MATHEMATICS

GRADE 6

TEACHER GUIDE

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MATHEMATICS GRADE 6 TEACHER GUIDE


MATHEMATICS
GRADE 4
TEACHER GUIDE


MATHEMATICS
GRADE 5
TEACHER GUIDE

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Mathematics

Grade 6

Teacher Guide

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Mathematics Teacher Guide Grade 6

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Mathematical background

Although a number symbol such as 357 is written by writing the three digits 3, 5 and 7, the number represented by the symbol 357 is not "three five seven" or "3 and 5 and 7", but 300 + 50 + 7. This is what is meant by "understanding place value". This should be made clear from the outset and emphasised whenever possible. Language constructions like "break down a number into its place value parts" and learning aids like place value cards were invented for this purpose and are prescribed to promote understanding of place value.

On a theoretical level (*intended for teachers only*), a distinction can be made between the "face value" of a digit in a number symbol, the "numerical value" or number (place value part) represented by the digit, and the place value of the position occupied by the digit. For example, in 357 the **face value** of the symbol "5" is 5. However, the symbol "5" represents the number 50, hence its **numerical value** is 50. The symbol "5" is in the tens position, a fact that is sometimes expressed by saying that the **place value** of the digit (actually the place value of the position it occupies) is tens (note the plural).

Resources

Two resources are absolutely critical for the work in this unit:

- Counting apparatus: wooden or plastic cubes and rods, or sticks and stick bundles, or other suitable apparatus
- Place value cards, all of the same colour, for units, tens, hundreds, thousands, ten thousands and preferably for hundred thousands too.

Each learner should have a set of counters (cubes and rods / sticks and bundles) and a set of place value cards. A master copy for place value cards for learners is provided on pages 420 to 426 in the Addendum at the back of this Teacher Guide. In addition, you should have a set of large place value cards for demonstration purposes, such as those provided on pages 427 to 440 of the Addendum.

1.1 Count and represent numbers

Critical knowledge and skills

It is critical that learners understand that “counting” doesn’t just mean counting objects one by one, but that it also includes structured counting in groups of ten, hundred, thousand, etc.

Mathematical notes

The structure of the diagram on page 3 of the Learner Book may be represented as shown below:

100	100	100	100	100
100	100	100	100	100
100	100	100	100	100
100	100	100	100	100
100	40			
100	7			

Do not demonstrate the above to learners before they have seriously engaged with question 1. Learners should preferably come to observe the structure of the diagram by themselves.

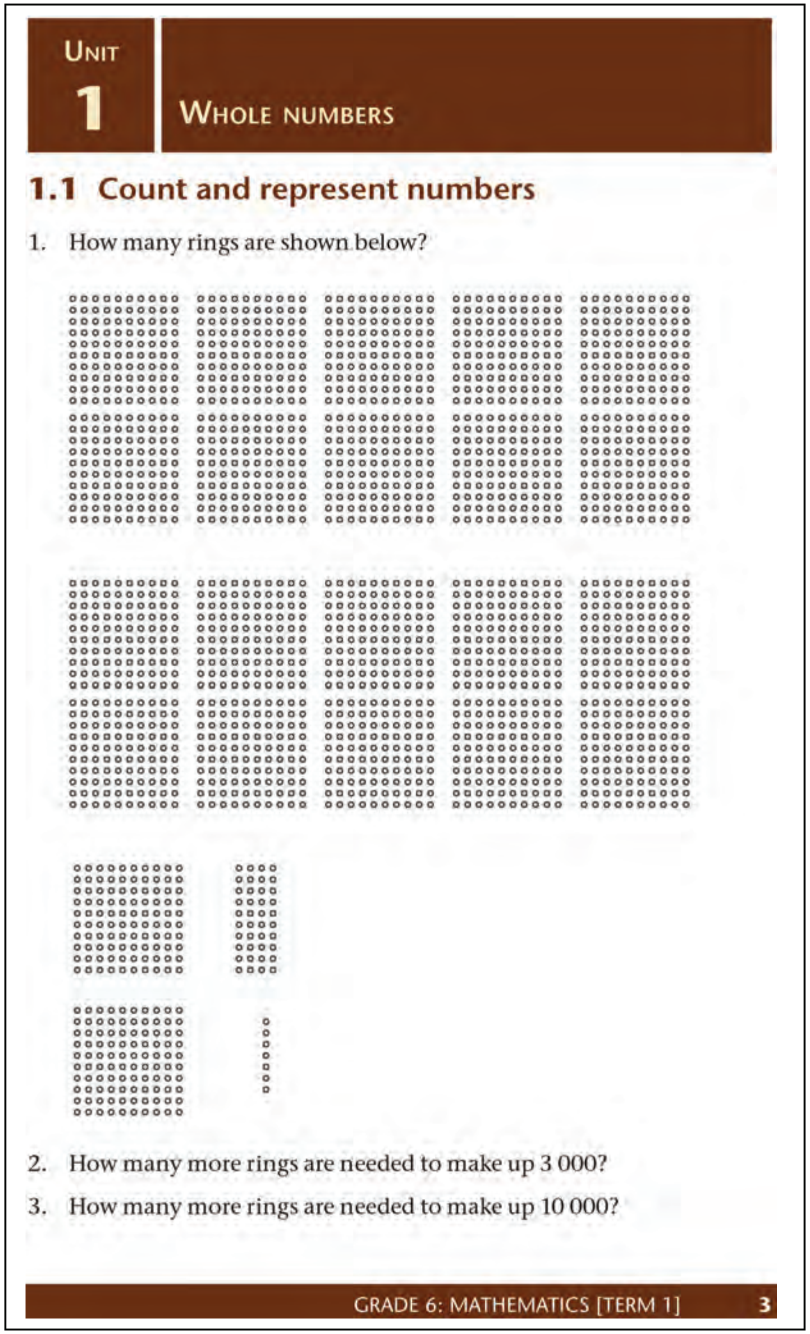
Teaching guidelines

Observe how learners approach question 1. Learners who try to count one by one need support. Suggest to learners that they should consider how many rings there are in each of the rectangular/square arrays, and how many rectangular/square arrays there are.

It is important that learners observe the structure of the diagram, for example that there are two equal blocks of 10 square arrays of rings in the upper part of the diagram. You may ask learners how many rings there are in each of these two blocks.

Answers

1. 2 247
2. 753
3. 7 753



UNIT 1 WHOLE NUMBERS

1.1 Count and represent numbers

1. How many rings are shown below?
2. How many more rings are needed to make up 3 000?
3. How many more rings are needed to make up 10 000?

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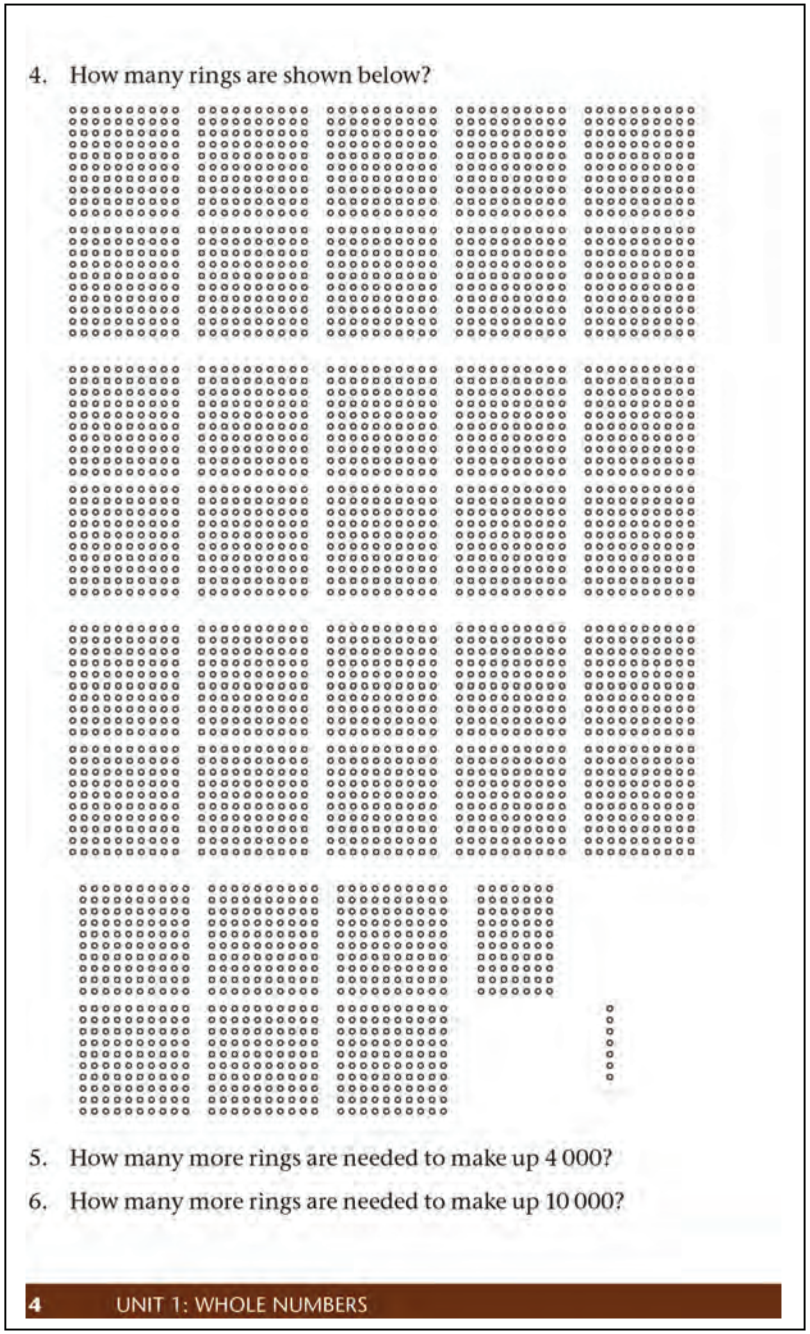
Teaching guidelines

If learners struggle to make progress with question 1, you may show them how the diagram in question 1 can be represented with numbers (as shown under "Mathematical notes" on the previous page), and suggest that they make a similar representation of the diagram in question 4. This will force them to analyse the structure of the diagram and hence determine the number of rings without endless counting.

Answers

4. 3 677
5. 323
6. 6 323



4. How many rings are shown below?

5. How many more rings are needed to make up 4 000?

6. How many more rings are needed to make up 10 000?

1.2 The place value parts of whole numbers

Teaching guidelines

It is highly desirable that learners have some opportunities to build numbers with place value cards, even if cards are only available for smaller numbers. This will empower them to make sense of the diagrams in the Learner Book. To help learners to engage with the shaded passage, you may build the number 485 627 or some other number on the board with large place value cards.

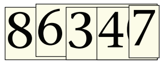
Place value cards are an indispensable tool to help learners to distinguish, in their own minds, between number symbols and the numbers themselves. It is important to use place value cards correctly. The basic place value card activity is to ask learners to "show" a number with cards. When learners are asked to show a number, for example 357, they should select and hold up the $\boxed{300}$, $\boxed{50}$ and $\boxed{7}$ cards, not the $\boxed{3}$, $\boxed{5}$ and $\boxed{7}$ cards.

To demonstrate the connection between building a number with place value cards and writing the number in expanded notation, you may write a number on the board and put its card presentation on top of it, for example:

Write:

86 347

Then put the cards on top of the writing:



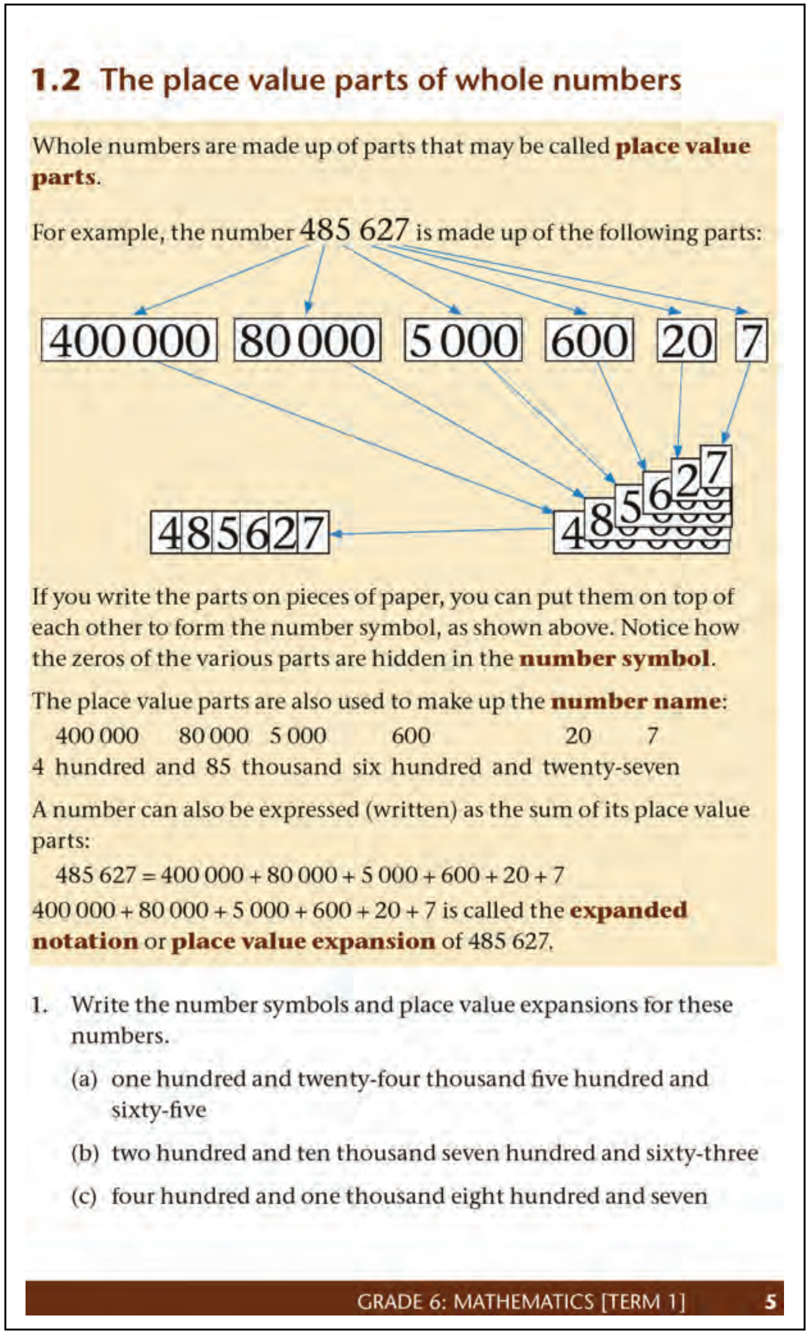
Then pull the cards apart and place them to the right of the written number:

86 347 $\boxed{80000}$ $\boxed{6000}$ $\boxed{300}$ $\boxed{40}$ $\boxed{7}$

86 347 = 80 000 + 6 000 + 300 + 40 + 7

Answers

1. (a) 124 565 = 100 000 + 20 000 + 4 000 + 500 + 60 + 5
   (b) 210 763 = 200 000 + 10 000 + 700 + 60 + 3
   (c) 401 807 = 400 000 + 1 000 + 800 + 7
   (d) 602 484 = 600 000 + 2 000 + 400 + 80 + 4
   (e) 106 558 = 100 000 + 6 000 + 500 + 50 + 8
   (f) 711 313 = 700 000 + 10 000 + 1 000 + 300 + 10 + 3



1.2 The place value parts of whole numbers

Whole numbers are made up of parts that may be called **place value parts**.

For example, the number 485 627 is made up of the following parts:

If you write the parts on pieces of paper, you can put them on top of each other to form the number symbol, as shown above. Notice how the zeros of the various parts are hidden in the **number symbol**.

The place value parts are also used to make up the **number name**:

400 000 80 000 5 000 600 20 7
4 hundred and 85 thousand six hundred and twenty-seven

A number can also be expressed (written) as the sum of its place value parts:

485 627 = 400 000 + 80 000 + 5 000 + 600 + 20 + 7

400 000 + 80 000 + 5 000 + 600 + 20 + 7 is called the **expanded notation** or **place value expansion** of 485 627.

1. Write the number symbols and place value expansions for these numbers.
   (a) one hundred and twenty-four thousand five hundred and sixty-five
   (b) two hundred and ten thousand seven hundred and sixty-three
   (c) four hundred and one thousand eight hundred and seven

Teaching guidelines

Show and remind learners that we use four place value cards to build the number 4 583:

4000 500 80 3 4583

We use three place value cards to build the number 4 083:

4000 80 3 4083

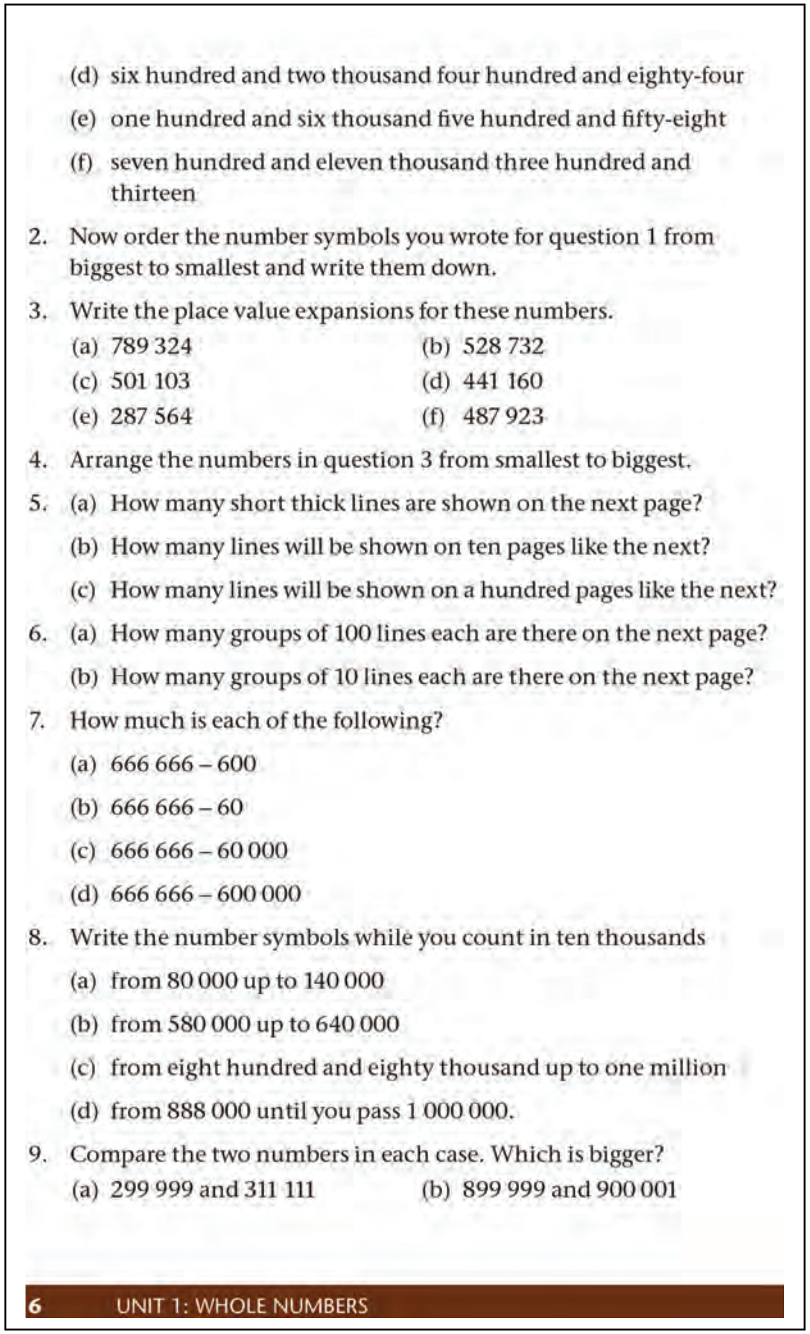
Representing the number 666 666 with place value cards may assist learners to solve question 7.

Answers

1. (d)–(f) See previous page.
2. 711 313 602 484 401 807 210 763 124 565 106 558
3. (a) 700 000 + 80 000 + 9 000 + 300 + 20 + 4
   (b) 500 000 + 20 000 + 8 000 + 700 + 30 + 2
   (c) 500 000 + 1 000 + 100 + 3
   (d) 400 000 + 40 000 + 1 000 + 100 + 60
   (e) 200 000 + 80 000 + 7 000 + 500 + 60 + 4
   (f) 400 000 + 80 000 + 7 000 + 900 + 20 + 3
4. 287 564 441 160 487 923 501 103 528 732 789 324

5.–6. See next page.

7. (a) 666 066 (b) 666 606 (c) 606 666 (d) 66 666
8. (a) 80 000 90 000 100 000 110 000
   120 000 130 000 140 000
   (b) 580 000 590 000 600 000 610 000
   620 000 630 000 640 000
   (c) 880 000 890 000 900 000 910 000
   920 000 930 000 940 000 950 000
   960 000 970 000 980 000 990 000 1 000 000
   (d) 888 000 898 000 908 000 918 000
   928 000 938 000 948 000 958 000
   968 000 978 000 988 000 998 000 1 008 000
9. (a) 311 111 (b) 900 001



(d) six hundred and two thousand four hundred and eighty-four

(e) one hundred and six thousand five hundred and fifty-eight

(f) seven hundred and eleven thousand three hundred and thirteen

2. Now order the number symbols you wrote for question 1 from biggest to smallest and write them down.
3. Write the place value expansions for these numbers.
   (a) 789 324 (b) 528 732
   (c) 501 103 (d) 441 160
   (e) 287 564 (f) 487 923
4. Arrange the numbers in question 3 from smallest to biggest.
5. (a) How many short thick lines are shown on the next page?
   (b) How many lines will be shown on ten pages like the next?
   (c) How many lines will be shown on a hundred pages like the next?
6. (a) How many groups of 100 lines each are there on the next page?
   (b) How many groups of 10 lines each are there on the next page?
7. How much is each of the following?
   (a) 666 666 – 600
   (b) 666 666 – 60
   (c) 666 666 – 60 000
   (d) 666 666 – 600 000
8. Write the number symbols while you count in ten thousands
   (a) from 80 000 up to 140 000
   (b) from 580 000 up to 640 000
   (c) from eight hundred and eighty thousand up to one million
   (d) from 888 000 until you pass 1 000 000.
9. Compare the two numbers in each case. Which is bigger?
   (a) 299 999 and 311 111 (b) 899 999 and 900 001

6 UNIT 1: WHOLE NUMBERS

Teaching guidelines

Questions 5 and 6 on Learner Book page 5 refer to the arrays on Learner Book page 7. For your convenience the questions are repeated here.

5. (a) How many short thick lines are shown on the next page?
   (b) How many lines will be shown on ten pages like the next?
   (c) How many lines will be shown on a hundred pages like the next?
6. (a) How many groups of 100 lines each are there on the next page?
   (b) How many groups of 10 lines each are there on the next page?

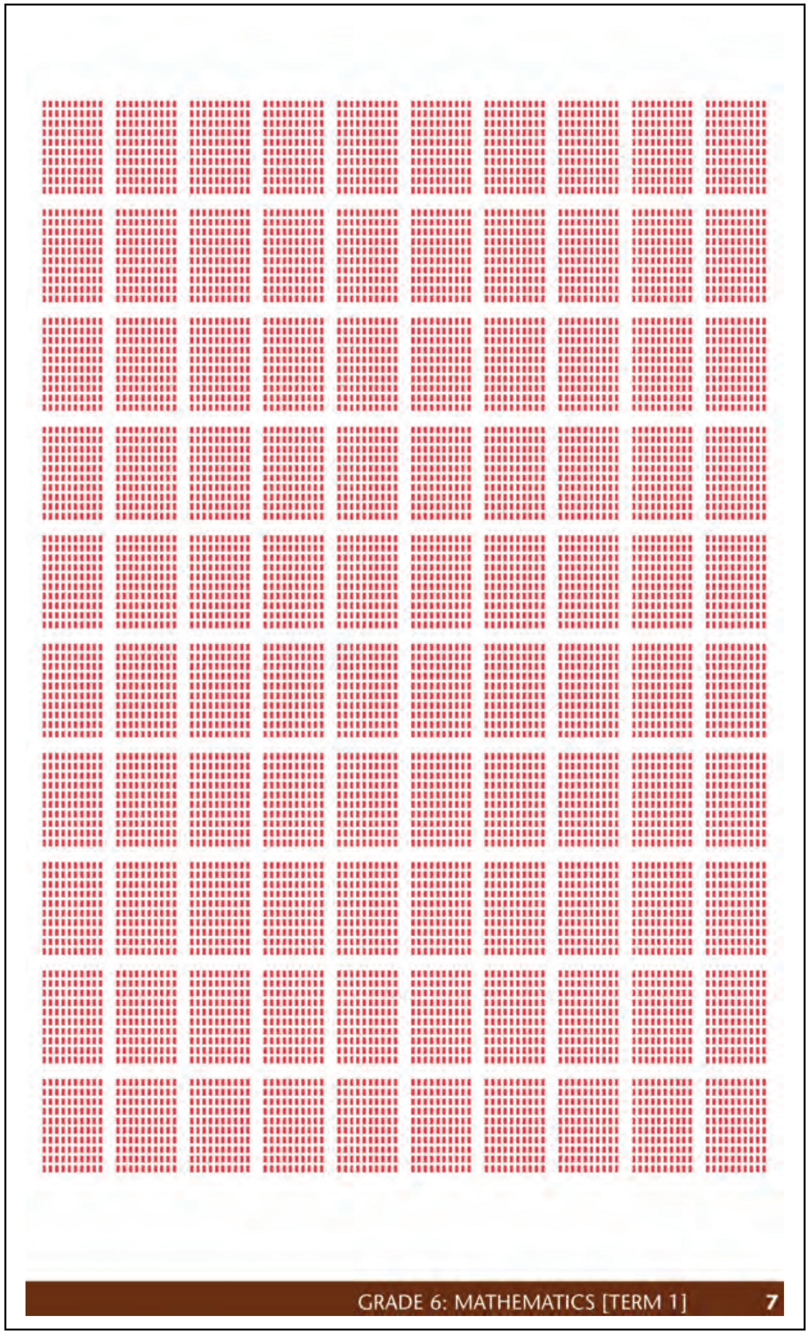
Answers

5. (a) 10 000 (b) 100 000 (c) 1 000 000
6. (a) 100 (b) 1 000

Extension

Many other questions can be asked with reference to this diagram, for example:

1. How many lines will be left if 10 lines are removed from each group of 100?
2. How many lines will be left if 37 lines are removed from each group of 100?
3. How many lines will be left if . . . . lines are removed from each group of 100?
4. There are 10 rows of 10 rectangular groups of lines on the diagram.
   How many lines will be left if 3 rectangular groups are removed from each row?



GRADE 6: MATHEMATICS [TERM 1] 7

1.3 Arrange numbers in order on number lines

Teaching guidelines

You may let learners do questions 1 and 2 in class; then let them do questions 3 and 4 at home.

You may have to demonstrate part of 1(a) on the board to ensure that learners do not waste time on figuring out what they have to do.

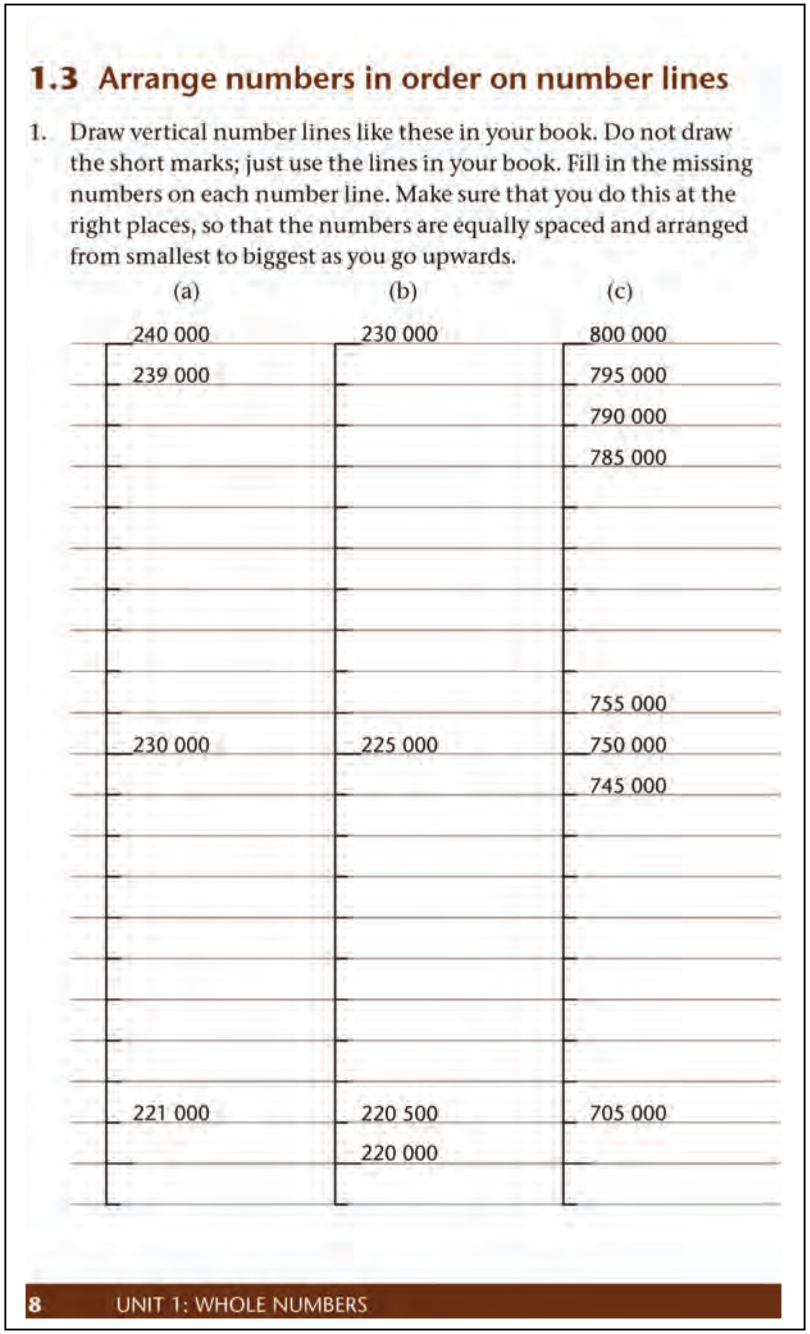
Answers

1. The numbers must be equally spaced and arranged from smallest to biggest as you move upwards.

   (a) 219 000 220 000 221 000 222 000 223 000 224 000
   225 000 226 000 227 000 228 000 229 000 230 000
   231 000 232 000 233 000 234 000 235 000 236 000
   237 000 238 000 239 000 240 000

   (b) 219 500 220 000 220 500 221 000 221 500 222 000
   222 500 223 000 223 500 224 000 224 500 225 000
   225 500 226 000 226 500 227 000 227 500 228 000
   228 500 229 000 229 500 230 000

   (c) 695 000 700 000 705 000 710 000 715 000 720 000
   725 000 730 000 735 000 740 000 745 000 750 000
   755 000 760 000 765 000 770 000 775 000 780 000
   785 000 790 000 795 000 800 000



1.3 Arrange numbers in order on number lines

1. Draw vertical number lines like these in your book. Do not draw the short marks; just use the lines in your book. Fill in the missing numbers on each number line. Make sure that you do this at the right places, so that the numbers are equally spaced and arranged from smallest to biggest as you go upwards.

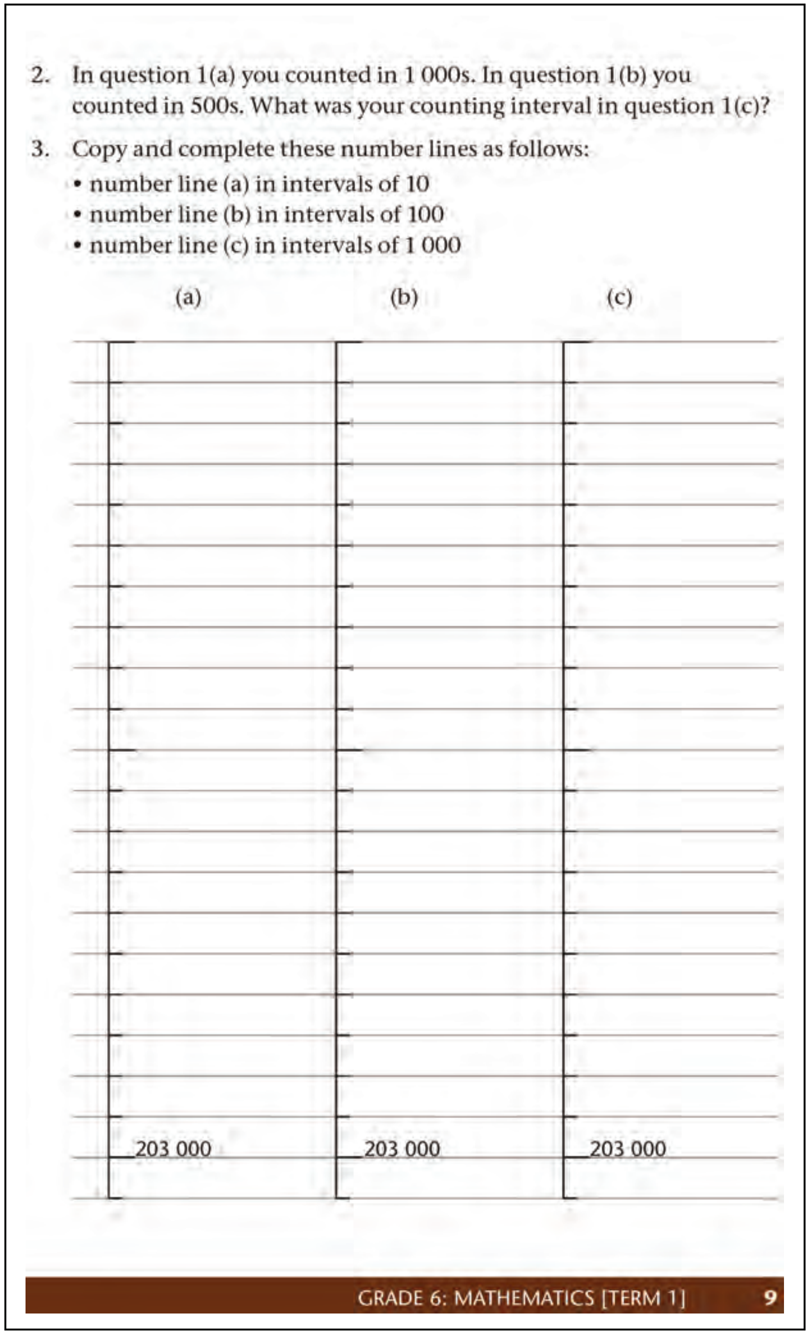
8 UNIT 1: WHOLE NUMBERS

Answers

2. 5 000s

3.

(a)	(b)	(c)
203 200	205 000	223 000
203 190	204 900	222 000
203 180	204 800	221 000
203 170	204 700	220 000
203 160	204 600	219 000
203 150	204 500	218 000
203 140	204 400	217 000
203 130	204 300	216 000
203 120	204 200	215 000
203 110	204 100	214 000
203 100	204 000	213 000
203 090	203 900	212 000
203 080	203 800	211 000
203 070	203 700	210 000
203 060	203 600	209 000
203 050	203 500	208 000
203 040	203 400	207 000
203 030	203 300	206 000
203 020	203 200	205 000
203 010	203 100	204 000
203 000	203 000	203 000
202 990	202 900	202 000



2. In question 1(a) you counted in 1 000s. In question 1(b) you counted in 500s. What was your counting interval in question 1(c)?

3. Copy and complete these number lines as follows:
   - number line (a) in intervals of 10
   - number line (b) in intervals of 100
   - number line (c) in intervals of 1 000

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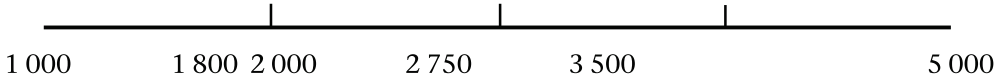
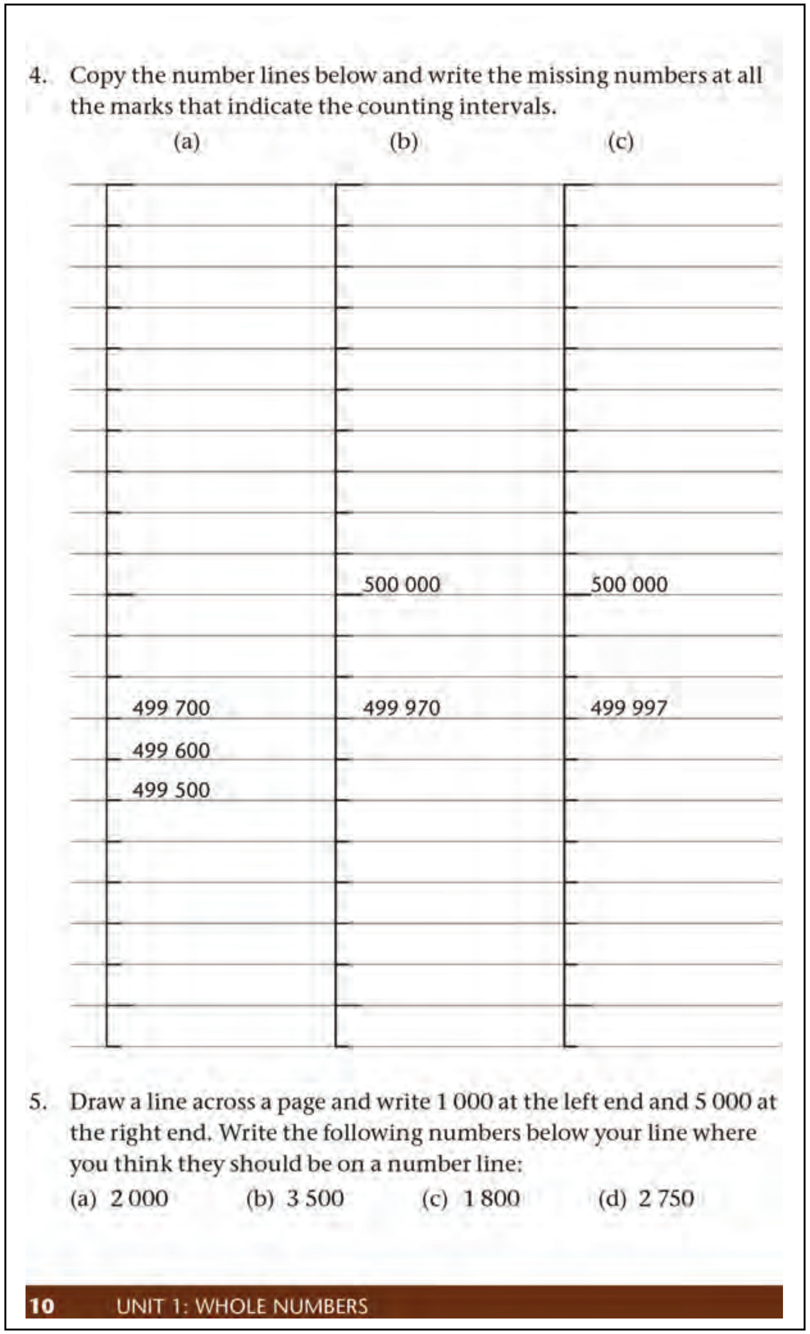
Answers

4.

(a)	(b)	(c)
501 000	500 100	500 010
500 900	500 090	500 009
500 800	500 080	500 008
500 700	500 070	500 007
500 600	500 060	500 006
500 500	500 050	500 005
500 400	500 040	500 004
500 300	500 030	500 003
500 200	500 020	500 002
500 100	500 010	500 001
500 000	500 000	500 000
499 900	499 990	499 999
499 800	499 980	499 998
499 700	499 970	499 997
499 600	499 960	499 996
499 500	499 950	499 995
499 400	499 940	499 994
499 300	499 930	499 993
499 200	499 920	499 992
499 100	499 910	499 991
499 000	499 900	499 990
498 900	499 890	499 989

5. Example:





4. Copy the number lines below and write the missing numbers at all the marks that indicate the counting intervals.

5. Draw a line across a page and write 1 000 at the left end and 5 000 at the right end. Write the following numbers below your line where you think they should be on a number line:

(a) 2 000 (b) 3 500 (c) 1 800 (d) 2 750

10 UNIT 1: WHOLE NUMBERS

1.4 Factors and multiples

Critical knowledge

When we multiply two numbers, for example 7 × 8 = 56, the answer is called the **product** of the two numbers.

The two numbers are called **factors** of the product. (The product may have other factors too, for example 14, 28, 2 and 4 in the case of 56.)

A product is a **multiple** of each of its factors.

The purpose of the shaded passage and questions 4 and 5 is mainly to develop knowledge of the above vocabulary.

Teaching guidelines

You may bring to learners' attention that:

- counting in groups of two produces the multiples of two
- counting in groups of three produces the multiples of three
- counting in groups of four produces the multiples of four
- counting in groups of five produces the multiples of five, etc.

Answers

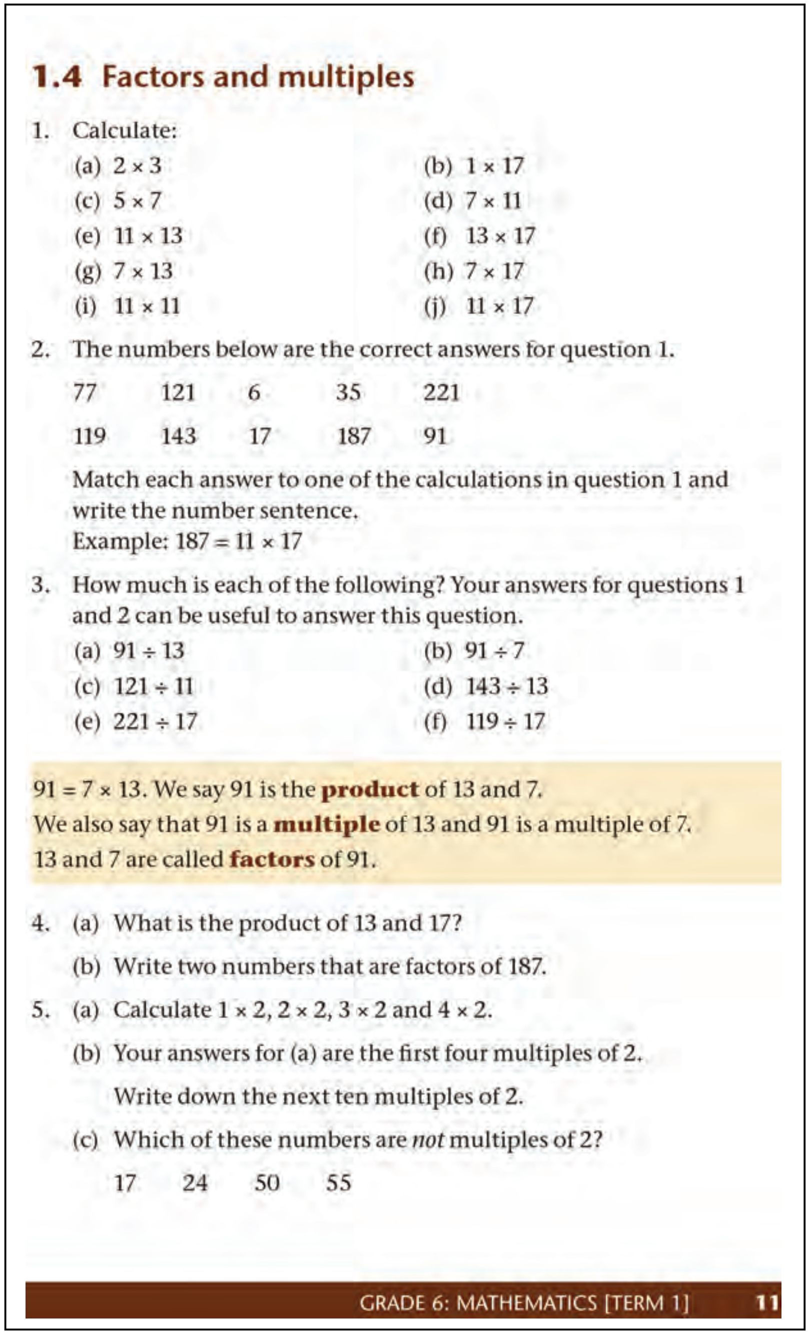
1. (a) 6 (b) 17 (c) 35 (d) 77
(e) 143 (f) 221 (g) 91 (h) 119
(i) 121 (j) 187

2. 77 = 7 × 11 121 = 11 × 11 6 = 2 × 3 35 = 5 × 7
221 = 13 × 17 119 = 7 × 17 143 = 11 × 13 17 = 1 × 17
187 = 11 × 17 91 = 7 × 13

3. (a) 7 (b) 13 (c) 11
(d) 11 (e) 13 (f) 7

4. (a) 221 (b) 11 and 17

5. (a) 1 × 2 = 2; 2 × 2 = 4; 3 × 2 = 6; 4 × 2 = 8
(b) 10; 12; 14; 16; 18; 20; 22; 24; 26; 28
(c) 17 and 55



1.4 Factors and multiples

1. Calculate:
(a) 2 × 3 (b) 1 × 17
(c) 5 × 7 (d) 7 × 11
(e) 11 × 13 (f) 13 × 17
(g) 7 × 13 (h) 7 × 17
(i) 11 × 11 (j) 11 × 17

2. The numbers below are the correct answers for question 1.

77 121 6 35 221
119 143 17 187 91

Match each answer to one of the calculations in question 1 and write the number sentence.
Example: 187 = 11 × 17

3. How much is each of the following? Your answers for questions 1 and 2 can be useful to answer this question.
(a) 91 ÷ 13 (b) 91 ÷ 7
(c) 121 ÷ 11 (d) 143 ÷ 13
(e) 221 ÷ 17 (f) 119 ÷ 17

91 = 7 × 13. We say 91 is the **product** of 13 and 7.
We also say that 91 is a **multiple** of 13 and 91 is a multiple of 7.
13 and 7 are called **factors** of 91.

4. (a) What is the product of 13 and 17?
(b) Write two numbers that are factors of 187.

5. (a) Calculate 1 × 2, 2 × 2, 3 × 2 and 4 × 2.
(b) Your answers for (a) are the first four multiples of 2.
Write down the next ten multiples of 2.
(c) Which of these numbers are *not* multiples of 2?

17 24 50 55

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Notes on questions

Note that question 7 does not only serve the purpose of identifying the prime numbers smaller than 100. It also provides extensive practice in identifying multiples of 2, 3, 5 and 7, and hence is a **Mental Mathematics** activity.

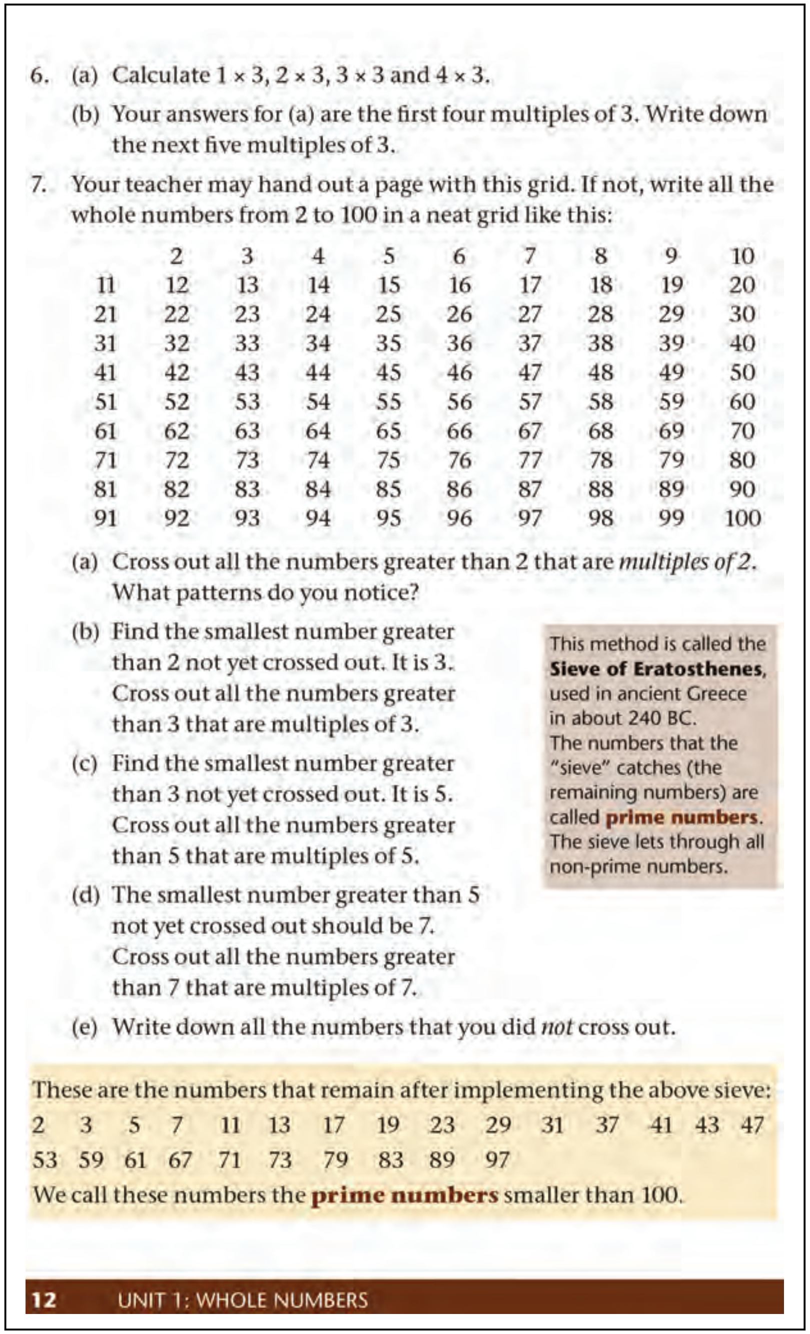
Point out that, for example, numbers that are "greater than 7" do not include the number 7: a number cannot be greater than itself.

Teaching guidelines

Let learners check their answer for question 7(e) by comparing their list of numbers to the list given in the shaded passage at the bottom of the page.

Answers

6. (a) 1 × 3 = 3; 2 × 3 = 6; 3 × 3 = 9; 4 × 3 = 12
   (b) 15; 18; 21; 24; 27
7. (a) All even numbers greater than 2 are crossed out.
   (b) Multiples of 3 greater than 3 are crossed out: 9, 15, 21, 27, 33, 39, 45, 51, 57, 63, 69, 75, 81, 87, 93, 99.
   (c) Multiples of 5 greater than 5 are crossed out: 25, 35, 55, 65, 85, 95.
   (d) Multiples of 7 greater than 7 are crossed out: 49, 77, 91.
   (e) Numbers not crossed out: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.



6. (a) Calculate 1 × 3, 2 × 3, 3 × 3 and 4 × 3.
   (b) Your answers for (a) are the first four multiples of 3. Write down the next five multiples of 3.
7. Your teacher may hand out a page with this grid. If not, write all the whole numbers from 2 to 100 in a neat grid like this:

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

   (a) Cross out all the numbers greater than 2 that are *multiples of 2*. What patterns do you notice?
   (b) Find the smallest number greater than 2 not yet crossed out. It is 3. Cross out all the numbers greater than 3 that are multiples of 3.
   (c) Find the smallest number greater than 3 not yet crossed out. It is 5. Cross out all the numbers greater than 5 that are multiples of 5.
   (d) The smallest number greater than 5 not yet crossed out should be 7. Cross out all the numbers greater than 7 that are multiples of 7.
   (e) Write down all the numbers that you did *not* cross out.

This method is called the **Sieve of Eratosthenes**, used in ancient Greece in about 240 BC. The numbers that the "sieve" catches (the remaining numbers) are called **prime numbers**. The sieve lets through all non-prime numbers.

These are the numbers that remain after implementing the above sieve:
2 3 5 7 11 13 17 19 23 29 31 37 41 43 47
53 59 61 67 71 73 79 83 89 97
We call these numbers the **prime numbers** smaller than 100.

