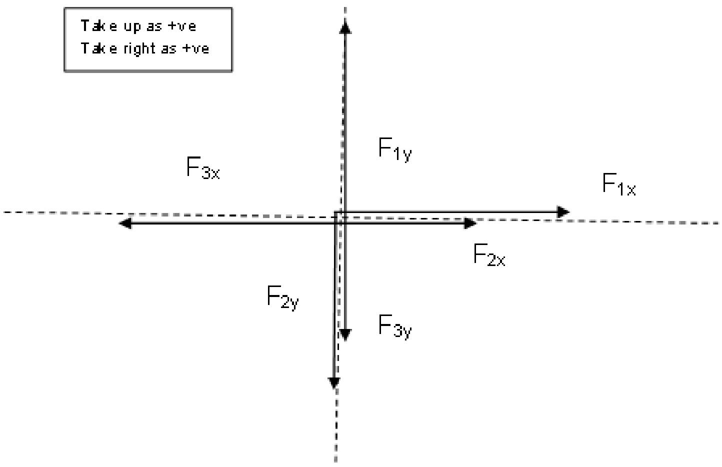
Vertical (F_y)

(F_y) = $F_3 \sin\Phi$

= 90 sin20º

= 30,78N (down)

Step 2: Hence, the new situation is:



Step 3:

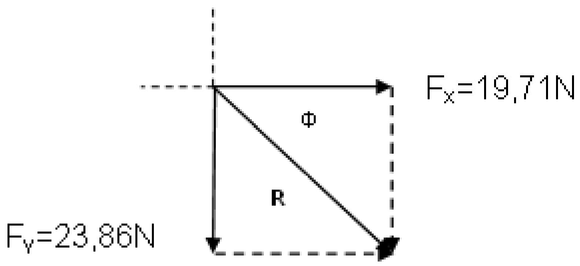
Thus, the sum of horizontal components = 64,28+40+ (-84,57)

= 19,71N to the right

And the sum of vertical components = 69,28+30,78+ (-76,20)

= 23,46N downwards

Step 4:



The resultant of the two vectors (at right angles to each other) is:

$\mathbf{R}^2 = R_x^2 + R_y^2$

$= 19,71^2+23,86^2$

$= 30,64N$

$\tan \Phi = 23,86/19,71$

$\Phi = 50,47^o$

Thus, the resultant of the three forces is 30,95N in a bearing of 50,47º from the horizontal.

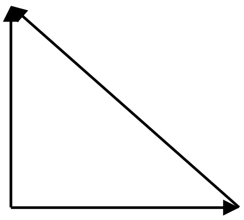
Exam questions

QUESTION 1: MULTIPLE-CHOICE QUESTIONS

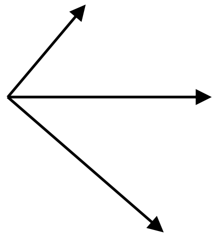
Four options are provided as possible answers to the following questions. Each question has only ONE correct answer. Write only the letter (A–D) next to the question number (1.1–1.10) in the ANSWER BOOK, for example 1.11 D.

1.1 Consider the following vector diagrams. Which ONE of these vector diagrams represents a zero resultant?

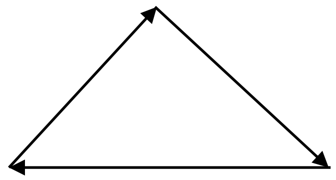
A



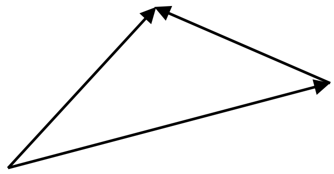
B



C



D



(2)

1.2 If the resultant of two forces acting at a point is zero, the forces …

A are of equal magnitude and act perpendicular to each other.

B are of different magnitudes, but act in opposite directions.

C are of equal magnitude and act in the same direction.

D are of equal magnitude, but act in opposite directions. (2)

1.3 Two forces of magnitudes 3 N and 4 N respectively act on a body. The maximum magnitude of the resultant of these forces is …

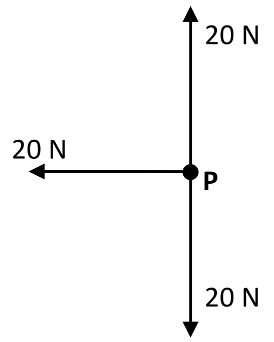
A 12 N.

B 7 N.

C 5 N.

D 1 N. (2)

1.4 Three forces of magnitude 20 N each act on object **P** as shown below.



The resultant force on object **P** is …

A zero.

B 20 N to the left.

C 20 N upwards.

D 20 N downwards. (2)

1.5 Two forces of magnitudes 15 N and 20 N act at a point on an object. Which one of the following magnitudes CANNOT be the resultant of these forces?

