MATHEMATICS

Grade 7 - Term 2

CAPS

Learner Book

Revised edition

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UKUQONDA institute

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CHAPTER 6

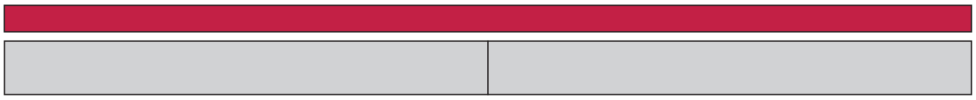
Fractions

6.1 Measuring accurately with parts of a unit

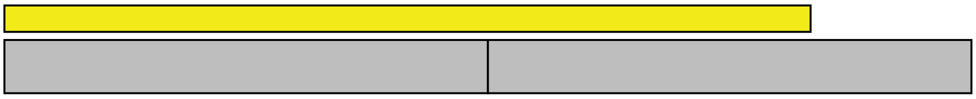
A STRANGE MEASURING UNIT

In this activity, you will measure lengths with a unit called a *greystick*. The grey measuring stick below is exactly one greystick long. You will use this stick to measure different objects.

The red bar below is exactly two greysticks long.



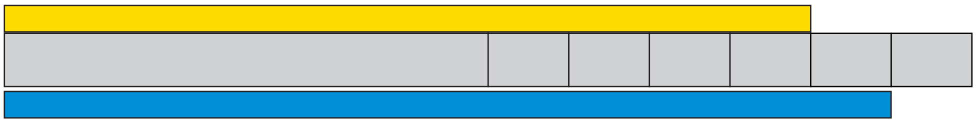
As you can see, the yellow bar below is longer than one greystick but shorter than two greysticks.



To try to measure the yellow bar accurately, we will divide one greystick into six equal parts:
So each of these parts is **one sixth** of a greystick.

This greystick ruler is divided into seven equal parts:
Each part is **one seventh** of a greystick.

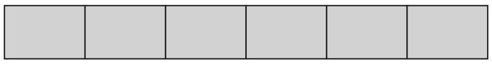
1. Do you think one can say the yellow bar is **one and four sixths of a greystick** long?

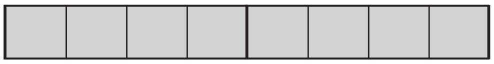


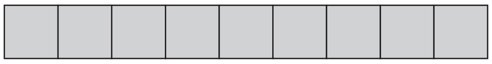
2. Describe the length of the blue bar, on the previous page, in words.

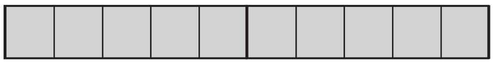
3. In each case below, say what the smaller parts of the greystick may be called. Write your answers in words.

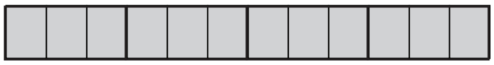
(a) 

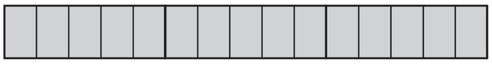
(b) 

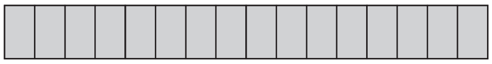
(c) 

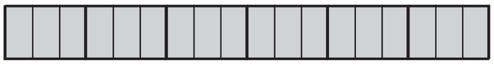
(d) 

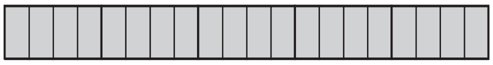
(e) 

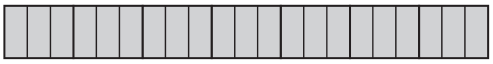
(f) 

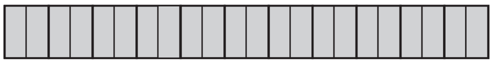
(g) 

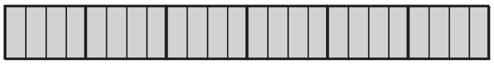
(h) 

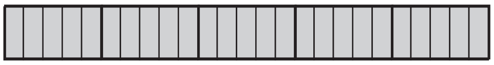
(i) 

(j) 

(k) 

(l) 

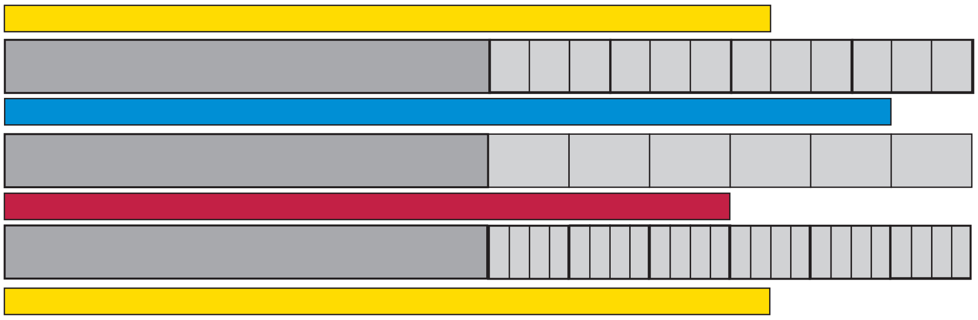
(m) 

(n) 

How did you find out what to call the small parts?

Write all your answers to the following questions *in words*.

4. (a) How long is the upper yellow bar?

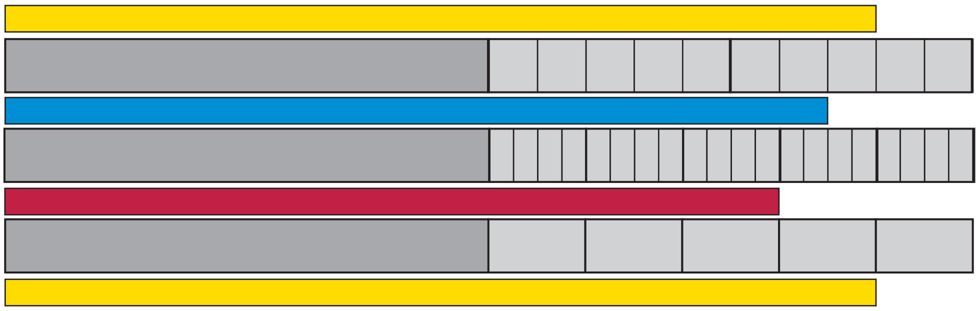


(b) How long is the lower yellow bar?

5. (a) How long is the blue bar above?
(b) How long is the red bar above?

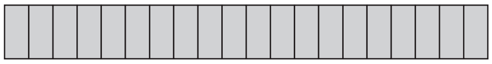
6. (a) How many twelfths of a greystick is the same length as one sixth of a greystick?
(b) How many twenty-fourths is the same length as one sixth of a greystick?
(c) How many twenty-fourths is the same length as seven twelfths of a greystick?

7. (a) How long is the upper yellow bar on the following page?
(b) How long is the lower yellow bar on the following page?



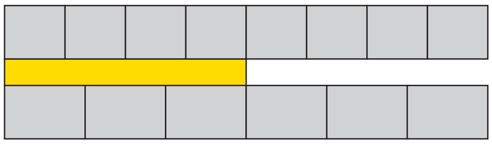
(c) How long is the blue bar?
(d) How long is the red bar?

8. (a) How many fifths of a greystick is the same length as 12 twentieths of a greystick?
   (b) How many fourths (or quarters) of a greystick is the same length as 15 twentieths of a greystick?



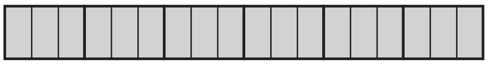
DESCRIBE THE SAME LENGTH IN DIFFERENT WAYS

Two fractions may describe the same length. You can see here that three sixths of a greystick is the same as four eighths of a greystick.



> When two fractions describe the same portion we say they are **equivalent**.

1. (a) What can each small part on this greystick be called?



   (b) How many eighteenths is one sixth of the greystick?
   (c) How many eighteenths is one third of the greystick?
   (d) How many eighteenths is five sixths of the greystick?

2. (a) Write (in words) the names of four different fractions that are all equivalent to three quarters.
   (b) Which equivalents for two thirds can you find on the greysticks?

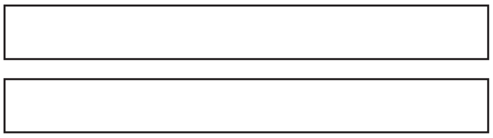
3. The information that two thirds is equivalent to four sixths, to six ninths and to eight twelfths is written in the second row of the table on the following page. Copy the table and complete the other rows of the table in the same way.

thirds	fourths	fifths	sixths	eighths	ninths	tenths	twelfths	twentieths
1								
2	–	–	4	–	6	–	8	–
–	3							
–	–	1						
–	–	2						
–	–	3						
–	–	4						

4. Copy and complete this table in the same way as the table in question 3.

fifths	tenths	fifteenths	twentieths	twenty-fifths	fiftieths	hundredths
1						
2						
3						
4						
5						
6						
7						

5. Copy the greysticks. Draw on them to show that three fifths and nine fifteenths are equivalent. Draw freehand; you need not measure and draw accurately.



6. Copy and complete these tables in the same way as the table in question 4.

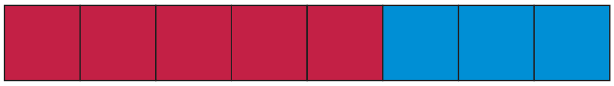
eighths	sixteenths	twenty-fourths
1		
2		
3		
4		
5		
6		
7		
8		
9		

twenty-fourths	sixths	twelfths	eighteenths
	1		
	2		
	3		
	4		
	5		
	6		
	7		
	8		
	9		

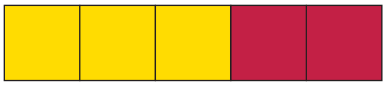
7. (a) How much is five twelfths plus three twelfths?
   (b) How much is five twelfths plus one quarter?
   (c) How much is five twelfths plus three quarters?
   (d) How much is one third plus one quarter? It may help if you work with the equivalent fractions in twelfths.

6.2 The fraction notation

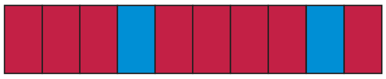
This strip is divided into eight equal parts. Five eighths of this strip is red.



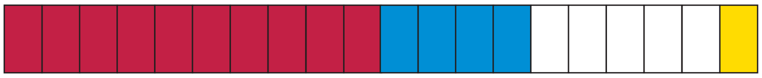
1. What part of the strip is blue?



2. What part of this strip is yellow?
3. What part of the strip is red?
4. What part of this strip is coloured blue and what part is coloured red?



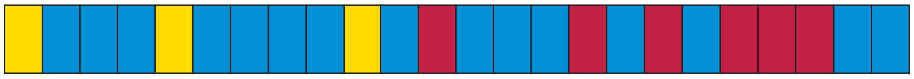
5. (a) What part of this strip is blue, what part is red and what part is white?



   (b) Express your answer differently with equivalent fractions.

6. A certain strip is not shown here. Two ninths of the strip is blue, and three ninths of the strip is green. The rest of the strip is red. What part of the strip is red?

7. What part of the strip below is yellow, what part is blue, and what part is red?



The number of parts in a fraction is called the **numerator** of the fraction. For example, the numerator in five sixths is five.

The type of part in a fraction is called the **denominator**. It is the name of the parts that are being referred to and it is determined by the size of the part. For example, sixths is the denominator in five sixths.

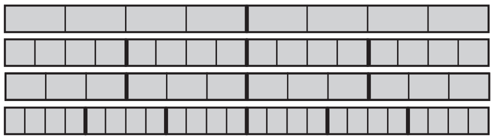
> $^5/_6$ is a short way to write five sixths.
>
> We may also write $\frac{5}{6}$.
>
> Even when we write $^5/_6$ or $\frac{5}{6}$, we still say "five sixths".
>
> $/_6$ and $\frac{}{6}$ are short ways to write *sixths*.

To **enumerate** means "to find the number of".
To **denominate** means "to give a name to".

The numerator (number of parts) is written above the line of the fraction: $\frac{\textit{numerator}}{\dots}$

The denominator is indicated by a number written below the line: $\frac{\dots}{\textit{denominator}}$

8. Consider the fraction three quarters. It can be written as $\frac{3}{4}$.



   (a) Multiply both the numerator and the denominator by two to form a new fraction. Is the new fraction equivalent to $\frac{3}{4}$? You may check on the diagram.

   (b) Multiply both the numerator and the denominator by three to form a new fraction. Is the new fraction equivalent to $\frac{3}{4}$?

   (c) Multiply both the numerator and the denominator by four to form a new fraction. Is the new fraction equivalent to $\frac{3}{4}$?

   (d) Multiply both the numerator and the denominator by six to form a new fraction. Is the new fraction equivalent to $\frac{3}{4}$?

6.3 Adding fractions

BIGGER AND SMALLER PARTS

Gertie was asked to solve this problem:

> *A team of road-builders built* $\frac{8}{12}$ *km of road in one week, and* $\frac{10}{12}$ *km in the next week. What is the total length of road that they built in the two weeks?*

She thought like this to solve the problem:

> $\frac{8}{12}$ *is* ***eight twelfths*** *and* $\frac{10}{12}$ *is* ***ten twelfths****, so altogether it is* ***eighteen twelfths****.*
>
> *I can write* $\frac{18}{12}$ *or "18 twelfths".*
>
> *I can also say 12 twelfths of a kilometre is 1 kilometre, so* ***18 twelfths*** *is* ***1 km and 6 twelfths of a kilometre****.*
>
> *This I can write as* $1\frac{6}{12}$*. It is the same as* $1\frac{1}{2}$ *km.*

Gertie was also asked the question: How much is $4\frac{5}{9} + 2\frac{7}{9}$?

She thought like this to answer it:

> $4\frac{5}{9}$ *is four wholes and five ninths, and* $2\frac{7}{9}$ *is two wholes and seven ninths.*
>
> *So altogether it is six wholes and 12 ninths. But 12 ninths is nine ninths (one whole) and three ninths, so I can say it is seven wholes and three ninths.*
>
> *I can write* $7\frac{3}{9}$.

1. Would Gertie be wrong if she said her answer was $7\frac{1}{3}$?
2. Senthereng has $4\frac{7}{12}$ bottles of cooking oil. He gives $1\frac{5}{12}$ bottles to his friend Willem. How much oil does Senthereng have left?
3. Margaret has $5\frac{5}{8}$ bottles of cooking oil. She gives $3\frac{7}{8}$ bottles to her friend Naledi. How much oil does Margaret have left?
4. Calculate each of the following:

(a) $4\frac{2}{7} - 3\frac{6}{7}$
(b) $3\frac{6}{7} + \frac{3}{7}$
(c) $3\frac{6}{7} + 1\frac{4}{5}$

(d) $4\frac{3}{8} - 2\frac{4}{5}$
(e) $1\frac{3}{10} - \frac{2}{3}$
(f) $3\frac{5}{10} - 1\frac{1}{2}$

(g) $\frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8}$
(h) $6\frac{2}{5} + 2\frac{1}{4} - \frac{1}{2}$

(i) $\frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8}$

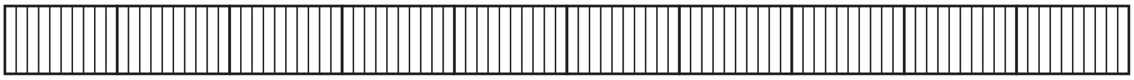
(j) $2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7}$

(k) $(4\frac{2}{7} + 1\frac{4}{7}) - 2\frac{1}{3}$
(l) $(2\frac{7}{10} + 3\frac{2}{5}) - (1\frac{2}{5} + 3\frac{7}{10})$

5. Neo's report had five chapters. The first chapter was $\frac{3}{4}$ of a page, the second chapter was $2\frac{1}{2}$ pages, the third chapter was $3\frac{3}{4}$ pages, the fourth chapter was three pages and the fifth chapter was $1\frac{1}{2}$ pages. How many pages was Neo's report in total?

6.4 Tenths and hundredths (percentages)

1. (a) 100 children each get three biscuits. How many biscuits is this in total?
   (b) 500 sweets are shared equally between 100 children. How many sweets does each child get?
2. The picture below shows a strip of licorice. The very small pieces can easily be broken off on the thin lines. How many very small pieces are shown in the picture?

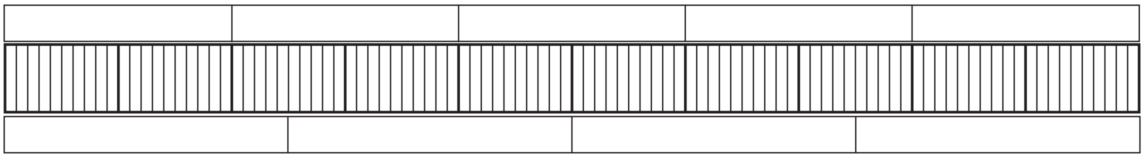


3. Gatsha runs a spaza shop. He sells strips of licorice like the above for R2 each.
   (a) What is the cost of one very small piece of licorice, when you buy from Gatsha?
   (b) Jonathan wants to buy one fifth of a strip of licorice. How much should he pay?
   (c) Batseba eats 25 very small pieces. What part of a whole strip of licorice is this?

Each small piece of the above strip is **one hundredth** of the whole strip.

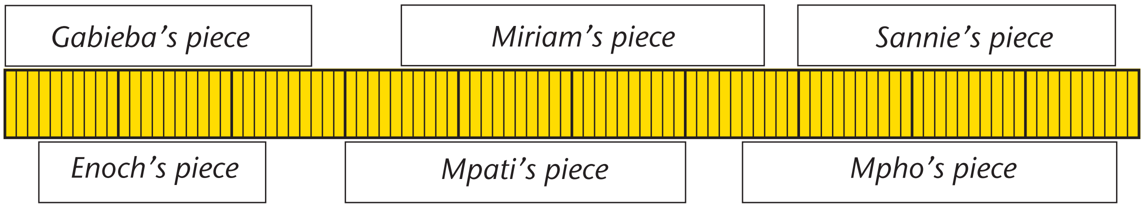
4. (a) Why can each small piece be called *one hundredth* of the whole strip?
   (b) How many hundredths is the same as one tenth of the strip?

Gatsha often sells parts of licorice strips to customers. He uses a "quarters marker" and a "fifths marker" to cut off the pieces correctly from full strips. His two markers are shown below, next to a full strip of licorice.



5. (a) How many hundredths is the same as two fifths of the whole strip?
   (b) How many tenths is the same as $\frac{2}{5}$ of the whole strip?
   (c) How many hundredths is the same as $\frac{3}{4}$ of the whole strip?
   (d) Freddie bought $\frac{60}{100}$ of a strip. How many fifths of a strip is this?
   (e) Jamey bought part of a strip for R1,60. What part of a strip did she buy?

6. Gatsha, the owner of the spaza shop, sold pieces of yellow licorice to different children. Their pieces are shown below. How much (what part of a whole strip) did each of them get?



7. The yellow licorice shown above costs R2,40 (240 cents) for a strip. How much does each of the children have to pay? Round off the amounts to the nearest cent.

8. (a) How much is $\frac{1}{100}$ of 300 cents?
   (b) How much is $\frac{7}{100}$ of 300 cents?
   (c) How much is $\frac{25}{100}$ of 300 cents?
   (d) How much is $\frac{1}{4}$ of 300 cents?
   (e) How much is $\frac{40}{100}$ of 300 cents?
   (f) How much is $\frac{2}{5}$ of 300 cents?

9. Explain why your answers for questions 8(e) and 8(f) are the same.

Another word for **hundredth** is **per cent**. Instead of saying Miriam received **32 hundredths** of a licorice strip, we can say Miriam received **32 per cent** of a licorice strip. The symbol for per cent is %.

10. How much is 80% of each of the following?

(a) R500 (b) R480 (c) R850 (d) R2 400

11. How much is 8% of each of the amounts in 10(a), (b), (c) and (d)?

12. How much is 15% of each of the amounts in 10(a), (b), (c) and (d)?

13. Building costs of houses increased by 20%. What is the new building cost for a house that previously cost R110 000 to build?

14. The value of a car decreases by 30% after one year. If the price of a new car is R125 000, what is the value of the car after one year?

15. Investigate which denominators of fractions can easily be converted to powers of 10.

6.5 Thousandths, hundredths and tenths

MANY EQUAL PARTS

1. In a camp for refugees, 50 kg of sugar must be shared equally between 1 000 refugees. How much sugar will each refugee get? Keep in mind that 1 kg is 1 000 g. You can give your answer in grams.

