Grade 12
Physical Sciences

Mechanics
(Vertical Projectile Motion)

Lesson Description

In this lesson we will:

- Learn that all projectiles fall freely under gravity and accelerate at "g" whether they are moving up or down.
- Use simple equations of motion to describe their motion.
- Look at using graphs of displacement, velocity and acceleration vs. time to show the flight of projectiles.
- Discuss using a structured approach to solve projectile motion questions and graphs.

Key Concepts

This lesson focuses on the following key concepts:

- Force of gravity
- Gravitational acceleration
- Equations of motion

Terminology & Definitions:

- **Acceleration due to gravity:** is the acceleration experienced by a projectile due to the gravitational force that acts on it.
- **Gravitational force:** is the force of attraction between any two objects in the universe. In the case of an object on earth, it will be the force with which the earth, as the bigger object, will attract the object towards its centre. This gravitational force is called the weight of an object and is measured in Newton (N).
- **Height:** the vertical displacement of an object.

Explanations & Notes:

The Gravitational Force

The gravitational force acting on an object during its motion is found by using the following formula:

$$F_g = ma$$

Where: F_g – gravitational force acting on object

m – mass of object

a – acceleration due to gravity (9,8 m.s^{-2})

Projectile Motion

Vertical projectile motion is the motion of an object in an up or down direction. An object in an up or down motion is called a projectile. The only force that acts on a projectile is the force due to gravity (gravitational force). This force is equal to the weight of the object. Since the movement of a projectile is in an up or down motion, its displacement is in a vertical direction and, therefore, the symbol used for it is Δy. All projectiles experience the same acceleration during motion. This acceleration is acceleration due to gravity, which is 9,8 m.s^{-2} in a downward direction (towards the centre of the earth).

When an object moves in an up or down motion, it is said to be moving in one dimension. The following equations are used in solving vertical projectile motion problems:

$$V_f = V_i + a\Delta t$$

$$V_f^2 = V_i^2 + 2a\Delta y$$

$$\Delta y = V_i\Delta t + \tfrac{1}{2}a\Delta t^2$$

$$\Delta y = \tfrac{1}{2}(V_f + V_i)\Delta t$$

Notice that Δy is used for the displacement since the object moves in a vertical direction.

Graphs of projectile motion

The acceleration due to gravity for a projectile is constant and equal to g = 9,8 m.s^{-2} downwards. This applies when a projectile is moving up or down. We must assign a direction to motion. It is usual to make up positive and, therefore, g = − 9,8 m.s^{-2}. The acceleration versus time graph for this motion is always the same as shown below:
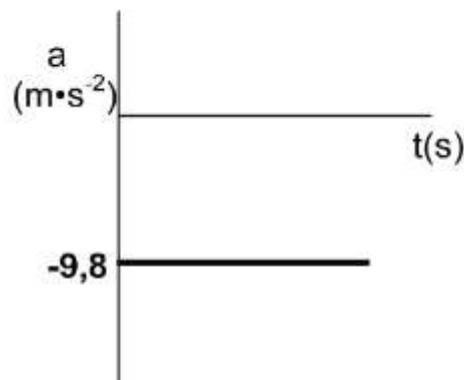
A stone is thrown upwards from a particular height, h, above the ground. A sketch of the position versus time for the motion of the stone is drawn below:
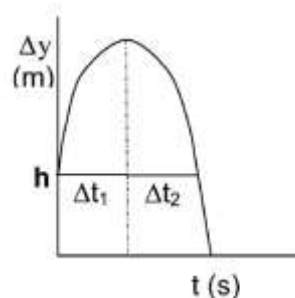
Notice from the graph that the time the stone takes to reach its maximum height above the ground, Δt_1, equals Δt_2, the time it takes to fall from its maximum height and return to the point of projection, h. The graph below is the velocity versus time graph of the stone thrown up from h:
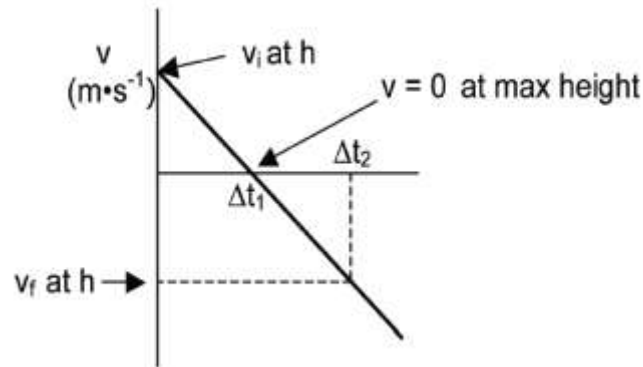
Notice from the graph that when the stone reaches its maximum height, its velocity becomes zero. Also notice that the velocity with which the stone reaches the point of projection equals the velocity with which it was projected. However, the two velocities are in opposing directions. Thus, if the velocity of the stone in the upwards direction is taken to be positive, its velocity as it falls back to its point of projection will be negative as shown in the graph.

