

A Guide to Differential Calculus

Teaching Approach

Calculus forms an integral part of the Mathematics Grade 12 syllabus and its applications in everyday life is widespread and important in every aspect, from being able to determine the maximum expansion and contraction of bridges to determining the maximum volume or maximum area given a function. It is important for students to be made aware of the uses of calculus over the wide spread of subjects and to get to grips with the ultimate application of calculus. Hence the first five videos give an in depth look at the reasons why calculus was developed.

The next emphasis is put on average gradient (average rate of change) in comparison to determining the gradient at a point or the rate of change at a certain value. The link between gradient at a point and the derivative is important as it is the reasons behind taking the derivative and setting it equal to zero to determine the maximum/minimum volume, area or distance.

Linking calculus with motion, distance, speed and acceleration is highlighted. Students must understand why the derivative of the distance with respect to time gives the speed/velocity at a specific time and the derivative of the speed gives the acceleration at a specific time. This topic will be made clear if we look at the average gradient of a distance time graph, namely distance divide by time (m/s). Calculating the limit of the average gradient as the time tends to zero, leads us to the derivative at a certain time, which is nothing else than the velocity at a certain time.

The formulae on the formula page are restricted to the formula for determining the gradient/derivative of a function at a point using first/basic principles and the average gradient formula:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$Ave\ m = \frac{y_2 - y_1}{x_2 - x_1}$$

Students and teachers are reminded that all area (total surface area), perimeter and volume formulae for prisms, spheres, pyramids and hemispheres are not necessarily given – any of these formulae is applicable in calculus when determining the maximum/minimum volume or area.

Hints on solving questions involving calculus

- Know the difference between average gradient/rate of change and gradient at a point (rate of change at a certain time, derivative at a point, instantaneous velocity etc) and use the correct formula – incorrect formula has a zero mark allocation

- Know that there are two ways to determine the rate of change at a certain time or the gradient at a point or the derivative at a point namely: first/basic principles and using differential rules. Never use basic/first principles unless specifically asked to do so.
- Ensure that you know the applications of calculus:
 - Sketching cubic functions (calculating x - and y - cuts and the turning points) and finding the equation, if the sketch is given
 - Determining the equation of a tangent to a curve
 - Determining the maximum/ minimum volume/area/distance (first derivative = 0)
 - Determining the point of inflection (second derivative = 0)
 - Being able to sketch a derivative function from a function and vice versa (even if the equation of the function is not given)
 - Know where the function increases/decreases and the role of the gradient in determining whether a function is increasing/decreasing over an interval.
 - Know the Total Surface area, volume and perimeter formulae of especially prisms and pyramids

Video Summaries

Some videos have a 'PAUSE' moment, at which point the teacher or learner can choose to pause the video and try to answer the question posed or calculate the answer to the problem under discussion. Once the video starts again, the answer to the question or the right answer to the calculation is given.

Mindset suggests a number of ways to use the video lessons. These include:

- Watch or show a lesson as an introduction to a lesson
- Watch or show a lesson after a lesson, as a summary or as a way of adding in some interesting real-life applications or practical aspects
- Design a worksheet or set of questions about one video lesson. Then ask learners to watch a video related to the lesson and to complete the worksheet or questions, either in groups or individually
- Worksheets and questions based on video lessons can be used as short assessments or exercises
- Ask learners to watch a particular video lesson for homework (in the school library or on the website, depending on how the material is available) as preparation for the next days lesson; if desired, learners can be given specific questions to answer in preparation for the next day's lesson

1. Introducing Calculus

This video gives a brief introduction to Calculus by looking at where Calculus is used in different spheres of life and the history of Calculus. It also looks at the reasons why Calculus was invented.

2. Why Calculus?

We briefly recap the maximisation problem that we started in the previous lesson as well as the fact that an intuitive solution is contradicted by the reality of our exploration. To understand the problem better we introduce some graphing software that draws the graph of the function that describes the problem.

3. Finding the Tangent I

In this lesson we come to the realisation that to determine the co-ordinates of the turning point it would be useful if we could determine the point on the graph at which the tangent to the graph has a gradient of zero.

4. Finding the Tangent II

In this lesson we determine the gradient of a line through a point of interest on a curve and another point on the curve which we bring increasingly closer to the point of interest. With time we begin to observe patterns.

5. Introducing the Derivative Function

We continue with the numerical exploration that we started in the previous lesson and extend it to other functions.

6. Working with the Derivatives Function

In this lesson we introduce the notion of a limit and use this to develop rules for differentiation of functions.

7. Determining the Derivatives using First Principles

In this lesson we continue with calculating the derivative of functions using first or basic principles. In the first example the function is a two term and in the second example the function is a fraction.

