

# CHAPTER 1

*Vectors In Two Dimensions*

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# 1 INTRODUCTION

In Grade 10 you learnt about vectors in one dimension. Now we will take these concepts further and learn about vectors in two dimensions as well as components of vectors.

## DEFINITION: Vector

*A vector is a physical quantity with both magnitude and direction.*

As a very short recap, a vector has both a magnitude and a direction. There are many physical quantities, like forces, that are well described by vectors (called or known as vector quantities). We often use arrows to represent vectors visually because the length of the arrow can be related to the magnitude and the arrowhead can indicate the direction. We will talk about the head, tail and magnitude of a vector when using arrows to represent them. Figure 1 shows a vector (the arrow). The magnitude is indicated by the length and the labels show the the tail and the head of the vector. The direction of the vector is indicated by the direction in which the arrow is pointing.

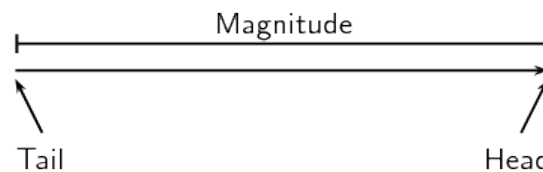


Figure 1: A diagram showing the different parts of a vector

When we write the symbol for a physical quantity represented by a vector we draw an arrow over it to signify that it is a vector. If the arrow is left out then we are referring only to the magnitude of the vector quantity.

## KEY CONCEPTS

- Theorem of Pythagoras - Mathematics, Grade 10 , Analytical geometry
- Units and unit conversions - Physical Sciences, Grade 10 Science skills
- Equations - Mathematics, Grade 10 , Equations and inequalities
- Trigonometry - Mathematics, Grade 10 , Trigonometry
- Graphs - Mathematics, Grade 10 , Functions and graphs

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## 2 RESULTANT OF PERPENDICULAR VECTORS

In Grade 10 you learnt about the resultant vector in one dimension, we are going to extend this to two dimensions. As a reminder, if you have a number of vectors (think forces for now) acting at the same time you can represent the result of all of them together with a single vector known as the resultant. The resultant vector will have the **same** effect as all the vectors adding together.

We will focus on examples involving forces but it is very **important** to remember that this applies to all physical quantities that can be described by vectors like forces, displacements, accelerations, velocities and more.

### 2.1 Vectors on the Cartesian plane

The first thing to make a note of is that in Grade 10 we worked with vectors all acting in a line, on a single axis. We are now going to go further and start to deal with two dimensions. We can represent this by using the Cartesian plane which consists of two perpendicular (at a right angle) axes. The axes are a  $x$ -axis and a  $y$ -axis. We normally draw the  $x$ -axis from left to right (horizontally) and the  $y$ -axis up and down (vertically).

We can draw vectors on the Cartesian plane. For example, if we have a force,  $\vec{F}$ , of magnitude 2 N acting in the positive  $x$ -direction we can draw it as a vector on the Cartesian plane as shown in Figure 2.

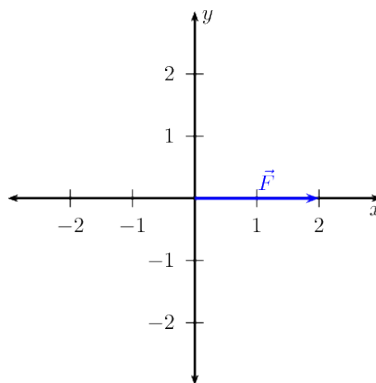


Figure 2: A vector with a magnitude of 2 N in the positive  $x$ -direction on a Cartesian plane

Notice that the length of the vector as measured using the axes is 2, the magnitude specified. A vector doesn't have to start at the origin but can be placed anywhere on the Cartesian plane. Where a vector starts on the plane doesn't affect the physical quantity as long as the magnitude and direction remain the same. That means that all of the vectors in the diagram below can represent the same force. This property is known as *equality of vectors*.

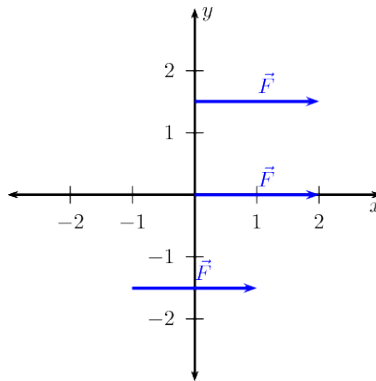


Figure 3: The same force at multiple starting locations

In Figure 3 the vectors have the same magnitude because the arrows are the same **length** and they have the same **direction**. They are all parallel to the  $x$ -direction and parallel to each other.

This applies equally in the  $y$ -direction. For example, if we have a force,  $\vec{F}$ , of magnitude 2,5 N acting in the positive  $y$ -direction we can draw it as a vector on the Cartesian plane as in Figure 4.

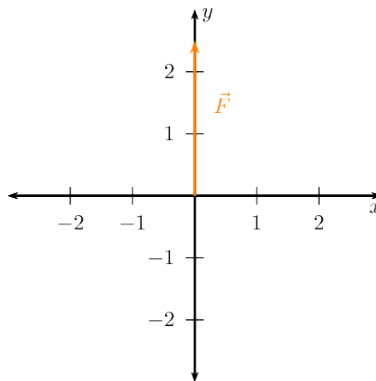


Figure 4: A 2,5 N vector in the positive  $y$ -direction

Just as in the case of the  $x$ -direction, a vector doesn't have to start at the origin but can be placed anywhere on the Cartesian plane. All of the vectors in Figure 5 below can represent the same force.

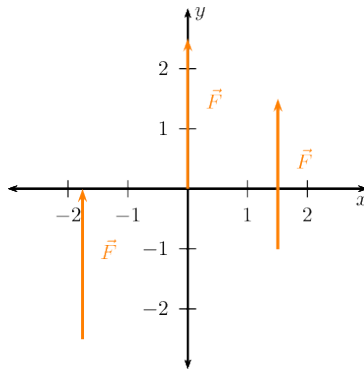


Figure 5: The same force at multiple starting locations

The Figure 6 shows an example of four force vectors, two vectors that are parallel to each other and the  $y$ -axis as well as two that are parallel to each other and the  $x$ -axis.

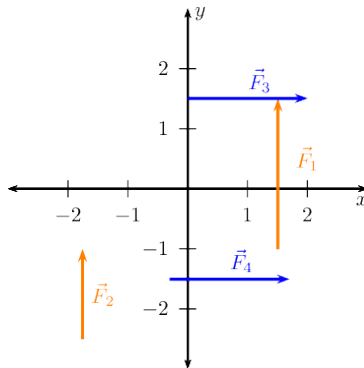


Figure 6: Perpendicular vectors in the positive  $x$  and  $y$  direction

To emphasise that the vectors are perpendicular you can see in Figure 7 that when originating from the same point the vector are at right angles.

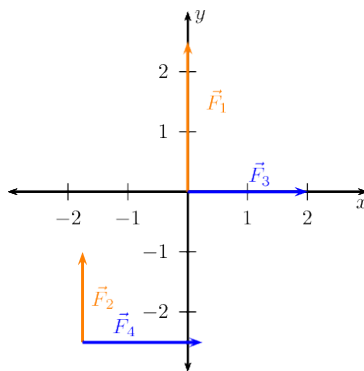


Figure 7: Perpendicular vectors with the same point of origin



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Vectors in two dimensions are not always parallel to an axis. We might know that a force acts at an angle to an axis so we still know the direction of the force and if we know the magnitude we can draw the force vector. For example, we can draw  $\vec{F}_1 = 2\text{ N}$  acting at  $45^\circ$  to the positive  $x$ -direction as in Figure 8.

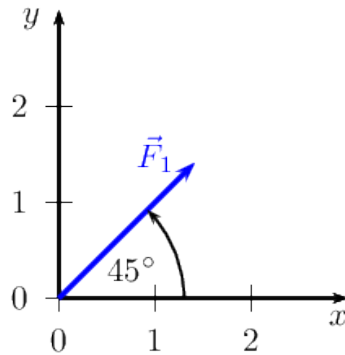


Figure 8: A  $\vec{F}_1 = 2\text{ N}$  force acting at  $45^\circ$  on a Cartesian plane

We always specify the angle as being anti-clockwise from the positive  $x$ -axis. So if we specified a negative angle we would measure it clockwise from the  $x$ -axis. For example,  $\vec{F}_1 = 2\text{ N}$  acting at  $-45^\circ$  to the positive  $x$ -direction as in Figure 9.

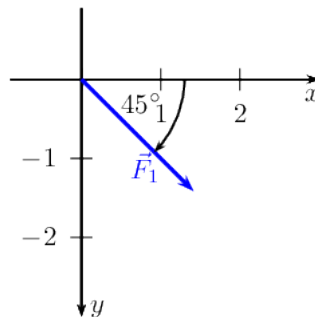


Figure 9: A  $\vec{F}_1 = 2\text{ N}$  force acting at  $-45^\circ$

We can use many other ways of specifying the direction of a vector. The direction just needs to be unambiguous. We have used the Cartesian coordinate system and an angle with the  $x$ -axis so far but there are other common ways of specifying direction that you need to be aware of and comfortable to handle.

## 2.2 Compass directions

We can use compass directions when appropriate to specify the direction of a vector. For example, if we were describing the forces of tectonic plates (the sections of the earth's crust that move) to talk about the forces involved in earthquakes we could talk about the force that the moving plates exert on each other.

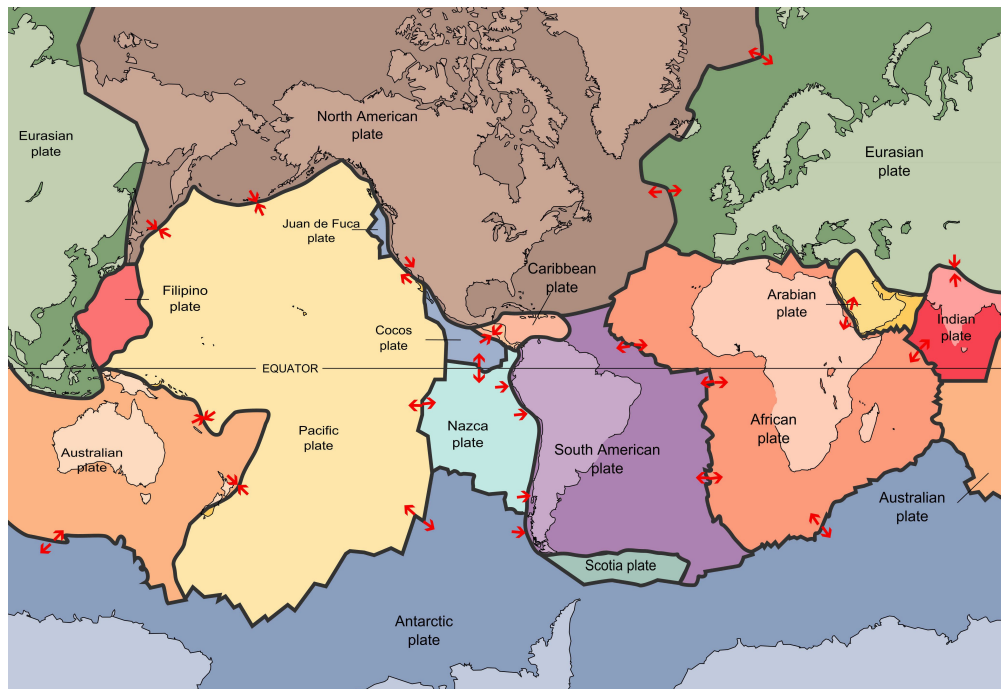


Figure 10: A map of the 15 major tectonic plates that make up the Earth's crust

The four cardinal directions are North, South, East and West when using a compass as shown in Figure 11.



Figure 11: A sketch of the compass directions

When specifying a direction of a vector we can use the directions of a compass given by name, North or South. If the direction is directly between two directions we can combine the names, for example North-East is half-way between North and East. This can only happen for directions at right angles to each other, you cannot say North-South as it is ambiguous.

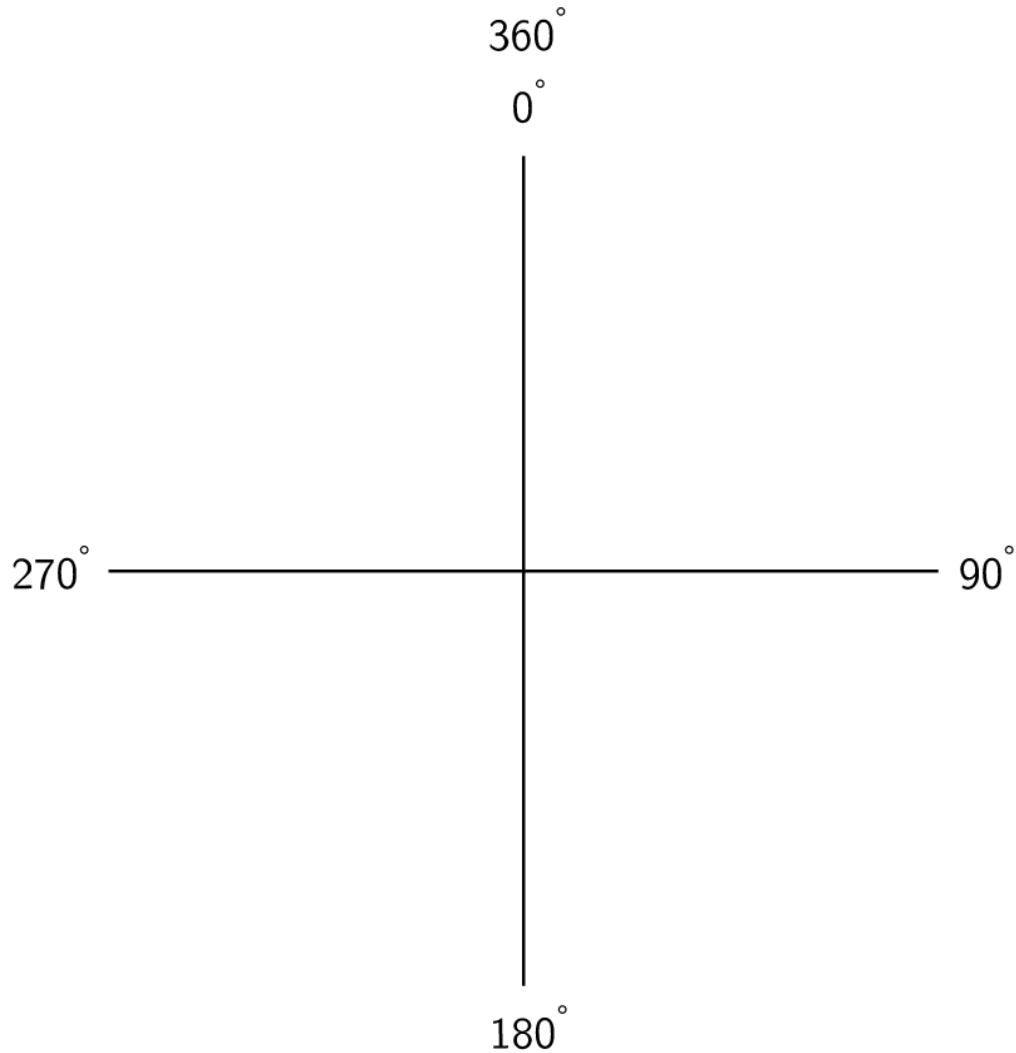
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## 2.3 Bearings

Another way of using the compass to specify direction in a numerical way is to use bearings. A bearing is an angle, usually measured clockwise from North.

**Note** that this is different to the Cartesian plane where angles are anti- or counter-clockwise from the positive  $x$ -direction.