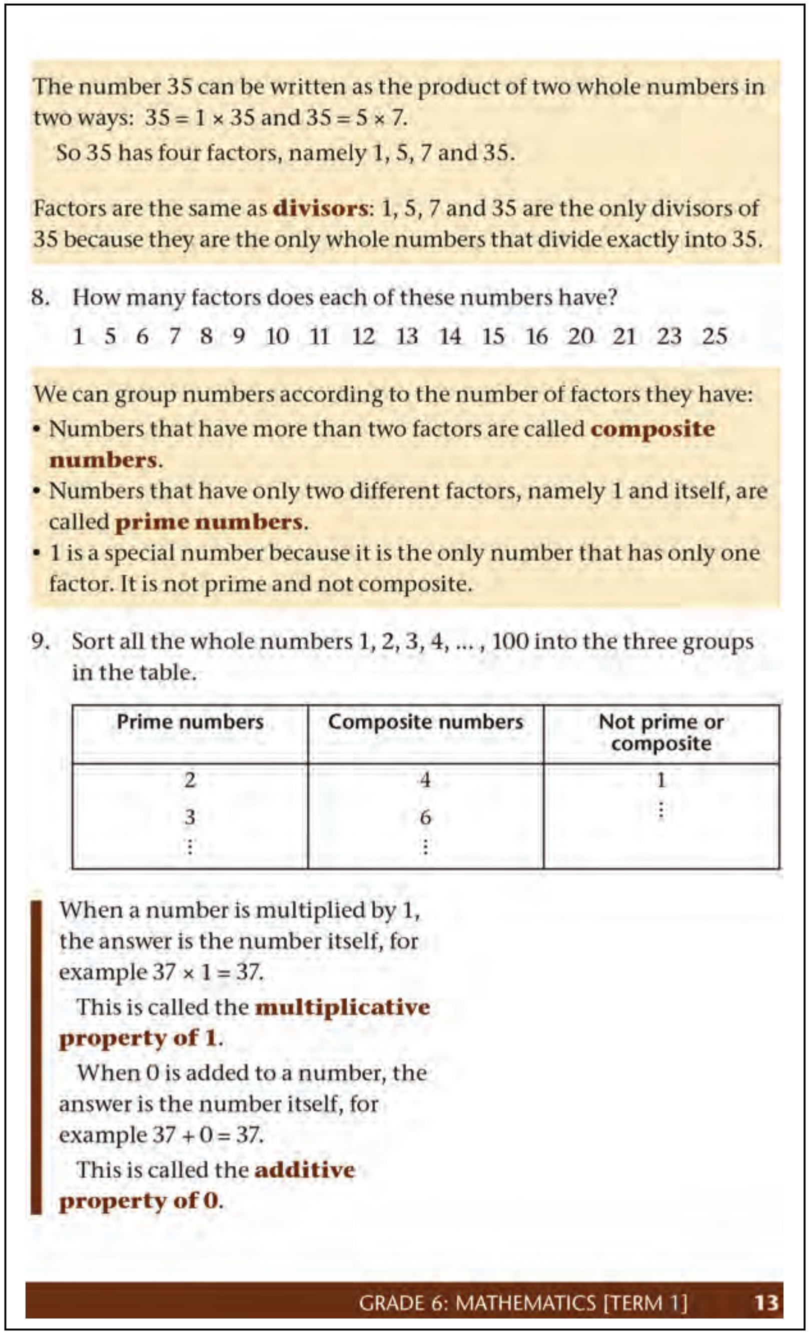
12 UNIT 1: WHOLE NUMBERS

Answers

8. 1: 1 factor 5: 2 factors 6: 4 factors 7: 2 factors 8: 4 factors
9: 3 factors 10: 4 factors 11: 2 factors 12: 6 factors 13: 2 factors
14: 4 factors 15: 4 factors 16: 5 factors 20: 6 factors 21: 4 factors
23: 2 factors 25: 3 factors

9.

Prime numbers			**Composite numbers**								**Not prime or composite**
2	29	67	4	20	33	45	56	68	80	91	1
3	31	71	6	21	34	46	57	69	81	92	
5	37	73	8	22	35	48	58	70	82	93	
7	41	79	9	24	36	49	60	72	84	94	
11	43	83	10	25	38	50	62	74	85	95	
13	47	89	12	26	39	51	63	75	86	96	
17	53	97	14	27	40	52	64	76	87	98	
19	59		15	28	42	54	65	77	88	99	
23	61		16	30	44	55	66	78	90	100	
			18	32							



The number 35 can be written as the product of two whole numbers in two ways: 35 = 1 × 35 and 35 = 5 × 7.

So 35 has four factors, namely 1, 5, 7 and 35.

Factors are the same as **divisors**: 1, 5, 7 and 35 are the only divisors of 35 because they are the only whole numbers that divide exactly into 35.

8. How many factors does each of these numbers have?
1 5 6 7 8 9 10 11 12 13 14 15 16 20 21 23 25

We can group numbers according to the number of factors they have:

- Numbers that have more than two factors are called **composite numbers**.
- Numbers that have only two different factors, namely 1 and itself, are called **prime numbers**.
- 1 is a special number because it is the only number that has only one factor. It is not prime and not composite.

9. Sort all the whole numbers 1, 2, 3, 4, ... , 100 into the three groups in the table.

Prime numbers	Composite numbers	Not prime or composite
2	4	1
3	6	⋮
⋮	⋮	

When a number is multiplied by 1, the answer is the number itself, for example 37 × 1 = 37.

This is called the **multiplicative property of 1**.

When 0 is added to a number, the answer is the number itself, for example 37 + 0 = 37.

This is called the **additive property of 0**.

GRADE 6: MATHEMATICS [TERM 1] 13

1.5 Rounding off

Mathematical notes

Different forms of “rounding off” form part of cultural practices.

Distances on road maps are normally given to the nearest kilometre, the length of a person is normally stated to the nearest centimetre, and in carpentry lengths are normally given to the nearest millimetre.

In everyday life, a person’s age is often rounded off to full years. This rounding is normally not done to the nearest full year but to the number of completed years: an age of 12 years and 9 months is rounded off to 12 years – the person cannot claim to be 13 yet. This is not rounding off to the nearest full year, but rounding down to the full year less than the actual age.

In contrast, when we round off numbers, we round off to the nearest unit (whole number or multiple of 5, 10, 100, etc.), even if it is bigger than the actual number.

Teaching guidelines

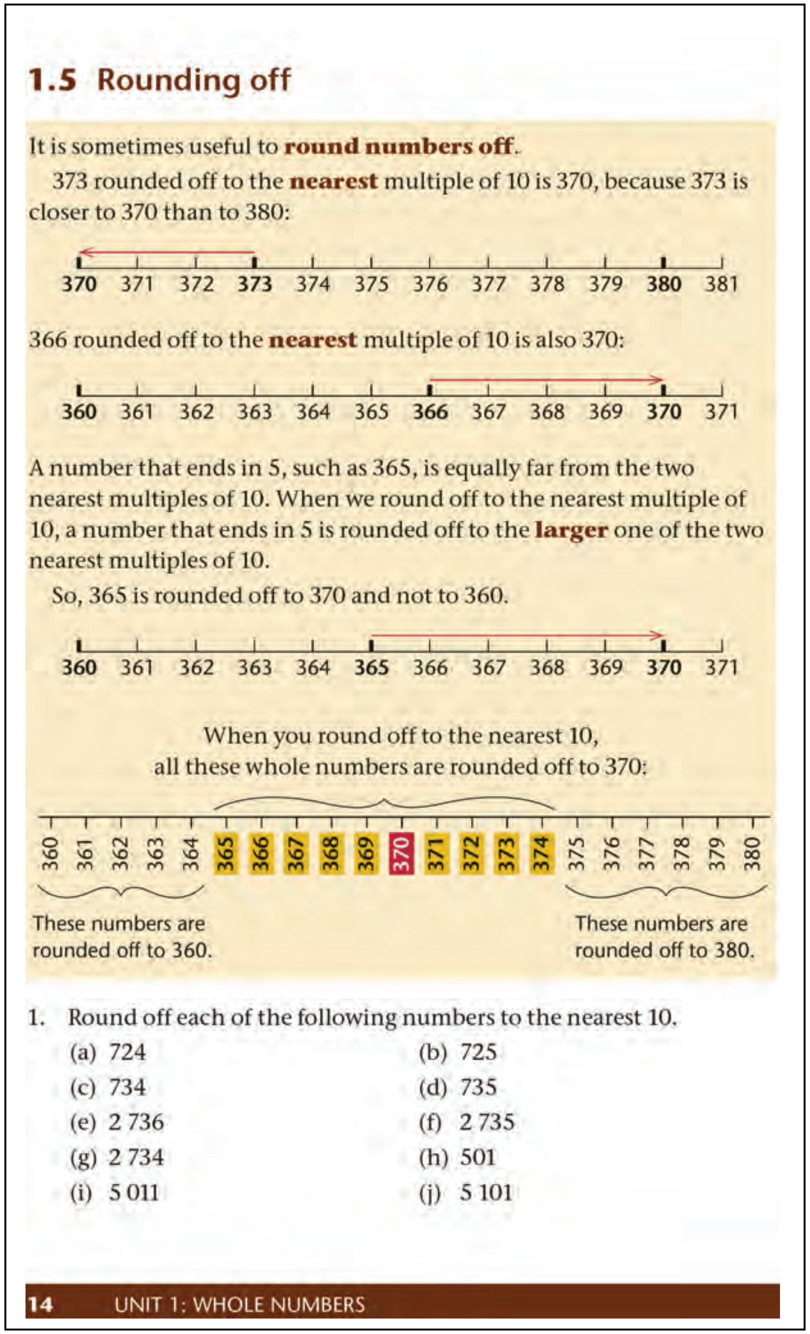
You may also ask learners to measure the width and the length of a book with a ruler, first accurate to the nearest centimetre, and then accurate to the nearest millimetre. The measurement accurate to the nearest centimetre corresponds to rounding off the measurement in millimetre to the nearest multiple of 10.

Drawings and an explanation can be used to link the above activity to the diagrams on pages 14 and 15 of the Learner Book, for example:

All these lines are 5 cm long, to the nearest centimetre.

Answers

1. (a) 720 (b) 730 (c) 730 (d) 740
(e) 2 740 (f) 2 740 (g) 2 730 (h) 500
(i) 5 010 (j) 5 100



1.5 Rounding off

It is sometimes useful to **round numbers off**.

373 rounded off to the **nearest** multiple of 10 is 370, because 373 is closer to 370 than to 380:

366 rounded off to the **nearest** multiple of 10 is also 370:

A number that ends in 5, such as 365, is equally far from the two nearest multiples of 10. When we round off to the nearest multiple of 10, a number that ends in 5 is rounded off to the **larger** one of the two nearest multiples of 10.

So, 365 is rounded off to 370 and not to 360.

When you round off to the nearest 10, all these whole numbers are rounded off to 370:

These numbers are rounded off to 360.

These numbers are rounded off to 380.

1. Round off each of the following numbers to the nearest 10.
(a) 724 (b) 725
(c) 734 (d) 735
(e) 2 736 (f) 2 735
(g) 2 734 (h) 501
(i) 5 011 (j) 5 101

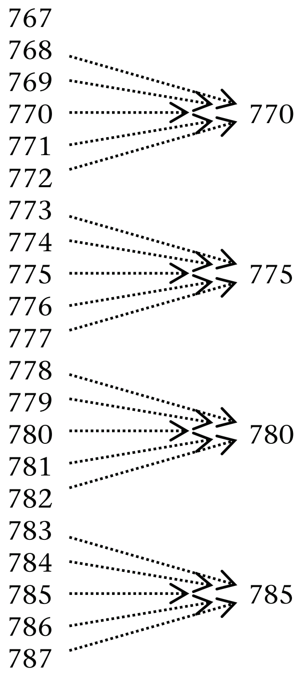
Teaching guidelines

The two diagrams with the multiple arrows may be helpful to learners who at this stage are still challenged by rounding off. To ensure that learners take note of these diagrams, you may ask them to redraw these and other diagrams at home – but do not use valuable class time for this purpose.

You may also write two columns of numbers on the board as shown on the right. Then show with arrows which of the numbers on the left are rounded off to 770, which to 775, etc., if they are rounded off to the nearest 5. You may use the same layout to show rounding off to the nearest 10 or to any other number.

In this way you can also write additional exercises for learners on the board.

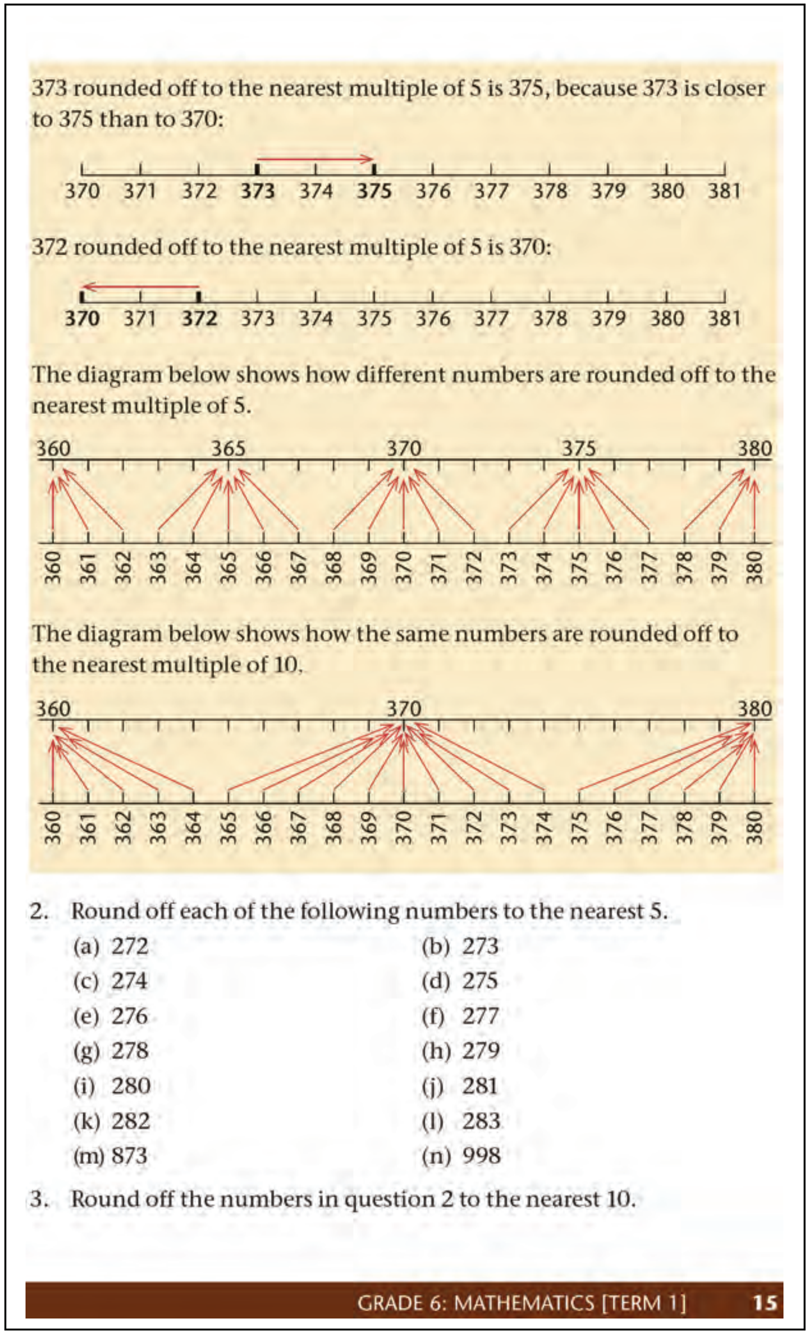
Note that by representing rounding off in this way, it can be linked to rounding off in measuring length (see Section 11.4 on pages 291 to 293 of the Learner Book).



Answers

2. (a) 270 (b) 275 (c) 275 (d) 275
(e) 275 (f) 275 (g) 280 (h) 280
(i) 280 (j) 280 (k) 280 (l) 285
(m) 875 (n) 1 000

3. (a) 270 (b) 270 (c) 270 (d) 280
(e) 280 (f) 280 (g) 280 (h) 280
(i) 280 (j) 280 (k) 280 (l) 280
(m) 870 (n) 1 000



373 rounded off to the nearest multiple of 5 is 375, because 373 is closer to 375 than to 370:

372 rounded off to the nearest multiple of 5 is 370:

The diagram below shows how different numbers are rounded off to the nearest multiple of 5.

The diagram below shows how the same numbers are rounded off to the nearest multiple of 10.

2. Round off each of the following numbers to the nearest 5.
(a) 272 (b) 273
(c) 274 (d) 275
(e) 276 (f) 277
(g) 278 (h) 279
(i) 280 (j) 281
(k) 282 (l) 283
(m) 873 (n) 998

3. Round off the numbers in question 2 to the nearest 10.

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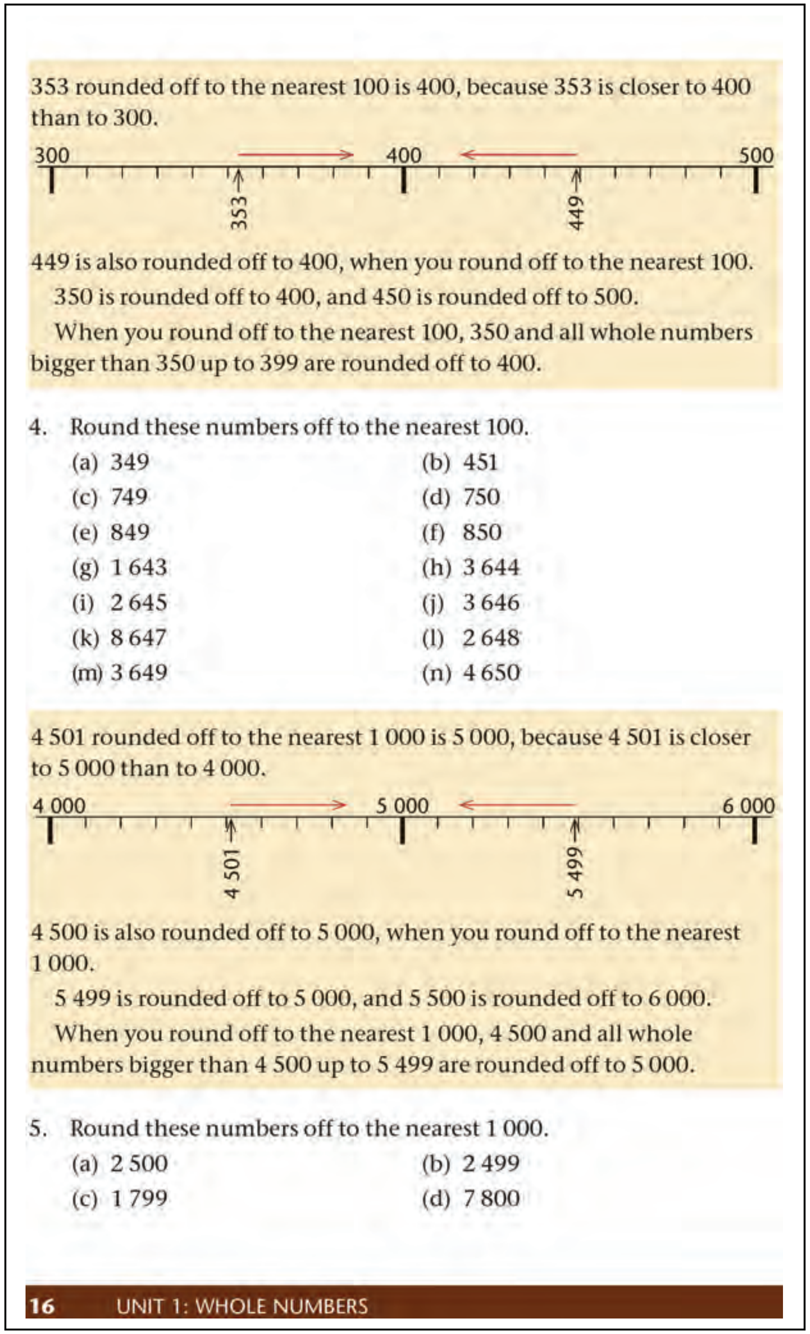
Teaching guidelines

You cannot take for granted that learners will understand diagrams like those in the shaded passages. Ask learners to all look at the diagram at the top, and ask them what the red arrow on the left tells us about the numbers 353 and 400. Take answers from some learners, but then ask different learners to say what the red arrow on the right indicates. To check to what extent learners in your class actually engage adequately with the diagrams, you may ask all learners to write, on a loose piece of paper, what the two red arrows on the number line in the lower shaded passage indicate. Take in the papers.

Answers

4.	(a) 300	(b) 500	(c) 700	(d) 800
	(e) 800	(f) 900	(g) 1 600	(h) 3 600
	(i) 2 600	(j) 3 600	(k) 8 600	(l) 2 600
	(m) 3 600	(n) 4 700		
5.	(a) 3 000	(b) 2 000	(c) 2 000	(d) 8 000



353 rounded off to the nearest 100 is 400, because 353 is closer to 400 than to 300.

449 is also rounded off to 400, when you round off to the nearest 100.

350 is rounded off to 400, and 450 is rounded off to 500.

When you round off to the nearest 100, 350 and all whole numbers bigger than 350 up to 399 are rounded off to 400.

4. Round these numbers off to the nearest 100.

(a) 349	(b) 451
(c) 749	(d) 750
(e) 849	(f) 850
(g) 1 643	(h) 3 644
(i) 2 645	(j) 3 646
(k) 8 647	(l) 2 648
(m) 3 649	(n) 4 650

4 501 rounded off to the nearest 1 000 is 5 000, because 4 501 is closer to 5 000 than to 4 000.

4 500 is also rounded off to 5 000, when you round off to the nearest 1 000.

5 499 is rounded off to 5 000, and 5 500 is rounded off to 6 000.

When you round off to the nearest 1 000, 4 500 and all whole numbers bigger than 4 500 up to 5 499 are rounded off to 5 000.

5. Round these numbers off to the nearest 1 000.

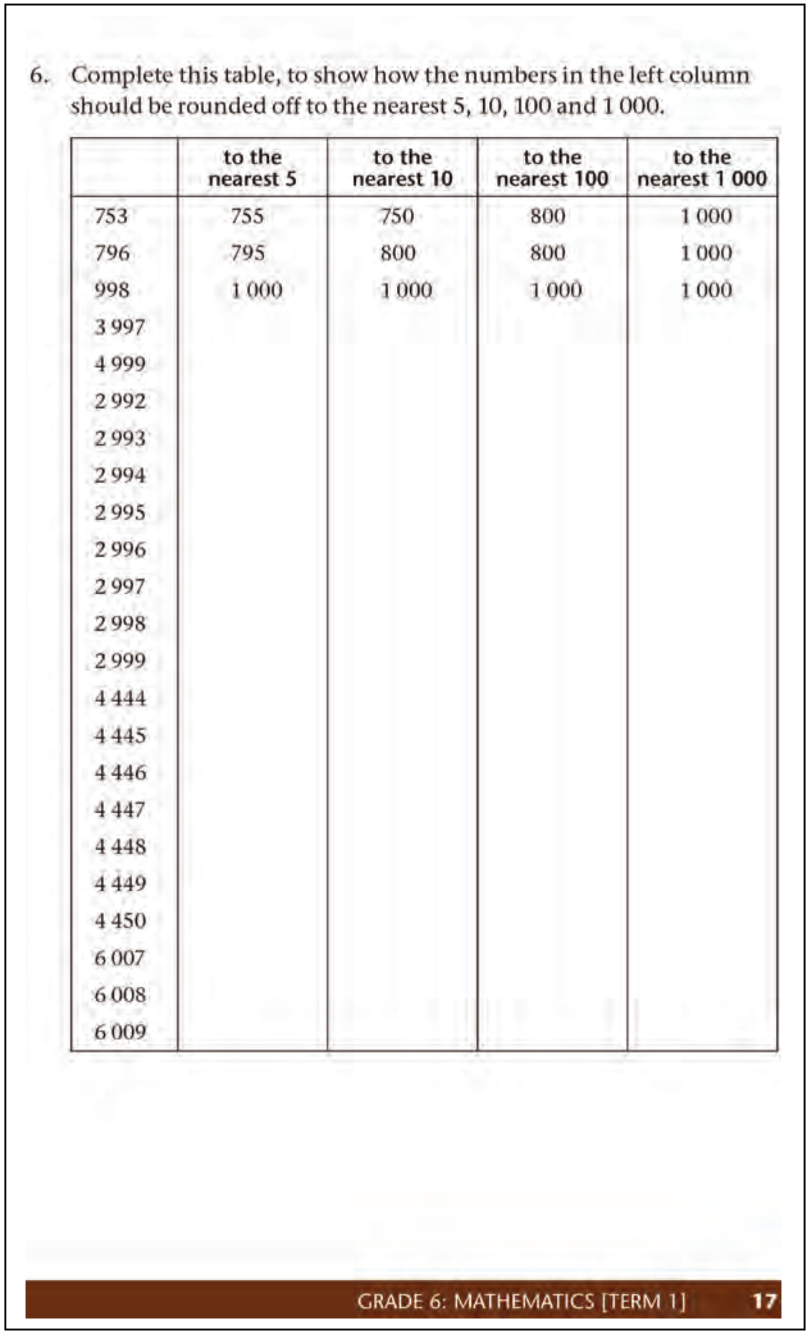
(a) 2 500	(b) 2 499
(c) 1 799	(d) 7 800

16 UNIT 1: WHOLE NUMBERS

Answers

6.

	to the nearest 5	to the nearest 10	to the nearest 100	to the nearest 1 000
753	755	750	800	1 000
796	795	800	800	1 000
998	1 000	1 000	1 000	1 000
3 997	3 995	4 000	4 000	4 000
4 999	5 000	5 000	5 000	5 000
2 992	2 990	2 990	3 000	3 000
2 993	2 995	2 990	3 000	3 000
2 994	2 995	2 990	3 000	3 000
2 995	2 995	3 000	3 000	3 000
2 996	2 995	3 000	3 000	3 000
2 997	2 995	3 000	3 000	3 000
2 998	3 000	3 000	3 000	3 000
2 999	3 000	3 000	3 000	3 000
4 444	4 445	4 440	4 400	4 000
4 445	4 445	4 450	4 400	4 000
4 446	4 445	4 450	4 400	4 000
4 447	4 445	4 450	4 400	4 000
4 448	4 450	4 450	4 400	4 000
4 449	4 450	4 450	4 400	4 000
4 450	4 450	4 450	4 500	4 000
6 007	6 005	6 010	6 000	6 000
6 008	6 010	6 010	6 000	6 000
6 009	6 010	6 010	6 000	6 000



6. Complete this table, to show how the numbers in the left column should be rounded off to the nearest 5, 10, 100 and 1 000.

	to the nearest 5	to the nearest 10	to the nearest 100	to the nearest 1 000
753	755	750	800	1 000
796	795	800	800	1 000
998	1 000	1 000	1 000	1 000
3 997				
4 999				
2 992				
2 993				
2 994				
2 995				
2 996				
2 997				
2 998				
2 999				
4 444				
4 445				
4 446				
4 447				
4 448				
4 449				
4 450				
6 007				
6 008				
6 009				

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Grade 6 Term 1 Unit 2 Number sentences

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
2.1 Equivalence	The concepts of calculation plan, number sentence and equivalence	18 to 22
2.2 Writing number sentences	Using number sentences in Mental Mathematics	22 to 24
2.3 Solve and complete number sentences	Open number sentences	24

CAPS time allocation	3 hours
CAPS page references	20 and 217 to 222

The whole of Section 2.2 is **Mental Mathematics**.

Mathematical background

This unit addresses a variety of critically-important mathematical concepts that form the basis of algebra:

- A number sentence is a statement about **numbers**, for example 3 × 12 + 5 × 12 = 8 × 12.
  A number sentence is a **sentence**. The verb is =, "equals", "is equal to" or "is equivalent to".
- 98 – 20 + 12 × 2 and 98 – (20 + 12) × 2 are **expressions**, and can also be referred to as **calculation plans**.
- A calculation plan is a description of calculations that are to be executed in a certain sequence, governed by certain generally accepted conventions that are described on page 22 of this Teacher Guide.
- A true number sentence with calculation plans on both sides, such as 3 × 12 + 5 × 12 = 8 × 12, is a **statement of equivalence**. It states that the two different calculation plans will produce the same number, which in this case is 96.
- Some numbers in a calculation plan may be unspecified. For example, the symbol □ in the calculation plan 2 × □ + 5 is a placeholder for any number that may be specified. A calculation plan with a placeholder can also be called a **formula**.
- Simple calculation plans with placeholders, such as 2 × □ + 5, can also be represented with flow diagrams, for example: $\rightarrow \boxed{\times 2} \rightarrow \boxed{+ 5} \rightarrow$
- A number sentence that contains unspecified numbers is called an **open sentence**. Some open sentences are given below:
  2 × □ + 5 = 21 This open sentence is only true if the unspecified number (the unknown) □ is taken to be 8.
  3 × □ + 5 = 25 – 2 × □ This open sentence is only true for □ = 4.
- The open sentence □ + ∇ = 10 is true for various pairs of values of □ and ∇, for example □ = 4 and ∇ = 6, and □ = 2 and ∇ = 8.
- Open sentences that are only true for some values of the unknowns are also referred to as **equations**.
- Some open sentences are true for any (all) numbers. For example, 🟥 (🟦 + 🟨) = 🟥 × 🟦 + 🟥 × 🟨 is true for any three numbers in place of the red, blue and yellow squares respectively. This is the distributive property, which is represented with letter symbols from Grade 7 onwards: $a(b + c) = ab + ac$.
- An open sentence that is true for any values of the placeholders, with a calculation plan on each side of the equal sign, can be called an **identity**.

2.1 Equivalence

Teaching guidelines

Ask learners to describe how they will calculate the total cost of 3 goats at R423 each. Take some responses (of the calculation plans, not the answer) and then write the two calculation plans 423 + 423 + 423 and 3 × 423 on the board. Ask learners not to do the calculations, but to clarify to themselves whether they think the two calculation plans will produce the same result.

Inform learners that different calculation plans that produce the same result are called **equivalent calculation plans**.

Let learners then do question 1.

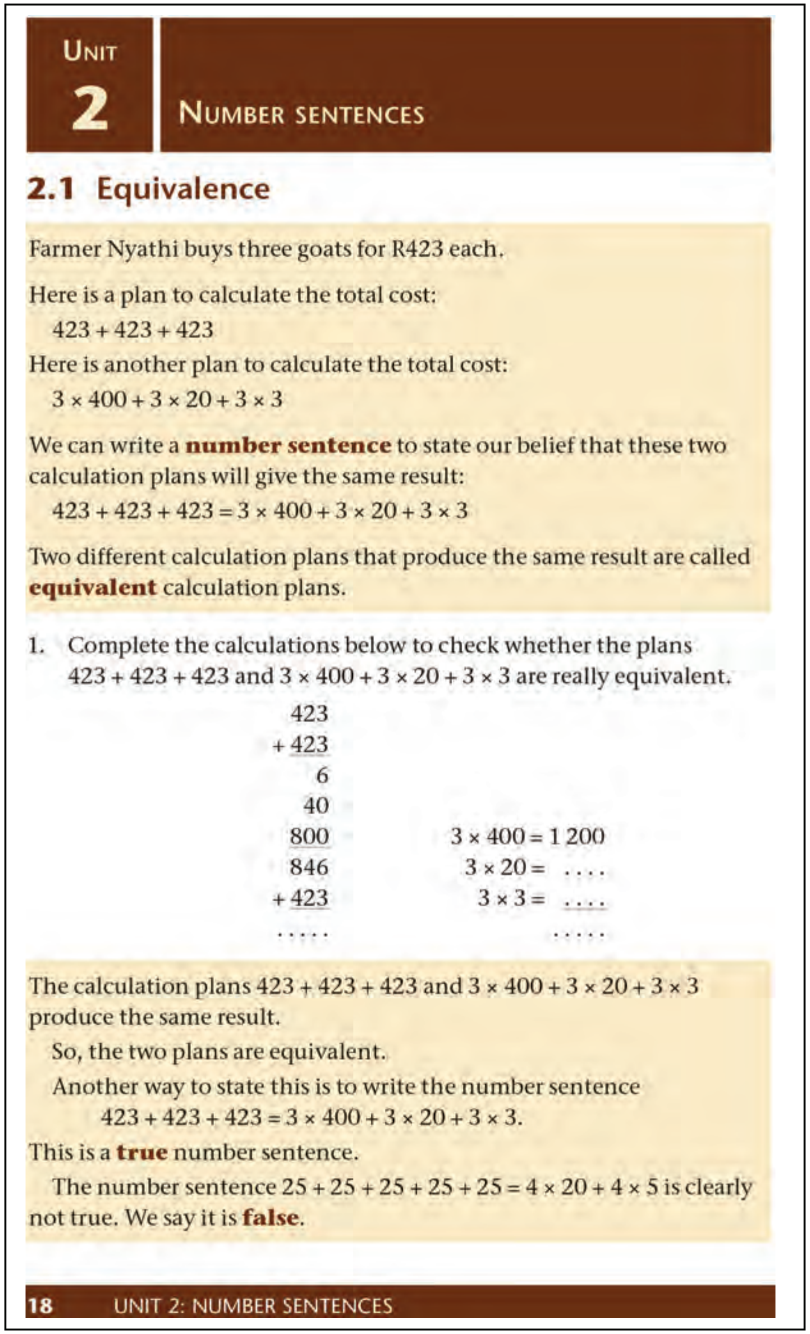
Possible misconceptions

The disempowering misconception that the equal sign means "and the answer is", is unfortunately very common. Learners with this misconception cannot make sense of statements of equivalence. The work on this page is specifically designed to help learners to come to understand the equal sign in a different way, namely "is equivalent to". In statements of equivalence the expression (calculation plan) on the right-hand side is not the answer obtained by executing the plan on the left-hand side. The calculation plans on the left-hand side and right-hand side specify two different sets of calculations. The equal sign means that when the two different plans are executed, the answers will be the same.

Answers

1. 1 269

   The calculation plans are equivalent.



UNIT 2

NUMBER SENTENCES

2.1 Equivalence

Farmer Nyathi buys three goats for R423 each.

Here is a plan to calculate the total cost:

423 + 423 + 423

Here is another plan to calculate the total cost:

3 × 400 + 3 × 20 + 3 × 3

We can write a **number sentence** to state our belief that these two calculation plans will give the same result:

423 + 423 + 423 = 3 × 400 + 3 × 20 + 3 × 3

Two different calculation plans that produce the same result are called **equivalent** calculation plans.

1. Complete the calculations below to check whether the plans 423 + 423 + 423 and 3 × 400 + 3 × 20 + 3 × 3 are really equivalent.

423	
+ 423	
6	
40	
800	3 × 400 = 1 200
846	3 × 20 = ...
+ 423	3 × 3 = ...
.....	.....

The calculation plans 423 + 423 + 423 and 3 × 400 + 3 × 20 + 3 × 3 produce the same result.

So, the two plans are equivalent.

Another way to state this is to write the number sentence

423 + 423 + 423 = 3 × 400 + 3 × 20 + 3 × 3.

This is a **true** number sentence.

The number sentence 25 + 25 + 25 + 25 + 25 = 4 × 20 + 4 × 5 is clearly not true. We say it is **false**.

Notes on questions

Questions 2 to 4 are intended to develop the idea of equivalent statements. Question 5 is critical as it leads into the next topic and further questions.

Answers

2. (a) True (b) False; 14 × 53 + 6 × 53 = 20 × 53
   (c) True (d) False; 96 + 36 = 100 + 32
   (e) True (f) True

3. (a) 10 × 37 = 370
   (c) 50 + 25 = 75
   (e) 10 × 76 = 760
   (f) 5 × 600 + 5 × 80 + 5 × 3 = 3 000 + 400 + 15 = 3 415

4. (a) 10 × 158 = 1 580
   (b) 10 × 47 = 470
   (c) 100 × 47 = 4 700

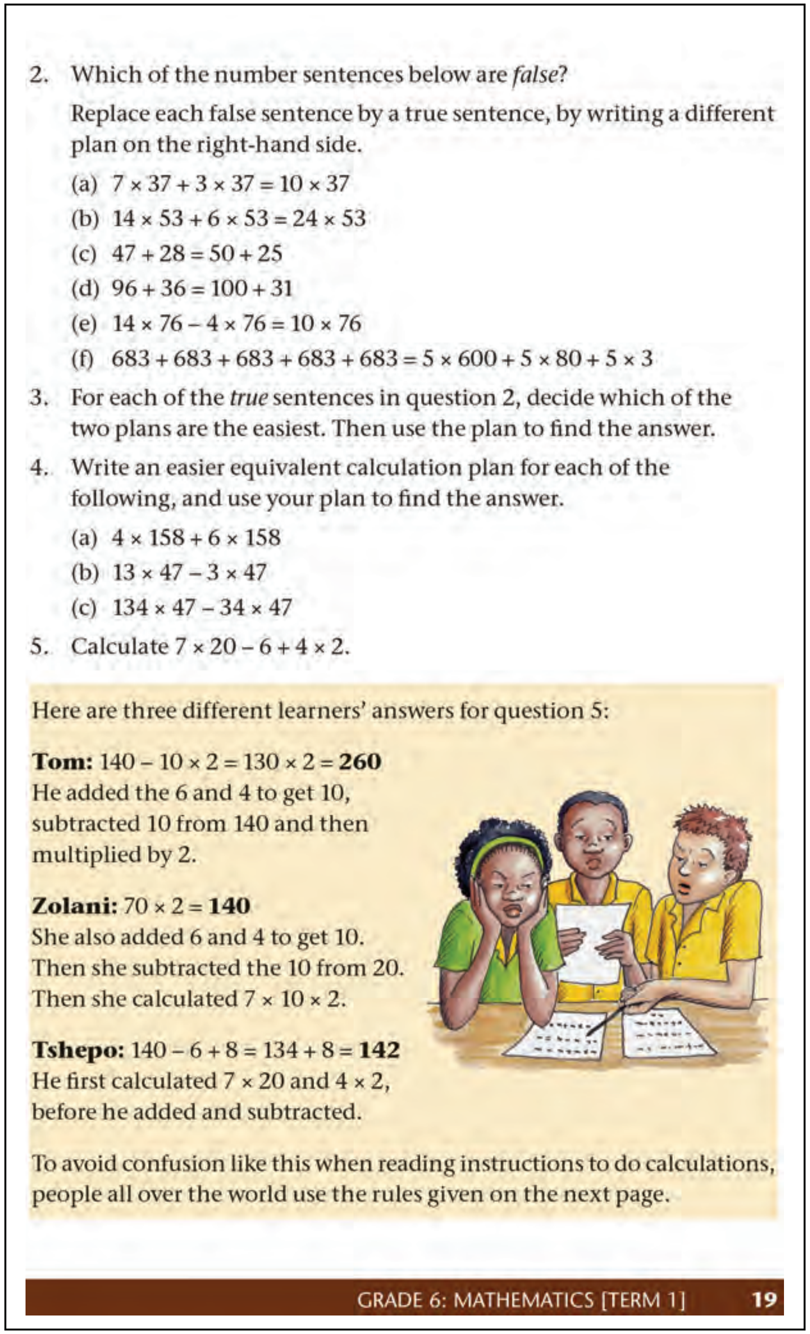
5. 142

Teaching guidelines

The purpose of question 5 and the shaded passage is to develop an awareness that calculation plans may be interpreted differently by different people, unless they all conform to agreed-upon interpretations. This provides motivation for the three conventions that are introduced on the next page.

Take feedback from all learners on their answers to question 5. If their answers differ, you may take a vote for the different likely answers 260, 140 and 142, and other answers that learners may give.

Ensure that learners understand that the different answers are produced because different people execute the given operations in different orders.



2. Which of the number sentences below are *false*?

   Replace each false sentence by a true sentence, by writing a different plan on the right-hand side.

   (a) 7 × 37 + 3 × 37 = 10 × 37
   (b) 14 × 53 + 6 × 53 = 24 × 53
   (c) 47 + 28 = 50 + 25
   (d) 96 + 36 = 100 + 31
   (e) 14 × 76 – 4 × 76 = 10 × 76
   (f) 683 + 683 + 683 + 683 + 683 = 5 × 600 + 5 × 80 + 5 × 3

3. For each of the *true* sentences in question 2, decide which of the two plans are the easiest. Then use the plan to find the answer.

4. Write an easier equivalent calculation plan for each of the following, and use your plan to find the answer.

   (a) 4 × 158 + 6 × 158
   (b) 13 × 47 – 3 × 47
   (c) 134 × 47 – 34 × 47

5. Calculate 7 × 20 – 6 + 4 × 2.

Here are three different learners' answers for question 5:

Tom: 140 – 10 × 2 = 130 × 2 = **260**
He added the 6 and 4 to get 10, subtracted 10 from 140 and then multiplied by 2.

Zolani: 70 × 2 = **140**
She also added 6 and 4 to get 10. Then she subtracted the 10 from 20. Then she calculated 7 × 10 × 2.

Tshepo: 140 – 6 + 8 = 134 + 8 = **142**
He first calculated 7 × 20 and 4 × 2, before he added and subtracted.

To avoid confusion like this when reading instructions to do calculations, people all over the world use the rules given on the next page.

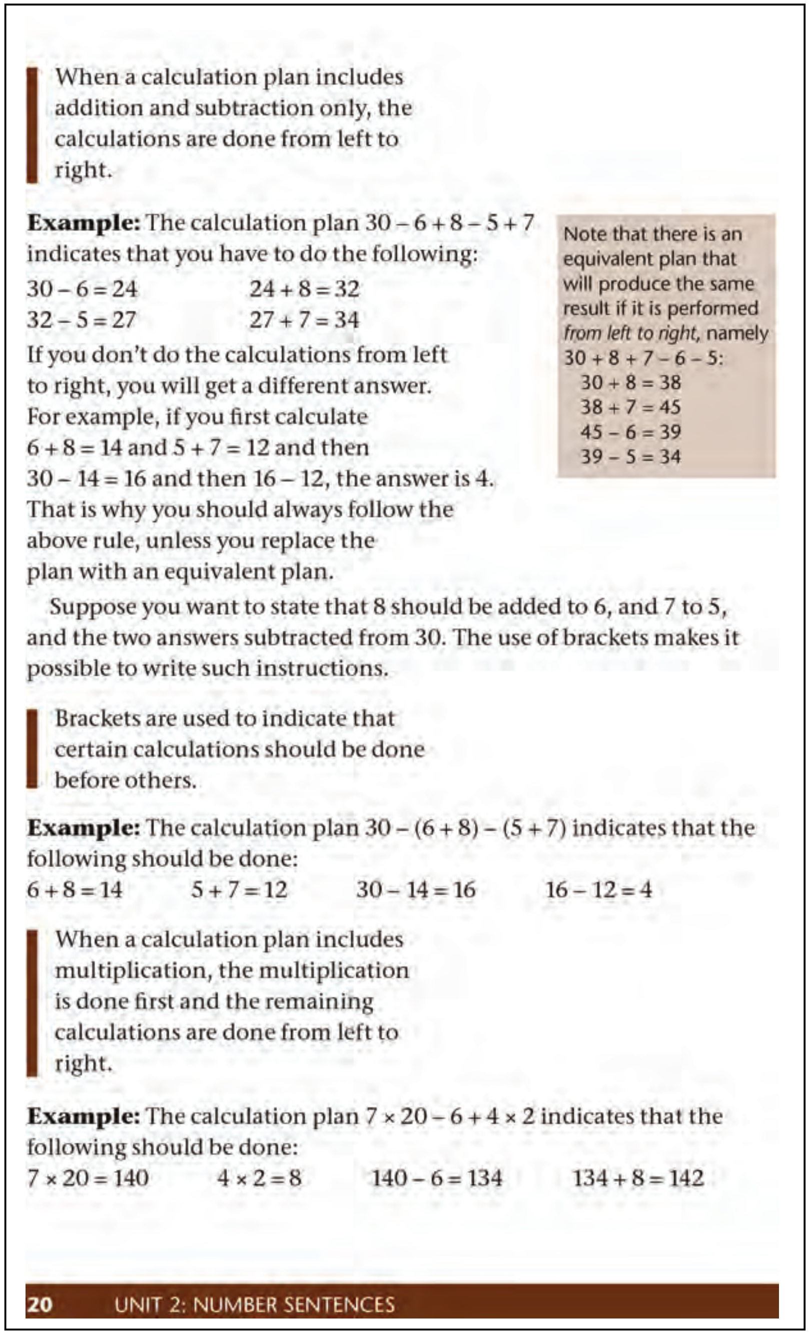
GRADE 6: MATHEMATICS [TERM 1] 19

Mathematical notes

A calculation plan describes which operations we must perform on which numbers in what order.

In mathematics there are generally agreed-upon conventions about the order in which we must perform the operations. Here are some important conventions:

- Brackets in a calculation plan indicate that the operations within the brackets should be performed first.
  For example, when the calculation plan 98 – (20 + 12) × 2 is executed, 20 + 12 should be calculated first: 98 – (20 + 12) × 2 = 98 – 32 × 2.
- In a calculation plan that contains no brackets, divisions and multiplications are performed first (before addition and subtraction), from left to right as they occur.
  For example, when 98 – 20 + 12 × 2 is executed, 12 × 2 is calculated first:
  98 – 20 + 12 × 2 = 98 – 20 + 24.
  When 15 + 24 ÷ 3 × 2 is calculated, 24 ÷ 3 is done first: 15 + 24 ÷ 3 × 2 = 15 + 8 × 2.
  The multiplication must be done next, then the addition: 15 + 8 × 2 = 15 + 16 = 31.
- Additions and subtractions that are not in brackets are performed last, from left to right in the order in which they occur.
  For example, when 98 – 20 + 24 is calculated, the subtraction 98 – 20 is done first:
  98 – 20 + 24 = 78 + 24 = 102.



When a calculation plan includes addition and subtraction only, the calculations are done from left to right.

Example: The calculation plan 30 – 6 + 8 – 5 + 7 indicates that you have to do the following:

30 – 6 = 24 24 + 8 = 32

32 – 5 = 27 27 + 7 = 34

If you don't do the calculations from left to right, you will get a different answer. For example, if you first calculate 6 + 8 = 14 and 5 + 7 = 12 and then 30 – 14 = 16 and then 16 – 12, the answer is 4. That is why you should always follow the above rule, unless you replace the plan with an equivalent plan.

Note that there is an equivalent plan that will produce the same result if it is performed *from left to right*, namely 30 + 8 + 7 – 6 – 5:
30 + 8 = 38
38 + 7 = 45
45 – 6 = 39
39 – 5 = 34

Suppose you want to state that 8 should be added to 6, and 7 to 5, and the two answers subtracted from 30. The use of brackets makes it possible to write such instructions.

Brackets are used to indicate that certain calculations should be done before others.

Example: The calculation plan 30 – (6 + 8) – (5 + 7) indicates that the following should be done:

6 + 8 = 14 5 + 7 = 12 30 – 14 = 16 16 – 12 = 4

When a calculation plan includes multiplication, the multiplication is done first and the remaining calculations are done from left to right.

Example: The calculation plan 7 × 20 – 6 + 4 × 2 indicates that the following should be done:

7 × 20 = 140 4 × 2 = 8 140 – 6 = 134 134 + 8 = 142

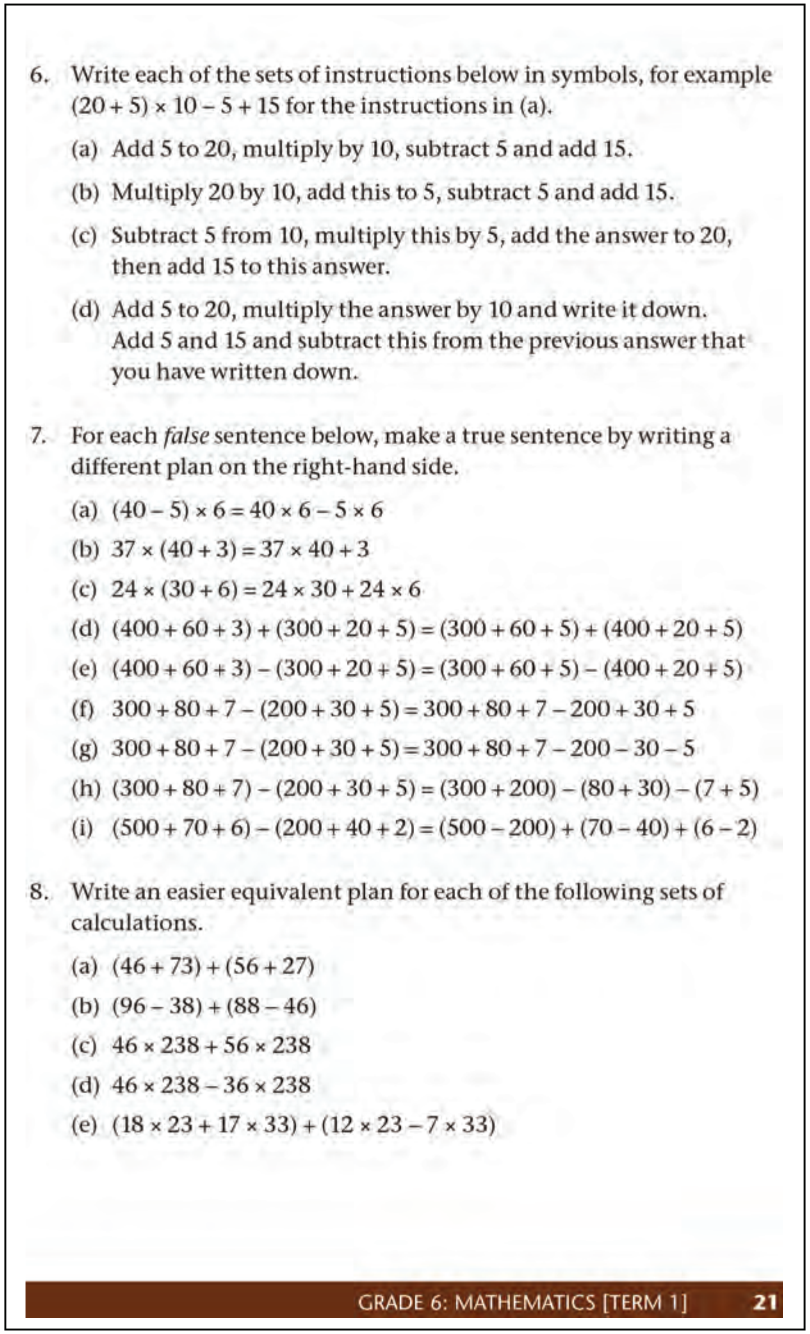
20 UNIT 2: NUMBER SENTENCES

Notes on questions

These questions provide practice in applying the conventions mentioned on the previous page, when writing and reading calculation plans.

Answers

6. (a) $(20 + 5) \times 10 - 5 + 15$ (b) $5 + (20 \times 10) - 5 + 15$
   (c) $20 + (10 - 5) \times 5 + 15$ (d) $(20 + 5) \times 10 - (5 + 15)$
7. (a) True
   (b) False; $37 \times (40 + 3) = 37 \times 40 + 37 \times 3$
   (c) True
   (d) False; $(400 + 60 + 3) + (300 + 20 + 5) = (300 + 60 + 5) + (400 + 20 + 3)$
   (e) False; $(400 + 60 + 3) - (300 + 20 + 5) = (400 - 300) + (50 - 20) + (13 - 5)$
   (f) False; $300 + 80 + 7 - (200 + 30 + 5) = 300 + 80 + 7 - 200 - 30 - 5$
   (g) True
   (h) False; $(300 + 80 + 7) - (200 + 30 + 5) = (300 - 200) + (80 - 30) + (7 - 5)$
   (i) True
8. The following are possible plans:
   (a) $(73 + 27) + (40 + 50) + (6 + 6)$
   (b) $(96 - 46) + (88 - 38)$
   (c) $(46 + 56) \times 238$ or $(100 + 2) \times 238$ or $(100 \times 238) + (2 \times 238)$
   (d) $(46 - 36) \times 238$
   (e) $(30 \times 23) + (10 \times 33)$



6. Write each of the sets of instructions below in symbols, for example $(20 + 5) \times 10 - 5 + 15$ for the instructions in (a).
   (a) Add 5 to 20, multiply by 10, subtract 5 and add 15.
   (b) Multiply 20 by 10, add this to 5, subtract 5 and add 15.
   (c) Subtract 5 from 10, multiply this by 5, add the answer to 20, then add 15 to this answer.
   (d) Add 5 to 20, multiply the answer by 10 and write it down. Add 5 and 15 and subtract this from the previous answer that you have written down.
7. For each *false* sentence below, make a true sentence by writing a different plan on the right-hand side.
   (a) $(40 - 5) \times 6 = 40 \times 6 - 5 \times 6$
   (b) $37 \times (40 + 3) = 37 \times 40 + 3$
   (c) $24 \times (30 + 6) = 24 \times 30 + 24 \times 6$
   (d) $(400 + 60 + 3) + (300 + 20 + 5) = (300 + 60 + 5) + (400 + 20 + 5)$
   (e) $(400 + 60 + 3) - (300 + 20 + 5) = (300 + 60 + 5) - (400 + 20 + 5)$
   (f) $300 + 80 + 7 - (200 + 30 + 5) = 300 + 80 + 7 - 200 + 30 + 5$
   (g) $300 + 80 + 7 - (200 + 30 + 5) = 300 + 80 + 7 - 200 - 30 - 5$
   (h) $(300 + 80 + 7) - (200 + 30 + 5) = (300 + 200) - (80 + 30) - (7 + 5)$
   (i) $(500 + 70 + 6) - (200 + 40 + 2) = (500 - 200) + (70 - 40) + (6 - 2)$
8. Write an easier equivalent plan for each of the following sets of calculations.
   (a) $(46 + 73) + (56 + 27)$
   (b) $(96 - 38) + (88 - 46)$
   (c) $46 \times 238 + 56 \times 238$
   (d) $46 \times 238 - 36 \times 238$
   (e) $(18 \times 23 + 17 \times 33) + (12 \times 23 - 7 \times 33)$

Notes on questions

Question 9 is of a special nature. No specific numbers are mentioned. The question is designed to promote awareness of the **distributive property** of multiplication and addition, and how it can be represented in a general way, i.e. applying to all numbers.

To investigate whether each open number sentence is true or false, a learner should choose specific numbers that may be hidden behind the stickers. (Choosing numbers for placeholders in calculation plans is called substitution.) For example, for 9(a) a learner may decide to hide the number 2 behind the red stickers, the number 3 behind the blue stickers and the number 4 behind the green stickers. When the two calculation plans are executed, different answers are obtained (see below). This demonstrates that sentence (a) is false.