8. Determining the Derivative using Differential Rules

We look at the second way of determining the derivative, namely using differential rules. We also look at the steps to take before the derivative of a function can be determined.

9. Sketching a Cubic Function

We go through the stages of drawing the graph of a third degree function step by step. We also use this lesson to answer a typical examination question in which we determine the equation of a tangent to a graph.

10. Exploring the Rate of Change

In this lesson we explore how the gradient of the tangent to a function (derivative) and the rate of change of a function are related.

11. Determining the Point of Inflection

We define the point of inflection. We then proceed to highlight two possible ways to determine the point of inflection of a curve.

12. Optimisation

In this lesson we explore how the gradient of the tangent to a function (derivative) relates to maximizing or minimizing a function. We look at how calculus is applied in maximizing/minimizing volumes, areas and distances.

Resource Material

Resource materials are a list of links available to teachers and learners to enhance their experience of the subject matter. They are not necessarily CAPS aligned and need to be used with discretion.

1. Introducing Calculus	http://www.slideshare.net/samiul11/calculus-in-real-life	This slide show gives us an in depth look at the wide range of uses of calculus.
	http://www.teach-nology.com/teachers/subject_matter/math/calculus/	This article highlights the history of Calculus, its origin and uses.
	http://www.analyzemath.com/calculus.html	This site gives us an in depth tutorial on all the topics dealt with in Calculus.
2. Why Calculus?	http://academic.sun.ac.za/mathed/malati/Grade12.pdf	This site deals with introductory notes on calculus that will assist the weaker learners with pre-Calculus questions.
3. Finding the Tangent I	http://www.curriki.org/xwiki/bin/download/Coll_siyavula/MathematicsChapter40DifferentialCalculus/MathematicsGrade12Ch40DifferentialCalculus.pdf	This site gives comprehensive notes and good examples on a variety of sections within calculus, which includes modelling, limits average gradient, rate of change and much more.

4. Finding the tangent II	http://cnx.org/content/m39270/1.1/	The notes and video in this site deals with introduction to calculus, average gradient and limits.
	http://www.youtube.com/watch?v=7Ufv0X5y0fc	This is a YouTube video (Mindset) that deals with a revision on Calculus questions.
5. Introducing the Derivative Function	http://www.education.gov.za/LinkClick.aspx?fileticket=l%2BFVxMrqH0U%3D&	Gives explanations and examples on determining the derivative using first principles
	http://tutorial.math.lamar.edu/Classes/Calcl/DiffFormulas.aspx	Examples on determining the derivative and the differential rules
	http://tutorial.math.lamar.edu/ProblemsNS/Calcl/DerivativeInterp.aspx	Examples on finding the derivative using differential rules
6. Working with the Derivative Function	http://www.education.gov.za/LinkClick.aspx?fileticket=l%2BFVxMrqH0U%3D&	Gives explanations and examples on determining the derivative using first principles
	http://tutorial.math.lamar.edu/Classes/Calcl/DiffFormulas.aspx	Examples on determining the derivative and the differential rules.
7. Determining the Derivatives using First Principles	http://www.education.gov.za/LinkClick.aspx?fileticket=l%2BFVxMrqH0U%3D&	Gives explanations and examples on determining the derivative using differential rules.
	http://tutorial.math.lamar.edu/Classes/Calcl/DefnOfDerivative.aspx	Examples on determining the derivative using first principles
8. Determining the Derivative using Differential Rules	http://www.education.gov.za/LinkClick.aspx?fileticket=l%2BFVxMrqH0U%3D&	Gives explanations and examples on determining the derivative using differential rules.
	http://tutorial.math.lamar.edu/Classes/Calcl/DiffFormulas.aspx	Examples on determining the derivative and the differential rules.
9. Sketching a Cubic Function	http://tutorial.math.lamar.edu/Classes/Calcl/TangentsRates.aspx	Examples in this video deals with finding the equation of a tangent to a curve, amongst other calculations.
	http://tutorial.math.lamar.edu/Classes/Calcl/ShapeofGraphPtII.aspx	Deals with important aspects of graph sketching.
10.Exploring the Rate of Change	http://tutorial.math.lamar.edu/Classes/Calcl/ShapeofGraphPtII.aspx	Deals with the application of the second derivative.
11.Determining the Point of Inflection	https://www.math.ucdavis.edu/~koubba/CalcOneDIRECTORY/maxmindirectory/MaxMin.html	This worksheet gives us an many examples of maxima and minima
	http://www.youtube.com/watch?v=3aVT9d_RTsk	This video gives an example on how to determine the second derivative.
	http://tutorial.math.lamar.edu/Classes/Calcl/RelatedRates.aspx	This tutorial deals with examples on rate of change.
12.Optimisation	http://cnx.org/content/m39273/1.1/	This video deals with optimization examples.
	http://www.education.com/study-help/article/optimization_answer/	This video gives us more optimization examples

