A Guide to Vectors 2 Dimensions

Teaching Approach

For Grade 11, it will help to begin with concepts of displacement and velocity that the students may have done in Grade 10. Videos 1 and 2 will remind them how vectors can be added, and what a resultant vector is.

CAPS for Grade 11 emphasises force as a vector quantity; this can help you because you can ask students to push or pull objects and they can sense in their arms that force has both magnitude and direction, and most importantly, that two forces in different directions can result in a force in a new direction. For example, if two students pull a desk in directions that are at right angles to each other, the desk will move in a direction that is in-between the two directions they are pulling. They can watch this principle at work as they use the forces board in Lesson 3: two strings pulling sideways in different directions produce a resultant force that acts vertically upward.

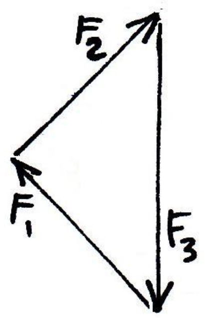
The videos have only a few questions in them so make use of the task video to ensure that the students exercise the ideas you are teaching them through the Mindset lesson videos. Where the video shows a PAUSE sign, stop the video. Repeat the question for the class, pause the video at the relevant graphic, and give them enough time to work on the question. By "enough time", we mean about 5 to 10 minutes. This could be time enough for them to draw sketches and discuss the problem with a partner.

Learning difficulties and misconceptions

1. In Lesson 3, you will need a forces board, similar to the one you see here. You fix a sheet of paper to the board and the students trace the position and direction of the strings by drawing on the paper.
   But the lengths of the strings have nothing to do with the tension forces in the strings. The tension force is shown by the **length of the vector arrow** that **represents** the force. So a short string may have large force on it and the long string may have a small force on it. Only the length of the **arrow** matters.

2. After you have marked the positions of the strings, and drawn the lengths of the vector arrows to represent the forces, you may find that your vectors don't quite meet up to form a closed polygon. This is probably due to some hidden friction force that prevents the strings moving quite freely. Check that the pulley-wheels spin easily, and that the weights don't rub against the board.
3. Students find it strange that we can represent forces by vectors — arrows of a certain length and direction — and then treat them as if we were doing geometry and trigonometry. They will get used to the idea after doing a few diagrams and calculations.

But watch for the following source of confusion: in geometry we label the vertices (the corners) of a triangle with capital letters, so we write triangle ABC, and we can write about "side AB". When working with vectors, the whole vector gets **one** label, such as vector F_1. In this diagram, F_1, F_2 and F_3 are three vectors, **not** the vertices of the triangle. So we could not write "side F_1F_2 "because it would have no meaning.



Video Summaries

Some videos have a 'PAUSE' moment, at which point the teacher or learner can choose to pause the video and try to answer the question posed or calculate the answer to the problem under discussion. Once the video starts again, the answer to the question or the right answer to the calculation is given

Mindset suggests a number of ways to use the video lessons. These include:

- Watch or show a lesson as an introduction to a lesson
- Watch or show a lesson after a lesson, as a summary or as a way of adding in some interesting real-life applications or practical aspects
- Design a worksheet or set of questions about one video lesson. Then ask learners to watch a video related to the lesson and to complete the worksheet or questions, either in groups or individually
- Worksheets and questions based on video lessons can be used as short assessments or exercises
- Ask learners to watch a particular video lesson for homework (in the school library or on the website, depending on how the material is available) as preparation for the next day's lesson; if desired, learners can be given specific questions to answer in preparation for the next day's lesson

1. Relative Velocity Vectors

The lesson recaps Grade 10 vectors in one dimension, and then introduces motion in two dimensions. A boat full of children is crossing a river. The boat has a velocity but the river has its own velocity. Where does the boat end up?

2. Vectors of Relative Motion

In lesson 2 the boatman finds a way to steer the boat so that its resultant velocity is straight across the river. Students use Pythagoras's theorem and trig ratios to calculate the magnitude of the resultant vector.

3. Forces in Equilibrium

This lesson looks at the classic demonstration of forces in equilibrium, using a forces board. The concept of the resultant force and the polygon of forces emerge from the activity. Again, the students have to deal with the addition of vectors.

4. Calculating Vectors using Pythagoras

In this lesson the students see that while they must add vectors to find a resultant vector, when the vectors form a right-angled triangle, then Pythagoras can be used to calculate the magnitudes of the vectors.

5. Resolution of Vectors into Components

Resolve a vector into its component vectors in a right-angled co-ordinate system. If the angle θ between the vector R and one of the axes is known, the component vectors are $R\cos\theta$ along the x-axis and $R\sin\theta$ along the y-axis.