You may decide to hold questions 9 and 10 back until learners have completed Section 2.3, which will provide them with some experience of substitution.

Answers

9. (a) and (c) are false. Possible examples:

   (a) 2 × (3 + 4) = 2 × 7 = 14 and 2 × 3 + 4 = 6 + 4 = 10

   (c) (2 + 2) × (3 + 4) = 4 × 7 = 28 and 2 × 3 + 2 × 4 = 6 + 8 = 14

10. (b) and (d) are true, (a) and (c) are false.

2.2 Writing number sentences

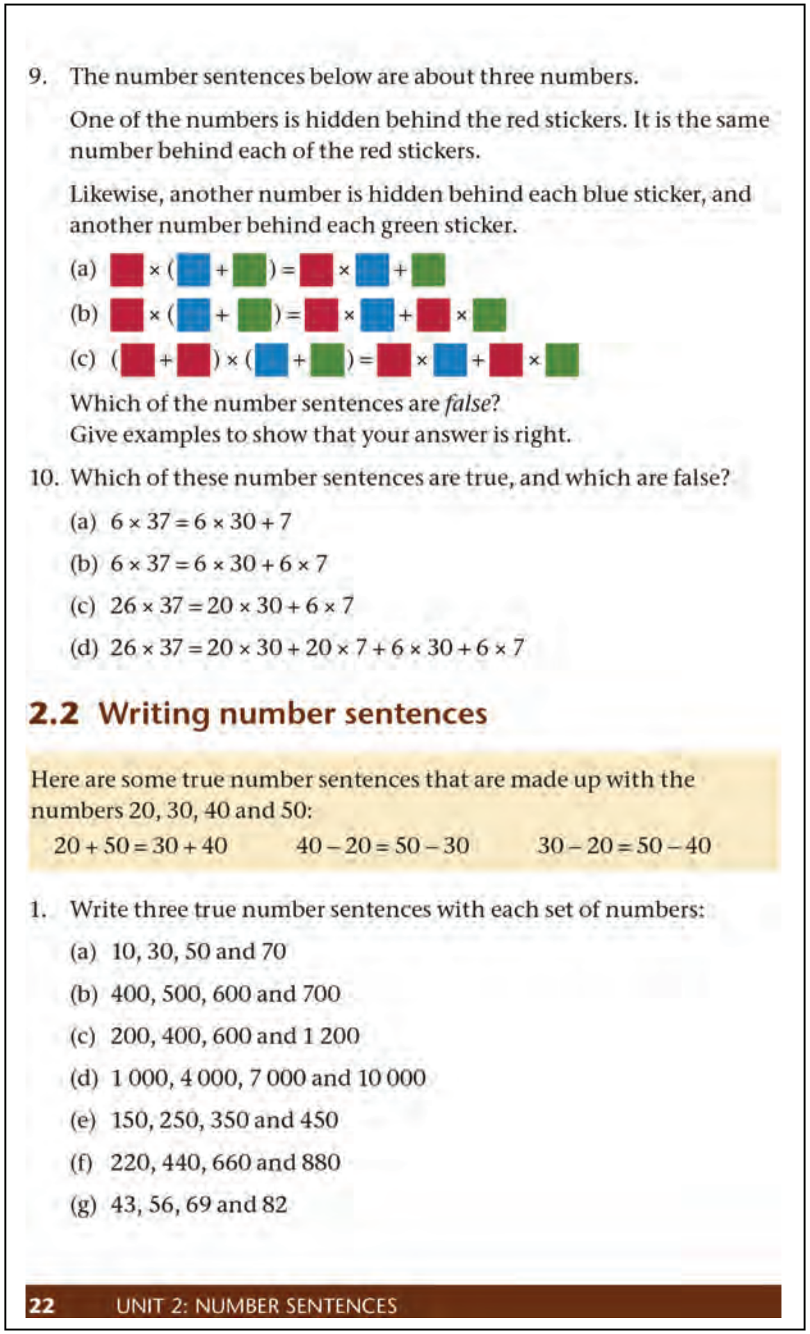
Notes on questions

Do some examples similar to those in the shaded passage on the board, to help learners to understand what is required in question 1. Learners may enjoy doing question 1 because it is somewhat like a puzzle. It requires recall of number facts and calculations with small numbers, because learners have to try out different combinations until they find combinations with which they can build a true number sentence, for example 70 + 10 and 50 + 30 for question 1(a).

Answers

1. Examples:

(a)	50 – 10 = 70 – 30	10 + 70 = 30 + 50	70 – 50 = 30 – 10
(b)	400 + 700 = 500 + 600	600 – 400 = 700 – 500	500 – 400 = 700 – 600
(c)	1 200 – 600 = 200 + 400	200 + 600 = 1 200 – 400	400 + 600 = 1 200 – 200
(d)	7 000 + 4 000 = 10 000 + 1 000	10 000 – 4 000 = 7 000 – 1 000	
	10 000 – 7 000 = 4 000 – 1 000		
(e)	450 – 250 = 350 – 150	450 + 150 = 350 + 250	450 – 350 = 250 – 150
(f)	880 + 220 = 440 + 660	880 – 440 = 660 – 220	880 – 660 = 440 – 220
(g)	43 + 82 = 56 + 69	82 – 56 = 69 – 43	82 – 69 = 56 – 43



9. The number sentences below are about three numbers.

   One of the numbers is hidden behind the red stickers. It is the same number behind each of the red stickers.

   Likewise, another number is hidden behind each blue sticker, and another number behind each green sticker.

   (a) 🟥 × (🟦 + 🟩) = 🟥 × 🟦 + 🟩

   (b) 🟥 × (🟦 + 🟩) = 🟥 × 🟦 + 🟥 × 🟩

   (c) (🟥 + 🟥) × (🟦 + 🟩) = 🟥 × 🟦 + 🟥 × 🟩

   Which of the number sentences are *false*?
   Give examples to show that your answer is right.

10. Which of these number sentences are true, and which are false?

    (a) 6 × 37 = 6 × 30 + 7

    (b) 6 × 37 = 6 × 30 + 6 × 7

    (c) 26 × 37 = 20 × 30 + 6 × 7

    (d) 26 × 37 = 20 × 30 + 20 × 7 + 6 × 30 + 6 × 7

2.2 Writing number sentences

Here are some true number sentences that are made up with the numbers 20, 30, 40 and 50:

20 + 50 = 30 + 40 40 – 20 = 50 – 30 30 – 20 = 50 – 40

1. Write three true number sentences with each set of numbers:

   (a) 10, 30, 50 and 70

   (b) 400, 500, 600 and 700

   (c) 200, 400, 600 and 1 200

   (d) 1 000, 4 000, 7 000 and 10 000

   (e) 150, 250, 350 and 450

   (f) 220, 440, 660 and 880

   (g) 43, 56, 69 and 82

22 UNIT 2: NUMBER SENTENCES

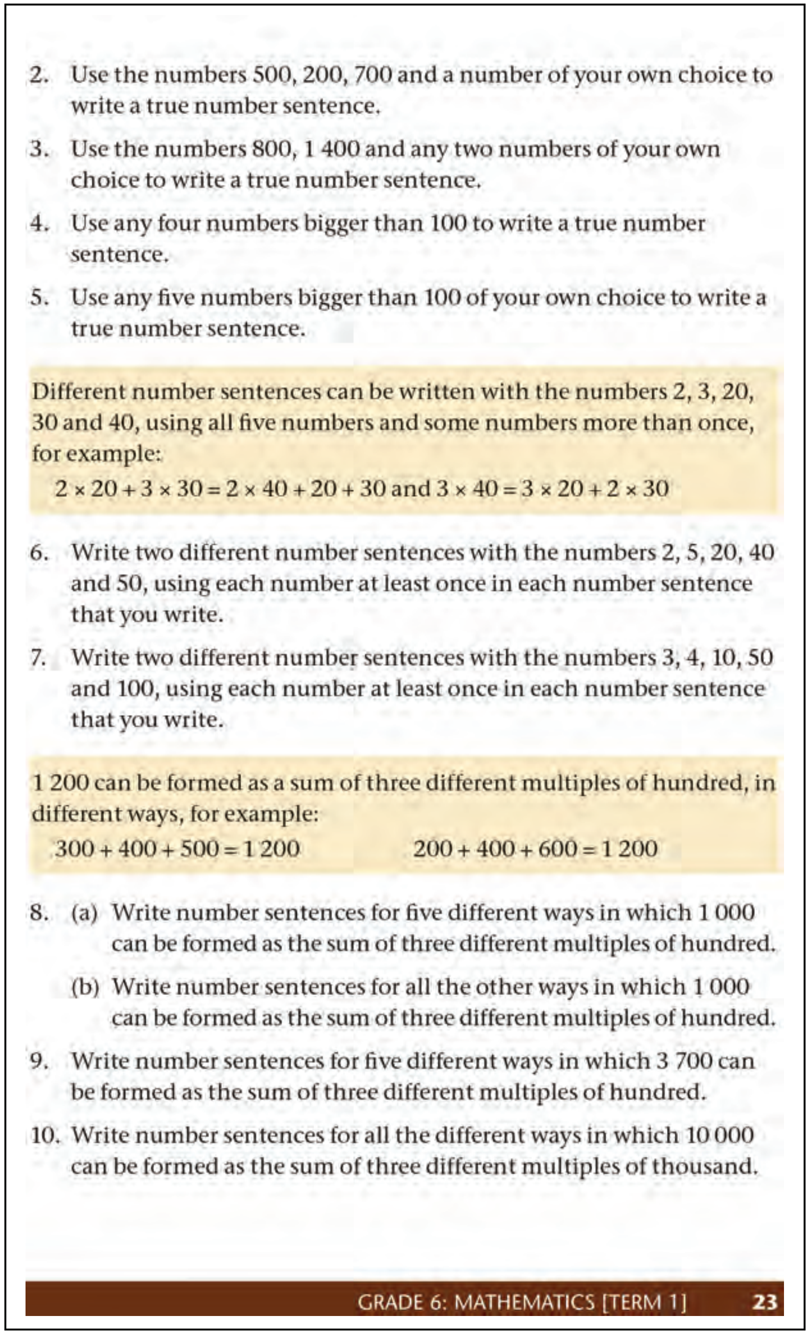
Teaching guidelines

Questions 2 to 5 are quite easy, because learners can choose one of the numbers. Questions 6 and 7 are much more demanding than questions 2 to 5. Tell learners that they may only use the given numbers in questions 6 and 7, though they may use some of the numbers more than once. You may use the examples in the shaded passage for this purpose.

The purpose of all the questions in this section is to provide extensive practice in Mental Mathematics: recalling number facts and doing computations with small numbers.

Answers

2. Learners' own answers, e.g. 700 – 500 = 200
3. Learners' own answers, e.g. 850 + 1 400 = 3 050 – 800
4. Learners' own answers, e.g. 6 000 – 150 = 7 000 – 1 150
5. Learners' own answers, e.g. 200 + 300 + 400 = 1 000 – 100
6. Example:
   50 × 2 + 2 × 40 = 5 × 40 – 20 (= 180)
   5 × 40 – 2 × 20 = 2 × 5 + 50 + 5 × 20 (= 160)
7. Example:
   4 × 10 + 4 × 50 = 3 × 50 + (100 – 10) (= 240)
   (10 – 4) × 3 + 50 + 100 = 50 – 3 × 4 + 3 × 10 + 100 (= 168)
8. (a) and (b):
   700 + 200 + 100
   400 + 100 + 500
   600 + 100 + 300
   300 + 500 + 200
   100 + 600 + 300
   100 + 200 + 700
   200 + 400 + 400
   600 + 300 + 100
   300 + 500 + 200
   500 + 400 + 100
   100 + 700 + 200
   100 + 100 + 800
   200 + 300 + 500
   100 + 400 + 500
   500 + 200 + 300
   500 + 300 + 200
   700 + 100 + 200
   200 + 500 + 300
   200 + 200 + 600
   100 + 300 + 600
   200 + 500 + 300
9. Examples:
   1 800 + 1 700 + 200
   1 400 + 1 100 + 1 200
   1 900 + 1 100 + 700
   1 600 + 1 300 + 800
   1 300 + 1 500 + 900
10. 7 000 + 2 000 + 1 000
    5 000 + 2 000 + 3 000
    3 000 + 5 000 + 2 000
    6 000 + 3 000 + 1 000
    4 000 + 1 000 + 5 000



2. Use the numbers 500, 200, 700 and a number of your own choice to write a true number sentence.
3. Use the numbers 800, 1 400 and any two numbers of your own choice to write a true number sentence.
4. Use any four numbers bigger than 100 to write a true number sentence.
5. Use any five numbers bigger than 100 of your own choice to write a true number sentence.

Different number sentences can be written with the numbers 2, 3, 20, 30 and 40, using all five numbers and some numbers more than once, for example:

2 × 20 + 3 × 30 = 2 × 40 + 20 + 30 and 3 × 40 = 3 × 20 + 2 × 30

6. Write two different number sentences with the numbers 2, 5, 20, 40 and 50, using each number at least once in each number sentence that you write.
7. Write two different number sentences with the numbers 3, 4, 10, 50 and 100, using each number at least once in each number sentence that you write.

1 200 can be formed as a sum of three different multiples of hundred, in different ways, for example:

300 + 400 + 500 = 1 200   200 + 400 + 600 = 1 200

8. (a) Write number sentences for five different ways in which 1 000 can be formed as the sum of three different multiples of hundred.
   (b) Write number sentences for all the other ways in which 1 000 can be formed as the sum of three different multiples of hundred.
9. Write number sentences for five different ways in which 3 700 can be formed as the sum of three different multiples of hundred.
10. Write number sentences for all the different ways in which 10 000 can be formed as the sum of three different multiples of thousand.

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Answers

11. Examples:

    2 600 – 2 200 = 400
    3 800 – 3 400 = 400
    1 500 – 1 100 = 400
    1 900 – 1 500 = 400
    5 700 – 5 300 = 400
    and many more possibilities

12. Examples:

    7 000 – 4 000 = 3 000
    9 000 – 6 000 = 3 000
    4 000 – 1 000 = 3 000
    5 000 – 2 000 = 3 000
    8 000 – 5 000 = 3 000
    and many more possibilities

2.3 Solve and complete number sentences

Teaching guidelines

This section is about solving open number sentences by inspection, utilising knowledge of number bonds. Learners need to learn to choose a number even if they are not sure it will make the number sentence true, and then test it.

You may demonstrate this with an example such as 650 + ... = 1 100.

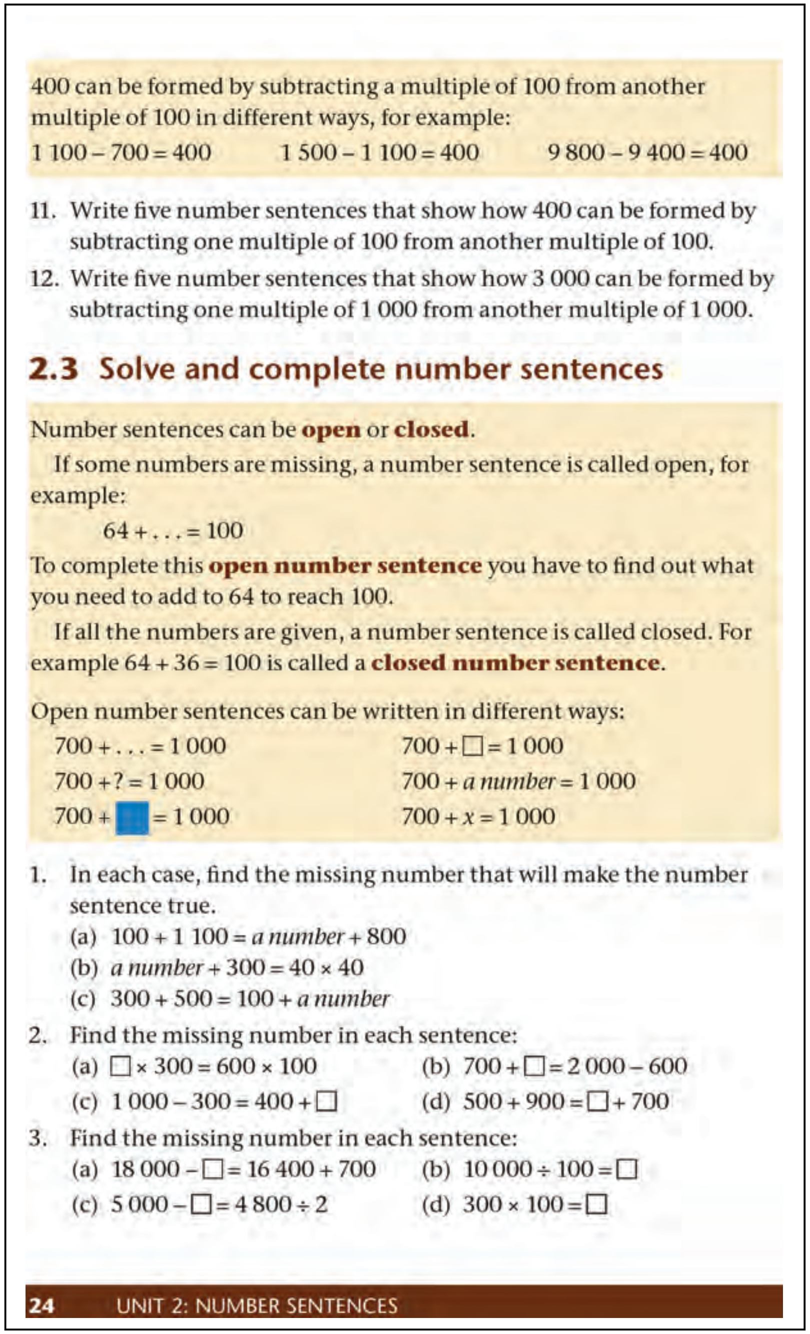
Write this open number sentence on the board, then say something like: "*I think 250 may make it true, so let me check.*" Then write 650 + 250 and ask learners to state what the answer is. It is 900. Write it on the board.

Ask learners to suggest another number. Add the number they suggest to 650 and write the answer on the board. Continue until 450 is suggested.

For example:
650 + ... = 1 100
650 + 250 = 900
650 + 550 = 1 200
⋮
650 + 450 = 1 100

Answers

1. (a) 400 (b) 1 300 (c) 700
2. (a) 200 (b) 700 (c) 300 (d) 700
3. (a) 900 (b) 100 (c) 2 600 (d) 30 000



400 can be formed by subtracting a multiple of 100 from another multiple of 100 in different ways, for example:
1 100 – 700 = 400 1 500 – 1 100 = 400 9 800 – 9 400 = 400

11. Write five number sentences that show how 400 can be formed by subtracting one multiple of 100 from another multiple of 100.
12. Write five number sentences that show how 3 000 can be formed by subtracting one multiple of 1 000 from another multiple of 1 000.

2.3 Solve and complete number sentences

Number sentences can be **open** or **closed**.

If some numbers are missing, a number sentence is called open, for example:

64 + . . . = 100

To complete this **open number sentence** you have to find out what you need to add to 64 to reach 100.

If all the numbers are given, a number sentence is called closed. For example 64 + 36 = 100 is called a **closed number sentence**.

Open number sentences can be written in different ways:
700 + . . . = 1 000
700 + ? = 1 000
700 + ■ = 1 000
700 + □ = 1 000
700 + *a number* = 1 000
700 + *x* = 1 000

1. In each case, find the missing number that will make the number sentence true.
   (a) 100 + 1 100 = *a number* + 800
   (b) *a number* + 300 = 40 × 40
   (c) 300 + 500 = 100 + *a number*
2. Find the missing number in each sentence:
   (a) □ × 300 = 600 × 100
   (b) 700 + □ = 2 000 – 600
   (c) 1 000 – 300 = 400 + □
   (d) 500 + 900 = □ + 700
3. Find the missing number in each sentence:
   (a) 18 000 – □ = 16 400 + 700
   (b) 10 000 ÷ 100 = □
   (c) 5 000 – □ = 4 800 ÷ 2
   (d) 300 × 100 = □

Grade 6 Term 1 Unit 3 Whole numbers: Addition and subtraction

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
3.1 Basic addition and subtraction facts and skills	The concepts of sum and difference; Mental Mathematics	25 to 28
3.2 Mental calculation techniques	Mental Mathematics	29 to 33
3.3 Subtraction and addition are inverses	Mental Mathematics	34 to 35
3.4 Rounding off and rearranging	Subskills for multi-digit addition and subtraction	36 to 38
3.5 Subtraction with place value parts	Subskills for multi-digit addition and subtraction	39 to 40
3.6 The vertical column notation for addition	Revision of breaking down and building up in columns	41 to 42
3.7 The vertical column notation for subtraction	Revision of breaking down and building up in columns	43 to 44
3.8 Practise addition and subtraction	Word problems	45
3.9 Using a calculator	Learning to use a calculator effectively	46 to 53
3.10 Apply your knowledge	Word problems using a calculator	53 to 54

CAPS time allocation	7 hours
CAPS page references	13 to 15 and 222 to 225

Mathematical background

Calculations with multi-digit numbers are done by breaking the task down into separate smaller tasks. For example, the single task 254 + 538 can be broken down into smaller tasks as follows:

Single task:	254 + 538		254	
Three separate tasks:	= (200 + 50 + 4) + (500 + 30 + 8)	(The numbers are broken down into their place value parts.)	$\underline{538}$	
Executing the three separate tasks:	= (200 + 500) + (50 + 30) + (4 + 8)	(The rearrangement can be done because addition is commutative and associative.)	700	(200 + 500)
	= 700 + 80 + 12		80	(50 + 30)
	= 700 + 90 + 2		$\underline{12}$	(4 + 8)
Building up the answer:	= 792		792	

The second and third columns above show two different ways in which exactly the same thinking (method) can be recorded in writing.

Learners can only use break-down, rearrange and build-up methods effectively if they are fluent in mental arithmetic, i.e. if they know the addition and subtraction bonds for units, and for multiples of ten and hundred well, or can quickly reconstruct these facts.

It is critical to understand that the vertical column notation is just a different way to set out the work for the same method of addition that learners have been using up to now. The mathematical thinking involved is exactly the same.

Resources

Calculators, place value cards

3.1 Basic addition and subtraction facts and skills

Mathematical notes

Addition and subtraction relate to different problem types (different kinds of situations in reality), including the following:

- **Summation**, for example adding the lengths of the three types of fencing described in the shaded passage.
- **Increase and decrease**, for example the removal of 580 m of the 1 572 m fence described in the shaded passage.
- **Comparison by difference**. With reference to the shaded passage, you may ask learners what the difference in length of the Type A and Type B fencing along the stretch of road is.

Awareness of the different problem types can strengthen learners' capacity to engage effectively with word problems. For example, learners who think of addition only as "making more" (increase) may not realise that addition is required in a situation that requires summation. Learners who only think of subtraction as "taking away" (decrease) may not realise that the difference between two quantities can be found by subtraction. Note that the text in the second part of the shaded passage deliberately moves from the *decrease* meaning of subtraction to the *difference* meaning.

Teaching guidelines

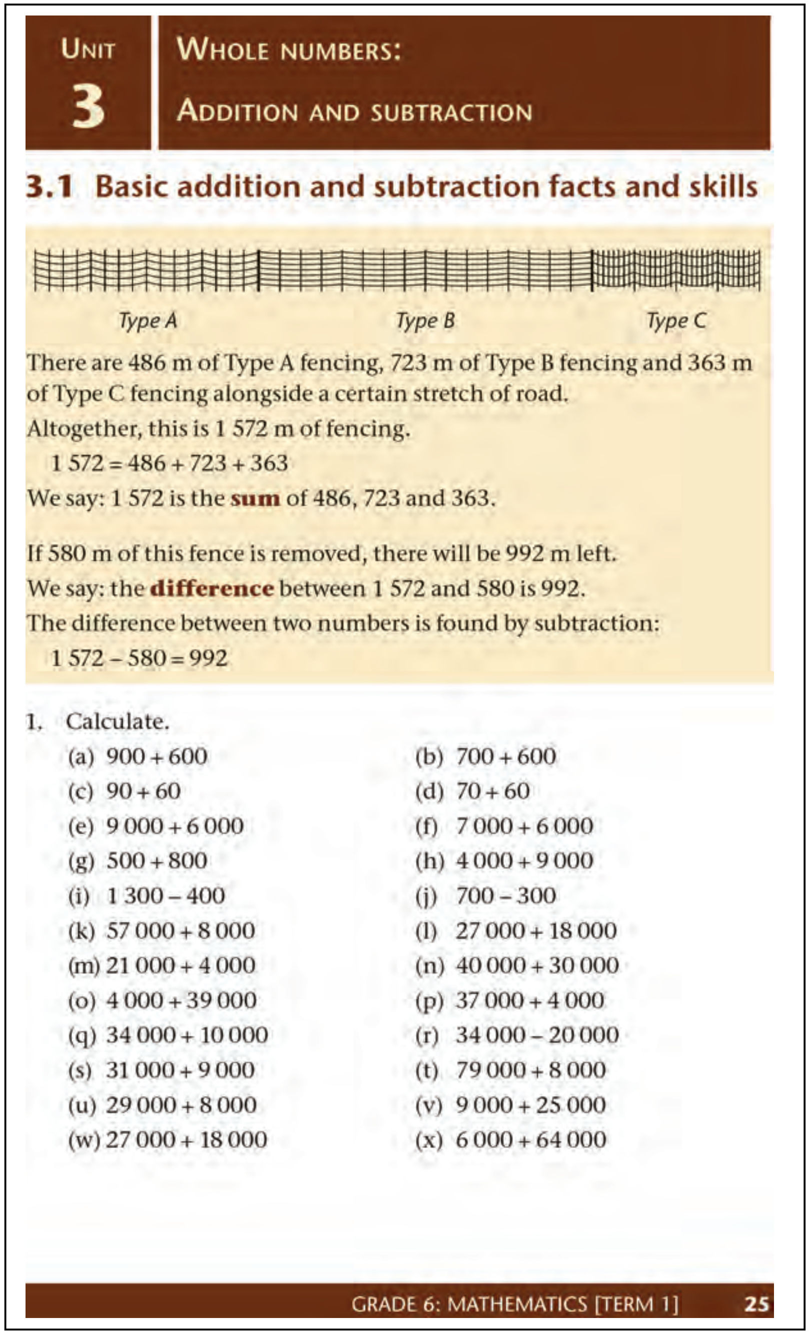
The shaded passage can be used as the basis for a teacher-led class discussion, to help learners to relate addition and subtraction to different problem types (different kinds of situations in reality). To help learners to engage with the context, you may make a rough copy of the diagram of the fence on the board.

Question 1 serves as a diagnostic activity, to provide you with an opportunity to get a sense of the level of learners' basic addition and subtraction knowledge and skills. It may be given as a **baseline assessment**, for learners to hand in. If learners do not perform well, this can be used as motivation for the work that follows.

Answers

1.

(a) 1 500	(b) 1 300	(c) 150	(d) 130
(e) 15 000	(f) 13 000	(g) 1 300	(h) 13 000
(i) 900	(j) 400	(k) 65 000	(l) 45 000
(m) 25 000	(n) 70 000	(o) 43 000	(p) 41 000
(q) 44 000	(r) 14 000	(s) 40 000	(t) 87 000
(u) 37 000	(v) 34 000	(w) 45 000	(x) 70 000



UNIT 3 WHOLE NUMBERS: ADDITION AND SUBTRACTION

3.1 Basic addition and subtraction facts and skills

There are 486 m of Type A fencing, 723 m of Type B fencing and 363 m of Type C fencing alongside a certain stretch of road.

Altogether, this is 1 572 m of fencing.

1 572 = 486 + 723 + 363

We say: 1 572 is the **sum** of 486, 723 and 363.

If 580 m of this fence is removed, there will be 992 m left.

We say: the **difference** between 1 572 and 580 is 992.

The difference between two numbers is found by subtraction:

1 572 − 580 = 992

1. Calculate.

(a) 900 + 600
(b) 700 + 600
(c) 90 + 60
(d) 70 + 60
(e) 9 000 + 6 000
(f) 7 000 + 6 000
(g) 500 + 800
(h) 4 000 + 9 000
(i) 1 300 − 400
(j) 700 − 300
(k) 57 000 + 8 000
(l) 27 000 + 18 000
(m) 21 000 + 4 000
(n) 40 000 + 30 000
(o) 4 000 + 39 000
(p) 37 000 + 4 000
(q) 34 000 + 10 000
(r) 34 000 − 20 000
(s) 31 000 + 9 000
(t) 79 000 + 8 000
(u) 29 000 + 8 000
(v) 9 000 + 25 000
(w) 27 000 + 18 000
(x) 6 000 + 64 000

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Notes on questions

Questions 4(b) and 5 are intended to strengthen understanding of the link between addition and reality. The questions serve to highlight addition as **summation**, which is a different meaning of addition than increasing (**adding on**). This is important because learners sometimes have a limited view of addition as only meaning to make something bigger by adding on.

Teaching guidelines

If learners have difficulty in "reading" the lines in questions 3 to 5, you may tell them that the blue and red coloured sections all consist of ten spaces each, i.e. units of ten.

Note that the acquisition of mental mathematics (knowledge of number facts and skills to produce number facts) and the development and strengthening of number concept go hand in hand: good number concept contributes to good mental mathematics, and *vice versa*.

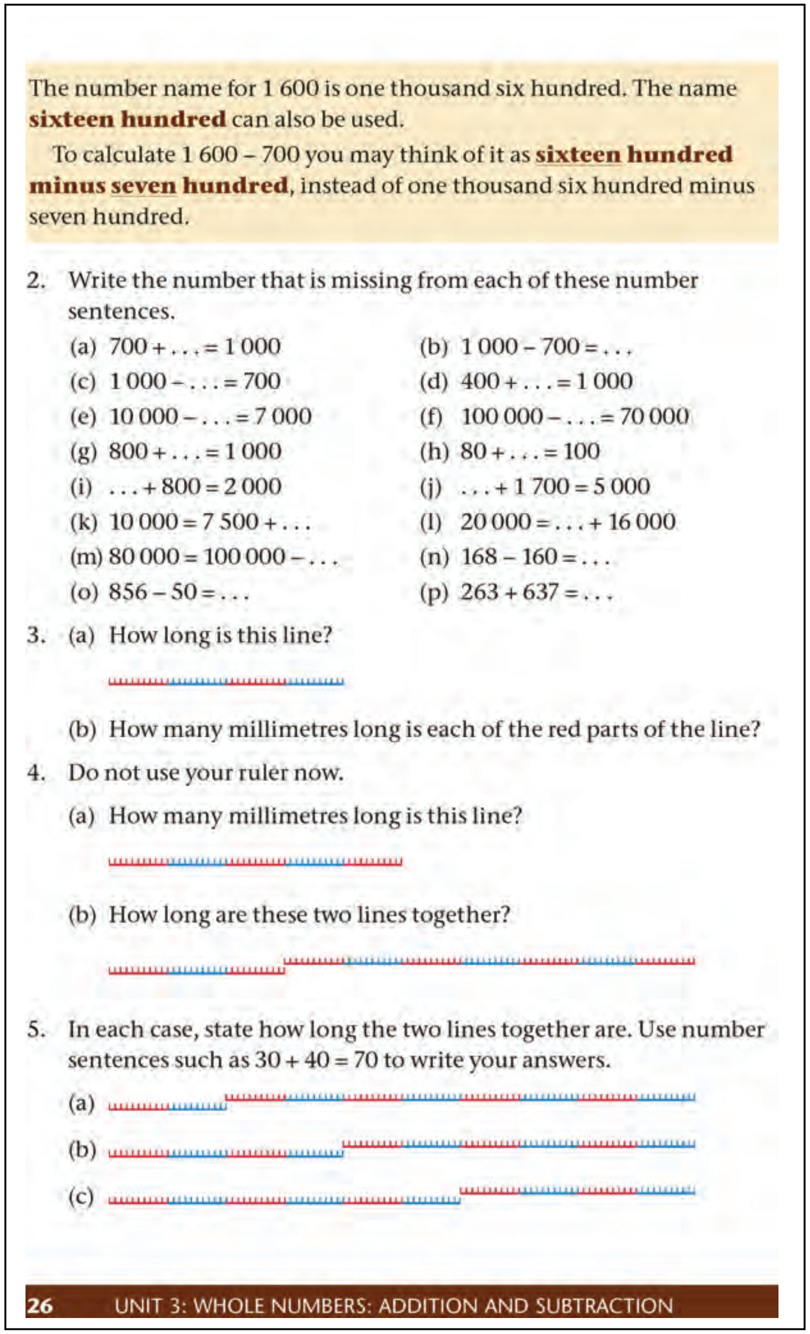
Answers

2. (a) 300 (b) 300 (c) 300 (d) 600
(e) 3 000 (f) 30 000 (g) 200 (h) 20
(i) 1 200 (j) 3 300 (k) 2 500 (l) 4 000
(m) 20 000 (n) 8 (o) 806 (p) 900

3. (a) 40 mm (b) 20 mm

4. (a) 50 mm (b) 30 mm + 70 mm = 100 mm

5. (a) 20 + 80 = 100
(b) 40 + 60 = 100
(c) 60 + 40 = 100



The number name for 1 600 is one thousand six hundred. The name **sixteen hundred** can also be used.

To calculate 1 600 – 700 you may think of it as **sixteen hundred minus seven hundred**, instead of one thousand six hundred minus seven hundred.

2. Write the number that is missing from each of these number sentences.
(a) 700 + . . . = 1 000
(b) 1 000 – 700 = . . .
(c) 1 000 – . . . = 700
(d) 400 + . . . = 1 000
(e) 10 000 – . . . = 7 000
(f) 100 000 – . . . = 70 000
(g) 800 + . . . = 1 000
(h) 80 + . . . = 100
(i) . . . + 800 = 2 000
(j) . . . + 1 700 = 5 000
(k) 10 000 = 7 500 + . . .
(l) 20 000 = . . . + 16 000
(m) 80 000 = 100 000 – . . .
(n) 168 – 160 = . . .
(o) 856 – 50 = . . .
(p) 263 + 637 = . . .

3. (a) How long is this line?
(b) How many millimetres long is each of the red parts of the line?

4. Do not use your ruler now.
(a) How many millimetres long is this line?
(b) How long are these two lines together?

5. In each case, state how long the two lines together are. Use number sentences such as 30 + 40 = 70 to write your answers.
(a)
(b)
(c)

26 UNIT 3: WHOLE NUMBERS: ADDITION AND SUBTRACTION

Teaching guidelines

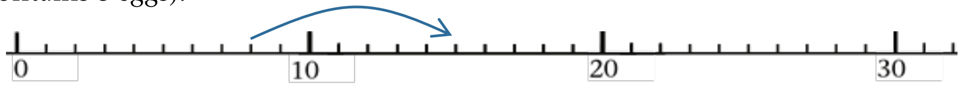
The short shaded passage serves as an example for question 6. You may alert learners to the commuted versions 8 000 + 1 000, 7 000 + 2 000, etc. and state that commutations need not be repeated when answering question 6.

Mathematical notes

The number line provides a way to visualise numbers and operations, and may hence help learners to keep track of their own thinking.

Note that the number line can be used in two different ways as a visualisation of addition, for example for 8 + 7:

Addition as increasing a given quantity (e.g. putting 7 eggs in a basket that already contains 8 eggs):



Addition as combining two quantities (e.g. calculating the total number of learners in a class with 8 girls and 7 boys):

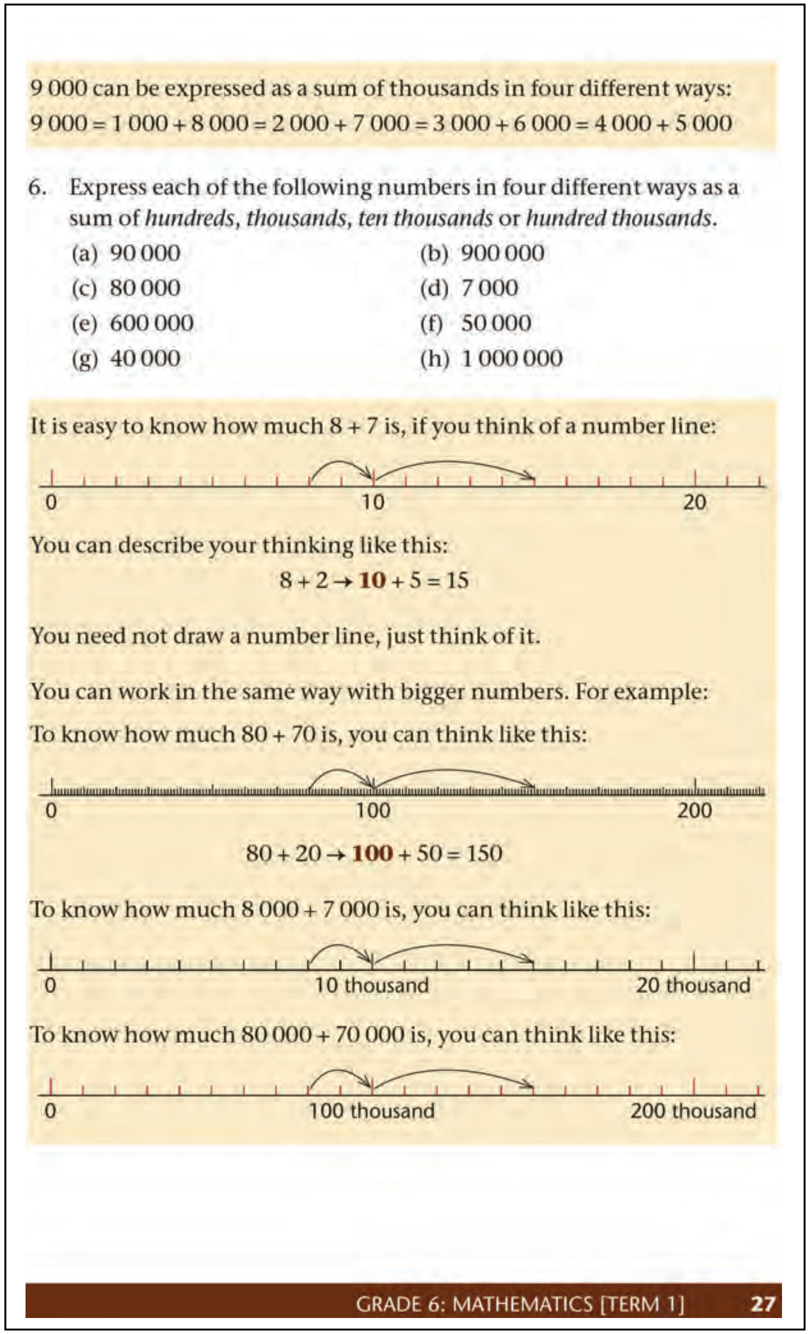
Possible misconceptions

When mental calculation techniques are explained by means of diagrams and written representations, as in the shaded passages on pages 27 and 29 or on the board, learners may form the false impression that they are required to apply these techniques in writing.

Impress on learners that they should try to apply these techniques without writing, in other words they should apply the techniques mentally.

Answers

6. There are various possibilities. Some examples are given below.
   (a) 90 000 = 30 000 + 40 000 + 9 000 + 6 000 + 4 000 + 700 + 300
   (b) 900 000 = 500 000 + 300 000 + 40 000 + 50 000 + 6 000 + 3 000 + 800 + 200
   (c) 80 000 = 50 000 + 20 000 + 9 000 + 700 + 300
   (d) 7 000 = 3 000 + 2 000 + 900 + 800 + 300
   (e) 600 000 = 400 000 + 80 000 + 70 000 + 40 000 + 9 000 + 500 + 500
   (f) 50 000 = 20 000 + 10 000 + 9 000 + 6 000 + 4 000 + 900 + 100
   (g) 40 000 = 20 000 + 10 000 + 9 000 + 600 + 300 + 100
   (h) 1 000 000 = 800 000 + 100 000 + 60 000 + 30 000 + 7 000 + 2 400 + 600



9 000 can be expressed as a sum of thousands in four different ways:
9 000 = 1 000 + 8 000 = 2 000 + 7 000 = 3 000 + 6 000 = 4 000 + 5 000

6. Express each of the following numbers in four different ways as a sum of *hundreds, thousands, ten thousands* or *hundred thousands*.
   (a) 90 000 (b) 900 000
   (c) 80 000 (d) 7 000
   (e) 600 000 (f) 50 000
   (g) 40 000 (h) 1 000 000

It is easy to know how much 8 + 7 is, if you think of a number line:

0 10 20

You can describe your thinking like this:

8 + 2 → **10** + 5 = 15

You need not draw a number line, just think of it.

You can work in the same way with bigger numbers. For example:

To know how much 80 + 70 is, you can think like this:

0 100 200

80 + 20 → **100** + 50 = 150

To know how much 8 000 + 7 000 is, you can think like this:

0 10 thousand 20 thousand

To know how much 80 000 + 70 000 is, you can think like this:

0 100 thousand 200 thousand

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Notes on questions

Questions 7 and 9 are designed for learners to take responsibility for their own learning. Learners are asked to identify which of the calculations they cannot yet do quickly, and then spend some time to do the calculations in questions 8 and 9.

Answers

7. (a) 300 (b) 700 (c) 1 500 (d) 15 000
(e) 150 (f) 150 000 (g) 12 000 (h) 24 000
(i) 16 000 (j) 160 000 (k) 140 000 (l) 60 000

8. Answers as for question 7 above.

9. (a) 1 100 (b) 110 000 (c) 1 300 000 (d) 17 000
(e) 54 000 (f) 600 000 (g) 120 000 (h) 120 000
(i) 220 000 (j) 14 000

Teaching guidelines with respect to word problems

A major reason why learners often underperform when they tackle word problems is that they do not try to understand the described situation before deciding what calculations to do and/or writing a number sentence.

Encourage learners to spend some time reading the description of the situation and trying to understand it before trying to get an answer. A rough sketch may often help learners to develop an understanding of the situation. Drawing a rough number line can often be very useful as a support for interpreting a problem situation.

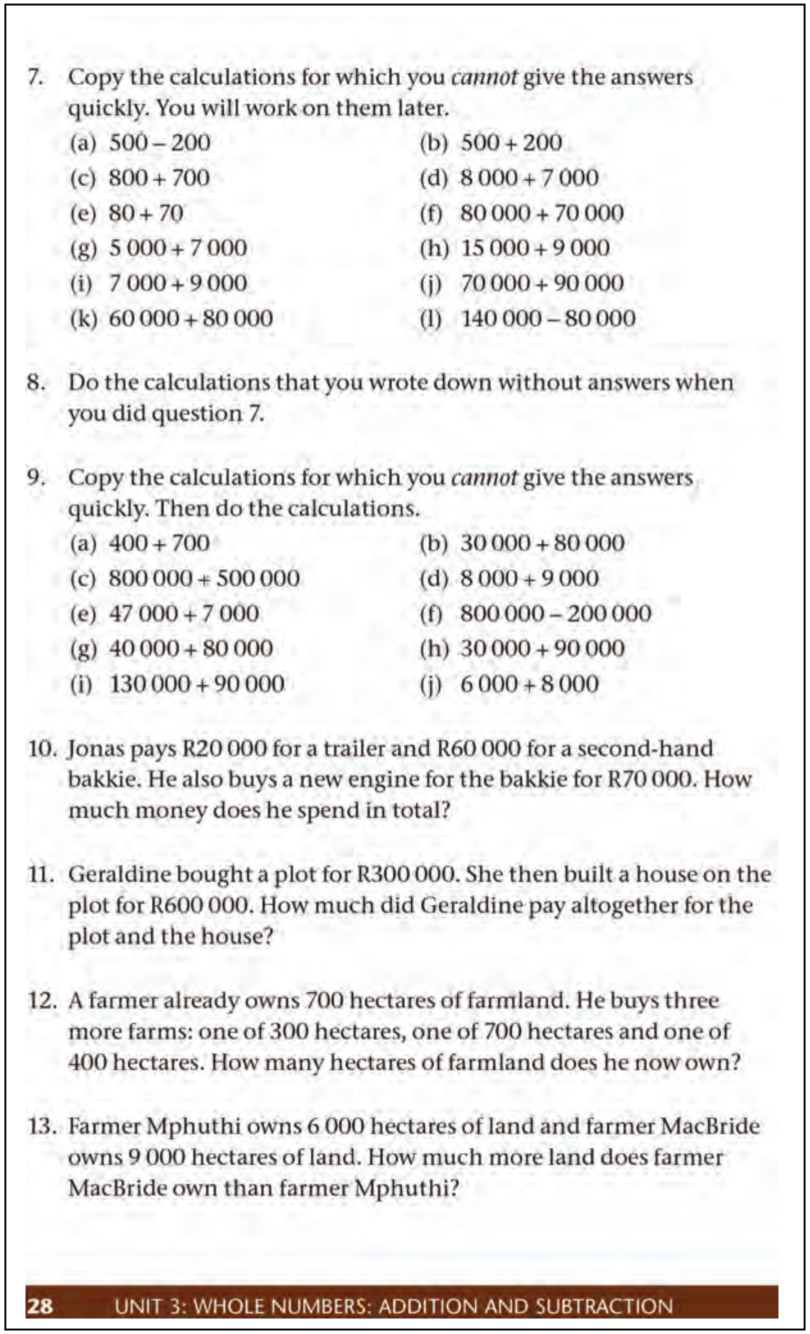
While this may take up much time at first, it is a good investment if learners develop the habit of engaging thoroughly with the given problem situation before trying to produce the answer.

The following are two other very useful habits that will increase learners' proficiency at solving word problems:

- the habit of estimating the solution before doing any calculations
- the habit of checking the answer, once the calculations have been completed, by checking whether it makes sense in the real situation.

Answers

10. R20 000 + R60 000 + R70 000 = R150 000
11. R300 000 + R600 000 = R900 000
12. 700 + 300 + 700 + 400 = 2 100 hectares
13. 9 000 – 6 000 = 3 000 hectares more



7. Copy the calculations for which you *cannot* give the answers quickly. You will work on them later.
(a) 500 – 200 (b) 500 + 200
(c) 800 + 700 (d) 8 000 + 7 000
(e) 80 + 70 (f) 80 000 + 70 000
(g) 5 000 + 7 000 (h) 15 000 + 9 000
(i) 7 000 + 9 000 (j) 70 000 + 90 000
(k) 60 000 + 80 000 (l) 140 000 – 80 000

8. Do the calculations that you wrote down without answers when you did question 7.

9. Copy the calculations for which you *cannot* give the answers quickly. Then do the calculations.
(a) 400 + 700 (b) 30 000 + 80 000
(c) 800 000 + 500 000 (d) 8 000 + 9 000
(e) 47 000 + 7 000 (f) 800 000 – 200 000
(g) 40 000 + 80 000 (h) 30 000 + 90 000
(i) 130 000 + 90 000 (j) 6 000 + 8 000

10. Jonas pays R20 000 for a trailer and R60 000 for a second-hand bakkie. He also buys a new engine for the bakkie for R70 000. How much money does he spend in total?

11. Geraldine bought a plot for R300 000. She then built a house on the plot for R600 000. How much did Geraldine pay altogether for the plot and the house?

12. A farmer already owns 700 hectares of farmland. He buys three more farms: one of 300 hectares, one of 700 hectares and one of 400 hectares. How many hectares of farmland does he now own?

13. Farmer Mphuthi owns 6 000 hectares of land and farmer MacBride owns 9 000 hectares of land. How much more land does farmer MacBride own than farmer Mphuthi?

28 UNIT 3: WHOLE NUMBERS: ADDITION AND SUBTRACTION

3.2 Mental calculation techniques

Critical knowledge and skills

It is critical that learners are able to reconstruct basic number facts they cannot recall, otherwise they will consistently resort to the time-consuming practice of drawing stripes and counting, which will inhibit their progress in Mathematics enormously.

To be able to form facts they cannot recall, learners need:

- ready knowledge of some addition, subtraction and multiplication facts
- knowledge and skills to quickly form new facts from known facts.

For example, a learner may have to calculate 344 + 557 but may not immediately know how much 40 + 50 is. If this learner knows that 40 + 40 = 80, he or she can find the value of 40 + 50 by adding 10 to 80.

Teaching guidelines

Ensure that learners understand *why* they need to be fluent in mental calculations, i.e. why they have to know some basic number facts by heart and be able to quickly and without writing produce the answers in cases where they do not recall the facts.

The reason is that if you cannot do the calculations with the place value parts quickly, a calculation with multi-digit numbers becomes a long and tedious process.

You may demonstrate this by doing a calculation on the board, for example 4 386 + 3 569:

4 000	300	80	6
+3 000	+500	+60	+9

If you do not know quickly that 6 + 9 = 15 and 80 + 60 = 140, it will take you very long to calculate 4 386 + 3 569.

Learners who do not accept the challenge to learn to do calculations reasonably fast cannot appreciate the need to be able to do mental calculations fluently.

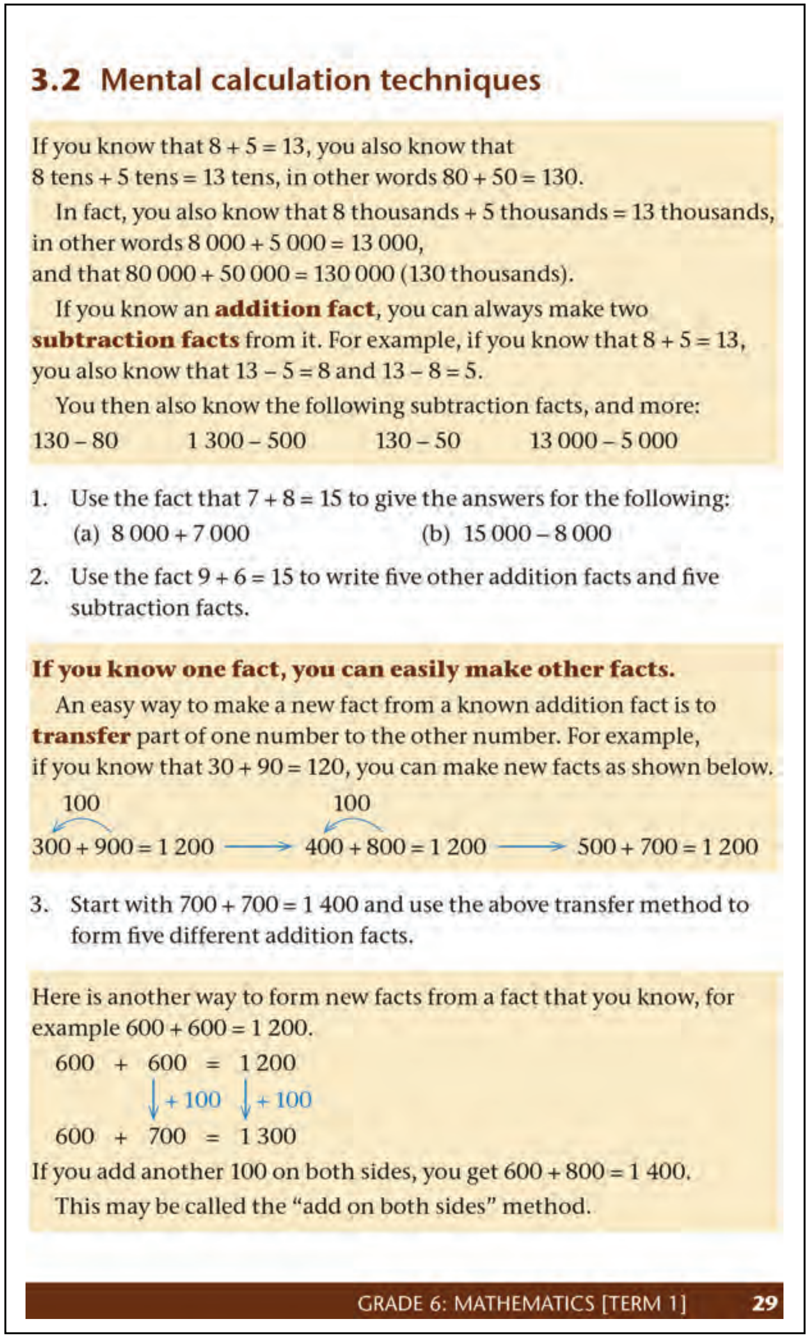
Answers

1. (a) 15 000 (b) 7 000

2. There are more possibilities than the examples below. Consider all learners' answers.

90 + 60 = 150	150 – 60 = 90
900 + 600 = 1 500	1 500 – 900 = 600
9 000 + 6 000 = 15 000	15 000 – 6 000 = 9 000
90 000 + 60 000 = 150 000	150 000 – 90 000 = 60 000
900 000 + 600 000 = 1 500 000	1 500 000 – 600 000 = 900 000

3. Examples:
   1 000 + 400 = 1 400
   800 + 600 = 1 400
   300 + 1 100 = 1 400
   900 + 500 = 1 400
   200 + 1 200 = 1 400



3.2 Mental calculation techniques

If you know that 8 + 5 = 13, you also know that 8 tens + 5 tens = 13 tens, in other words 80 + 50 = 130.

In fact, you also know that 8 thousands + 5 thousands = 13 thousands, in other words 8 000 + 5 000 = 13 000, and that 80 000 + 50 000 = 130 000 (130 thousands).

If you know an **addition fact**, you can always make two **subtraction facts** from it. For example, if you know that 8 + 5 = 13, you also know that 13 – 5 = 8 and 13 – 8 = 5.

You then also know the following subtraction facts, and more:

130 – 80   1 300 – 500   130 – 50   13 000 – 5 000

1. Use the fact that 7 + 8 = 15 to give the answers for the following:
   (a) 8 000 + 7 000 (b) 15 000 – 8 000
2. Use the fact 9 + 6 = 15 to write five other addition facts and five subtraction facts.