2. How much is each of the following?

(a) one tenth of R6 000 (b) one hundredth of R6 000
(c) one thousandth of R6 000 (d) ten hundredths of R6 000
(e) 100 thousandths of R6 000 (f) seven hundredths of R6 000
(g) 70 thousandths of R6 000 (h) seven thousandths of R6 000

3. Calculate.

(a) $\frac{3}{10} + \frac{5}{8}$ (b) $3\frac{3}{10} + 2\frac{4}{5}$ (c) $\frac{3}{10} + \frac{7}{100}$
(d) $\frac{3}{10} + \frac{70}{100}$ (e) $\frac{3}{10} + \frac{7}{1\,000}$ (f) $\frac{3}{10} + \frac{70}{1\,000}$

4. Calculate.

(a) $\frac{3}{10} + \frac{7}{100} + \frac{4}{1\,000}$ (b) $\frac{3}{10} + \frac{70}{100} + \frac{400}{1\,000}$
(c) $\frac{6}{10} + \frac{20}{100} + \frac{700}{1\,000}$ (d) $\frac{2}{10} + \frac{5}{100} + \frac{4}{1\,000}$

5. In each case investigate whether the statement is true or not, and give reasons for your final decision.

(a) $\frac{1}{10} + \frac{23}{100} + \frac{346}{1\,000} = \frac{6}{10} + \frac{3}{100} + \frac{46}{1\,000}$ (b) $\frac{1}{10} + \frac{23}{100} + \frac{346}{1\,000} = \frac{7}{10} + \frac{2}{100} + \frac{6}{1\,000}$
(c) $\frac{1}{10} + \frac{23}{100} + \frac{346}{1\,000} = \frac{6}{10} + \frac{7}{100} + \frac{6}{1\,000}$ (d) $\frac{676}{1\,000} = \frac{6}{10} + \frac{7}{100} + \frac{6}{1\,000}$

6.6 Fraction of a fraction

FORM PARTS OF PARTS

1. (a) How much is one fifth of R60?
   (b) How much is three fifths of R60?

2. How much is seven tenths of R80? (You may first work out how much one tenth of R80 is.)

3. In Britain the unit of currency is pound sterling, in Western Europe it is the euro, and in Botswana it is the pula.
   (a) How much is two fifths of 20 pula?
   (b) How much is two fifths of 20 euro?
   (c) How much is two fifths of 12 pula?

4. Why was it so easy to calculate two fifths of 20, but difficult to calculate two fifths of 12?

There is a way to make it easy to calculate something like three fifths of R4. You just change the rands to cents!

5. Calculate each of the following. You may change the rands to cents to make it easier.
   (a) three eighths of R2,40
   (b) seven twelfths of R6
   (c) two fifths of R21
   (d) five sixths of R3

6. You will now do some calculations about secret objects.
   (a) How much is three tenths of 40 secret objects?
   (b) How much is three eighths of 40 secret objects?

The secret objects in question 6 are fiftieths of a rand.

7. (a) How many fiftieths is three tenths of 40 fiftieths?
   (b) How many fiftieths is five eighths of 40 fiftieths?

8. (a) How many twentieths of a kilogram is the same as $\frac{3}{4}$ of a kilogram?
   (b) How much is one fifth of 15 rands?
   (c) How much is one fifth of 15 twentieths of a kilogram?
   (d) So, how much is one fifth of $\frac{3}{4}$ of a kilogram?

9. (a) How much is $\frac{1}{12}$ of 24 fortieths of some secret object?
   (b) How much is $\frac{7}{12}$ of 24 fortieths of the secret object?

10. Do you agree that the answers for the previous question are two fortieths and 14 fortieths? If you disagree, explain why you disagree.

11. (a) How much is $\frac{1}{5}$ of 80?

(b) How much is $\frac{3}{5}$ of 80?

(c) How much is $\frac{1}{40}$ of 80?

(d) How much is $\frac{24}{40}$ of 80?

(e) Explain why $\frac{3}{5}$ of 80 is the same as $\frac{24}{40}$ of 80.

12. Look again at your answers for questions 9(b) and 11(e). How much is $\frac{7}{12}$ of $\frac{3}{5}$? Explain your answer.

The secret object in question 9 was an envelope with R160 in it.

After the work you did in questions 9, 10 and 11, you know that:

- $\frac{24}{40}$ and $\frac{3}{5}$ are just two ways to describe the same thing, and
- $\frac{7}{12}$ of $\frac{3}{5}$ is therefore the same as $\frac{7}{12}$ of $\frac{24}{40}$.

It is easy to calculate $\frac{7}{12}$ of $\frac{24}{40}$: one twelfth of 24 is 2, so seven twelfths of 24 is 14, so seven twelfths of 24 fortieths is 14 fortieths.

$\frac{3}{8}$ of $\frac{2}{3}$ can be calculated in the same way. But one eighth of $\frac{2}{3}$ is a slight problem, so it would be better to use some equivalent of $\frac{2}{3}$. The equivalent should be chosen so that it is easy to calculate one eighth of it; so it would be nice if the numerator could be eight.

$\frac{8}{12}$ is equivalent to $\frac{2}{3}$, so instead of calculating $\frac{3}{8}$ of $\frac{2}{3}$ we may calculate $\frac{3}{8}$ of $\frac{8}{12}$.

13. (a) Calculate $\frac{3}{8}$ of $\frac{8}{12}$.

(b) So, how much is $\frac{3}{8}$ of $\frac{2}{3}$?

14. In each case replace the second fraction by a suitable equivalent, and then calculate.

(a) How much is $\frac{3}{4}$ of $\frac{5}{8}$?

(b) How much is $\frac{7}{10}$ of $\frac{2}{3}$?

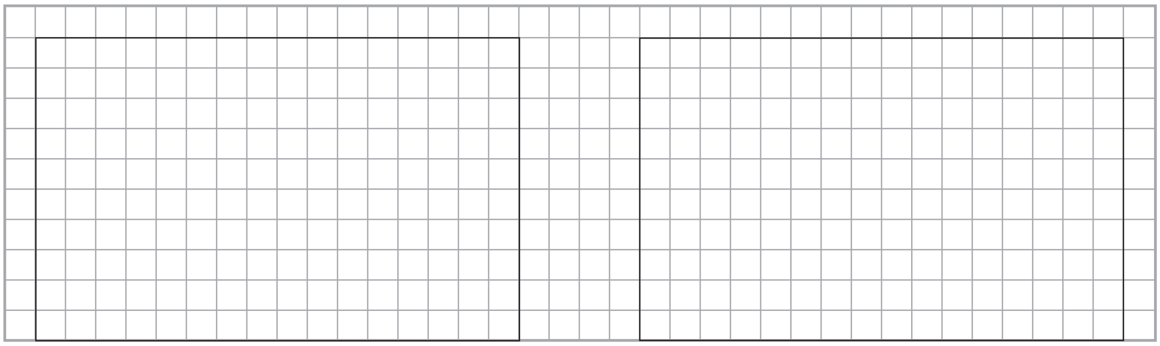
(c) How much is $\frac{2}{3}$ of $\frac{1}{2}$?

(d) How much is $\frac{3}{5}$ of $\frac{3}{5}$?

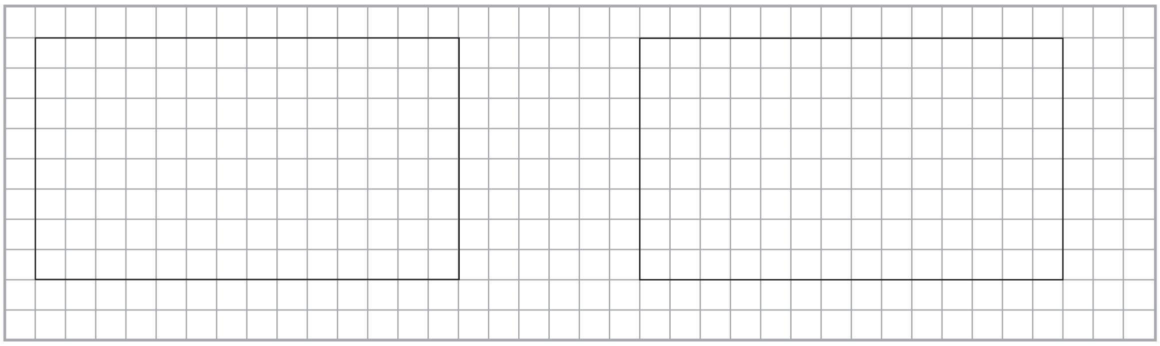
6.7 Multiplying with fractions

PARTS OF RECTANGLES AND PARTS OF PARTS

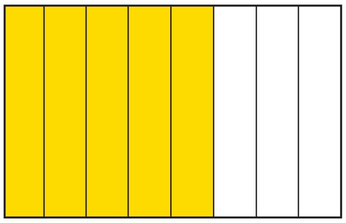
1. (a) Copy the rectangles below. Divide the rectangle on the left into eighths by drawing vertical lines. Lightly shade the left three eighths of the rectangle.
   (b) Divide the rectangle on the right into fifths drawing horizontal lines. Lightly shade the upper two fifths of the rectangle.

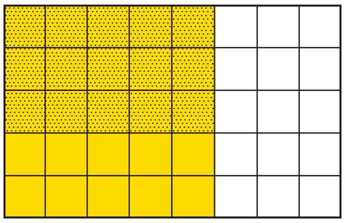


2. (a) Copy the rectangles below. Shade four sevenths of the rectangle on the left below.
   (b) Shade 16 twenty-eighths of the rectangle on the right below.



3. (a) What part of each big rectangle below is coloured yellow?

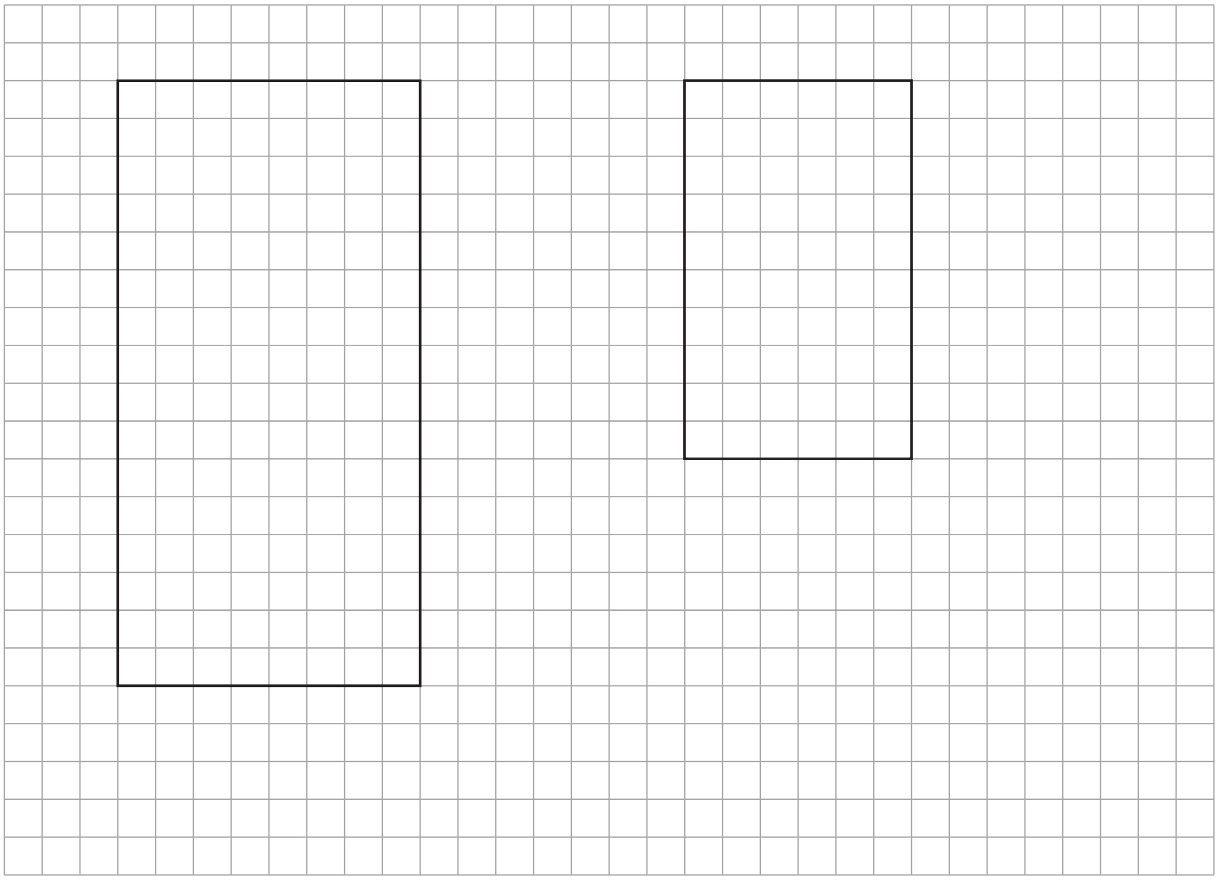




   (b) What part of the *yellow* part of the rectangle on the right is dotted?
   (c) Into how many squares is the whole rectangle on the right divided?
   (d) What part of the whole rectangle on the right is yellow *and* dotted?

4. On grid paper make diagrams to help you to figure out how much each of the following is:

   (a) $\frac{3}{4}$ of $\frac{5}{8}$ (b) $\frac{2}{3}$ of $\frac{4}{5}$



Here is something you can do with the fractions $\frac{3}{4}$ and $\frac{5}{8}$:
multiply the two numerators and make this the numerator of a new fraction.
Also multiply the two denominators, and make this the denominator of a new fraction $\frac{3 \times 5}{4 \times 8} = \frac{15}{32}$.

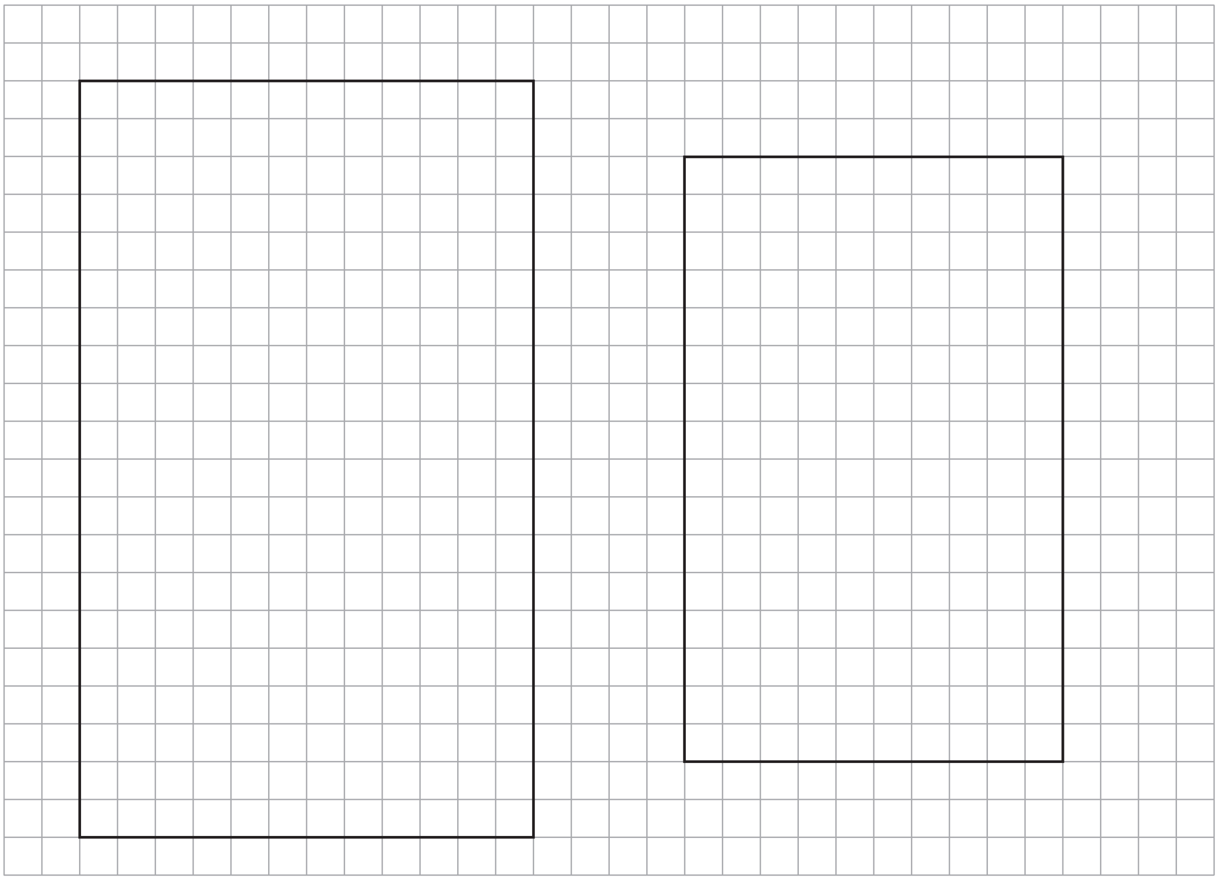
5. Compare the above with what you did in question 14(a) of section 6.6 and in question 4(a) at the top of this page. What do you notice about $\frac{3}{4}$ of $\frac{5}{8}$ and $\frac{3 \times 5}{4 \times 8}$?

6. (a) Alan has five heaps of eight apples each. How many apples is that in total?
   (b) Sean has ten heaps of six quarter apples each. How many apples is that in total?

> Instead of saying $\frac{5}{8}$ of R40 or $\frac{5}{8}$ of $\frac{2}{3}$ of a floor surface,
> we may say $\frac{5}{8}$ × R40 or $\frac{5}{8} \times \frac{2}{3}$ of a floor surface.

7. Use the diagrams below to figure out how much each of the following is:

(a) $\frac{3}{10} \times \frac{5}{6}$ (b) $\frac{2}{5} \times \frac{7}{8}$



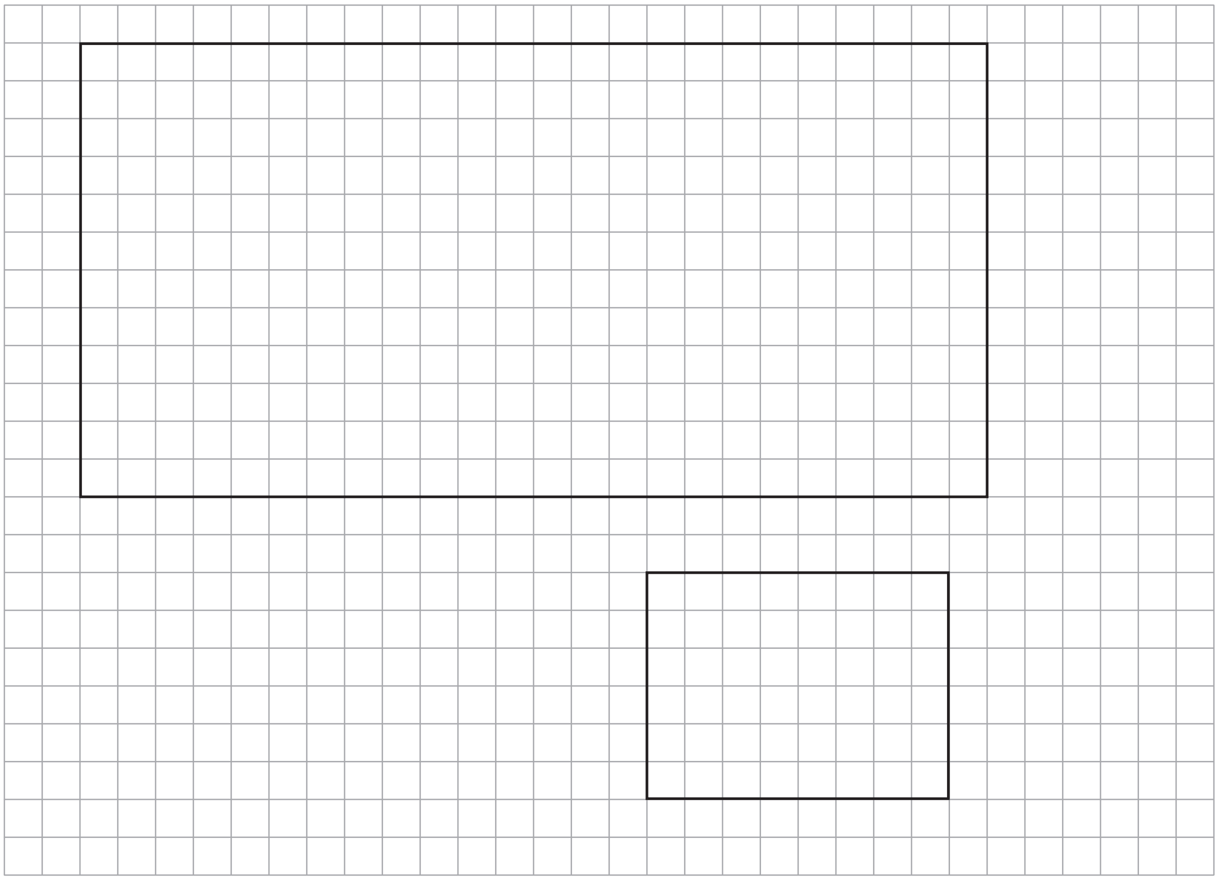
8. (a) Perform the calculations $\frac{\text{numerator} \times \text{numerator}}{\text{denominator} \times \text{denominator}}$ for $\frac{3}{10}$ and $\frac{5}{6}$ and compare the answer to your answer for question 7(a).

(b) Do the same for $\frac{2}{5}$ and $\frac{7}{8}$.

9. Perform the calculations $\frac{\text{numerator} \times \text{numerator}}{\text{denominator} \times \text{denominator}}$ for

(a) $\frac{5}{6}$ and $\frac{7}{12}$ (b) $\frac{3}{4}$ and $\frac{2}{3}$

10. Use the diagrams on the following page to check whether the formula $\frac{\text{numerator} \times \text{numerator}}{\text{denominator} \times \text{denominator}}$ produces the correct answers for $\frac{5}{6} \times \frac{7}{12}$ and $\frac{3}{4} \times \frac{2}{3}$.