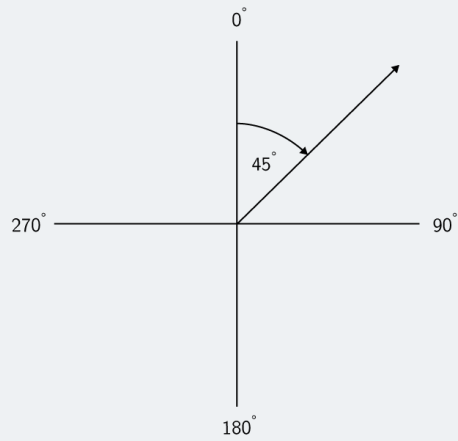
Using the compass with angle bearing:



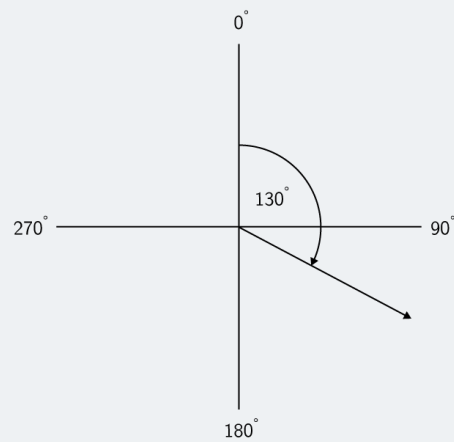
## EXAMPLES

### Measuring from $0^\circ$ :

1. force bearing at  $45^\circ$

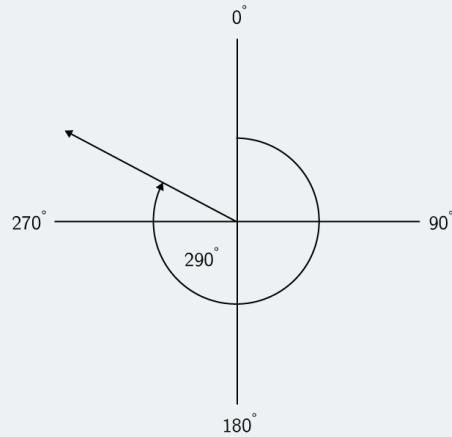


2. force bearing at  $130^\circ$

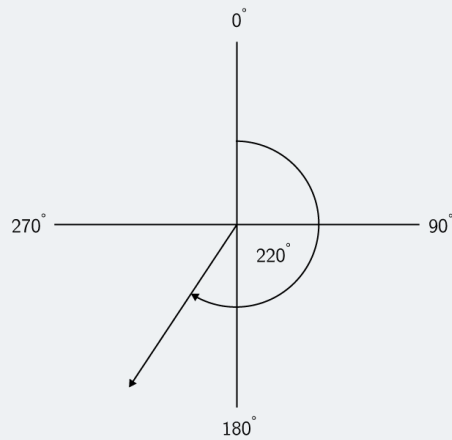


## EXAMPLES (continued)

### 3. force bearing at $290^\circ$



### 4. force bearing at $220^\circ$



## 2.4 The resultant vector

In Grade 10 you learnt about adding vectors together in one dimension. The same principle can be applied for vectors in two dimensions. The following examples show addition of vectors. Vectors that are parallel can be shifted to fall on a line. Vectors falling on the same line are called *co-linear* vectors. To add co-linear vectors we use the tail-to-head method you learnt in Grade 10. In Figure 12 we remind you of the approach of adding co-linear vectors to get a resultant vector.

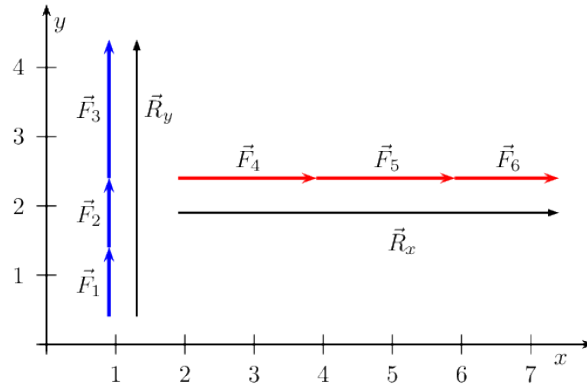


Figure 12: Adding co-linear vectors to get a resultant vector

In Figure 12 the blue vectors are in the  $y$ -direction and the red vectors are in the  $x$ -direction. The two black vectors represent the resultants of the co-linear vectors graphically.

What we have done is implement the tail-to-head method of vector addition for the vertical set of vectors and the horizontal set of vectors.

### WORKED EXAMPLE 1: REVISION: HEAD-TO-TAIL ADDITION IN ONE DIMENSION

#### QUESTION

Use the graphical head-to-tail method to determine the resultant force on a rugby player if two players on his team are pushing him forwards with forces of  $\vec{F}_1 = 600 \text{ N}$  and  $\vec{F}_2 = 900 \text{ N}$  respectively and two players from the opposing team are pushing him backwards with forces of  $\vec{F}_3 = 1\,000 \text{ N}$  and  $\vec{F}_4 = 650 \text{ N}$  respectively.

## WORKED EXAMPLE 1: REVISION: HEAD-TO-TAIL ADDITION IN ONE DIMENSION (continued)

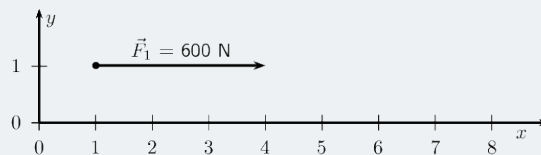
### SOLUTION

#### Step 1: Choose a scale and a reference direction

Let's choose a scale of  $100 \text{ N} : 0,5 \text{ cm}$  and for our diagram we will define the positive direction as *to the right*.

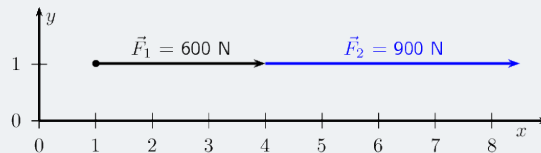
#### Step 2: Choose one of the vectors and draw it as an arrow of the correct length in the correct direction

We will start with drawing the vector  $\vec{F}_1 = 600 \text{ N}$  pointing in the positive direction. Using our scale of  $0,5 \text{ cm} : 100 \text{ N}$ , the length of the arrow must be  $3 \text{ cm}$  pointing to the right.



#### Step 3: Take the next vector and draw it starting at the arrowhead of the previous vector

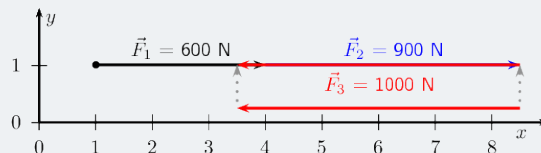
The next vector is  $\vec{F}_2 = 900 \text{ N}$  in the same direction as  $\vec{F}_1$ . Using the scale, the arrow should be  $4,5 \text{ cm}$  long and pointing to the right.



#### Step 4: Take the next vector and draw it starting at the arrowhead of the previous vector

The next vector is  $\vec{F}_3 = 1\,000 \text{ N}$  in the opposite direction. Using the scale, this arrow should be  $5 \text{ cm}$  long and point to the *left*.

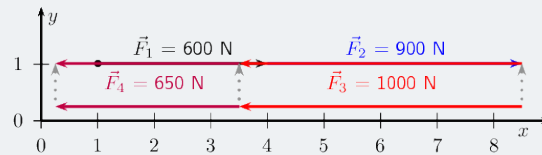
**Note:** We are working in one dimension so this arrow would be drawn on top of the first vectors to the left. This will get confusing so we'll draw it next to the actual line as well to show you what it looks like.



### WORKED EXAMPLE 1: REVISION: HEAD-TO-TAIL ADDITION IN ONE DIMENSION (continued)

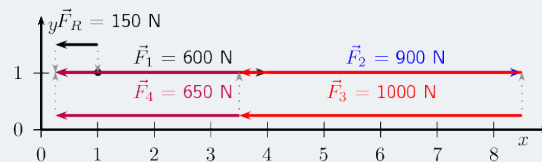
#### Step 5: Take the next vector and draw it starting at the arrowhead of the previous vector

The fourth vector is  $\vec{F}_4 = 650 \text{ N}$  in the opposite direction. Using the scale, this arrow must be 3,25 cm long and point to the left.



#### Step 6: Draw the resultant, measure its length and find its direction

We have now drawn all the force vectors that are being applied to the player. The resultant vector is the arrow which starts at the tail of the first vector and ends at the head of the last drawn vector.



The resultant vector measures 0,75 cm which, using our scale is equivalent to 150 N and points to the left (or the negative direction or the direction the opposing team members are pushing in).

## 2.5 Magnitude of the resultant of vectors at right angles

We apply the same principle to vectors that are at right angles or perpendicular to each other.

### 2.5.1 Sketching tail-to-head method

The tail of the one vector is placed at the head of the other but in two dimensions the vectors may not be co-linear. The approach is to draw all the vectors, one at a time. For the first vector begin at the origin of the Cartesian plane, for the second vector draw it from the head of the first vector. The third vector should be drawn from the head of the second and so on. Each vector is drawn from the head of the vector that preceded it. The order doesn't matter as the resultant will be the same for any order in which the vectors are drawn.

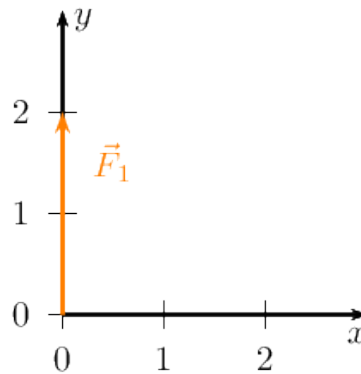


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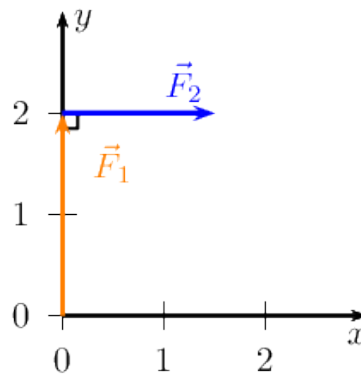
Let us apply this procedure to two vectors:

- $\vec{F}_1 = 2 \text{ N}$  in the positive  $y$ -direction
- $\vec{F}_2 = 1,5 \text{ N}$  in the positive  $x$ -direction

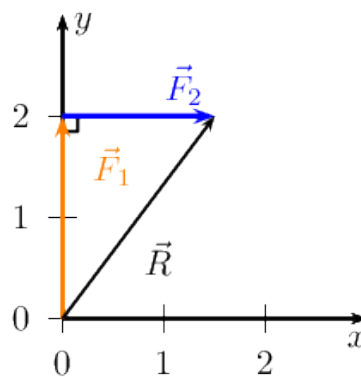
We first draw a Cartesian plane with the first vector originating at the origin:



The next step is to take the second vector and draw it from the head of the first vector:



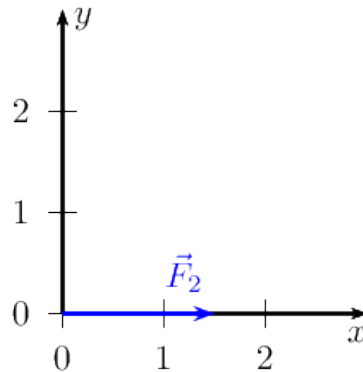
The resultant,  $\vec{R}$ , is the vector connecting the tail of the first vector drawn to the head of the last vector drawn:



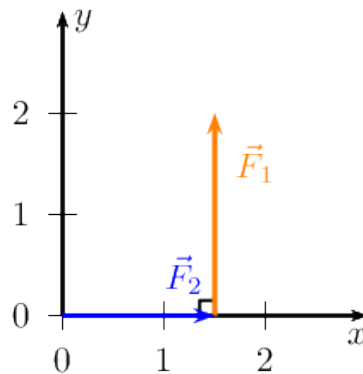
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It is important to remember that the order in which we draw the vectors doesn't matter. If we had drawn them in the opposite order we would have the same resultant,  $\vec{R}$ . We can repeat the process to demonstrate this:

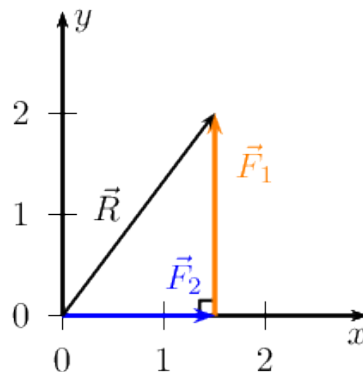
We first draw a Cartesian plane with the second vector originating at the origin:



The next step is to take the other vector and draw it from the head of the vector we have already drawn:



The resultant,  $\vec{R}$ , is the vector connecting the tail of the first vector drawn to the head of the last vector drawn (the vector from the start point to the end point):



## WORKED EXAMPLE 2: SKETCHING VECTORS USING TAIL-TO-HEAD

### QUESTION

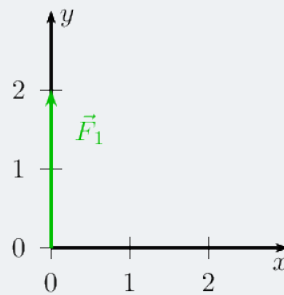
Sketch the resultant of the following force vectors using the tail-to-head method:

- $\vec{F}_1 = 2 \text{ N}$  in the positive  $y$ -direction
- $\vec{F}_2 = 1,5 \text{ N}$  in the positive  $x$ -direction
- $\vec{F}_3 = 1,3 \text{ N}$  in the negative  $y$ -direction
- $\vec{F}_4 = 1 \text{ N}$  in the negative  $x$ -direction

### SOLUTION

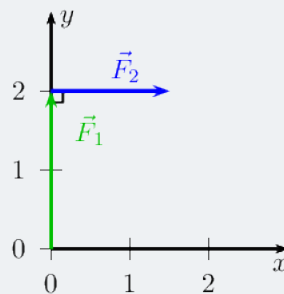
#### Step 1: Draw the Cartesian plane and the first vector

First draw the Cartesian plane and force,  $\vec{F}_1$  starting at the origin.



#### Step 2: Draw the second vector

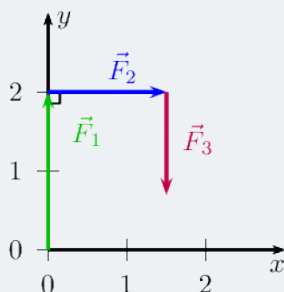
Starting at the head of the first vector we draw the tail of the second vector.



## WORKED EXAMPLE 2: SKETCHING VECTORS USING TAIL-TO-HEAD (continued)

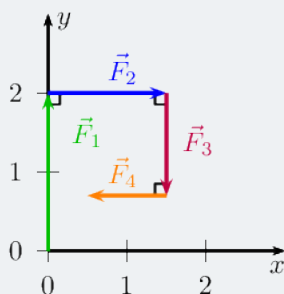
### Step 3: Draw the third vector

Starting at the head of the second vector we draw the tail of the third vector.



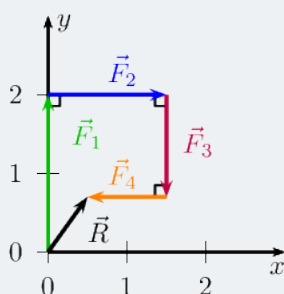
### Step 4: Draw the fourth vector

Starting at the head of the third vector we draw the tail of the fourth vector.



### Step 5: Draw the resulting vector

Starting at the origin draw the resultant vector to the head of the fourth vector.