Resource Material

1. Relative Velocity Vectors	http://cnx.org/content/m14030/latest/	A resource on relative velocity in two dimensions.
	http://www.examsolutions.net/maths-revision/mechanics/vectors/relative-velocity/tutorial-1.php	A video introducing relative velocity.
2. Vectors of Relative Motion	https://www.youtube.com/watch?v=nxF47gTE2yU	This video discusses relative motion.
	http://www.physicstutorials.org/home/mechanics/1d-kinematics/relative-motion	Relative motion with examples.
3. Forces in Equilibrium	http://fiziknota.blogspot.com/2009/05/analysing-forces-in-equilibrium.html	Analyzing forces in equilibrium.
	https://www.uwstout.edu/physics/upload/Force-Equilibrium.pdf	A laboratory activity demonstrating the conditions under which an object is held in static equilibrium due to concurrent forces.
	http://www.physics.arizona.edu/physics/gdresources/documents/03_Vectors.pdf	A study of vectors in the context of force
4. Calculating Vectors using Pythagoras	http://www.mathsisfun.com/algebra/vectors.html	All about calculating vectors.
5. Resolution of Vectors into Components	http://www.wikihow.com/Resolve-a-Vector-Into-Components	How to resolve a vector into components.

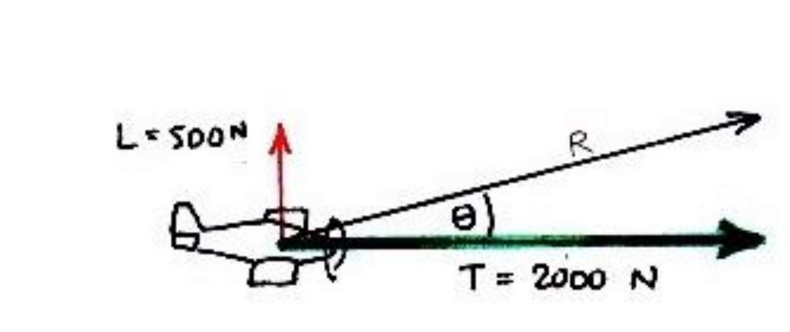
Task

Question 1

Question one. You remember the aeroplane that accelerated and climbed? Here's the aeroplane again . . . look at the sketch.

The thrust vector is 2 000 N and the lift vector is 500 N.

Do a scale drawing of the vectors and measure the resultant R and the angle of climb ɵ. In your drawing, use a scale of 1 cm to 200 N. You'll need a protractor to measure the angle ɵ.



Question 2

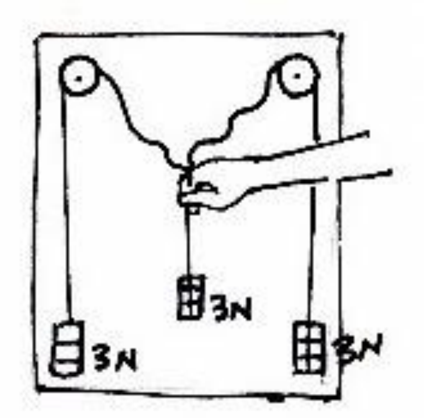
2.1 Work out the resultant force on the aeroplane using the theorem of Pythagoras.

2.2 Pythagoras does not work for all triangles. Why can you use Pythagoras on this triangle?

Question 3

Here is the forces board again, and you can see a hand holding the middle weight up.

[Use task video Fig 2]



The weights on the three strings are each 3N. When the weight in the middle can hang freely, in what directions will the strings lie?