11. Calculate each of the following:

(a) $\frac{1}{2}$ of $\frac{1}{3}$ of R60 (b) $\frac{2}{7}$ of $\frac{2}{9}$ of R63 (c) $\frac{4}{3}$ of $\frac{2}{5}$ of R45

12. (a) John normally practises soccer for three quarters of an hour every day. Today he practised for only half his usual time. How long did he practise today?

(b) A bag of peanuts weighs $\frac{3}{8}$ of a kilogram. What does $\frac{3}{4}$ of a bag weigh?

(c) Calculate the mass of $7\frac{1}{2}$ packets of sugar if one packet has a mass of $\frac{3}{4}$ kg.

6.8 Ordering and comparing fractions

1. Order the following from the smallest to the biggest:

(a) $\frac{7}{16}$; $\frac{3}{8}$; $\frac{11}{24}$; $\frac{5}{12}$; $\frac{23}{48}$ (b) $\frac{703}{1\,000}$; $\frac{13}{20}$; $\frac{7}{10}$; 73%; $\frac{71}{100}$

2. Order the following from the biggest to the smallest:

(a) $\frac{41}{60}$; $\frac{19}{30}$; $\frac{7}{10}$; $\frac{11}{15}$; $\frac{17}{20}$ (b) $\frac{23}{24}$; $\frac{2}{3}$; $\frac{7}{8}$; $\frac{19}{20}$; $\frac{5}{6}$

3. Use the symbols = , > or < to make the following true:

(a) $\frac{7}{17}$ ☐ $\frac{21}{51}$ (b) $\frac{1}{17}$ ☐ $\frac{1}{19}$

WORKSHEET

1. Do the calculations given below. Rewrite each question in the common fraction notation. Then write the answer in words and in the common fraction notation.
   (a) three twentieths + five twentieths (b) five twelfths + 11 twelfths
   (c) three halves + five quarters (d) three fifths + three tenths

2. Complete the equivalent fractions.
   (a) $\frac{5}{7} = \frac{\square}{49}$ (b) $\frac{9}{11} = \frac{\square}{33}$ (c) $\frac{15}{10} = \frac{3}{\square}$
   (d) $\frac{1}{9} = \frac{4}{\square}$ (e) $\frac{45}{18} = \frac{\square}{2}$ (f) $\frac{4}{5} = \frac{\square}{35}$

3. Do the calculations given below. Rewrite each question in words. Then write the answer in words and in the common fraction notation.
   (a) $\frac{3}{10} + \frac{7}{30}$ (b) $\frac{2}{5} + \frac{7}{12}$
   (c) $\frac{1}{100} + \frac{7}{10}$ (d) $\frac{3}{5} - \frac{3}{8}$
   (e) $2\frac{3}{10} + 5\frac{9}{10}$

4. Joe earns R5 000 per month. His salary increases by 12%. What is his new salary?

5. Ahmed earned R7 500 per month. At the end of a certain month, his employer raised his salary by 10%. However, one month later his employer had to decrease his salary again by 10%. What was Ahmed's salary then?

6. Calculate each of the following and simplify the answer to its lowest form:
   (a) $\frac{13}{20} - \frac{2}{5}$ (b) $3\frac{24}{100} - 1\frac{2}{10}$
   (c) $5\frac{9}{11} - 2\frac{1}{4}$ (d) $\frac{2}{3} + \frac{4}{7}$

7. Evaluate.
   (a) $\frac{1}{2} \times 9$ (b) $\frac{3}{5} \times \frac{10}{27}$ (c) $\frac{2}{3} \times 15$ (d) $\frac{2}{3} \times \frac{3}{4}$

8. Calculate.
   (a) $2\frac{2}{3} \times 2\frac{2}{3}$ (b) $8\frac{2}{5} \times 3\frac{1}{3}$
   (c) $(\frac{1}{3} + \frac{1}{2}) \times \frac{6}{7}$ (d) $\frac{2}{3} \times \frac{1}{2} \times \frac{3}{4}$
   (e) $\frac{5}{6} + \frac{2}{3} \times \frac{1}{5}$ (f) $\frac{3}{4} - \frac{2}{5} \times \frac{5}{6}$

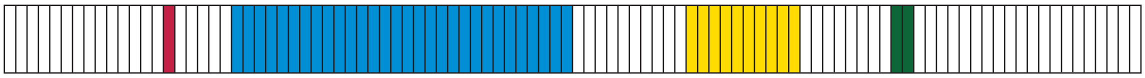
CHAPTER 7

The decimal notation for fractions

7.1 Other symbols for tenths and hundredths

TENTHS AND HUNDREDTHS AGAIN ...

1. (a) What part of the rectangle below is coloured yellow?



   (b) What part of the rectangle is red? What part is blue? What part is green, and what part is not coloured?

> 0,1 is another way to write $\frac{1}{10}$ and
> 0,01 is another way to write $\frac{1}{100}$.
> 0,1 and $\frac{1}{10}$ are different notations for the same number.
> $\frac{1}{10}$ is called the **(common) fraction notation** and 0,1 is called the **decimal notation**.

2. Write the answers for 1(a) and (b) in decimal notation.

3. Three tenths and seven hundredths of a rectangle is coloured red, and two tenths and six hundredths of the rectangle is coloured brown. What part of the rectangle (how many tenths and how many hundredths) is not coloured? Write your answer in fraction notation and in decimal notation.

4. On Monday, Steve ate three tenths and seven hundredths of a strip of licorice. On Tuesday, Steve ate two tenths and five hundredths of a strip of licorice. How much licorice did he eat on Monday and Tuesday together? Write your answer in fraction notation and in decimal notation.

5. Lebogang's answer for question 4 is *five tenths and 12 hundredths*. Susan's answer is *six tenths and two hundredths*. Who is right, or are they both wrong?

The same quantity can be expressed in different ways in tenths and hundredths.

For example, *three tenths and 17 hundredths* can be expressed as *two tenths and 27 hundredths* or *four tenths and seven hundredths*.

All over the world, people have agreed to keep the number of hundredths in such statements below ten. This means that the normal way of expressing the above quantity is *four tenths and seven hundredths*.

Written in decimal notation, four tenths and seven hundredths is 0,47. This is read as *nought comma four seven* and NOT *nought comma forty-seven*.

6. What is the decimal notation for each of the following numbers?

(a) $3\frac{7}{10}$ (b) $4\frac{19}{100}$ (c) $\frac{47}{10}$ (d) $\frac{4}{100}$

... AND THOUSANDTHS

0,001 is another way of writing $\frac{1}{1\,000}$.

1. What is the decimal notation for each of the following?

(a) $\frac{7}{1\,000}$ (b) $\frac{9}{1\,000}$ (c) $\frac{147}{1\,000}$ (d) $\frac{999}{1\,000}$

2. Write the following numbers in the decimal notation:

(a) $2 + \frac{3}{10} + \frac{7}{100} + \frac{4}{1\,000}$ (b) $12 + \frac{1}{10} + \frac{4}{1\,000}$

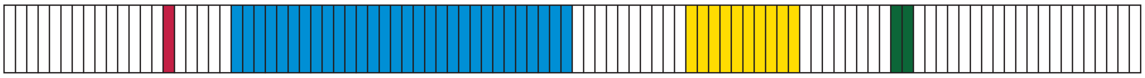
(c) $2 + \frac{4}{1\,000}$ (d) $67\frac{123}{1\,000}$

(e) $34\frac{61}{1\,000}$ (f) $654\frac{3}{1\,000}$

7.2 Percentages and decimal fractions

HUNDREDTHS, PERCENTAGES AND DECIMALS

1. The rectangle below is divided into small parts.



(a) How many of these small parts are there in the rectangle? And in one tenth of the rectangle?
(b) What part of the rectangle is blue? What part is green? What part is red?

Instead of *six hundredths*, you may say six *per cent*.
It means the same.

Ten per cent of the rectangle above is yellow.

2. Use the word "per cent" to say what part of the rectangle is green. What part is red?

3. What percentage of the rectangle is blue? What percentage is white?

We do not say: "How many per cent of the rectangle is green?" We say: "What percentage of the rectangle is green?"

The symbol % is used for "per cent".

Instead of writing "17 per cent", you may write 17%.

Per cent means *hundredths*. The symbol % is a bit like the symbol $\frac{}{100}$.

4. (a) How much is 1% of R400? (In other words: How much is $\frac{1}{100}$ or 0,01 of R400?)
(b) How much is 37% of R400?
(c) How much is 37% of R700?

5. (a) 25 apples are shared equally between 100 people. What fraction of the apples does each person get? Write your answer as a common fraction and as a decimal fraction.
(b) How much is 1% (one hundredth) of 25?
(c) How much is 8% of 25?
(d) How much is 8% of 50? And how much is 0,08 of 50?

0,37 and 37% and $\frac{37}{100}$ are different symbols for the same thing: *37 hundredths.*

6. Express each of the following in three ways:
- in the *decimal notation*,
- in the *percentage notation*, and
- if possible, in the *common fraction notation, using hundredths*.

(a) three tenths
(b) seven hundredths
(c) 37 hundredths
(d) seven tenths
(e) three quarters
(f) seven eighths

7. (a) How much is three tenths of R200 and seven hundredths of R200 altogether?
(b) How much is $\frac{37}{100}$ of R200?

(c) How much is 0,37 of R200?

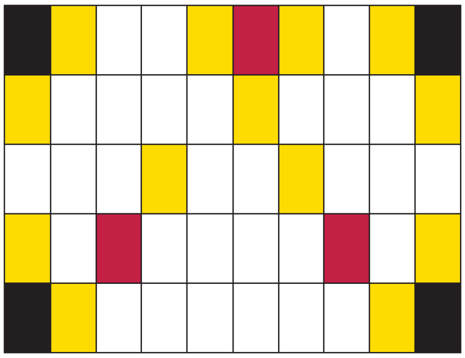
(d) And how much is 37% of R200?

8. Express each of the following in three ways:
   - in the *decimal notation*,
   - in the *percentage notation*, and
   - in the *common fraction notation, using hundredths*.

(a) 20 hundredths
(b) 50 hundredths
(c) 25 hundredths
(d) 75 hundredths

9. (a) Jan eats a quarter of a watermelon. What percentage of the watermelon is this?
   (b) Sibu drinks 75% of the milk in a bottle. What fraction of the milk is this?
   (c) Jeminah uses 0,75 (seven tenths and five hundredths) of the paint in a tin. What percentage of the paint does she use?

10. The floor of a large room is shown alongside. What part of the floor is covered in each of the four colours? Copy the table below. Express your answer in four ways:
   (a) in the *common fraction notation, using hundredths*,
   (b) in the *decimal notation*,
   (c) in the *percentage notation*, and
   (d) if possible, *in the common fraction notation, as tenths* ***and*** *hundredths* (for example $\frac{3}{10} + \frac{4}{100}$).



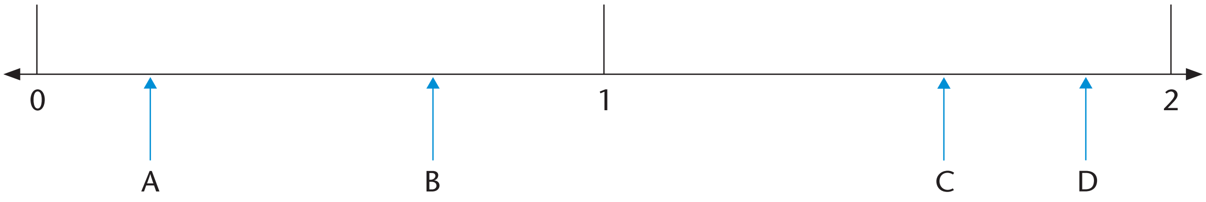
	(a)	(b)	(c)	(d)
white				
red				
yellow				
black				

7.3 Decimal measurements

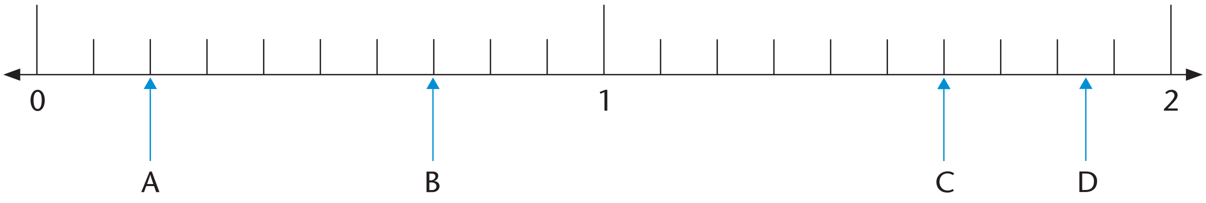
MEASURING ON A NUMBER LINE

1. Read the lengths at the marked points (A to D) for the number lines on the next two pages. Give your answers as accurate as possible in decimal notation.

(a)

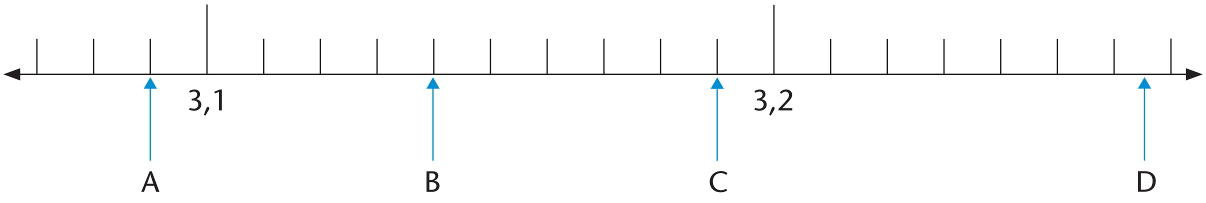


(b)

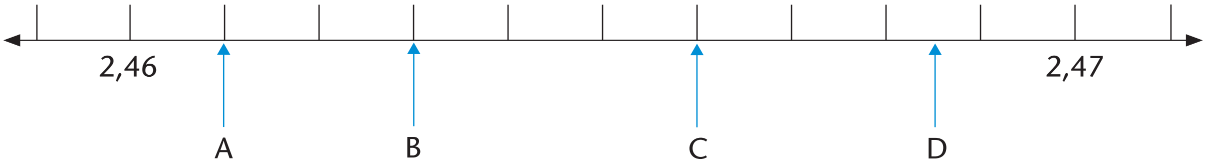


(c)

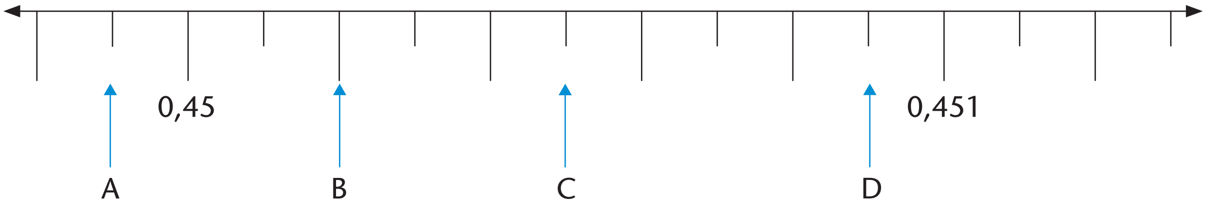
(d)



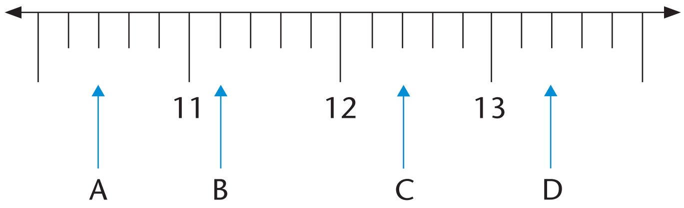
(e)



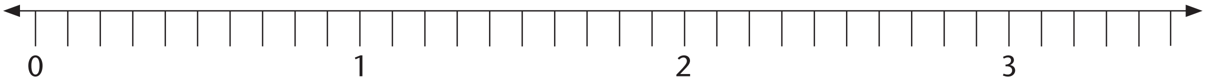
(f)



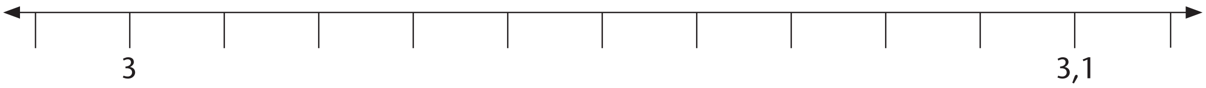
(g)



2. Copy the number line below and show the following numbers on it:
   - (a) 0,6
   - (b) 1,2
   - (c) 1,85
   - (d) 2,3
   - (e) 2,65
   - (f) 3,05
   - (g) 0,08



3. Copy the number line below and show the following numbers on it:
   - (a) 3,06
   - (b) 3,08
   - (c) 3,015
   - (d) 3,047
   - (e) 3,005



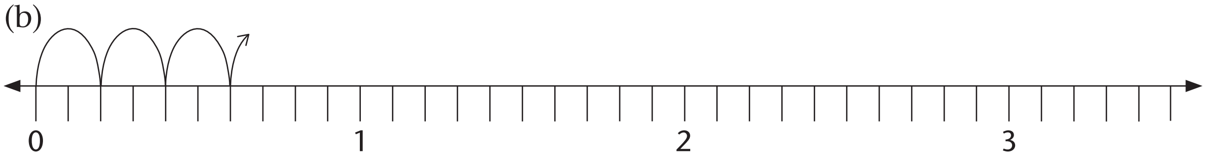
7.4 More decimal concepts

DECIMAL JUMPS

Copy the number lines below. Write the next ten numbers in the number sequences and show your number sequences, as far as possible, on the number lines.

1. (a) 0,2; 0,4; 0,6; ...

   (b)

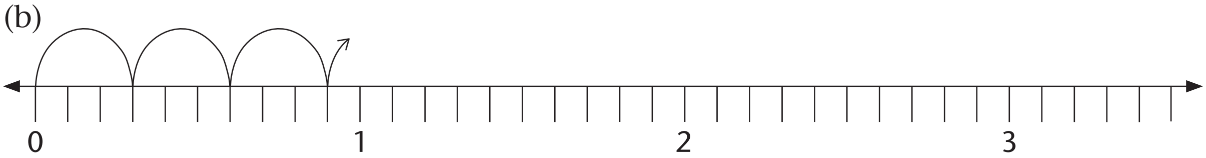


   (c) How many 0,2s are there in 1?

   (d) Write 0,2 as a common fraction.

2. (a) 0,3; 0,6; 0,9; ...

   (b)

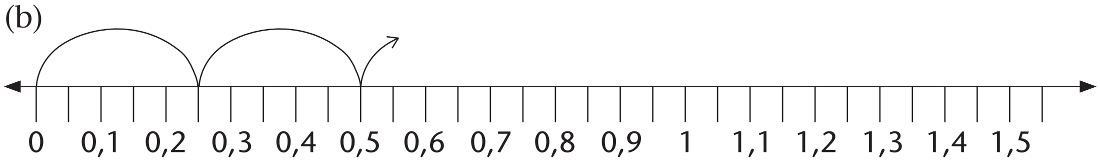


   (c) How many 0,3s are there in 3?

   (d) Write 0,3 as a common fraction.

3. (a) 0,25; 0,5; ...

   (b)



   (c) How many 0,25s are there in 1?

   (d) Write 0,25 as a common fraction.

> A calculator can be programmed to do the same operation over and over again.
>
> For example, press 0,1 $\boxed{+}$ $\boxed{=}$ (do not press CLEAR or any other operation). Press the $\boxed{=}$ key repeatedly and see what happens.
>
> The calculator counts in 0,1s.

4. You can check your answers for questions 1 to 3 with a calculator. Program the calculator to help you.

5. Write the next five numbers in the number sequences:

   (a) 9,3; 9,2; 9,1; ...

   (b) 0,15; 0,14; 0,13; 0,12; ...

6. Check your answers with a calculator. Program the calculator to help you.

PLACE VALUE

1. Write each of the following as one number:

   (a) 2 + 0,5 + 0,07
   (b) 2 + 0,5 + 0,007
   (c) 2 + 0,05 + 0,007
   (d) 5 + 0,4 + 0,03 + 0,001
   (e) 5 + 0,04 + 0,003 + 0,1
   (f) 5 + 0,004 + 0,3 + 0,01

We can write 3,784 in expanded notation as 3,784 = 3 + 0,7 + 0,08 + 0,004.
We can also name these parts as follows:

- the 3 represents the **units**
- the 7 represents the **tenths**
- the 8 represents the **hundredths**
- the 4 represents the **thousandths**

> We say: the **value** of the 7 is seven tenths but the **place value** of the 7 is tenths, because any digit **in that place** will represent the number of tenths.

> For example, in 2,536 the **value** of the three is 0,03, and its **place value** is hundredths, because the value of the **place where it stands** is hundredths.

2. Now write the value (in decimal fractions) and the place value of each of the underlined digits.

(a) 2,345 (b) 4,678 (c) 1,953
(d) 34,856 (e) 564,34 (f) 0,987

7.5 Ordering and comparing decimal fractions

FROM BIGGEST TO SMALLEST AND SMALLEST TO BIGGEST

1. Order the following numbers from biggest to smallest. Explain your method.

0,8 0,05 0,5 0,15 0,465 0,55 0,75 0,4 0,62

2. Below are the results of some of the 2012 London Olympic events. Copy and complete the tables. In each case, order them from first to last place.