Task

Question 1

Consider the function: $f(x) = x^2$. Calculate the average gradient of $f(x)$ between :

1.1 (1; 1) and (2; 4)

1.2 (5; $f(5)$) and (3; $f(3)$)

Question 2

Calculate:

2.1 $\lim_{x \rightarrow 3} (2x + 1)$

2.2 $\lim_{x \rightarrow 3} \left(\frac{x^2 - 9}{x - 3} \right)$

Question 3

Use the gradient formula $\text{gradient}(m) = \frac{f(x+h) - f(x)}{h}$ to determine the gradients in terms of h for any two points on the functions:

3.1 $f(x) = x^2$

3.2 $f(x) = x^3$

3.3 $f(x) = x$

3.4 $f(x) = a$ $a = \text{constant}$

3.5 $f(x) = x^2 + x$.

Question 4

Determine the derivative of each of the following functions by using first principles:

4.1 $f(x) = 5$

4.2 $f(x) = -4x^2$

4.3 $f(x) = x^2 + x$

Question 5

Determine the derivative of the following function by using first principles:

$$f(x) = \frac{1}{x^2}$$

Question 6

Find the value of x that will give a maximum volume and determine the maximum volume if $V(x) = 4x^3 - 80x^2 + 400x$

Question 7

Given the function $f(x) = 2x^3 + 4x^2 + 2x$. Determine the points on the curve where the gradient is equal to 10.

Question 8

Determine $\frac{d}{dx}$ of $f(x)$ in each of the following if:

8.1 $f(x) = 3(x^2 - 2)(x + 4)$

8.2 $f(x) = \frac{8x^3 - 3x^2 + x + 1}{2x}$

8.3 $f(x) = \frac{4}{\sqrt{x}} + \frac{6}{\sqrt[3]{x^2}}$

8.4 $f(x) = \frac{x^2 - 3x - 4}{x + 1}$

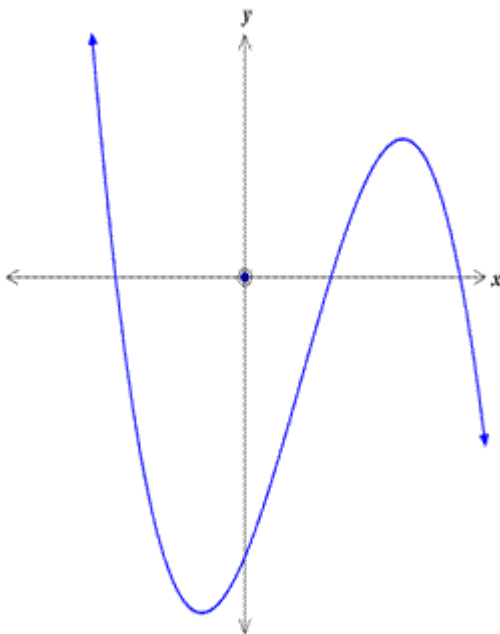
Question 9

9.1 Make a neat sketch graph of $f(x) = x^3 - 9x^2 + 24x - 20$, clearly indicate all turning points and points of intersection with the axes.

9.2 Make a neat sketch graph of $f(x) = -x^3 + 4x^2 - 4x$ and then determine the equation of the tangent to $f(x)$ at $x=3$

Question 10

The graph below shows the curve of $f(x) = -x^3 + 4x^2 + 11x - 30$



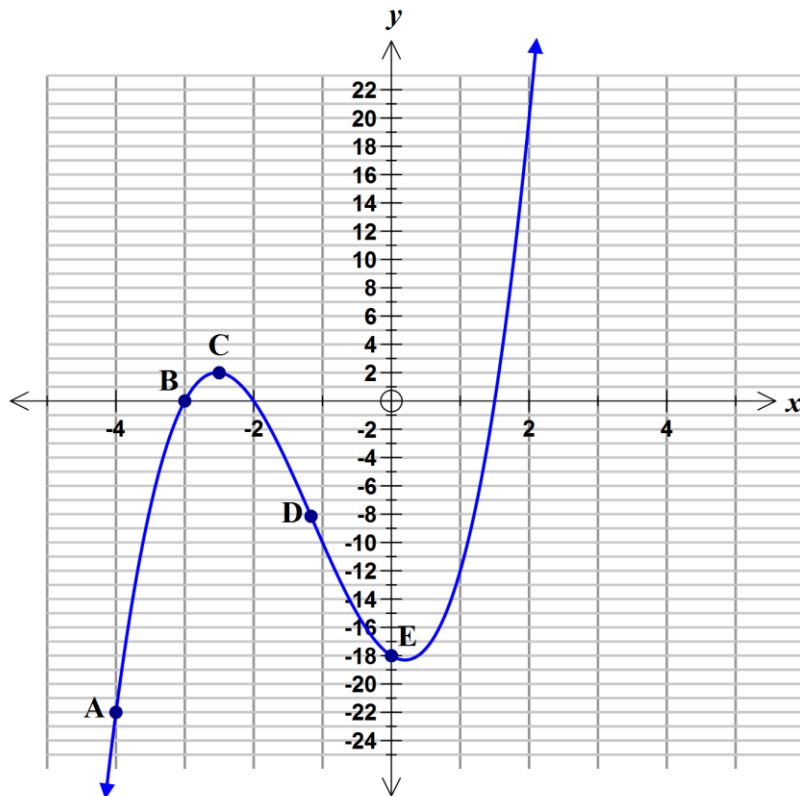
10.1 Determine the coordinates of the point of inflection of the graph.