If you know one fact, you can easily make other facts.

An easy way to make a new fact from a known addition fact is to **transfer** part of one number to the other number. For example, if you know that 30 + 90 = 120, you can make new facts as shown below.

$$300 + \overset{100}{900} = 1\,200 \longrightarrow 400 + \overset{100}{800} = 1\,200 \longrightarrow 500 + 700 = 1\,200$$

3. Start with 700 + 700 = 1 400 and use the above transfer method to form five different addition facts.

Here is another way to form new facts from a fact that you know, for example 600 + 600 = 1 200.

$$\begin{array}{ccccc} 600 & + & 600 & = & 1\,200 \\ & & \downarrow +100 & & \downarrow +100 \\ 600 & + & 700 & = & 1\,300 \end{array}$$

If you add another 100 on both sides, you get 600 + 800 = 1 400.

This may be called the "add on both sides" method.

Teaching guidelines

Questions 4, 6, 7 and 8 provide practice in forming number facts by starting with doubles.

Encourage learners not to focus only on additions with an answer of 1 600 when they do question 8, but also sums other than 1 600 that can be formed by starting with 800 + 800 = 1 600, for example 800 + 900 = 1 700.

Before learners do question 8, it may be helpful to demonstrate how subtraction facts can be produced from known addition facts. For example, if you know that 800 + 900 = 1 700, you also know that 1 700 – 800 = 900 and 1 700 – 900 = 800.

Answers

4. Examples: 700 + 800 = 1 500 700 + 900 = 1 600
700 + 1 000 = 1 700 700 + 600 = 1 300 720 + 730 = 1 450

5. Examples:

200 + 300 = 500 500 – 300 = 200 500 – 200 = 300
2 000 + 3 000 = 5 000 5 000 – 3 000 = 2 000 5 000 – 2 000 = 3 000
200 + 500 = 700 700 – 500 = 200 700 – 200 = 500
30 + 40 = 70 70 – 40 = 30 70 – 30 = 40
300 + 400 = 700 700 – 400 = 300 700 – 300 = 400
400 + 900 = 1 300 1 300 – 900 = 400 1 300 – 400 = 900

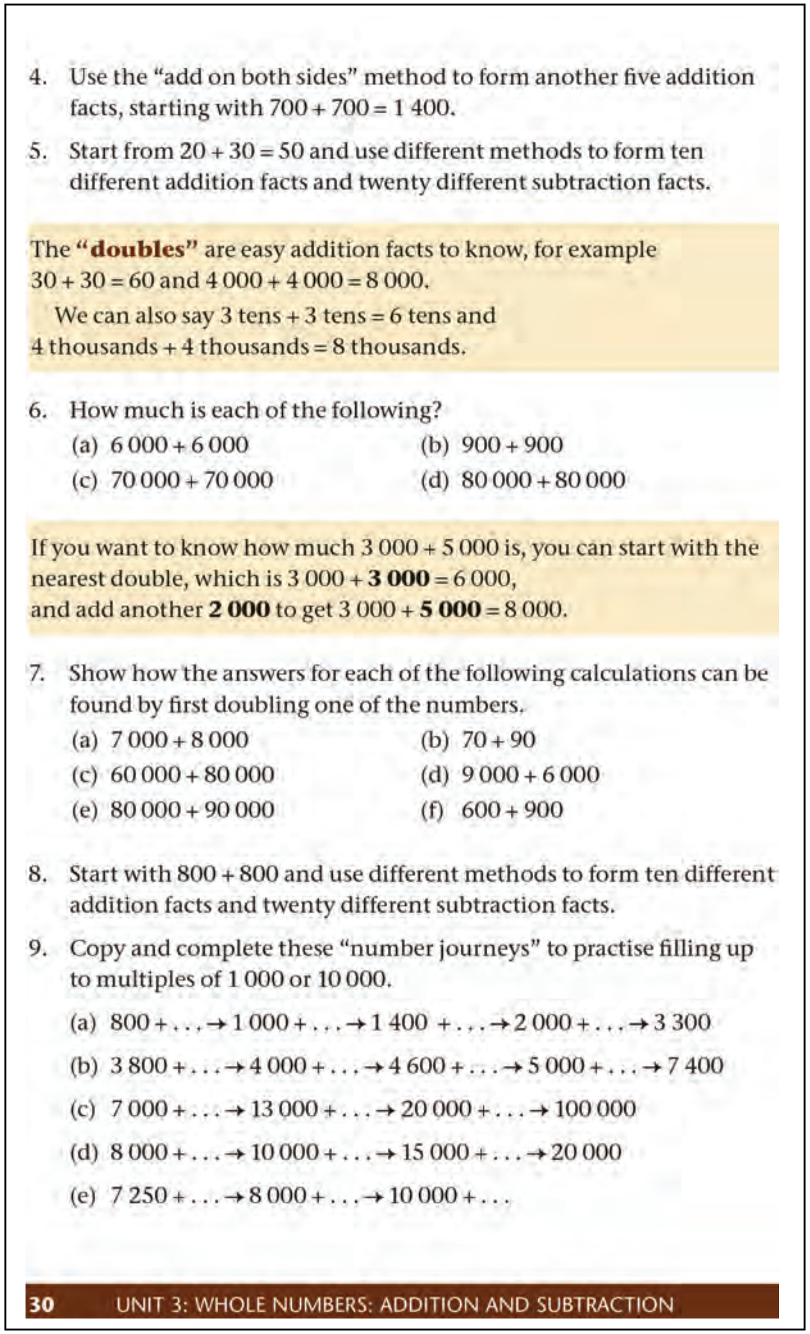
6. (a) 12 000 (b) 1 800 (c) 140 000 (d) 160 000

7. (a) 7 000 + 7 000 → 14 000 + 1 000 = 15 000
(b) 70 + 70 → 140 + 20 = 160
(c) 60 000 + 60 000 → 120 000 + 20 000 = 140 000
(d) 6 000 + 6 000 → 12 000 + 3 000 = 15 000 or 9 000 + 9 000 → 18 000 – 3 000 = 15 000
(e) 80 000 + 80 000 → 160 000 + 10 000 = 170 000
(f) 600 + 600 → 1 200 + 300 = 1 500

8. Examples:

800 + 900 = 1 700 1 700 – 800 = 900 1 700 – 900 = 800
700 + 800 = 1 500 1 500 – 800 = 700 1 500 – 700 = 800
800 + 1 000 = 1 800 1 800 – 800 = 1 000 1 800 – 1 000 = 800
800 + 700 = 1 500 1 500 – 700 = 800 1 500 – 800 = 700
800 + 600 = 1 400 1 400 – 600 = 800 1 400 – 800 = 600

9. (a) 800 + 200 → 1 000 + 400 → 1 400 + 600 → 2 000 + 1 300 → 3 300
(b) 3 800 + 200 → 4 000 + 600 → 4 600 + 400 → 5 000 + 2 400 → 7 400
(c) 7 000 + 6 000 → 13 000 + 7 000 → 20 000 + 80 000 → 100 000
(d) 8 000 + 2 000 → 10 000 + 5 000 → 15 000 + 5 000 → 20 000
(e) 7 250 + 750 → 8 000 + 2 000 → 10 000 + (e.g.) 3 500 → (e.g.) 13 500



4. Use the "add on both sides" method to form another five addition facts, starting with 700 + 700 = 1 400.

5. Start from 20 + 30 = 50 and use different methods to form ten different addition facts and twenty different subtraction facts.

The **"doubles"** are easy addition facts to know, for example 30 + 30 = 60 and 4 000 + 4 000 = 8 000.

We can also say 3 tens + 3 tens = 6 tens and 4 thousands + 4 thousands = 8 thousands.

6. How much is each of the following?
(a) 6 000 + 6 000 (b) 900 + 900
(c) 70 000 + 70 000 (d) 80 000 + 80 000

If you want to know how much 3 000 + 5 000 is, you can start with the nearest double, which is 3 000 + **3 000** = 6 000, and add another **2 000** to get 3 000 + **5 000** = 8 000.

7. Show how the answers for each of the following calculations can be found by first doubling one of the numbers.
(a) 7 000 + 8 000 (b) 70 + 90
(c) 60 000 + 80 000 (d) 9 000 + 6 000
(e) 80 000 + 90 000 (f) 600 + 900

8. Start with 800 + 800 and use different methods to form ten different addition facts and twenty different subtraction facts.

9. Copy and complete these "number journeys" to practise filling up to multiples of 1 000 or 10 000.
(a) 800 + . . . → 1 000 + . . . → 1 400 + . . . → 2 000 + . . . → 3 300
(b) 3 800 + . . . → 4 000 + . . . → 4 600 + . . . → 5 000 + . . . → 7 400
(c) 7 000 + . . . → 13 000 + . . . → 20 000 + . . . → 100 000
(d) 8 000 + . . . → 10 000 + . . . → 15 000 + . . . → 20 000
(e) 7 250 + . . . → 8 000 + . . . → 10 000 + . . .

30 UNIT 3: WHOLE NUMBERS: ADDITION AND SUBTRACTION

Notes on questions

Question 10 draws attention to **finding a difference** as one of the meanings of subtraction. This avoids limited understanding of subtraction as being only

- **taking away** (making smaller), for example: "I have 10 marbles and give 3 to my friend. How many marbles do I have left?" or
- **calculating a shortfall**, for example: "John wants to buy a toy that costs R50 but he only has R30. How much money does he need to buy the toy?"

The technique of "filling up" (see shaded passage) shows how to use a shortfall in subtraction.

Learners may be challenged by question 10. You may suggest that they make a drawing to show Jan and Tebogo's positions at 30 m and 45 m from the starting point.

Teaching guidelines

For the sake of speed it is important that learners learn to calculate with as little writing as possible, in other words that they learn:

- to do some calculations (e.g. calculations like those in questions 11 and 12) mentally, without writing
- to do calculations with multi-digit numbers with as little writing as possible, for example by using the vertical column formats for addition, subtraction and multiplication.

While they are still *learning* techniques for doing calculations mentally, for example to think of movements on the number line and filling up multiples of ten, hundred, thousand and higher powers of ten, learners may have to write and make drawings. **In these questions learners should draw number lines quickly, freehand without using a ruler.**

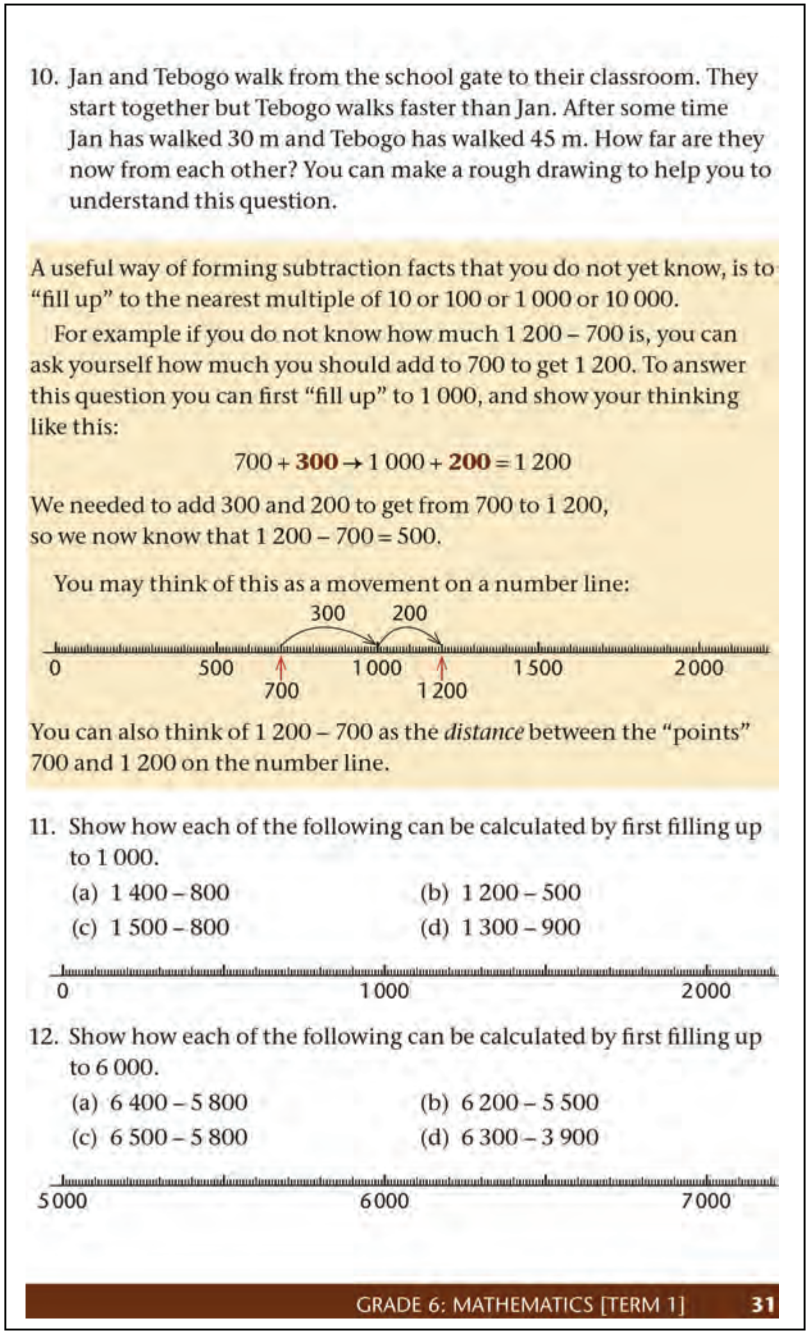
However, when learners have completed questions 11 and 12, you may give them some more similar calculations, for example 8 300 – 700 and 7 400 – 3 600, and challenge them to do the calculations with no or little writing. Learners may find this hard, but it is an excellent way of strengthening their mental capacities and number concept.

Answers

10. 15 m

11. (a) 800 + 200 → 1 000 + 400 = 1 400, so 1 400 – 800 → 200 + 400 = 600
    (b) 500 + 500 → 1 000 + 200 = 1 200, so 1 200 – 500 → 500 + 200 = 700
    (c) 800 + 200 → 1 000 + 500 = 1 500, so 1 500 – 800 → 200 + 500 = 700
    (d) 900 + 100 → 1 000 + 300 = 1 300, so 1 300 – 900 → 100 + 300 = 400

12. (a) 5 800 + 200 → 6 000 + 400 = 6 400, so 6 400 – 5 800 → 200 + 400 = 600
    (b) 5 500 + 500 → 6 000 + 200 = 6 200, so 6 200 – 5 500 → 500 + 200 = 700
    (c) 5 800 + 200 → 6 000 + 500 = 6 500, so 6 500 – 5 800 → 200 + 500 = 700
    (d) 3 900 + 2 100 → 6 000 + 300 = 6 300, so 6 300 – 3 900 → 2 100 + 300 = 2 400



10. Jan and Tebogo walk from the school gate to their classroom. They start together but Tebogo walks faster than Jan. After some time Jan has walked 30 m and Tebogo has walked 45 m. How far are they now from each other? You can make a rough drawing to help you to understand this question.

A useful way of forming subtraction facts that you do not yet know, is to "fill up" to the nearest multiple of 10 or 100 or 1 000 or 10 000.

For example if you do not know how much 1 200 – 700 is, you can ask yourself how much you should add to 700 to get 1 200. To answer this question you can first "fill up" to 1 000, and show your thinking like this:

700 + **300** → 1 000 + **200** = 1 200

We needed to add 300 and 200 to get from 700 to 1 200, so we now know that 1 200 – 700 = 500.

You may think of this as a movement on a number line:

You can also think of 1 200 – 700 as the *distance* between the "points" 700 and 1 200 on the number line.

11. Show how each of the following can be calculated by first filling up to 1 000.
    (a) 1 400 – 800
    (b) 1 200 – 500
    (c) 1 500 – 800
    (d) 1 300 – 900

12. Show how each of the following can be calculated by first filling up to 6 000.
    (a) 6 400 – 5 800
    (b) 6 200 – 5 500
    (c) 6 500 – 5 800
    (d) 6 300 – 3 900

Notes on questions

Ask learners to try to find the answers for questions 13 and 14 by thinking of movements on the number line, but without drawing the number line. They may then check their answers by drawing number lines.

Questions 15 and 16 provide for practice in mental calculations. The combination of questions 15 and 16 is designed to encourage learners to take responsibility for their own learning with respect to number facts. Question 15 helps them to identify some facts they do not know, and question 16 provides them with the opportunity to engage with the facts they do not yet know offhand.

The shaded passage provides motivation for questions 17 to 21 on the next page.

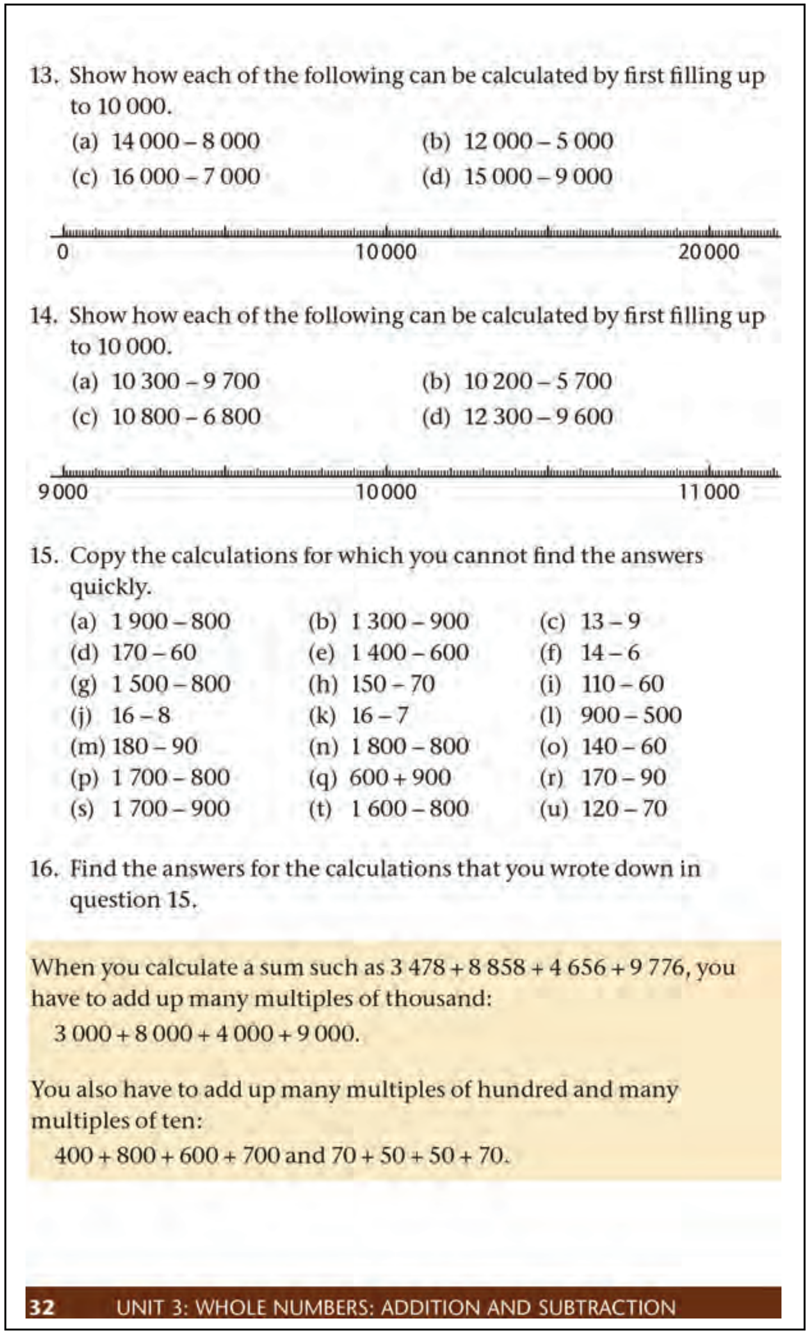
Answers

13. (a) 8 000 + 2 000 → 10 000 + 4 000 = 14 000, so 14 000 – 8 000 → 2 000 + 4 000 = 6 000
(b) 5 000 + 5 000 → 10 000 + 2 000 = 12 000, so 12 000 – 5 000 → 5 000 + 2 000 = 7 000
(c) 7 000 + 3 000 → 10 000 + 6 000 = 16 000, so 16 000 – 7 000 → 3 000 + 6 000 = 9 000
(d) 9 000 + 1 000 → 10 000 + 5 000 = 15 000, so 15 000 – 9 000 → 1 000 + 5 000 = 6 000

14. (a) 9 700 + 300 → 10 000 + 300 → 10 300, so 10 300 – 9 700 → 300 + 300 = 600
(b) 5 700 + 300 → 6 000 + 4 000 → 10 000 + 200 = 10 200, so 10 200 – 5 700 → 300 + 4 000 + 200 = 4 500
(c) 6 800 + 200 → 7 000 + 3 000 → 10 000 + 800 = 10 800, so 10 800 – 6 800 → 200 + 3 000 + 800 = 4 000
(d) 9 600 + 400 → 10 000 + 2 300 = 12 300, so 12 300 – 9 600 = 400 + 2 300 = 2 700

15. Learners copy the number sentences for which they cannot find the answers quickly.
(a) 1 900 – 800 = 1 100
(b) 1 300 – 900 = 400
(c) 13 – 9 = 4
(d) 170 – 60 = 110
(e) 1 400 – 600 = 800
(f) 14 – 6 = 8
(g) 1 500 – 800 = 700
(h) 150 – 70 = 80
(i) 110 – 60 = 50
(j) 16 – 8 = 8
(k) 16 – 7 = 9
(l) 900 – 500 = 400
(m) 180 – 90 = 90
(n) 1 800 – 800 = 1 000
(o) 140 – 60 = 80
(p) 1 700 – 800 = 900
(q) 600 + 900 = 1 500
(r) 170 – 90 = 80
(s) 1 700 – 900 = 800
(t) 1 600 – 800 = 800
(u) 120 – 70 = 50

16. See question 15 above.



13. Show how each of the following can be calculated by first filling up to 10 000.
(a) 14 000 – 8 000
(b) 12 000 – 5 000
(c) 16 000 – 7 000
(d) 15 000 – 9 000

14. Show how each of the following can be calculated by first filling up to 10 000.
(a) 10 300 – 9 700
(b) 10 200 – 5 700
(c) 10 800 – 6 800
(d) 12 300 – 9 600

15. Copy the calculations for which you cannot find the answers quickly.
(a) 1 900 – 800
(b) 1 300 – 900
(c) 13 – 9
(d) 170 – 60
(e) 1 400 – 600
(f) 14 – 6
(g) 1 500 – 800
(h) 150 – 70
(i) 110 – 60
(j) 16 – 8
(k) 16 – 7
(l) 900 – 500
(m) 180 – 90
(n) 1 800 – 800
(o) 140 – 60
(p) 1 700 – 800
(q) 600 + 900
(r) 170 – 90
(s) 1 700 – 900
(t) 1 600 – 800
(u) 120 – 70

16. Find the answers for the calculations that you wrote down in question 15.

When you calculate a sum such as 3 478 + 8 858 + 4 656 + 9 776, you have to add up many multiples of thousand:

3 000 + 8 000 + 4 000 + 9 000.

You also have to add up many multiples of hundred and many multiples of ten:

400 + 800 + 600 + 700 and 70 + 50 + 50 + 70.

Mathematical notes

The words **sum**, **difference**, **product** and **quotient** form a family, and all of them are used in two related but different meanings.

We may refer to “30 + 40 + 70” as a sum. When used in this way, *sum* indicates a **calculation plan** that requires addition only. This is the way in which *sum* is used in questions 19, 20 and 22.

We may also say “140 is the sum of 30, 40 and 70”. When used in this way, *sum* indicates the **answer** that is obtained when addition is performed. This is the way in which *sum* is used in questions 17 and 18.

Teaching guidelines

Encourage learners to try to do question 23 mentally, without writing. This will strengthen their number concept. For checking purposes learners may draw rough number lines (freehand, without using rulers).

Answers

17. (a) 560 (b) 3 700 (c) 45 000 (d) 440 000

18. (a) 450 000 (b) 450 000 (c) 450 000

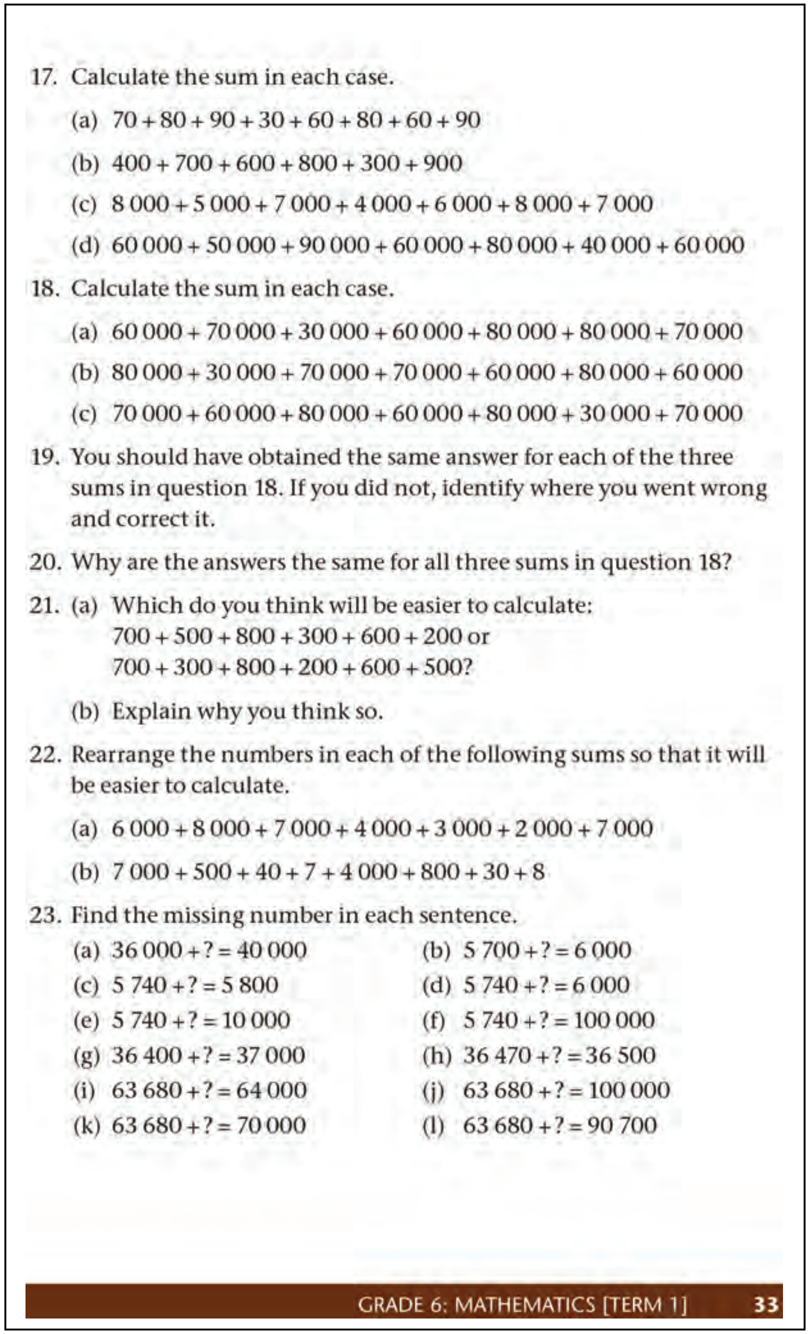
19. Learners check and correct their answers.

20. The numbers are the same – they were simply added in a different order.

21. (a) 700 + 300 + 800 + 200 + 600
    (b) The numbers next to each other add up to an easier multiple of ten (1 000 in this case).

22. (a) 6 000 + 4 000 + 8 000 + 2 000 + 7 000 + 3 000 + 7 000
    (b) 7 000 + 4 000 + 800 + 500 + 40 + 30 + 8 + 7

23. (a) 4 000 (b) 300 (c) 60 (d) 260
    (e) 4 260 (f) 94 260 (g) 600 (h) 30
    (i) 320 (j) 36 320 (k) 6 320 (l) 27 020



17. Calculate the sum in each case.
    (a) 70 + 80 + 90 + 30 + 60 + 80 + 60 + 90
    (b) 400 + 700 + 600 + 800 + 300 + 900
    (c) 8 000 + 5 000 + 7 000 + 4 000 + 6 000 + 8 000 + 7 000
    (d) 60 000 + 50 000 + 90 000 + 60 000 + 80 000 + 40 000 + 60 000

18. Calculate the sum in each case.
    (a) 60 000 + 70 000 + 30 000 + 60 000 + 80 000 + 80 000 + 70 000
    (b) 80 000 + 30 000 + 70 000 + 70 000 + 60 000 + 80 000 + 60 000
    (c) 70 000 + 60 000 + 80 000 + 60 000 + 80 000 + 30 000 + 70 000

19. You should have obtained the same answer for each of the three sums in question 18. If you did not, identify where you went wrong and correct it.

20. Why are the answers the same for all three sums in question 18?

21. (a) Which do you think will be easier to calculate:
    700 + 500 + 800 + 300 + 600 + 200 or
    700 + 300 + 800 + 200 + 600 + 500?
    (b) Explain why you think so.

22. Rearrange the numbers in each of the following sums so that it will be easier to calculate.
    (a) 6 000 + 8 000 + 7 000 + 4 000 + 3 000 + 2 000 + 7 000
    (b) 7 000 + 500 + 40 + 7 + 4 000 + 800 + 30 + 8

23. Find the missing number in each sentence.
    (a) 36 000 + ? = 40 000 (b) 5 700 + ? = 6 000
    (c) 5 740 + ? = 5 800 (d) 5 740 + ? = 6 000
    (e) 5 740 + ? = 10 000 (f) 5 740 + ? = 100 000
    (g) 36 400 + ? = 37 000 (h) 36 470 + ? = 36 500
    (i) 63 680 + ? = 64 000 (j) 63 680 + ? = 100 000
    (k) 63 680 + ? = 70 000 (l) 63 680 + ? = 90 700

GRADE 6: MATHEMATICS [TERM 1] 33

3.3 Subtraction and addition are inverses

Teaching guidelines

Do questions 1(a) and 2(a) on the board:

250 + 450 = 700 means that 700 – 450 = 250 and 700 – 250 = 450

3 678 – 600 = 3 078 means that 3 078 + 600 = 3 678 and 3 678 – 3 078 = 600

Ask learners to do at least two of the other sub-questions of questions 1 and 2.

Once learners have completed question 4, demonstrate how the difference between 7 000 and 16 000 can be shown on the number line, as in the second shaded passage. Note that learners do not have to draw number lines in question 5.

Answers

1. (a) 700 – 250 = 450 700 – 450 = 250
   (b) 1 000 – 367 = 633 1 000 – 633 = 367
   (c) 5 000 – 2 480 = 2 520 5 000 – 2 520 = 2 480
   (d) 92 291 – 64 753 = 27 538 92 291 – 27 538 = 64 753
2. (a) 600 + 3 078 = 3 678 (b) 3 608 + 70 = 3 678
   (c) 3 000 + 678 = 3 678 (d) 3 070 + 608 = 3 678
3. (a) 3 678 – 3 078 = 600 (b) 3 678 – 3 608 = 70
   (c) 3 678 – 678 = 3 000 (d) 3 678 – 3 070 = 608
4. (a) 8 302 (b) 382 (c) 8 002 (d) 380
5. (a) 5 000 (b) 50 000 (c) 35 000 (d) 8 000
   (e) 19 000 (f) 105 000 (g) 9 000 (h) 59 000

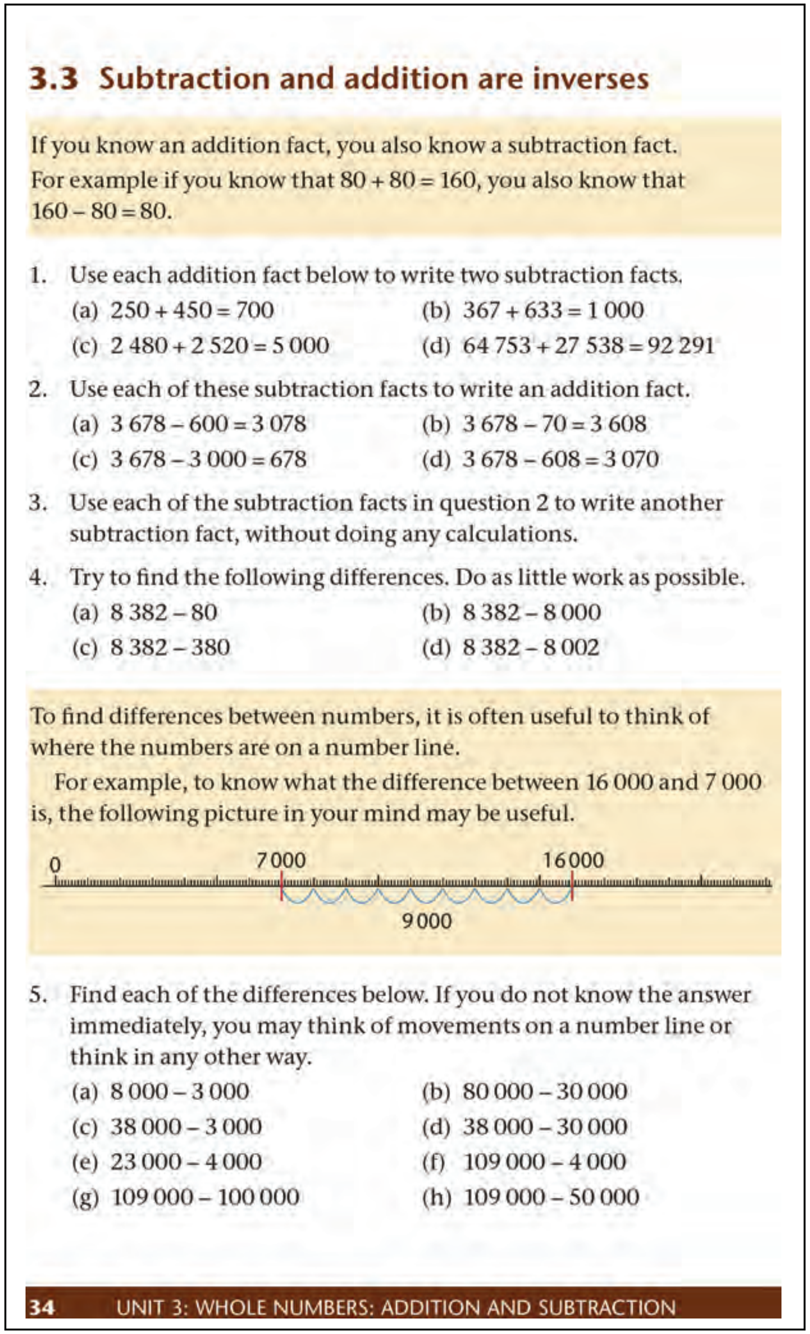
Mathematical notes

Brackets are used in calculation plans to specify that certain calculations should be performed before others, irrespective of all other conventions (multiply and divide before add and subtract, working from left to right – see page 20 of the Learner Book). For example, when executing the calculation plan 5 000 – (3 000 – 500 – 20), the following calculations should be performed in the order stated here:

3 000 – 500 = 2 500 2 500 – 20 = 2 480 5 000 – 2 480 = 2 520

However, instead of actually performing the calculation plan 5 000 – (3 000 – 500 – 20), you may replace it with the equivalent plan 5 000 – 3 000 + 500 + 20, which is executed as follows:

5 000 – 3 000 = 2 000 2 000 + 500 = 2 500 2 500 + 20 = 2 520



3.3 Subtraction and addition are inverses

If you know an addition fact, you also know a subtraction fact. For example if you know that 80 + 80 = 160, you also know that 160 – 80 = 80.

1. Use each addition fact below to write two subtraction facts.
   (a) 250 + 450 = 700 (b) 367 + 633 = 1 000
   (c) 2 480 + 2 520 = 5 000 (d) 64 753 + 27 538 = 92 291
2. Use each of these subtraction facts to write an addition fact.
   (a) 3 678 – 600 = 3 078 (b) 3 678 – 70 = 3 608
   (c) 3 678 – 3 000 = 678 (d) 3 678 – 608 = 3 070
3. Use each of the subtraction facts in question 2 to write another subtraction fact, without doing any calculations.
4. Try to find the following differences. Do as little work as possible.
   (a) 8 382 – 80 (b) 8 382 – 8 000
   (c) 8 382 – 380 (d) 8 382 – 8 002

To find differences between numbers, it is often useful to think of where the numbers are on a number line.

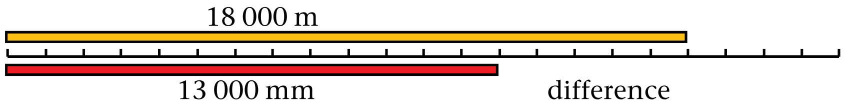
For example, to know what the difference between 16 000 and 7 000 is, the following picture in your mind may be useful.

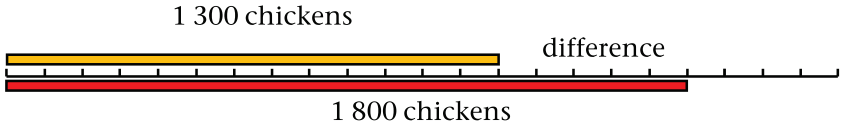
5. Find each of the differences below. If you do not know the answer immediately, you may think of movements on a number line or think in any other way.
   (a) 8 000 – 3 000 (b) 80 000 – 30 000
   (c) 38 000 – 3 000 (d) 38 000 – 30 000
   (e) 23 000 – 4 000 (f) 109 000 – 4 000
   (g) 109 000 – 100 000 (h) 109 000 – 50 000

34 UNIT 3: WHOLE NUMBERS: ADDITION AND SUBTRACTION

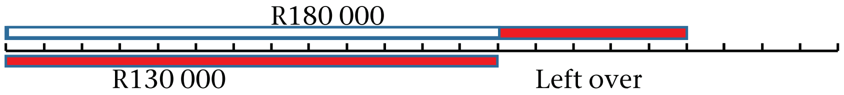
Mathematical notes

Questions 6 and 11 require subtraction to find the **difference** between two quantities.



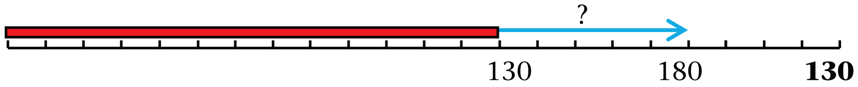


Question 8 requires subtraction to establish how much is left over if a given amount is **taken away** from another given amount.



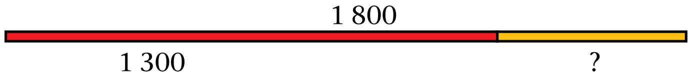
180 000 – 130 000 = ?

Question 9 requires subtraction to find a **shortfall**.



130 + ? = 180

Question 12 requires subtraction to calculate the value of **one part of a given sum**, if the other parts are given.



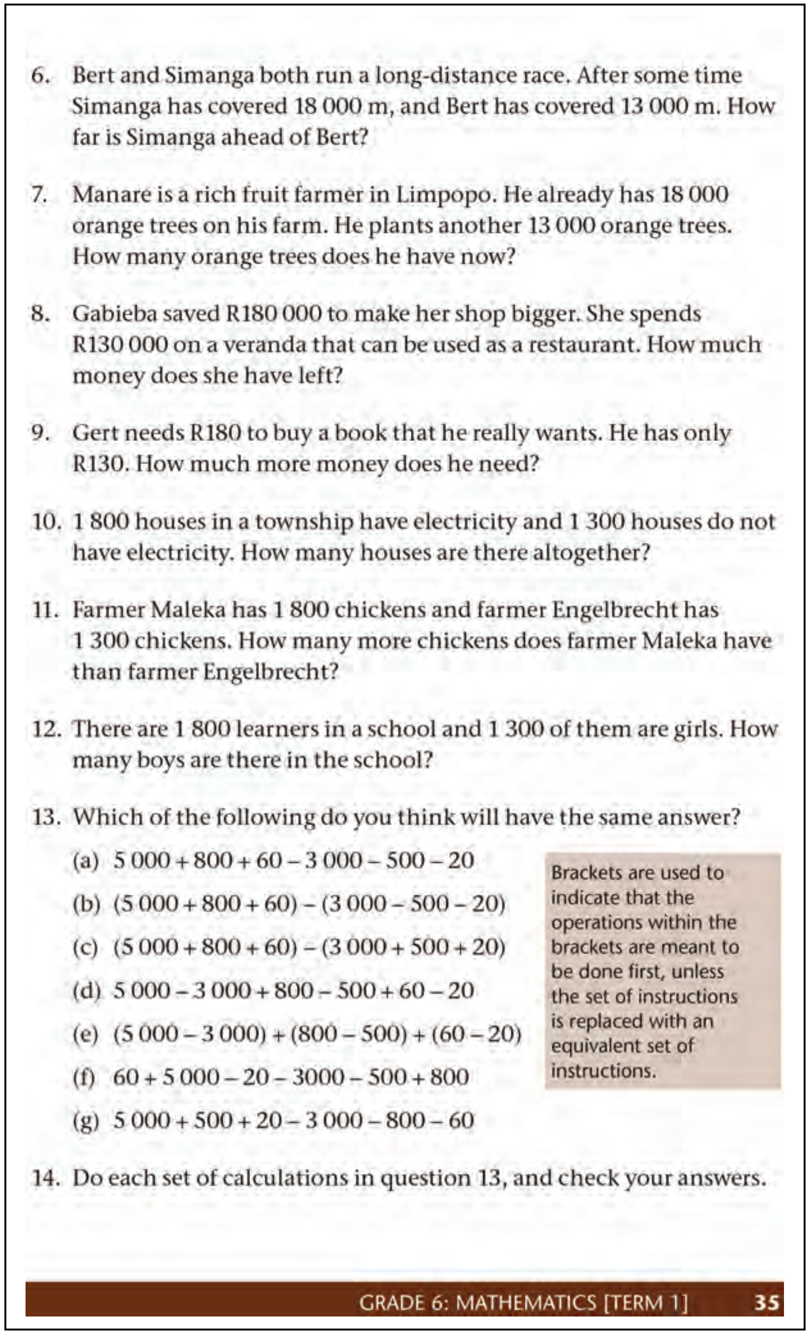
1 800 = 1 300 + ?

To summarise, learners should be alerted to the different problem types (kinds of situations), in which subtraction can be used to find certain information (different **meanings** of subtraction).

In question 13, note that additions and subtractions are to be performed from left to right in the order given, unless otherwise indicated by brackets. Also see "Mathematical notes" on the previous page of this Teacher Guide.

Answers

6. 5 000 m 7. 31 000 orange trees 8. R50 000
9. R50 10. 3 100 houses 11. 500 chickens
12. 500 boys 13. (a) and (c) and (d) and (e) and (f)
14. (a) 2 340 (b) 3 380 (c) 2 340 (d) 2 340 (e) 2 340 (f) 2 340 (g) 1 660



6. Bert and Simanga both run a long-distance race. After some time Simanga has covered 18 000 m, and Bert has covered 13 000 m. How far is Simanga ahead of Bert?

7. Manare is a rich fruit farmer in Limpopo. He already has 18 000 orange trees on his farm. He plants another 13 000 orange trees. How many orange trees does he have now?

8. Gabieba saved R180 000 to make her shop bigger. She spends R130 000 on a veranda that can be used as a restaurant. How much money does she have left?

9. Gert needs R180 to buy a book that he really wants. He has only R130. How much more money does he need?

10. 1 800 houses in a township have electricity and 1 300 houses do not have electricity. How many houses are there altogether?

11. Farmer Maleka has 1 800 chickens and farmer Engelbrecht has 1 300 chickens. How many more chickens does farmer Maleka have than farmer Engelbrecht?

12. There are 1 800 learners in a school and 1 300 of them are girls. How many boys are there in the school?

13. Which of the following do you think will have the same answer?
    (a) 5 000 + 800 + 60 – 3 000 – 500 – 20
    (b) (5 000 + 800 + 60) – (3 000 – 500 – 20)
    (c) (5 000 + 800 + 60) – (3 000 + 500 + 20)
    (d) 5 000 – 3 000 + 800 – 500 + 60 – 20
    (e) (5 000 – 3 000) + (800 – 500) + (60 – 20)
    (f) 60 + 5 000 – 20 – 3000 – 500 + 800
    (g) 5 000 + 500 + 20 – 3 000 – 800 – 60

Brackets are used to indicate that the operations within the brackets are meant to be done first, unless the set of instructions is replaced with an equivalent set of instructions.

14. Do each set of calculations in question 13, and check your answers.

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3.4 Rounding off and rearranging

Teaching guidelines

Expose learners to the various uses of rounding off, including:

- making quick estimates in real-life situations, as described in the shaded passage
- making quick estimates to check whether accurately calculated answers are realistic in terms of problem contexts, as demonstrated in questions 3, 4 and 8 on the next page.

Please refer to the notes and guidelines about rounding off on page 14 of the Learner Book.

Explain to learners that "approximately" (used in the shaded passage on page 36) means "close to".

When estimating how much several items together will cost, people often **round up,** for example:

John will buy:

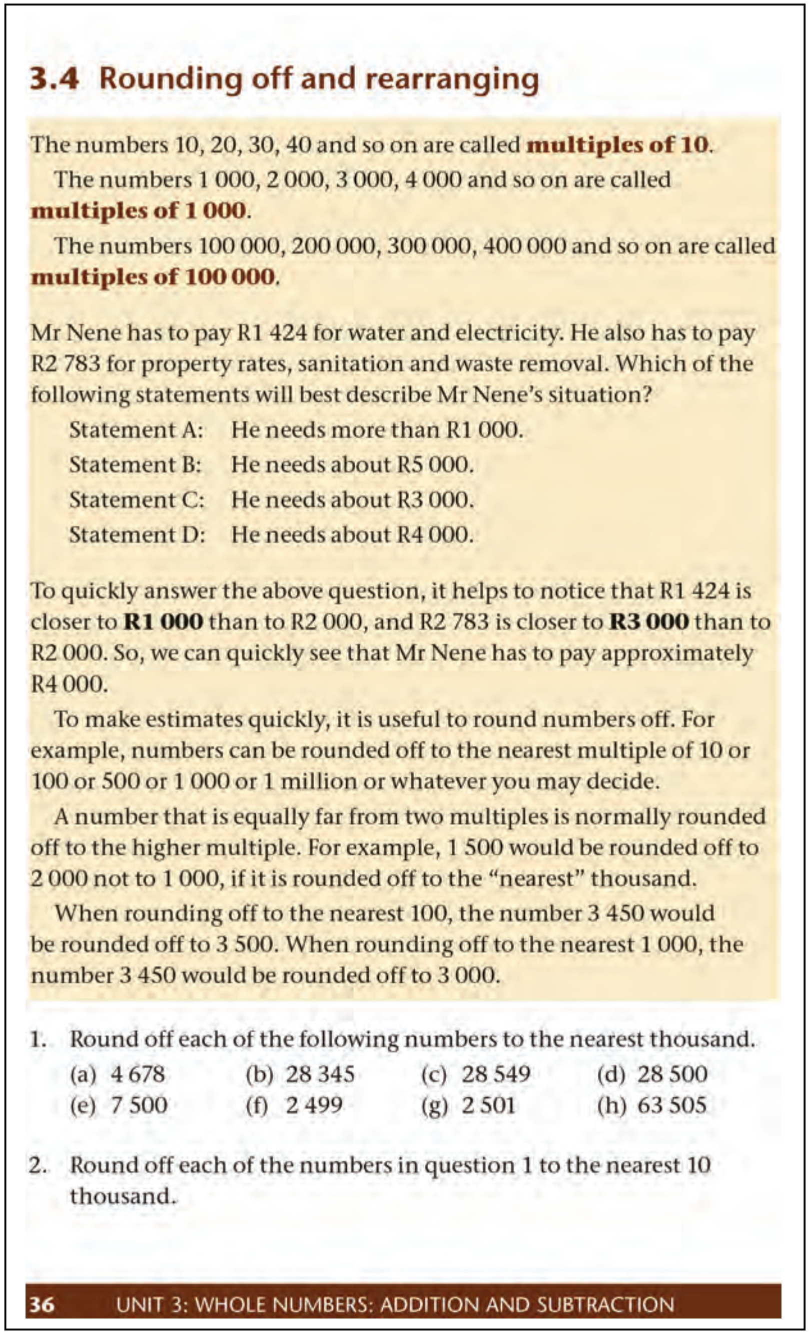
- beans for R13,45
- milk for R12,35
- cheese for R26,95.

To quickly know how much money he will need, John may calculate 15 + 15 + 30 = 60. When doing so, he rounds the prices up to the nearest R5.

In a different context, **rounding down** may be more appropriate, for example: A school has 5 small buses, which can carry 23, 36, 44, 47 and 27 passengers respectively. When asked to state how many passengers the buses can transport all together, the principal calculated 20 + 30 + 40 + 40 + 20 and said "at least 150 but actually a few more". He rounded all the numbers down to the next lower multiple of ten.

Answers

1.	(a) 5 000	(b) 28 000	(c) 29 000	(d) 29 000
	(e) 8 000	(f) 2 000	(g) 3 000	(h) 64 000
2.	(a) 0	(b) 30 000	(c) 30 000	(d) 30 000
	(e) 10 000	(f) 0	(g) 0	(h) 60 000



3.4 Rounding off and rearranging

The numbers 10, 20, 30, 40 and so on are called **multiples of 10**.

The numbers 1 000, 2 000, 3 000, 4 000 and so on are called **multiples of 1 000**.

The numbers 100 000, 200 000, 300 000, 400 000 and so on are called **multiples of 100 000.**

Mr Nene has to pay R1 424 for water and electricity. He also has to pay R2 783 for property rates, sanitation and waste removal. Which of the following statements will best describe Mr Nene's situation?

Statement A: He needs more than R1 000.
Statement B: He needs about R5 000.
Statement C: He needs about R3 000.
Statement D: He needs about R4 000.

To quickly answer the above question, it helps to notice that R1 424 is closer to **R1 000** than to R2 000, and R2 783 is closer to **R3 000** than to R2 000. So, we can quickly see that Mr Nene has to pay approximately R4 000.

To make estimates quickly, it is useful to round numbers off. For example, numbers can be rounded off to the nearest multiple of 10 or 100 or 500 or 1 000 or 1 million or whatever you may decide.

A number that is equally far from two multiples is normally rounded off to the higher multiple. For example, 1 500 would be rounded off to 2 000 not to 1 000, if it is rounded off to the "nearest" thousand.

When rounding off to the nearest 100, the number 3 450 would be rounded off to 3 500. When rounding off to the nearest 1 000, the number 3 450 would be rounded off to 3 000.

1. Round off each of the following numbers to the nearest thousand.
   (a) 4 678 (b) 28 345 (c) 28 549 (d) 28 500
   (e) 7 500 (f) 2 499 (g) 2 501 (h) 63 505

2. Round off each of the numbers in question 1 to the nearest 10 thousand.

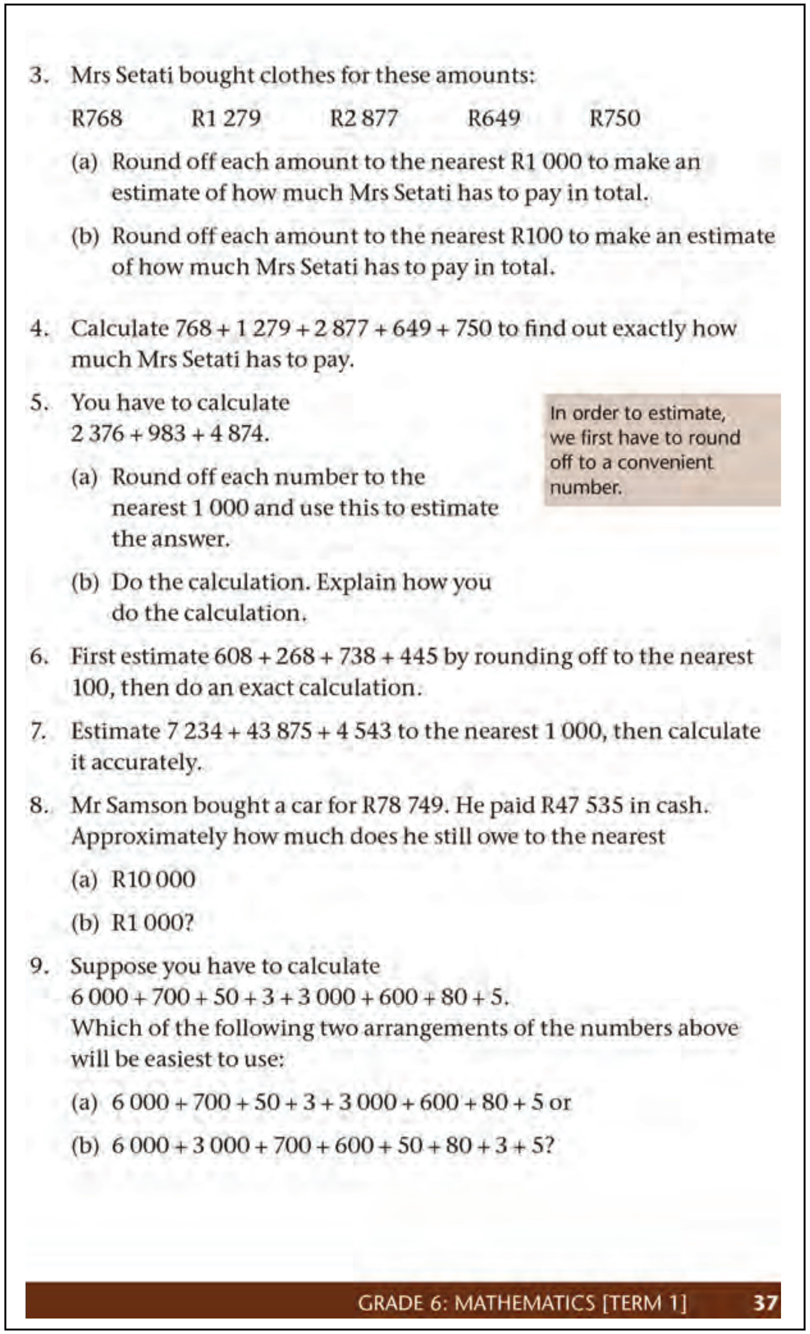
Mathematical notes

This section and the next two (Sections 3.4 and 3.5) bring together a variety of skills in preparation for adding and subtracting multi-digit numbers in columns in Sections 3.6 and 3.7:

- rounding off
- estimation
- breaking numbers down into their place value parts (expanded notation)
- replacing place value parts with other expansions more convenient for the purpose at hand.