GRAPH OF VELOCITY VS TIME
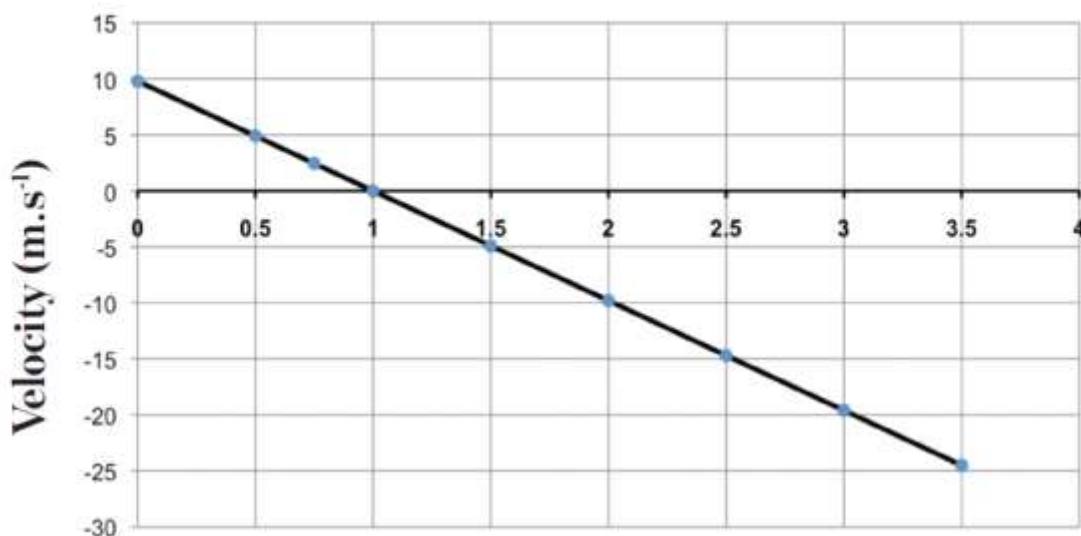
Answering Vertical Projectile Motion Questions

ALWAYS draw a diagram of the situation and enter all the numerical values onto your diagram. Remember to SELECT A DIRECTION AS POSITIVE OR NEGATIVE!

Equations of motion are equations that are used to describe the motion of a body while experiencing a force as a function of time. These equations apply only to bodies moving linearly (unilaterally) i.e. in one direction with a constant acceleration. The body's motion is considered between two time points, i.e. from one initial time points and its final point in time.

Motion can be describe in different ways:

- Words – when your friend explains his first experience in driving a car and tells you in detail how he struggles to pull off and stop.
- Diagrams – when you draw a sketch to explain am specific movement.
- Graphs – we use three different graphs.
    - Velocity-time graph
    - Acceleration-time graph
    - Position-time graph

METHOD FOR ANSWERING THESE QUESTIONS:

STEP 1:   Write down all the information given in the question.