### WORKED EXAMPLE 3: SKETCHING VECTORS USING TAIL-TO-HEAD

#### QUESTION

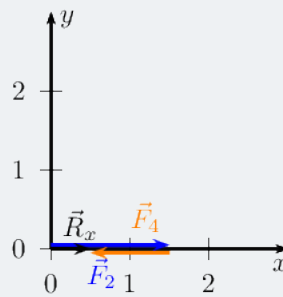
Sketch the resultant of the following force vectors using the tail-to-head method by first determining the resultant in the  $x$ - and  $y$ -directions:

- $\vec{F}_1 = 2 \text{ N}$  in the positive  $y$ -direction
- $\vec{F}_2 = 1,5 \text{ N}$  in the positive  $x$ -direction
- $\vec{F}_3 = 1,3 \text{ N}$  in the negative  $y$ -direction
- $\vec{F}_4 = 1 \text{ N}$  in the negative  $x$ -direction

#### SOLUTION

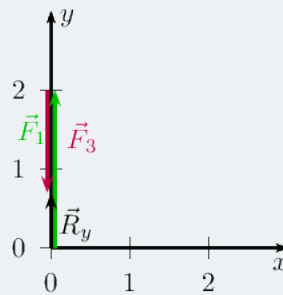
##### Step 1: First determine $\vec{R}_x$

First draw the Cartesian plane with the vectors in the  $x$ -direction.



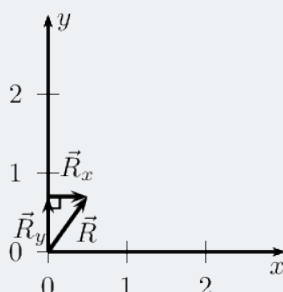
##### Step 2: Secondly determine $\vec{R}_y$

Next we draw the Cartesian plane with the vectors in the  $y$ -direction.



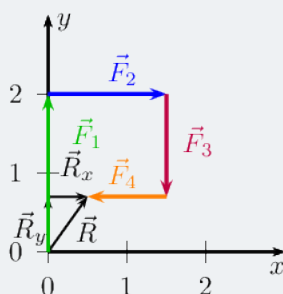
### WORKED EXAMPLE 3: SKETCHING VECTORS USING TAIL-TO-HEAD (continued)

**Step 3: Draw the resultant vectors,  $\vec{R}_y$  and  $\vec{R}_x$  head-to-tail**



**Step 4: Comparison of results**

To double check, we can replot all the vectors again as we did in the previous worked example to see that the outcome is the same.



## 2.5.2 Sketching tail-to-tail method

In this method we draw the two vectors with their tails on the origin. Then we draw a line parallel to the first vector from the head of the second vector and vice versa. Where the parallel lines intersect is the head of the resultant vector that will also start at the origin. We will only deal with perpendicular vectors but this procedure works for any vectors.

### FACT

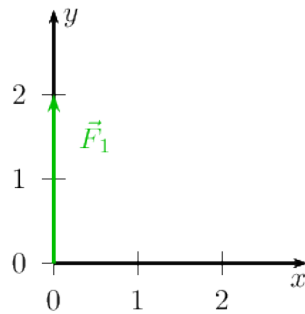
*When dealing with more than two vectors the procedure is repetitive. First find the resultant of any two of the vectors to be added. Then use the same method to add the resultant from the first two vectors with a third vector. This new resultant is then added to the fourth vector and so on, until there are no more vectors to be added.*

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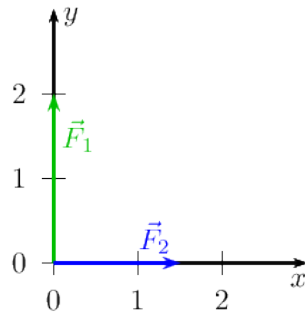
Let us apply this procedure to the same two vectors we used to illustrate the head-to-tail method:

- $\vec{F}_1 = 2 \text{ N}$  in the positive  $y$ -direction
- $\vec{F}_2 = 1,5 \text{ N}$  in the positive  $x$ -direction

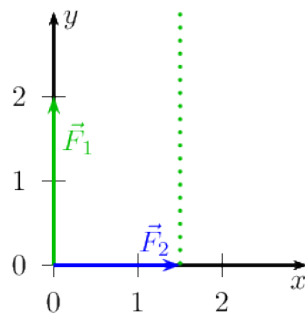
We first draw a Cartesian plane with the first vector originating at the origin:



Then we add the second vector but also originating from the origin so that the vectors are drawn tail-to-tail:

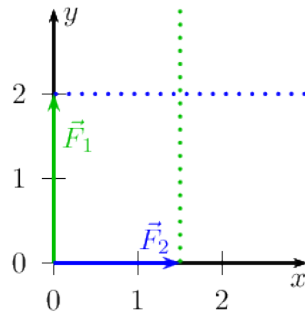


Now we draw a line parallel to  $\vec{F}_1$  from the head of  $\vec{F}_2$  :

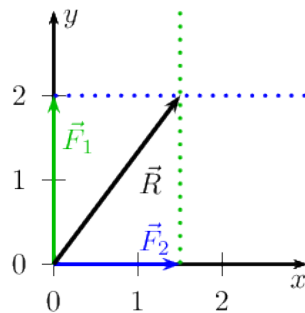


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Next we draw a line parallel to  $\vec{F}_2$  from the head of  $\vec{F}_1$  :



Where the two lines intersect is the head of the resultant vector which will originate at the origin so:



You might be asking what you would do if you had more than 2 vectors to add together. In this case all you need to do is first determine  $\vec{R}_x$  by adding all the vectors that are parallel to the  $x$ -direction and  $\vec{R}_y$  by adding all the vectors that are parallel to the  $y$ -direction. Then you use the tail-to-tail method to find the resultant of  $\vec{R}_x$  and  $\vec{R}_y$ .

## 2.6 Closed vector diagrams

A closed vector diagram is a set of vectors drawn on the Cartesian using the tail-to-head method and that has a resultant with a magnitude of zero. This means that if the first vector starts at the origin the last vector drawn must end at the origin. The vectors form a closed polygon, no matter how many of them are drawn. Here are a few examples of closed vector diagrams:



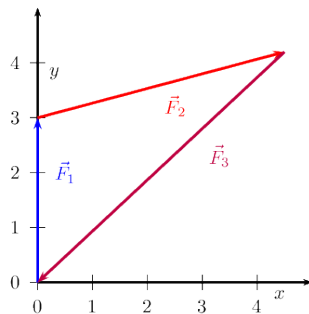


Figure 13: 3 force vectors drawn head-to-tail

In Figure 13 there were 3 force vectors. When drawn tail-to-head with the first force starting at the origin the last force drawn ends at the origin. The resultant would have a magnitude of zero. The resultant is drawn from the tail of the first vector to the head of the final vector. In the diagram below there are 4 vectors that also form a closed vector diagram.

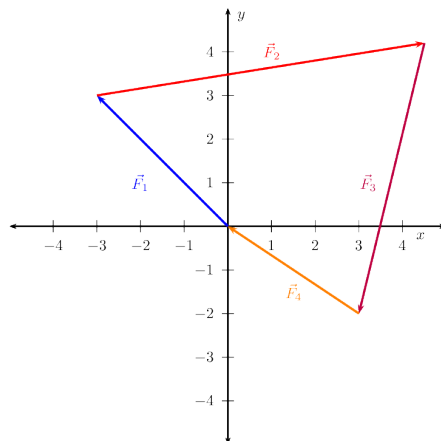


Figure 14: 4 force vectors drawn head-to-tail

In Figure 14 there are 4 vectors, the shape is a 4-sided polygon. Any polygon made up of vectors drawn tail-to-head will be a closed vector diagram because a polygon has no gaps.

## 2.7 Using Pythagoras' theorem to find magnitude

If we wanted to know the resultant of the three blue vectors and the three red vectors in Figure 12 we can use the resultant vectors in the  $x$ - and  $y$  directions to determine this.

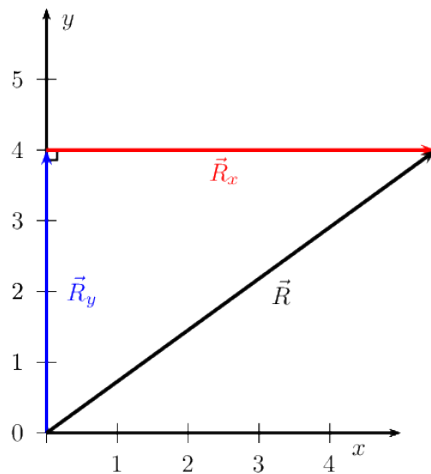


Figure 15: Finding the resultant

The black arrow represents the resultant of the vectors  $\vec{R}_x$  and  $\vec{R}_y$ . We can find the magnitude of this vector using the theorem of Pythagoras because the three vectors form a right angle triangle. If we had drawn the vectors to scale we would be able to measure the magnitude of the resultant as well.

What we've actually sketched out already is our approach to finding the resultant of many vectors using components so remember this example when we get there a little later.

#### WORKED EXAMPLE 4: FINDING THE MAGNITUDE OF THE RESULTANT

##### QUESTION

A similar force vectors figure such as in Figure 12 have the following magnitudes: 1 N, 1 N, 2 N for  $y$ -vectors and 2 N, 2 N and 1, 5 N for the  $x$ -vectors. Determine the magnitude of the resultant.

##### SOLUTION

###### Step 1: Determine the resultant of the vectors parallel to the $y$ -axis

The resultant of the vectors parallel to the  $y$ -axis is found by adding the magnitudes (lengths) of three vectors because they all point in the same direction. The answer is  $\vec{R}_y = 1\text{ N} + 1\text{ N} + 2\text{ N} = 4\text{ N}$  in the positive  $y$ -direction.

###### Step 2: Determine the resultant of the vectors parallel to the $x$ -axis

The resultant of the vectors parallel to the  $x$ -axis is found by adding the magnitudes (lengths) of three vectors because they all point in the same direction. The answer is  $\vec{R}_x = 2\text{ N} + 2\text{ N} + 1,5\text{ N} = 5,5\text{ N}$  in the positive  $x$ -direction.

#### WORKED EXAMPLE 4: FINDING THE MAGNITUDE OF THE RESULTANT (continued)

##### Step 3: Determine the magnitude of the resultant

We have a right angled triangle. We also know the length of two of the sides. Using Pythagoras we can find the length of the third side. From what we know about resultant vectors this length will be the magnitude of the resultant vector. The resultant is:

$$\begin{aligned}R_x^2 + R_y^2 &= R^2 \text{ (Pythagoras' theorem)} & (1) \\(5,5)^2 + (4)^2 &= R^2 \\R &= 6,8\end{aligned}$$

##### Step 4: Quote the final answer

Magnitude of the resultant: 6,8 N

**Note:** we did not determine the resultant vector in the worked example above because we only determined the magnitude. A vector needs a **magnitude** and a **direction**. We did not determine the direction of the resultant vector.

## 2.8 Graphical techniques

In Grade 10 you learnt how to add vectors in one dimension graphically.

We can expand these ideas to include vectors in two-dimensions. The following worked example shows this.

#### WORKED EXAMPLE 5: FINDING THE MAGNITUDE OF THE RESULTANT IN TWO DIMENSIONS GRAPHICALLY

##### QUESTION

Given two vectors,  $\vec{R}_y = 4$  N in the positive  $y$ -direction and  $\vec{R}_x = 3$  N in the positive  $x$ -direction, use the tail-to-head method to find the resultant of these vectors graphically.

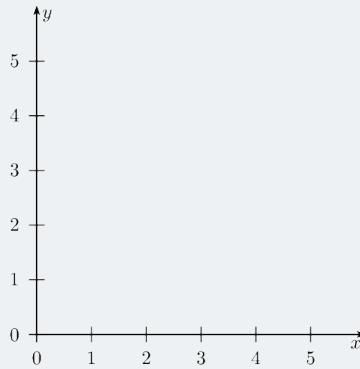
##### SOLUTION

##### Step 1: Choose a scale and draw axes

The vectors we have do not have very big magnitudes so we can choose simple scale, we can use 1 N : 1 cm as our scale for the drawing.

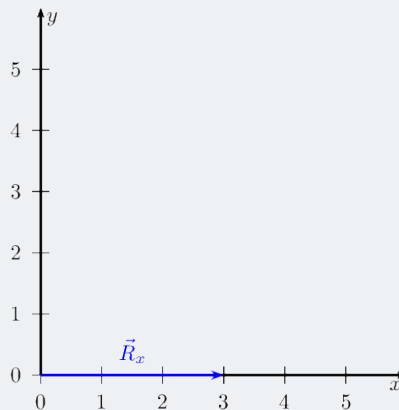
Then we draw axes that the vector diagram should fit in. The largest vector has length 4 N and both vectors are in the positive direction so we can draw axes from the origin to 5 and expect the vectors to fit.

**WORKED EXAMPLE 5: FINDING THE MAGNITUDE OF THE RESULTANT IN TWO DIMENSIONS GRAPHICALLY**  
(continued)



**Step 2: Draw  $\vec{R}_x$**

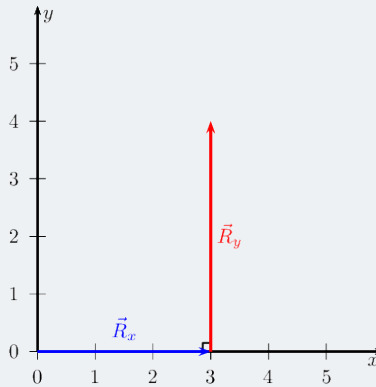
The magnitude of  $\vec{R}_x$  is 3 N so the arrow we need to draw must be 3 cm long. The arrow must point in the positive  $x$ -direction.



**WORKED EXAMPLE 5: FINDING THE MAGNITUDE OF THE RESULTANT IN TWO DIMENSIONS GRAPHICALLY**  
(continued)

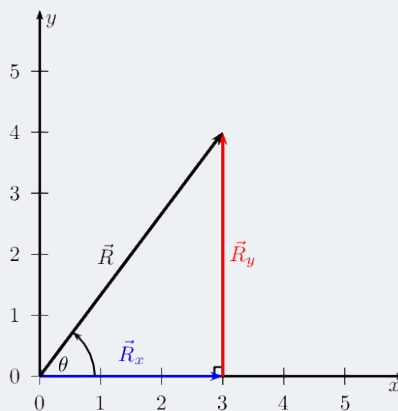
**Step 3: Draw  $\vec{R}_y$**

The length of  $\vec{R}_y$  is 4 so the arrow we need to draw must be 4 cm long. The arrow must point in the positive  $y$ -direction. The important fact to note is that we are implementing the head-to-tail method so the vector must start at the end (head) of  $\vec{R}_x$ .



**Step 4: Draw the resultant vector,  $\vec{R}$**

The resultant vector is the vector from the tail of the first vector we drew directly to the head of the last vector we drew. This means we need to draw a vector from the tail of  $\vec{R}_x$  to the head of  $\vec{R}_y$ .



**Step 5: Measure the resultant,  $\vec{R}$**

We are solving the problem graphically so we now need to measure the magnitude of the vector and use the scale we chose to convert our answer from the diagram to the actual result. In the last diagram the resultant,  $\vec{R}$  is 5 cm long therefore the magnitude of the vector is 5 N. The direction of the resultant,  $\theta$ , we need to measure from the diagram using a protractor. The angle that the vector makes with the  $x$ -axis is  $53^\circ$ .

**Step 6: Quote the final answer**

$\vec{R}$  is 5 N at  $53^\circ$  from the positive  $x$ -direction.

In the case where you have to find the resultant of more than two vectors first apply the tail-to-head method to all the vectors parallel to the one axis and then all the vectors parallel to the other axis. For example, you would first calculate  $\vec{R}_y$  from all the vectors parallel to the  $y$ -axis and then  $\vec{R}_x$  from all the vectors parallel to the  $x$ -axis. After that you apply the same procedure as in the previous worked example to get the final resultant.