Answers

3. (a) R1 000 R1 000 R3 000 R1 000 R1 000
Estimated total: R7 000
(b) R800 R1 300 R2 900 R600 R800
Estimated total: R6 400
4. 6 323
5. (a) 2 000 + 1 000 + 5 000 = 8 000
(b) 8 233. Learners have to explain how they did the calculation.
6. Estimated answer: 600 + 300 + 700 + 400 = 2 000
Calculated answer: 2 059
7. Estimated answer: 7 000 + 44 000 + 5 000 = 56 000
Calculated answer: 55 652
8. (a) R30 000
(b) R31 000
9. (b) 6 000 + 3 000 + 700 + 600 + 50 + 80 + 3 + 5



3. Mrs Setati bought clothes for these amounts:

R768 R1 279 R2 877 R649 R750

(a) Round off each amount to the nearest R1 000 to make an estimate of how much Mrs Setati has to pay in total.

(b) Round off each amount to the nearest R100 to make an estimate of how much Mrs Setati has to pay in total.

4. Calculate 768 + 1 279 + 2 877 + 649 + 750 to find out exactly how much Mrs Setati has to pay.

5. You have to calculate 2 376 + 983 + 4 874.

(a) Round off each number to the nearest 1 000 and use this to estimate the answer.

(b) Do the calculation. Explain how you do the calculation.

In order to estimate, we first have to round off to a convenient number.

6. First estimate 608 + 268 + 738 + 445 by rounding off to the nearest 100, then do an exact calculation.

7. Estimate 7 234 + 43 875 + 4 543 to the nearest 1 000, then calculate it accurately.

8. Mr Samson bought a car for R78 749. He paid R47 535 in cash. Approximately how much does he still owe to the nearest

(a) R10 000

(b) R1 000?

9. Suppose you have to calculate
6 000 + 700 + 50 + 3 + 3 000 + 600 + 80 + 5.
Which of the following two arrangements of the numbers above will be easiest to use:

(a) 6 000 + 700 + 50 + 3 + 3 000 + 600 + 80 + 5 or

(b) 6 000 + 3 000 + 700 + 600 + 50 + 80 + 3 + 5?

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Possible misconceptions

Learners have already begun adding and subtracting in columns in Grade 5. The column format simplifies the recording of calculations. But especially with larger numbers, learners may lose sight of the actual mathematical actions and logic involved. When working in columns, it is very easy to stop thinking of the digits in the various place value positions as representing multiples of ten, hundred, thousand, etc., and to start thinking of them as single-digit numbers. This may be referred to as "losing sight of place value".

The work in this section, and specifically the work on this page, is intended to promote the maintenance of the understanding of place value when doing addition and subtraction.

Answers

10. (a) (9 000 + 4 000) + (700 + 600) + (80 + 60) + (7 + 5)
    (b) (50 000 + 30 000) + (7 000 + 4 000) + (600 + 400) + (60 + 30) + (7 + 4)
    (c) (900 + 400 + 300) + (80 + 30 + 30) + (6 + 5 + 4)

11. (a) 13 000 + 1 300 + 140 + 12 = 14 452
    (b) 80 000 + 11 000 + 1 000 + 90 + 11 = 92 101
    (c) 1 600 + 140 + 15 = 1 755

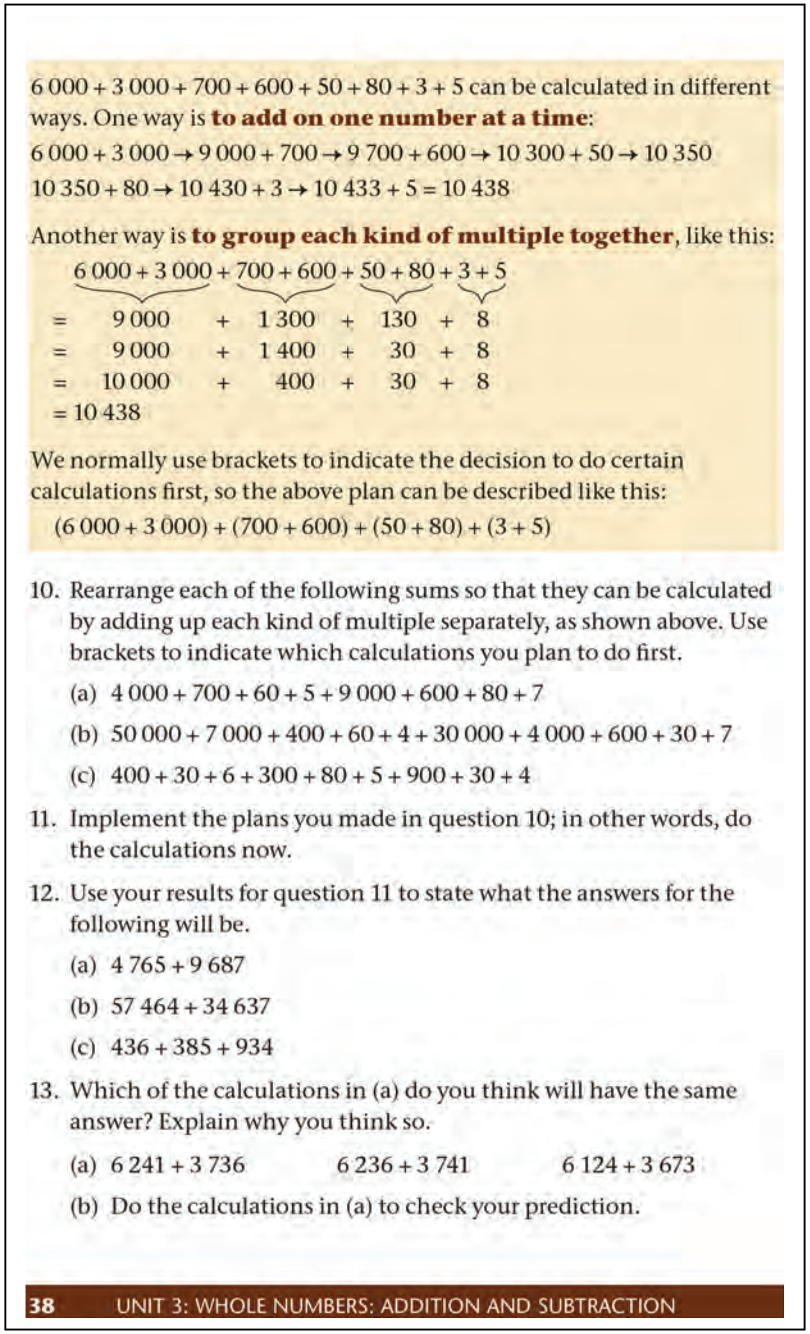
12. (a) 14 452 (b) 92 101 (c) 1 755

13. (a) 6 241 + 3 736 and 6 236 + 3 741 (Consider learners' explanations.)
    (b) 6 241 + 3 736 = 9 977 and 6 236 + 3 741 = 9 977

Possible misconceptions

When learners do not understand the need for and the logic of replacement ("borrowing") when doing subtraction, they tend to make mistakes such as "subtracting the smaller digit from the larger digit" when recording in columns, as in the three examples below.

$$\begin{array}{r} 334 \\ -\underline{276} \\ 142 \end{array} \qquad \begin{array}{r} 7\,682 \\ -\underline{3\,785} \\ 4\,103 \end{array} \qquad \begin{array}{r} 60\,524 \\ -\underline{59\,278} \\ 19\,354 \end{array}$$



6 000 + 3 000 + 700 + 600 + 50 + 80 + 3 + 5 can be calculated in different ways. One way is **to add on one number at a time**:

6 000 + 3 000 → 9 000 + 700 → 9 700 + 600 → 10 300 + 50 → 10 350
10 350 + 80 → 10 430 + 3 → 10 433 + 5 = 10 438

Another way is **to group each kind of multiple together**, like this:

$$\begin{array}{lcccccccc} & \underbrace{6\,000+3\,000} & + & \underbrace{700+600} & + & \underbrace{50+80} & + & \underbrace{3+5} \\ = & 9\,000 & + & 1\,300 & + & 130 & + & 8 \\ = & 9\,000 & + & 1\,400 & + & 30 & + & 8 \\ = & 10\,000 & + & 400 & + & 30 & + & 8 \\ = 10\,438 \end{array}$$

We normally use brackets to indicate the decision to do certain calculations first, so the above plan can be described like this:

(6 000 + 3 000) + (700 + 600) + (50 + 80) + (3 + 5)

10. Rearrange each of the following sums so that they can be calculated by adding up each kind of multiple separately, as shown above. Use brackets to indicate which calculations you plan to do first.
    (a) 4 000 + 700 + 60 + 5 + 9 000 + 600 + 80 + 7
    (b) 50 000 + 7 000 + 400 + 60 + 4 + 30 000 + 4 000 + 600 + 30 + 7
    (c) 400 + 30 + 6 + 300 + 80 + 5 + 900 + 30 + 4
11. Implement the plans you made in question 10; in other words, do the calculations now.
12. Use your results for question 11 to state what the answers for the following will be.
    (a) 4 765 + 9 687
    (b) 57 464 + 34 637
    (c) 436 + 385 + 934
13. Which of the calculations in (a) do you think will have the same answer? Explain why you think so.
    (a) 6 241 + 3 736 6 236 + 3 741 6 124 + 3 673
    (b) Do the calculations in (a) to check your prediction.

3.5 Subtraction with place value parts

Teaching guidelines

Please read the note on “Possible misconceptions” at the bottom of the previous page.

The activities in Section 3.5 are specifically intended to develop understanding of the need for and the logic of suitable replacement of the place value expansions of numbers during subtraction. In order to monitor the effectiveness of working through this section, it could be very useful to give a diagnostic test at the beginning (*diagnostic assessment, not to be used for formal assessment purposes*). You may use the instrument given on the next page for this purpose – learners should be given about 20 minutes to complete this diagnostic test.

It is critical that learners understand *why* it is sometimes necessary to replace the place value expansion of the bigger number with a different expansion.

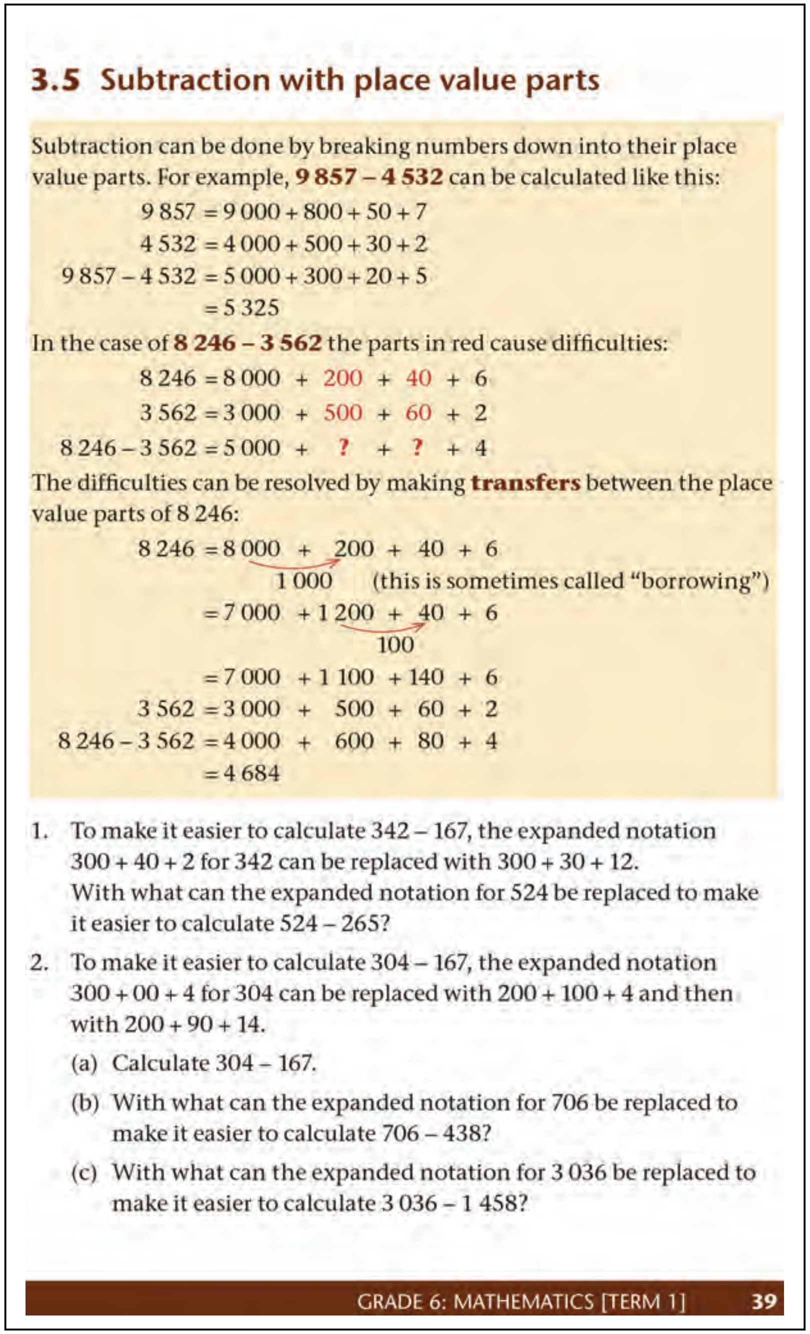
The calculation of 8 246 – 3 562 in the shaded passage may be used as an example to explain this. It may actually help to link the numbers to a context, for example to suggest that there are 8 246 elephants in a game reserve, and then that 3 562 of them are taken away to another game reserve. Point out that one can think of the elephants in the game reserve as a group of 8 000, a group of 200, a group of 40 and a group of 6, as shown on the left below. Point out that the 3 562 elephants that must be taken to another game reserve can be thought of as a group of 3 000, a group of 500, a group of 60 and a group of 2. Then ask the questions below, and preferably write them on the board.

8 000 elephants	*Can the group of 3 000 be taken from these 8 000 elephants?*
200 elephants	*Can the group of 500 be taken from these 200 elephants?*
40 elephants	*Can the group of 60 be taken from these 40 elephants?*
6 elephants	*Can the group of 2 be taken from these 6 elephants?*

Allow discussion on these questions, and ask learners to make suggestions on how the elephants could be rearranged so that there are enough in each group to take away from.

Answers

1. 400 + 110 + 14
2. (a) 200 + 90 + 14 – (100 + 60 + 7) = 100 + 30 + 7 = 137
   (b) 600 + 90 + 16
   (c) 2 000 + 900 + 120 + 16



3.5 Subtraction with place value parts

Subtraction can be done by breaking numbers down into their place value parts. For example, **9 857 – 4 532** can be calculated like this:

9 857 = 9 000 + 800 + 50 + 7
4 532 = 4 000 + 500 + 30 + 2
9 857 – 4 532 = 5 000 + 300 + 20 + 5
= 5 325

In the case of **8 246 – 3 562** the parts in red cause difficulties:

8 246 = 8 000 + 200 + 40 + 6
3 562 = 3 000 + 500 + 60 + 2
8 246 – 3 562 = 5 000 + **?** + **?** + 4

The difficulties can be resolved by making **transfers** between the place value parts of 8 246:

8 246 = 8 000 + 200 + 40 + 6
1 000 (this is sometimes called “borrowing”)
= 7 000 + 1 200 + 40 + 6
100
= 7 000 + 1 100 + 140 + 6
3 562 = 3 000 + 500 + 60 + 2
8 246 – 3 562 = 4 000 + 600 + 80 + 4
= 4 684

1. To make it easier to calculate 342 – 167, the expanded notation 300 + 40 + 2 for 342 can be replaced with 300 + 30 + 12.
   With what can the expanded notation for 524 be replaced to make it easier to calculate 524 – 265?
2. To make it easier to calculate 304 – 167, the expanded notation 300 + 00 + 4 for 304 can be replaced with 200 + 100 + 4 and then with 200 + 90 + 14.
   (a) Calculate 304 – 167.
   (b) With what can the expanded notation for 706 be replaced to make it easier to calculate 706 – 438?
   (c) With what can the expanded notation for 3 036 be replaced to make it easier to calculate 3 036 – 1 458?

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Answers

3. 5 000 + 1 100 + 120 + 11
4. (a) 28 000 – 12 000 = 16 000 (b) 85 000 – 53 000 = 32 000
   (c) 65 000 – 21 000 = 44 000 (d) 30 000 – 15 000 = 15 000
5. (b) and (c)
6. 437
7. 47 235 = 236 + 46 999 49 531 = 532 + 48 999
   46 999 – 32 876 = 14 123 48 999 – 23 845 = 25 154
   so, 47 235 – 32 876 so, 49 531 – 23 845
   = 14 123 + 236 = 25 154 + 532
   = 14 359 = 25 686
8. 88 354 – 52 768 = 35 586 and 76 423 – 52 678 = 23 745
9. (a) 15 365 (b) 32 635 (c) 43 218 (d) 14 227
10. (a) 27 689 (b) 85 324 (c) 64 504 (d) 29 679

A diagnostic assessment that may be done at the beginning of Section 3.5

Calculate the following. Use a pen and show all your work.

A. 334 – 276 B. 408 – 276 C. 132 + 276 D. 58 + 276
E. 7 682 – 3 785 F. 60 524 – 59 278
G. Ben has 334 goats and sells 276 of them. How many goats does Ben have left?

The correct answers:

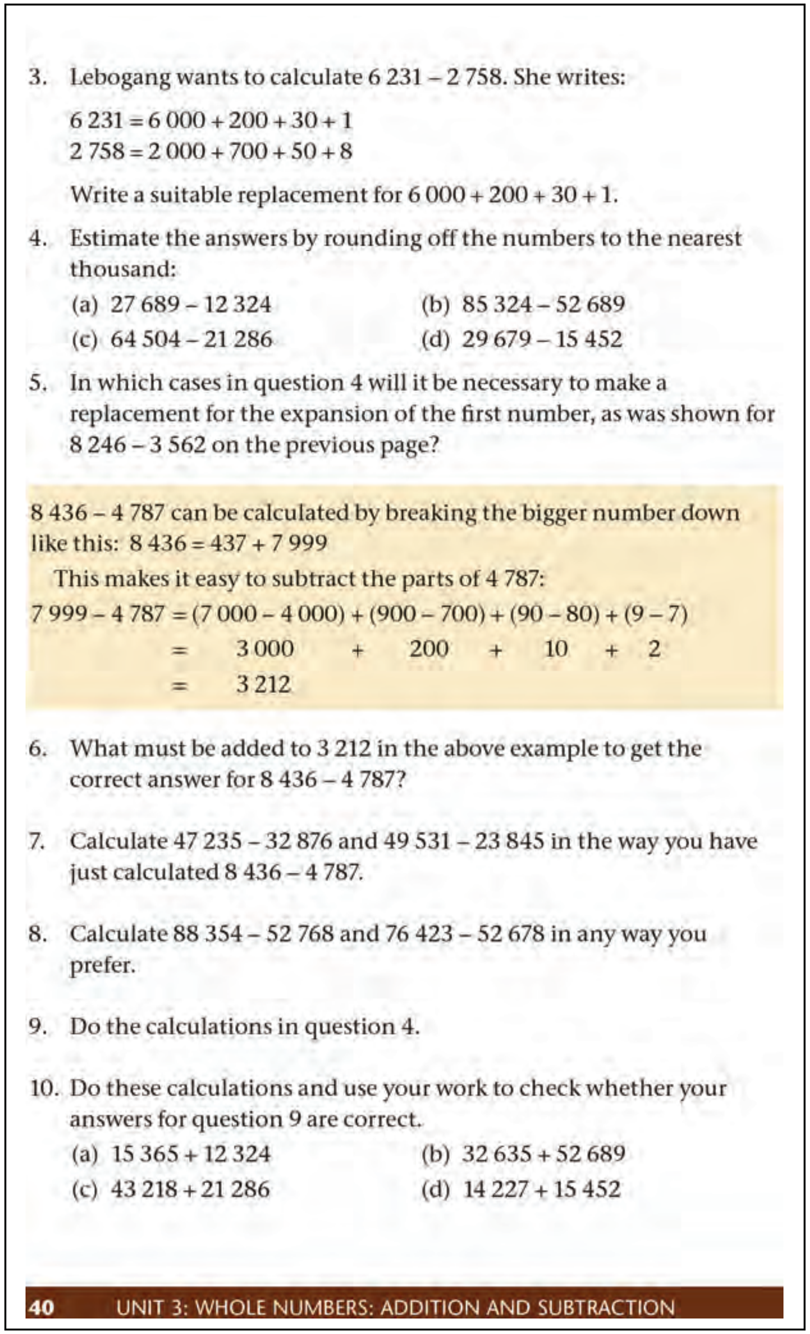
A. 58 B. 132 C. 408 D. 334
E. 3 897 F. 1 246 G. 58 goats

A possible set of answers that indicate serious lack of understanding of subtraction:

A. 132 B. 272 C. 408 D. 334
E. 4 103 F. 19 354 G. 58 goats (or 146 goats)

One way to support learners who give these answers is to ask them to compare their answers for questions A and G, and reconsider their answers for A. You may also ask them what information their (correct) answers for questions C and D provide, with respect to their answers for A and B.

A similar assessment instrument is given at the beginning of Section 3.7, to monitor possible improvement as a result of doing the work in Sections 3.5 and 3.6.



3. Lebogang wants to calculate 6 231 – 2 758. She writes:
   6 231 = 6 000 + 200 + 30 + 1
   2 758 = 2 000 + 700 + 50 + 8
   Write a suitable replacement for 6 000 + 200 + 30 + 1.
4. Estimate the answers by rounding off the numbers to the nearest thousand:
   (a) 27 689 – 12 324 (b) 85 324 – 52 689
   (c) 64 504 – 21 286 (d) 29 679 – 15 452
5. In which cases in question 4 will it be necessary to make a replacement for the expansion of the first number, as was shown for 8 246 – 3 562 on the previous page?

8 436 – 4 787 can be calculated by breaking the bigger number down like this: 8 436 = 437 + 7 999

This makes it easy to subtract the parts of 4 787:

7 999 – 4 787 = (7 000 – 4 000) + (900 – 700) + (90 – 80) + (9 – 7)
= 3 000 + 200 + 10 + 2
= 3 212

6. What must be added to 3 212 in the above example to get the correct answer for 8 436 – 4 787?
7. Calculate 47 235 – 32 876 and 49 531 – 23 845 in the way you have just calculated 8 436 – 4 787.
8. Calculate 88 354 – 52 768 and 76 423 – 52 678 in any way you prefer.
9. Do the calculations in question 4.
10. Do these calculations and use your work to check whether your answers for question 9 are correct.
    (a) 15 365 + 12 324 (b) 32 635 + 52 689
    (c) 43 218 + 21 286 (d) 14 227 + 15 452

3.6 The vertical column notation for addition

Mathematical notes

Doing addition in columns and doing subtraction in columns are not different methods than the break-down and build-up methods that learners have used previously. Working in columns is simply an alternative format for setting out the work, and it has the advantage that it can be abbreviated by not recording all the thinking steps.

The transition from addition and subtraction by breaking down and building up as learners have done it up to now to the so-called "column methods" is not a change of method, it is a change of formatting style and a reduction in the extent to which the actual mathematical steps (thinking) are recorded in writing.

Teaching guidelines

It is critically important not to rush into teaching the shortest possible column format ("finished product") for addition by breaking down, rearranging and building up.

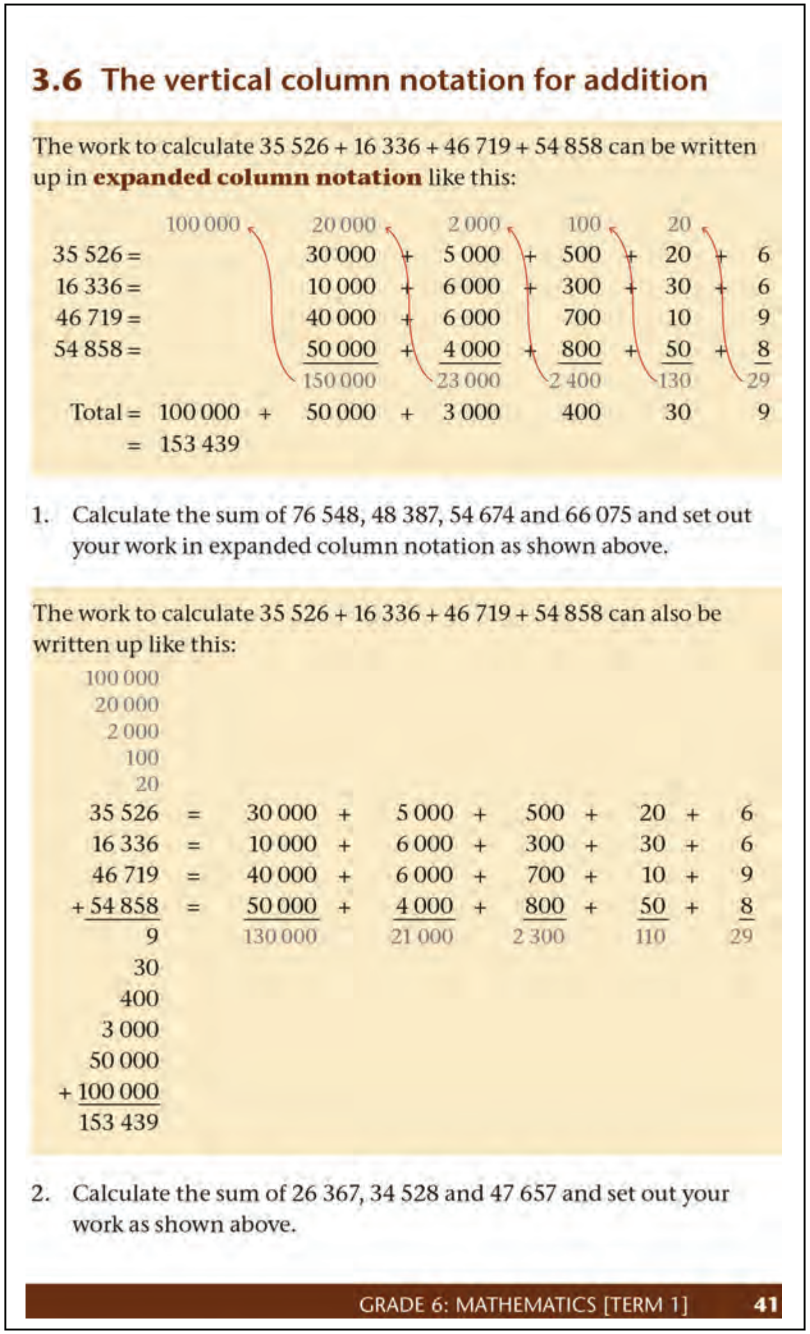
Rushing to this format too quickly will aggravate the risk that learners will suspend making sense of the numbers and the actions they take with respect to the numbers.

$$\begin{array}{r} \scriptstyle 1\ 2\ 2\ 1\ \\ 35\,526 \\ 16\,336 \\ 46\,719 \\ +\underline{54\,858} \\ 153\,439 \end{array}$$

The shaded passages show two ways in which you can explain the logic of adding in this way. Write it on the board. You may also decide not to demonstrate these two ways of documenting column addition on the board, but to let learners read it for themselves while they are doing questions 1 and 2.

Answers

1. Learners should use the format illustrated in the first shaded passage. Answer: 245 684
2. Learners should use the format illustrated in the second shaded passage. Answer: 108 552



3.6 The vertical column notation for addition

The work to calculate 35 526 + 16 336 + 46 719 + 54 858 can be written up in **expanded column notation** like this:

	100 000	20 000	2 000	100	20	
35 526 =		30 000 +	5 000 +	500 +	20 +	6
16 336 =		10 000 +	6 000 +	300 +	30 +	6
46 719 =		40 000 +	6 000 +	700 +	10 +	9
54 858 =		50 000 +	4 000 +	800 +	50 +	8
		150 000	23 000	2 400	130	29
Total =	100 000 +	50 000 +	3 000	400	30	9
=	153 439					

1. Calculate the sum of 76 548, 48 387, 54 674 and 66 075 and set out your work in expanded column notation as shown above.

The work to calculate 35 526 + 16 336 + 46 719 + 54 858 can also be written up like this:

100 000						
20 000						
2 000						
100						
20						
35 526	=	30 000 +	5 000 +	500 +	20 +	6
16 336	=	10 000 +	6 000 +	300 +	30 +	6
46 719	=	40 000 +	6 000 +	700 +	10 +	9
+ 54 858	=	50 000 +	4 000 +	800 +	50 +	8
9		130 000	21 000	2 300	110	29
30						
400						
3 000						
50 000						
+ 100 000						
153 439						

2. Calculate the sum of 26 367, 34 528 and 47 657 and set out your work as shown above.

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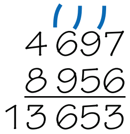
Mathematical notes

Traditionally, addition with carrying was set out as shown on the right for 4 697 + 8 956. The blue marks, from right to left, actually indicate 10, 100 and 1 000. When the marks are read as “1”, “1” and “1”, and the thinking for the tens column is “1 + 9 + 5 = 15”, for the hundreds column “1 + 6 + 9 = 16” and for the thousands column “1 + 4 + 8 = 13”, learners’ awareness and understanding of place value and of the actual numbers *four thousand six hundred and ninety-seven* and *eight thousand nine hundred and fifty-six* may be seriously undermined.

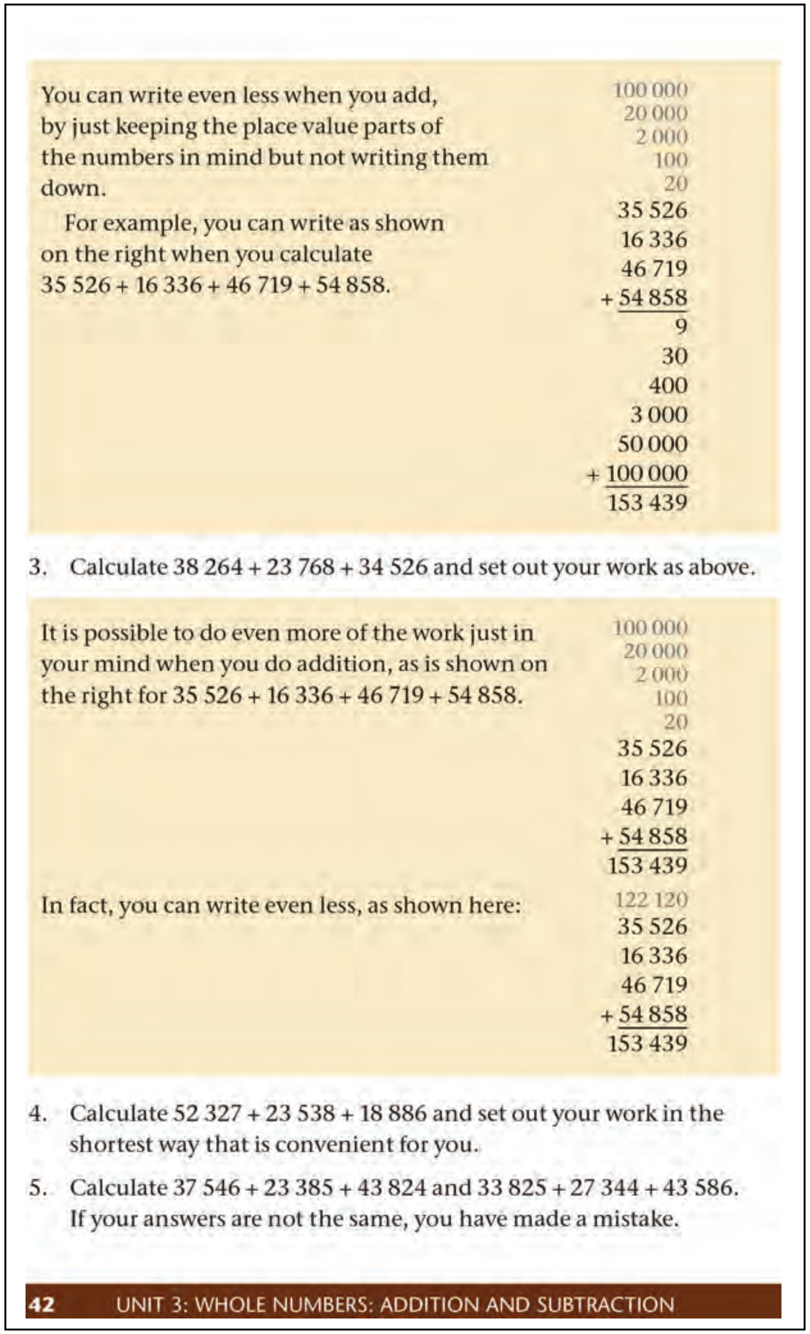


Teaching guidelines

With a view to maintain learners’ awareness and understanding of place value and of the actual numbers involved, the transition from separate recording of the column totals (part answers) to the traditional condensed form of the column format is introduced gradually through the phases demonstrated in the shaded passages.

Answers

3. Learners must use the recording format demonstrated in the first shaded passage. Answer: 96 558
4. 94 751
5. 37 546 + 23 385 + 43 824 = 104 755
   33 825 + 27 344 + 43 586 = 104 755



You can write even less when you add, by just keeping the place value parts of the numbers in mind but not writing them down.

For example, you can write as shown on the right when you calculate 35 526 + 16 336 + 46 719 + 54 858.

100 000
20 000
2 000
100
20
35 526
16 336
46 719
+ 54 858
9
30
400
3 000
50 000
+ 100 000
153 439

3. Calculate 38 264 + 23 768 + 34 526 and set out your work as above.

It is possible to do even more of the work just in your mind when you do addition, as is shown on the right for 35 526 + 16 336 + 46 719 + 54 858.

100 000
20 000
2 000
100
20
35 526
16 336
46 719
+ 54 858
153 439

In fact, you can write even less, as shown here:

122 120
35 526
16 336
46 719
+ 54 858
153 439

4. Calculate 52 327 + 23 538 + 18 886 and set out your work in the shortest way that is convenient for you.
5. Calculate 37 546 + 23 385 + 43 824 and 33 825 + 27 344 + 43 586. If your answers are not the same, you have made a mistake.

42 UNIT 3: WHOLE NUMBERS: ADDITION AND SUBTRACTION

3.7 The vertical column notation for subtraction

Teaching guidelines

This section is a continuation of the work on subtraction done in Section 3.5. Please read the teaching guidelines for page 39 of the Learner Book again. It may be necessary to again explain to learners why it is sometimes necessary to replace the place value expansion of the bigger number when doing subtraction.

For learners who performed poorly in the diagnostic assessment recommended in Section 3.5, it may be useful to repeat such an assessment at this stage to monitor whether the work done in Sections 3.5 and 3.6 has impacted positively on their commitment to make sense of their actions and of the numbers involved when they are doing calculations.

The instrument below is equivalent to the instrument given in Section 3.5.

A diagnostic assessment that may be done at the beginning of Section 3.7

Calculate the following. Use a pen and show all your work.

A. 743 – 689   B. 835 – 689   C. 146 + 689   D. 54 + 689
E. 8 314 – 3 567   F. 80 324 – 79 981
G. Ben has 743 goats and sells 689 of them. How many goats does Ben have left?

The correct answers:

A. 54   B. 146   C. 835   D. 743
E. 4 747   F. 343   G. 54 goats

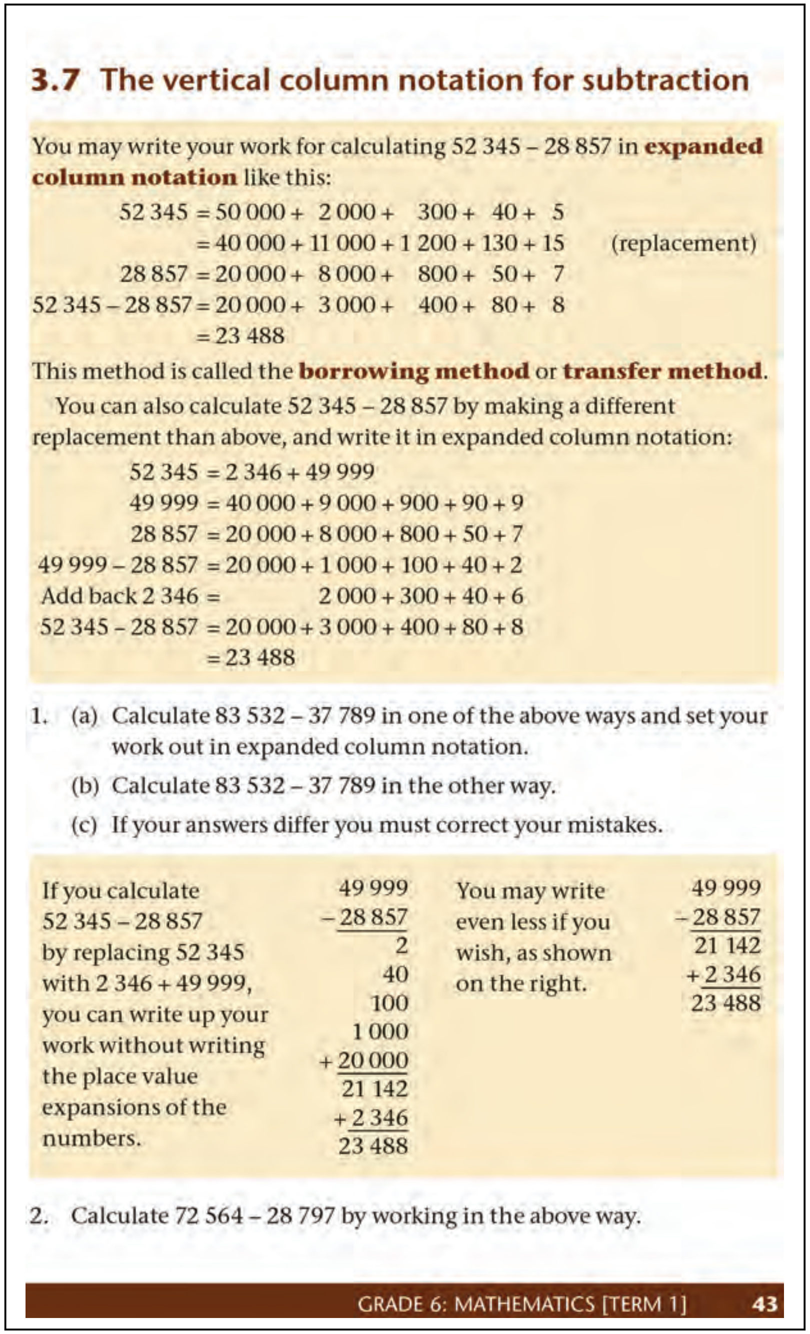
A possible set of answers that indicate serious lack of understanding of subtraction:

A. 146   B. 254   C. 835   D. 743
E. 5 253   F. 19 663   G. 54 goats (or 146 goats)

One way to support learners who give these answers, is to ask them to compare their answers for questions A and G, and reconsider their answer for A. You may also ask them what information their (correct) answers for questions C and D provide, with respect to their answers for A and B.

Answers

1. (a)–(c) 45 743
2. 43 767



3.7 The vertical column notation for subtraction

You may write your work for calculating 52 345 – 28 857 in **expanded column notation** like this:

52 345 = 50 000 + 2 000 + 300 + 40 + 5
= 40 000 + 11 000 + 1 200 + 130 + 15 (replacement)
28 857 = 20 000 + 8 000 + 800 + 50 + 7
52 345 – 28 857 = 20 000 + 3 000 + 400 + 80 + 8
= 23 488

This method is called the **borrowing method** or **transfer method**.

You can also calculate 52 345 – 28 857 by making a different replacement than above, and write it in expanded column notation:

52 345 = 2 346 + 49 999
49 999 = 40 000 + 9 000 + 900 + 90 + 9
28 857 = 20 000 + 8 000 + 800 + 50 + 7
49 999 – 28 857 = 20 000 + 1 000 + 100 + 40 + 2
Add back 2 346 = 2 000 + 300 + 40 + 6
52 345 – 28 857 = 20 000 + 3 000 + 400 + 80 + 8
= 23 488

1. (a) Calculate 83 532 – 37 789 in one of the above ways and set your work out in expanded column notation.
   (b) Calculate 83 532 – 37 789 in the other way.
   (c) If your answers differ you must correct your mistakes.

If you calculate 52 345 – 28 857 by replacing 52 345 with 2 346 + 49 999, you can write up your work without writing the place value expansions of the numbers.

$$\begin{array}{r} 49\,999 \\ -\underline{28\,857} \\ 2 \\ 40 \\ 100 \\ 1\,000 \\ +\underline{20\,000} \\ 21\,142 \\ +\underline{2\,346} \\ 23\,488 \end{array}$$

You may write even less if you wish, as shown on the right.

$$\begin{array}{r} 49\,999 \\ -\underline{28\,857} \\ 21\,142 \\ +\underline{2\,346} \\ 23\,488 \end{array}$$

2. Calculate 72 564 – 28 797 by working in the above way.

Teaching guidelines

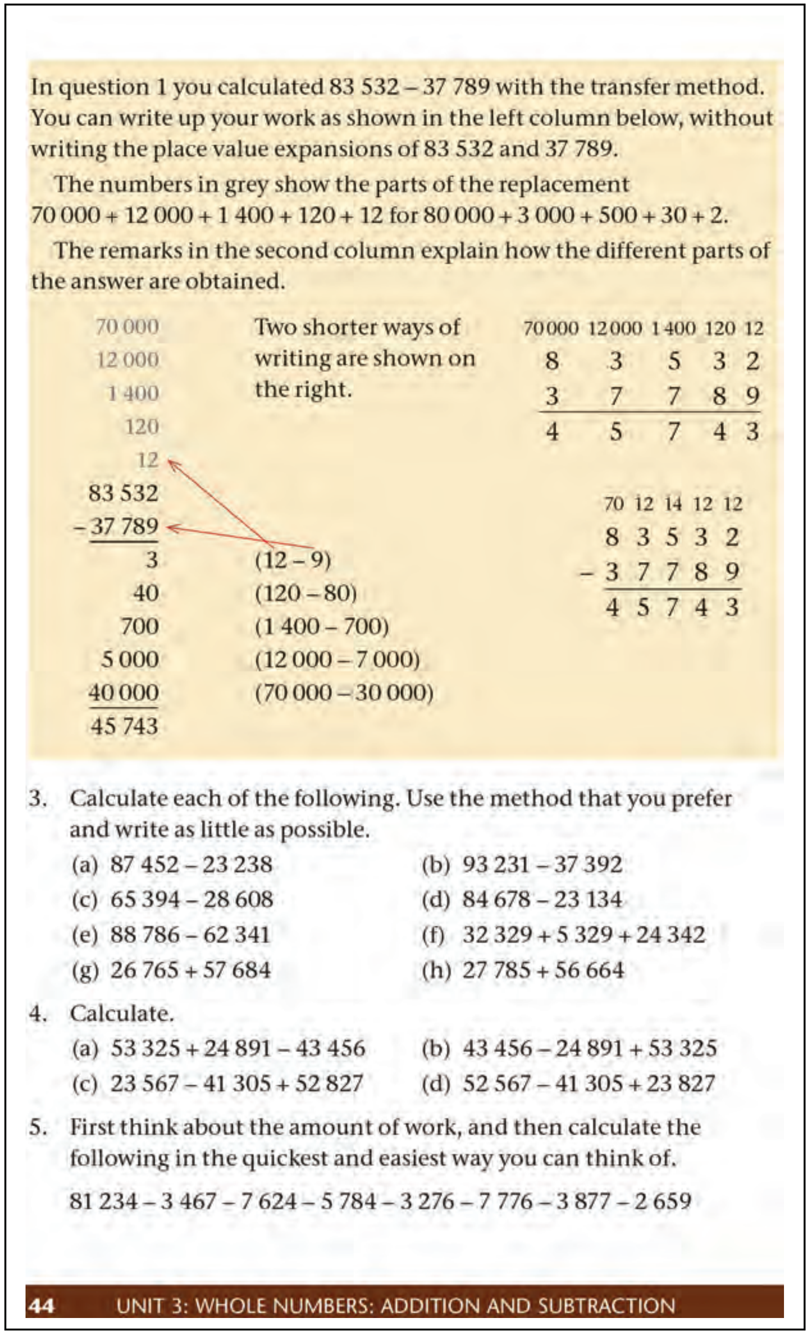
Work through the calculation of 83 532 – 37 789 (as shown in the shaded passage) with the class. Use the calculation to show learners that there is a difference between producing an answer and explaining the thinking used in arriving at the answer.

Answers

3. (a) 64 214 (b) 55 839 (c) 36 786 (d) 61 544
(e) 26 445 (f) 62 000 (g) 84 449 (h) 84 449

4. (a) 34 760 (b) 71 890 (c) 35 089 (d) 35 089

5. 3 467 + 7 624 + 5 784 + 3 276 + 7 776 + 3 877 + 2 659 = 34 463
81 234 – 34 463 = 46 771



In question 1 you calculated 83 532 – 37 789 with the transfer method. You can write up your work as shown in the left column below, without writing the place value expansions of 83 532 and 37 789.

The numbers in grey show the parts of the replacement 70 000 + 12 000 + 1 400 + 120 + 12 for 80 000 + 3 000 + 500 + 30 + 2.

The remarks in the second column explain how the different parts of the answer are obtained.

70 000 12 000 1 400 120 12	Two shorter ways of writing are shown on the right.
83 532	
– 37 789	
3	(12 – 9)
40	(120 – 80)
700	(1 400 – 700)
5 000	(12 000 – 7 000)
40 000	(70 000 – 30 000)
45 743	

$$\begin{array}{ccccc} 70000 & 12000 & 1400 & 120 & 12 \\ 8 & 3 & 5 & 3 & 2 \\ 3 & 7 & 7 & 8 & 9 \\ \hline 4 & 5 & 7 & 4 & 3 \end{array}$$

$$\begin{array}{rccccc} & 70 & 12 & 14 & 12 & 12 \\ & 8 & 3 & 5 & 3 & 2 \\ - & 3 & 7 & 7 & 8 & 9 \\ \hline & 4 & 5 & 7 & 4 & 3 \end{array}$$

3. Calculate each of the following. Use the method that you prefer and write as little as possible.
   (a) 87 452 – 23 238 (b) 93 231 – 37 392
   (c) 65 394 – 28 608 (d) 84 678 – 23 134
   (e) 88 786 – 62 341 (f) 32 329 + 5 329 + 24 342
   (g) 26 765 + 57 684 (h) 27 785 + 56 664

4. Calculate.
   (a) 53 325 + 24 891 – 43 456 (b) 43 456 – 24 891 + 53 325
   (c) 23 567 – 41 305 + 52 827 (d) 52 567 – 41 305 + 23 827

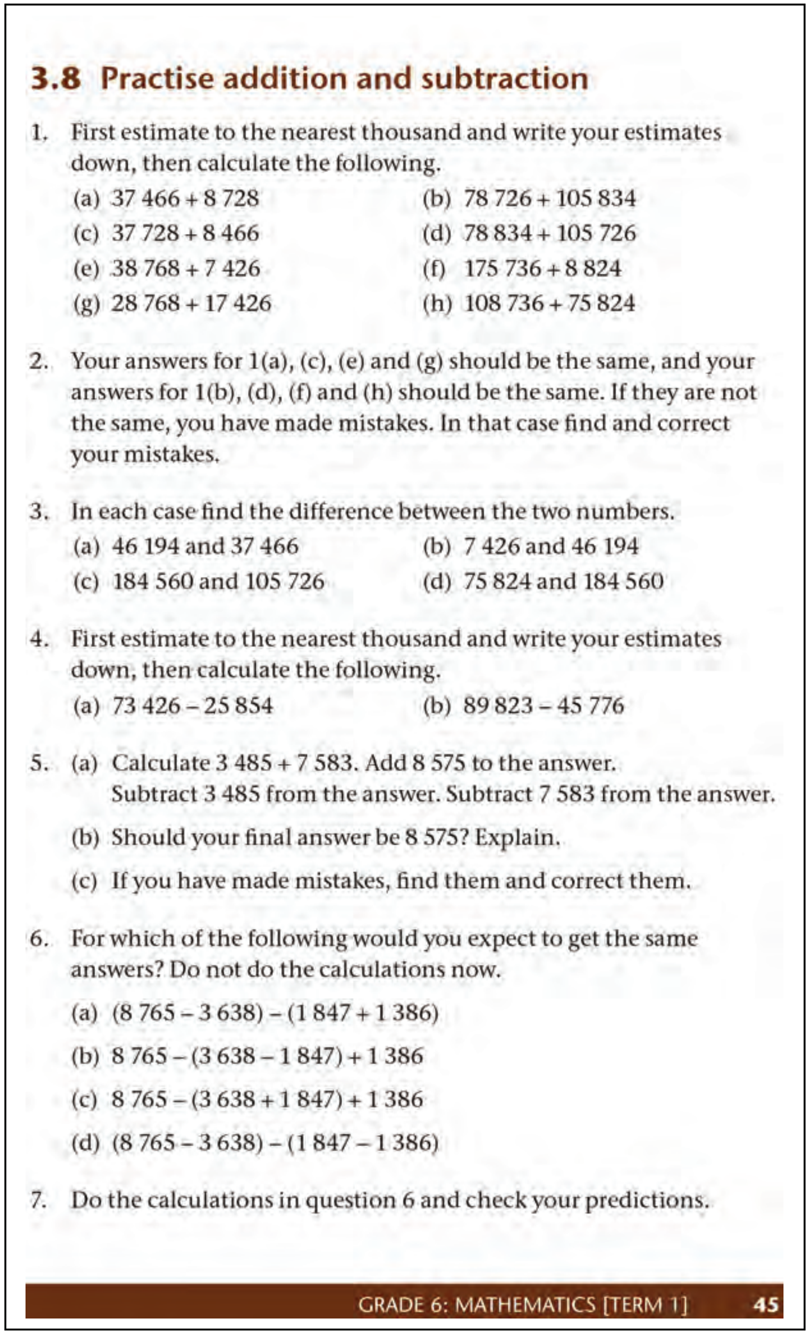
5. First think about the amount of work, and then calculate the following in the quickest and easiest way you can think of.

   81 234 – 3 467 – 7 624 – 5 784 – 3 276 – 7 776 – 3 877 – 2 659

3.8 Practise addition and subtraction

Answers

1. (a) 46 000 46 194
   (b) 185 000 184 560
   (c) 46 000 46 194
   (d) 185 000 184 560
   (e) 46 000 46 194
   (f) 185 000 184 560
   (g) 46 000 46 194
   (h) 185 000 184 560
2. Learners check and correct their mistakes.
3. (a) 8 728
   (b) 38 768
   (c) 78 834
   (d) 108 736
4. (a) Estimated: 47 000 Calculated: 47 572
   (b) Estimated: 44 000 Calculated: 44 047
5. (a) 8 575
   (b) Yes, because the two numbers in the initial sum (i.e. 3 485 and 7 583) to which 8 575 was added, were subtracted again.
   (c) Learners check and correct their calculations.
6. (c) and (d)
7. (a) 1 894 (b) 8 360 (c) 4 666 (d) 4 666



3.8 Practise addition and subtraction

1. First estimate to the nearest thousand and write your estimates down, then calculate the following.
   (a) 37 466 + 8 728 (b) 78 726 + 105 834
   (c) 37 728 + 8 466 (d) 78 834 + 105 726
   (e) 38 768 + 7 426 (f) 175 736 + 8 824
   (g) 28 768 + 17 426 (h) 108 736 + 75 824

2. Your answers for 1(a), (c), (e) and (g) should be the same, and your answers for 1(b), (d), (f) and (h) should be the same. If they are not the same, you have made mistakes. In that case find and correct your mistakes.

3. In each case find the difference between the two numbers.
   (a) 46 194 and 37 466 (b) 7 426 and 46 194
   (c) 184 560 and 105 726 (d) 75 824 and 184 560

4. First estimate to the nearest thousand and write your estimates down, then calculate the following.
   (a) 73 426 – 25 854 (b) 89 823 – 45 776

5. (a) Calculate 3 485 + 7 583. Add 8 575 to the answer.
   Subtract 3 485 from the answer. Subtract 7 583 from the answer.
   (b) Should your final answer be 8 575? Explain.
   (c) If you have made mistakes, find them and correct them.