(a) Women: Long jump – Final

Name	Country	Distance	Place
Anna Nazarova	RUS	6,77 m	
Brittney Reese	USA	7,12 m	
Elena Sokolova	RUS	7,07 m	
Ineta Radevica	LAT	6,88 m	
Janay DeLoach	USA	6,89 m	3rd
Lyudmila Kolchanova	RUS	6,76 m	

(b) Women: 400 m hurdles – Final

Name	Country	Time	Place
Georganne Moline	USA	53,92 s	
Kaliese Spencer	JAM	53,66 s	4th
Lashinda Demus	USA	52,77 s	
Natalya Antyukh	RUS	52,70 s	
T'erea Brown	USA	55,07 s	
Zuzana Hejnová	CZE	53,38 s	

(c) Men: 110 m hurdles – Final

Name	**Country**	**Time**	**Place**
Aries Merritt	USA	12,92 s	
Hansle Parchment	JAM	13,12 s	
Jason Richardson	USA	13,04 s	
Lawrence Clarke	GBR	13,39 s	
Orlando Ortega	CUB	13,43 s	
Ryan Brathwaite	BAR	13,40 s	

(d) Men: Javelin – Final

Name	**Country**	**Distance**	**Place**
Andreas Thorkildsen	NOR	82,63 m	
Antti Ruuskanen	FIN	84,12 m	
Keshorn Walcott	TRI	84,58 m	
Oleksandr Pyatnytsya	UKR	84,51 m	
Tero Pitkämäki	FIN	82,80 m	
Vítezslav Veselý	CZE	83,34 m	

3. In each case, give a number that falls between the two numbers.
   (This means you may give *any* number that falls anywhere between the two numbers.)
   (a) 3,5 and 3,7 (b) 3,9 and 3,11 (c) 3,1 and 3,2

4. How many numbers are there between 3,1 and 3,2?

5. Copy and fill in <, > or =.
   (a) 0,4 ☐ 0,52 (b) 0,4 ☐ 0,32 (c) 2,61 ☐ 2,7
   (d) 2,4 ☐ 2,40 (e) 2,34 ☐ 2,567 (f) 2,34 ☐ 2,251

7.6 Rounding off

Just as whole numbers can be rounded off to the nearest 10, 100 or 1 000, decimal fractions can be rounded off to the nearest whole number or to one, two, three etc. digits after the comma. A decimal fraction is rounded off to the number whose value is closest to it. Therefore 13,24 rounded off to one decimal place is 13,2 and 13,26 rounded off to one decimal place is 13,3. A decimal ending in a 5 is an equal distance from the two numbers to which it can be rounded off. Such decimals are rounded off to the biggest number, so 13,15 rounded off to one decimal place becomes 13,2.

SAYING IT NEARLY BUT NOT QUITE

1. Round each of the following numbers off to the nearest whole number:
   7,6 18,3 204,5 1,89 0,9 34,7 11,5 0,65
2. Round each of the following numbers off to one decimal place:
   7,68 18,93 21,47 0,643 0,938 1,44 3,81
3. Round each of the following numbers off to two decimal places:
   3,432 54,117 4,809 3,762 4,258 10,222 9,365 299,996

ROUND OFF TO HELP YOU CALCULATE

1. John and three of his brothers sell an old bicycle for R44,65. How can the brothers share the money fairly?
2. A man buys 3,75 m of wood at R11,99 per metre. What does the wood cost him?
3. Estimate the answers of each of the following by rounding off the numbers:
   (a) 89,3 × 3,8
   (b) 227,3 + 71,8 – 28,6

7.7 Addition and subtraction with decimal fractions

MENTAL CALCULATIONS

1. Copy and complete the number chain.

34,123	→	+ 20	→	54,123	→		→	454,123	→		→	454,023
												↓
												↓
422,011	←		←	452,011	←		←	452,021	←		←	452,023
↓												
↓												
222,011	→		→	222,211	→		→	222,231	→		→	222,232
												↓
												↓
222,489	←		←	222,482	←		←	222,422	←		←	222,222

When you add or subtract decimal fractions, you can change them to common fractions to make the calculation easier.

For example:

$0,4 + 0,5$

$= \frac{4}{10} + \frac{5}{10}$

$= \frac{9}{10}$

$= 0,9$

2. Calculate each of the following:

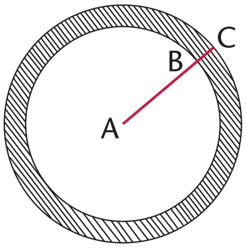
(a) $0,7 + 0,2$
(b) $0,7 + 0,4$
(c) $1,3 + 0,8$
(d) $1,35 + 0,8$
(e) $0,25 + 0,7$
(f) $0,25 + 0,07$
(g) $3 - 0,1$
(h) $3 - 0,01$
(i) $2,4 - 0,5$

SOME REAL-LIFE PROBLEMS

1. The owner of an internet café looks at her bank statement at the end of the day. She finds the following amounts paid into her account: R281,45; R39,81; R104,54 and R9,80. How much money was paid into her account on that day?
2. At the beginning of a journey the odometer in a car reads: 21 589,4. At the end of the journey the odometer reads: 21 763,7. What distance was travelled?
3. At an athletics competition, an athlete runs the 100 m race in 12,8 seconds. The announcer says that the athlete has broken the previous record by 0,4 seconds. What was the previous record?
4. In a surfing competition, five judges give each contestant a mark out of 10. The highest and the lowest marks are ignored and the other three marks are totalled. Work out each contestant's score and place the contestants in order from first to last.

A:	7,5	8	7	8,5	7,7	B:	8,5	8,5	9,1	8,9	8,7
C:	7,9	8,1	8,1	7,8	7,8	D:	8,9	8,7	9	9,3	9,1

5. A pipe is measured accurately. AC = 14,80 mm and AB = 13,97 mm.
How thick is the pipe (BC)?



6. Mrs Mdlankomo buys three packets of mincemeat.
The packets weigh 0,356 kg, 1,201 kg and 0,978 kg respectively.
What do they weigh together?

7.8 Multiplication and decimal fractions

THE POWER OF TEN

1. (a) Copy and complete the multiplication table.

×	1 000	100	10	1	0,1	0,01	0,001
6	6 000		60			0,06	
6,4		640					
0,5					0,05		
4,78	4 780		47,8				
41,2	41 200						

(b) Is it correct to say that "multiplication makes bigger"? When does multiplication make bigger?

(c) Formulate rules for multiplying with 10; 100; 1 000; 0,1; 0,01 and 0,001. Can you explain the rules?

(d) Now use your rules to calculate each of the following:
0,5 × 10   0,3 × 100   0,42 × 10   0,675 × 100

2. (a) Copy and complete the division table.

÷	1	10	100	1 000
6	6	0,6	0,06	
6,4	6,4			
0,5			0,005	
4,78				
41,2				

(b) Formulate rules for dividing with 10; 100 and 1 000. Can you explain the rules?

(c) Now use your rules to calculate each of the following:
0,5 ÷ 10   0,3 ÷ 100   0,42 ÷ 10

3. Copy and complete the following statement:
Multiplying with 0,1 is the same as dividing by ...

Now discuss it with a partner or explain to him or her why this is so.

4. Copy and fill in the missing numbers:

1,23456	→	× 10	→	12,3456	→		→	123,456	→		→	12 345,6
												↓
												↓
1 234,56	←		←	12 345,6	←		←	123 456	←		←	1 234 560
↓												
↓												
123,456	→		→	1,23456	→		→	0,123456	→		→	123 456

What does multiplying a decimal number with a whole number mean?

What does something like 4 × 0,5 mean?

What does something like 0,5 × 4 mean?

4 × 0,5 means four groups of $\frac{1}{2}$, which is $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$, which is 2.

0,5 × 4 means $\frac{1}{2}$ of 4, which is 2.

A real-life example where we would find this is:

$$\begin{aligned} 6 \times 0{,}42 \text{ kg} &= 6 \times \frac{42}{100} \\ &= (6 \times 42) \div 100 \\ &= 252 \div 100 \\ &= 2{,}52 \text{ kg} \end{aligned}$$

What really happens is that we convert 6 × 0,42 to the product of two whole numbers, do the calculation and then convert the answer to a decimal fraction again (÷ 100).

MULTIPLYING DECIMALS WITH WHOLE NUMBERS

1. Calculate each of the following. Use fraction notation to help you.
   (a) 0,3 × 7 (b) 0,21 × 91 (c) 8 × 0,4
2. Estimate the answers to each of the following and then calculate:
   (a) 0,4 × 7 (b) 0,55 × 7
   (c) 12 × 0,12 (d) 0,601 × 2
3. Make a rule for multiplying with decimals. Explain your rule to a partner.

What does multiplying a decimal with a decimal mean?

For example, what does 0,32 × 0,87 mean?

If you buy 0,32 m of ribbon and each metre costs R0,87, you can write it as 0,32 × 0,87.

$0{,}32 \times 0{,}87 = \frac{32}{100} \times \frac{87}{100}$ [Write as common fractions]

$= \frac{32 \times 87}{10\,000}$ [Multiplication of two fractions]

$= \frac{2\,784}{10\,000}$ [The product of the whole numbers 32 × 87]

$= 0{,}2784$ [Convert to a decimal by dividing the product by 10 000]

The product of two decimals is thus converted to the product of whole numbers and then converted back to a decimal.

The product of two decimals and the product of two whole numbers with the same digits differ only in terms of the place value of the products, in other words the position of the decimal comma. It can also be determined by estimating and checking.

MULTIPLYING DECIMALS WITH DECIMALS

1. Calculate each of the following. Use fraction notation to help you.
   (a) 0,6 × 0,4 (b) 0,06 × 0,4 (c) 0,06 × 0,04

Mandla uses this method to multiply decimals with decimals:

0,84 × 0,6 = (84 ÷ 100) × (6 ÷ 10)
= (84 × 6) ÷ (100 × 10)
= 504 ÷ 1 000
= 0,504

2. Calculate the following using Mandla's method:
   (a) 0,4 × 0,7 (b) 0,4 × 7 (c) 0,04 × 0,7

7.9 Division and decimal fractions

Look carefully at the following three methods of calculation:

1. 0,6 ÷ 2 = 0,3 [six tenths ÷ 2 = three tenths]
2. 12,4 ÷ 4 = 3,1 [(12 units + four tenths) ÷ 4]
   = (12 units ÷ 4) + (four tenths ÷ 4)
   = three units + one tenth
   = 3,1

3. 2,8 ÷ 5 = 28 tenths ÷ 5
= 25 tenths ÷ 5 and three tenths ÷ 5
= five tenths and (three tenths ÷ 5) [three tenths cannot be divided by 5]
= five tenths and (30 hundredths ÷ 5) [three tenths = 30 hundredths]
= five tenths and six hundredths
= 0,56

DIVIDING DECIMALS BY WHOLE NUMBERS

1. Complete.

(a) 8,4 ÷ 2
= (8........ + 4 tenths) ÷ 2
= (8........ ÷ 2) + (.............)
= 4........ + ... tenths
= .....

(b) 3,4 ÷ 4
= (3 units + 4 tenths) ÷ 4
= (32........ + 20.............) ÷ 4
= (.......... ÷ 4) + (.............. ÷ 4)
= .... + ... hundredths
= ....

2. Calculate each of the following in the shortest possible way:
   - (a) 0,08 ÷ 4
   - (b) 14,4 ÷ 12
   - (c) 8,4 ÷ 7
   - (d) 4,5 ÷ 15
   - (e) 1,655 ÷ 5
   - (f) 0,225 ÷ 25

3. A grocer buys 15 kg of bananas for R99,90. What do the bananas cost per kilogram?

4. Given 26,8 ÷ 4 = 6,7. Write down the answers to the following without calculating:
   - (a) 268 ÷ 4
   - (b) 0,268 ÷ 4

5. Given 128 ÷ 8 = 16. Write down the answers to the following without calculating:
   - (a) 12,8 ÷ 8
   - (b) 1,28 ÷ 8

6. John buys 0,45 m of chain. The chain costs R20 per metre. What does John pay for the chain?

7. You may use a calculator for this question.
Anna buys a packet of mincemeat. It weighs 0,215 kg. The price for the mincemeat is R42,95 per kilogram. What does she pay for her packet of mincemeat? (Give a sensible answer.)

Chapter 8
Functions and relationships 1

8.1 Constant and variable quantities

LOOK FOR CONNECTIONS BETWEEN QUANTITIES

1. (a) How many fingers does a person who is 14 years old have?
   (b) How many fingers does a person who is 41 years old have?
   (c) Does the number of fingers on a person's hand depend on their age? Explain.

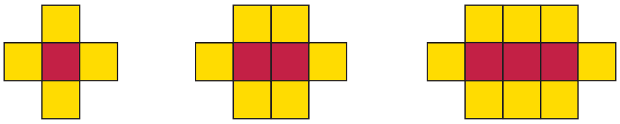
There are two quantities in the above situation: **age** and the **number of fingers** on a person's hand. The number of fingers remains the same, irrespective of a person's age. So we say the number of fingers is a **constant** quantity. However, age changes, or varies, so we say age is a **variable** quantity.

2. Now consider each situation below. For each situation, state whether one quantity influences the other. If it does, try to say *how* the one quantity will influence the other quantity. Also say whether there is a constant in the situation.

   (a) The number of calls you make and the amount of airtime left on your cell phone
   (b) The number of houses to be built and the number of bricks required
   (c) The number of learners at a school and the duration of the Mathematics period

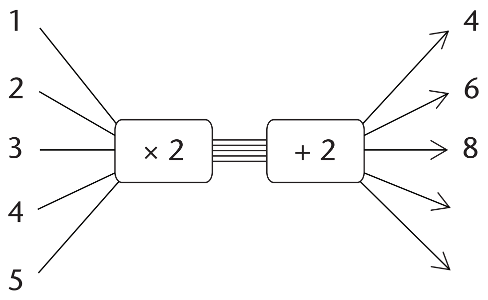
If one variable quantity is influenced by another, we say there is a **relationship** between the two variables. It is sometimes possible to find out what value of the one quantity, in other words, what number is linked to a specific value of the other variable.

3. Consider the following arrangements:



(a) How many yellow squares are there if there is only one red square?
(b) How many yellow squares are there if there are two red squares?
(c) How many yellow squares are there if there are three red squares?
(d) Copy and complete the flow diagram below by filling in the missing numbers.

Can you see the connection between the arrangements above and the flow diagram? We can also describe the relationship between the red and yellow squares in words.



In words:

The number of yellow squares is found by multiplying the number of red squares by 2 and then adding 2 to the answer.

Input numbers
(Number of red squares)

Output numbers
(Number of yellow squares)

(e) How many yellow squares will there be if there are 10 red squares?
(f) How many yellow squares will be there if there are 21 red squares?

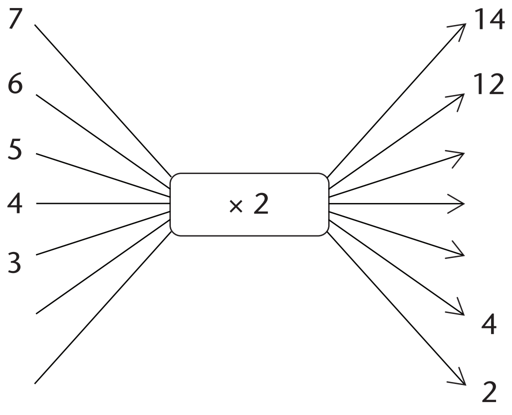
8.2 Different ways to describe relationships

COMPLETE SOME FLOW DIAGRAMS AND TABLES OF VALUES

A relationship between two quantities can be shown with a flow diagram. In a flow diagram we cannot show all the numbers, so we show only some.

1. Copy the flow diagram below and calculate the missing input and output numbers.

(a)



Each **input number** in a flow diagram has a corresponding **output number**. The first (top) input number corresponds to the first output number. The second input number corresponds to the second output number, and so on.

We say × 2 is the **operator**.

(b) What types of numbers are given as input numbers?

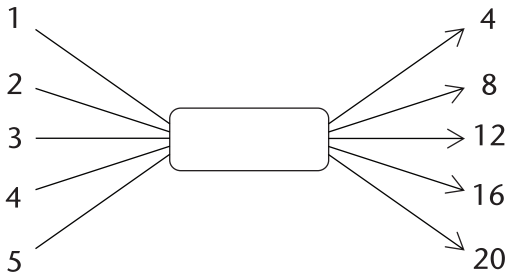
(c) In the previous flow diagram, the output number 14 corresponds to the input number 7. Copy and complete the following sentences in the same way:

In the relationship shown in the previous flow diagram, the output number ...... corresponds to the input number 5.

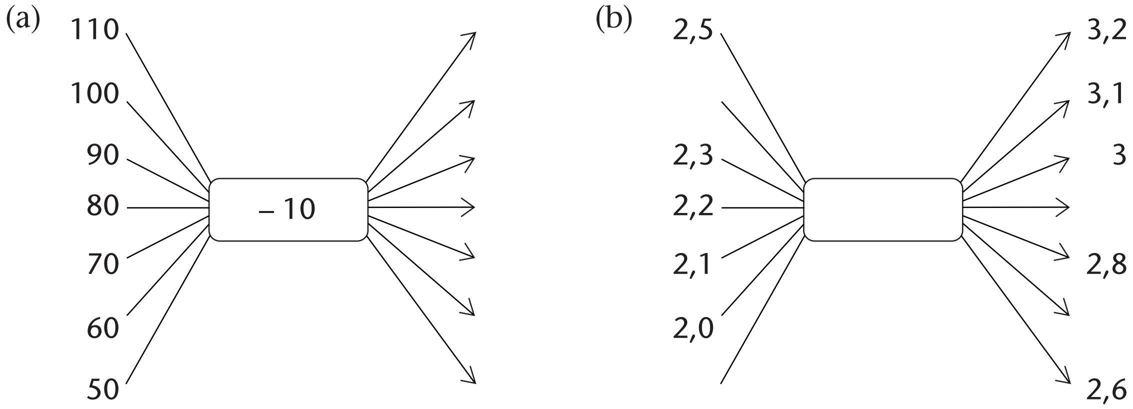
The input number ...... corresponds to the output number 6.

If more places are added to the flow diagram, the input number ...... will correspond to the output number 40.

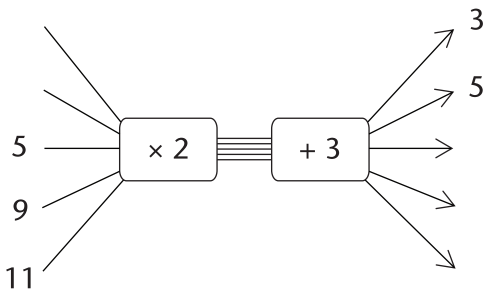
2. Copy and complete this flow diagram by writing the appropriate operator, and then write the rule for completing this flow diagram in words.



3. Copy and complete the flow diagrams below. You have to find out what the operator for (b) is and fill it in yourself.



4. Copy and complete the flow diagram:



A completed flow diagram shows two kinds of information:

- It shows what calculations are done to produce the output numbers.
- It shows which output number is connected to which input number.

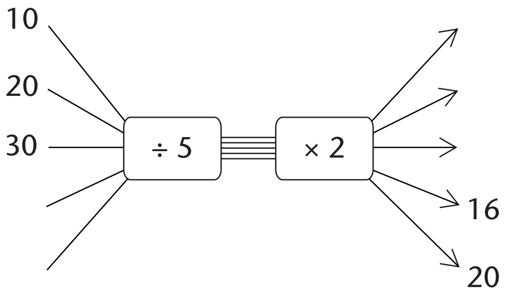
The flow diagram that you completed in question 4 shows the following information:

- Each input number is multiplied by 2 and then 3 is added to produce the output numbers.
- It shows which output number is connected to which input number.

The relationship between the input and output numbers can also be shown in a table:

Input numbers	0	1	5	9	11
Output numbers	3	5	13	21	25

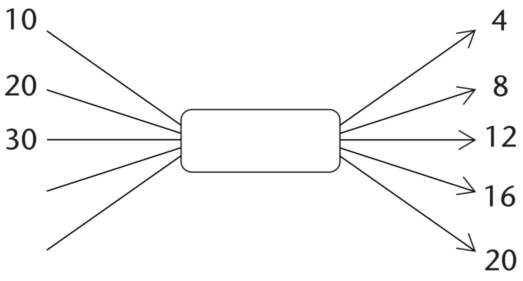
5. (a) Copy and complete the flow diagram, then describe in words how the output numbers below can be calculated.



(b) Copy and complete the table below to show which output numbers are connected to which input numbers in the above flow diagram.

Input numbers					
Output numbers					

(c) Copy the flow diagram below and fill in the appropriate operator.



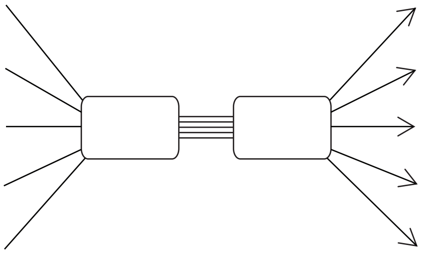
(d) The flow diagrams in question 5(a) and 5(c) have different operators, but they produce the same output values for the same input values. Explain.