10.2 Determine the coordinates of the turning points of the graph

10.3 For which values of x will $f(x)$ decrease and for which values of x will $f(x)$ increase?

Question 11

Consider the graph of $f(x) = 2x^3 + 7x^2 - 3x - 18$ sketched below:



11.1 Use the words increasing, decreasing and constant to describe what is happening to the function value :

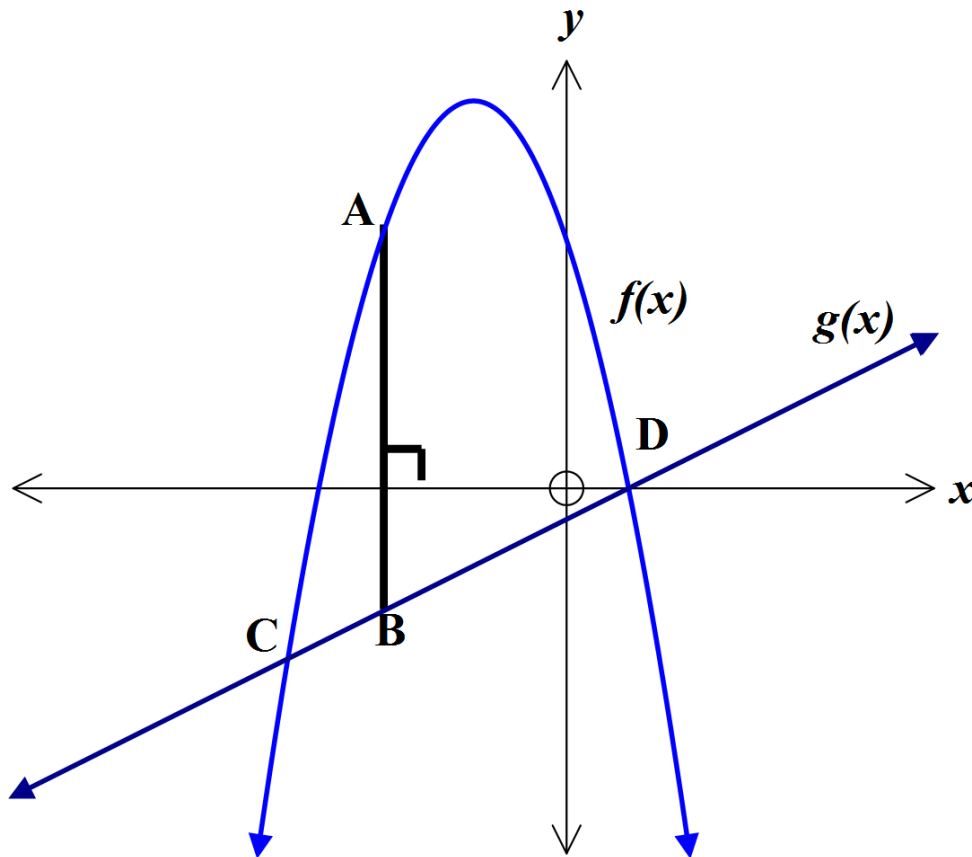
- In the interval AB
- In the interval DE

11.2 The coordinates of A is $(-4; -22)$ and B is $(-3; 0)$

- Determine the average rate of change between points A and B
- Determine the rate of change at point B

Question 12

A parabola, $f(x) = -2x^2 - 6x + 8$ and a line graph, $g(x) = x - 1$ are sketched. The graphs intersect at points C and D. Line AB is parallel to the y-axis with point A on the parabola and point B is on the straight line. Determine the maximum length of line AB between points C and D.