#### WORKED EXAMPLE 6: FINDING THE MAGNITUDE OF THE RESULTANT IN TWO DIMENSIONS GRAPHICALLY

##### QUESTION

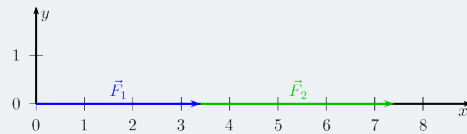
Given the following three force vectors, determine the resultant force:

- $\vec{F}_1 = 3,4$  N in the positive  $x$ -direction
- $\vec{F}_2 = 4$  N in the positive  $x$ -direction
- $\vec{F}_3 = 3$  N in the negative  $y$ -direction

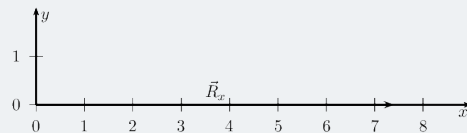
##### SOLUTION

###### Step 1: Determine $\vec{R}_x$

First we determine the resultant of all the vectors that are parallel to the  $x$ -axis. There are two vectors  $\vec{F}_1$  and  $\vec{F}_2$  that we need to add. We do this using the tail-to-head method for co-linear vectors.



The single vector,  $\vec{R}_x$ , that would give us the same outcome is:



###### Step 2: Determine $\vec{R}_y$

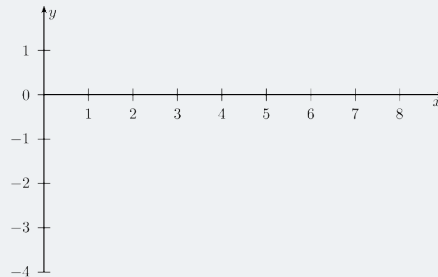
There is only one vector in the  $y$ -direction,  $\vec{F}_3$ , therefore  $\vec{R}_y = \vec{F}_3$ .

###### Step 3: Choose a scale and draw axes

The vectors we have do not have large magnitudes so we can choose simple scale of 1 N : 1 cm for the drawing.

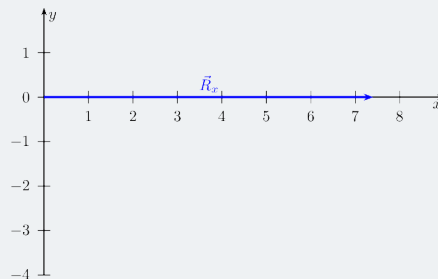
**WORKED EXAMPLE 6: FINDING THE MAGNITUDE OF THE RESULTANT IN TWO DIMENSIONS GRAPHICALLY**  
(continued)

Then we draw axes that the diagram should fit on. The longest vector has length a length of 7,4 N. We need our axes to extend just further than the vectors aligned with each axis. Our axes need to start at the origin and go beyond 7,4 N in the positive  $x$ -direction and further than 3 N in the negative  $y$ -direction. Our scale choice of 1 N : 1 cm means that our axes actually need to extend 7,4 cm in the positive  $x$ -direction and further than 3 cm in the negative  $y$ -direction.



**Step 4: Draw  $\vec{R}_x$**

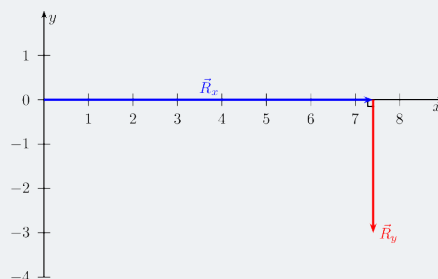
The magnitude of  $\vec{R}_x$  is 7,4 N so the arrow we need to draw must be 7,4 cm long. The arrow must point in the positive  $x$ -direction.



**WORKED EXAMPLE 6: FINDING THE MAGNITUDE OF THE RESULTANT IN TWO DIMENSIONS GRAPHICALLY**  
(continued)

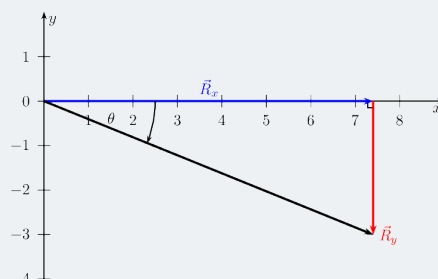
**Step 5: Draw  $\vec{R}_y$**

The magnitude of  $\vec{R}_y$  is 3 N so the arrow we need to draw must be 3 cm long. The arrow must point in the negative  $y$ -direction. The important fact to note is that we are implementing the head-to-tail method so the vector must start at the end (head) of  $\vec{R}_x$ .



**Step 6: Draw the resultant vector,  $\vec{R}$**

The resultant vector is the vector from the tail of the first vector we drew directly to the head of the last vector we drew. This means we need to draw a vector from the tail of  $\vec{R}_x$  to the head of  $\vec{R}_y$ .



**Step 7: Measure the resultant,  $\vec{R}$**

We are solving the problem graphically so we now need to measure the magnitude of the vector and use the scale we chose to convert our answer from the diagram to the actual result. In the last diagram the resultant,  $\vec{R}$  is 8,0 cm long therefore the magnitude of the vector is 8,0 N.

The direction of the resultant we need to measure from the diagram using a protractor. The angle that the vector makes with the  $x$ -axis is  $22^\circ$ .

**Step 8: Quote the final answer**

$\vec{R}$  is 8,0 N at  $-22^\circ$  from the positive  $x$ -direction.



## WORKED EXAMPLE 7: FINDING THE RESULTANT IN TWO DIMENSIONS GRAPHICALLY

### QUESTION

Given the following three force vectors, determine the resultant force:

- $\vec{F}_1 = 2,3$  N in the positive  $x$ -direction
- $\vec{F}_2 = 4$  N in the positive  $y$ -direction
- $\vec{F}_3 = 3,3$  N in the negative  $y$ -direction
- $\vec{F}_4 = 2,1$  N in the negative  $y$ -direction

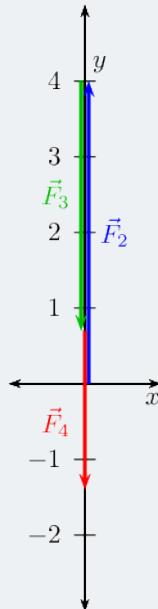
### SOLUTION

#### Step 1: Determine $\vec{R}_x$

There is only one vector in the  $x$ -direction,  $\vec{F}_1$ , therefore  $\vec{R}_x = \vec{F}_1$ .

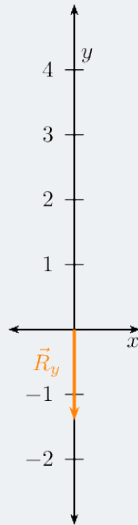
#### Step 2: Determine $\vec{R}_y$

Then we determine the resultant of all the vectors that are parallel to the  $y$ -axis. There are three vectors  $\vec{F}_2$ ,  $\vec{F}_3$  and  $\vec{F}_4$  that we need to add. We do this using the tail-to-head method for co-linear vectors.



### WORKED EXAMPLE 7: FINDING THE RESULTANT IN TWO DIMENSIONS GRAPHICALLY (continued)

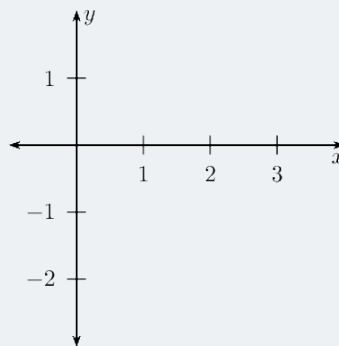
The single vector,  $\vec{R}_y$ , that would give us the same effect is:



#### Step 3: Choose a scale and draw axes

We choose a scale 1 N : 1 cm for the drawing.

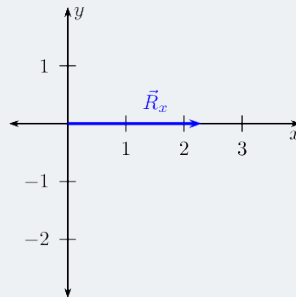
Then we draw axes that the diagram should fit in. We need our axes to extend just further than the vectors aligned with each axis. Our axes need to start at the origin and go beyond 2,3 N in the positive  $x$ -direction and further than 1,4 N in the negative  $y$ -direction. Our scale choice of 1 N = 1 cm means that our axes needs to extend further than 2,3 cm in the positive  $x$ -direction and further than 1,4 cm in the negative  $y$ -direction



### WORKED EXAMPLE 7: FINDING THE RESULTANT IN TWO DIMENSIONS GRAPHICALLY (continued)

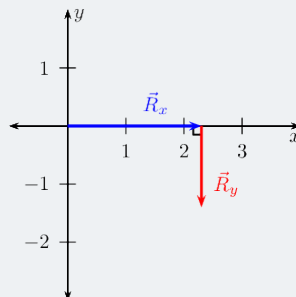
#### Step 4: Draw $\vec{R}_x$

The magnitude of  $\vec{R}_x$  is 2,3 N so the arrow we need to draw must be 2,3 cm long. The arrow must point in the positive  $x$ -direction.



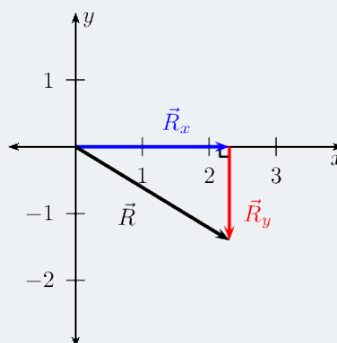
#### Step 5: Draw $\vec{R}_y$

The magnitude of  $\vec{R}_y$  is 1,4 N so the arrow we need to draw must be 1,4 cm long. The arrow must point in the negative  $y$ -direction. The important fact to note is that we are implementing the head-to-tail method so the vector must start at the end (head) of  $\vec{R}_x$ .



#### Step 6: Draw the resultant vector, $\vec{R}$

The resultant vector is the vector from the tail of the first vector we drew directly to the head of the last vector we drew. This means we need to draw a vector from the tail of  $\vec{R}_x$  to the head of  $\vec{R}_y$ .



### WORKED EXAMPLE 7: FINDING THE RESULTANT IN TWO DIMENSIONS GRAPHICALLY (continued)

#### Step 7: Measure the resultant, $\vec{R}$

We are solving the problem graphically so we now need to measure the magnitude of the vector and use the scale we chose to convert our answer from the diagram to the magnitude of the vector. In the last diagram the resultant,  $\vec{R}$  is 2,7 cm long therefore the magnitude of the vector is 2,7 N.

The direction of the resultant we need to measure from the diagram using a protractor. The angle that the vector makes with the  $x$ -axis is 31 degrees.

#### Step 8: Quote the final answer

$\vec{R}$  is 2,7 N at  $-31^\circ$  from the positive  $x$ -direction.

### WORKED EXAMPLE 8: FINDING THE RESULTANT IN TWO DIMENSIONS GRAPHICALLY

#### QUESTION

A number of tugboats are trying to manoeuvre a submarine in the harbour but they are not working as a team. Each tugboat is exerting a different force on the submarine.



Given the following force vectors, determine the resultant force:

- $\vec{F}_1 = 3,4 \text{ kN}$  in the positive  $x$ -direction
- $\vec{F}_2 = 4\,000 \text{ N}$  in the positive  $y$ -direction
- $\vec{F}_3 = 300 \text{ N}$  in the negative  $y$ -direction
- $\vec{F}_4 = 7 \text{ kN}$  in the negative  $y$ -direction

## WORKED EXAMPLE 8: FINDING THE RESULTANT IN TWO DIMENSIONS GRAPHICALLY (continued)

### SOLUTION

#### Step 1: Convert to consistent S.I. units

To use the graphical method of finding the resultant we need to work in the same units. Strictly speaking in this problem all the vectors are in newtons but they have different factors which will affect the choice of scale. These need to be taken into account and the simplest approach is to convert them all to a consistent unit and factor. We could use kN or N, the choice does not matter. We will choose kN. Remember that k represents a factor of  $\times 10^3$ .

$\vec{F}_1$  and  $\vec{F}_4$  do not require any adjustment because they are both in kN. To convert N to kN we use:

$$\begin{aligned} \text{kN} &= \times 10^3 \\ \frac{\text{N}}{\text{kN}} &= \frac{1}{\times 10^3} \\ \text{N} &= \times 10^{-3} \text{ kN} \end{aligned}$$

To convert the magnitude of  $\vec{F}_2$  to kN :

$$\begin{aligned} \vec{F}_2 &= 4\,000 \text{ N} \\ \vec{F}_2 &= 4\,000 \times 10^{-3} \text{ kN} \\ \vec{F}_2 &= 4 \text{ kN} \end{aligned}$$

Therefore  $\vec{F}_2 = 4 \text{ kN}$  in the positive  $y$ -direction.

To convert the magnitude of  $\vec{F}_3$  to kN :

$$\begin{aligned} \vec{F}_3 &= 300 \text{ N} \\ \vec{F}_3 &= 300 \times 10^{-3} \text{ kN} \\ \vec{F}_3 &= 0,3 \text{ kN} \end{aligned}$$

Therefore  $\vec{F}_3 = 0,3 \text{ kN}$  in the negative  $y$ -direction. So:

- $\vec{F}_1 = 3,4 \text{ kN}$  in the positive  $x$ -direction
- $\vec{F}_2 = 4 \text{ kN}$  in the positive  $y$ -direction
- $\vec{F}_3 = 0,3 \text{ kN}$  in the negative  $y$ -direction
- $\vec{F}_4 = 7 \text{ kN}$  in the negative  $y$ -direction

### WORKED EXAMPLE 8: FINDING THE RESULTANT IN TWO DIMENSIONS GRAPHICALLY (continued)

#### Step 2: Choose a scale and draw axes

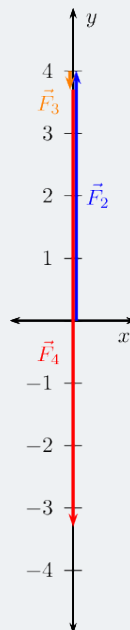
The vectors we have do have very big magnitudes so we need to choose a scale that will allow us to draw them in a reasonable space, we can use 1 kN : 1 cm as our scale for the drawings.

#### Step 3: Determine $\vec{R}_x$

There is only one vector in the  $x$ -direction,  $\vec{F}_1$ , therefore  $\vec{R}_x = \vec{F}_1$ .

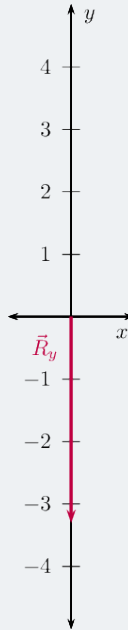
#### Step 4: Determine $\vec{R}_y$

Then we determine the resultant of all the vectors that are parallel to the  $y$ -axis. There are three vectors  $\vec{F}_2$ ,  $\vec{F}_3$  and  $\vec{F}_4$  that we need to add. We do this using the tail-to-head method for co-linear vectors.



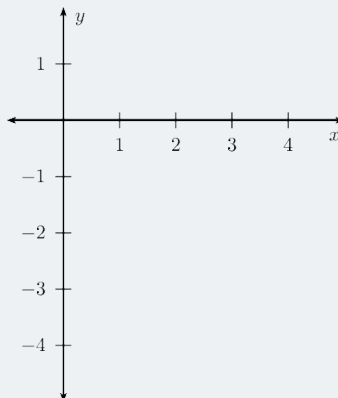
### WORKED EXAMPLE 8: FINDING THE RESULTANT IN TWO DIMENSIONS GRAPHICALLY (continued)

The single vector,  $\vec{R}_y$ , that would give us the same outcome is:



#### Step 5: Draw axes

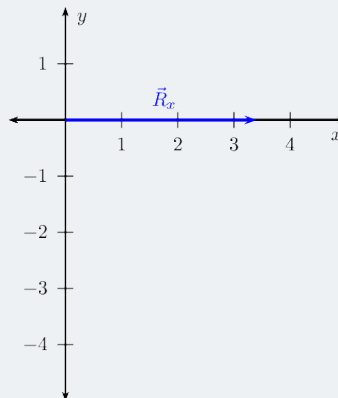
Then we draw axes that the diagram should fit on. We need our axes to extend just further than the vectors aligned with each axis. Our axes need to start at the origin and go beyond 3,4 kN in the positive  $x$ -direction and further than 3,3 kN in the negative  $y$ -direction. Our scale choice of 1 kN : 1 cm means that our axes actually need to extend 3,4 cm in the positive  $x$ -direction and further than 3,3 cm in the negative  $y$ -direction



### WORKED EXAMPLE 8: FINDING THE RESULTANT IN TWO DIMENSIONS GRAPHICALLY (continued)

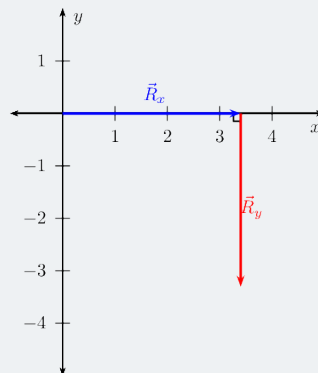
#### Step 6: Draw $\vec{R}_x$

The length of  $\vec{R}_x$  is 3,4 kN so the arrow we need to draw must be 3,4 cm long. The arrow must point in the positive  $x$ -direction.