6. The rule for converting temperature given in degrees Celsius to degrees Fahrenheit is given as: "Multiply the degrees Celsius by 1,8 and then add 32."

   (a) Check whether the table below was completed correctly. If you find a mistake, note what it is and correct it.

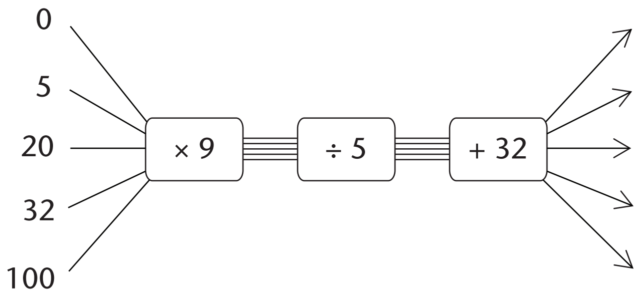
Temperature in degrees Celsius	0	5	20	32	100
Temperature in degrees Fahrenheit	32	41	68		212

   (b) Copy and complete the flow diagram to represent the information in (a).



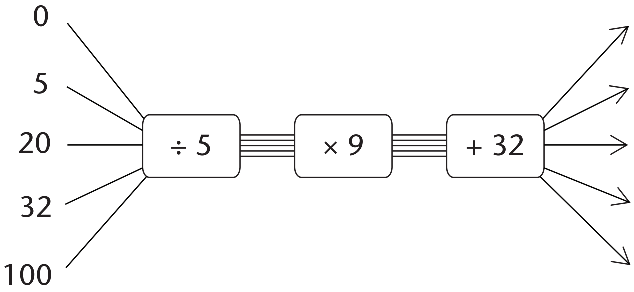
7. Another rule for converting temperature given in degrees Celsius to degrees Fahrenheit is given as: "Multiply the degrees Celsius by 9, then divide the answer by 5 and then add 32 to the answer."

   (a) Copy and complete the flow diagram below.



   (b) Why do you think the flow diagrams in questions 6(b) and 7(a) produce the same output numbers for the same input numbers, even though they have different operators?

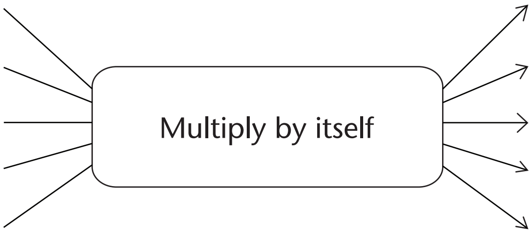
   (c) Copy and complete the flow diagram on the next page. Does this flow diagram give the same output values as the flow diagram in question 7(a)? Explain.



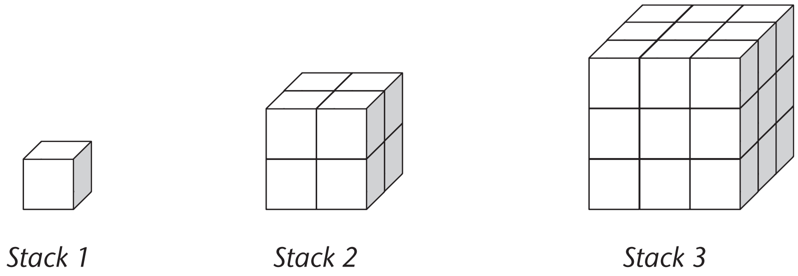
8. The rule for calculating the area of a square is: "Multiply the length of a side of the square by itself."
   (a) Copy and complete the table below.

Length of side	4	6		10	
Area of square			64		144

   (b) Copy and complete the flow diagram to represent the information in the table.



9. (a) The pattern below shows stacks of building blocks. The number of blocks in each stack is dependent on the number of the stack.



Copy and complete the table below to represent the relationship between the number of blocks and the number of the stack.

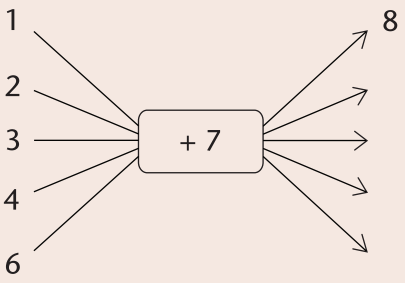
Stack number	1	2	3	4	5	6	7	8
Number of blocks	1	8						

   (b) Describe in words how the output values can be calculated.

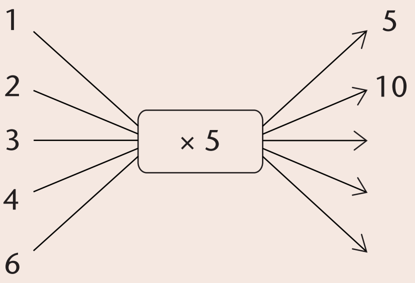
EXTENSION: LINKING FLOW DIAGRAMS, TABLES OF VALUES, AND RULES

1. Copy and complete the flow diagrams below.

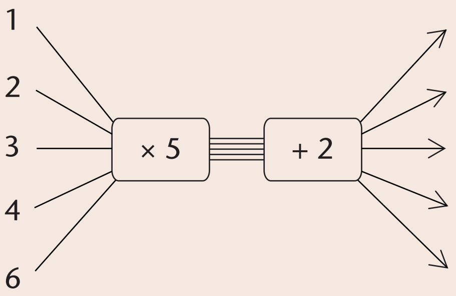
(a)



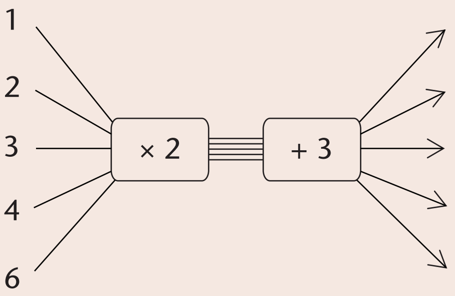
(b)



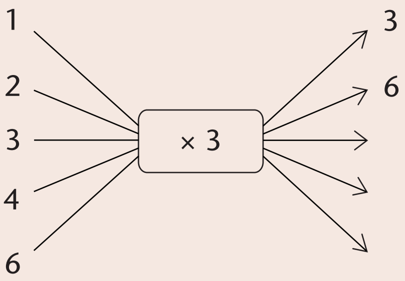
(c)



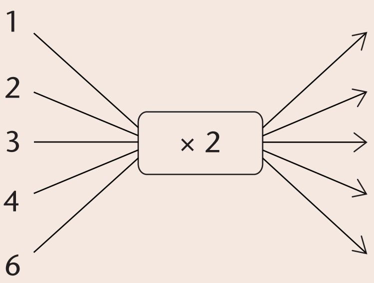
(d)



(e)



(f)



2. Calculate the differences between the consecutive output numbers and compare them to the differences between the consecutive input numbers. Consider the operator of the flow diagram. What do you notice?

3. Determine the rule to calculate the missing output numbers in this relationship and copy and complete the table:

Input numbers	1	2	3	4	5	7	10
Output numbers	9	16	23				

CHAPTER 9
Perimeter and area of 2D shapes

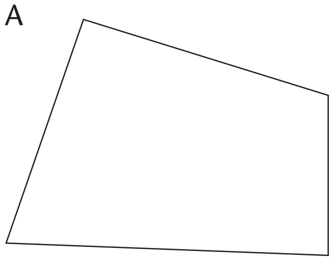
9.1 Perimeter of polygons

The **perimeter** of a shape is the total distance around the shape, or the lengths of its sides added together. Perimeter (*P*) is measured in units such as millimetres (mm), centimetres (cm) and metres (m).

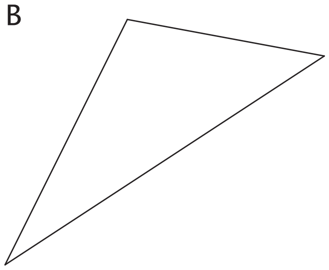
MEASURING PERIMETERS

1. (a) Use a compass and/or a ruler to measure the length of each side in figures A to C. For each figure, write the measurements in millimetres.
   (b) Write down the perimeter of each figure.

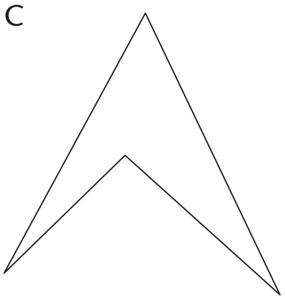
A



B

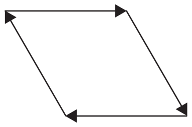


C

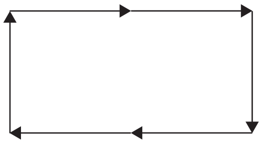


2. The following shapes consist of arrows that are equal in length.
   (a) What is the perimeter of each shape in number of arrows?
   (b) If each arrow is 30 mm long, what is the perimeter of each shape in millimetres?

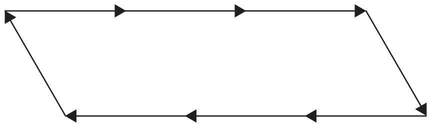
A



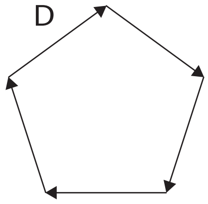
B



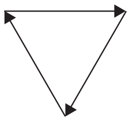
C



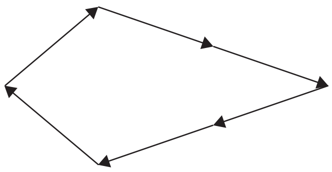
D



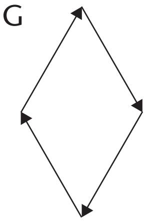
E



F



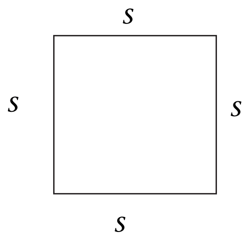
G



9.2 Perimeter formulae

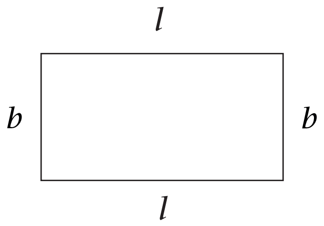
If the sides of a square are all s units long:

Perimeter of square $= s + s + s + s$
$= 4 \times s$
or $P = 4s$



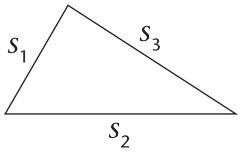
If the length of a rectangle is l units and the breadth (width) is b units:

Perimeter of rectangle $= l + l + b + b$
$= 2 \times l + 2 \times b$
$= 2l + 2b$
or $P = 2(l + b)$



A triangle has three sides, so:

Perimeter of triangle $= s_1 + s_2 + s_3$
or $P = s_1 + s_2 + s_3$



APPLYING PERIMETER FORMULAE

1. Calculate the perimeter of a square if the length of one of its sides is 17,5 cm.
2. One side of an equilateral triangle is 32 cm. Calculate the triangle's perimeter.
3. Calculate the length of one side of a square if the perimeter of the square is 7,2 m. (Hint: $4s = ?$ Therefore $s = ?$)
4. Two sides of a triangle are 2,5 cm each. Calculate the length of the third side if the triangle's perimeter is 6,4 cm.
5. A rectangle is 40 cm long and 25 cm wide. Calculate its perimeter.
6. Calculate the perimeter of a rectangle that is 2,4 m wide and 4 m long.
7. The perimeter of a rectangle is 8,88 m. How long is the rectangle if it is 1,2 m wide?
8. Do the necessary calculations and then copy and complete the tables. (All the measurements refer to rectangles.)

	Length	Breadth	Perimeter
(a)	74 mm	30 mm	
(c)		1,125 cm	6,25 cm
(e)	7,5 m	3,8 m	

	Length	Breadth	Perimeter
(b)	25 mm		90 mm
(d)	5,5 cm		22 cm
(f)		2,5 m	12 m

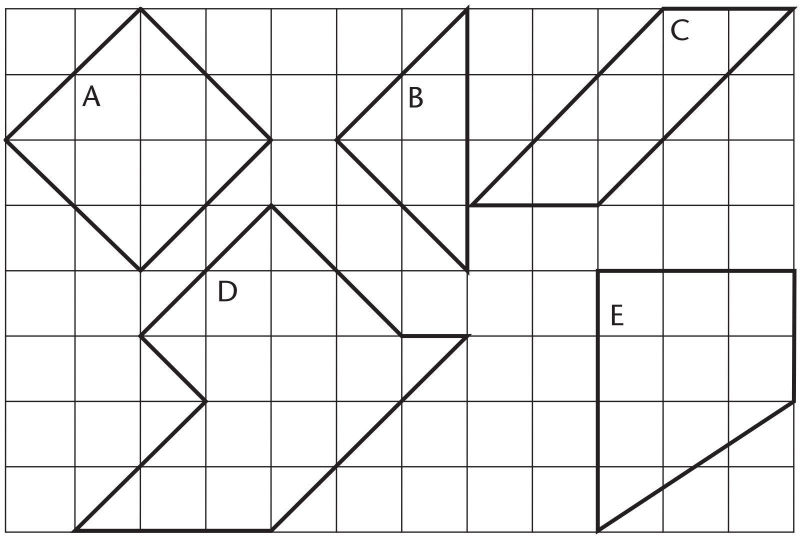
9.3 Area and square units

The **area** of a shape is the size of the flat surface surrounded by the border (perimeter) of the shape.

Usually, area (A) is measured in square units, such as square millimetres (mm^2), square centimetres (cm^2) and square metres (m^2).

SQUARE UNITS TO MEASURE AREA

1. In your book, write down the area of figures A to E below by counting the square units. (Remember to add halves or smaller parts of squares.)



A is .... square units.

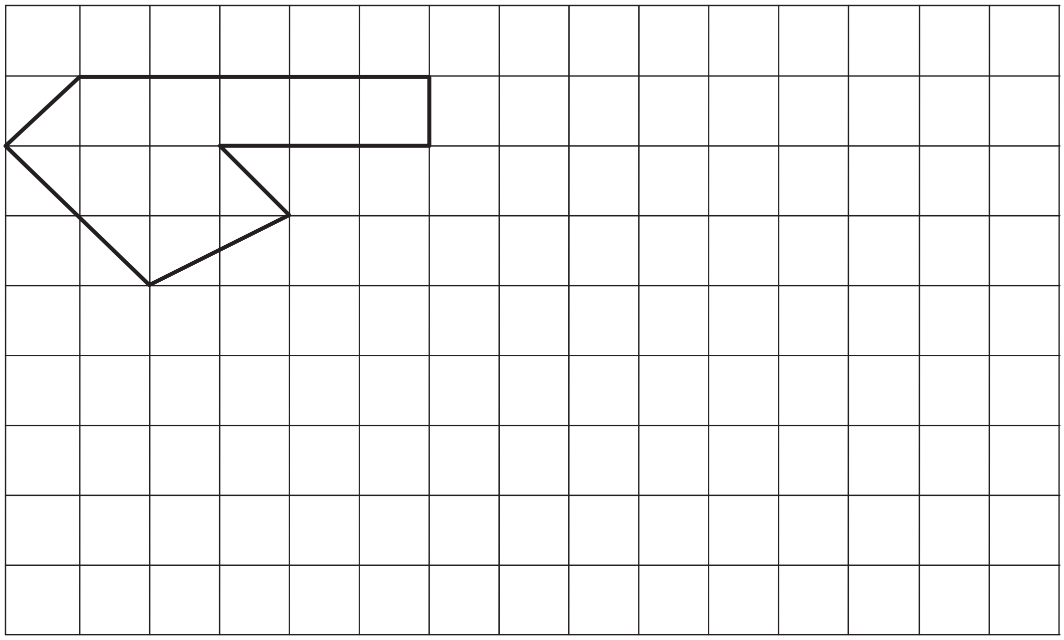
B is .... square units.

C is .... square units.

D is .... square units.

E is .... square units.

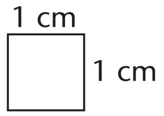
2. Each square in the grid below measures 1 cm^2 (1 cm × 1 cm).

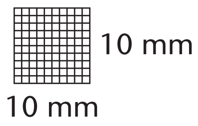


(a) What is the area of the shape drawn on the grid?

(b) Trace the grid, then draw two shapes of your own. The shapes should have the same area, but different perimeters.

CONVERSION OF UNITS





The figure on the right shows a square with sides of 1 cm. The area of the square is one square centimetre (1 cm^2).

How many squares of 1 mm by 1 mm (1 mm^2) would fit into the 1 cm^2 square? Copy and complete: 1 cm^2 = ... mm^2

To change cm^2 to mm^2:

1 cm^2 = 1 cm × 1 cm
= 10 mm × 10 mm
= 100 mm^2

Similarly, to change mm^2 to cm^2:

1 mm^2 = 1 mm × 1 mm
= 0,1 cm × 0,1 cm
= 0,01 cm^2

We can use the same method to convert between other square units too. Copy and complete:

From m^2 to cm^2:	**From cm^2 to m^2:**
1 m^2 = 1 m × 1 m	1 cm^2 = 1 cm × 1 cm
= .... cm × .... cm	= 0,01 m × 0,01 m
= .... cm^2	= .... m^2

> So, to convert between m^2, cm^2 and mm^2 you do the following:
> - cm^2 to mm^2 → multiply by 100
> - m^2 to cm^2 → multiply by 10 000
> - mm^2 to cm^2 → divide by 100
> - cm^2 to m^2 → divide by 10 000

Do the necessary calculations in your exercise book. Then copy the conversions and fill in your answers.

1. (a) 5 m^2 = ... cm^2
   (b) 5 cm^2 = ... mm^2
   (c) 20 cm^2 = ... m^2
   (d) 20 mm^2 = ... cm^2

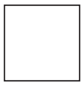
2. (a) 25 m^2 = ... cm^2
   (b) 240 000 cm^2 = ... m^2
   (c) 460,5 mm^2 = ... cm^2
   (d) 0,4 m^2 = ... cm^2
   (e) 12 100 cm^2 = ... m^2
   (f) 2,295 cm^2 = ... mm^2

9.4 Area of squares and rectangles

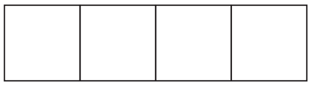
INVESTIGATING THE AREA OF SQUARES AND RECTANGLES

1. Each of the following four figures is divided into squares of equal size, namely 1 cm by 1 cm.

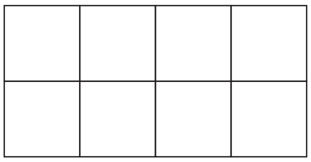
A



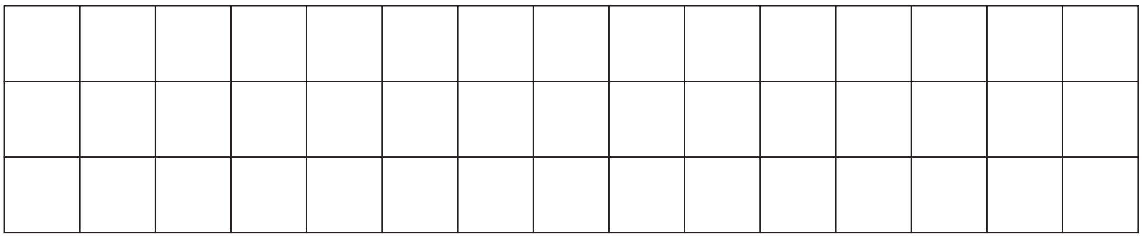
B



C

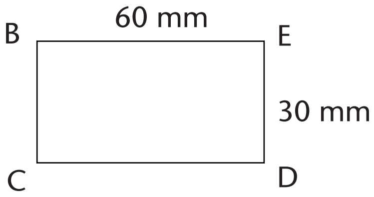


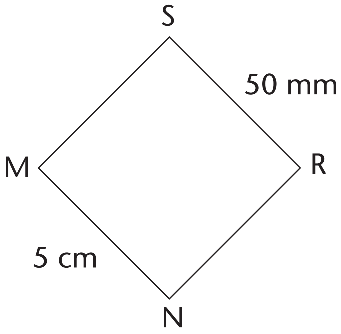
D



   (a) Give the area of each figure in square centimetres (cm²).
   (b) Is there a shorter method to work out the area of each figure? Explain.

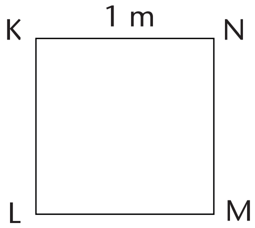
2. Figure BCDE is a rectangle and MNRS is a square.





   (a) How many cm² (1 cm × 1 cm) would fit into rectangle BCDE?
   (b) How many mm² (1 mm × 1 mm) would fit into rectangle BCDE?
   (c) What is the area of square MNRS in cm²?
   (d) What is the area of square MNRS in mm²?

3. Figure KLMN is a square with sides of 1 m.



   (a) How many squares with sides of 1 cm would fit along the length of the square?
   (b) How many squares with sides of 1 cm would fit along the breadth of the square?
   (c) How many squares (cm²) would therefore fit into the whole square?
   (d) Copy and complete: 1 m² = ... cm²

> A quick way of calculating the number of squares that would fit into a rectangle is to multiply the number of squares that would fit along its length by the number of squares that would fit along its breadth.