STEP 2:   Identify which formula to use, i.e. identify the known and unknown quantities.

STEP 3:   Substitute into the equation.

STEP 4:   Interpret the answer.

Use of equations of motion in the vertical direction

A projectile is an object that is given an initial velocity by shooting, throwing, etc. and once launched, the only force acting on it is the force due to gravity. In the absence of air resistance, the object is free falling.

Terminal velocity is reached when the downward force of gravity and the upward force of air resistance are equal, and now the object falls at a constant velocity as a result of there being no resultant force acting in on the object.

Equations of motion (Linear Motion)

$$v_f = v_i + a\Delta t$$

$$\Delta x = v_i\Delta t + \tfrac{1}{2}\, a\Delta t^2$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

Equations of motion (Vertical Projectile Motion)

- **Substitute "g" for "a" and "Δy" for "Δx" in projectile motion problems**

$$v_f = v_i + g\Delta t$$

$$\Delta y = v_i\Delta t + \tfrac{1}{2}\, g\Delta t^2$$

$$v_f^2 = v_i^2 + 2g\Delta y$$

All objects are attracted to the earth with a gravitational force called **weight**.

All objects will accelerated towards with earth with a constant acceleration called the **gravitational acceleration (g)** which has a value of 9,8 m.s^{-2}.

Gravitational acceleration (g) is **ALWAYS** downwards no matter whether the object is being thrown up or falling down.

Important facts about Vertical Projectile Motion:

- At the greatest height of the upward motion, $v_f = 0$ m.s^{-1}.
- The object will take the same time to reach its greatest height from point of upwards launch as the time taken to fall back to point of launch, i.e. $t_{up} = t_{down}$.

- Can have motion described by a single set of equations of motion for the upward and downward motion.
- If the object is being released from rest or being dropped, its initial velocity is 0 m.s^{-1}.

- If the object is being thrown upwards, it must start with a maximum velocity and as it moves up, the velocity decreases until it stops.
- As these are vectors, direction is important.

QUESTIONS: (Vertical Projectile Motion)

1. An object is thrown upwards with a velocity of 24,5 m.s^{-1} and takes 5 s to return to the thrower's hand. Use the information to sketch the following graphs:
   (a)   Displacement vs time
   (b)   Velocity vs time
   (c)   Acceleration vs time

2. A boy standing on top of a building throws a ball vertically upwards from a position 3,5 m above the ground with an initial velocity of 10 m.s^{-1}. Ignore the effects of air resistance and answer the following questions:
   (a)   What is the magnitude and direction of the acceleration of the ball?
   (b)   Calculate the maximum height reached by the ball above the ground.
   (c)   What was the velocity of the ball at its maximum height?
   (d)   Calculate the time taken by the ball to reach its maximum height.
   (e)   How much time did the ball take to reach its maximum position and back to the position from which it was thrown?
   (f)   Calculate the total time taken by the ball to reach the ground.
   (g)   Calculate the velocity with which the ball hits the ground.
   (h)   Draw a rough sketch of the velocity-time graph. Show relevant points on your velocity and time axis.

3. A boy stands at the edge of a cliff. He throws a stone vertically upwards with an initial velocity of 10 m.s⁻¹. The stone strikes the ground at a point below the cliff after 3,5 seconds. The velocity-time graph below was obtained from measurements made during the motion of the stone.