A. 35 N

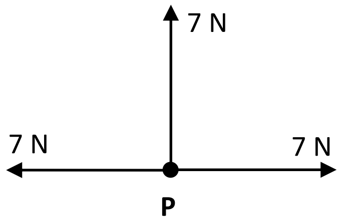
B. 10 N

C. 4 N

D. 18 N

(2)

1.6 Three forces, each of magnitude 7 N, act on object **P** as shown.



The resultant force on object **P** is …

A zero.

B 7 N to the left.

C 7 N upwards.

D 7 N downwards. (2)

1.7 Two forces of magnitude 50 N and 70 N respectively act on a body. The maximum magnitude of the resultant force on the body is ...

A 20 N.

B 60 N.

C 120 N.

D 140 N. (2)

1.8 Two forces of magnitudes 8 N and 6 N are added to each other. Which of the following values CANNOT be a resultant of these two forces?

A 2 N

B 3 N

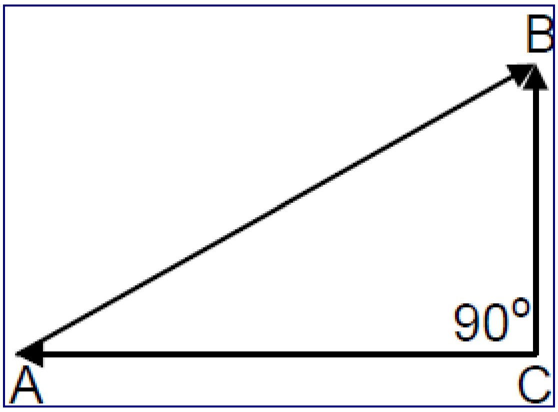
C 14 N

D 16 N (2)

1.9 You can replace two forces, P and Q, with a single force of 7 N. If the magnitude of force P is 3 N, which one of the following can be the magnitude of force Q?

A 2 N
B 3 N
C 8 N
D 13N (2)

1.10 Consider the following vector diagram:



The vector which represents the resultant of the other two, is …

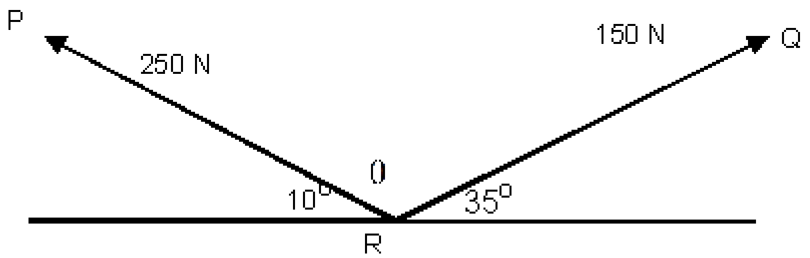
A. AB.
B. AC.
C. CB.
D. BA.

(2)

STRUCTURED QUESTION

QUESTION 1 (Grade11 KZN MARCH 2015)

The diagram below shows TWO forces **P** and **Q** of magnitude 250 N and 150 N respectively acting at a point **R**.



1.1 Calculate the horizontal and vertical components of vector P. (4)

1.2 Calculate the vector sum of horizontal components of P and Q. (3)

1.3 The vector sum of the vertical components of these forces is 129,45 N.

Using the vector sums of the horizontal and vertical components of P and Q, draw a labeled force vector diagram to show the resultant force acting on the point R. (3)

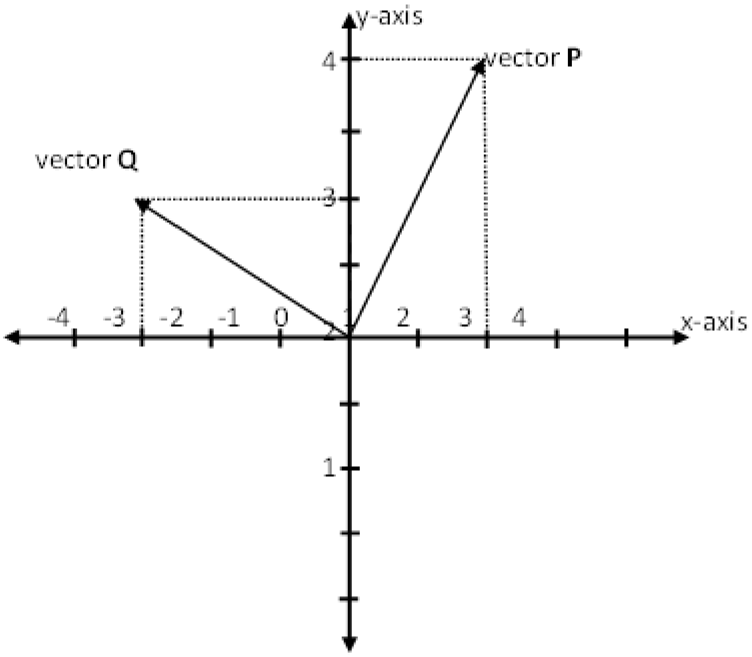
1.4 Calculate the magnitude of the resultant of forces P and Q. (3)

1.5 Calculate the direction (measured clockwise from the positive Y axis) of the resultant of vectors P and Q. (3)

1.6 If vector P was fixed but the direction of vector Q could be changed, for which value of ϴ will the resultant force have a maximum value? (1)

QUESTION 2 (FS CONTROL TEST TERM 1 – 2014)

Force vectors **P** and **Q** were drawn to scale on the Cartesian plane shown below.