#### Step 7: Draw $\vec{R}_y$

The length of  $\vec{R}_y$  is 3,3 kN so the arrow we need to draw must be 3,3 cm long. The arrow must point in the negative  $y$ -direction. The important fact to note is that we are implementing the head-to-tail method so the vector must start at the end (head) of  $\vec{R}_x$ .

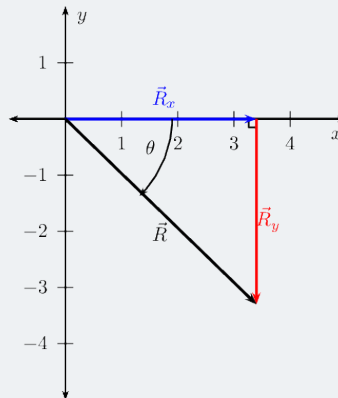




### WORKED EXAMPLE 8: FINDING THE RESULTANT IN TWO DIMENSIONS GRAPHICALLY (continued)

#### Step 8: Draw the resultant vector, $\vec{R}$

The resultant vector is the vector from the tail of the first vector we drew directly to the head of the last vector we drew. This means we need to draw a vector from the tail of  $\vec{R}_x$  to the head of  $\vec{R}_y$ .



#### Step 9: Measure the resultant, $\vec{R}$

We are solving the problem graphically so we now need to measure the magnitude of the vector and use the scale we chose to convert our answer from the diagram to the actual result. In the last diagram the resultant,  $\vec{R}$  is 4,7 cm long therefore the magnitude of the vector is 4,7 kN.

The direction of the resultant we need to measure from the diagram using a protractor. The angle that the vector makes with the  $x$ -axis is  $44^\circ$ .

#### Step 10: Quote the final answer

$\vec{R}$  is 4,7 kN at  $-44^\circ$  from the positive  $x$ -direction.

## 2.9 Algebraic addition and subtraction of vectors

In Grade 10 you learnt about addition and subtraction of vectors in one dimension. The following worked example provides a refresher of the concepts.

## WORKED EXAMPLE 9: ADDING VECTORS ALGEBRAICALLY

### QUESTION

A force of 5 N to the right is applied to a crate. A second force of 2 N to the left is also applied to the crate. Calculate algebraically the resultant of the forces applied to the crate.

### SOLUTION

#### Step 1: Draw a sketch

A simple sketch will help us understand the problem.



#### Step 2: Decide which method to use to calculate the resultant

Remember that force is a vector. Since the forces act along a straight line (i.e. the  $x$ -direction), we can use the algebraic technique of vector addition.

#### Step 3: Choose a positive direction

Choose the **positive** direction to be to the right. This means that the **negative** direction is to the left.

Rewriting the problem using the choice of a positive direction gives us a force of 5 N in the positive  $x$ -direction and force of 2 N in the negative  $x$ -direction being applied to the crate.

#### Step 4: Now define our vectors algebraically

$$\vec{F}_1 = 5 \text{ N}$$

$$\vec{F}_2 = -2 \text{ N}$$

#### Step 5: Add the vectors

Thus, the resultant force is:

$$\vec{F}_1 + \vec{F}_2 = (5) + (-2) \quad (2)$$

$$= 3 \text{ N} \quad (3)$$

#### Step 6: Quote the resultant

Remember that in this case a positive force means to the right: 3 N to the right.

We can now expand on this work to include vectors in two dimensions.

## WORKED EXAMPLE 10: ALGEBRAIC SOLUTION IN TWO DIMENSIONS

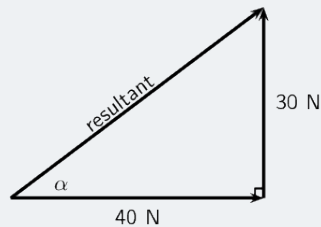
### QUESTION

A force of 40 N in the positive  $x$ -direction acts simultaneously (at the same time) to a force of 30 N in the positive  $y$ -direction. Calculate the magnitude of the resultant force.

### SOLUTION

#### Step 1: Draw a rough sketch

As before, the rough sketch looks as follows:



#### Step 2: Determine the length of the resultant

Note that the triangle formed by the two force vectors and the resultant vector is a right-angle triangle. We can thus use the Theorem of Pythagoras to determine the length of the resultant. Let  $R$  represent the length of the resultant vector. Then:

$$F_x^2 + F_y^2 = R^2 \text{Pythagoras' theorem} \quad (4)$$

$$(40)^2 + (30)^2 = R^2 \quad (5)$$

$$R = 50 \text{ N} \quad (6)$$

#### Step 3: Quote the resultant

The magnitude of the resultant force is then 50 N.

## 2.10 Direction

For two dimensional vectors we have only covered finding the magnitude of vectors algebraically. We also need to know the direction. For vectors in one dimension this was simple. We chose a positive direction and then the resultant was either in the positive or in the negative direction. In Grade 10 you learnt about the different ways to specify direction. We will now look at using trigonometry to determine the direction of the resultant vector.

We can use simple trigonometric identities to calculate the direction. We can calculate the direction of the resultant in the previous worked example.

### WORKED EXAMPLE 11: DIRECTION OF THE RESULTANT

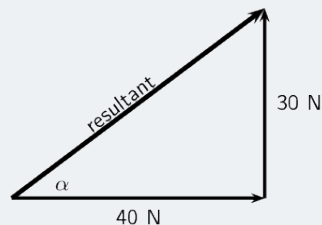
#### QUESTION

A force of 40 N in the positive  $x$ -direction acts simultaneously (at the same time) to a force of 30 N in the positive  $y$ -direction. Calculate the magnitude of the resultant force.

#### SOLUTION

##### Step 1: Magnitude

We determined the magnitude of the resultant vector in the previous worked example to be 50 N. The sketch of the situation is:



##### Step 2: Determine the direction of the resultant

To determine the direction of the resultant force, we calculate the angle  $\alpha$  between the resultant force vector and the positive  $x$ -axis, by using simple trigonometry:

$$\tan \alpha = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\tan \alpha = \frac{30}{40}$$

$$\alpha = \tan^{-1}(0,75)$$

$$\alpha = 36,87^\circ$$

##### Step 3: Quote the resultant

The resultant force is then 50 N at  $36,9^\circ$  to the positive  $x$ -axis.

## 3 COMPONENTS OF VECTORS

In the discussion of vector addition we saw that a number of vectors acting together can be combined to give a single vector (the resultant). In much the same way a single vector can be broken down into a number of vectors which when added give that original vector. These vectors which sum to the original are called **components** of the original vector. The process of breaking a vector into its components is called **resolving into components**.

In practise it is most useful to resolve a vector into components which are at right angles to one another, usually horizontal and vertical. Think about all the problems we've solved so far. If we have vectors parallel to the  $x$ - and  $y$ -axes problems are straightforward to solve.

Any vector can be resolved into a horizontal and a vertical component. If  $\vec{R}$  is a vector, then the horizontal component of  $\vec{R}$  is  $\vec{R}_x$  and the vertical component is  $\vec{R}_y$ .

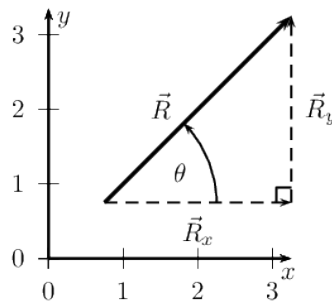


Figure 16: A force divided into its  $x$ - and  $y$ -direction components

When resolving into components that are parallel to the  $x$ - and  $y$ -axes we are always dealing with a right-angled triangle. This means that we can use trigonometric identities to determine the magnitudes of the components (we know the directions because they are aligned with the axes).

From the triangle in the diagram above we know that

$$\begin{aligned}\cos(\theta) &= \frac{R_x}{R} \\ \frac{R_x}{R} &= \cos(\theta) \\ R_x &= R \cos(\theta)\end{aligned}$$

and

$$\begin{aligned}\sin \theta &= \frac{R_y}{R} \\ \frac{R_y}{R} &= \sin(\theta) \\ R_y &= R \sin(\theta)\end{aligned}$$

$$R_x = R \cos(\theta)$$

$$R_y = R \sin(\theta)$$

**Note** that the angle is measured counter-clockwise from the positive  $x$ -axis.

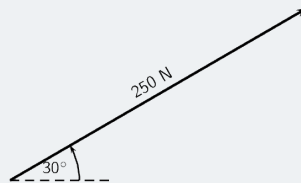
## WORKED EXAMPLE 12: RESOLVING A VECTOR INTO COMPONENTS

### QUESTION

A force of 250 N acts at an angle of  $30^\circ$  to the positive  $x$ -axis. Resolve this force into components parallel to the  $x$ - and  $y$ -axes.

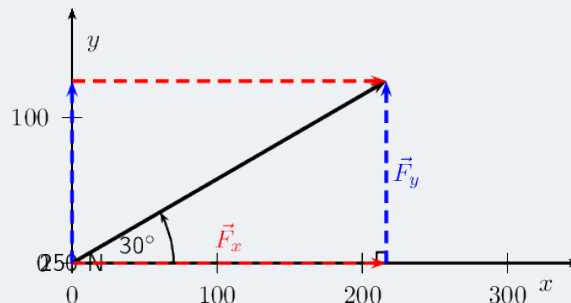
### SOLUTION

#### Step 1: Draw a rough sketch of the original vector



#### Step 2: Determine the vector components

Next we resolve the force into components parallel to the axes. Since these directions are perpendicular to one another, the components form a right-angled triangle with the original force as its hypotenuse.



Notice how the two components acting together give the original vector as their resultant.

#### Step 3: Determine the magnitudes of the component vectors

$$\begin{aligned} F_y &= 250 \sin(30^\circ) \\ &= 125 \text{ N} \end{aligned}$$

and

$$\begin{aligned} F_x &= 250 \cos(30^\circ) \\ &= 216,5 \text{ N} \end{aligned}$$

Remember  $F_x$  and  $F_y$  are the magnitudes of the components.  $\vec{F}_x$  is in the positive  $x$ -direction and  $\vec{F}_y$  is in the positive  $y$ -direction.

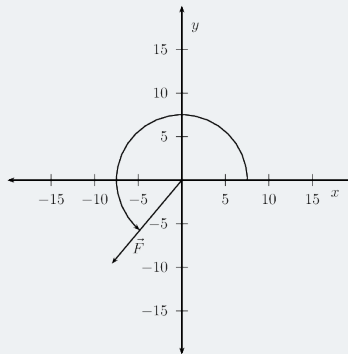
### WORKED EXAMPLE 13: RESOLVING A VECTOR INTO COMPONENTS

#### QUESTION

A force of 12,5 N acts at an angle of  $230^\circ$  to the positive  $x$ -axis. Resolve this force into components parallel to the  $x$ - and  $y$ -axes.

#### SOLUTION

##### Step 1: Draw a rough sketch of the original vector



##### Step 2: Determine the vector components

Next we resolve the force into components parallel to the axes. Since these directions are perpendicular to one another, the components form a right-angled triangle with the original force as its hypotenuse

Now we can use trigonometry to calculate the magnitudes of the components of the original force:

$$\begin{aligned}F_y &= 12,5 \sin(230^\circ) \\ &= -9,58 \text{ N}\end{aligned}$$

and

$$\begin{aligned}F_x &= 12,5 \cos(230^\circ) \\ &= -8,03 \text{ N}\end{aligned}$$

**Notice** that by using the full angle we actually get the correct signs for the components if we use the standard Cartesian coordinates.  $\vec{F}_x$  is in the negative  $x$ -direction and  $\vec{F}_y$  is in the negative  $y$ -direction.

### 3.1 Vector addition using components

Components can also be used to find the resultant of vectors. This technique can be applied to both graphical and algebraic methods of finding the resultant.

The method is straightforward:

1. make a rough sketch of the problem;
2. find the horizontal and vertical components of each vector;
3. find the sum of all horizontal components,  $\vec{R}_x$ ;
4. find the sum of all the vertical components,  $\vec{R}_y$
5. then use them to find the resultant,  $\vec{R}$ .

Consider the two vectors,  $\vec{F}_1$  and  $\vec{F}_2$ , in Figure 17 , together with their resultant,  $\vec{R}$ .

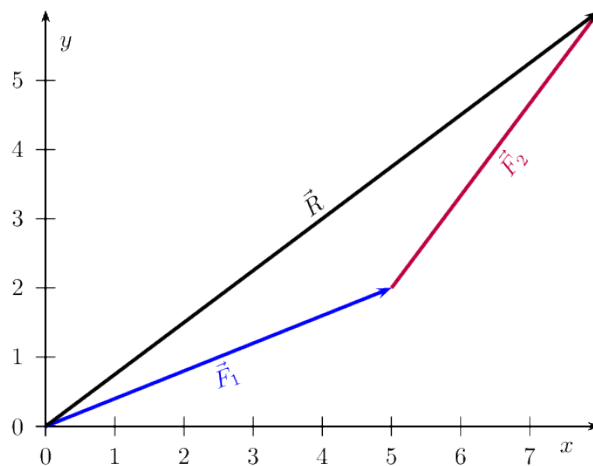


Figure 17: An example of two vectors being added to give a resultant

Each vector in Figure 17 can be broken down into one component in the  $x$ -direction (horizontal) and one in the  $y$ -direction (vertical). These components are two vectors which when added give you the original vector as the resultant. This is shown in Figure 18 below.



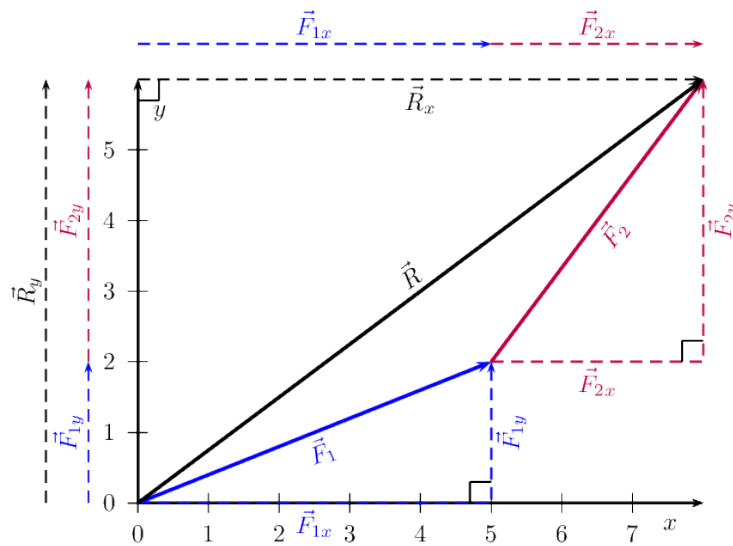


Figure 18: Adding vectors using components

We can see that:

$$\vec{F}_1 = \vec{F}_{1x} + \vec{F}_{1y}$$

$$\vec{F}_2 = \vec{F}_{2x} + \vec{F}_{2y}$$

$$\vec{R} = \vec{R}_x + \vec{R}_y$$

$$\text{But, } \vec{R}_x = \vec{F}_{1x} + \vec{F}_{2x}$$

$$\text{and } \vec{R}_y = \vec{F}_{1y} + \vec{F}_{2y}$$

In summary, addition of the  $x$ -components of the two original vectors gives the  $x$ -component of the resultant. The same applies to the  $y$ -components. So if we just added all the components together we would get the **same answer!** This is another important property of vectors.