Question 13

The average mass of a baby in the first twenty days of life is given by the equation

$$m(t) = \frac{t^3}{648} - \frac{t^2}{36} + 3 \text{ where } m \text{ is the mass in kilogram and } t \text{ is the time in days.}$$

- 13.1 What is the average mass of a baby at birth, according to the equation?
- 13.2 For a short period of time after birth it is usual for a baby to lose mass, When, according the equation, does the baby's mass reach a minimum?
- 13.3 After how many days would the baby's mass once again be the same as its mass at birth?

Task Answers

Question 1

1.1

$$\text{Ave } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 1}{2 - 1} = 3$$

1.2 $f(x) = x^2$

$$x = 5 \therefore y = 25 \quad (5; 25)$$

$$x = 3 \therefore y = 9 \quad (3; 9)$$

$$\text{Ave } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{25 - 9}{5 - 3} = 8$$

Question 2

2.1

$$\lim_{x \rightarrow 3} (2x + 1) = 2(3) + 1 = 7$$

2.2

$$\lim_{x \rightarrow 3} \left(\frac{x^2 - 9}{x - 3} \right) = \lim_{x \rightarrow 3} \frac{(x - 3)(x + 3)}{x - 3} = \lim_{x \rightarrow 3} x + 3 = 3 + 3 = 6$$

Question 3

3.1

$$f(x) = x^2$$

$$\begin{aligned} \text{gradient } (m) &= \frac{f(x+h) - f(x)}{h} \\ &= \frac{(x+h)^2 - x^2}{h} \\ &= \frac{x^2 + 2xh + h^2 - x^2}{h} \\ &= \frac{2xh + h^2}{h} = \frac{h(2x+h)}{h} = 2x + h \end{aligned}$$

3.2

$$f(x) = x^3$$

$$\begin{aligned} \text{gradient } (m) &= \frac{f(x+h) - f(x)}{h} \\ &= \frac{(x+h)^3 - x^3}{h} \\ &= \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \\ &= \frac{3x^2h + 3xh^2 + h^3}{h} = \frac{h(3x^2 + 3xh + h^2)}{h} \\ &= 3x^2 + 3xh + h^2 \end{aligned}$$

3.3

$$f(x) = x$$

$$\begin{aligned} \text{gradient } (m) &= \frac{f(x+h) - f(x)}{h} \\ &= \frac{x+h-x}{h} \\ &= \frac{h}{h} = 1 \end{aligned}$$

3.4

$$f(x) = a$$

$$\begin{aligned} \text{gradient } (m) &= \frac{f(x+h) - f(x)}{h} \\ &= \frac{a-a}{h} \\ &= \frac{0}{h} = 0 \end{aligned}$$

3.5

$$f(x) = x^2 + x$$

$$\begin{aligned} \text{gradient } (m) &= \frac{f(x+h) - f(x)}{h} \\ &= \frac{(x+h)^2 + (x+h) - (x^2 + x)}{h} \\ &= \frac{x^2 + 2xh + h^2 + x + h - x^2 - x}{h} \\ &= \frac{2xh + h^2 + h}{h} = \frac{h(2x + h + 1)}{h} = 2x + 1 \end{aligned}$$

Question 4

4.1 $f(x) = 5$

$$\begin{aligned} \therefore f'(x) &= \lim_{h \rightarrow h} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow h} \frac{5-5}{h} \\ &= \lim_{h \rightarrow h} \frac{0}{h} \\ &= 0 \end{aligned}$$

4.2

$$f(x) = -4x^2$$

$$\therefore f'(x) = \lim_{h \rightarrow h} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = -4x^2 \dots\dots\dots f(x+h) = -4(x+h)^2$$

$$= -4(x^2 + 2xh + h^2)$$

$$= -4x^2 - 8xh - 4h^2$$

$$= \lim_{h \rightarrow h} \frac{-4x^2 - 8xh - 4h^2 - (-4x^2)}{h}$$

$$= \lim_{h \rightarrow h} \frac{-4x^2 - 8xh - 4h^2 + 4x^2}{h}$$

$$= \lim_{h \rightarrow h} \frac{-8xh - 4h^2}{h}$$

$$= \lim_{h \rightarrow h} \frac{h(-8x - 4h)}{h}$$

$$= \lim_{h \rightarrow h} -8x - 4h = -8x$$

4.3

$$f(x) = x^2 + x$$

$$\therefore f'(x) = \lim_{h \rightarrow h} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = x^2 + x \dots\dots\dots f(x+h) = (x+h)^2 + x + h$$