6. For which of the following would you expect to get the same answers? Do not do the calculations now.
   (a) (8 765 – 3 638) – (1 847 + 1 386)
   (b) 8 765 – (3 638 – 1 847) + 1 386
   (c) 8 765 – (3 638 + 1 847) + 1 386
   (d) (8 765 – 3 638) – (1 847 – 1 386)

7. Do the calculations in question 6 and check your predictions.

3.9 Using a calculator

Teaching guidelines

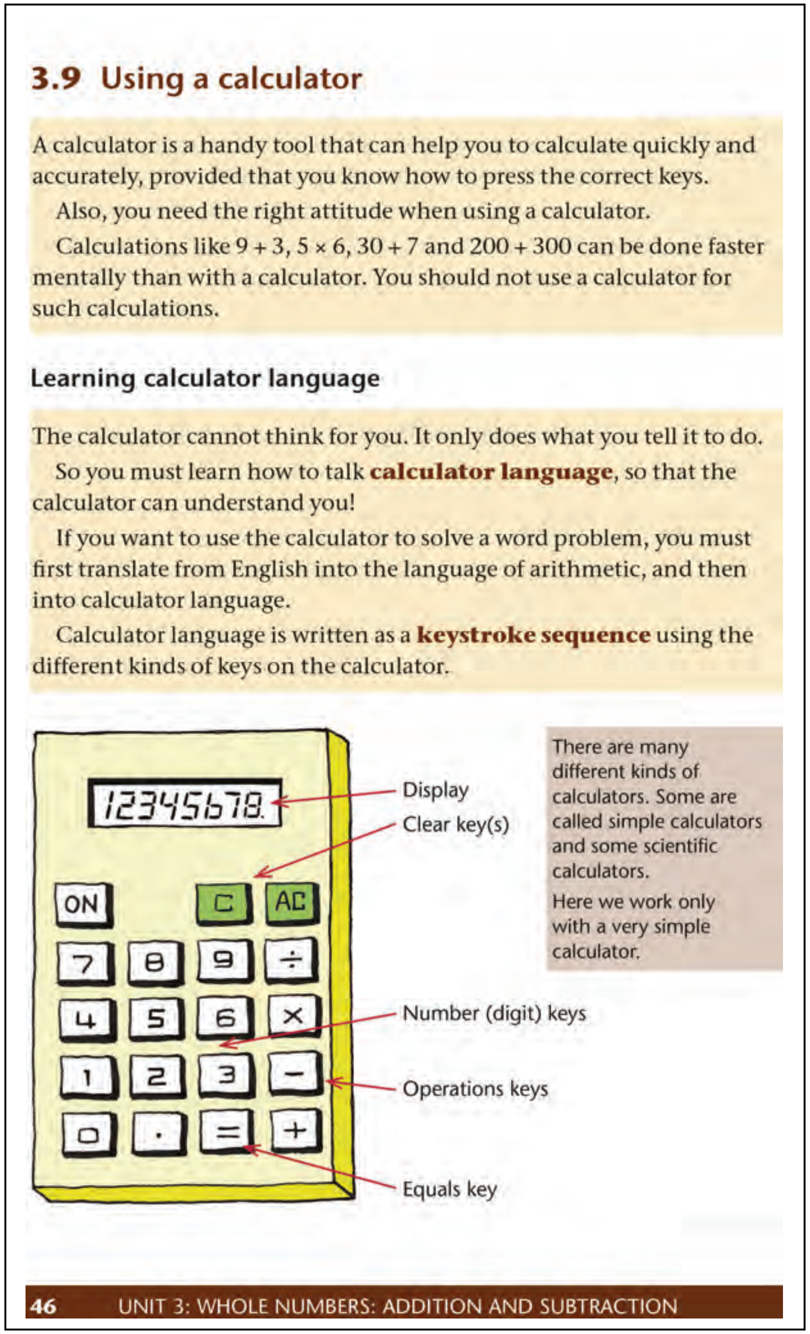
If it is used with good judgement, the calculator can be a powerful **mathematical tool** for learners. It can enhance their capacities to engage with numbers and with situations that involve numbers.

However, if not used with the right attitude, the calculator can undermine learners' capacities to engage with numbers. Learners need to understand that they **should be able to do any calculations without a calculator**. The calculator does not make it unnecessary to know basic number facts and to be able to do calculations with small numbers quickly and correctly (mental mathematics).

Discuss this thoroughly with learners when you introduce the calculator.

The calculator can also be a powerful **tool for learning mathematics**. For example, working with a calculator can contribute to the following:

- understanding of the meaning of calculation plans
- the idea of equivalent calculation plans
- the idea of inverse operations
- learning to estimate the results of calculations (by providing a facility to quickly check how far estimates differ from the accurate results).



3.9 Using a calculator

A calculator is a handy tool that can help you to calculate quickly and accurately, provided that you know how to press the correct keys.

Also, you need the right attitude when using a calculator.

Calculations like 9 + 3, 5 × 6, 30 + 7 and 200 + 300 can be done faster mentally than with a calculator. You should not use a calculator for such calculations.

Learning calculator language

The calculator cannot think for you. It only does what you tell it to do.

So you must learn how to talk **calculator language**, so that the calculator can understand you!

If you want to use the calculator to solve a word problem, you must first translate from English into the language of arithmetic, and then into calculator language.

Calculator language is written as a **keystroke sequence** using the different kinds of keys on the calculator.

There are many different kinds of calculators. Some are called simple calculators and some scientific calculators.

Here we work only with a very simple calculator.

46 UNIT 3: WHOLE NUMBERS: ADDITION AND SUBTRACTION

Teaching guidelines

It is quite easy to use the calculator to do one operation at a time, and learners can learn to do this without consciously thinking of a keystroke sequence.

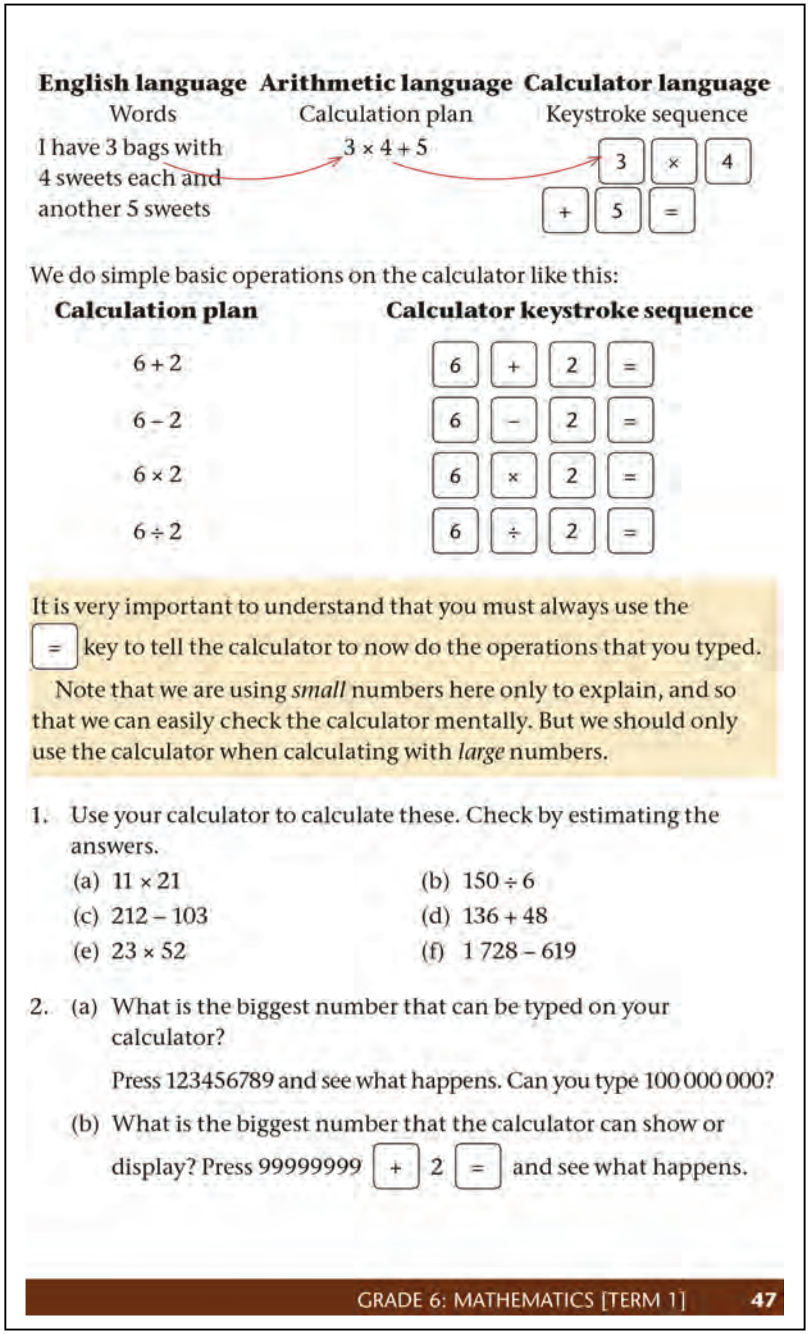
However, the idea of a keystroke sequence will empower learners to use the calculator effectively for performing calculations that involve more than one operation. Hence it will be useful to let learners write down some keystroke sequences for calculation plans with more than one operation, as shown in the examples at the top of the page in the Learner Book.

The keystroke sequence for 3 × 4 + 5 is $\boxed{3}\ \boxed{\times}\ \boxed{4}\ \boxed{+}\ \boxed{5}\ \boxed{=}$.

Let learners write keystroke sequences for some similar calculation plans, for example 5 × 4 – 7 and 9 × 2 + 8.

Answers

1. (a) 231 (b) 25 (c) 109
   (d) 184 (e) 1 196 (f) 1 109
2. (a) Practical activity
   (b) Practical activity



English language	**Arithmetic language**	**Calculator language**
Words	Calculation plan	Keystroke sequence
I have 3 bags with 4 sweets each and another 5 sweets	3 × 4 + 5	$\boxed{3}\ \boxed{\times}\ \boxed{4}\ \boxed{+}\ \boxed{5}\ \boxed{=}$

We do simple basic operations on the calculator like this:

Calculation plan	**Calculator keystroke sequence**
6 + 2	$\boxed{6}\ \boxed{+}\ \boxed{2}\ \boxed{=}$
6 – 2	$\boxed{6}\ \boxed{-}\ \boxed{2}\ \boxed{=}$
6 × 2	$\boxed{6}\ \boxed{\times}\ \boxed{2}\ \boxed{=}$
6 ÷ 2	$\boxed{6}\ \boxed{\div}\ \boxed{2}\ \boxed{=}$

It is very important to understand that you must always use the $\boxed{=}$ key to tell the calculator to now do the operations that you typed.

Note that we are using *small* numbers here only to explain, and so that we can easily check the calculator mentally. But we should only use the calculator when calculating with *large* numbers.

1. Use your calculator to calculate these. Check by estimating the answers.
   (a) 11 × 21 (b) 150 ÷ 6
   (c) 212 – 103 (d) 136 + 48
   (e) 23 × 52 (f) 1 728 – 619

2. (a) What is the biggest number that can be typed on your calculator?
   Press 123456789 and see what happens. Can you type 100 000 000?
   (b) What is the biggest number that the calculator can show or display? Press 99999999 $\boxed{+}$ 2 $\boxed{=}$ and see what happens.

Teaching guidelines

Once learners have completed question 3, you need to introduce the work on correcting mistakes (questions 4 to 7). You may start by writing a calculation plan such as the following on the board and ask learners to calculate it with their calculators:

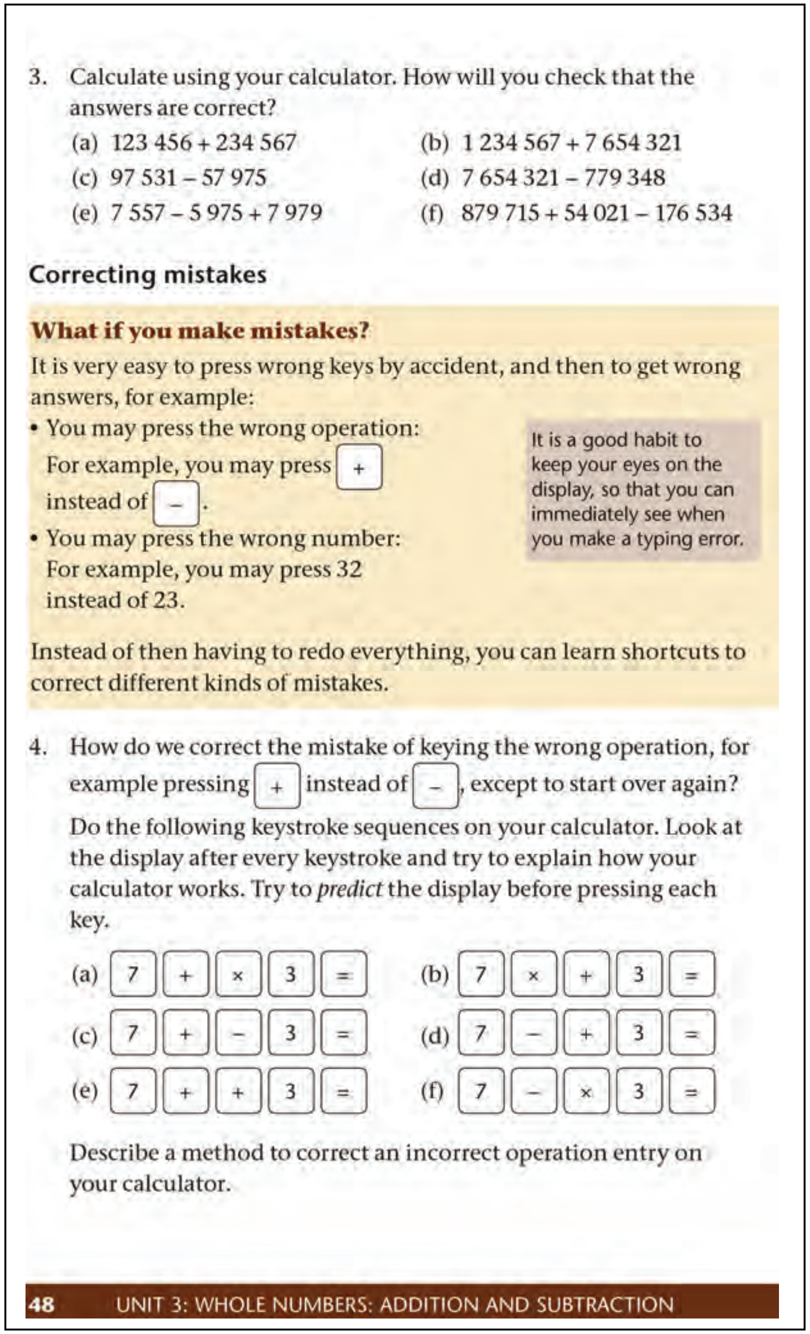
$$7 + 5 - 3 + 8 - 3 - 1 + 7$$

It is quite likely that some learners will have made mistakes. You can use the mistakes to motivate the work in questions 4 to 7. If all learners get it right, you can congratulate them but then demonstrate on the board how mistakes can easily be made when using the calculator.

Answers

3. (a) 358 023 (b) 8 888 888
   (c) 39 556 (d) 6 874 973
   (e) 9 561 (f) 757 202
4. Most simple calculators will execute these calculation plans and will give the answers below. (Some scientific calculators will give a syntax error message.)
   (a) $7 \times 3 = 21$ (b) $7 + 3 = 10$
   (c) $7 - 3 = 4$ (d) $7 + 3 = 10$
   (e) $7 + 3 = 10$ (f) $7 \times 3 = 21$

   If you enter an incorrect operation on your calculator, you can correct it by immediately entering the correct operation.



3. Calculate using your calculator. How will you check that the answers are correct?
   (a) 123 456 + 234 567 (b) 1 234 567 + 7 654 321
   (c) 97 531 – 57 975 (d) 7 654 321 – 779 348
   (e) 7 557 – 5 975 + 7 979 (f) 879 715 + 54 021 – 176 534

Correcting mistakes

What if you make mistakes?

It is very easy to press wrong keys by accident, and then to get wrong answers, for example:

- You may press the wrong operation:
  For example, you may press $\boxed{+}$ instead of $\boxed{-}$.
- You may press the wrong number:
  For example, you may press 32 instead of 23.

It is a good habit to keep your eyes on the display, so that you can immediately see when you make a typing error.

Instead of then having to redo everything, you can learn shortcuts to correct different kinds of mistakes.

4. How do we correct the mistake of keying the wrong operation, for example pressing $\boxed{+}$ instead of $\boxed{-}$, except to start over again?

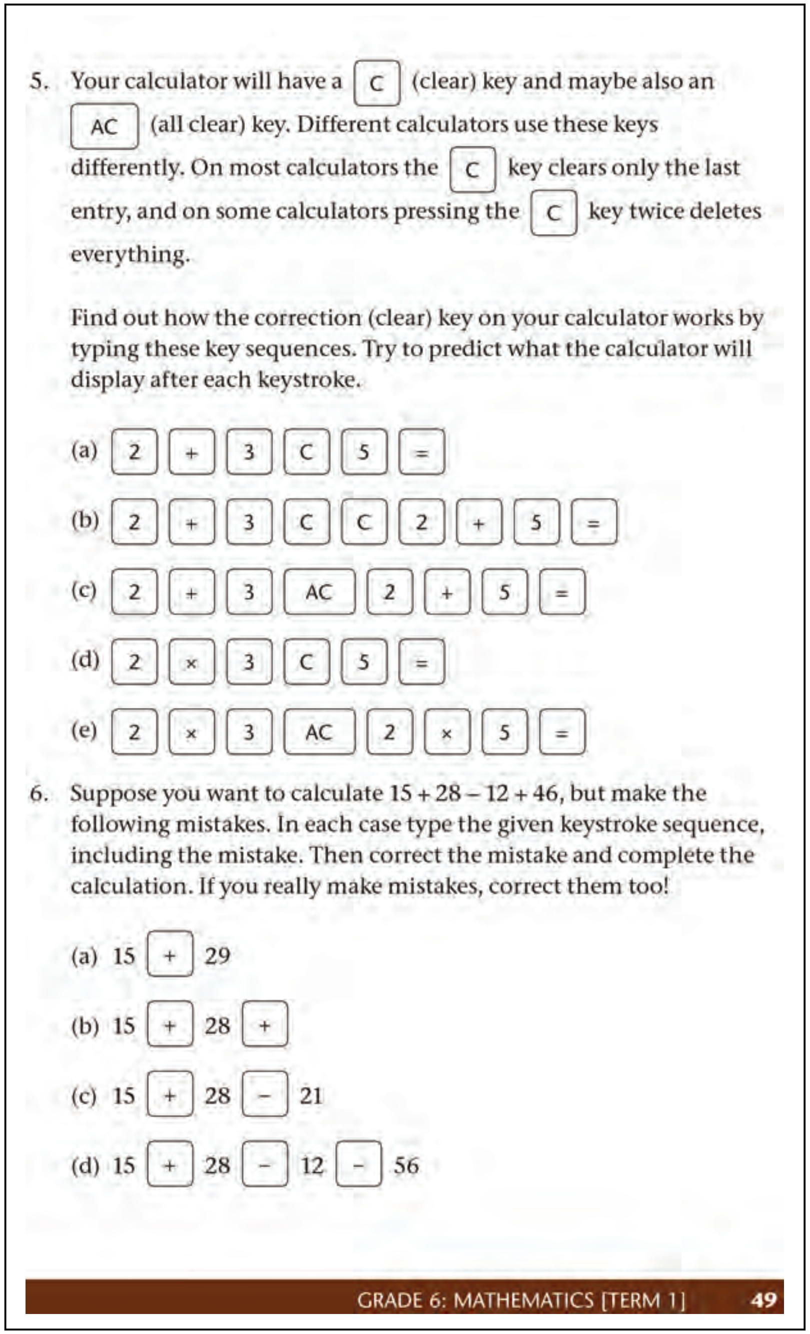
   Do the following keystroke sequences on your calculator. Look at the display after every keystroke and try to explain how your calculator works. Try to *predict* the display before pressing each key.

   (a) $\boxed{7}\ \boxed{+}\ \boxed{\times}\ \boxed{3}\ \boxed{=}$ (b) $\boxed{7}\ \boxed{\times}\ \boxed{+}\ \boxed{3}\ \boxed{=}$
   (c) $\boxed{7}\ \boxed{+}\ \boxed{-}\ \boxed{3}\ \boxed{=}$ (d) $\boxed{7}\ \boxed{-}\ \boxed{+}\ \boxed{3}\ \boxed{=}$
   (e) $\boxed{7}\ \boxed{+}\ \boxed{+}\ \boxed{3}\ \boxed{=}$ (f) $\boxed{7}\ \boxed{-}\ \boxed{\times}\ \boxed{3}\ \boxed{=}$

   Describe a method to correct an incorrect operation entry on your calculator.

Answers

5.–6. Practical activities



5. Your calculator will have a $\boxed{\text{C}}$ (clear) key and maybe also an $\boxed{\text{AC}}$ (all clear) key. Different calculators use these keys differently. On most calculators the $\boxed{\text{C}}$ key clears only the last entry, and on some calculators pressing the $\boxed{\text{C}}$ key twice deletes everything.

   Find out how the correction (clear) key on your calculator works by typing these key sequences. Try to predict what the calculator will display after each keystroke.

   (a) $\boxed{2}\ \boxed{+}\ \boxed{3}\ \boxed{\text{C}}\ \boxed{5}\ \boxed{=}$

   (b) $\boxed{2}\ \boxed{+}\ \boxed{3}\ \boxed{\text{C}}\ \boxed{\text{C}}\ \boxed{2}\ \boxed{+}\ \boxed{5}\ \boxed{=}$

   (c) $\boxed{2}\ \boxed{+}\ \boxed{3}\ \boxed{\text{AC}}\ \boxed{2}\ \boxed{+}\ \boxed{5}\ \boxed{=}$

   (d) $\boxed{2}\ \boxed{\times}\ \boxed{3}\ \boxed{\text{C}}\ \boxed{5}\ \boxed{=}$

   (e) $\boxed{2}\ \boxed{\times}\ \boxed{3}\ \boxed{\text{AC}}\ \boxed{2}\ \boxed{\times}\ \boxed{5}\ \boxed{=}$

6. Suppose you want to calculate 15 + 28 – 12 + 46, but make the following mistakes. In each case type the given keystroke sequence, including the mistake. Then correct the mistake and complete the calculation. If you really make mistakes, correct them too!

   (a) $15\ \boxed{+}\ 29$

   (b) $15\ \boxed{+}\ 28\ \boxed{+}$

   (c) $15\ \boxed{+}\ 28\ \boxed{-}\ 21$

   (d) $15\ \boxed{+}\ 28\ \boxed{-}\ 12\ \boxed{-}\ 56$

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Answers

7. Adding and subtracting the same number results in zero and will undo the incorrect operation/calculation.

Teaching guidelines

Questions 8 and 9 are about an important skill which is worth developing as a habit: to maintain some control over the accuracy of your work **by first estimating the answer** when using the calculator for calculations with large numbers.

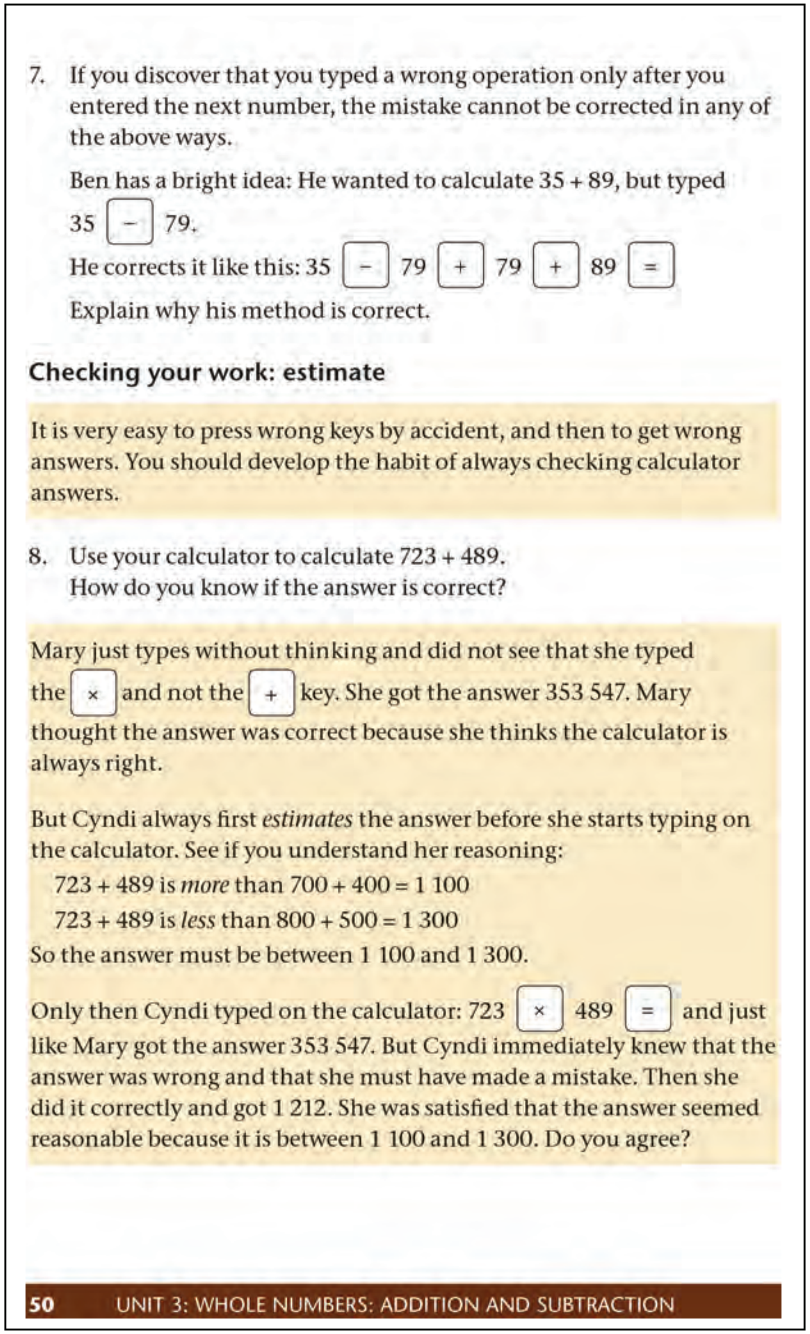
Demonstrate the work done by Mary and Cyndi on the board, using the examples given in the shaded passage, or other examples.

Possible misconceptions

Learners may think that making an estimate is the same thing as trying to guess what the exact answer is. It is not. An estimate is not intended to be the correct accurate answer; it is only meant to be an approximation of the answer.

Answers

8. 1 212



7. If you discover that you typed a wrong operation only after you entered the next number, the mistake cannot be corrected in any of the above ways.

   Ben has a bright idea: He wanted to calculate 35 + 89, but typed 35 $\boxed{-}$ 79.

   He corrects it like this: 35 $\boxed{-}$ 79 $\boxed{+}$ 79 $\boxed{+}$ 89 $\boxed{=}$

   Explain why his method is correct.

Checking your work: estimate

It is very easy to press wrong keys by accident, and then to get wrong answers. You should develop the habit of always checking calculator answers.

8. Use your calculator to calculate 723 + 489.
   How do you know if the answer is correct?

Mary just types without thinking and did not see that she typed the $\boxed{\times}$ and not the $\boxed{+}$ key. She got the answer 353 547. Mary thought the answer was correct because she thinks the calculator is always right.

But Cyndi always first *estimates* the answer before she starts typing on the calculator. See if you understand her reasoning:

723 + 489 is *more* than 700 + 400 = 1 100

723 + 489 is *less* than 800 + 500 = 1 300

So the answer must be between 1 100 and 1 300.

Only then Cyndi typed on the calculator: 723 $\boxed{\times}$ 489 $\boxed{=}$ and just like Mary got the answer 353 547. But Cyndi immediately knew that the answer was wrong and that she must have made a mistake. Then she did it correctly and got 1 212. She was satisfied that the answer seemed reasonable because it is between 1 100 and 1 300. Do you agree?

Answers

9. (a) 3 456 + 4 567 is more than 3 000 + 4 000 = 7 000, but less than 4 000 + 5 000 = 9 000. Answer: 8 023
   (b) 34 567 + 45 678 is more than 79 000, but less than 81 000. Answer: 80 245
   (c) 34 × 56 is more than 1 500, but less than 2 400. Answer: 1 906
   (d) 678 × 234 is more than 120 000 but less than 210 000. Answer: 158 652
   (e) 123 456 + 257 257 is more than 370 000, but less than 390 000. Answer: 380 713
   (f) 34 527 + 426 426 is more than 450 00 but less than 470 000. Answer: 460 953

Teaching guidelines

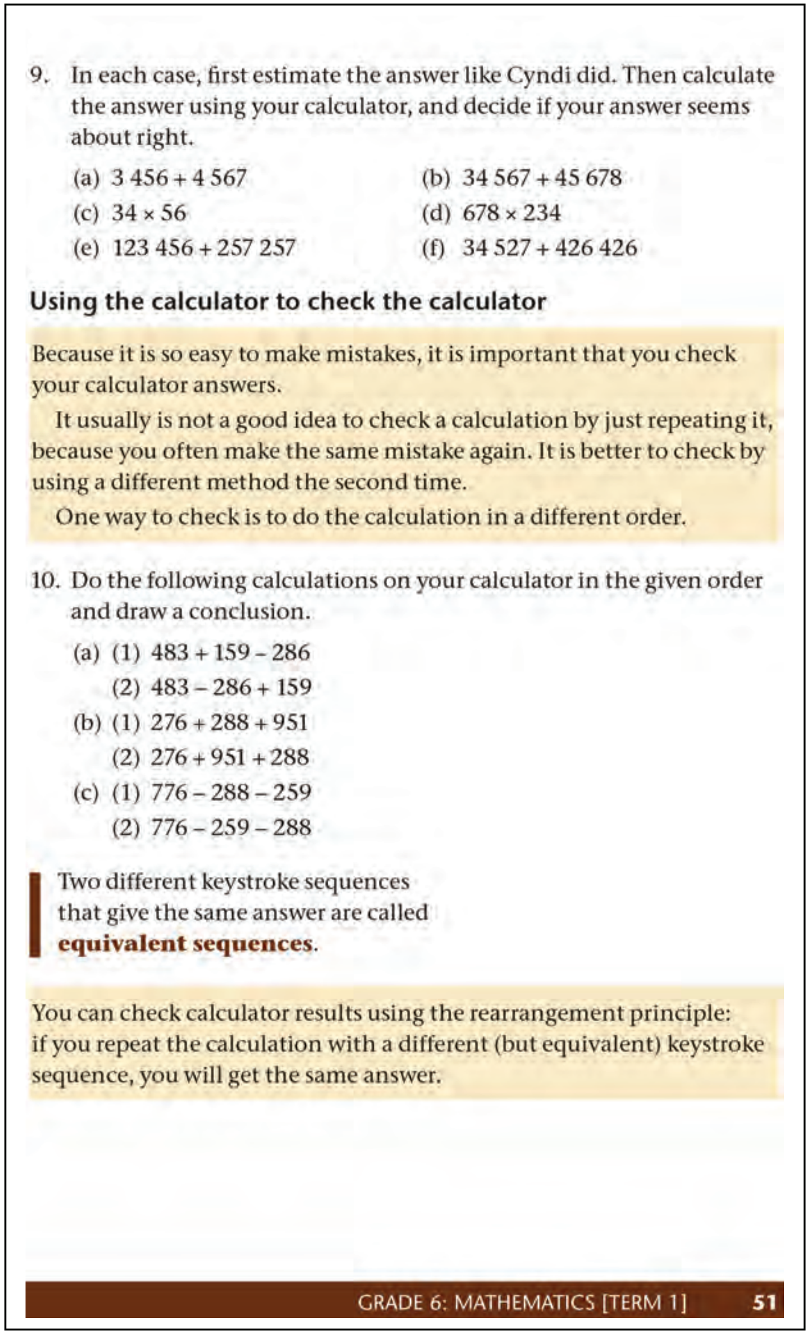
Using the calculator to check calculations that you have done on the calculator, as described on page 51 of the Learner Book, is useful for two reasons:

- It provides learners with another tool to exercise quality control on the calculations they do with the calculator.
- It provides another experience with equivalent calculation plans and properties of operations.

Answers

10. (a) (1) 483 + 159 – 286 = 356
        (2) 483 – 286 + 159 = 356
    (b) (1) 276 + 288 + 951 = 1 515
        (2) 276 + 951+ 288 = 1 515
    (c) (1) 776 – 288 – 259 = 229
        (2) 776 – 259 – 288 = 229

    If you repeat a calculation with a different but equivalent keystroke sequence (e.g. a different order), you get the same answer. So it is a way to check your answer.



9. In each case, first estimate the answer like Cyndi did. Then calculate the answer using your calculator, and decide if your answer seems about right.

(a) 3 456 + 4 567	(b) 34 567 + 45 678
(c) 34 × 56	(d) 678 × 234
(e) 123 456 + 257 257	(f) 34 527 + 426 426

Using the calculator to check the calculator

Because it is so easy to make mistakes, it is important that you check your calculator answers.

It usually is not a good idea to check a calculation by just repeating it, because you often make the same mistake again. It is better to check by using a different method the second time.

One way to check is to do the calculation in a different order.

10. Do the following calculations on your calculator in the given order and draw a conclusion.
    (a) (1) 483 + 159 – 286
        (2) 483 – 286 + 159
    (b) (1) 276 + 288 + 951
        (2) 276 + 951 + 288
    (c) (1) 776 – 288 – 259
        (2) 776 – 259 – 288

Two different keystroke sequences that give the same answer are called **equivalent sequences**.

You can check calculator results using the rearrangement principle: if you repeat the calculation with a different (but equivalent) keystroke sequence, you will get the same answer.

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Answers

11. (a) 27 504 (b) 9 932
    (c) 20 932 (d) 3 360
    (e) 125 187 (f) 59 465

Teaching guidelines

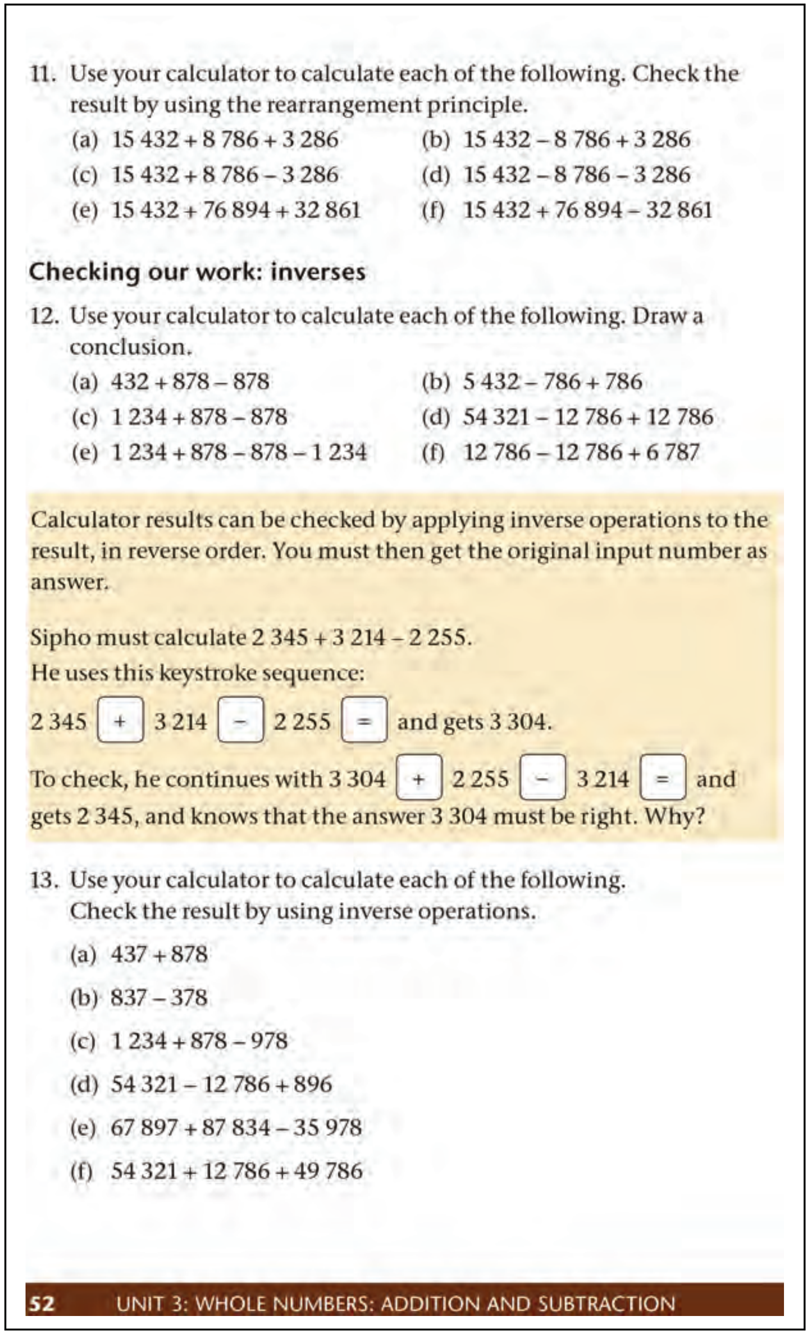
Checking by using inverses is not only useful as a technique to check work done on the calculator, it also provides learners with a useful experience of the idea of inverse operations.

Answers

12. (a) 432 (b) 5 432 (c) 1 234
    (d) 54 321 (e) 0 (f) 6 787

    If you apply the inverse operations in the reverse order to the calculator answer, you will get the original input number as an answer. This is therefore a way to check your work.

13. (a) 1 315 (b) 459 (c) 1 134
    (d) 42 431 (e) 119 753 (f) 116 893



11. Use your calculator to calculate each of the following. Check the result by using the rearrangement principle.
    (a) 15 432 + 8 786 + 3 286 (b) 15 432 – 8 786 + 3 286
    (c) 15 432 + 8 786 – 3 286 (d) 15 432 – 8 786 – 3 286
    (e) 15 432 + 76 894 + 32 861 (f) 15 432 + 76 894 – 32 861

Checking our work: inverses

12. Use your calculator to calculate each of the following. Draw a conclusion.
    (a) 432 + 878 – 878 (b) 5 432 – 786 + 786
    (c) 1 234 + 878 – 878 (d) 54 321 – 12 786 + 12 786
    (e) 1 234 + 878 – 878 – 1 234 (f) 12 786 – 12 786 + 6 787

Calculator results can be checked by applying inverse operations to the result, in reverse order. You must then get the original input number as answer.

Sipho must calculate 2 345 + 3 214 – 2 255.
He uses this keystroke sequence:

2 345 $\boxed{+}$ 3 214 $\boxed{-}$ 2 255 $\boxed{=}$ and gets 3 304.

To check, he continues with 3 304 $\boxed{+}$ 2 255 $\boxed{-}$ 3 214 $\boxed{=}$ and gets 2 345, and knows that the answer 3 304 must be right. Why?

13. Use your calculator to calculate each of the following. Check the result by using inverse operations.
    (a) 437 + 878
    (b) 837 – 378
    (c) 1 234 + 878 – 978
    (d) 54 321 – 12 786 + 896
    (e) 67 897 + 87 834 – 35 978
    (f) 54 321 + 12 786 + 49 786

Mathematical notes

There is one important difference between arithmetic language and calculator language. Some simple calculators are not programmed to interpret and perform multiplication first in a calculation plan where multiplication is not specified first. For example, the keystroke sequence $\boxed{3}\ \boxed{+}\ \boxed{4}\ \boxed{\times}\ \boxed{5}\ \boxed{=}$ will not produce the correct answer for the calculation plan 3 + 4 × 5 because the calculator is not programmed to "know" that in this case multiplication is to be performed first. The calculator will calculate (3 + 4) × 5 when the keystroke sequence $\boxed{3}\ \boxed{+}\ \boxed{4}\ \boxed{\times}\ \boxed{5}\ \boxed{=}$ is entered, and produce the answer 35.

While it is not necessary to address this issue when learners begin to use calculators in class, you will have to address it at some stage. Scientific calculators, however, are programmed to interpret calculation plans as we do, hence they will correctly perform 3 + 4 × 5 when the keystroke sequence $\boxed{3}\ \boxed{+}\ \boxed{4}\ \boxed{\times}\ \boxed{5}\ \boxed{=}$ is entered.

Answers

14. Individual work. Answer in shaded passage.

15. (a) 3 360 (b) 9 932 (c) 27 504
(d) 20 932 (e) 5 606 (f) 4 276

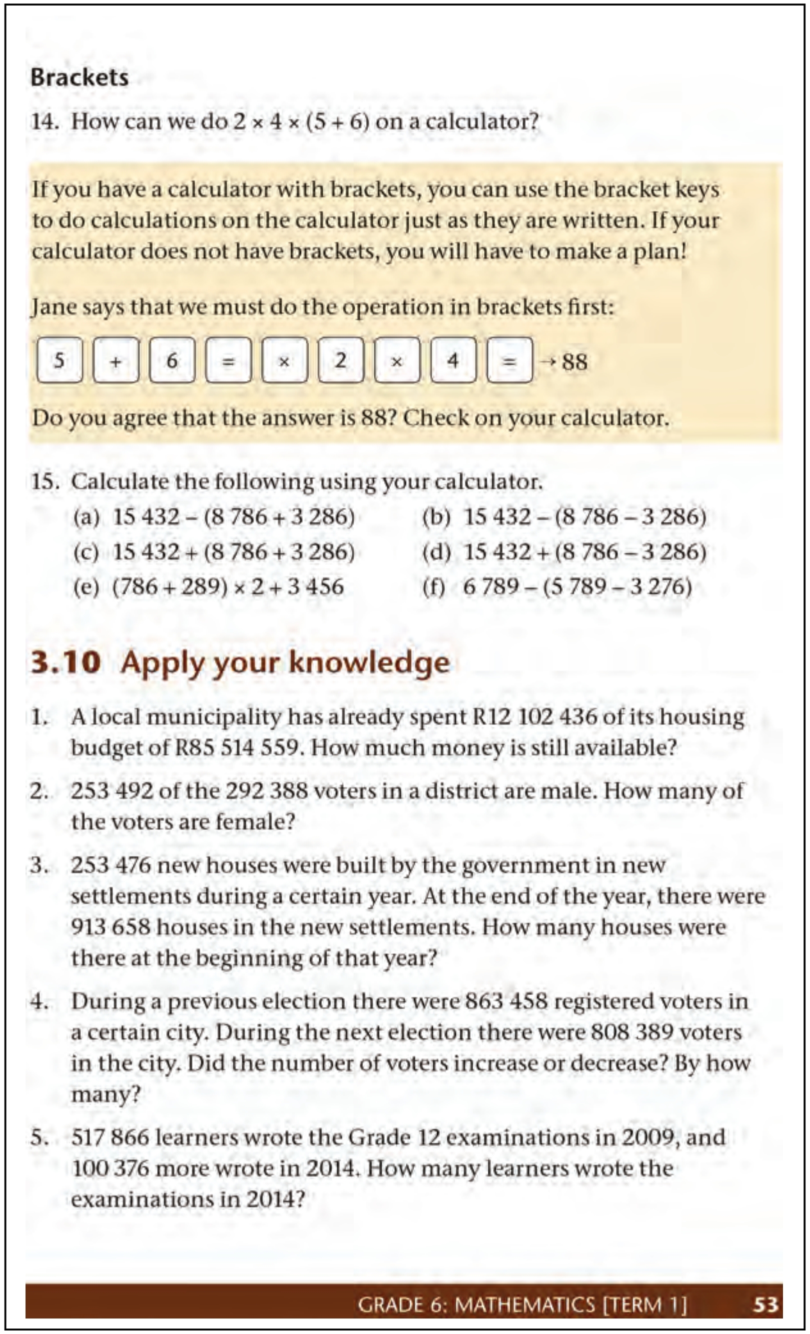
3.10 Apply your knowledge

Teaching guidelines

Please see the notes on the next page.

Answers

1. R73 412 123
2. 38 896 voters are female.
3. 660 182 houses
4. The number of voters decreased by 55 069.
5. 618 242 learners



Brackets

14. How can we do 2 × 4 × (5 + 6) on a calculator?

If you have a calculator with brackets, you can use the bracket keys to do calculations on the calculator just as they are written. If your calculator does not have brackets, you will have to make a plan!

Jane says that we must do the operation in brackets first:

$\boxed{5}\ \boxed{+}\ \boxed{6}\ \boxed{=}\ \boxed{\times}\ \boxed{2}\ \boxed{\times}\ \boxed{4}\ \boxed{=} \rightarrow 88$

Do you agree that the answer is 88? Check on your calculator.

15. Calculate the following using your calculator.
    (a) 15 432 – (8 786 + 3 286) (b) 15 432 – (8 786 – 3 286)
    (c) 15 432 + (8 786 + 3 286) (d) 15 432 + (8 786 – 3 286)
    (e) (786 + 289) × 2 + 3 456 (f) 6 789 – (5 789 – 3 276)

3.10 Apply your knowledge

1. A local municipality has already spent R12 102 436 of its housing budget of R85 514 559. How much money is still available?
2. 253 492 of the 292 388 voters in a district are male. How many of the voters are female?
3. 253 476 new houses were built by the government in new settlements during a certain year. At the end of the year, there were 913 658 houses in the new settlements. How many houses were there at the beginning of that year?
4. During a previous election there were 863 458 registered voters in a certain city. During the next election there were 808 389 voters in the city. Did the number of voters increase or decrease? By how many?
5. 517 866 learners wrote the Grade 12 examinations in 2009, and 100 376 more wrote in 2014. How many learners wrote the examinations in 2014?

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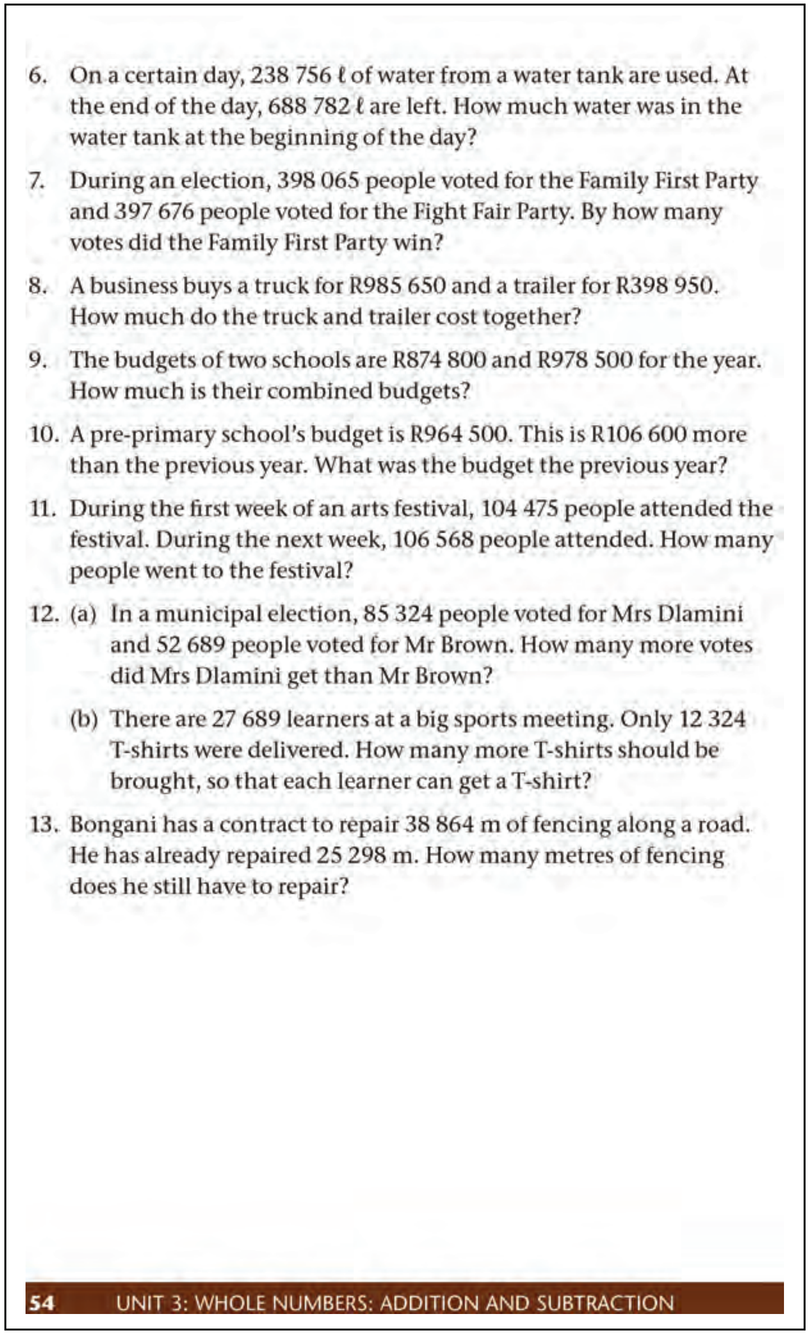
Notes on questions

The word problems in Section 3.10 are about situations of the following types:

- Decreasing a quantity, result unknown: question 1
  initial quantity – decrease = ?
- Finding the missing component in a given combination of quantities: question 2.
  one component + ? = total
- Increasing a quantity, initial quantity unknown: questions 3 and 10
  ? + increase = result
- Decreasing a quantity, decrease unknown: question 4
  initial quantity – ? = result
- Combining two quantities: questions 5, 8, 9 and 11
  one quantity + another quantity = ?
- Decreasing a quantity by a given amount, initial quantity unknown: question 6
  ? – decrease = result
- Finding the difference between two quantities: questions 7 and 12(a)
  one quantity – another quantity = ?
- Establishing how much more than a given quantity is needed to attain a given larger quantity (establishing a "shortfall"): questions 12(b) and 13.
  initial quantity + ? = given end quantity

Answers

6. 927 538 ℓ
7. 389 votes
8. R1 384 600
9. R1 853 300
10. R857 900
11. 211 043 people
12. (a) 32 635 more votes
    (b) 15 365 T-shirts
13. 13 566 m



6. On a certain day, 238 756 ℓ of water from a water tank are used. At the end of the day, 688 782 ℓ are left. How much water was in the water tank at the beginning of the day?
7. During an election, 398 065 people voted for the Family First Party and 397 676 people voted for the Fight Fair Party. By how many votes did the Family First Party win?
8. A business buys a truck for R985 650 and a trailer for R398 950. How much do the truck and trailer cost together?
9. The budgets of two schools are R874 800 and R978 500 for the year. How much is their combined budgets?
10. A pre-primary school's budget is R964 500. This is R106 600 more than the previous year. What was the budget the previous year?
11. During the first week of an arts festival, 104 475 people attended the festival. During the next week, 106 568 people attended. How many people went to the festival?
12. (a) In a municipal election, 85 324 people voted for Mrs Dlamini and 52 689 people voted for Mr Brown. How many more votes did Mrs Dlamini get than Mr Brown?
    (b) There are 27 689 learners at a big sports meeting. Only 12 324 T-shirts were delivered. How many more T-shirts should be brought, so that each learner can get a T-shirt?
13. Bongani has a contract to repair 38 864 m of fencing along a road. He has already repaired 25 298 m. How many metres of fencing does he still have to repair?

54 UNIT 3: WHOLE NUMBERS: ADDITION AND SUBTRACTION

Grade 6 Term 1 Unit 4 Common fractions

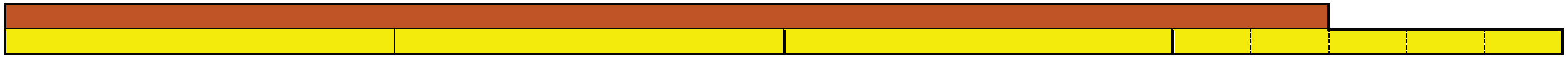
Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
4.1 Dividing into fraction parts	Understanding the concept of fractions and equivalent fractions	55 to 59
4.2 Measuring lengths accurately	Working with fractions as sub-units of measurement	60 to 62
4.3 Comparing and ordering fractions	Comparing and ordering fractions using fraction strips and representing fractions on a number line	63 to 65
4.4 Hundredths	Working with hundredths in preparation for the decimal notation of fractions	65 to 67
4.5 Adding and subtracting fractions	Adding and subtracting fractions using equivalent fractions	68 to 69
4.6 Problem solving	Solving word problems involving fractions	70 to 71

CAPS time allocation	10 hours
CAPS page references	16 and 226 to 227

Mathematical background

Fractions were most likely invented to facilitate accurate measurement in cases where the commonly used standard unit of measurement could not provide an exact description of a quantity. This is evident if we look at the Latin names of our existing units of measurement, i.e. centimetre (hundredth of a metre) and millimetre (thousandth of a metre).

Look at the brown strip below. If we measure it with this yellow strip  as a unit, its length is 3 and 2 fifths of the yellow unit.



This example demonstrates how **fractions are used as measures**.

Fractions help us in other ways too. Mathematically, the fraction concept is very important to our understanding of decimals, because **the place value parts after the decimal comma are fractions**. For example, the expanded notation for the number 23,47 is $20 + 3 + \frac{4}{10} + \frac{7}{100}$ or 2 tens + 3 units + 4 tenths + 7 hundredths.

Fractions are also used to describe parts of collections and parts of non-physical quantities, for example “3 eighths of the learners in a school” or “63 hundredths of the available marks”. In the latter case, we usually use the percentage notation (%) for hundredths.