Graph of velocity versus time
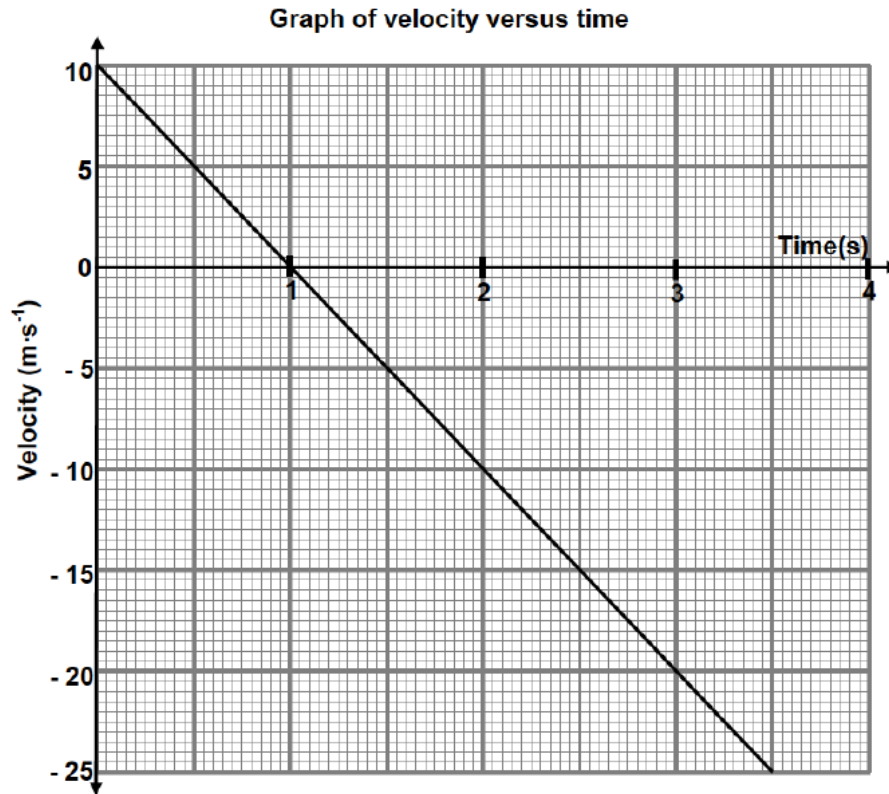


Use the information on the graph to answer the following questions:

(a) Calculate the acceleration of the stone between times $t = 2$ s and $t = 3$ s.

(b) At which time(s) is the stone moving at a speed of 5 m.s⁻¹?

(c) After how many seconds does the stone reach its highest point?

(d) Determine the height of the cliff from which the stone was thrown.

(e) Using the top of the cliff as the initial position of the stone, sketch the position-time graph for motion of the stone from its highest point until it reaches the ground. Only indicate relevant time values on the x-axis.

4. Calculate how fast a ball will be moving if it is allowed to fall for 2 seconds from a stationary point.

5. Calculate how far up a ball will rise if it is thrown upwards at 19,6 m.s⁻¹.

6. A supervisor, 1,8 m tall, visits a construction site. A brick resting at the edge of a roof 50 m above the ground suddenly falls. At the instant when the brick has fallen 30 m the supervisor sees the brick coming down directly towards him from above.
   Ignore the effects of friction and take the downwards motion as positive.
   (a)   Calculate the speed of the brick after it has fallen 30 m.
   (b)   The average reaction time of a human being is 0,4 s. With the aid of a suitable calculation, determine whether the supervisor will be able to avoid being hit by the brick.