3.1 First do a rough sketch of the strings as you think they will lie when the forces are acting on them

3.2 Sketch the force vectors acting at the knot in the middle. What do you know about these three vectors?

3.3 The angles between the strings are equal. Think of a reason why that is so.

3.4 Now do a scale drawing of the vectors, using a scale of 3 cm to 1 N.

3.5 Draw the resultant that will balance the force that acts vertically downward.

3.6 Draw the resultant and two other vectors head to tail to show how the two vectors together cause the resultant.

3.7 Draw the polygon of forces that act around the knot. In this case, the polygon has three sides.

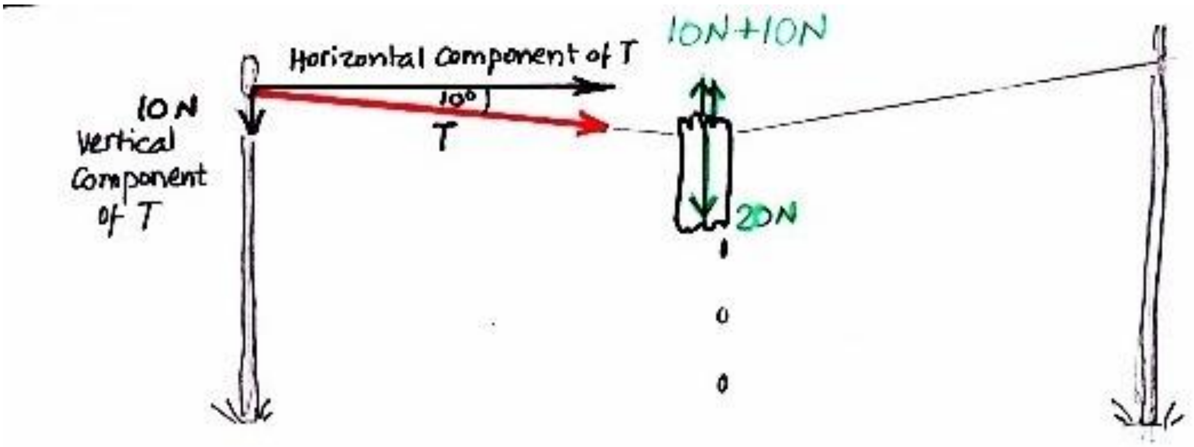
3.8 What are the angles between the vectors?

Question 4

In this question you must explain why wet washing on a line can pull down the poles of the washing line. Look at the diagram.

The line is made of wire, that won't stretch. When there is no washing on the wire, the wire is almost straight. But now look at this diagram [below]. . . when a wet towel that weighs 20 N is hung in the middle of the line, the poles bend a little and the wire makes an angle of 10 ˚ with the horizontal.

Vector T is the tension in the wire. T has two component vectors, a vertical component of 10 N and a horizontal component that you must work out. The vertical downward component of 10 N is caused by one half of the weight of the towel. The other half of the towel's weight causes another vertical component at the other end of the wire

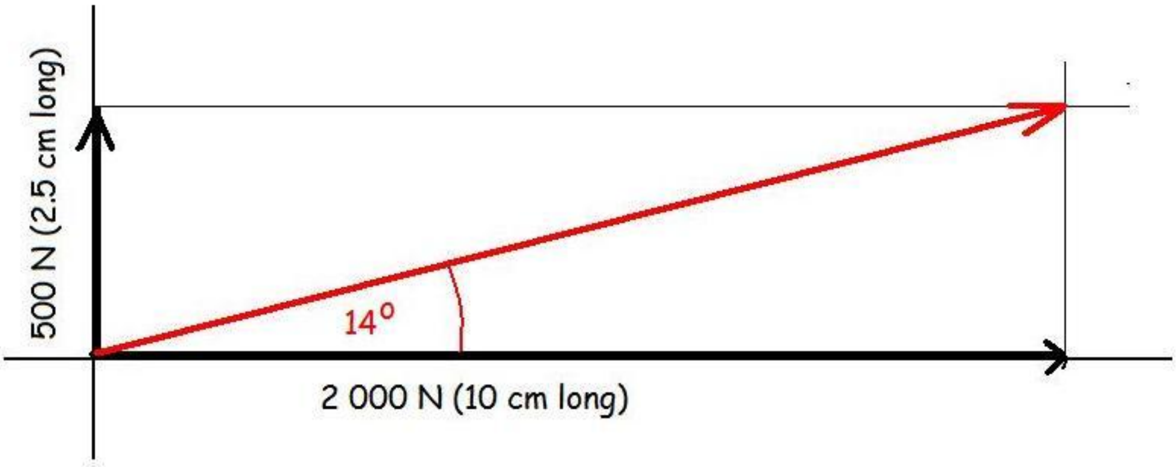


4.1 First of all, sketch the situation for yourself. On your sketch fill in all the information you have about this system.
4.2 What is the horizontal component of the tension in the wire? Here's a hint . . . if you can find a right-angled triangle in your sketch, then you can use a trig function. Try the sine or tan functions. You'll need a calculator.
4.3 Compare the magnitude of the horizontal force that tries to bend the pole with the 10 N vertical force that helps to hold the towel up.
4.4 Use the same method to work this out: if you made the wire tighter, so that it sagged only <u>half</u> as much, that is, only 5 ˚, would you have to <u>double</u> the horizontal force?
4.5 Work out the tension T in the wire when the wire sags at an angle of 10 ˚ and the wire must hold up 10 N of the weight of the wet towel. You will need a different trig function for this.

Task Answers

Question 1

You should be able to find that the angle is 14°, measuring with your protractor.
Make sure that the angle between vector L and vector T is a right angle. Use your protractor to check.