$$= (x^2 + 2xh + h^2) + x + h$$

$$= x^2 + 2xh + h^2 + x + h$$

$$= \lim_{h \rightarrow h} \frac{x^2 + 2xh + h^2 + x + h - (x^2 + x)}{h}$$

$$= \lim_{h \rightarrow h} \frac{x^2 + 2xh + h^2 + x + h - x^2 - x}{h}$$

$$= \lim_{h \rightarrow h} \frac{2xh + h^2 + h}{h}$$

$$= \lim_{h \rightarrow h} \frac{h(2x + h + 1)}{h}$$

$$= \lim_{h \rightarrow h} 2x + h + 1 = 2x + 1$$

Question 5

$$f(x) = \frac{1}{x^2}$$

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = \frac{1}{x^2} \dots\dots\dots f(x+h) = \frac{1}{(x+h)^2}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

$$\frac{1}{(x+h)^2} - \frac{1}{x^2} = \frac{x^2 - (x+h)^2}{x^2(x+h)^2} = \frac{x^2 - x^2 - 2xh - h^2}{x^2(x+h)^2}$$

$$= \frac{-2xh - h^2}{x^2(x+h)^2}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{-2xh - h^2}{x^2(x+h)^2}}{h} = \lim_{h \rightarrow 0} \frac{h(-2x - h)}{x^2(x+h)^2} \times \frac{1}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-2x - h}{x^2(x+h)^2}$$

$$= \frac{-2x - 0}{x^2(x+0)^2} = \frac{-2x}{x^4} = \frac{-2}{x^3}$$

Question 6

$$V(x) = 4x^3 - 80x^2 + 400x$$

$$V'(x) = 12x^2 - 160x + 400$$

$$\text{for maximum, } V'(x) = 0$$

$$\therefore 12x^2 - 160x + 400 = 0$$

$$\therefore 3x^2 - 40x + 100 = 0$$

$$\therefore (3x - 10)(x - 10) = 0$$

$$\therefore x = \frac{10}{3} \text{ or } x = 10$$

$$V\left(\frac{10}{3}\right) = 4\left(\frac{10}{3}\right)^3 - 80\left(\frac{10}{3}\right)^2 + 400\left(\frac{10}{3}\right) = 592,6\text{cm}^3$$

$$V(10) = 4(10)^3 - 80(10)^2 + 400(10) = 0\text{cm}^3$$

$$\therefore \text{Volume}_{\max} = 592,6\text{cm}^3$$

Question 7

Given the function $f(x) = 2x^3 + 4x^2 + 2x$

Determine the points on the curve where the gradient is equal to 10

$$f(x) = 2x^3 + 4x^2 + 2x$$

$$m = f'(x) = 6x^2 + 8x + 2 = 10$$

$$\therefore 6x^2 + 8x + 2 = 10$$

$$\therefore 6x^2 + 8x - 8 = 0$$

$$\div 2 \quad \therefore 3x^2 + 4x - 4 = 0$$

$$(3x - 2)(x + 2) = 0$$

$$\therefore x = \frac{2}{3} \text{ or } x = -2$$

$$x = \frac{2}{3} \quad \therefore y = 2\left(\frac{2}{3}\right)^3 + 4\left(\frac{2}{3}\right)^2 + 2\left(\frac{2}{3}\right) = \frac{100}{27} \quad \therefore \left(\frac{2}{3}; \frac{100}{27}\right)$$

$$x = -2 \quad \therefore y = 2(-2)^3 + 4(-2)^2 + 2(-2) = -4 \quad \therefore (-2; -4)$$

Question 8

8.1

$$f(x) = 3(x^2 - 2)(x + 4)$$

$$= 3(x^3 + 4x^2 - 2x - 8)$$

$$= 3x^3 + 12x^2 - 6x - 24$$

$$\therefore \frac{d[f(x)]}{dx} = 9x^2 + 24x - 6$$

8.2

$$f(x) = \frac{8x^3 - 3x^2 + x + 1}{2x}$$

$$= \frac{8x^3}{2x} - \frac{3x^2}{2x} + \frac{x}{2x} + \frac{1}{2x}$$

$$= 4x^2 - \frac{3}{2}x + \frac{1}{2} + \frac{1}{2}x^{-1}$$

$$\frac{d[f(x)]}{dx} = 8x - \frac{3}{2} - \frac{1}{2}x^{-2} \text{ or } 8x - \frac{3}{2} - \frac{1}{2x^2}$$