7. A ball is released from a certain height. The velocity-time graph below represents the motion of the ball as it bounces vertically on a concrete floor. The interaction time of the ball with the floor is negligibly small and thus ignored.
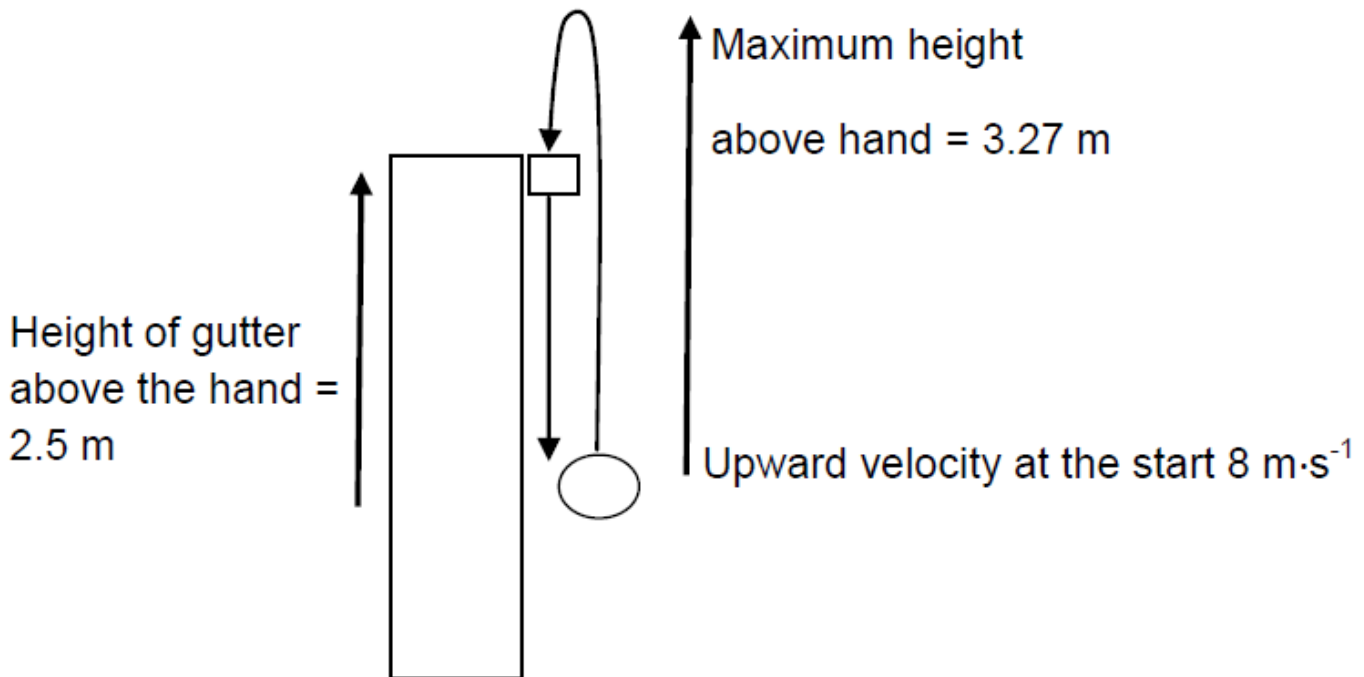

Graph of velocity versus time
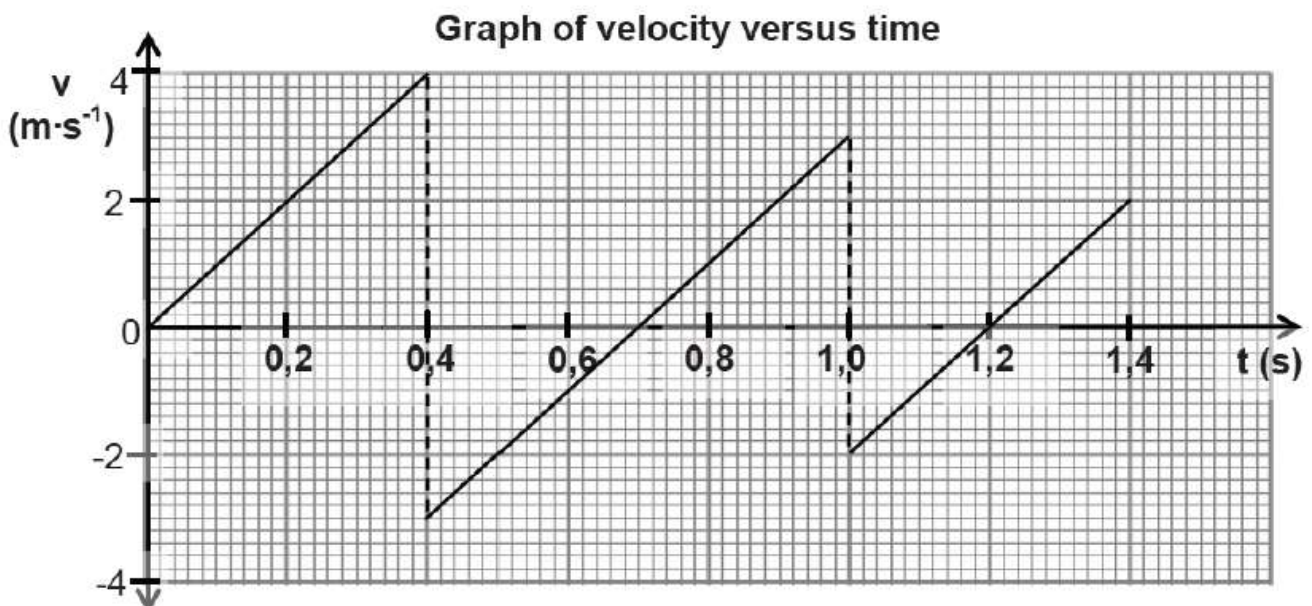
(a) Describe the changes, if any, in velocity and acceleration of the ball from t = 0 s and t = 0,4 s.