2.1. Define the term *resultant* of a vector. (2)

2.2. From the graph, without using a scale drawing, CALCULATE the (no units are required):

2.2.1 Magnitude of the horizontal component of vector **P** (1)

2.2.2 Magnitude of the horizontal component of the resultant of vectors **P** and **Q** (2)

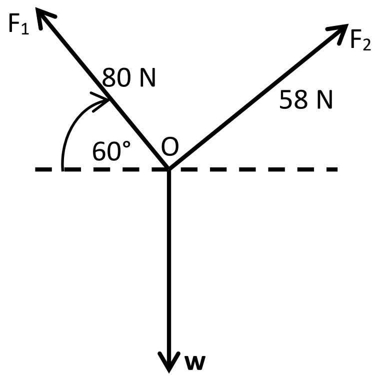
2.2.3 Magnitude of the vertical component of the resultant of vector **P** and **Q** (2)

2.2.4 Resultant of vectors **P** and **Q**. (6)

[13]

QUESTION 3(Fs CONTROL TEST TERM 1 – 2015)

Three forces, $\mathbf{F_1}$, $\mathbf{F_2}$ and **w**, act on point **O** as shown in the diagram below.



3.1 Define the term *resultant of forces*. (2)

3.2 By means of an accurate scale drawing, determine the vertical component of $\mathbf{F_1}$. Use a scale where 10 N is represented by 10 mm. (5)

3.3 The horizontal and vertical components of $\mathbf{F_2}$ are equal to 40 N and 42 N respectively.

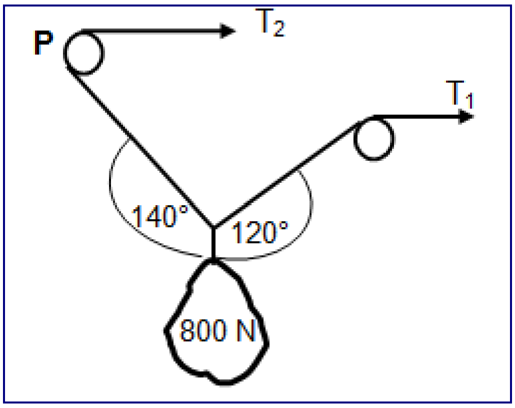
3.3.1 Prove with calculations that the horizontal components of the forces are in equilibrium. (3)

3.3.2 Calculate the magnitude and direction of force **w**. (2)

[12]

QUESTION 4

The diagram below shows a rope and pulley arrangement of a device being used to lift an 800 N object. Assume that the ropes are light and inextensible and also that the pulley is light and frictionless.



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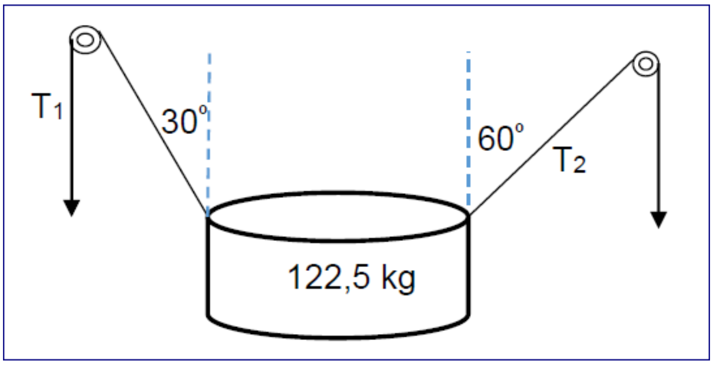
2.1 Magnitudes of the tensions **T_1** and **T_2** (7)

2.2 Magnitude and direction of the reaction force at pulley **P** (4)

QUESTION 5 (EC NOV 2016)

The diagram below shows a rope and pulley system of a device being used to lift a 122,5 kg container upwards at a constant velocity. Assume that the ropes are light and inextensible and the pulley is frictionless.



5.1 Calculate the weight of the container. (3)

5.2 The system is moving upwards at a constant velocity as indicated above.

5.2.1 Draw a vector diagram of all forces acting on the container and indicate the angles represented in the diagram. (5)

5.2.2 Determine the magnitudes of the forces **T1** and **T2**. (7)

5.3 The system is moving upwards at a constant velocity.

5.3.1 What does the statement above tell us about forces acting on the container? (2)

5.3.2 Which Newton's law support your answer in QUESTION 5.3.1? (2)