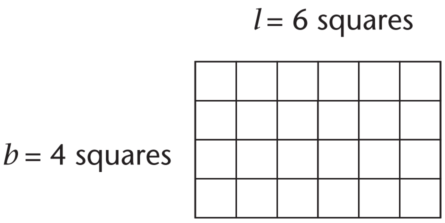
FORMULAE: AREA OF RECTANGLES AND SQUARES

In the rectangle on the right:

Number of squares = squares along the length × squares along the breadth

$= 6 \times 4$

$= 24$



From this we can deduce the following:

> **Area of rectangle** = length of rectangle × breadth of rectangle
>
> $A = l \times b$
>
> (where A is the area in square units, l is the length and b is the breadth)
>
> **Area of square** = length of side × length of side
>
> $A = l \times l$
>
> $= l^2$
>
> (where A is the area in square units, and l is the length of a side)

The units of the values used in the calculations must be the same. Remember:

- 1 m = 100 cm and 1 cm = 10 mm
- 1 cm^2 = 1 cm × 1 cm = 10 mm × 10 mm = 100 mm^2
- 1 m^2 = 1 m × 1 m = 100 cm × 100 cm = 10 000 cm^2
- 1 mm^2 = 1 mm × 1 mm = 0,1 cm × 0,1 cm = 0,01 cm^2
- 1 cm^2 = 1 cm × 1 cm = 0,01 m × 0,01 m = 0,0001 m^2

Examples

1. Calculate the area of a rectangle with a length of 50 mm and a breadth of 3 cm. Give the answer in cm^2.

   Solution:

   Area of rectangle $= l \times b$

   $= (50 \times 30)\ \text{mm}^2$ or $A = (5 \times 3)\ \text{cm}^2$

   $= 1\,500\ \text{mm}^2$ or $= 15\ \text{cm}^2$

2. Calculate the area of a square bathroom tile with a side of 150 mm.

   Solution:

$$\begin{aligned}\text{Area of square tile} &= l \times l\\ &= (150 \times 150)\ \text{mm}^2\\ &= 22\,500\ \text{mm}^2\end{aligned}$$

   The area is therefore 22 500 mm^2 (or 225 cm^2).

3. Calculate the length of a rectangle if its area is 450 cm^2 and its width is 150 mm.

   Solution:

$$\begin{aligned}\text{Area of rectangle} &= l \times b\\ 450 &= l \times 15\\ 30 \times 15 &= l \times 15 \qquad \text{or} \qquad 450 \div 15 = l\\ 30 &= l \qquad\qquad\qquad\qquad\quad 30 = l\end{aligned}$$

   The length is therefore 30 cm (or 300 mm).

APPLYING THE FORMULAE

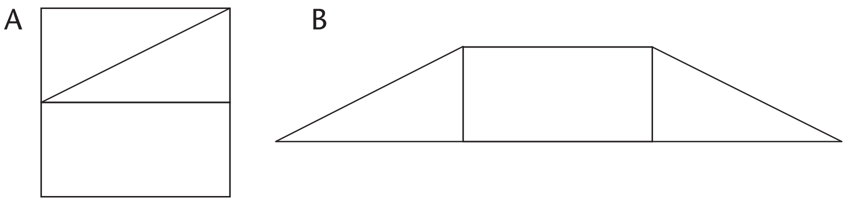
1. Calculate the area of each of the following shapes:
   (a) a rectangle with sides of 12 cm and 9 cm
   (b) a square with sides of 110 mm (answer in cm^2)
   (c) a rectangle with sides of 2,5 cm and 105 mm (answer in mm^2)
   (d) a rectangle with a length of 8 cm and a perimeter of 24 cm

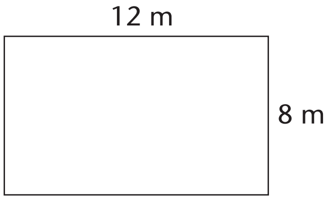
2. A rugby field has a length of 100 m (goal post to goal post) and a breadth of 69 m.
   (a) What is the area of the field (excluding the area behind the goal posts)?
   (b) What would it cost to plant new grass on that area at a cost of R45/m^2?
   (c) Another unit for area is the hectare (ha). It is mainly used for measuring land. The size of 1 ha is the equivalent of 100 m × 100 m. Is a rugby field greater or smaller than 1 ha? Explain your answer.

3. Do the necessary calculations and then copy and complete the table below. (All the measurements refer to rectangles.)

	Length	**Breadth**	**Area**
(a)	m	8 m	120 m^2
(b)	120 mm	mm	60 cm^2
(c)	3,5 m	4,3 m	m^2
(d)	2,3 cm	cm	2,76 cm^2
(e)	5,2 m	460 cm	m^2

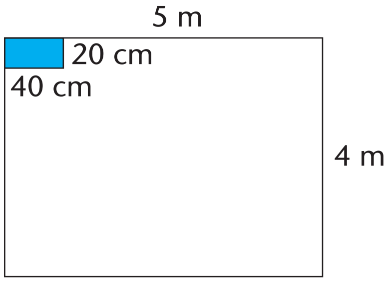
4. Figure A is a square with sides of 20 mm. It is cut as shown in A and the parts are combined to form figure B. Calculate the area of figure B.





5. Margie plants a vegetable patch measuring 12 m × 8 m.
   (a) What is the area of the vegetable patch?
   (b) She plants carrots on half of the patch, and tomatoes and potatoes on a quarter of the patch each. Calculate the area covered by each type of vegetable?
   (c) How much will she pay to put fencing around the patch? The fencing costs R38/m.

6. Mr Allie has to tile a kitchen floor measuring 5 m × 4 m. The blue tiles he uses each measure 40 cm × 20 cm.



   (a) How many tiles does Mr Allie need?
   (b) The tiles are sold in boxes containing 20 tiles. How many boxes should he buy?

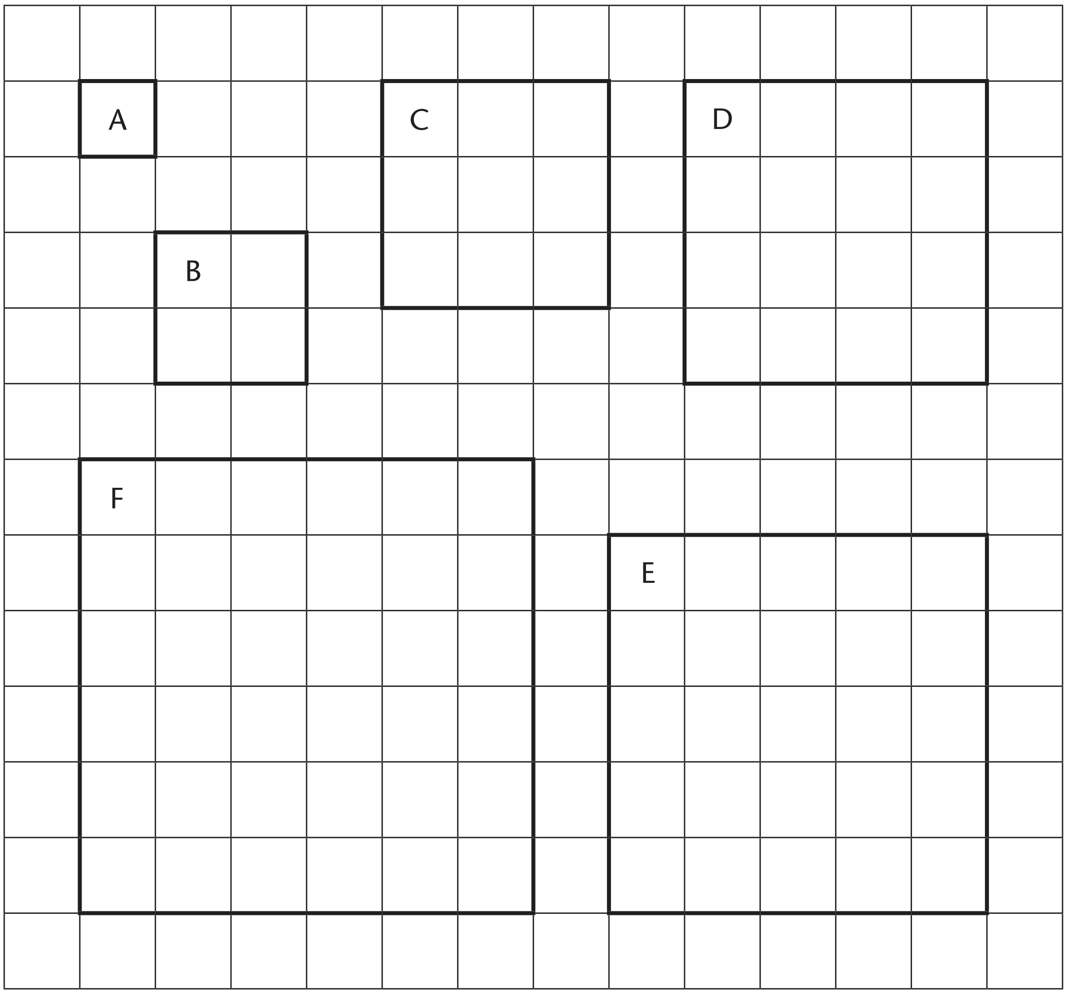
Copy the grid on page 141. The size of each square making up the grid is 1 cm × 1 cm.

7. (a) For each square drawn on the grid, label the lengths of its sides.
   (b) Write down the area of each square. (Write the answer inside the square.)

8. (a) Write the sidelengths and the areas of the squares in a table like this.

Length of each side in cm	1	2	3	4	5	6
Area of square in cm^2						
Perimeter in cm						

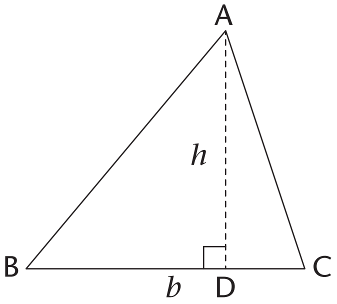
   (b) By how much does the perimeter increase when the length of the sides increase by 1 cm?
   (c) By how much does the area increase when the length of the sides increase by 1 cm?



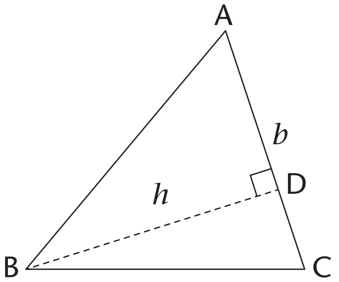
9.5 Area of triangles

HEIGHTS AND BASES OF A TRIANGLE

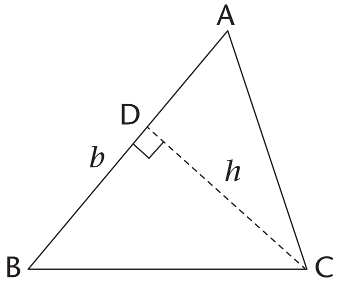
The **height (*h*)** of a triangle is a perpendicular line segment drawn from a vertex to its opposite side. The opposite side, which forms a right angle with the height, is called the **base (*b*)** of the triangle. Any triangle has three heights and three bases.



AD = height
BC = base

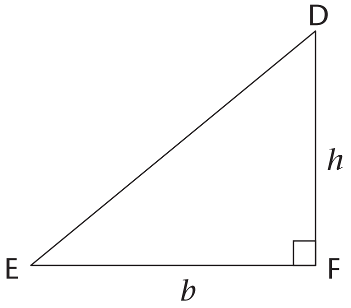


BD = height
AC = base

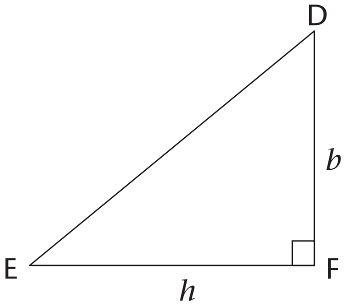


CD = height
AB = base

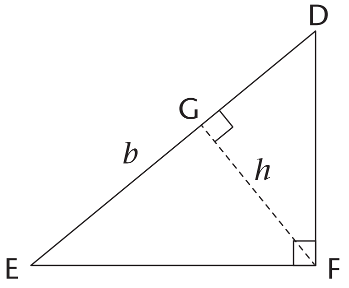
In a right-angled triangle, two sides are already at right angles:



DF = height
EF = base

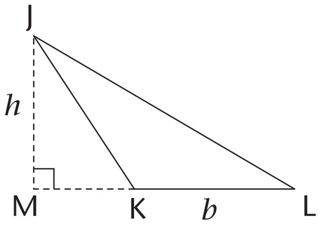


EF = height
DF = base

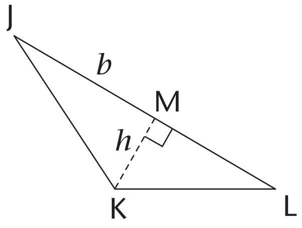


FG = height
DE = base

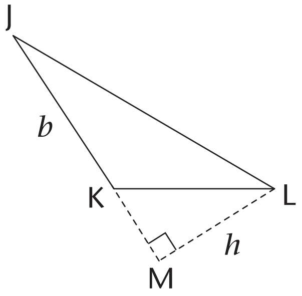
Sometimes a base must be extended outside of the triangle in order to draw the perpendicular height. This is shown in the first and third triangles below. Note that the extended part does not form part of the base's measurement:



JM = height
KL = base

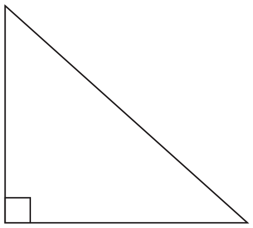


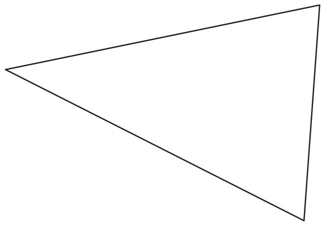
KM = height
JL = base

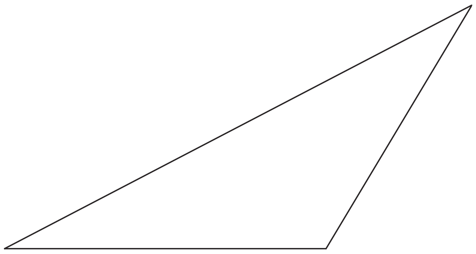


LM = height
JK = base

1. Copy the following triangles. Draw any height in each of the triangles. Label the height (*h*) and base (*b*) on each triangle.



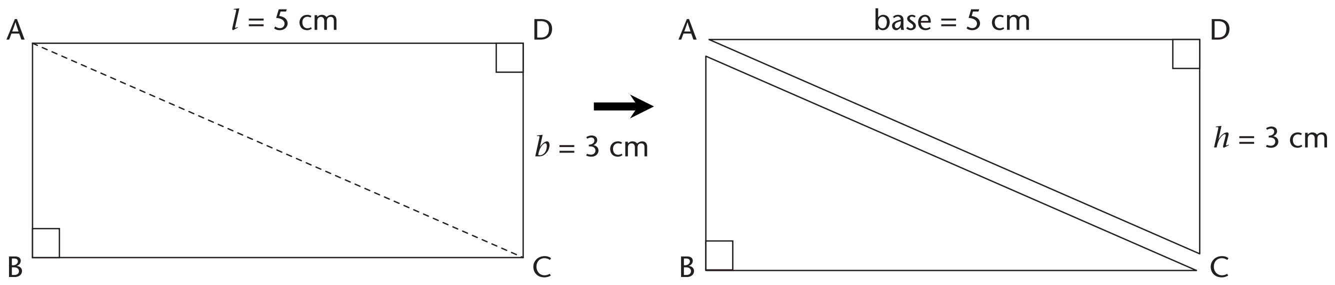




2. Label another set of heights and bases on each triangle of question 1.

FORMULA: AREA OF A TRIANGLE

ABCD is a rectangle with length = 5 cm and breadth = 3 cm. When A and C are joined, it creates two triangles that are equal in area: ΔABC and ΔADC.



$$\text{Area of rectangle} = l \times b$$

$$\text{Area of } \Delta ABC \text{ (or } \Delta ADC) = \frac{1}{2} \text{ (Area of rectangle)}$$

$$= \frac{1}{2}(l \times b)$$

In rectangle ABCD, AD is its length and CD is its breadth.
But look at ΔADC. Can you see that AD is a base and CD is its height?

So instead of saying:

$$\text{Area of } \Delta ADC \text{ or any other triangle} = \frac{1}{2}(l \times b)$$

we say:

$$\textbf{Area of a triangle} = \frac{1}{2}(\text{base} \times \text{height})$$

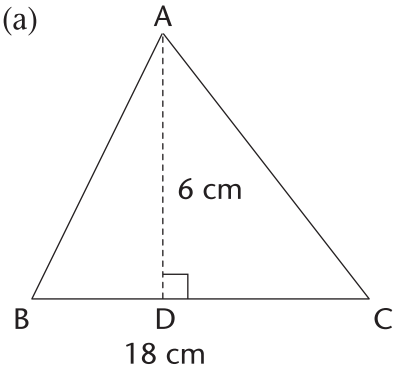
$$= \frac{1}{2}(b \times h)$$

In the formula for the area of a triangle, b means "base" and not "breadth", and h means perpendicular height.

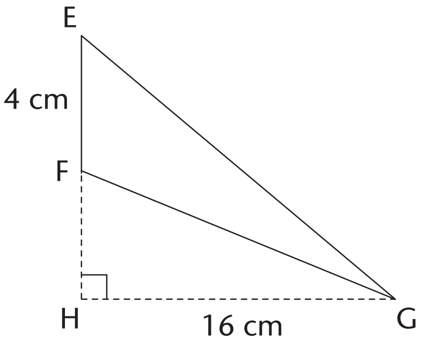
APPLYING THE AREA FORMULA

1. Use the formula to calculate the areas of the following triangles: ΔABC, ΔEFG, ΔJKL and ΔMNP.

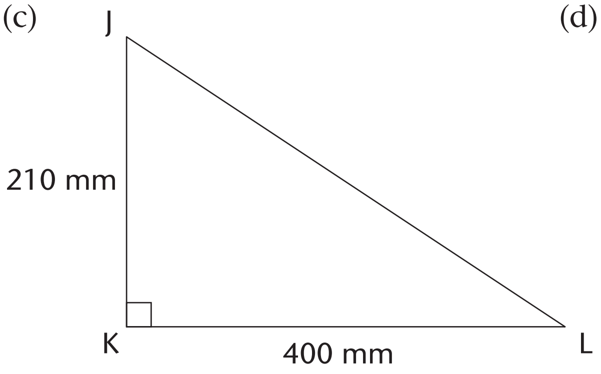
(a)



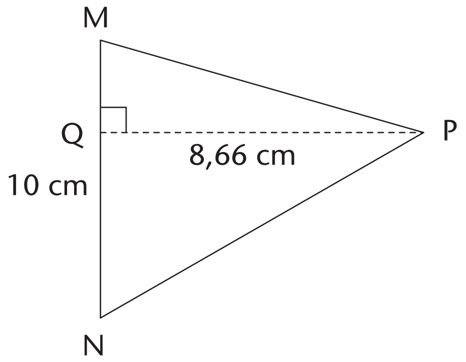
(b)



(c)

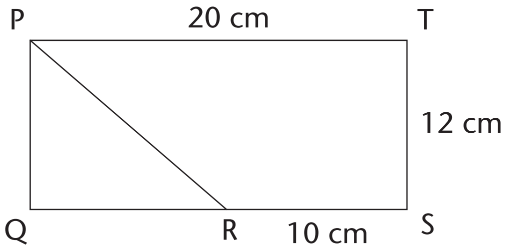


(d)

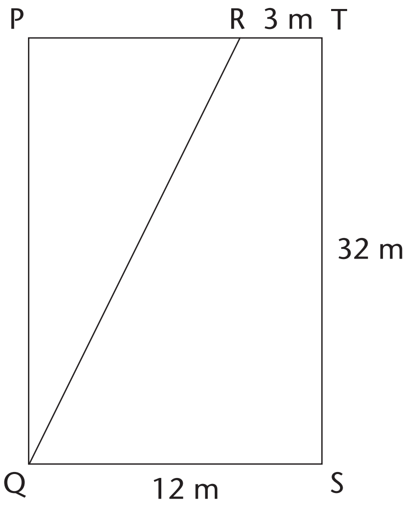


2. PQST is a rectangle in each case below. Calculate the area of ΔPQR each time.

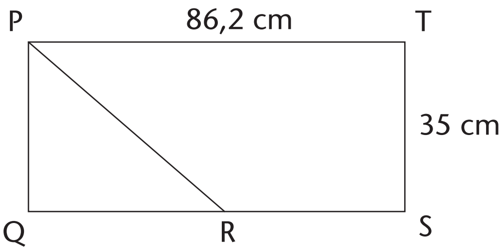
(a)



(b)



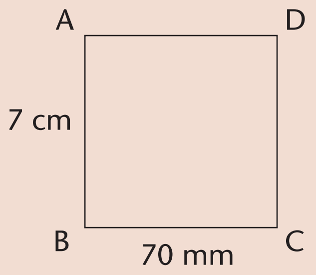
(c) R is the midpoint of QS.