(b) Without using the equations of motion, calculate the height from which the ball has been dropped initially.

(c) Use the given velocity-time graph for the motion of the ball to sketch the corresponding position-time graph for the time interval 0 s to 0,7 s.

8. A cross-bow (bow and arrow) is used to shoot an arrow vertically upwards into the air from the top of an 80 m high platform. The arrow reaches a height of 15 m above the platform and then falls to the ground below. Ignore the effects of air friction.
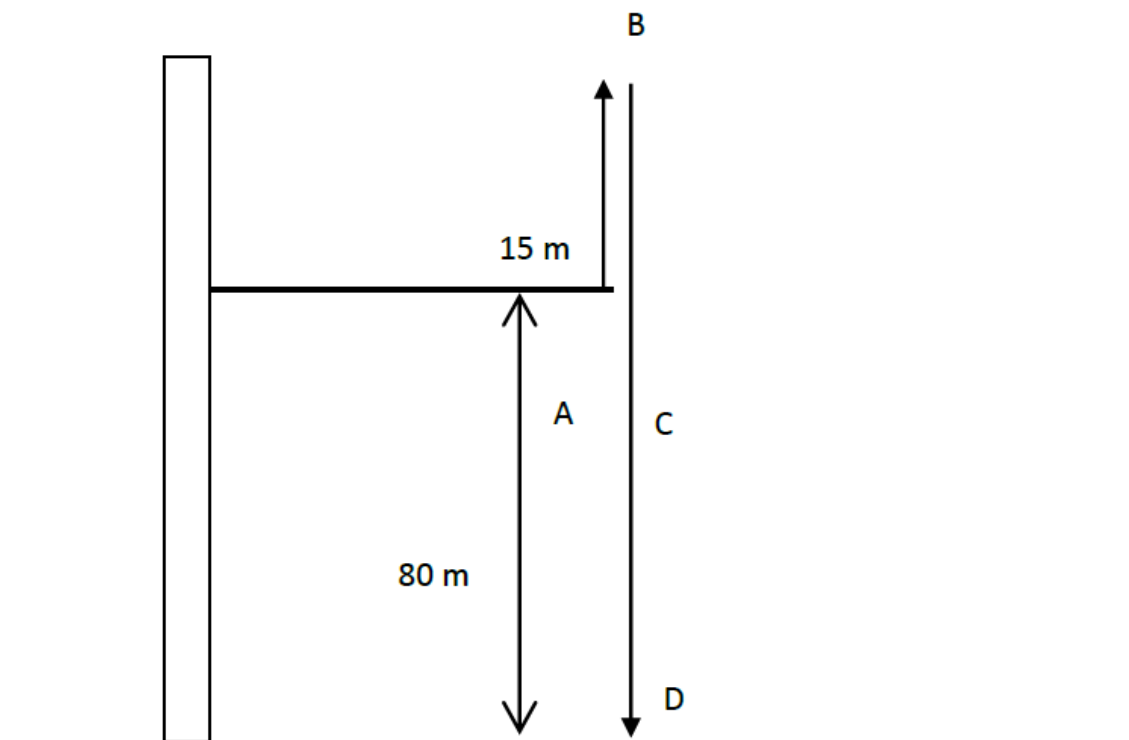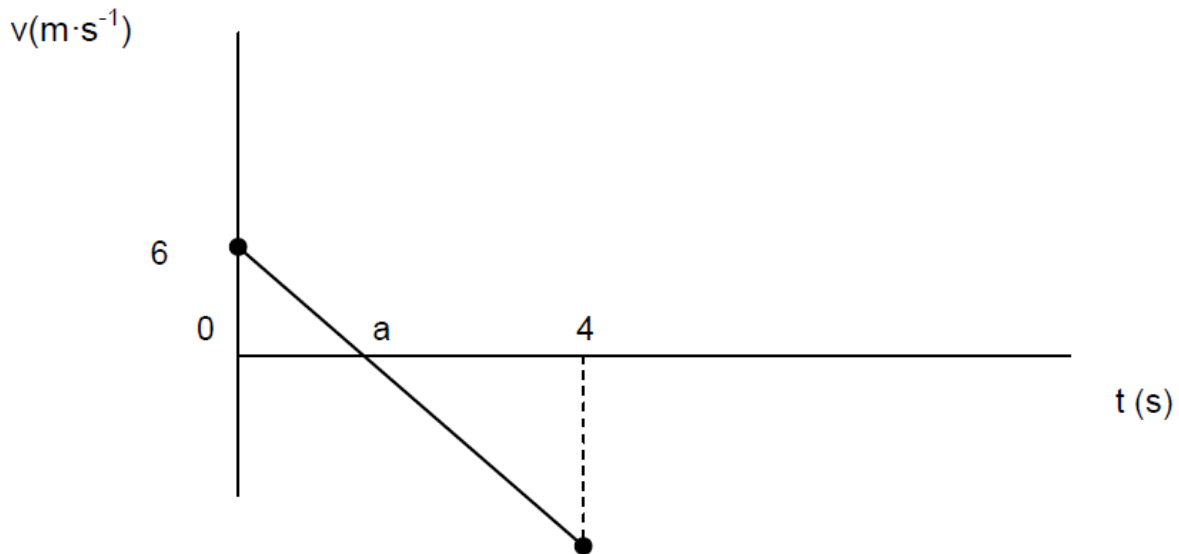

(a) Calculate the magnitude of the velocity of the arrow at the instant it is shot up into the air from the top of the platform.

(b) Calculate the time it takes for the arrow to reach the ground from the moment it is shot upwards.

9. Sandile throws a small metal ball of mass 10 g vertically up into the air. The ball accidentally lands in the gutter of a building. It remains in the gutter for 0,5 s during which time it rolls a few centimetres in the gutter, and then falls through a hole in the gutter back to the original position in Sandile's hand. The upward velocity with which the ball left Sandile's hand was 8 m.s^{-1}. When the ball finally falls back into his hand, the velocity is 7 m.s^{-1} downward. Ignore friction as well as all horizontal movement and answer the following questions:

Height of gutter above the hand = 2.5 m

Maximum height above hand = 3.27 m

Upward velocity at the start 8 m·s^{-1}

(a) At what speed would the ball have fallen into Sandile's hand if the ball had not fallen into the gutter?

(b) The maximum height that the ball reaches above Sandile's hand is 3,27 m. Prove that this is correct by using an equation of motion and not energy principles.