Question 2

2.1 $R^2 = 500^2 + 2000^2$. $\therefore R = \sqrt{500^2 + 2000^2}$ $\therefore R = \sqrt{250000 + 4\,000\,000}$

$\therefore R = 2061.6$ newtons

2.2 This is a right-angled triangle. The lift vector L is vertical and the thrust vector T is horizontal.

Question 3

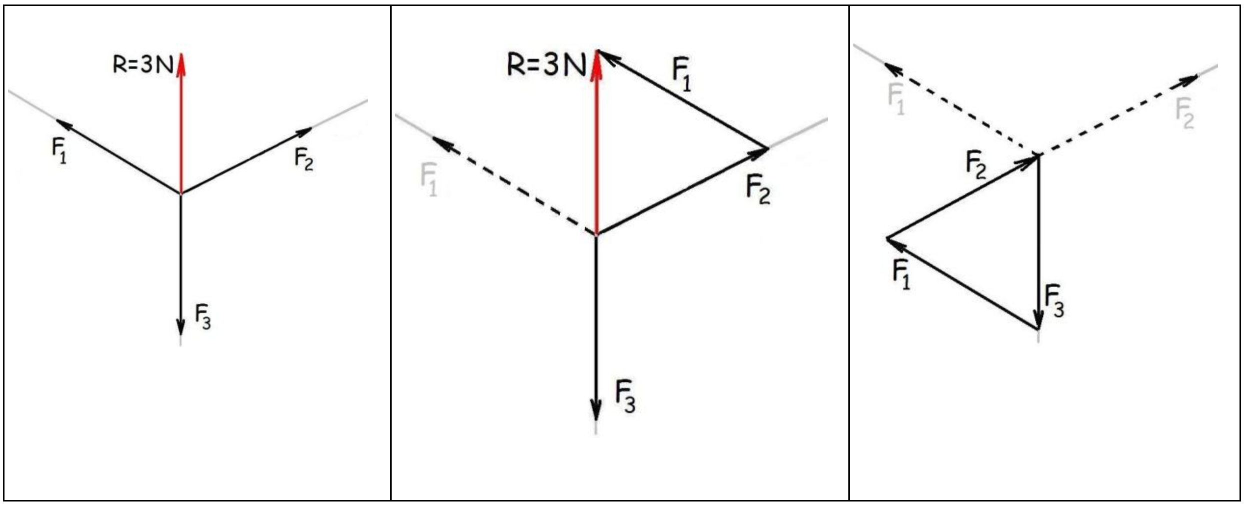
3.1 The sketch should just show the directions of the strings. The strings may be any length

3.2 The sketch will show three vectors the same length. They are equal in length because the tension force in each string is 3 N.

3.3 The forces in all directions are equal – each force is 3 N. If one force were bigger, it would move the knot in the direction of the bigger force and the angle between the remaining two forces would become smaller.

3.4 The vectors will be 3 cm long and at 120° to each other.

3.5. Resultant must be 3 N upward.	3.6. Vector F1 can be moved parallel to itself. Adding vector F1 to vector F2 gives the resultant R	3.7. F1, F2 and F3 can be added head to tail and they form a closed polygon.



3.5 The resultant will be 3 cm long and point upwards.

3.6 Move vector F_1 parallel to itself.

3.7 The polygon will be an equilateral triangle. Each interior angle will be 60°.

3.8 The angles must be 120° because there are only three angles, they are equal, and they must share 360° equally between them.

Question 4

4.1 The student's sketch will look much like the sketch in the video. The purpose of redrawing it is to become familiar with the situation and the quantities that are given.

4.2 $\frac{10}{H}$ = tan 10° 10 = H tan 10° $\frac{10}{\tan 10°}$ = H H = $\frac{10}{0.176}$

H = 56,7 N. The answer must be in newtons because H is a force.

4.3 56,7 N is almost six times the 10 N half-weight of the towel. (The towel weighs 20 N but each half of the wire holds up half the weight. That is why the diagram shows the upward force on the towel as 10 N + 10 N.)

4.4 $\frac{10}{H}$ = tan 5° 10 = H tan 5° $\frac{10}{\tan 5°}$ = H H = $\frac{10}{0.087}$

H = 11,3 N. So the answer is no, if you want to halve the angle between the wire and the horizontal, then you must increase the horizontal force component from 56,7 N to 114,3 N, which is **more** than double 56,7 N. So when you stretch a washing-line very tightly between the poles, you will put a very big force on the poles as soon as you put wet washing on the line. The force will be very much bigger than the weight of the washing.

4.5 $\frac{10}{T}$ = sin 10° 10 = T sin10° $\frac{10}{\sin 10°}$ = T T = $\frac{10}{0.174}$

T = 57,5 N The answer must be in newtons because T is a force.

You could have used $\frac{H}{T}$ = cos 10° but then if you'd calculated H incorrectly, your answer for T would also be incorrect. You were given the information that the downward component is 10 N, so use that instead.

Also do a common-sense check on your answer. When you look at the diagram, ask yourself if your answer for vector T could ever be shorter than the length of vector H?
No, it could not be shorter, because T is the hypotenuse of the triangle, and the hypotenuse is always longer than the other two sides.

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