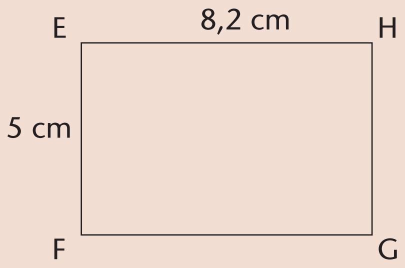


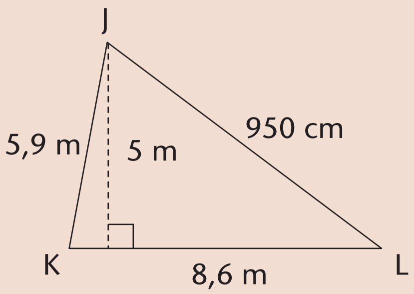
3. In ΔABC, the area is 42 m^2, and the perpendicular height is 16 m. Find the length of the base.

WORKSHEET

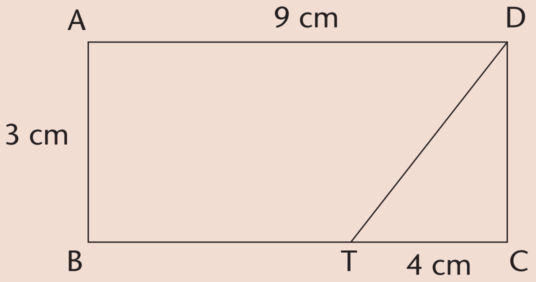
1. Calculate the perimeter (P) and area (A) of the following figures:







2. Figure ABCD is a rectangle:
   AB = 3 cm, AD = 9 cm and TC = 4 cm.



   (a) Calculate the perimeter of ABCD.

   (b) Calculate the area of ABCD.

   (c) Calculate the area of ΔDTC.

   (d) Calculate the area of ABTD.

Chapter 10

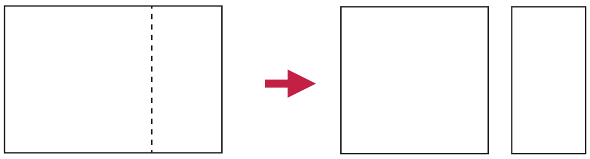
Surface area and volume of 3D objects

10.1 Surface area of cubes and rectangular prisms

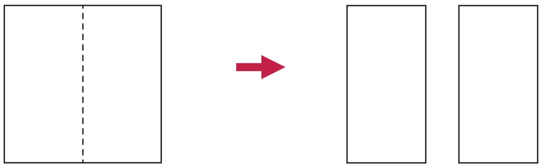
INVESTIGATING SURFACE AREA

1. Follow the instructions below to make a paper cube.

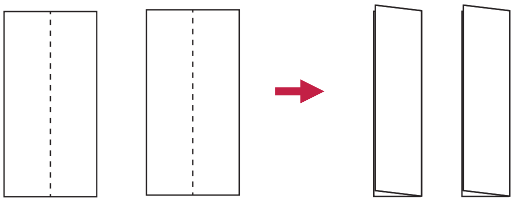
Step 1: Cut off part of an A4 sheet so that you are left with a square.



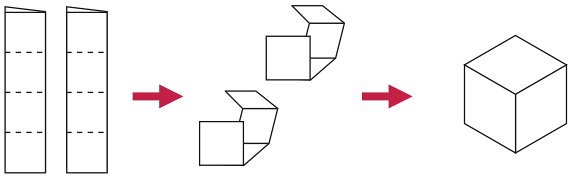
Step 2: Cut the square into two equal halves.



Step 3: Fold each half square lengthwise down the middle to form two double-layered strips.



Step 4: Fold each strip into four square sections, and put the two parts together to form a paper cube. Use sticky tape to keep it together.



2. Number each face of the cube. How many faces does the cube have?
3. Measure the side length of one face of the cube.
4. Calculate the area of one face of the cube.
5. Add up the areas of all the faces of the cube.

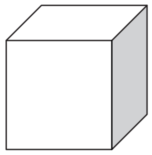
The **surface area** of an object is the sum of the areas of all its faces (or outer surfaces).

As for other areas, we measure surface area in square units, for example mm^2, cm^2 and m^2.

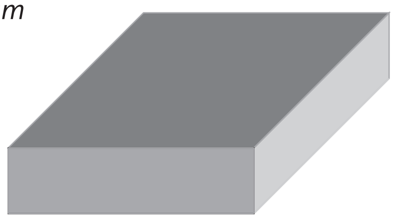
A cube has six identical square faces. A die (plural: dice) is an example of a cube.

A rectangular prism also has six faces, but its faces can be squares and/or rectangles. A matchbox is an example of a rectangular prism.

Cube



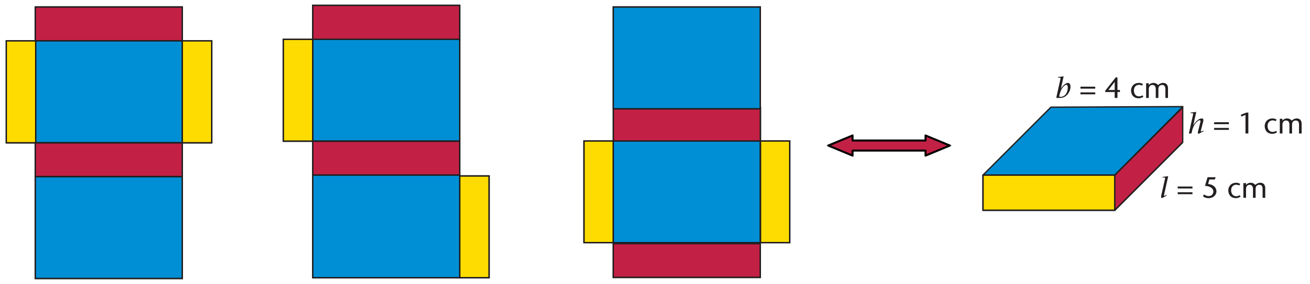
Rectangular prism

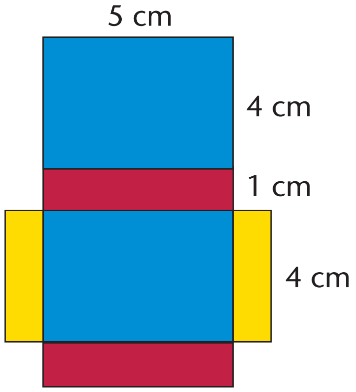


USING NETS OF RECTANGULAR PRISMS AND CUBES

It is sometimes easier to see all the faces of a rectangular prism or cube if we look at its net. A **net** of a prism is the figure obtained when cutting the prism along some of its edges, unfolding it and laying it flat.

1. Take a sheet of paper and wrap it around a matchbox so that it covers the whole box without going over the same place twice. Cut off extra bits of paper as necessary so that you have only the paper that covers each face of the matchbox.
2. Flatten the paper and draw lines where the paper has been folded. Your sheet might look like one of the following nets (there are also other possibilities):

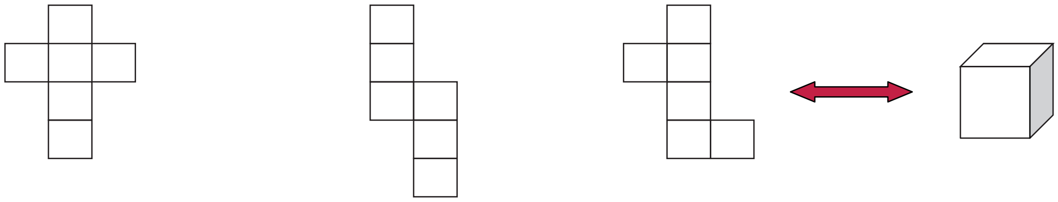




3. Notice that there are six rectangles in the net, each matching a rectangular face of the matchbox. Point to the three pairs of identical rectangles in each net.
4. Use the measurements given to work out the surface area of the prism. (Add up the areas of each face.)
5. Explain to a classmate why you think the following formula is or is not correct:

Surface area of a rectangular prism $= 2(l \times b) + 2(l \times h) + 2(b \times h)$

6. Here are three different nets of the same cube.



(a) Can you picture in your mind how the squares can fold up to make a cube?

(b) If the length of an edge of the cube is 1 cm, what is the area of one of its faces? What then is the area of all its six faces?

(c) Explain to a classmate why you think the following formula is or is not correct:

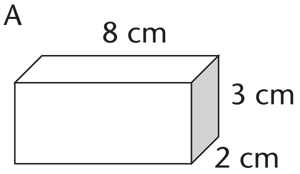
Surface area of a cube = $6(l \times l) = 6l^2$

(d) If the length of an edge of the cube above is 3 cm, what is the surface area of the cube?

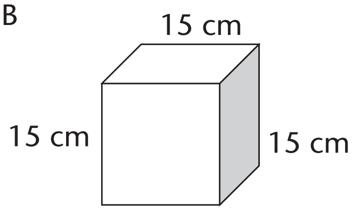
WORKING OUT SURFACE AREAS

1. Work out the surface areas of the following rectangular prisms and cubes.

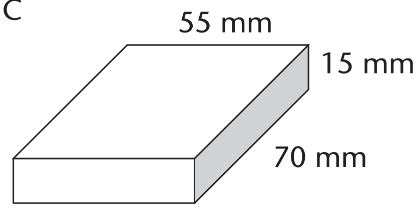
A



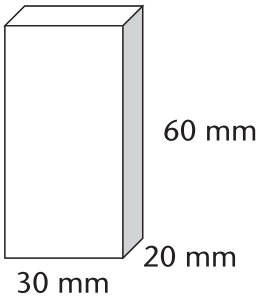
B



C



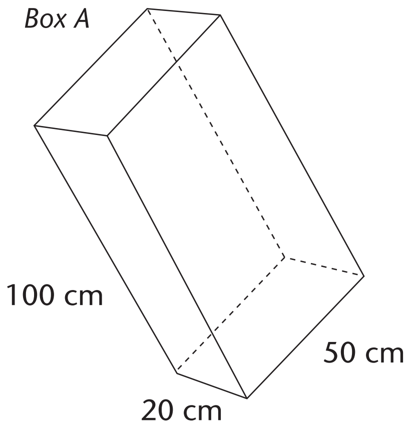
D

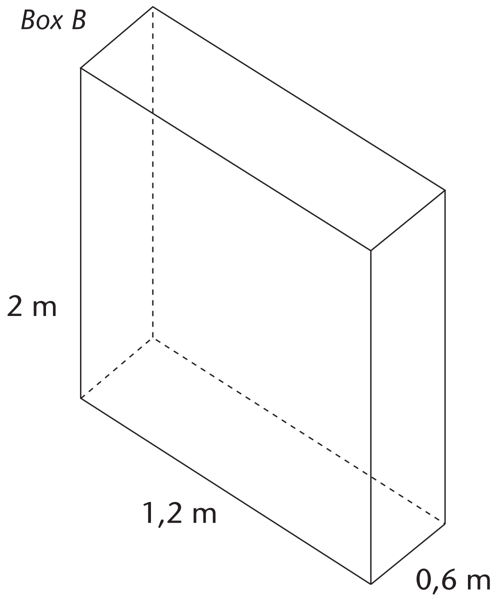


2. The two boxes on the following page are rectangular prisms. The boxes must be painted.

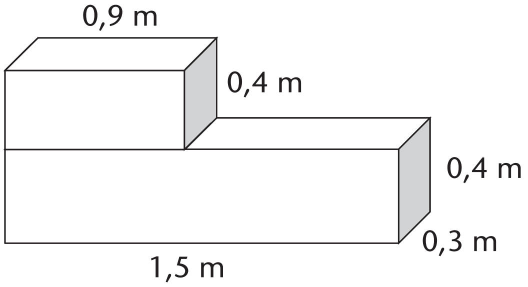
(a) Calculate the total surface area of Box A and of Box B.

(b) What will it cost to paint both boxes if the paint costs R1,34 per m²?

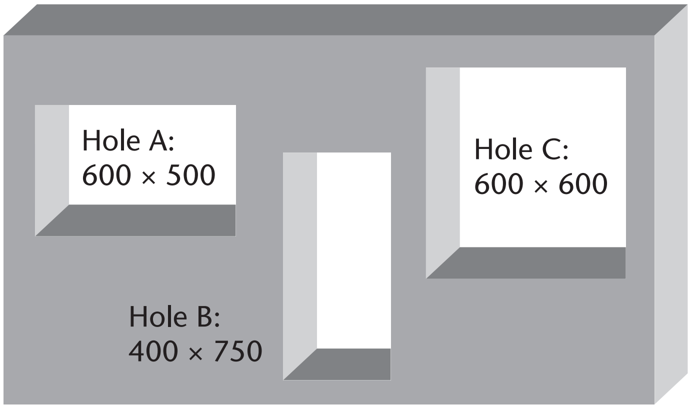




3. Two cartons, which are rectangular prisms, are glued together as shown. Calculate the surface area of this object. (Note which faces can be seen and which cannot be seen.)



4. This large plastic wall measures 3 m × 0,5 m × 1,5 m. It has to be painted for the Uyavula Literacy Project. The wall has three holes in it, labelled A, B and C, as shown. The holes go right through the wall. The measurements of the holes are in millimetres.



(a) Calculate the area of the front and back surfaces that must be painted.

(b) Calculate the area of the two side faces, as well as the top face.

(c) Calculate the total surface area of the wall, excluding the bottom and the inner surfaces where the holes are, because these will not be painted.

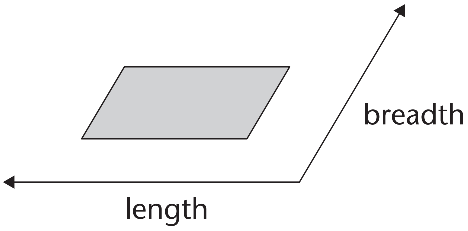
(d) What will it cost if the water-based paint costs R2,00 per m^2?

Remember from the previous chapter:
$1\ cm^2 = 100\ mm^2$
$1\ m^2 = 10\ 000\ cm^2$

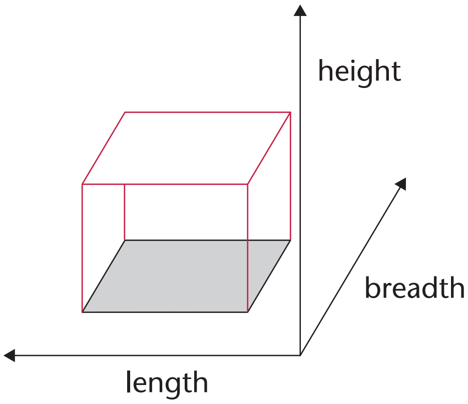
10.2 Volume of rectangular prisms and cubes

2D shapes are flat and have only two dimensions, namely length (l) and breadth (b). 3D objects have three dimensions, namely length (l), breadth (b) and height (h). You can think of a dimension as a direction in space. Look at these examples:

2D shape: rectangle



3D object: rectangular prism



3D objects therefore take up space in a way that 2D shapes do not. We can measure the amount of space that 3D objects take up.

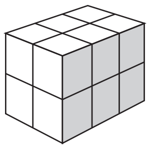
Every object in the real world is 3D. Even a sheet of paper is a 3D object. Its height is about 0,1 mm.

CUBES TO MEASURE AMOUNT OF SPACE

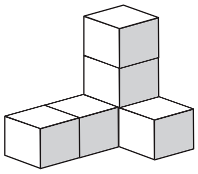
We can use cubes to measure the amount of space that an object takes up.

1. Identical toy building cubes were used to make the stacks shown below.

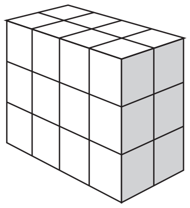
A



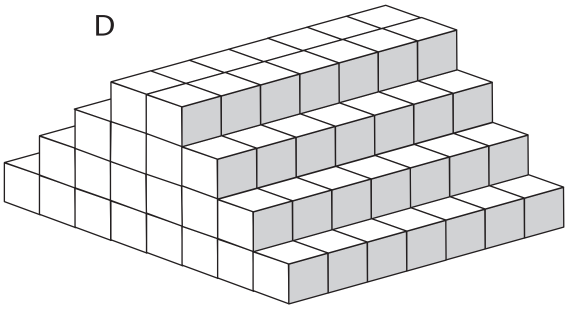
B



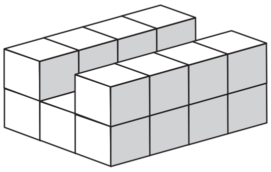
C



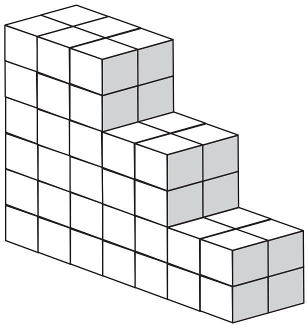
D



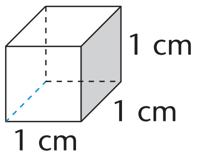
E



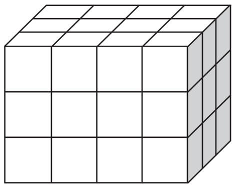
F



(a) Which stack takes up the least space?
(b) Which stack takes up the most space?
(c) Order the stacks from the one that takes up the least space to the one that takes up the most space. (Write the letters of the stacks.)



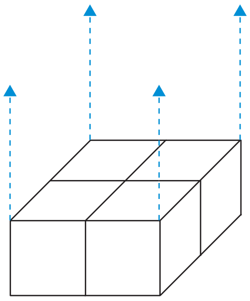
> The space (in all directions) occupied by a 3D object is called its **volume**.
>
> Cubes are the units we use to measure volume.
>
> A cube with edges of 1 cm (that is, 1 cm × 1 cm × 1 cm) has a volume of one cubic centimetre (1 cm^3).



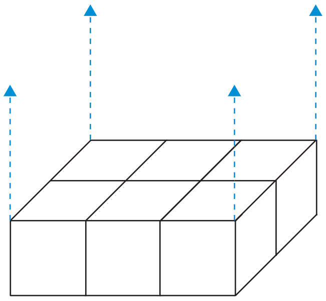
2. The figure on the right shows a rectangular prism made from 36 cubes, each with an edge length of 1 cm. The prism thus has a volume of 36 cubic centimetres (36 cm^3).

   (a) The stack is taken apart and all 36 cubes are stacked again to make a new rectangular prism with a base of four cubes (see A below.) How many layers of cubes will the new prism be? What is the height of the new prism?

A



B

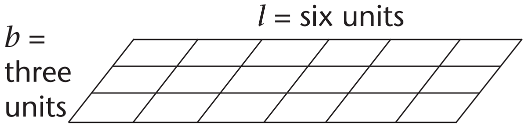


   (b) Repeat (a), but this time make a prism with a base of six cubes (see B above).
   (c) Which one of the rectangular prisms in questions (a) and (b) takes up the most space in all directions? (Which one has the greatest volume?)
   (d) What will be the volume of the prism in question (b) if there are seven layers of cubes altogether?
   (e) A prism is built with 48 cubes, each with an edge length of 1 cm. The base consists of eight layers. What is the height of the prism?

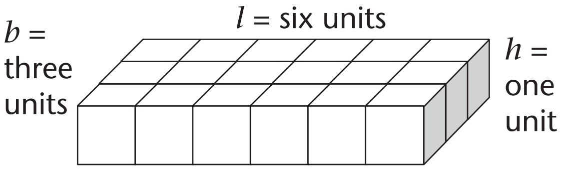
FORMULA TO CALCULATE VOLUME

You can think about the volume of a rectangular prism in the following way:

Step 1: Measure the area of the bottom face (also called the base) of a rectangular prism. For the prism given here: $A = l \times b = 6 \times 3 = 18$ square units.



Step 2: A layer of cubes, each one unit high, is placed on the flat base. The base now holds 18 cubes. It is 6 × 3 × 1 cubic units.