10. A helicopter is rising vertically at constant velocity.
When the helicopter is at a height of 100 m above the ground, a girl accidentally drops her camera out of the window of the helicopter. The velocity-time graph below represents the motion of the camera from the moment it is released from the helicopter until it strikes the ground. Ignore air resistance.

(a) What is the value of the slope (gradient) of the graph?

(b) Use the gradient to calculate the time a on the time axis.

(c) Which point of the path of the camera responds to time a?

(d) Use an equation of motion to calculate the magnitude of the velocity of the camera as it reaches the ground at 4 s.

(e) Use the graph to calculate the maximum height reached by the camera.

(f) Draw a rough displacement-time graph and an acceleration time graph to represent the motion of the camera from the moment it was released until it hit the ground. Time values must be shown but y-values need not be shown.

11. A hot-air balloon is rising upwards at a constant velocity of 5 m.s^{-1}. When the balloon is 100 m above the ground, a sandbag is dropped from it (see FIGURE 1). FIGURE 2 shows the path of the sandbag as it falls to the ground. Ignore air resistance.
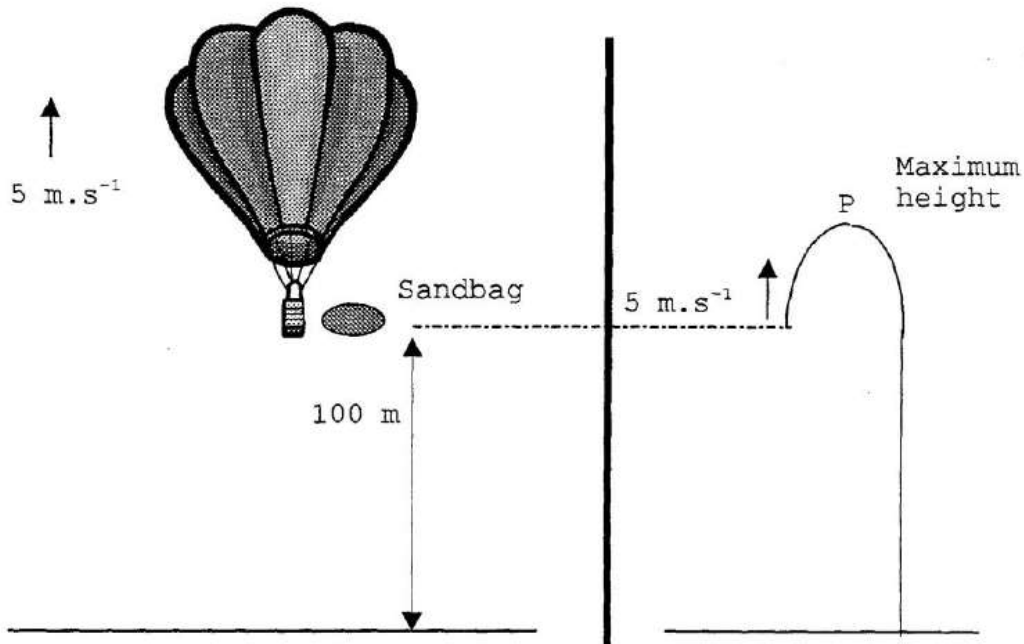

FIGURE 1   FIGURE 2 Motion of sandbag

(a) What is the acceleration of:
   i. The hot-air balloon while the sandbag is in it?
   ii. The sandbag the moment it is dropped from the hot-air balloon?

(b) Determine the maximum height P, above the ground, reached by the sandbag after is released from hot-air balloon.

(c) Calculate the time taken for the sandbag to reach this maximum height after it has been released.

(d) Calculate the total time taken for the sandbag to reach the ground after it has been released.

(e) Will the velocity of the hot-air balloon INCREASE, DECREASE or REMAIN THE SAME immediately after the sandbag has been released? Explain fully.