#### WORKED EXAMPLE 14: ADDING VECTORS USING COMPONENTS

##### QUESTION

If in Figure 18  $\vec{F}_1 = 5,385 \text{ N}$  at an angle of  $21,8^\circ$  to the horizontal and  $\vec{F}_2 = 5 \text{ N}$  at an angle of  $53,13^\circ$  to the horizontal, find the resultant force,  $\vec{R}$

##### SOLUTION

###### Step 1: Decide how to tackle the problem

The first thing we must realise is that the order that we add the vectors does not matter. Therefore, we can work through the vectors to be added in any order. We also draw up the following table to help us work through the problem:

WORKED EXAMPLE 14: ADDING VECTORS USING COMPONENTS (continued)

Vector	$x$ -component	$y$ -component	Total
$\vec{F}_1$			
$\vec{F}_2$			
Resultant			

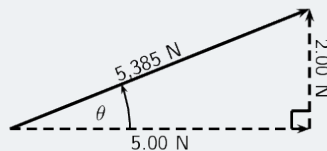
**Step 2: Resolve  $\vec{F}_1$  into components**

We find the components of  $\vec{F}_1$  by using known trigonometric ratios. First we find the magnitude of the vertical component,  $F_{1y}$  :

$$\begin{aligned}\sin(\theta) &= \frac{F_{1y}}{F_1} \\ \sin(21,8^\circ) &= \frac{F_{1y}}{5,385} \\ F_{1y} &= (\sin(21,8^\circ)) (5,385) \\ &= 2,00 \text{ N}\end{aligned}$$

Secondly we find the magnitude of the horizontal component,  $F_{1x}$  :

$$\begin{aligned}\cos(\theta) &= \frac{F_{1x}}{F_1} \\ \cos(21,8^\circ) &= \frac{F_{1x}}{5,385} \\ F_{1x} &= (\cos(21,8^\circ)) (5,385) \\ &= 5,00 \text{ N}\end{aligned}$$



The components give the sides of the right angle triangle, for which the original vector,  $\vec{F}_1$ , is the hypotenuse.

Vector	$x$ -component	$y$ -component	Resultant
$\vec{F}_1$	5,00 N	2,00 N	5,385 N
$\vec{F}_2$			
Resultant			

WORKED EXAMPLE 14: ADDING VECTORS USING COMPONENTS (continued)

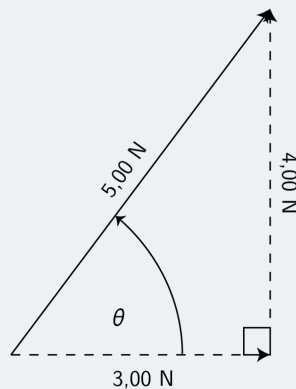
**Step 3: Resolve  $\vec{F}_2$  into components**

We find the components of  $\vec{F}_2$  by using known trigonometric ratios. First we find the magnitude of the vertical component,  $F_{2y}$  :

$$\begin{aligned}\sin(\theta) &= \frac{F_{2y}}{F_2} \\ \sin(53,13^\circ) &= \frac{F_{2y}}{5} \\ F_{2y} &= (\sin(53,13^\circ)) (5) \\ &= 4,00 \text{ N}\end{aligned}$$

Secondly we find the magnitude of the horizontal component,  $F_{2x}$  :

$$\begin{aligned}\cos(\theta) &= \frac{F_{2x}}{F_2} \\ \cos(53,13^\circ) &= \frac{F_{2x}}{5} \\ F_{2x} &= (\cos(53,13^\circ)) (5) \\ &= 3,00 \text{ N}\end{aligned}$$



Vector	$x$ -component	$y$ -component	Total
$\vec{F}_1$	5,00 N	2,00 N	5,385 N
$\vec{F}_2$	3,00 N	4,00 N	5 N
Resultant			

#### WORKED EXAMPLE 14: ADDING VECTORS USING COMPONENTS (continued)

##### Step 4: Determine the components of the resultant vector

Now we have all the components. If we add all the horizontal components then we will have the  $x$ -component of the resultant vector,  $\vec{R}_x$ . Similarly, we add all the vertical components then we will have the  $y$ -component of the resultant vector,  $\vec{R}_y$ .

$$\begin{aligned}R_x &= F_{1x} + F_{2x} \\ &= 5,00 \text{ N} + 3,00 \text{ N} \\ &= 8,00 \text{ N}\end{aligned}$$

Therefore,  $\vec{R}_x$  is 8 N to the right.

$$\begin{aligned}R_y &= F_{1y} + F_{2y} \\ &= 2,00 \text{ N} + 4,00 \text{ N} \\ &= 6,00 \text{ N}\end{aligned}$$

Therefore,  $\vec{R}_y$  is 6 N up.

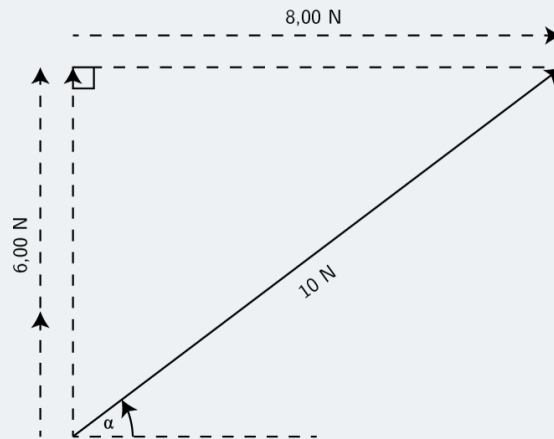
Vector	$x$ -component	$y$ -component	Total
$\vec{F}_1$	5,00 N	2,00 N	5,385 N
$\vec{F}_2$	3,00 N	4,00 N	5 N
Resultant	8,00 N	6,00 N	

##### Step 5: Determine the magnitude and direction of the resultant vector

Now that we have the components of the resultant, we can use the Theorem of Pythagoras to determine the magnitude of the resultant,  $R$ .

$$\begin{aligned}R^2 &= (R_y)^2 + (R_x)^2 \\ &= (6,00)^2 + (8,00)^2 \\ &= 100,00 \\ R &= 10,00 \text{ N}\end{aligned}$$

#### WORKED EXAMPLE 14: ADDING VECTORS USING COMPONENTS (continued)



The magnitude of the resultant,  $R$  is 10,00 N. So all we have to do is calculate its direction. We can specify the direction as the angle the vectors makes with a known direction. To do this you only need to visualise the vector as starting at the origin of a coordinate system. We have drawn this explicitly below and the angle we will calculate is labelled  $\alpha$ .

Using our known trigonometric ratios we can calculate the value of  $\alpha$

$$\begin{aligned}\tan \alpha &= \frac{6,00}{8,00} \\ \alpha &= \tan^{-1} \frac{6,00}{8,00} \\ \alpha &= 36,9^\circ\end{aligned}$$

#### Step 6: Quote the final answer

$\vec{R}$  is 10 N at an angle of  $36,9^\circ$  to the positive  $x$ -axis.

## WORKED EXAMPLE 15: RESULTANT USING COMPONENTS

### QUESTION

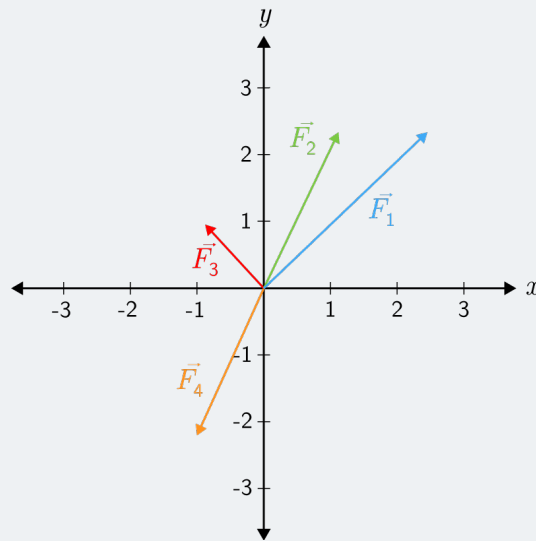
Determine, by resolving into components, the resultant of the following four forces acting at a point:

- $\vec{F}_1 = 3,5 \text{ N}$  at  $45^\circ$  to the positive  $x$ -axis.
- $\vec{F}_2 = 2,7 \text{ N}$  at  $63^\circ$  to the positive  $x$ -axis
- $\vec{F}_3 = 1,3 \text{ N}$  at  $127^\circ$  to the positive  $x$ -axis.
- $\vec{F}_4 = 2,5 \text{ N}$  at  $245^\circ$  to the positive  $x$ -axis.

### SOLUTION

#### Step 1: Sketch the problem

Draw all of the vectors on the Cartesian plane. This does not have to be precisely accurate because we are solving algebraically but vectors need to be drawn in the correct quadrant and with the correct relative positioning to each other.

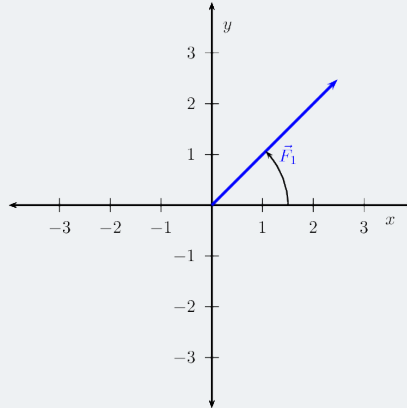


We are going to record the various components in a table to help us manage keep track of the calculation. For each vector we need to determine the components in the  $x$ - and  $y$ -directions.

Vector	$x$ -component	$y$ -component	Total
$\vec{F}_1$			3,5 N
$\vec{F}_2$			2,7 N
$\vec{F}_3$			1,3 N
$\vec{F}_4$			2,5 N
$\vec{R}$			

### WORKED EXAMPLE 15: RESULTANT USING COMPONENTS (continued)

#### Step 2: Determine components of $\vec{F}_1$



$$\sin(\theta) = \frac{F_{1y}}{F_1}$$

$$\sin(45^\circ) = \frac{F_{1y}}{3,5}$$

$$\begin{aligned} F_{1y} &= (\sin(45^\circ))(3,5) \\ &= 2,47 \text{ N} \end{aligned}$$

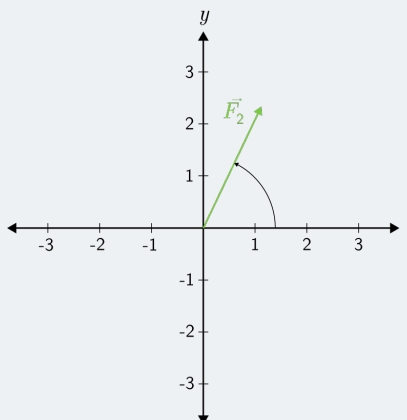
Secondly we find the magnitude of the horizontal component,  $F_{1x}$  :

$$\cos(\theta) = \frac{F_{1x}}{F_1}$$

$$\cos(45^\circ) = \frac{F_{1x}}{3,5}$$

$$\begin{aligned} F_{1x} &= (\cos(45^\circ))(3,5) \\ &= 2,47 \text{ N} \end{aligned}$$

#### Step 3: Determine components of $\vec{F}_2$



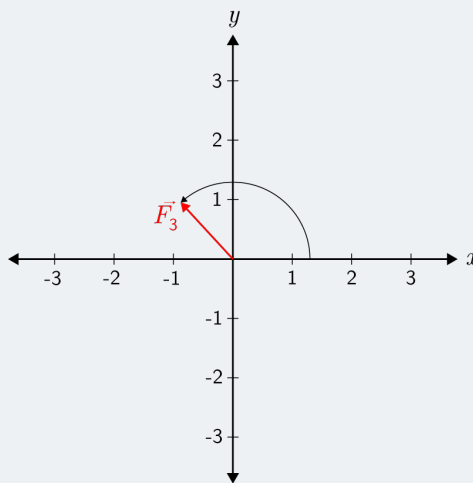
WORKED EXAMPLE 15: RESULTANT USING COMPONENTS (continued)

$$\begin{aligned}\sin(\theta) &= \frac{F_{2y}}{F_2} \\ \sin(63^\circ) &= \frac{F_{2y}}{2,7} \\ F_{2y} &= (\sin(63^\circ))(2,7) \\ &= 2,41 \text{ N}\end{aligned}$$

Secondly we find the magnitude of the horizontal component,  $F_{2x}$  :

$$\begin{aligned}\cos \theta &= \frac{F_{2x}}{F_2} \\ \cos(63^\circ) &= \frac{F_{2x}}{2,7} \\ F_{2x} &= (\cos(63^\circ))(2,7) \\ &= 1,23 \text{ N}\end{aligned}$$

**Step 4: Determine components of  $\vec{F}_3$**



$$\begin{aligned}\sin(\theta) &= \frac{F_{3y}}{F_3} \\ \sin(127^\circ) &= \frac{F_{3y}}{1,3} \\ F_{3y} &= (\sin 127^\circ)(1,3) \\ &= 1,04 \text{ N}\end{aligned}$$

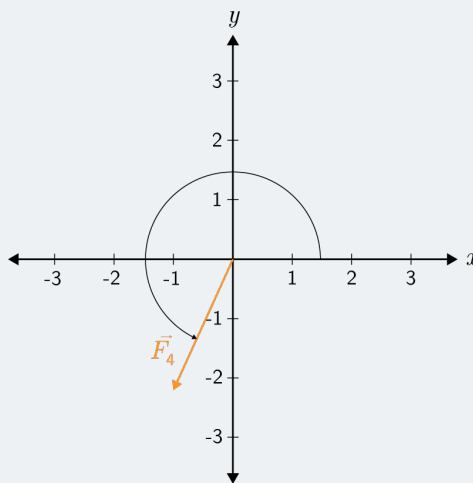
Secondly we find the magnitude of the horizontal component,  $F_{3x}$  :



WORKED EXAMPLE 15: RESULTANT USING COMPONENTS (continued)

$$\begin{aligned}\cos(\theta) &= \frac{F_{3x}}{F_3} \\ \cos(127^\circ) &= \frac{F_{3x}}{1,3} \\ F_{3x} &= (\cos 127^\circ)(1,3) \\ &= -0,78 \text{ N}\end{aligned}$$

**Step 5: Determine components of  $\vec{F}_4$**



$$\begin{aligned}\sin(\theta) &= \frac{F_{4y}}{F_4} \\ \sin(245^\circ) &= \frac{F_{4y}}{2,5} \\ F_{4y} &= (\sin(245^\circ))(2,5) \\ &= -2,27 \text{ N}\end{aligned}$$

Secondly we find the magnitude of the horizontal component,  $F_{4x}$  :

$$\begin{aligned}\cos(\theta) &= \frac{F_{4x}}{F_4} \\ \cos(245^\circ) &= \frac{F_{4x}}{2,5} \\ F_{4x} &= (\cos(245^\circ))(2,5) \\ &= -1,06 \text{ N}\end{aligned}$$

**WORKED EXAMPLE 15: RESULTANT USING COMPONENTS (continued)****Step 6: Determine components of resultant**

Sum the various component columns to determine the components of the resultant. Remember that if the component was negative don't leave out the negative sign in the summation.