8.3

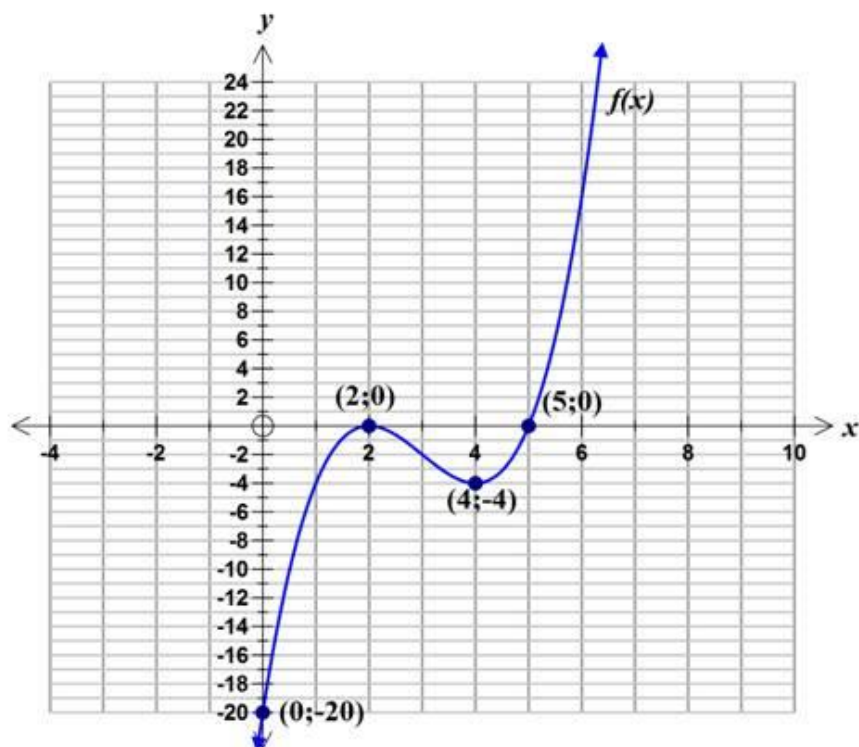
$$\begin{aligned} f(x) &= \frac{4}{\sqrt{x}} + \frac{6}{\sqrt[3]{x^2}} \\ &= \frac{4}{x^{\frac{1}{2}}} + \frac{6}{x^{\frac{2}{3}}} = 4x^{-\frac{1}{2}} + 6x^{-\frac{2}{3}} \\ \frac{d[f(x)]}{dx} &= 4 \times -\frac{1}{2} x^{-1,5} + 6 \times -\frac{2}{3} x^{-1\frac{2}{3}} \\ &= -2x^{-1,5} - 4x^{-1\frac{2}{3}} \end{aligned}$$

8.4

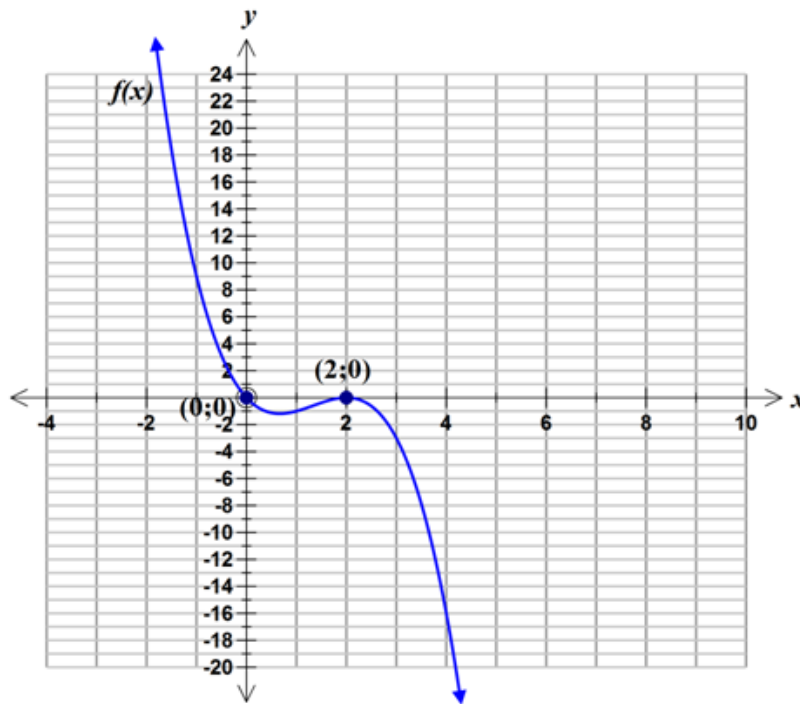
$$\begin{aligned} f(x) &= \frac{x^2 - 3x - 4}{x + 1} \\ &= \frac{(x+1)(x-4)}{x+1} \\ &= x - 4 \\ \therefore \frac{d[f(x)]}{dx} &= 1 \end{aligned}$$