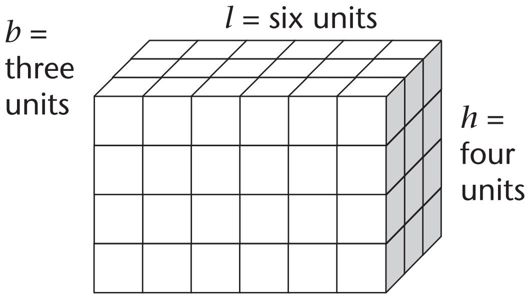


Step 3: Three more layers of cubes are added so that there are four layers altogether. The prism's height (h) is four units. The volume of the prism is:

$$V = (6 \times 3) \times 4$$

or V = Area of base × number of layers

$$= (l \times b) \times h$$



Therefore:

Volume of a rectangular prism = Area of base × height

$= l \times b \times h$

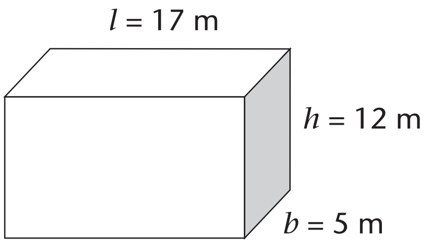
Volume of a cube = $l \times l \times l$ (edges are all the same length)

$= l^3$

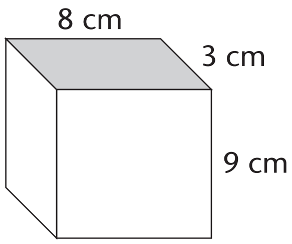
APPLYING THE FORMULAE

1. Calculate the volume of these prisms and cubes.

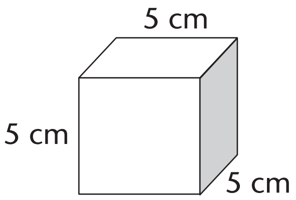
A



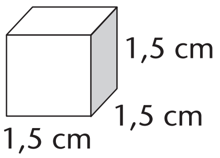
B



C



D



2. Calculate the volume of prisms with the following measurements:
   (a) $l = 7$ m, $b = 6$ m, $h = 6$ m
   (b) $l = 55$ cm, $b = 10$ cm, $h = 20$ cm
   (c) Surface of base = 48 m^2, $h = 4$ m
   (d) Surface of base = 16 mm^2, $h = 12$ mm

3. Calculate the volume of cubes with the following edge lengths:
   (a) 7 cm
   (b) 12 mm

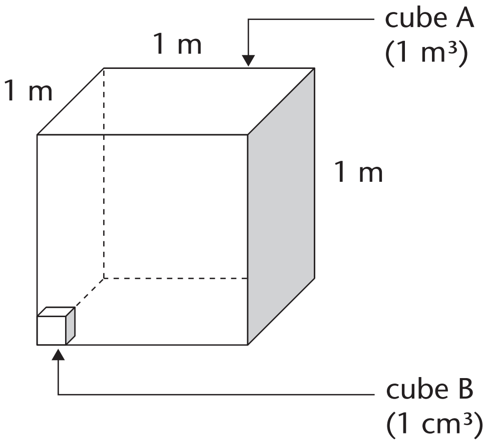
4. Calculate the volume of the following square-based prisms:
   (a) side of the base = 5 mm, $h = 12$ mm (b) side of the base = 11 m, $h = 800$ cm

5. The volume of a prism is 375 m^3. What is the height of the prism if its length is 8 m and its breadth is 15 m?

10.3 Converting between cubic units

CUBIC UNITS TO MEASURE VOLUME

This drawing shows a cube (A) with an edge length of 1 m. Also shown is a small cube (B) with an edge length of 1 cm.



How many small cubes can fit inside the large cube?

- 100 small cubes can fit along the length of the base of cube A (because there are 100 cm in 1 m).
- 100 small cubes can fit along the breadth of the base of cube A.
- 100 small cubes can fit along the height of cube A.

$$\begin{aligned} \text{Total number of 1 cm}^3 \text{ cubes in 1 m}^3 &= 100 \times 100 \times 100 \\ &= 1\,000\,000 \\ \therefore \quad 1 \text{ m}^3 &= 1\,000\,000 \text{ cm}^3 \end{aligned}$$

Work out how many mm^3 are equal to 1 cm^3. Copy and complete:

$$\begin{aligned} 1 \text{ cm}^3 &= 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm} \\ &= \ldots\ldots \text{ mm} \times \ldots\ldots \text{ mm} \times \ldots\ldots \text{ mm} \\ &= \ldots\ldots\ldots \text{ mm}^3 \end{aligned}$$

Cubic units:

1 m^3 = 1 000 000 cm^3
(multiply by 1 000 000 to change m^3 to cm^3)

1 cm^3 = 0,000001 m^3
(divide by 1 000 000 to change cm^3 to m^3)

1 cm^3 = 1 000 mm^3
(multiply by 1 000 to change cm^3 to mm^3)

1 mm^3 = 0,001 cm^3
(divide by 1 000 to change mm^3 to cm^3)

WORKING WITH CUBIC UNITS

1. Which unit, the cubic centimetre (cm^3) or the cubic metre (m^3), would be used to measure the volume of each of the following?
   - (a) a bar of soap
   - (b) a book
   - (c) a wooden rafter for a roof
   - (d) sand on a truck
   - (e) a rectangular concrete wall
   - (f) a die
   - (g) water in a swimming pool
   - (h) medicine in a syringe

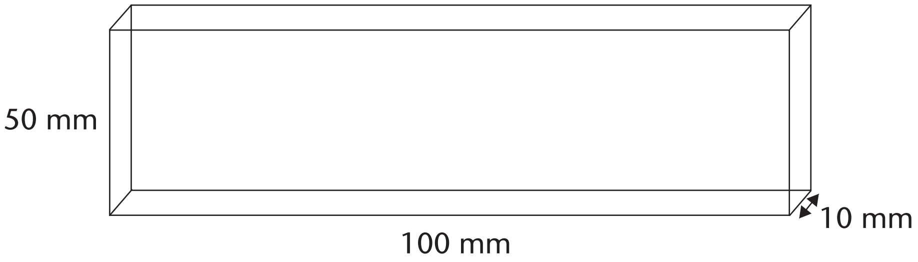
2. Write the following volumes in cm^3:
   - (a) 1 000 mm^3
   - (b) 3 000 mm^3
   - (c) 2 500 mm^3
   - (d) 4 450 mm^3
   - (e) 7 824 mm^3
   - (f) 50 mm^3

3. Write the following volumes in m^3:
   - (a) 1 000 000 cm^3
   - (b) 4 000 000 cm^3
   - (c) 1 500 000 cm^3
   - (d) 2 350 000 cm^3
   - (e) 500 000 cm^3
   - (f) 350 000 cm^3

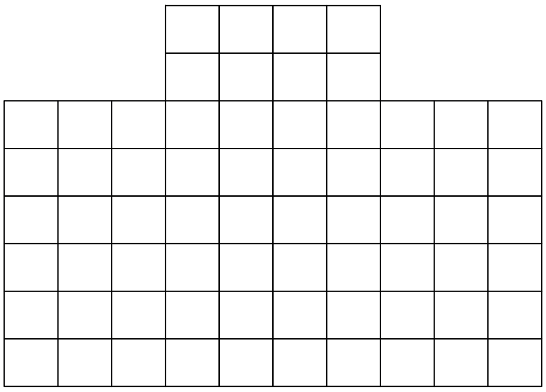
4. Write the following volumes in cm^3:
   - (a) 2 000 mm^3
   - (b) 4 120 mm^3
   - (c) 1,5 m^3
   - (d) 34 m^3
   - (e) 50 000 mm^3
   - (f) 2,23 m^3

5. A rectangular hole has been dug for a children's swimming pool. It is 7 m long, 4 m wide and 1 m deep. What is the volume of earth that has been dug out?

6. Calculate the volume of wood in the plank shown below. Answer in cm^3.



7. The drawing shows the base (viewed from below) of a stack built with 1 cm^3 cubes. The stack is 80 mm high everywhere.
   - (a) What is the volume of the stack?
   - (b) Copy and complete the following:
     Volume of stack = area of base ....



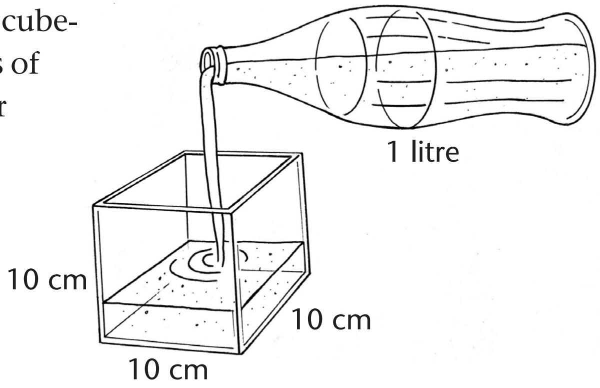
8. Calculate the volume of each of the following rectangular prisms:
   - (a) length = 20 cm; breadth = 15 cm; height = 10 cm
   - (b) length = 130 mm; breadth = 10 cm; height = 5 mm
   - (c) length = 1 200 cm; breadth = 5,5 m; height = 3 m
   - (d) length = 1,2 m; breadth = 2,25 m; height = 4 m
   - (e) area of base = 300 cm^2; height = 150 mm
   - (f) area of base = 12 m^2; height = 2,25 m

10.4 Volume and capacity

The space inside a container is called the internal volume, or **capacity**, of the container. Capacity is often measured in units of millilitres (ml), litres (ℓ) and kilolitres (kl). However, it can also be measured in cubic units.

EQUIVALENT UNITS FOR VOLUME AND CAPACITY

If the contents of a 1 ℓ bottle are poured into a cube-shaped container with internal measurements of 10 cm × 10 cm × 10 cm, it will fill the container exactly. Thus:



$(10 \text{ cm} \times 10 \text{ cm} \times 10 \text{ cm}) = 1\ \ell$

or $1\,000 \text{ cm}^3 = 1\ \ell$

Since $1\ \ell = 1\,000 \text{ ml}$

$1\,000 \text{ cm}^3 = 1\,000 \text{ ml}$ [$1\ \ell = 1\,000 \text{ cm}^3$]

$\therefore$ $1 \text{ cm}^3 = 1 \text{ ml}$ [divide both sides by 1 000]

Since $1 \text{ kl} = 1\,000\ \ell$

$= 1\,000 \times (1\,000 \text{ cm}^3)$ [$1\ \ell = 1\,000 \text{ cm}^3$]

$= 1\,000\,000 \text{ cm}^3$

$= 1 \text{ m}^3$ [$1\,000\,000 \text{ cm}^3 = 1 \text{ m}^3$]

This means that an object with a volume of 1 cm^3 will take up the same amount of space as 1 ml of water. Or an object with a volume of 1 m^3 will take up the space of 1 kl of water.

The following diagram shows the conversions in another way:

$$\begin{tikzcd}
1\text{ ml} \arrow[r] \arrow[r, bend left, "\times 1\,000"] & 1\,\ell\text{ (or }1\,000\text{ ml)} \arrow[r] \arrow[r, bend left, "\times 1\,000"] & 1\text{ kl (or }1\,000\,\ell) \\
1\text{ cm}^3 \arrow[r] \arrow[r, bend right, "\times 1\,000"'] & 1\,000\text{ cm}^3 \arrow[r] \arrow[r, bend right, "\times 1\,000"'] & 1\text{ m}^3\text{ (or }1\,000\,000\text{ cm}^3)
\end{tikzcd}$$

Conversion is the changing of something into something else. In this case, it refers to changes between equivalent units of measurement.

From the diagram on the previous page, you can see that:

- 1 ℓ = 1 000 ml; 1 ml = 0,001 ℓ
- 1 kl = 1 000 ℓ; 1 ℓ = 0,001 kl
- 1 ml = 1 cm^3
- 1 ℓ = 1 000 cm^3
- 1 kl = 1 000 000 cm^3 or 1 m^3

> Remember these conversions:
> 1 ml = 1 cm^3
> 1 kl = 1 m^3

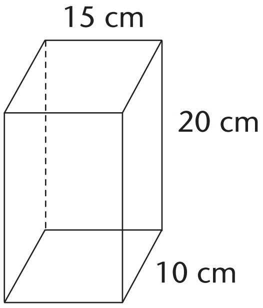
VOLUME AND CAPACITY CALCULATIONS

1. Write the following volumes in ml:
   (a) 2 000 cm^3
   (b) 250 cm^3
   (c) 1 ℓ
   (d) 4 ℓ
   (e) 2,5 ℓ
   (f) 6,85 ℓ
   (g) 0,5 ℓ
   (h) 0,5 cm^3

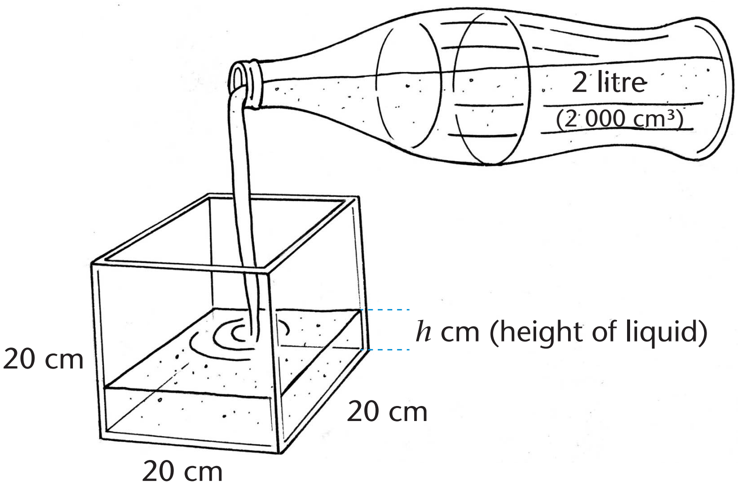
2. Write the following volumes in kl:
   (a) 2 000 ℓ
   (b) 2 500 ℓ
   (c) 5 m^3
   (d) 6 500 m^3
   (e) 3 000 000 cm^3
   (f) 1 423 000 cm^3
   (g) 20 ℓ
   (h) 2,5 ℓ

3. A glass can hold up to 250 ml of water. What is the capacity of the glass:
   (a) in ml?
   (b) in cm^3?

4. A vase is shaped like a rectangular prism. Its inside measurements are 15 cm × 10 cm × 20 cm. What is the capacity of the vase (in ml)?



5. A liquid is poured from a full 2 ℓ bottle into a glass tank with inside measurements of 20 cm by 20 cm by 20 cm.



   (a) What is the volume of the liquid when it is in the bottle?
   (b) What is the capacity of the bottle?
   (c) What is the volume of the liquid after it is poured into the tank?
   (d) What is the capacity of the tank?
   (e) How high does the liquid go in the tank?

In question 5 above, you should have found the following:

$$\begin{aligned} \text{Volume of liquid in tank} &= \text{Volume of liquid in bottle} \\ 20 \times 20 \times h \ \text{(liquid's height in tank)} &= 2\,000 \text{ cm}^3 \\ h &= \frac{2000}{(20 \times 20)} \\ &= 5 \text{ cm} \end{aligned}$$

Note: The capacity of the tank is 20 cm × 20 cm × 20 cm = 8 000 cm³ (8 ℓ). The volume of liquid in the bottle is 2 000 cm³ (2 ℓ).

WORKSHEET

1. Do the following unit conversions:

   (a) 2 348 cm^2 = ... m^2

   (b) 5,104 m^2 = ... cm^2

   (c) 1 m^3 = ... kl

   (d) 250 cm^3 = ... ml = ... ℓ

   (e) 0,5 kl = ... ℓ = ... ml

   (f) 6,850 ℓ = ... ml = ... cm^3

2. A rectangular prism measures 8 m × 4 m × 3 m. Calculate:

   (a) its surface area

   (b) its volume

3. A boy has 27 cubes, with edges of 20 mm. He uses these cubes to build one big cube.

   (a) What is the volume of the cube if he uses all 27 small cubes?

   (b) What is the edge length of the big cube?

   (c) What is the surface area of the big cube?

4. A glass tank has the following inside measurements: length = 250 mm, breadth = 120 mm and height = 100 mm. Calculate the capacity of the tank:

   (a) in cubic centimetres

   (b) in millilitres

   (c) in litres

5. Calculate the capacity of each of the following rectangular containers. The inside measurements have been given. Copy and complete the table.

	Length	**Breadth**	**Height**	**Capacity**
(a)	15 mm	8 mm	5 mm	cm^3
(b)	2 m	50 cm	30 cm	ℓ
(c)	3 m	2 m	1,5 m	kl

6. A water tank has a square base with internal edge lengths of 150 mm. What is the height of the tank when the maximum capacity of the tank is 11 250 cm^3?

CHAPTER 26
Probability

26.1 Possible and actual outcomes, and frequencies

WHAT CAN YOU EXPECT?

You will soon do an experiment. To do the experiment you need a bag like a plastic shopping bag or a brown paper bag. You also need three objects of the same size and shape, like three buttons, bottle tops or small square pieces of cardboard. The three objects must look different, for example they should have different colours such as yellow, red and blue. If you use cardboard squares, you can write "yellow", "red" and "blue" on them.

1. (a) Put your three objects in your bag. You will later draw one object out of the bag, without looking inside. Can you say whether the object that you will draw will be the yellow one, the blue one or the red one?
   (b) Discuss this with two classmates.

2. (a) Now draw an object out of the bag, write down its colour, and put it back.
   (b) You will soon do this 12 times. Can you say how many times you will draw each of the three colours? If you think you can, write down your prediction.
   (c) Compare your predictions with two classmates.
   (d) Can you think of any reason why you may draw blue more often than red or yellow, when you do the experiment described in (b)?

3. (a) Draw an object out of the bag, write down its colour, and put it back. Do this 12 times and write down the colour of the object each time.
   (b) Write your results in a table like the one below.

Outcome	Yellow	Red	Blue
Number of times obtained			

What you did in question 3 is called a **probability experiment**. Each time you drew an object out of the bag, you performed a **trial**.

Each time you performed a trial, three different things could have happened. These are called the **possible outcomes**.

Each time you performed a trial, one of the possible outcomes actually occurred. This is called the **actual outcome**.

The number of times that a specific outcome occurred during an experiment is called the **actual frequency** of that outcome.

4. (a) What were the possible outcomes in the experiment that you did in question 3?
   (b) How many trials did you perform in the experiment?
   (c) What was the actual outcome in the third trial that you performed?
   (d) What was the actual frequency of drawing a blue object during the 12 trials in the experiment that you did?

26.2 Relative frequencies

Thomas also did the experiment in question 3 on page 294 but he performed more trials and his results were as follows:

Outcome	Yellow	Red	Blue
Number of times obtained	5	7	8

1. (a) How many trials did Thomas perform in total?
   (b) What fraction of the trials produced yellow as an outcome?
   (c) What fraction of the trials produced red as an outcome?
   (d) What fraction of the trials produced blue as an outcome?

The fraction of the trials in an experiment that produce a specific outcome is called the **relative frequency** of that outcome.

$$\text{Relative frequency of an outcome} = \frac{\text{number of times the outcome occurred}}{\text{total number of trials}}$$

A relative frequency can be expressed as a common fraction, as a decimal or as a percentage. The relative frequencies in the results of the experiment Thomas did (question 1) were one quarter for yellow, seven twentieths for red and two fifths for blue. Expressed as percentages, the relative frequencies were 25%, 35% and 40%. The **range** of Thomas's relative frequencies, expressed as percentages, is 15% (40% – 25%).

2. (a) Use your calculator to calculate the relative frequencies that you obtained for the three different outcomes in the experiment you did in question 3 on page 294. Express them both as fractions and percentages.
   (b) Calculate the range of the relative frequencies of the three outcomes for the results of the experiment you did in question 3.
   (c) You will soon repeat the experiment with three possible outcomes and 12 trials that you did. Do you think the results will be the same as the first time you did the experiment?

3. (a) Join with three or four classmates to work as a team, and discuss question 2(c).
   (b) Assign the "names" A, B, C, D and E (if there are five of you) to the team members and copy and complete the table below for the experiment you did in question 3 on page 294. Give the relative frequencies as percentages. Note that to calculate the relative frequencies for the totals as percentages, you have to use your calculators.

	Actual frequencies			Relative frequencies %			Range
	Yellow	Red	Blue	Yellow	Red	Blue	
Experiment 1 by A							
Experiment 1 by B							
Experiment 1 by C							
Experiment 1 by D							
Experiment 1 by E							
Totals for experiment 1							

   (c) Which of the ranges is the smallest?

26.3 More trials and relative frequencies

WHAT HAPPENS WHEN YOU CONDUCT MANY TRIALS?

1. Join up with your teammates of the previous activity. Each of you will soon repeat the experiment you did previously. You will put a yellow object, a red object and a blue object in a bag, draw one object and note the colour. You will do this 12 times. This will be experiment 2.
   (a) Do you expect that the results will, in some ways, be the same as for experiment 1 in the previous section? Do not talk to your teammates yet. Form your own opinion, and also consider *why* you think the results will be different or the same.
   (b) Share your ideas with your teammates.

You will soon repeat the experiment and write the results in the rows for "experiment 2" on the table on the next page. You will repeat it once more and write the results in the rows for "experiment 3". If you have time left, you may repeat it once more as "experiment 4".

2. (a) Look at the table on the next page. Certain rows are for the outcomes that you and your teammates obtain. The shaded rows are for adding different sets of outcomes together. Think about what may happen and predict in what rows the ranges will be smaller than in other rows, and in what row the range will be the smallest of all. Copy the table.

(b) Share your ideas with your teammates.

3. (a) Copy the totals for “experiment 1” into the first row of the table. Do the experiment described in question 1 and enter the results in the rows for “experiment 2”. Calculate the relative frequencies and the range.
   (b) Add in the results of your teammates, add up the totals and calculate the relative frequencies and the range of the totals.

4. Repeat question 3, and enter the results in the rows for “experiment 3”.

		Actual frequencies			Relative frequencies %			Range
		Yellow	Red	Blue	Yellow	Red	Blue	
1	Totals for experiment 1							
2	Experiment 2 by A							
3	Experiment 2 by B							
4	Experiment 2 by C							
5	Experiment 2 by D							
6	Experiment 2 by E							
7	Totals for experiment 2							
8	Totals for experiments 1 and 2 combined							
9	Experiment 3 by A							
10	Experiment 3 by B							
11	Experiment 3 by C							
12	Experiment 3 by D							
13	Experiment 3 by E							
14	Totals for experiment 3							
15	Totals for experiments 1, 2 and 3 combined							
16	Experiment 4 by A							
17	Experiment 4 by B							
18	Experiment 4 by C							
19	Experiment 4 by D							
20	Experiment 4 by E							
21	Totals for experiment 4							
22	Totals for experiments 1, 2, 3 and 4 combined							