In everyday life and everyday language, we sometimes use words such as “half” and “quarter” to indicate *approximate* **parts of whole objects or collections**. People may, for example, refer to a “quarter of an apple” or “half a loaf of bread”. But this is generally not mathematically accurate language. Although this everyday use of fraction language is different from the mathematical use because the fraction words are not used to indicate precise parts, the everyday use provides us with a useful starting point for learning about fractions.

4.1 Dividing into fraction parts

Possible misconceptions

Learners' concept of fractions is often muddled by the idea that a fraction consists of or is made up of two numbers – for example, "3 fifths" is made up of the numbers 3 and 5 – without understanding the totally different *roles* of the two numbers. This misconception is often supported by misleading language, such as referring to $\frac{3}{5}$ as "three over five" instead of "3 fifths". Make sure that you use the proper fraction name, for example "3 fifths", as it supports learners' understanding of what the denominator of a fraction is.

Notes on questions

Question 1 is intended to refresh learners' knowledge of certain number facts that will help them to work effectively with questions 2 to 5.

Questions 2 to 5 involve division as equal sharing, which is an important context for the concept of fractions.

Teaching guidelines

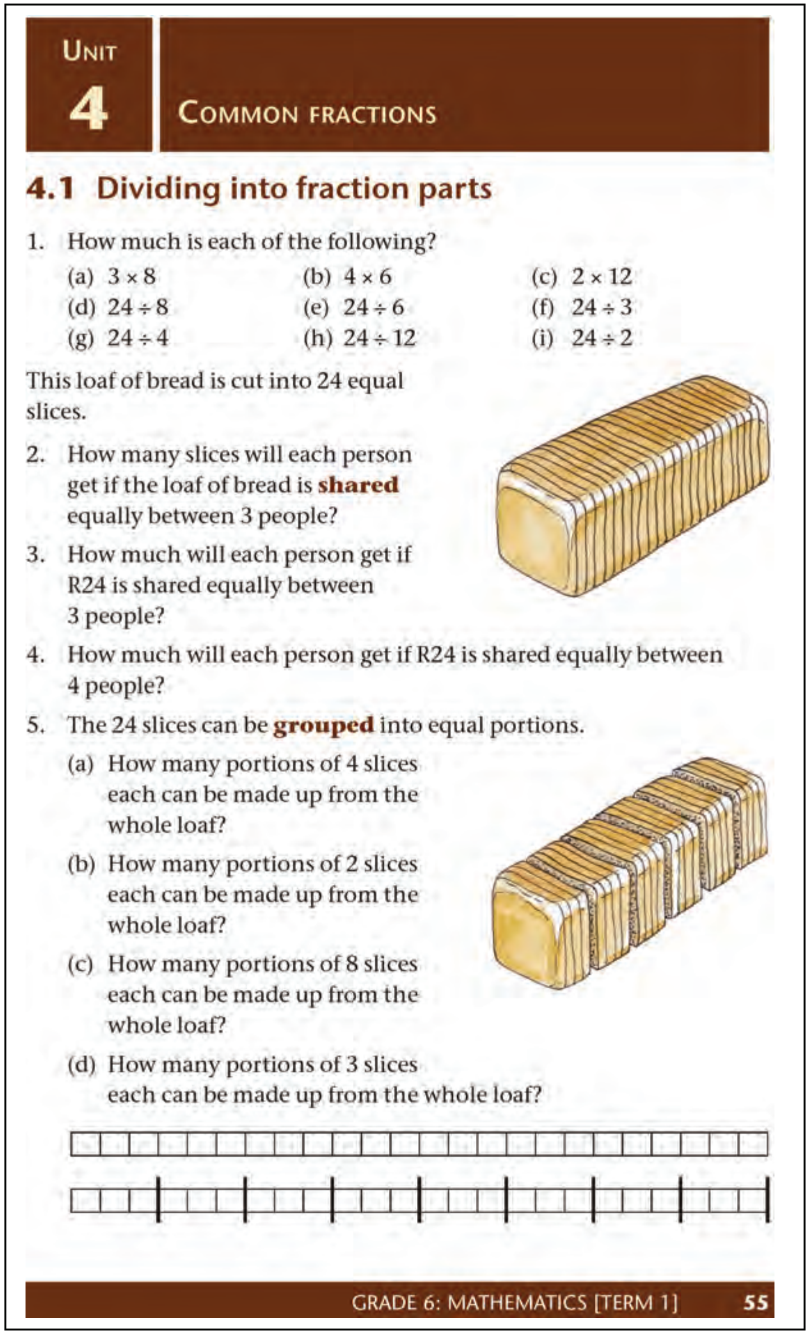
Learners may use the fraction strips at the bottom of the Learner Book page to help them answer the questions. Question 5(d) is represented by the bottom fraction strip. The fraction strip above it is more generally helpful.

Once learners have completed questions 2 to 5, ask them questions such as "What fraction of the loaf does each person get in question 2?", to develop their understanding of the link between division and fractions.

Asking questions such as these will promote their understanding of fractions as parts of wholes (the loaf) and parts of collections (the slices).

Answers

1. (a) 24 (b) 24 (c) 24
   (d) 3 (e) 4 (f) 8
   (g) 6 (h) 2 (i) 12
2. 8 slices
3. R8
4. R6
5. (a) 6 portions (b) 12 portions
   (c) 3 portions (d) 8 portions



UNIT 4

COMMON FRACTIONS

4.1 Dividing into fraction parts

1. How much is each of the following?
   (a) 3 × 8 (b) 4 × 6 (c) 2 × 12
   (d) 24 ÷ 8 (e) 24 ÷ 6 (f) 24 ÷ 3
   (g) 24 ÷ 4 (h) 24 ÷ 12 (i) 24 ÷ 2

This loaf of bread is cut into 24 equal slices.

2. How many slices will each person get if the loaf of bread is **shared** equally between 3 people?
3. How much will each person get if R24 is shared equally between 3 people?
4. How much will each person get if R24 is shared equally between 4 people?
5. The 24 slices can be **grouped** into equal portions.
   (a) How many portions of 4 slices each can be made up from the whole loaf?
   (b) How many portions of 2 slices each can be made up from the whole loaf?
   (c) How many portions of 8 slices each can be made up from the whole loaf?
   (d) How many portions of 3 slices each can be made up from the whole loaf?

GRADE 6: MATHEMATICS [TERM 1] 55

Critical knowledge and skills

Learners must be able to represent fractions diagrammatically by drawing fraction strips. They should be able to draw freehand and quickly enough so that it does not take up much time. This will provide them with a tool to think of fractions in terms of what they really are. It is very important though, that learners do not spend excessive time on drawing fraction strips accurately. These diagrams are not normally used for making measurements; they are only used to support thinking conceptually about fractions.

Possible misconceptions

The use of the proper fraction name, for example "3 fifths", supports learners in understanding what the denominator of a fraction is. Encourage learners not to be put off by the fact that the word "fifths" is a little difficult to pronounce – it simply *must* be said aloud for proper mathematical understanding.

Notes on questions

Questions 6, 7 and 8 will contribute to learners' awareness of equivalent fractions. You may use this opportunity to discuss equivalent fractions in class, but do note that the concept of equivalent fractions is dealt with in detail later on.

Answers

6. (a) 4 fifteenths of a loaf (b) 1 fifth of a loaf (c) They are the same.
(d) 10 fifteenths of a loaf (e) They are the same. (f) 1 fifth of a loaf

7. (a)

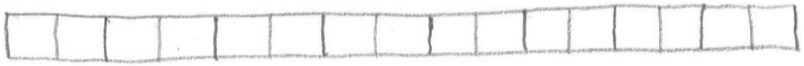
(b)

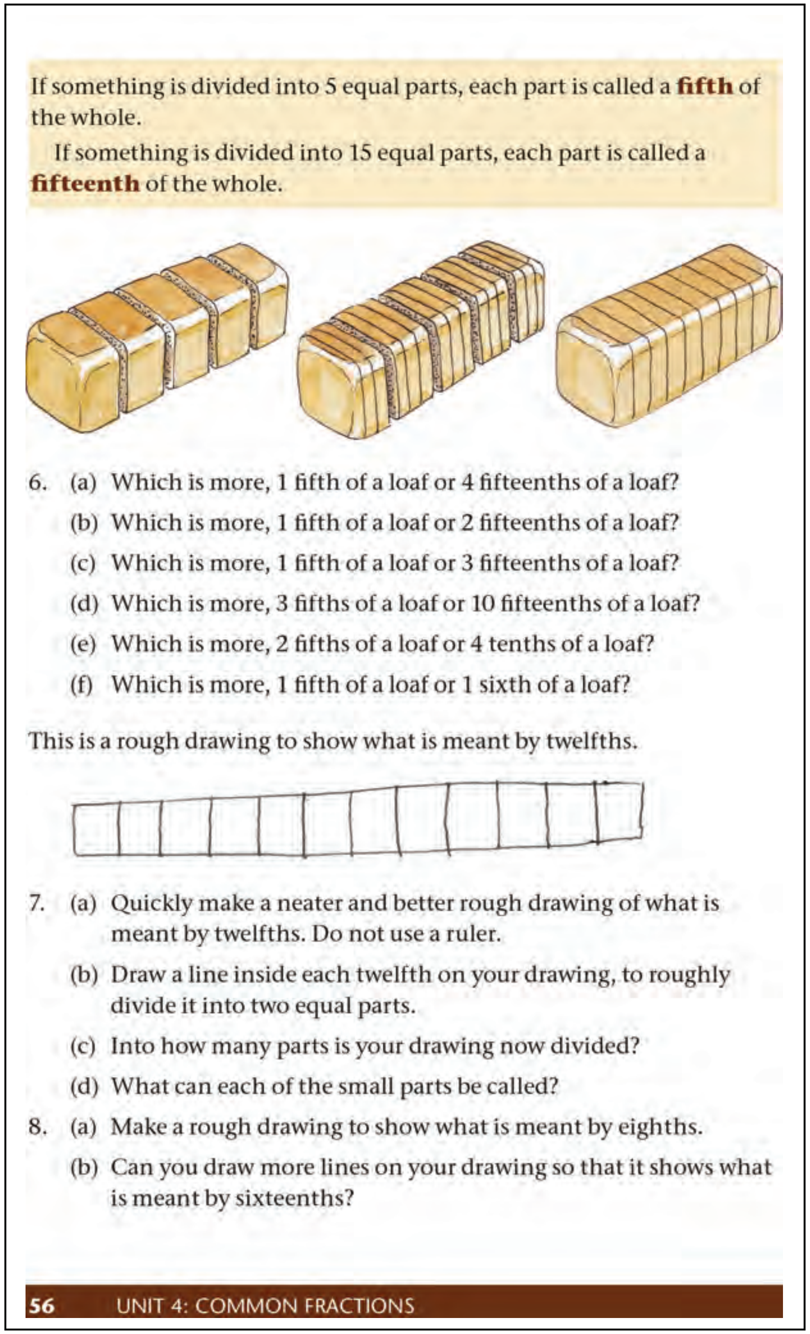
(c) 24 parts

(d) 1 twenty-fourth

8. (a)

(b)





If something is divided into 5 equal parts, each part is called a **fifth** of the whole.

If something is divided into 15 equal parts, each part is called a **fifteenth** of the whole.

6. (a) Which is more, 1 fifth of a loaf or 4 fifteenths of a loaf?
(b) Which is more, 1 fifth of a loaf or 2 fifteenths of a loaf?
(c) Which is more, 1 fifth of a loaf or 3 fifteenths of a loaf?
(d) Which is more, 3 fifths of a loaf or 10 fifteenths of a loaf?
(e) Which is more, 2 fifths of a loaf or 4 tenths of a loaf?
(f) Which is more, 1 fifth of a loaf or 1 sixth of a loaf?

This is a rough drawing to show what is meant by twelfths.

7. (a) Quickly make a neater and better rough drawing of what is meant by twelfths. Do not use a ruler.
(b) Draw a line inside each twelfth on your drawing, to roughly divide it into two equal parts.
(c) Into how many parts is your drawing now divided?
(d) What can each of the small parts be called?

8. (a) Make a rough drawing to show what is meant by eighths.
(b) Can you draw more lines on your drawing so that it shows what is meant by sixteenths?

56 UNIT 4: COMMON FRACTIONS

Teaching guidelines

Discuss with learners that when we talk about fraction parts of loaves of bread, these are only *approximate* fractions.

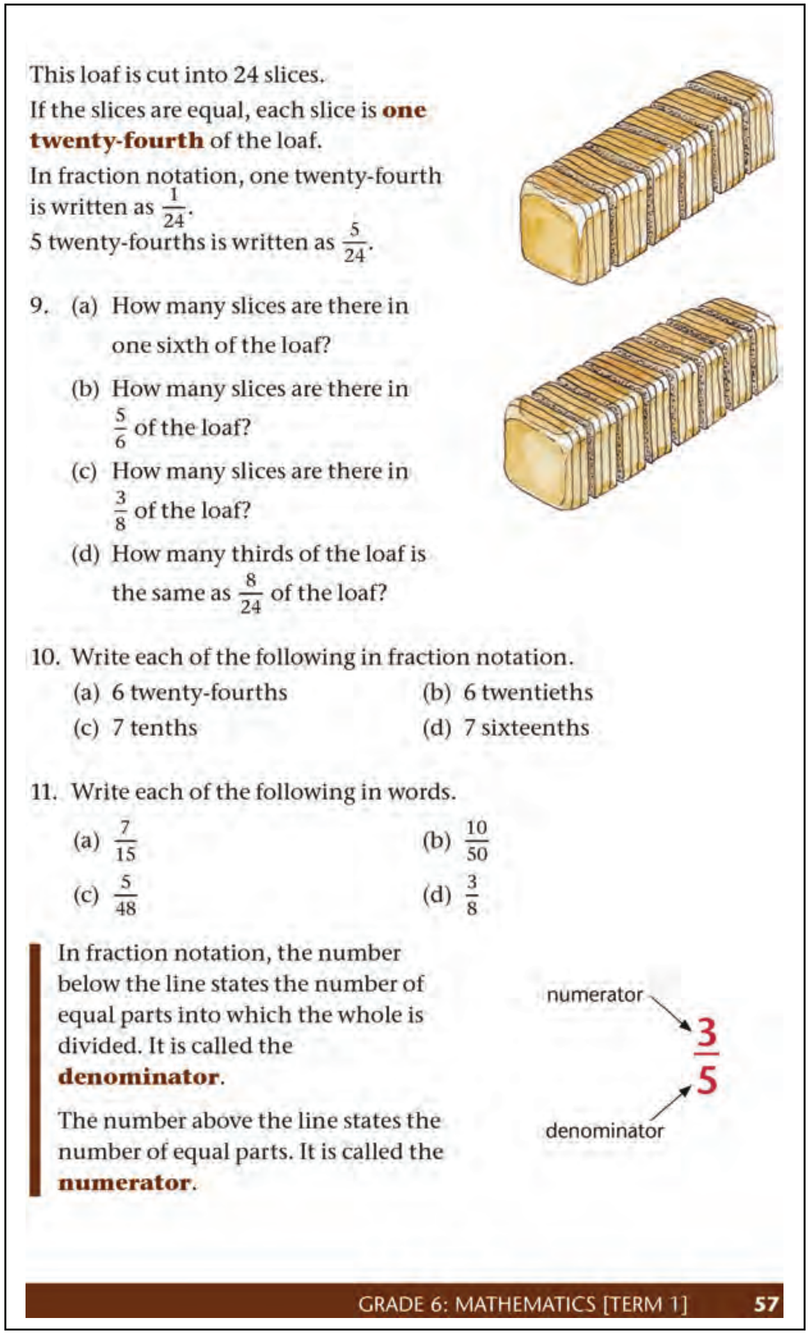
Once learners have completed questions 9 to 11, consolidate their understanding of the common fraction notation by explaining the meaning of the terms **numerator** and **denominator**. The denominator tells us the number of equal parts into which the whole is divided. The numerator tells us the **number** of equal parts with which we are dealing within the whole.

Possible misconceptions

We use fraction terminology almost daily, for example “a quarter of an apple” or “half a loaf of bread”. These terms refer to the *approximate* parts. For example, when people refer to a quarter of an apple, it is seldom exactly a quarter. Using this terminology on a daily basis will help develop learners’ knowledge of fractions. It may, however, weaken their understanding of the mathematical meaning of fractions as “*exact* fractional parts” of wholes, collections, quantities and units of measurement.

Answers

9. (a) 4 slices (b) 20 slices
   (c) 9 slices (d) 1 third
10. (a) $\frac{6}{24}$ (b) $\frac{6}{20}$
    (c) $\frac{7}{10}$ (d) $\frac{7}{16}$
11. (a) Seven fifteenths (b) Ten fiftieths
    (c) Five forty-eighths (d) Three eighths



This loaf is cut into 24 slices.
If the slices are equal, each slice is **one twenty-fourth** of the loaf.
In fraction notation, one twenty-fourth is written as $\frac{1}{24}$.
5 twenty-fourths is written as $\frac{5}{24}$.

9. (a) How many slices are there in one sixth of the loaf?
   (b) How many slices are there in $\frac{5}{6}$ of the loaf?
   (c) How many slices are there in $\frac{3}{8}$ of the loaf?
   (d) How many thirds of the loaf is the same as $\frac{8}{24}$ of the loaf?

10. Write each of the following in fraction notation.
    (a) 6 twenty-fourths (b) 6 twentieths
    (c) 7 tenths (d) 7 sixteenths

11. Write each of the following in words.
    (a) $\frac{7}{15}$ (b) $\frac{10}{50}$
    (c) $\frac{5}{48}$ (d) $\frac{3}{8}$

In fraction notation, the number below the line states the number of equal parts into which the whole is divided. It is called the **denominator**.

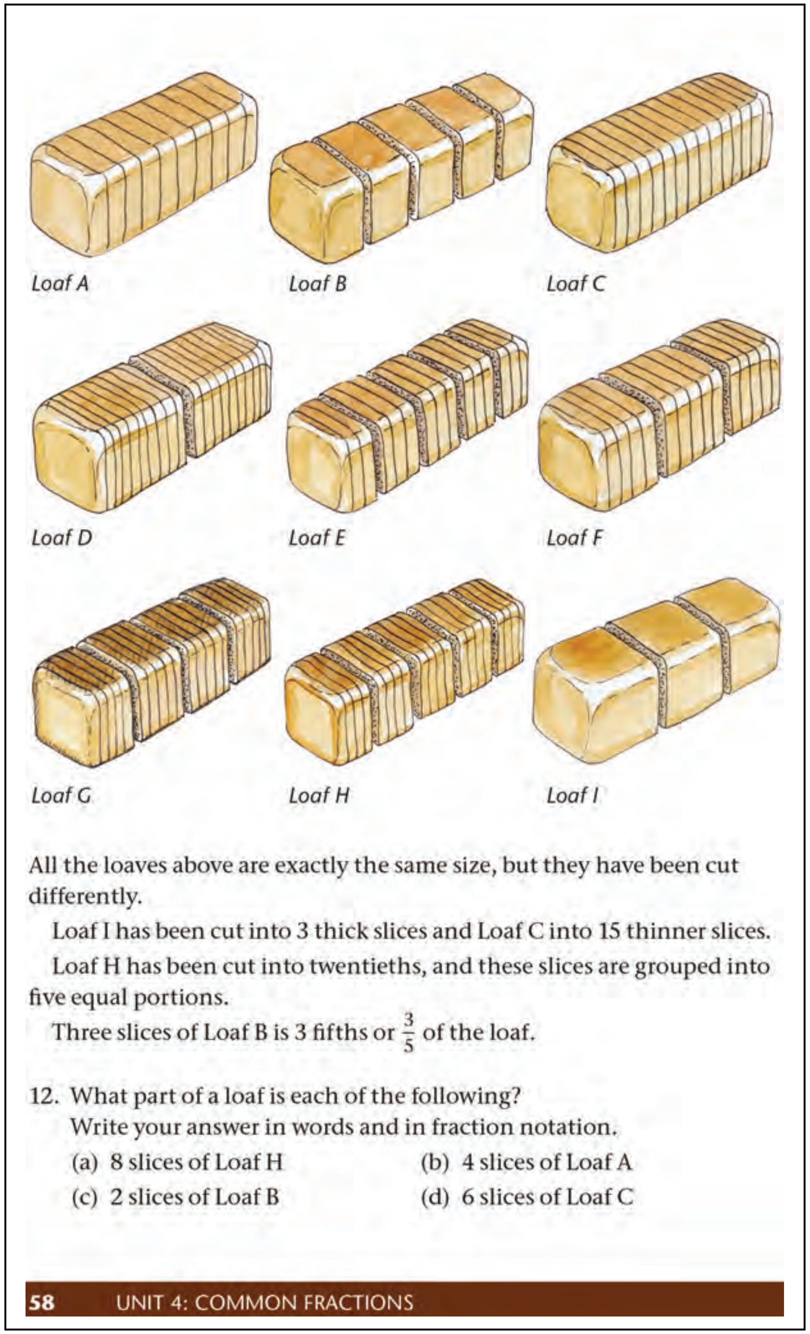
The number above the line states the number of equal parts. It is called the **numerator**.

Teaching guidelines

Discuss with the class that two fifths can look very different in real situations, for example in Loaves B, E and H. The amount of bread in each of the five parts of these loaves is the same. The number of slices, however, differs.

Answers

12. (a) $\frac{8}{20}$; eight twentieths (or $\frac{2}{5}$; two fifths)
    (b) $\frac{4}{10}$; four tenths (or $\frac{2}{5}$; two fifths)
    (c) $\frac{2}{5}$; two fifths
    (d) $\frac{6}{15}$; six fifteenths (or $\frac{2}{5}$; two fifths)



Loaf A *Loaf B* *Loaf C*

Loaf D *Loaf E* *Loaf F*

Loaf G *Loaf H* *Loaf I*

All the loaves above are exactly the same size, but they have been cut differently.

Loaf I has been cut into 3 thick slices and Loaf C into 15 thinner slices.

Loaf H has been cut into twentieths, and these slices are grouped into five equal portions.

Three slices of Loaf B is 3 fifths or $\frac{3}{5}$ of the loaf.

12. What part of a loaf is each of the following?
    Write your answer in words and in fraction notation.
    (a) 8 slices of Loaf H
    (b) 4 slices of Loaf A
    (c) 2 slices of Loaf B
    (d) 6 slices of Loaf C

Notes on questions

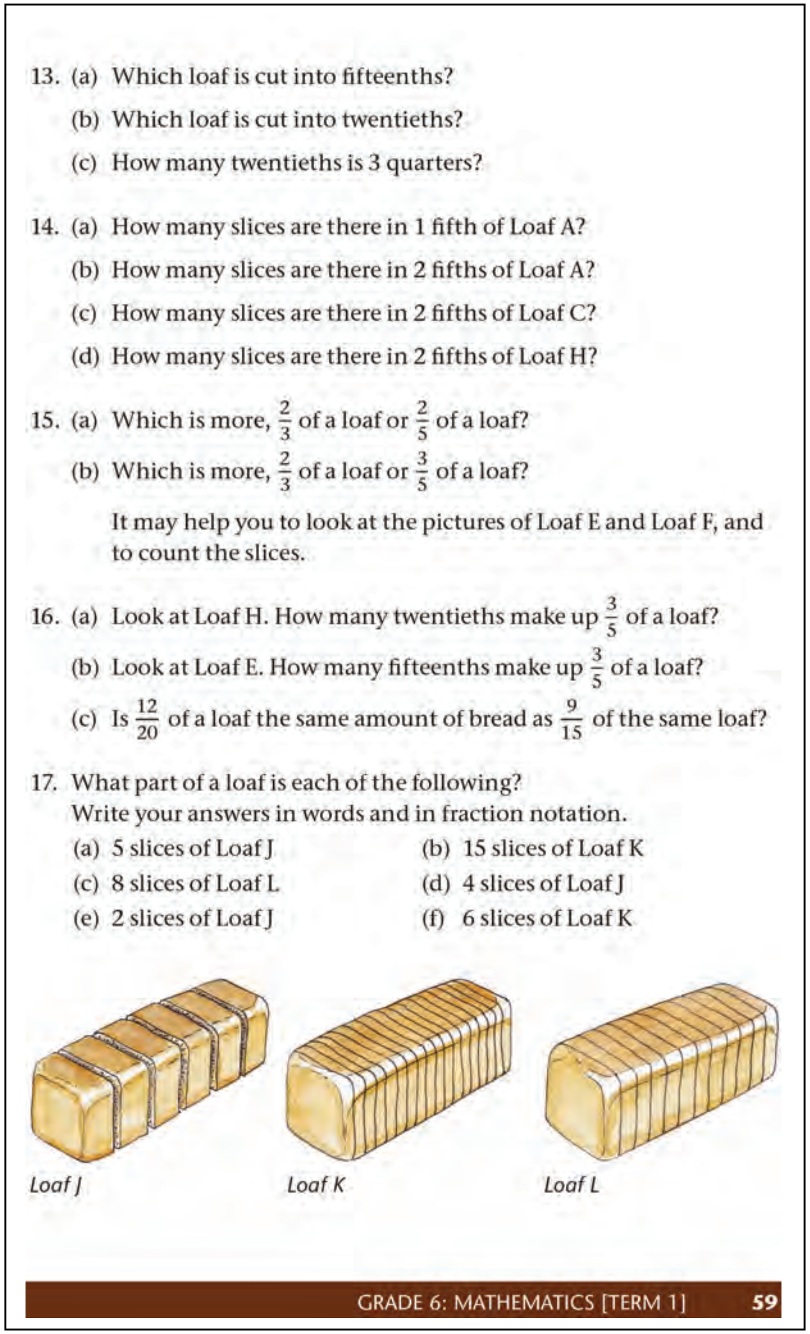
Note that question 14 supports learners in their understanding of equivalent fractions. Once learners have completed question 14, you may ask them whether or not the slices are the same size in the different loaves (they are not), and whether the amounts of bread described in (b), (c) and (d) are the same or not (they are).

Question 15 focuses on the meaning of the denominator. If learners experience difficulties with this question in spite of looking at the pictures of the loaves on the previous page, it may be because they consider the denominators without keeping the meaning of the denominators in mind. The denominator 5 indicates smaller parts (fifths) than the denominator 3 (thirds), although the number 5 is a bigger number than the number 3. A "bigger" denominator indicates smaller parts.

Questions 16 and 17 provide for further development of the concept of equivalent fractions. Once learners have completed question 17, ask them to write some of their answers in different ways.

Answers

13. (a) Loaves C, E and F (b) Loaves D, G and H
(c) 15 twentieths
14. (a) Two slices (b) Four slices
(c) Six slices (d) Eight slices
15. (a) $\frac{2}{3}$ of a loaf (b) $\frac{2}{3}$ of a loaf
16. (a) 12 twentieths ($\frac{12}{20}$) (b) 9 fifteenths ($\frac{9}{15}$)
(c) Yes, $\frac{12}{20} = \frac{9}{15}$
17. (a) Five sixths; $\frac{5}{6}$ (b) Fifteen eighteenths; $\frac{15}{18}$
(c) Eight twelfths; $\frac{8}{12}$ (d) Four sixths; $\frac{4}{6}$
(e) Two sixths; $\frac{2}{6}$ (f) Six eighteenths; $\frac{6}{18}$



13. (a) Which loaf is cut into fifteenths?
(b) Which loaf is cut into twentieths?
(c) How many twentieths is 3 quarters?

14. (a) How many slices are there in 1 fifth of Loaf A?
(b) How many slices are there in 2 fifths of Loaf A?
(c) How many slices are there in 2 fifths of Loaf C?
(d) How many slices are there in 2 fifths of Loaf H?

15. (a) Which is more, $\frac{2}{3}$ of a loaf or $\frac{2}{5}$ of a loaf?
(b) Which is more, $\frac{2}{3}$ of a loaf or $\frac{3}{5}$ of a loaf?

It may help you to look at the pictures of Loaf E and Loaf F, and to count the slices.

16. (a) Look at Loaf H. How many twentieths make up $\frac{3}{5}$ of a loaf?
(b) Look at Loaf E. How many fifteenths make up $\frac{3}{5}$ of a loaf?
(c) Is $\frac{12}{20}$ of a loaf the same amount of bread as $\frac{9}{15}$ of the same loaf?

17. What part of a loaf is each of the following?
Write your answers in words and in fraction notation.
(a) 5 slices of Loaf J (b) 15 slices of Loaf K
(c) 8 slices of Loaf L (d) 4 slices of Loaf J
(e) 2 slices of Loaf J (f) 6 slices of Loaf K

Loaf J *Loaf K* *Loaf L*

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4.2 Measuring lengths accurately

Mathematical notes

Understanding fractions as parts of units of measurement is critically important and provides the conceptual basis for understanding decimal fractions (addressed in Term 2). Using fractional units of measurement also provides an empowering context for understanding equivalent fractions, in the sense that the same length (or other quantity) can be expressed in different ways (particularly fractional parts of units of measurement).

As mentioned before, fractions were most likely invented to facilitate accurate measurement in cases where the commonly used standard unit of measurement could not provide an exact description of a quantity.

Notes on questions

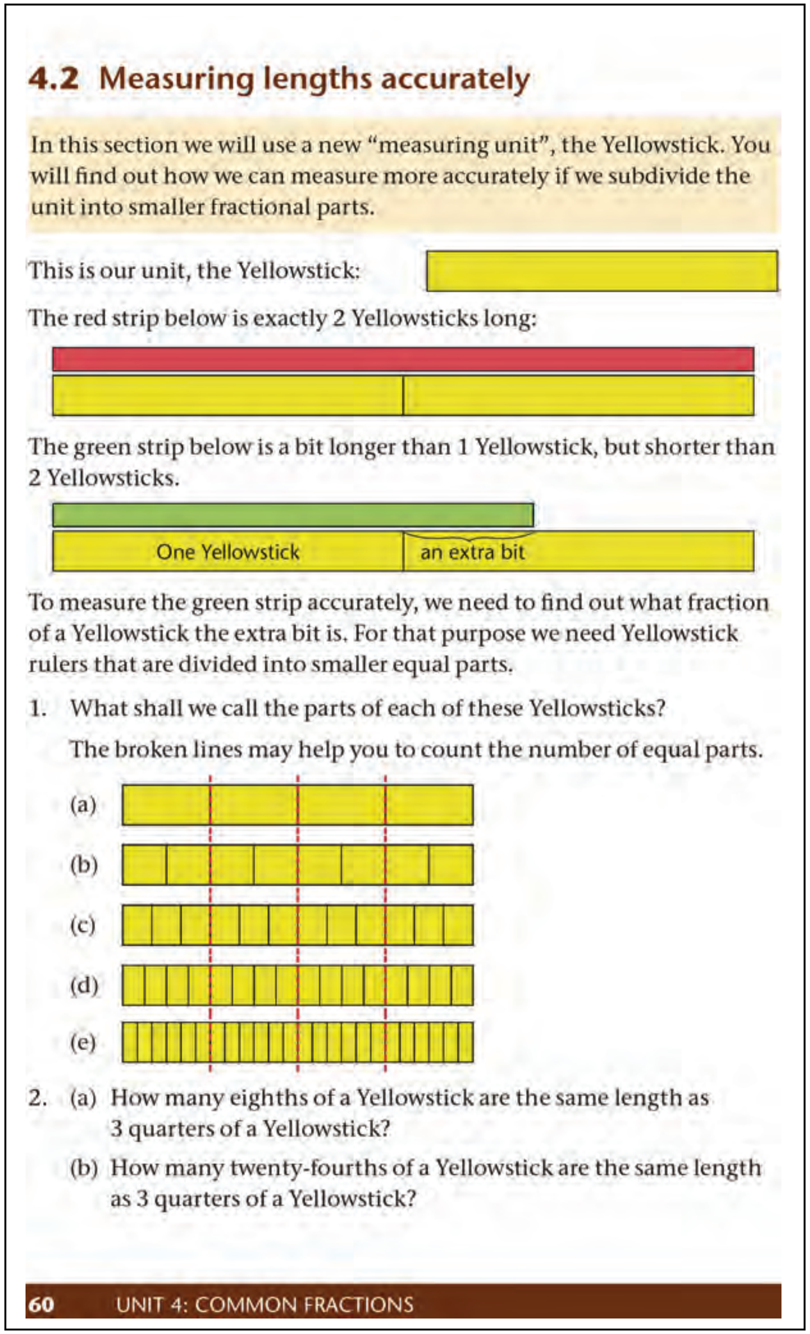
The measurement tasks in this section also serve as a development of the concept of equivalent fractions.

If learners have difficulty with question 1, remind them that they need to count the number of equal parts on each yellow strip, in order to know what the parts are called.

Question 2 promotes awareness of the possibility that the same length can be expressed in terms of different fractions and therefore, equivalent fractions. Learners can inspect the diagrams in question 1 to answer question 2. For example, to answer question 2(a), they can look at the diagrams in questions 1(a) and (b) and count how many eighths correspond to 3 quarters.

Answers

1. (a) Quarters (b) Eighths
   (c) Twelfths (d) Sixteenths
   (e) Twenty-fourths
2. (a) 6 eighths (b) 18 twenty-fourths



4.2 Measuring lengths accurately

In this section we will use a new "measuring unit", the Yellowstick. You will find out how we can measure more accurately if we subdivide the unit into smaller fractional parts.

This is our unit, the Yellowstick:

The red strip below is exactly 2 Yellowsticks long:

The green strip below is a bit longer than 1 Yellowstick, but shorter than 2 Yellowsticks.

To measure the green strip accurately, we need to find out what fraction of a Yellowstick the extra bit is. For that purpose we need Yellowstick rulers that are divided into smaller equal parts.

1. What shall we call the parts of each of these Yellowsticks?

   The broken lines may help you to count the number of equal parts.

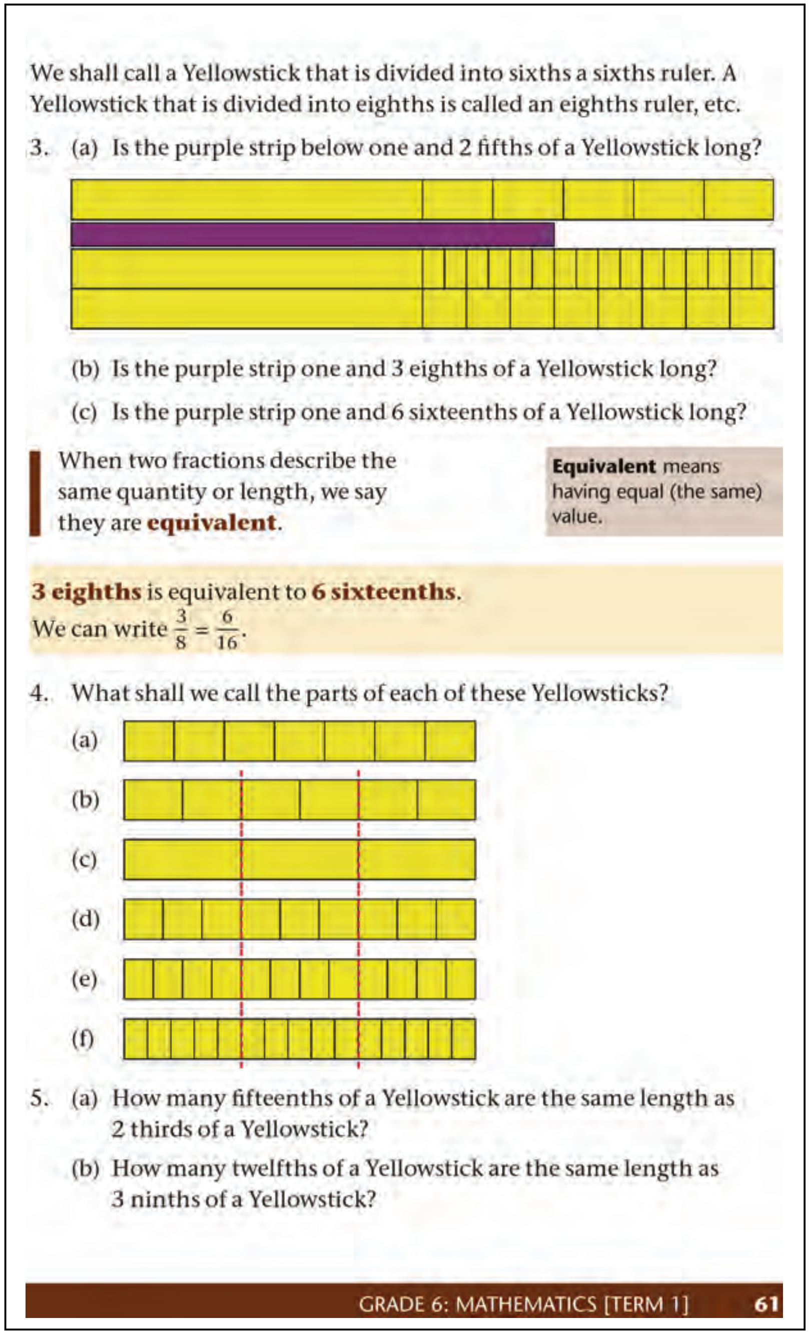
2. (a) How many eighths of a Yellowstick are the same length as 3 quarters of a Yellowstick?
   (b) How many twenty-fourths of a Yellowstick are the same length as 3 quarters of a Yellowstick?

Notes on questions

The purpose of questions 3(b) and (c) is to let learners experience the fact that 3 eighths and 6 sixteenths of a Yellowstick represent the same length.

Answers

3. (a) No, it is a bit shorter than 1 and 2 fifths.
(b) Yes (c) Yes
4. (a) Sevenths (b) Sixths
(c) Thirds (d) Ninths
(e) Twelfths (f) Fifteenths
5. (a) 10 fifteenths (b) 4 twelfths



We shall call a Yellowstick that is divided into sixths a sixths ruler. A Yellowstick that is divided into eighths is called an eighths ruler, etc.

3. (a) Is the purple strip below one and 2 fifths of a Yellowstick long?

(b) Is the purple strip one and 3 eighths of a Yellowstick long?

(c) Is the purple strip one and 6 sixteenths of a Yellowstick long?

When two fractions describe the same quantity or length, we say they are **equivalent**.

Equivalent means having equal (the same) value.

3 eighths is equivalent to **6 sixteenths**.
We can write $\frac{3}{8} = \frac{6}{16}$.

4. What shall we call the parts of each of these Yellowsticks?

(a)

(b)

(c)

(d)

(e)

(f)

5. (a) How many fifteenths of a Yellowstick are the same length as 2 thirds of a Yellowstick?

(b) How many twelfths of a Yellowstick are the same length as 3 ninths of a Yellowstick?

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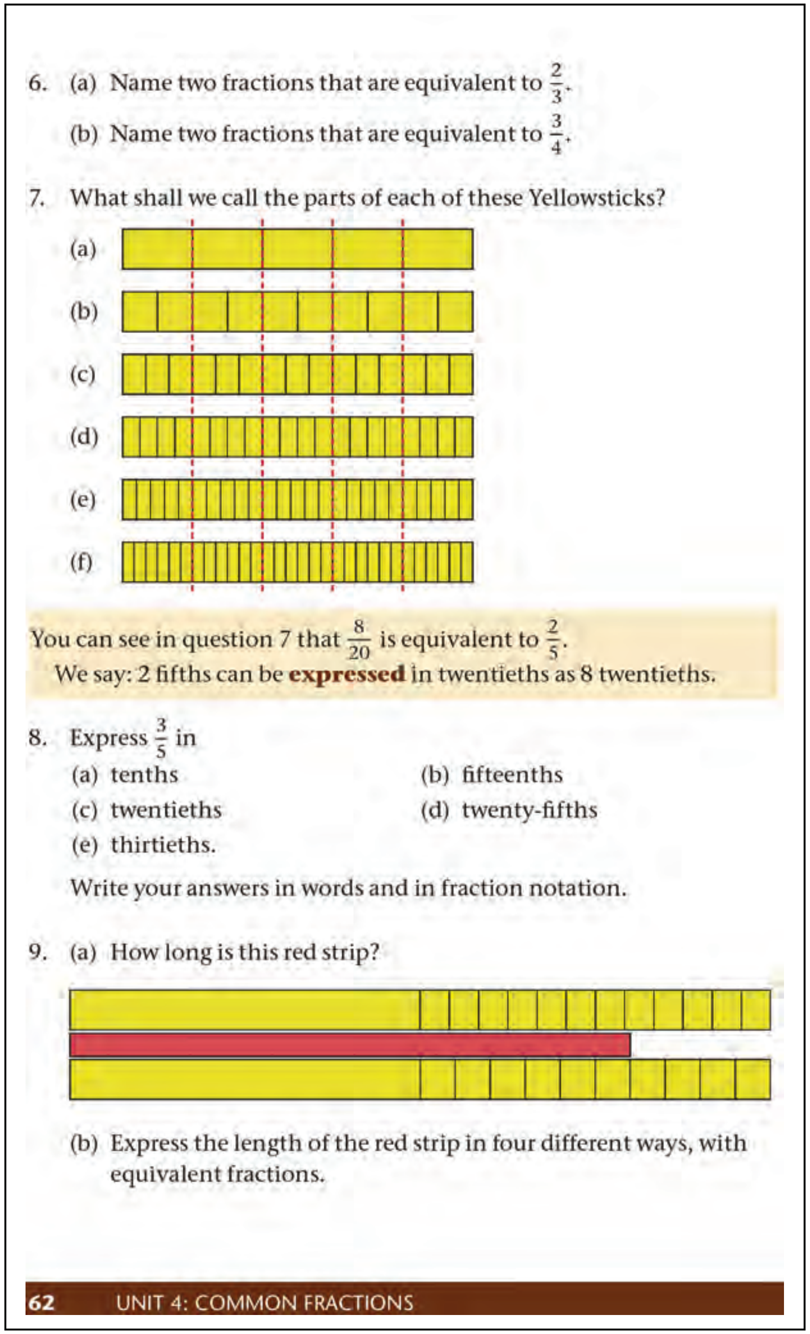
Teaching guidelines

Learners should work individually on these questions. Doing so will strengthen their knowledge and understanding of equivalent fractions.

For example, 2 fifths can also be expressed as 8 twentieths. At this point you may conduct a whole-class discussion and ask learners to use their answers from question 7 to express 2 fifths (or 3 or 4 or 5 fifths) in other ways too, for example 3 fifths may also be expressed as 9 fifteenths.

Answers

6. (a) $\frac{4}{6}$; $\frac{6}{9}$
   (b) Examples: $\frac{6}{8}$; $\frac{9}{12}$; $\frac{12}{16}$
7. (a) Fifths
   (b) Tenths
   (c) Fifteenths
   (d) Twentieths
   (e) Twenty-fifths
   (f) Thirtieths
8. (a) 6 tenths; $\frac{6}{10}$
   (b) 9 fifteenths; $\frac{9}{15}$
   (c) 12 twentieths; $\frac{12}{20}$
   (d) 15 twenty-fifths; $\frac{15}{25}$
   (e) 18 thirtieths; $\frac{18}{30}$
9. (a) $1\frac{6}{10}$ of a Yellowstick
   (b) $1\frac{3}{5}$; $1\frac{6}{10}$; $1\frac{9}{15}$; $1\frac{12}{20}$; $1\frac{15}{25}$



6. (a) Name two fractions that are equivalent to $\frac{2}{3}$.
   (b) Name two fractions that are equivalent to $\frac{3}{4}$.

7. What shall we call the parts of each of these Yellowsticks?

You can see in question 7 that $\frac{8}{20}$ is equivalent to $\frac{2}{5}$.
We say: 2 fifths can be **expressed** in twentieths as 8 twentieths.

8. Express $\frac{3}{5}$ in
   (a) tenths
   (b) fifteenths
   (c) twentieths
   (d) twenty-fifths
   (e) thirtieths.

   Write your answers in words and in fraction notation.

9. (a) How long is this red strip?

   (b) Express the length of the red strip in four different ways, with equivalent fractions.

62 UNIT 4: COMMON FRACTIONS

4.3 Comparing and ordering fractions

Teaching guidelines

When drawing fraction strips, it is best that learners draw the whole strip to begin with, so that they can physically experience the partitioning of the whole strip into equal parts afterwards. This physical experience of partitioning can support their understanding of fractions as the numbers that describe the size of **parts** of wholes.

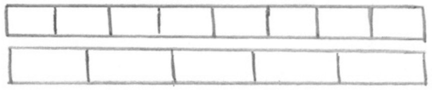
When drawing a fraction strip for an even number of parts, for example eighths, it helps to first draw the line that separates the whole strip into two halves. For quarters, eighths, sixteenths, etc. one can then continue to halve the sections, as shown below on the left.

For a number of parts that is an odd number and multiple of three (for example ninths), the first step could be to draw two lines to partition the whole strip approximately into thirds, as shown below in the middle.

Drawing a fifths-strip is slightly more difficult. It helps to draw a line that divides the whole strip into two parts, with the one part about one-and-a-half times as long as the other, as shown below on the right. You can quickly demonstrate this on the board.

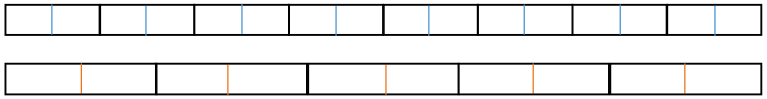
Answers

1. (a)



(b) $\frac{5}{8}$ of a loaf (c) $\frac{2}{5}$ of a loaf (d) $\frac{6}{8}$ of a loaf (e) $\frac{4}{5}$ of a loaf

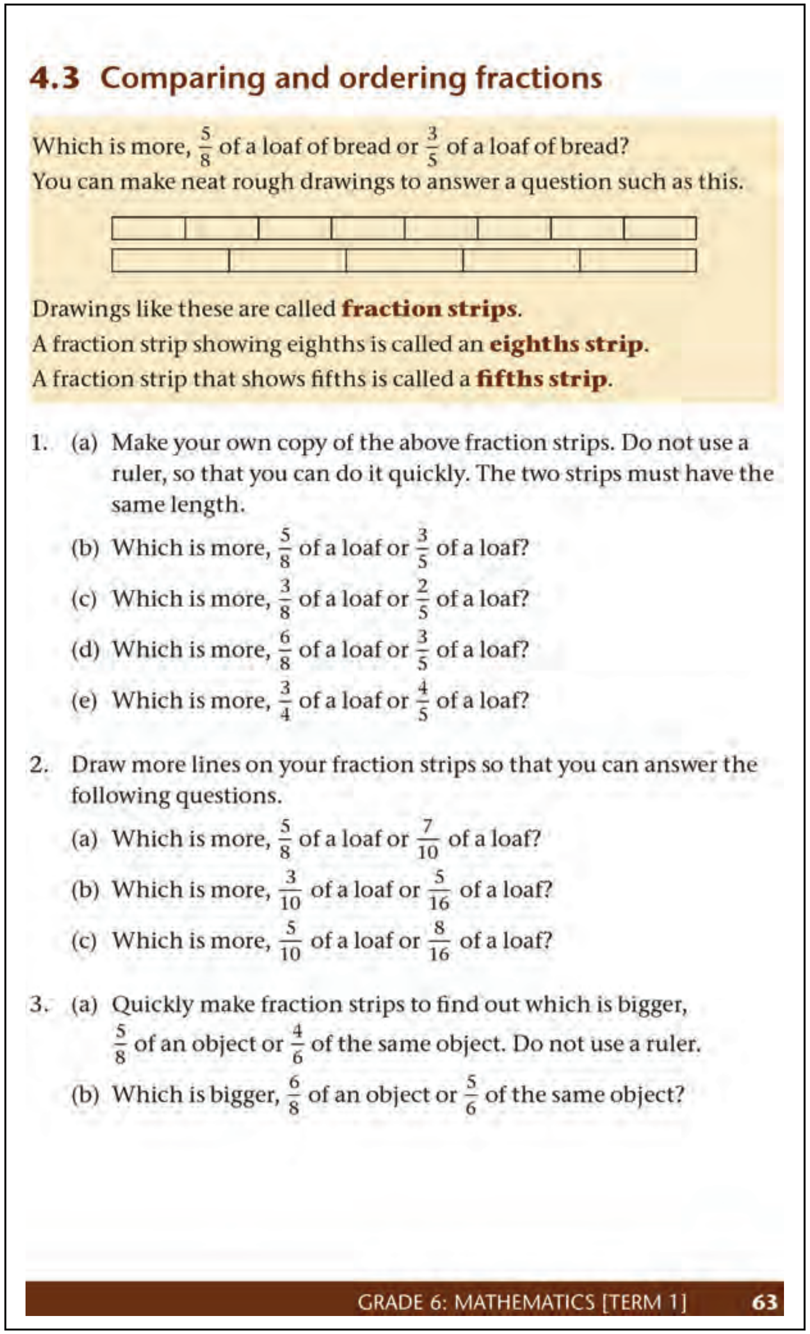
2. Learners draw additional lines on their freehand fraction strips. The strips below are only provided because they show the fraction parts more clearly.



(a) $\frac{7}{10}$ of a loaf (b) $\frac{5}{16}$ of a loaf

(c) They are the same: $\frac{5}{10} = \frac{1}{2}$ and $\frac{8}{16} = \frac{1}{2}$

3. (a) $\frac{4}{6}$ (b) $\frac{5}{6}$



4.3 Comparing and ordering fractions

Which is more, $\frac{5}{8}$ of a loaf of bread or $\frac{3}{5}$ of a loaf of bread?
You can make neat rough drawings to answer a question such as this.

Drawings like these are called **fraction strips**.
A fraction strip showing eighths is called an **eighths strip**.
A fraction strip that shows fifths is called a **fifths strip**.

1. (a) Make your own copy of the above fraction strips. Do not use a ruler, so that you can do it quickly. The two strips must have the same length.
   (b) Which is more, $\frac{5}{8}$ of a loaf or $\frac{3}{5}$ of a loaf?
   (c) Which is more, $\frac{3}{8}$ of a loaf or $\frac{2}{5}$ of a loaf?
   (d) Which is more, $\frac{6}{8}$ of a loaf or $\frac{3}{5}$ of a loaf?
   (e) Which is more, $\frac{3}{4}$ of a loaf or $\frac{4}{5}$ of a loaf?
2. Draw more lines on your fraction strips so that you can answer the following questions.
   (a) Which is more, $\frac{5}{8}$ of a loaf or $\frac{7}{10}$ of a loaf?
   (b) Which is more, $\frac{3}{10}$ of a loaf or $\frac{5}{16}$ of a loaf?
   (c) Which is more, $\frac{5}{10}$ of a loaf or $\frac{8}{16}$ of a loaf?
3. (a) Quickly make fraction strips to find out which is bigger, $\frac{5}{8}$ of an object or $\frac{4}{6}$ of the same object. Do not use a ruler.
   (b) Which is bigger, $\frac{6}{8}$ of an object or $\frac{5}{6}$ of the same object?

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Answers

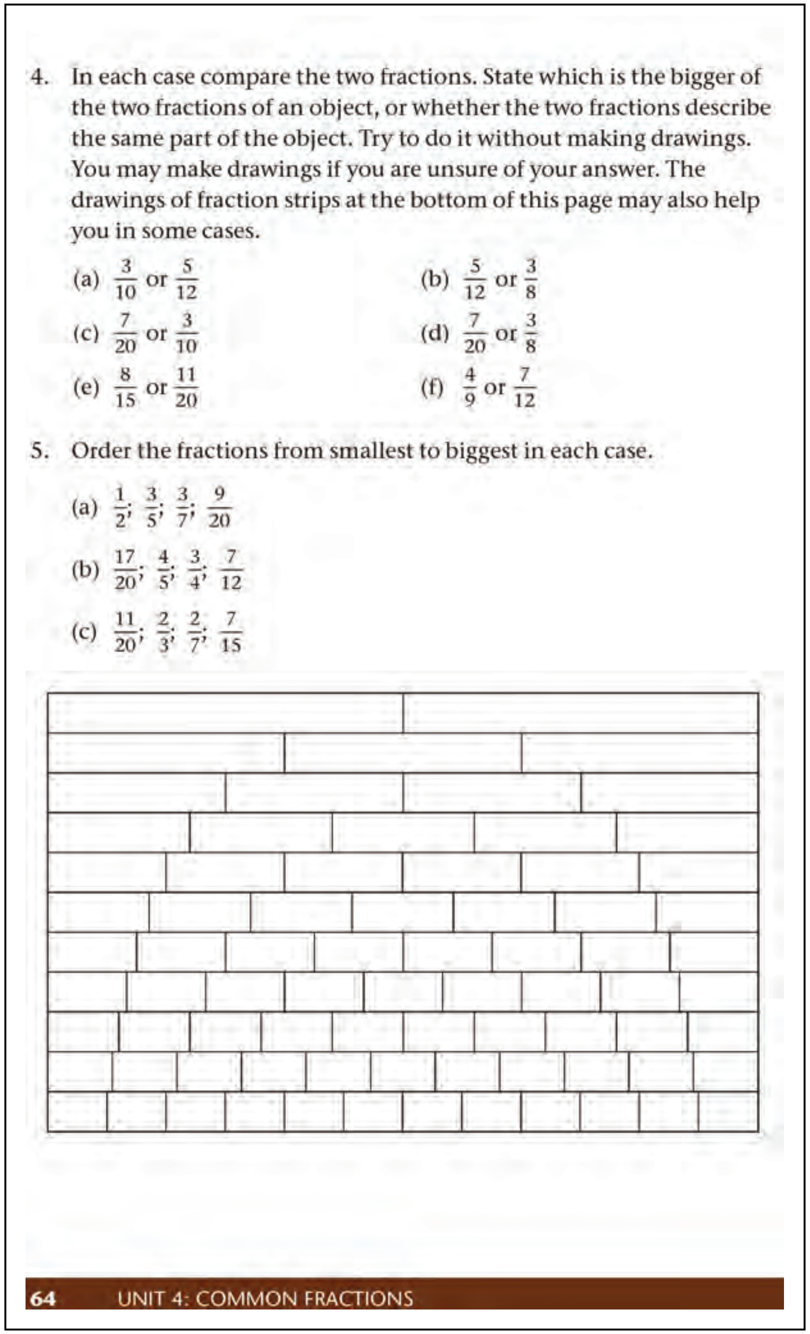
4. (a) $\frac{5}{12}$ (b) $\frac{5}{12}$ (c) $\frac{7}{20}$

(d) $\frac{3}{8}$ (e) $\frac{11}{20}$ (f) $\frac{7}{12}$

5. (a) $\frac{3}{7}$; $\frac{9}{20}$; $\frac{1}{2}$; $\frac{3}{5}$

(b) $\frac{7}{12}$; $\frac{3}{4}$; $\frac{4}{5}$; $\frac{17}{20}$

(c) $\frac{2}{7}$; $\frac{7}{15}$; $\frac{11}{20}$; $\frac{2}{3}$



4. In each case compare the two fractions. State which is the bigger of the two fractions of an object, or whether the two fractions describe the same part of the object. Try to do it without making drawings. You may make drawings if you are unsure of your answer. The drawings of fraction strips at the bottom of this page may also help you in some cases.