Vector	$x$ -component	$y$ -component	Total
$\vec{F}_1$	2,47 N	2,47 N	3,5 N
$\vec{F}_2$	1,23 N	2,41 N	2,7 N
$\vec{F}_3$	-0,78 N	1,04 N	1,3 N
$\vec{F}_4$	-1,06 N	-2,27 N	2,5 N
$\vec{R}$	1,86 N	3,65 N	

Now that we have the components of the resultant, we can use the Theorem of Pythagoras to determine the magnitude of the resultant,  $R$ .

$$\begin{aligned}R^2 &= (R_y)^2 + (R_x)^2 \\&= (1,86)^2 + (3,65)^2 \\&= 16,78 \\R &= 4,10 \text{ N}\end{aligned}$$

We can also determine the angle with the positive  $x$ -axis.

$$\begin{aligned}\tan(\alpha) &= \frac{1,86}{3,65} \\ \alpha &= \tan^{-1}\left(\frac{3,65}{1,86}\right) \\ \alpha &= 63,00^\circ\end{aligned}$$

**Step 7: Quote the final answer**

The resultant has a magnitude of 4,10 N at an angle of 27,00° to the positive  $x$ -direction.

## INFORMAL EXPERIMENT

### Force board

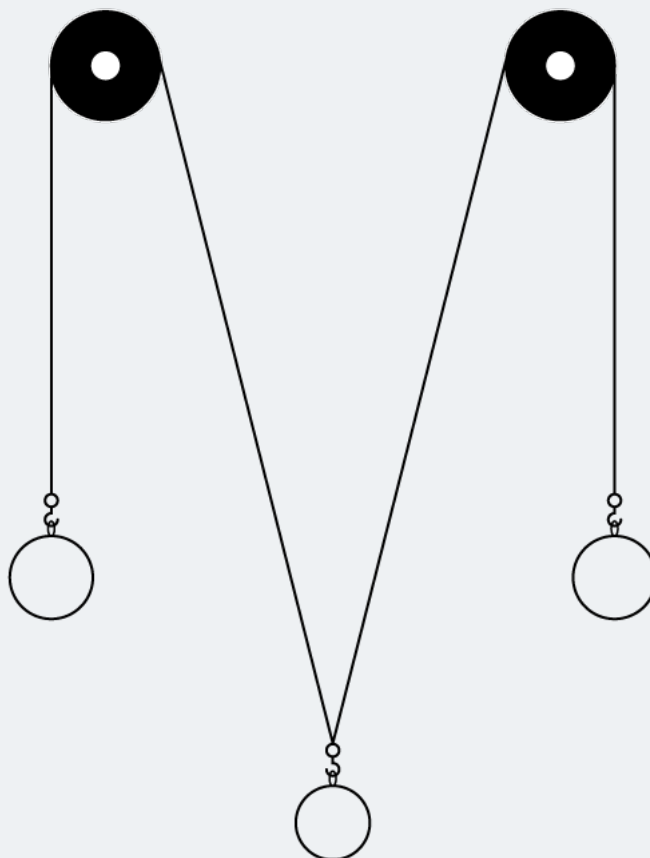
#### Aim

Determine the resultant of three non-linear forces using a force board.

#### Apparatus and materials

You will need:

- blank paper
- force board
- 4 spring balances
- assortment of weights
- gut or string
- four pulleys



## INFORMAL EXPERIMENT (continued)

### Method

Before beginning the detailed method think about the strategy. By connecting a cord to the ring, running it over a pulley and hanging weights off it you can get a force exerted on the ring. The more weights or heavier the weight you hang the greater the force. The force is in the direction of the cord. If you run a number of cords over pulleys you are exerting more forces, all in different directions, on the ring. So we have a system where we can change the magnitude and direction of the forces acting on the ring. By putting a spring balance between the cord and the ring, we can measure the force. Putting a piece of paper under the ring allows us to draw the directions and the readings on the spring balances allows us to measure the magnitude.

We are going to use this information to measure the forces acting on the rings and then we can determine the resultants graphically.

1. Set up the forceboard and place a piece of paper under the ring.
2. Set up four different forces by connecting a spring balance to the ring on the one side and some cord on the other side. Run the cord over a pulley and attach some weights to it. Work in a group to do this effectively.
3. Draw a line along each cord being careful not to move any of them.
4. Make a note of the reading on each spring balance.
5. Now remove the paper.
6. Working on the paper, draw each line back towards the where the centre of the ring had been. The lines should all intersect at a point. Make this point the centre of your Cartesian coordinate system.
7. Now choose an appropriate scale to relate the length of arrows to the readings on the spring balances. Using the appropriate spring balance and correct line on the paper, draw an arrow to represent each of the forces.

### Results

For two different choices of 3 of the force vectors we will determine the resultant. To determine the resultant we need to add the vectors together. The easiest way to do this is to replicate the vectors using a ruler and protractor and draw them tail-to-head.

### Conclusions and questions

Note the direction and the magnitudes of the resultants of the various combinations.

### INFORMAL EXPERIMENT (continued)

1. How does the calculated resultant compare to the vector that wasn't used to calculate the resultant in each case?
2. What general relationship should exist between the resultant and the fourth vector and why do you think this is the case?
3. Would this be the same if we had more or less forces in the problem? Justify your answer.

## 4 CHAPTER SUMMARY

- A vector has a magnitude and direction.
- Vectors can be used to represent many physical quantities that have a magnitude and direction, like forces.
- Vectors may be represented as arrows where the length of the arrow indicates the magnitude and the arrowhead indicates the direction of the vector.
- Vectors in two dimensions can be drawn on the Cartesian plane.
- Vectors can be added graphically using the head-to-tail method or the tail-to-tail method.
- A closed vector diagram is a set of vectors drawn on the Cartesian using the tail-to-head method and that has a resultant with a magnitude of zero.
- Vectors can be added algebraically using Pythagoras' theorem or using components.
- The direction of a vector can be found using simple trigonometric calculations.
- The components of a vector are a series of vectors that, when combined, give the original vector as their resultant.
- Components are usually created that align with the Cartesian coordinate axes. For a vector  $\vec{F}$  that makes an angle of  $\theta$  with the positive  $x$ -axis the  $x$ -component is  $\vec{R}_x = R \cos(\theta)$  and the  $y$ -component is  $\vec{R}_y = R \sin(\theta)$

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## 5 EXERCISES

### 5.1 Exercise 1

1. Draw the following force as a vector on the Cartesian plane using a scale 1 N : 1 cm.

$$\vec{F}_1 = 2 \text{ N at } 45^\circ \text{ to the positive } x\text{-axis.}$$

2. Draw the following force as a vector on the Cartesian plane using a scale 1 N : 2 cm.

$$\vec{F}_1 = 3 \text{ N at } -50^\circ \text{ to the positive } x\text{-axis.}$$

3. Draw the following force as a vector on the Cartesian plane using a scale 1 N : 0,5 cm.

$$\vec{F}_1 = 5 \text{ N at } -45^\circ \text{ to the positive } y\text{-axis.}$$

4. Draw the following force as a vector on the Cartesian plane using a scale 1 N : 0,5 cm.

$$\vec{F}_1 = 10 \text{ N at } 30^\circ \text{ to the positive } y\text{-axis.}$$

### 5.2 Exercise 2

1. Find the resultant in the  $x$ -direction,  $\vec{R}_x$  and  $y$ -direction,  $\vec{R}_y$  for the following forces. Use the scale 1 N : 1 cm.

$$\vec{F}_1 = 3 \text{ N, positive } x\text{-direction.}$$

$$\vec{F}_2 = 4 \text{ N, negative } x\text{-direction.}$$

$$\vec{F}_3 = 2 \text{ N, positive } x\text{-direction.}$$

$$\vec{F}_4 = 1,5 \text{ N, positive } y\text{-direction}$$

$$\vec{F}_5 = 3 \text{ N, negative } y\text{-direction.}$$

2. Find the resultant in the  $x$ -direction,  $\vec{R}_x$  and  $y$ -direction,  $\vec{R}_y$  for the following forces. Use the scale 1 N : 1 cm.

$$\vec{F}_1 = 4 \text{ N, positive } x\text{-direction.}$$

$$\vec{F}_2 = 2 \text{ N, negative } x\text{-direction.}$$

$$\vec{F}_3 = 3 \text{ N, negative } y\text{-direction.}$$

3. Find a force in the  $x$ -direction,  $F_x$  and  $y$ -direction,  $F_y$ , that you can add to the following forces to make the resultant in the  $x$ -direction,  $\vec{R}_x$  and  $y$ -direction,  $\vec{R}_y$  zero.

$$\vec{F}_1 = 0,5 \text{ N, negative } x\text{-direction.}$$

$$\vec{F}_2 = 1,5 \text{ N, positive } x\text{-direction.}$$

$$\vec{F}_3 = 2 \text{ N, positive } y\text{-direction.}$$

$$\vec{F}_4 = 3,5 \text{ N, negative } y\text{-direction.}$$

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4. Find a force in the  $x$ -direction,  $F_x$  and  $y$ -direction,  $F_y$ , that you can add to the following forces to make the resultant in the  $x$ -direction,  $\vec{R}_x$  and  $y$ -direction,  $\vec{R}_y$  zero.

$$\vec{F}_1 = 2,5 \text{ N, positive } x\text{-direction.}$$

$$\vec{F}_2 = 3,5 \text{ N, negative } y\text{-direction.}$$

$$\vec{F}_3 = 4 \text{ N, positive } y\text{-direction.}$$

5. Draw a force of 5 N bearing at  $45^\circ$ .  
Use scale 1 N : 1 cm.
6. Draw a force of 3 N bearing at  $90^\circ$ .  
Use scale 1 N : 1 cm.
7. Draw a force of 6,5 N bearing at  $230^\circ$ .  
Use scale 1 N : 1 cm.
8. Draw a force of 2,5 N bearing at  $310^\circ$ .  
Use scale 1 N : 1 cm.
9. Draw the following forces on the same axis.  
Use scale 10 N : 1 cm.  
 $\vec{F}_1 = 30 \text{ N bearing at } 110^\circ$   
 $\vec{F}_2 = 80 \text{ N bearing at } 90^\circ$

### 5.3 Exercise 3

1. Sketch the resultant of the following force vectors using the tail-to-head method. Use the scale 1 N : 1 cm.  
 $\vec{F}_1 = 5 \text{ N in positive } y\text{-direction}$   
 $\vec{F}_2 = 10 \text{ N in negative } x\text{-direction}$
2. Sketch the resultant of the following force vectors using the tail-to-head method. Use the scale 1 N : 1 cm.  
 $\vec{F}_1 = 6 \text{ N in positive } y\text{-direction}$   
 $\vec{F}_2 = 8 \text{ N in positive } x\text{-direction}$
3. Sketch the resultant of the following force vectors using the tail-to-head method. Use the scale 1 N : 1 cm.  
 $\vec{F}_1 = 7 \text{ N in positive } y\text{-direction}$   
 $\vec{F}_2 = 1 \text{ N in positive } x\text{-direction}$   
 $\vec{F}_3 = 8 \text{ N in negative } y\text{-direction}$   
 $\vec{F}_4 = 5 \text{ N in negative } x\text{-direction}$
4. Sketch the resultant of the following force vectors using the tail-to-head method. Use the scale 1 N : 1 cm.  
 $\vec{F}_1 = 10 \text{ N in positive } y\text{-direction}$   
 $\vec{F}_2 = 2 \text{ N in positive } x\text{-direction}$   
 $\vec{F}_3 = 4 \text{ N in negative } y\text{-direction}$   
 $\vec{F}_4 = 6 \text{ N in negative } x\text{-direction}$

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5. Sketch the resultant of the following force vectors using the tail-to-head method by first determining the resultant in the  $x$ - and  $y$ -directions. Use the scale 1 N : 1 cm.

$$\vec{F}_1 = 5 \text{ N in positive } y\text{-direction}$$

$$\vec{F}_2 = 3 \text{ N in positive } x\text{-direction}$$

$$\vec{F}_3 = 9 \text{ N in negative } y\text{-direction}$$

6. Sketch the resultant of the following force vectors using the tail-to-head method by first determining the resultant in the  $x$ - and  $y$ -directions. Use the scale 1 N : 1 cm.

$$\vec{F}_1 = 1 \text{ N in positive } x\text{-direction}$$

$$\vec{F}_2 = 8 \text{ N in positive } y\text{-direction}$$

$$\vec{F}_3 = 2 \text{ N in negative } x\text{-direction}$$

7. Sketch the resultant of the following force vectors using the tail-to-head method by first determining the resultant in the  $x$ - and  $y$ -directions. Use the scale 1 N : 1 cm.

$$\vec{F}_1 = 3 \text{ N in positive } x\text{-direction}$$

$$\vec{F}_2 = 8 \text{ N in positive } y\text{-direction}$$

$$\vec{F}_3 = 2 \text{ N in negative } x\text{-direction}$$

$$\vec{F}_4 = 5 \text{ N in negative } y\text{-direction}$$

## 5.4 Exercise 4

1. A force of 17 N in the positive  $x$ -direction acts simultaneously with a force of 23 N in the positive  $y$ -direction. Calculate the resultant force.

2. A force of 23,7 N in the negative  $x$ -direction acts simultaneously with a force of 9 N in the positive  $y$ -direction. Calculate the resultant force.

3. Four forces act simultaneously at a point, find the resultant if the forces are:

$$\vec{F}_1 = 2,3 \text{ N in the positive } x\text{-direction}$$

$$\vec{F}_2 = 4 \text{ N in the positive } y\text{-direction}$$

$$\vec{F}_3 = 3,3 \text{ N in the negative } y\text{-direction}$$

$$\vec{F}_4 = 2,1 \text{ N in the negative } y\text{-direction}$$

4. The following forces act simultaneously on a pole, if the pole suddenly snaps in which direction will it be pushed:

$$\vec{F}_1 = 2,3 \text{ N in the negative } x\text{-direction}$$

$$\vec{F}_2 = 11,7 \text{ N in the negative } y\text{-direction}$$

$$\vec{F}_3 = 6,9 \text{ N in the negative } y\text{-direction}$$

$$\vec{F}_4 = 1,9 \text{ N in the negative } y\text{-direction}$$



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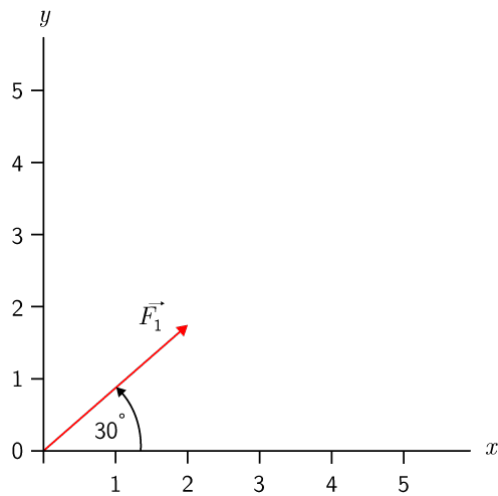
## 5.5 Exercise 5

1. A force of 250 N acts at an angle of  $30^\circ$  to the positive  $x$ -axis. Resolve this force into components parallel to the  $x$ - and  $y$ -axes.
2. A force of 12,5 N acts at an angle of  $230^\circ$  to the positive  $x$ -axis. Resolve this force into components parallel to the  $x$ - and  $y$ -axes.
3. Resolve each of the following vectors into components:
  - 3.1  $\vec{F}_1 = 5 \text{ N}$  at  $45^\circ$  to the positive  $x$ -axis
  - 3.2  $\vec{F}_2 = 15 \text{ N}$  at  $63^\circ$  to the positive  $x$ -axis.
  - 3.3  $\vec{F}_3 = 11,3 \text{ N}$  at  $127^\circ$  to the positive  $x$ -axis.
  - 3.4  $\vec{F}_4 = 125 \text{ N}$  at  $245^\circ$  to the positive  $x$ -axis.
4. Resolve each of the following vectors into components:
  - 4.1  $\vec{F}_1 = 11 \times 10^4 \text{ N}$  at  $33^\circ$  to the positive  $x$ -axis
  - 4.2  $\vec{F}_2 = 15 \text{ GN}$  at  $28^\circ$  to the positive  $x$ -axis.
  - 4.3  $\vec{F}_3 = 11,3 \text{ kN}$  at  $193^\circ$  to the positive  $x$ -axis.
  - 4.4  $\vec{F}_4 = 125 \times 10^5 \text{ N}$  at  $317^\circ$  to the positive  $x$ -axis.
5.  $\vec{F}_1 = 5,385 \text{ N}$  at an angle of  $21,8^\circ$  to the horizontal and  $\vec{F}_2 = 5 \text{ N}$  at an angle of  $53,13^\circ$  to the horizontal, find the resultant force,  $\vec{R}$