Question 9

9.1 $f(x) = x^3 - 9x^2 + 24x - 20$



9.2 $f(x) = -x^3 + 4x^2 - 4x$



$$f(x) = -x^3 + 4x^2 - 4x$$

if $x = 3$, then $y = -(3)^3 + 4(3)^2 - 4(3) = -3$

$\therefore (3; -3)$ is a point on the tangent

$$m = f'(x) = -3x^2 + 8x - 4$$

$$\therefore m_{x=3} = -3(3)^2 + 8(3) - 4 = -7$$

$$\therefore m_{\text{tangent}} = -7$$

$$\therefore y - y_1 = m(x - x_1)$$

$$y + 3 = -7(x - 3)$$

$$\therefore y = -7x + 18$$

Question 10

10.1

$$f(x) = -x^3 + 4x^2 + 11x - 30$$

$$f'(x) = -3x^2 + 8x + 11$$

$$f''(x) = -6x + 8 = 0$$

$$-6x = -8$$

$$\therefore x = \frac{8}{6} = \frac{4}{3}$$

$$y = f\left(\frac{4}{3}\right) = -\left(\frac{4}{3}\right)^3 + 4\left(\frac{4}{3}\right)^2 + 11\left(\frac{4}{3}\right) - 30 = -\frac{286}{27} = 10,6$$

$$\therefore \left(\frac{4}{3}; -10,6\right)$$

10.2

$$f(x) = -x^3 + 4x^2 + 11x - 30$$

$$f'(x) = -3x^2 + 8x + 11 = 0$$

$$3x^2 - 8x - 11 = 0$$

$$(3x - 11)(x + 1) = 0$$

$$\therefore x = \frac{11}{3} \text{ or } x = -1$$

$$y = f\left(\frac{11}{3}\right) = -\left(\frac{11}{3}\right)^3 + 4\left(\frac{11}{3}\right)^2 + 11\left(\frac{11}{3}\right) - 30 = \frac{400}{27} = 14,6$$

$$\therefore \left(\frac{11}{3}; 14,6\right)$$

$$y = f(-1) = -(-1)^3 + 4(-1)^2 + 11(-1) - 30 = -36$$

$$\therefore (-1; -36)$$

10.3 Decreasing: $x < -1$ and $x > \frac{11}{3}$

Increasing: $-1 < x < \frac{11}{3}$

Question 11

11.1

AB: The function is increasing

DE: The function is decreasing

11.2

$$\text{Average rate of change} = \frac{-22 - 0}{-4 - (-3)} = -1$$

$$f'(x) = 6x^2 + 14x - 3$$

$$\therefore \text{Rate of change at } B(-3; 0) = f'(-3) = 6(-3)^2 + 14(-3) - 3 = 9$$

Question 12

$$A(x; -2x^2 - 6x + 8) \text{ and } B(x; x - 1)$$

$$\therefore \text{distance } AB(D_{AB}) = -2x^2 - 6x + 8 - (x - 1)$$

$$= -2x^2 - 6x + 8 - x + 1 = -2x^2 - 7x + 9$$

$$\therefore D'_{AB}(x) = -4x - 7$$

$$\text{For max distance } D'_{AB}(x) = 0$$

$$\therefore -4x - 7 = 0 \therefore -4x = 7$$

$$\therefore x = \frac{7}{-4}$$

$$\text{max distance } D_{AB} = -2\left(\frac{7}{-4}\right)^2 - 7\left(\frac{7}{-4}\right) + 9 = \frac{121}{8} = 15,1 \text{ units}$$

Question 13

13.1

$$m(t) = \frac{t^3}{648} - \frac{t^2}{36} + 3$$

$$\text{At birth, } t = 0 \therefore m(0) = \frac{0}{648} - \frac{0}{36} + 3 = 3 \text{ kg}$$

13.2

$$m(t) = \frac{t^3}{648} - \frac{t^2}{36} + 3$$

$$m'(t) = \frac{3t^2}{648} - \frac{2t}{36} + 0 = \frac{t^2}{216} - \frac{t}{18}$$

$$\text{Min / Max..... } m'(t) = 0$$

$$\therefore \frac{t^2}{216} - \frac{t}{18} = 0$$

$$\times 216 \quad \therefore t^2 - 12t = 0$$

$$\therefore t(t - 12) = 0$$

$$\therefore t \neq 0 \therefore t = 12$$

$$\therefore \text{after 12 days}$$

13.3

$$m(t) = \frac{t^3}{648} - \frac{t^2}{36} + 3 = 3$$

$$\therefore \frac{t^3}{648} - \frac{t^2}{36} + 3 = 3$$

$$\therefore \frac{t^3}{648} - \frac{t^2}{36} = 0$$

$$\times 648 \quad \therefore t^3 - 18t^2 = 0$$

$$\therefore t^2(t - 18) = 0$$

$$\therefore t = 0 \therefore t = 18$$

$$\therefore \text{after 18 days}$$

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