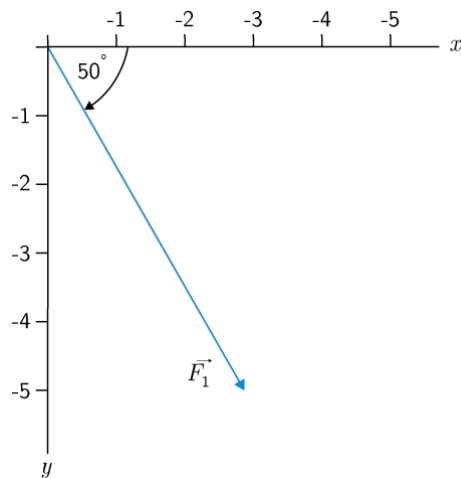
## 6 ANSWERS FOR EXERCISES

### 6.1 Exercise 1

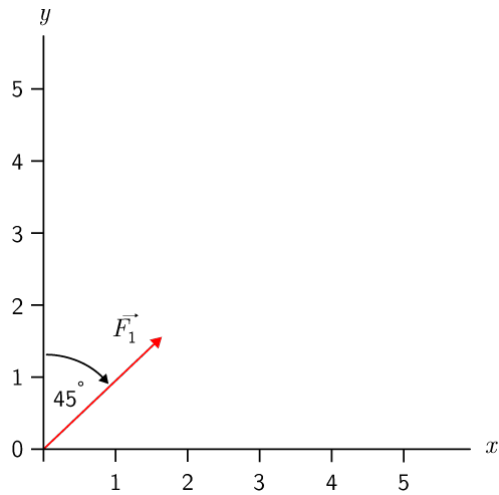
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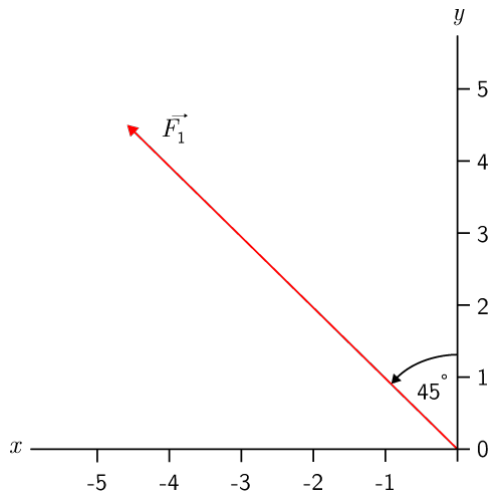
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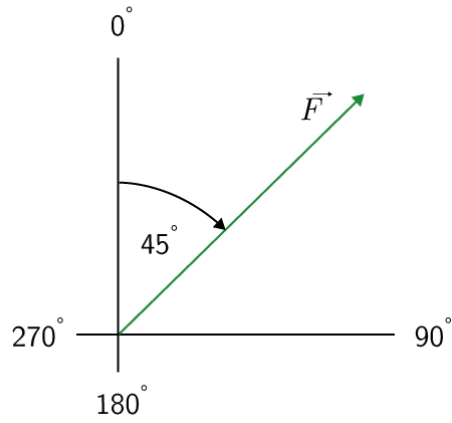


## 6.2 Exercise 2

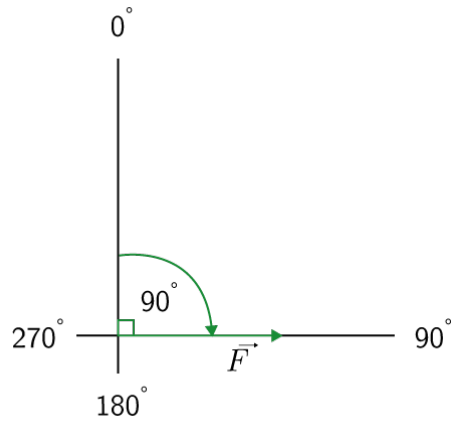
- $\vec{R}_x = 1 \text{ N}$ , in the positive  $x$ -direction.  
 $\vec{R}_y = 1,5 \text{ N}$ , in the negative  $y$ -direction.
- $\vec{R}_x = 2 \text{ N}$ , in the positive  $x$ -direction.  
 $\vec{R}_y = 3 \text{ N}$ , in the negative  $y$ -direction.
- $\vec{F}_x = 1 \text{ N}$ , in the negative  $x$ -direction.  
 $\vec{F}_y = 1,5 \text{ N}$ , in the positive  $y$ -direction.

4.  $\vec{F}_x = 2,5 \text{ N}$ , in the negative  $x$ -direction.  
 $\vec{F}_y = 0,5 \text{ N}$ , in the negative  $y$ -direction.

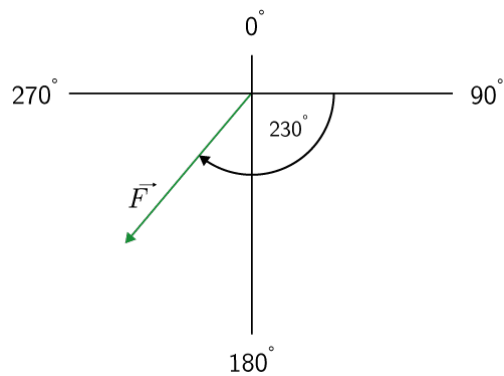
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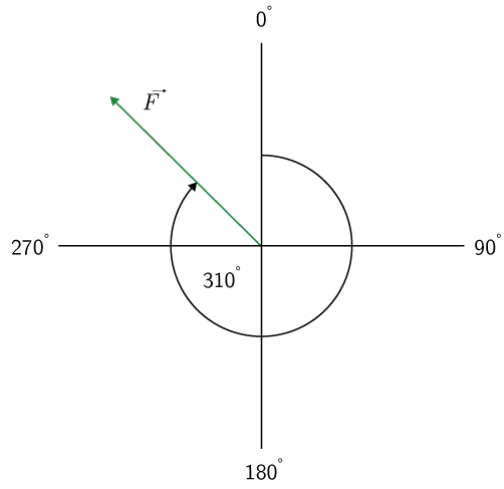
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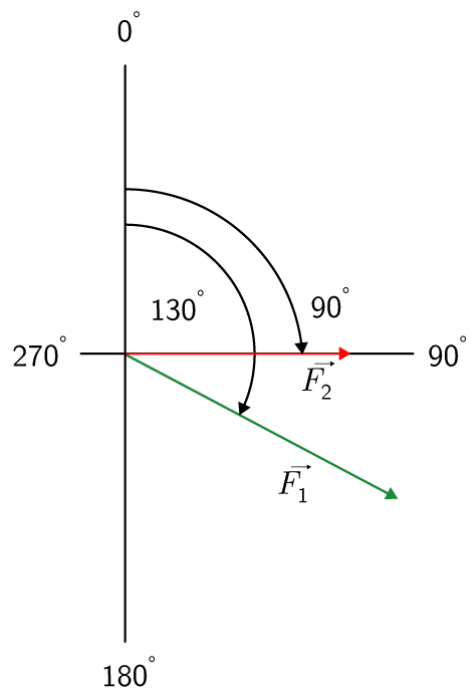
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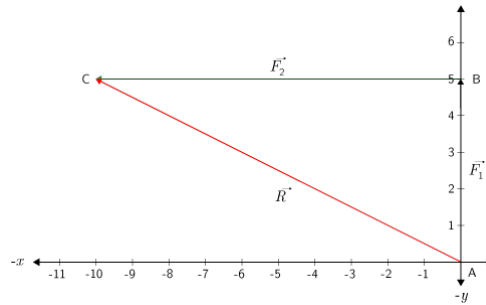


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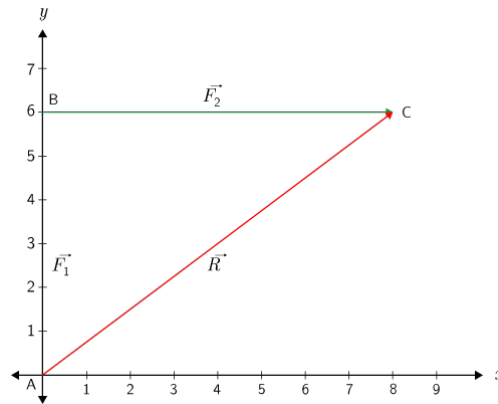


### 6.3 Exercise 3

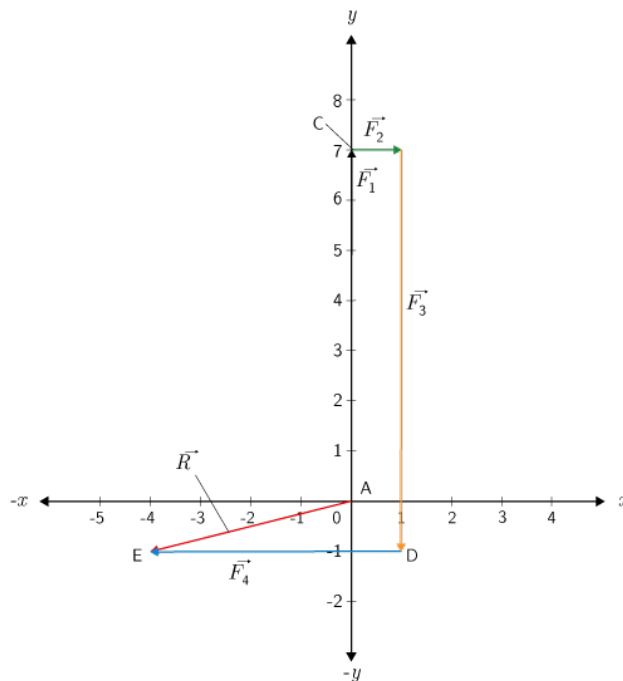
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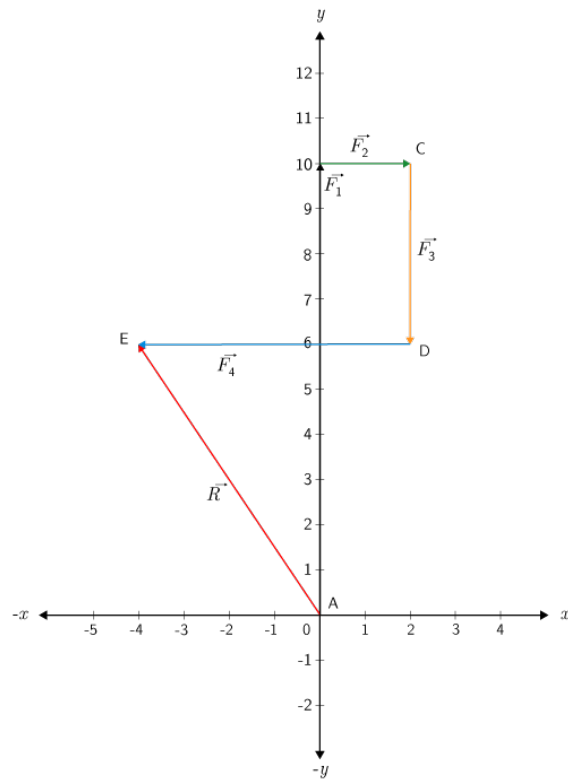
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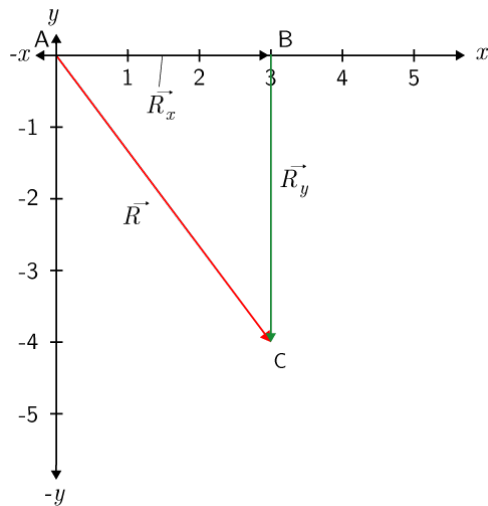
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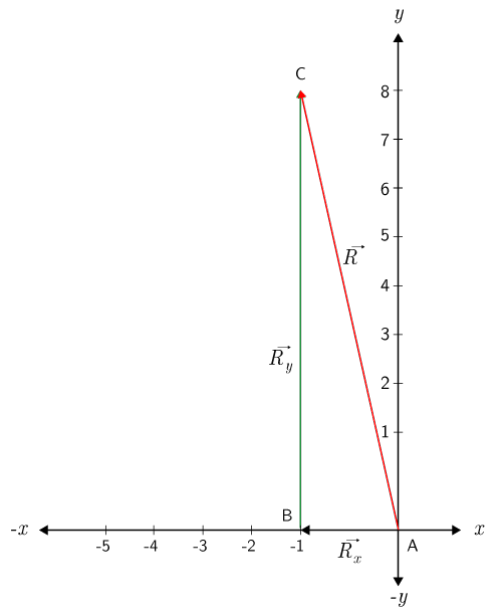
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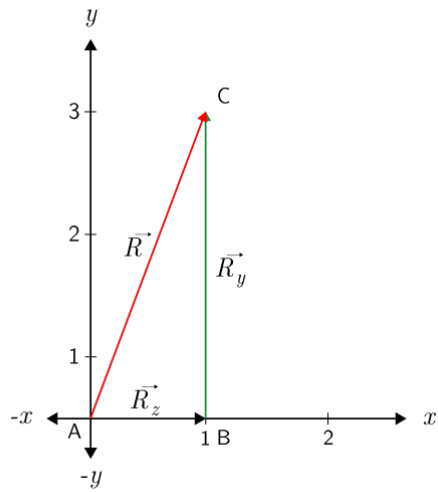
5.



6.



7.





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## 6.4 Exercise 4

1. 28,6 N at  $53,53^\circ$  to the positive  $x$ -axis.
2. 25,4 N at  $159,21^\circ$  to the positive  $x$ -axis.
3. 2,69 N at  $328,67^\circ$  to the positive  $x$ -axis.
4. 20,63 N at  $263,6^\circ$  to the positive  $x$ -axis.

## 6.5 Exercise 5

1.  $F_x = 216,51$  N and  $F_y = 125$  N
2.  $F_x = -8,03$  N and  $F_y = -9,58$  N
- 3.1.  $F_x = F_y = 3,54$  N
- 3.2.  $F_x = 6,81$  N and  $F_y = 13,37$  N
- 3.3.  $F_x = -6,80$  N and  $F_y = 9,02$  N
- 3.4.  $F_x = -113,29$  N and  $F_y = -52,83$  N
- 4.1.  $F_x = 9,22 \times 10^4$  N and  $F_y = 5,99 \times 10^4$  N
- 4.2.  $F_x = 13,2$  GN and  $F_y = 7,04$  GN
- 4.3.  $F_x = -11,01$  kN and  $F_y = -2,54$  kN
- 4.4.  $F_x = 9,14 \times 10^6$  N and  $F_y = -8,52 \times 10^6$  N
5. 10 N at  $36,9^\circ$  to the positive  $x$ -axis.