(a) $\frac{3}{10}$ or $\frac{5}{12}$ (b) $\frac{5}{12}$ or $\frac{3}{8}$

(c) $\frac{7}{20}$ or $\frac{3}{10}$ (d) $\frac{7}{20}$ or $\frac{3}{8}$

(e) $\frac{8}{15}$ or $\frac{11}{20}$ (f) $\frac{4}{9}$ or $\frac{7}{12}$

5. Order the fractions from smallest to biggest in each case.

(a) $\frac{1}{2}$; $\frac{3}{5}$; $\frac{3}{7}$; $\frac{9}{20}$

(b) $\frac{17}{20}$; $\frac{4}{5}$; $\frac{3}{4}$; $\frac{7}{12}$

(c) $\frac{11}{20}$; $\frac{2}{3}$; $\frac{2}{7}$; $\frac{7}{15}$

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Answers

6. A: (a) $\frac{1}{6}$ (b) $\frac{2}{3}$ (c) $\frac{5}{6}$

B: (a) $\frac{1}{6}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $1\frac{1}{6}$ (e) $1\frac{1}{3}$

(f) $1\frac{2}{3}$ (g) $1\frac{5}{6}$

C: (a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$ (d) $\frac{5}{8}$ (e) $\frac{7}{8}$

D: (a) $\frac{1}{10}$ (b) $\frac{1}{5}$ (c) $\frac{3}{10}$ (d) $\frac{2}{5}$ (e) $\frac{3}{5}$

(f) $\frac{7}{10}$ (g) $\frac{9}{10}$

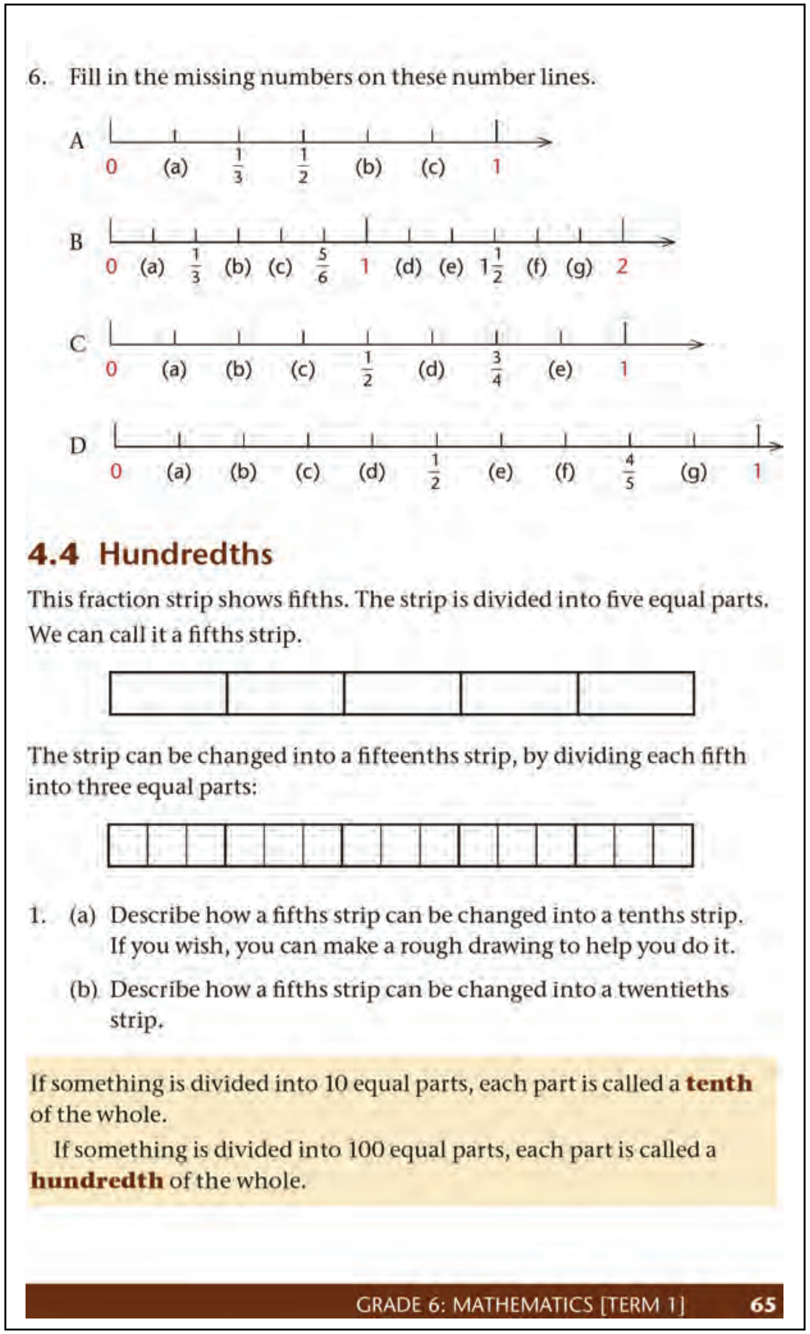
4.4 Hundredths

Mathematical notes

Decimal notation fractions include the use of tenths and hundredths. It is for this reason that learners need to know about tenths and hundredths before they move on to the decimal notation for fractions, which is introduced in Term 2.

Answers

1. (a) Divide each fifth into two equal parts.
   (b) Divide each fifth into four equal parts.



6. Fill in the missing numbers on these number lines.

4.4 Hundredths

This fraction strip shows fifths. The strip is divided into five equal parts. We can call it a fifths strip.

The strip can be changed into a fifteenths strip, by dividing each fifth into three equal parts:

1. (a) Describe how a fifths strip can be changed into a tenths strip. If you wish, you can make a rough drawing to help you do it.
   (b) Describe how a fifths strip can be changed into a twentieths strip.

If something is divided into 10 equal parts, each part is called a **tenth** of the whole.

If something is divided into 100 equal parts, each part is called a **hundredth** of the whole.

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Notes on questions

Question 2 is critical, since it provides learners with an opportunity to interpret the definition of hundredths given on the previous page of the Learner Book.

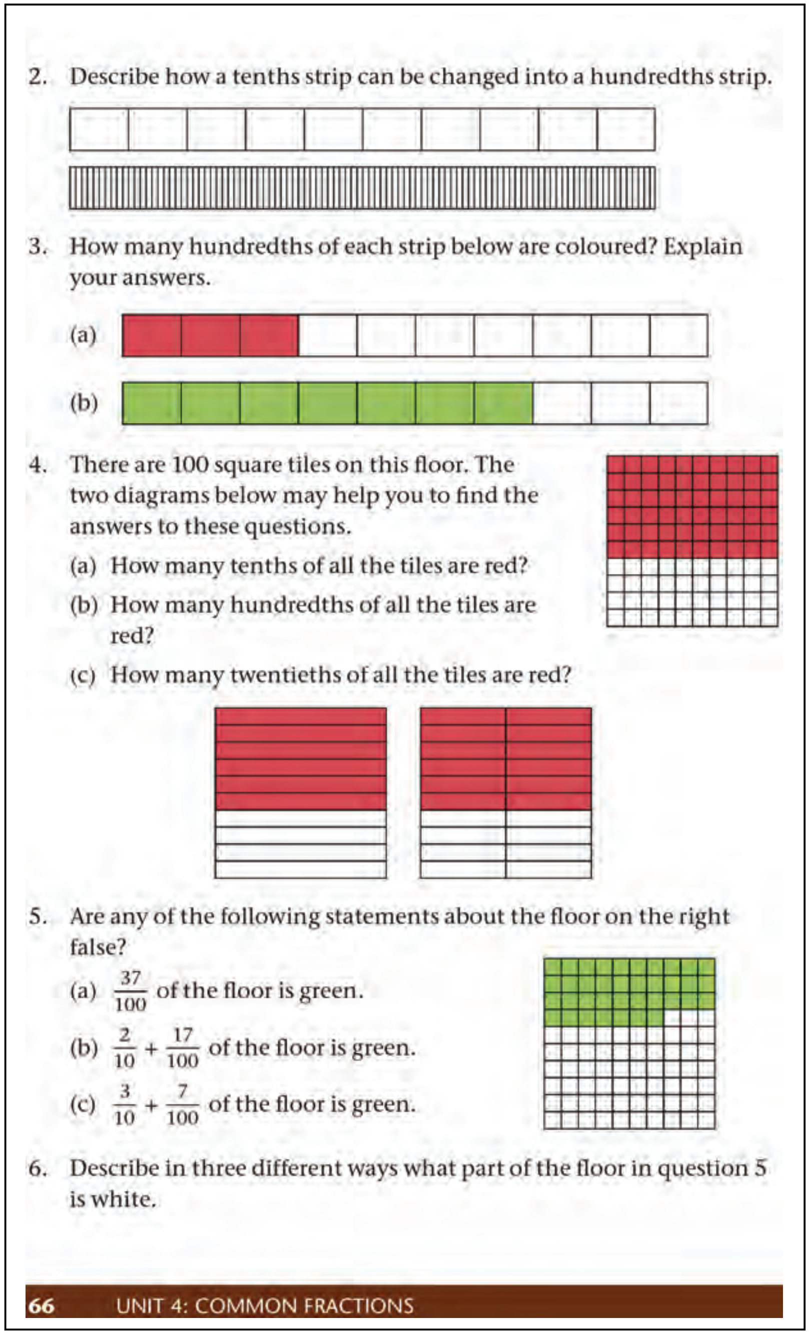
Learners should be able to do these questions on their own, given what they have done in the preceding sections. It is preferable that they do so, even if it takes some time. However, some guidelines for supporting learners are given below.

If learners have difficulty with question 2, you may ask them how many small equal parts there should be in each of 10 tenths, so that there will be 100 small equal parts in the strip as a whole.

If learners have difficulty with question 3, you may ask them how many hundredths are equal to one tenth – they may consult their work in question 2 to clarify this.

Answers

2. Divide each tenth into ten equal parts.
3. (a) 30 hundredths (b) 70 hundredths
4. (a) 6 tenths (b) 60 hundredths (c) 12 twentieths
5. None of them is false.
   (a) True (b) True (c) True
6. $\frac{63}{100}$ or $\frac{6}{10}+\frac{3}{100}$ or $\frac{5}{10}+\frac{13}{100}$ or $\frac{1}{2}+\frac{13}{100}$ of the floor is white.



2. Describe how a tenths strip can be changed into a hundredths strip.

3. How many hundredths of each strip below are coloured? Explain your answers.

   (a)

   (b)

4. There are 100 square tiles on this floor. The two diagrams below may help you to find the answers to these questions.
   (a) How many tenths of all the tiles are red?
   (b) How many hundredths of all the tiles are red?
   (c) How many twentieths of all the tiles are red?

5. Are any of the following statements about the floor on the right false?
   (a) $\frac{37}{100}$ of the floor is green.
   (b) $\frac{2}{10}+\frac{17}{100}$ of the floor is green.
   (c) $\frac{3}{10}+\frac{7}{100}$ of the floor is green.

6. Describe in three different ways what part of the floor in question 5 is white.

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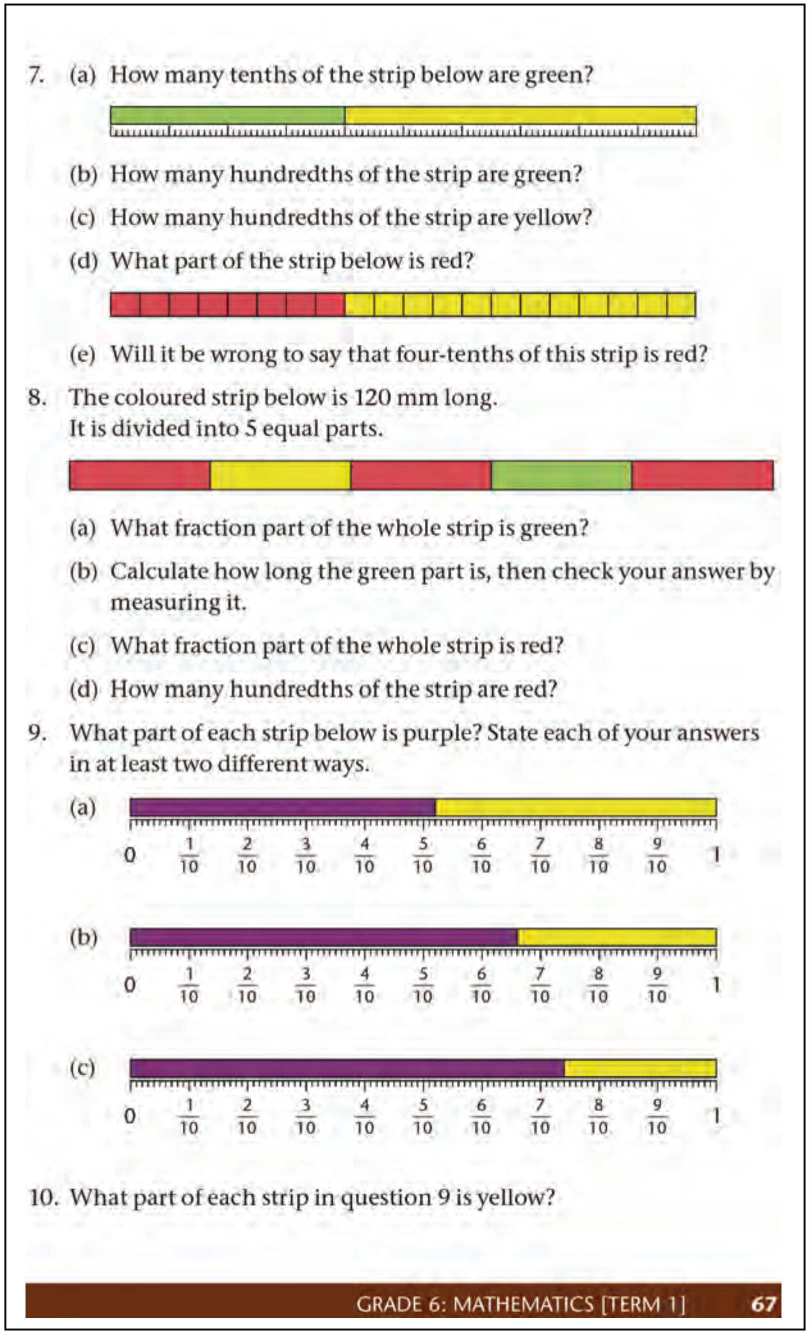
Answers

7. (a) 4 tenths
   (b) 40 hundredths
   (c) 60 hundredths
   (d) 8 twentieths or 4 tenths or 2 fifths
   (e) No
8. (a) 1 fifth
   (b) 24 mm
   (c) 3 fifths
   (d) 60 hundredths
9. (a) $\frac{52}{100}$; $\frac{1}{2} + \frac{2}{100}$
   (b) $\frac{66}{100}$; $\frac{6}{10} + \frac{6}{100}$; $\frac{3}{5} + \frac{6}{100}$
   (c) $\frac{74}{100}$; $\frac{7}{10} + \frac{4}{100}$; $\frac{37}{50}$
10. (a) $\frac{48}{100}$
   (b) $\frac{34}{100}$
   (c) $\frac{26}{100}$



7. (a) How many tenths of the strip below are green?

   (b) How many hundredths of the strip are green?

   (c) How many hundredths of the strip are yellow?

   (d) What part of the strip below is red?

   (e) Will it be wrong to say that four-tenths of this strip is red?

8. The coloured strip below is 120 mm long.
   It is divided into 5 equal parts.

   (a) What fraction part of the whole strip is green?

   (b) Calculate how long the green part is, then check your answer by measuring it.

   (c) What fraction part of the whole strip is red?

   (d) How many hundredths of the strip are red?

9. What part of each strip below is purple? State each of your answers in at least two different ways.

   (a)

   (b)

   (c)

10. What part of each strip in question 9 is yellow?

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4.5 Adding and subtracting fractions

Teaching guidelines

Go through the shaded passage with the class. **Do not give them any "recipes" for making the denominators the same.** Rather, show them how to make equivalent fractions. Take them through the sum of 5 twelfths and 4 twelfths (9 twelfths). Immediately below the shaded passage there is a diagram that takes them a step further. It shows the
5 twelfths and 1 third in yellow, and the answer in blue. Take them through the drawing and then ask them question 4. Give them time to think about why this is true. Ask one or two learners to explain. If they say it is false, give them an opportunity to share their thinking. You can point out flaws in their reasoning (if any), and ask for another view on it.

The important extra step in question 4 is that 9 twelfths is equivalent to 3 quarters.

Possible misconceptions

Some learners might add the denominators too when adding fractions because they are now used to counting "blocks" when working with fraction strips.

Learners may make this mistake because they think of addition as adding two strips together. For example, when calculating $\frac{1}{5}$ + $\frac{3}{5}$, learners may think of placing two strips next to each other, as shown below:

Strip A

Strip B

Because Strip A is used to represent one fifth and a separate Strip B is used to represent 3 fifths, learners may fall into the trap of adding the two strips together as shown below, and saying that the yellow parts are now 4 tenths of the *combined* strip.

Learners may then use the correct statement that the yellow parts are 4 tenths of the combined strip as the answer for $\frac{1}{5} + \frac{3}{5}$. Explain to them that the question is not about adding the whole strips that may be used to represent the two fractions, but adding the parts.

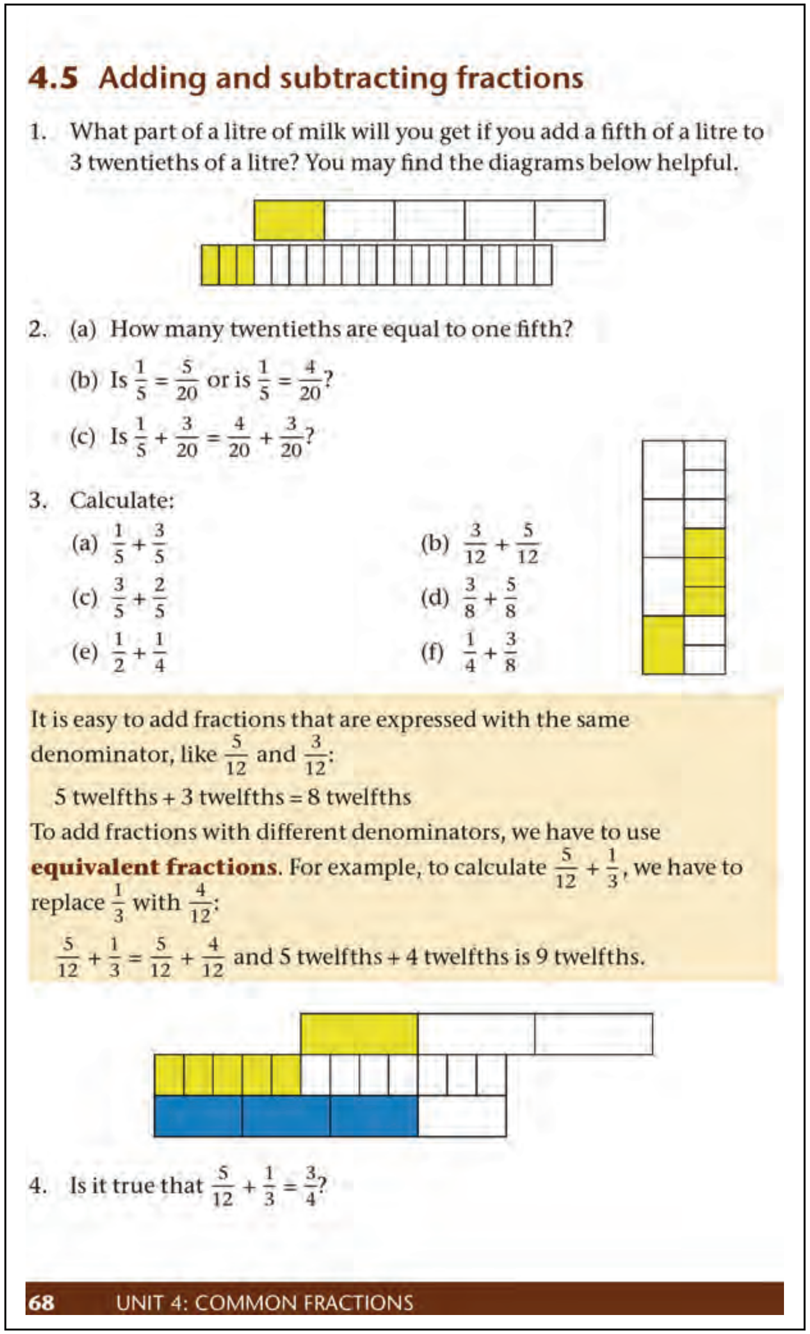
Answers

1. $\frac{7}{20}$

2. (a) 4 twentieths (b) $\frac{1}{5} = \frac{4}{20}$ (c) Yes; because $\frac{1}{5} + \frac{3}{20} = \frac{4}{20} + \frac{3}{20} = \frac{7}{20}$

3. (a) $\frac{4}{5}$ (b) $\frac{8}{12}$ (c) $\frac{5}{5} = 1$ (d) $\frac{8}{8} = 1$ (e) $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$ (f) $\frac{2}{8} + \frac{3}{8} = \frac{5}{8}$

4. Yes; because $\frac{5}{12} + \frac{1}{3} = \frac{5}{12} + \frac{4}{12} = \frac{9}{12} = \frac{3}{4}$



4.5 Adding and subtracting fractions

1. What part of a litre of milk will you get if you add a fifth of a litre to 3 twentieths of a litre? You may find the diagrams below helpful.

2. (a) How many twentieths are equal to one fifth?
   (b) Is $\frac{1}{5} = \frac{5}{20}$ or is $\frac{1}{5} = \frac{4}{20}$?
   (c) Is $\frac{1}{5} + \frac{3}{20} = \frac{4}{20} + \frac{3}{20}$?

3. Calculate:
   (a) $\frac{1}{5} + \frac{3}{5}$
   (b) $\frac{3}{12} + \frac{5}{12}$
   (c) $\frac{3}{5} + \frac{2}{5}$
   (d) $\frac{3}{8} + \frac{5}{8}$
   (e) $\frac{1}{2} + \frac{1}{4}$
   (f) $\frac{1}{4} + \frac{3}{8}$

It is easy to add fractions that are expressed with the same denominator, like $\frac{5}{12}$ and $\frac{3}{12}$:

5 twelfths + 3 twelfths = 8 twelfths

To add fractions with different denominators, we have to use **equivalent fractions**. For example, to calculate $\frac{5}{12} + \frac{1}{3}$, we have to replace $\frac{1}{3}$ with $\frac{4}{12}$:

$\frac{5}{12} + \frac{1}{3} = \frac{5}{12} + \frac{4}{12}$ and 5 twelfths + 4 twelfths is 9 twelfths.

4. Is it true that $\frac{5}{12} + \frac{1}{3} = \frac{3}{4}$?

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Teaching guidelines

Once learners have completed question 5, work through the shaded passage on the board. Go through the first method with the class. Then ask them to check Judy's method individually, and decide if she is correct. Ask if there is a third way to solve this problem. By then, the class should be confident enough to attempt question 6.

Answers

5. (a) $1\frac{7}{8}$ (b) $\frac{10}{6} = 1\frac{4}{6}$ or $1\frac{2}{3}$ (c) $\frac{9}{8} = 1\frac{1}{8}$

(d) 0 (e) $\frac{12}{8} = 1\frac{4}{8} = 1\frac{1}{2}$ (f) $\frac{4}{8} = \frac{1}{2}$

(g) $\frac{12}{16} = \frac{6}{8} = \frac{3}{4}$ (h) $\frac{9}{16}$ (i) $\frac{17}{20} + \frac{6}{20} - \frac{8}{20} = \frac{15}{20}$

(j) $\frac{16}{12} = 1\frac{4}{12}$ or $1\frac{1}{3}$ (k) $\frac{45}{16} = 2\frac{13}{16}$ (l) $\frac{16}{15} = 1\frac{1}{15}$

(m) $\frac{33}{8} + 4\frac{1}{8}$ (n) $\frac{27}{100}$ (o) $\frac{128}{100} = 1\frac{28}{100}$ or $1\frac{14}{50}$ or $1\frac{7}{25}$

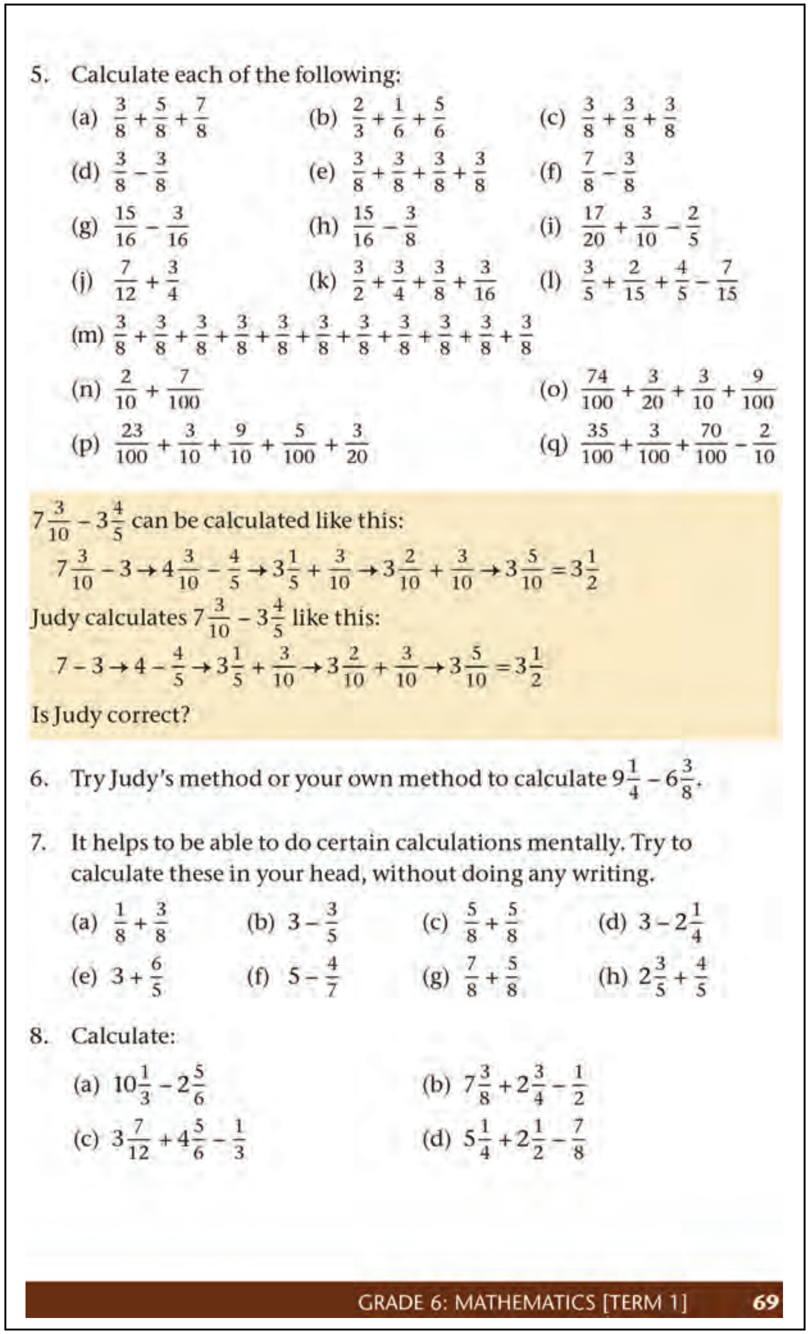
(p) $\frac{163}{100} = 1\frac{63}{100}$ (q) $\frac{88}{100}$ or $\frac{44}{50}$ or $\frac{22}{25}$

6. $9\frac{1}{4} - 6 \rightarrow 3\frac{1}{4} - \frac{3}{8} \rightarrow 2\frac{5}{8} + \frac{1}{4} \rightarrow 2\frac{7}{8}$ or $9\frac{1}{4} - 6\frac{3}{8} \rightarrow 8\frac{5}{4} - 6\frac{3}{8} \rightarrow 8\frac{10}{8} - 6\frac{3}{8} = 2\frac{7}{8}$

7. (a) $\frac{4}{8}$ or $\frac{1}{2}$ (b) $2\frac{2}{5}$ (c) $\frac{10}{8} = 1\frac{2}{8}$ or $1\frac{1}{4}$ (d) $\frac{3}{4}$

(e) $4\frac{1}{5}$ (f) $4\frac{3}{7}$ (g) $\frac{12}{8} = 1\frac{4}{8}$ or $1\frac{1}{2}$ (h) $3\frac{2}{5}$

8. (a) $7\frac{1}{2}$ (b) $9\frac{5}{8}$ (c) $8\frac{1}{12}$ (d) $6\frac{7}{8}$



5. Calculate each of the following:

(a) $\frac{3}{8} + \frac{5}{8} + \frac{7}{8}$ (b) $\frac{2}{3} + \frac{1}{6} + \frac{5}{6}$ (c) $\frac{3}{8} + \frac{3}{8} + \frac{3}{8}$

(d) $\frac{3}{8} - \frac{3}{8}$ (e) $\frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8}$ (f) $\frac{7}{8} - \frac{3}{8}$

(g) $\frac{15}{16} - \frac{3}{16}$ (h) $\frac{15}{16} - \frac{3}{8}$ (i) $\frac{17}{20} + \frac{3}{10} - \frac{2}{5}$

(j) $\frac{7}{12} + \frac{3}{4}$ (k) $\frac{3}{2} + \frac{3}{4} + \frac{3}{8} + \frac{3}{16}$ (l) $\frac{3}{5} + \frac{2}{15} + \frac{4}{5} - \frac{7}{15}$

(m) $\frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8}$

(n) $\frac{2}{10} + \frac{7}{100}$ (o) $\frac{74}{100} + \frac{3}{20} + \frac{3}{10} + \frac{9}{100}$

(p) $\frac{23}{100} + \frac{3}{10} + \frac{9}{10} + \frac{5}{100} + \frac{3}{20}$ (q) $\frac{35}{100} + \frac{3}{100} + \frac{70}{100} - \frac{2}{10}$

$7\frac{3}{10} - 3\frac{4}{5}$ can be calculated like this:

$7\frac{3}{10} - 3 \rightarrow 4\frac{3}{10} - \frac{4}{5} \rightarrow 3\frac{1}{5} + \frac{3}{10} \rightarrow 3\frac{2}{10} + \frac{3}{10} \rightarrow 3\frac{5}{10} = 3\frac{1}{2}$

Judy calculates $7\frac{3}{10} - 3\frac{4}{5}$ like this:

$7 - 3 \rightarrow 4 - \frac{4}{5} \rightarrow 3\frac{1}{5} + \frac{3}{10} \rightarrow 3\frac{2}{10} + \frac{3}{10} \rightarrow 3\frac{5}{10} = 3\frac{1}{2}$

Is Judy correct?

6. Try Judy's method or your own method to calculate $9\frac{1}{4} - 6\frac{3}{8}$.

7. It helps to be able to do certain calculations mentally. Try to calculate these in your head, without doing any writing.

(a) $\frac{1}{8} + \frac{3}{8}$ (b) $3 - \frac{3}{5}$ (c) $\frac{5}{8} + \frac{5}{8}$ (d) $3 - 2\frac{1}{4}$

(e) $3 + \frac{6}{5}$ (f) $5 - \frac{4}{7}$ (g) $\frac{7}{8} + \frac{5}{8}$ (h) $2\frac{3}{5} + \frac{4}{5}$

8. Calculate:

(a) $10\frac{1}{3} - 2\frac{5}{6}$ (b) $7\frac{3}{8} + 2\frac{3}{4} - \frac{1}{2}$

(c) $3\frac{7}{12} + 4\frac{5}{6} - \frac{1}{3}$ (d) $5\frac{1}{4} + 2\frac{1}{2} - \frac{7}{8}$

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4.6 Problem solving

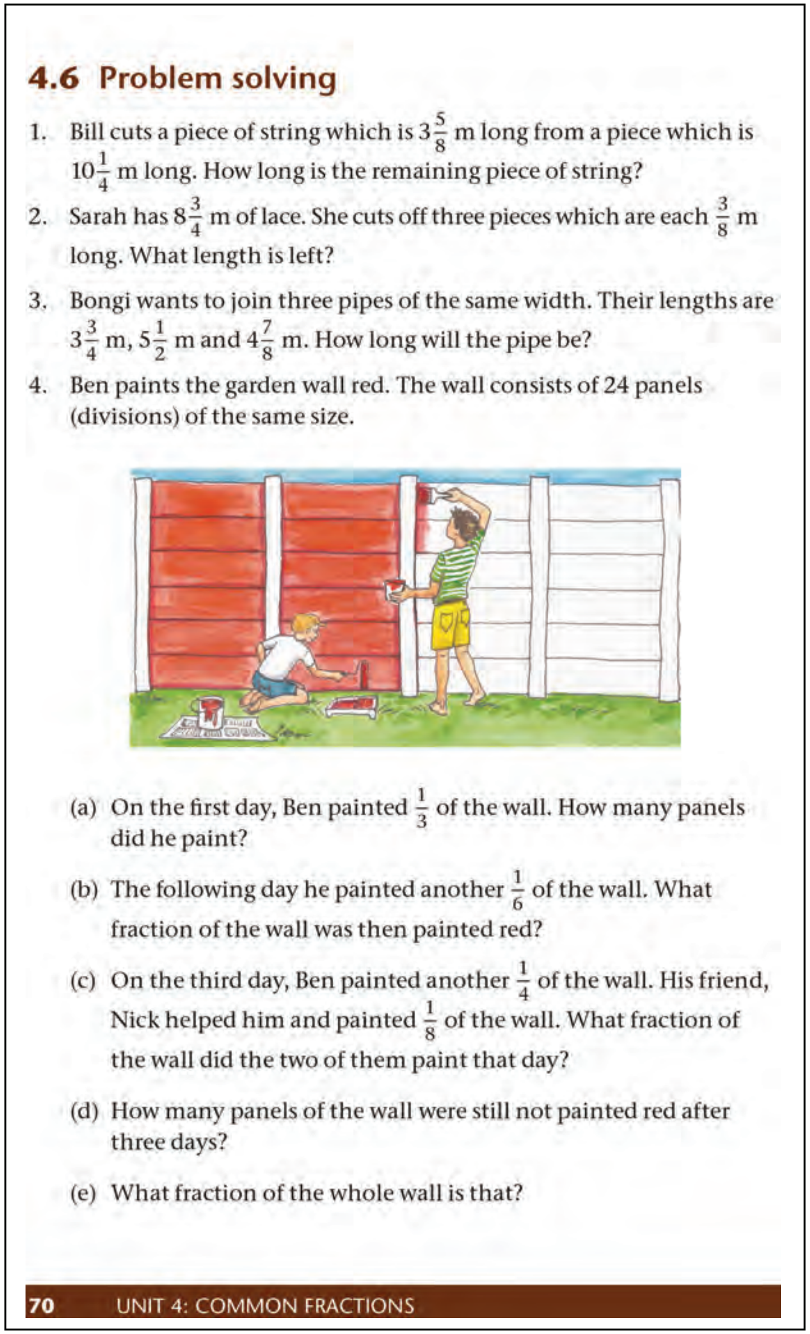
Teaching guidelines

In problem solving, learners should be encouraged to make drawings, sketches and diagrams and show their calculations. This is so that if their thinking is muddled, you can identify where the problem is and help them through it. Of course, if the class is consistently making an error, it is important for you to know this as well, as perhaps there is something they have not grasped in the preceding section(s).

Make sure learners understand that the panels referred to in question 4 are the rectangular strips of wood or concrete that were used to build the wall. There are six panels, stacked on top of each other, between each two upright poles.

Answers

1. $6\frac{5}{8}$ m
2. $7\frac{5}{8}$ m
3. $14\frac{1}{8}$ m
4. (a) Eight panels
   (b) $\frac{1}{2}$
   (c) $\frac{3}{8}$
   (d) Three panels
   (e) $\frac{1}{8}$



4.6 Problem solving

1. Bill cuts a piece of string which is $3\frac{5}{8}$ m long from a piece which is $10\frac{1}{4}$ m long. How long is the remaining piece of string?
2. Sarah has $8\frac{3}{4}$ m of lace. She cuts off three pieces which are each $\frac{3}{8}$ m long. What length is left?
3. Bongi wants to join three pipes of the same width. Their lengths are $3\frac{3}{4}$ m, $5\frac{1}{2}$ m and $4\frac{7}{8}$ m. How long will the pipe be?
4. Ben paints the garden wall red. The wall consists of 24 panels (divisions) of the same size.

   (a) On the first day, Ben painted $\frac{1}{3}$ of the wall. How many panels did he paint?
   (b) The following day he painted another $\frac{1}{6}$ of the wall. What fraction of the wall was then painted red?
   (c) On the third day, Ben painted another $\frac{1}{4}$ of the wall. His friend, Nick helped him and painted $\frac{1}{8}$ of the wall. What fraction of the wall did the two of them paint that day?
   (d) How many panels of the wall were still not painted red after three days?
   (e) What fraction of the whole wall is that?

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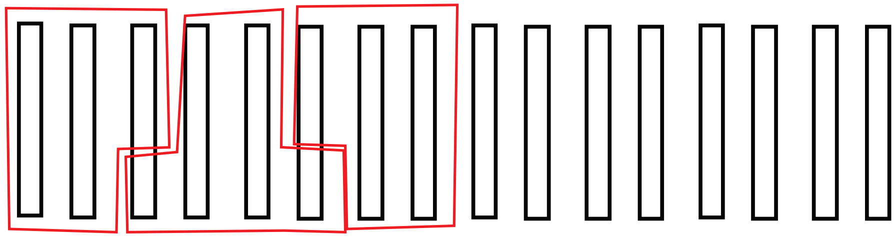
Teaching guidelines

Ensure that learners feel free to make rough drawings when they do questions 5 to 7.

If learners struggle with question 5, you can make a drawing like this on the board:



The drawing shows the sausages that three children will get. Let learners make and complete the drawing.

In question 7, ensure that learners understand that the third row is for the *fraction of the slab* that each learner gets.

Answers

5. Six children
6. 39 sausages
7.

Number of people who share	②	③	④	5	⑥	7	⑧
Number of blocks per person	12	8	6		4		3
Fraction per person	$\frac{1}{2}$ or	$\frac{1}{3}$ or	$\frac{1}{4}$ or	$\frac{1}{5}$ or	$\frac{1}{6}$ or	$\frac{1}{7}$ or $\frac{2}{14}$	$\frac{1}{8}$ or
Fraction written in another way	$\frac{12}{24}$	$\frac{8}{24}$	$\frac{6}{24}$	$\frac{2}{10}$	$\frac{4}{24}$		$\frac{3}{24}$

Note: Learners may use other equivalent fractions in the third and fourth rows.

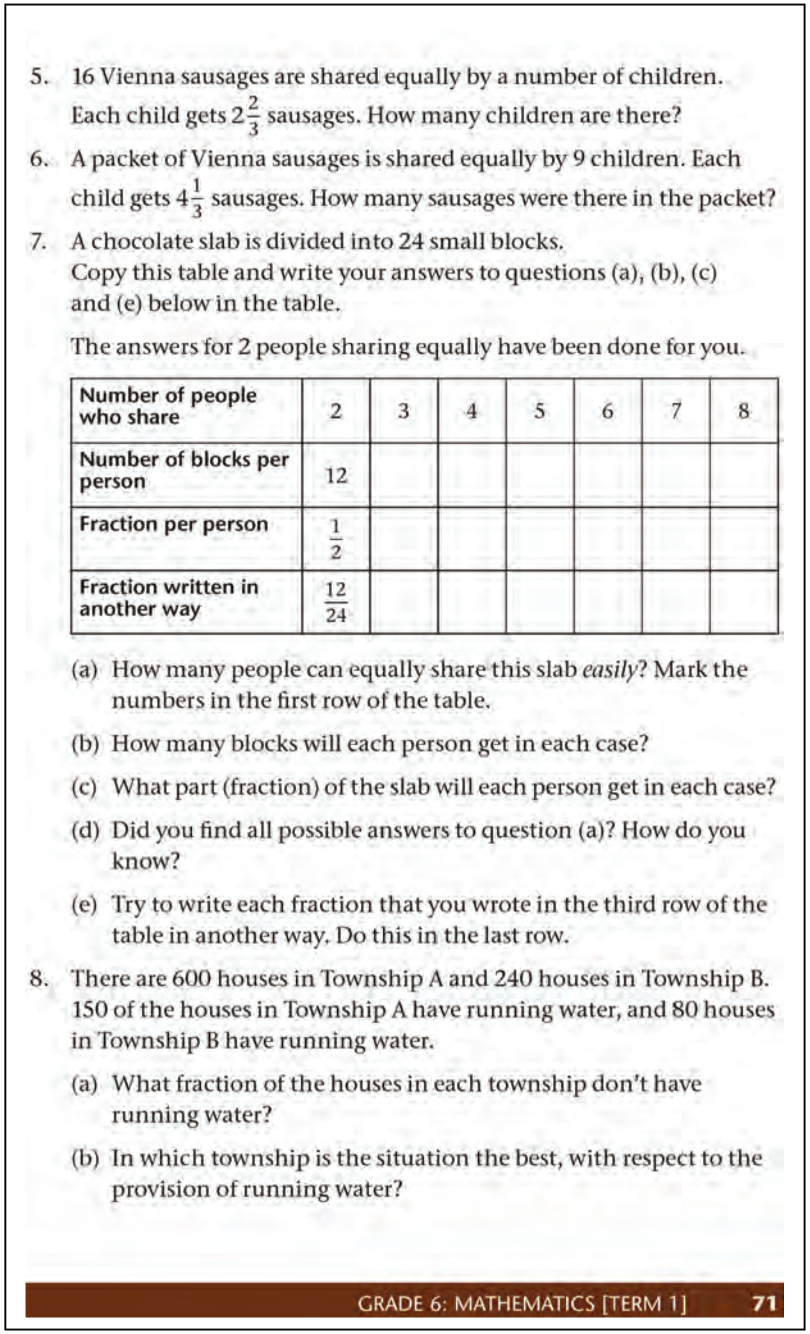
(d) The numbers that can easily be shared are the factors of 24, i.e. 2, 3, 4, 6 and 8.

8. (a) Note that the question asks what fraction of houses *don't* have running water.

Township A: $\frac{450}{600}$ Township B: $\frac{160}{240}$

(b) $\frac{450}{600} = \frac{45}{60} = \frac{3}{4}$ and $\frac{160}{240} = \frac{16}{24} = \frac{2}{3}$

The situation in Township B is best, because $\frac{2}{3} < \frac{3}{4}$. Two thirds being less than three quarters means that, relative to the total number of people living in each township, more people are provided with water in Township B than in Township A.



5. 16 Vienna sausages are shared equally by a number of children. Each child gets $2\frac{2}{3}$ sausages. How many children are there?
6. A packet of Vienna sausages is shared equally by 9 children. Each child gets $4\frac{1}{3}$ sausages. How many sausages were there in the packet?
7. A chocolate slab is divided into 24 small blocks. Copy this table and write your answers to questions (a), (b), (c) and (e) below in the table.

The answers for 2 people sharing equally have been done for you.

Number of people who share	2	3	4	5	6	7	8
Number of blocks per person	12						
Fraction per person	$\frac{1}{2}$						
Fraction written in another way	$\frac{12}{24}$						

(a) How many people can equally share this slab *easily*? Mark the numbers in the first row of the table.

(b) How many blocks will each person get in each case?

(c) What part (fraction) of the slab will each person get in each case?

(d) Did you find all possible answers to question (a)? How do you know?

(e) Try to write each fraction that you wrote in the third row of the table in another way. Do this in the last row.

8. There are 600 houses in Township A and 240 houses in Township B. 150 of the houses in Township A have running water, and 80 houses in Township B have running water.

(a) What fraction of the houses in each township don't have running water?

(b) In which township is the situation the best, with respect to the provision of running water?

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Grade 6 Term 1 Unit 5 Time

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
5.1 Introduction	Telling the time without clocks or watches (or cell phones)	72
5.2 Read, write and tell time	Reading, writing and telling the time in 12-hour and 24-hour time	73 to 74
5.3 Time intervals	Calculating time intervals in hours and minutes, and reading timetables	75 to 76
5.4 Time intervals on the stopwatch	Reading stopwatches and calculating time intervals in hours, minutes and seconds	77 to 78
5.5 Years, decades and centuries	Calculating time in days, weeks, months, years, decades and centuries	79 to 80
5.6 A short history of calendars	Linking calendars to the earth's orbit around the sun	81 to 83
5.7 Time zones	Understanding and calculating time zones	84 to 85

CAPS time allocation	4 hours
CAPS page references	27 and 228

Mathematical background

Learners deal with time and time-related issues every day. By now, Grade 6 learners should be able to read clocks and watches.

There are two issues that make the concept of time difficult. Firstly, time cannot be seen, touched or physically experienced like length, capacity or volume, area and mass. We measure time by looking at environmental changes, changes in the position of the hands of a clock, or the numbers on a clock face. Secondly, unlike the number system and other forms of measurement, the numbers do not get bigger forever. Instead, we measure time in modular units that are periodic, for example 1–60 seconds, 1–60 minutes, 1–24 hours and 1–365 days.

The topic of time also involves more than just reading clocks. Learners need to be able to:

- read, write and tell the time in 12-hour and 24-hour time
- calculate time intervals in hours and minutes, and read timetables
- read stopwatches and calculate time intervals in hours, minutes and seconds
- calculate time in days, weeks, months, years, decades and centuries
- read and interpret time zones.

Resources

12-hour and 24-hour clocks (analogue as well as digital), stopwatch, year calendars, world globe and large mirror

5.1 Introduction

Teaching guidelines

This section serves as a short introduction to Time. It is suggested that you combine Sections 5.1 and 5.2 into one lesson.

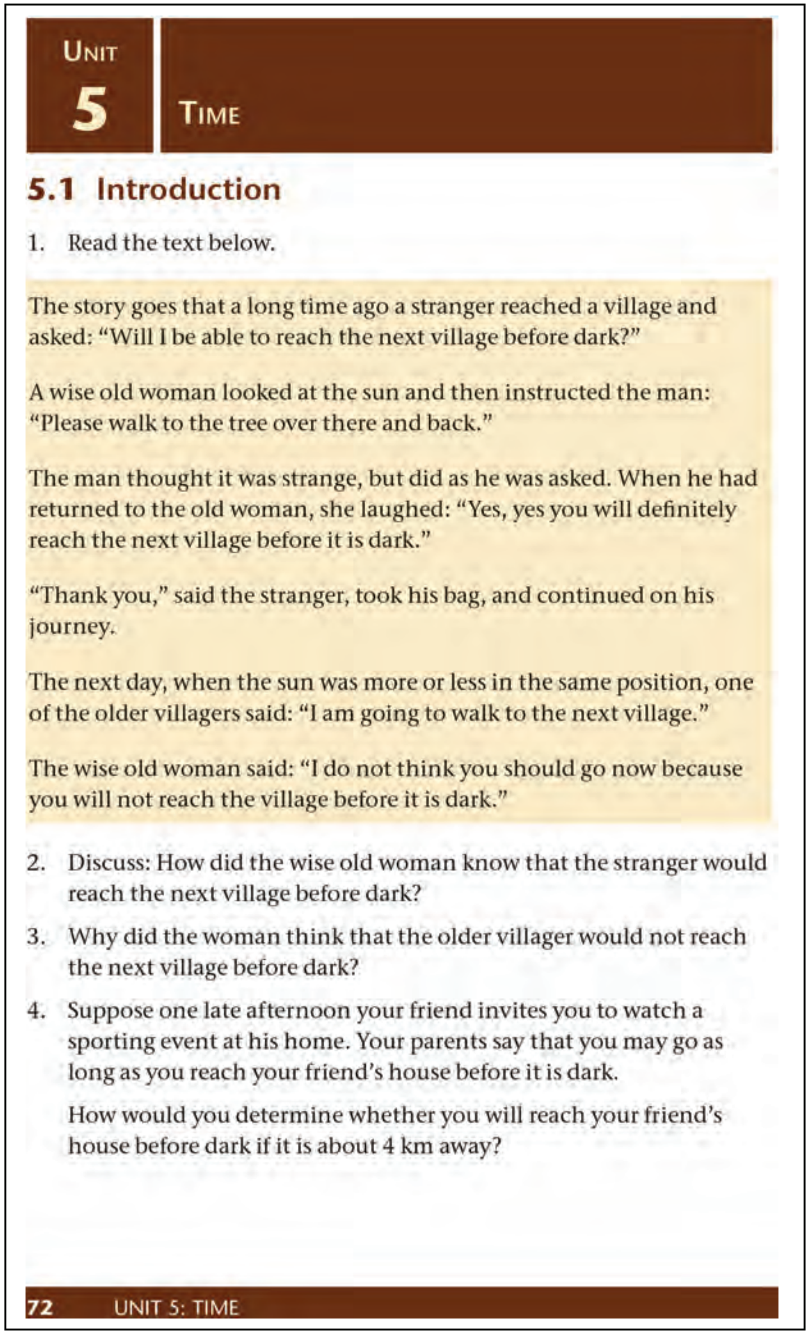
Start the lesson by asking learners how they work out the time if they don't have a clock, watch or cell phone. Ask them how people "told the time" before clocks and watches were invented.

Some learners will be able to read the story in the shaded passage aloud to the class, but keep in mind that not all learners will be able to do so. Discuss the following with the learners in preparing them for this task:

- The story begins with the words "The story goes ...". This means that this is a story that people tell.
- What is a stranger? The word "stranger" and the word "strange" appear in the first five lines. Some learners will think that the two words are related, but they have different meanings. The "stranger" refers to a person who has never been to the village before. The word "strange" refers to a peculiar or odd type of behaviour, way of thinking, or instruction such as "Please walk to the tree over there and back."
- What does "the sun was more or less in the same position" mean? It means that the sun was in roughly the same position in the sky as it was the day before. "More or less" means "approximately, not exactly".

Answers

2. The wise old woman judged his walking speed by seeing how quickly he could walk to the tree and back, and assessed how many daylight hours were left by looking at the position of the sun.
3. She knew that he walked more slowly than the stranger.
4. You could assess how many hours of daylight are left and estimate whether or not you can walk 4 km in that time. You could estimate your walking speed over a shorter distance and turn back if you think that you will not make it.



UNIT 5 TIME

5.1 Introduction

1. Read the text below.

> The story goes that a long time ago a stranger reached a village and asked: "Will I be able to reach the next village before dark?"
>
> A wise old woman looked at the sun and then instructed the man: "Please walk to the tree over there and back."
>
> The man thought it was strange, but did as he was asked. When he had returned to the old woman, she laughed: "Yes, yes you will definitely reach the next village before it is dark."
>
> "Thank you," said the stranger, took his bag, and continued on his journey.
>
> The next day, when the sun was more or less in the same position, one of the older villagers said: "I am going to walk to the next village."
>
> The wise old woman said: "I do not think you should go now because you will not reach the village before it is dark."

2. Discuss: How did the wise old woman know that the stranger would reach the next village before dark?
3. Why did the woman think that the older villager would not reach the next village before dark?
4. Suppose one late afternoon your friend invites you to watch a sporting event at his home. Your parents say that you may go as long as you reach your friend's house before it is dark.

   How would you determine whether you will reach your friend's house before dark if it is about 4 km away?

5.2 Read, write and tell time

Teaching guidelines

Begin this section by finding out what learners know about 12-hour and 24-hour time. Use the shaded passage to fill in any gaps in learners' knowledge. Use questions 1 and 2 to assess how much learners remember from Grade 5 about writing time in 12-hour and 24-hour notation. You can refer to Term 1 Unit 6 in the Grade 5 Learner Book.

Possible misconceptions

Learners may be confused about how to write midday and midnight in 12-hour and 24-hour time. You may need to clarify that midday is called 12 p.m. (post meridiem) and is written as 12:00 in 24-hour time.

Midnight is written as 12 a.m. (ante meridiem) and is written as 00:00 in 24-hour time. This is simply a convention that has been adopted for the sake of clarity. The first hour of a day is between 00:00 and 01:00, therefore the second hour is between 01:00 and 02:00. This implies that the twenty-fourth hour will be between 23:00 and 24:00, therefore 24:00 will be referred to as 0:00.

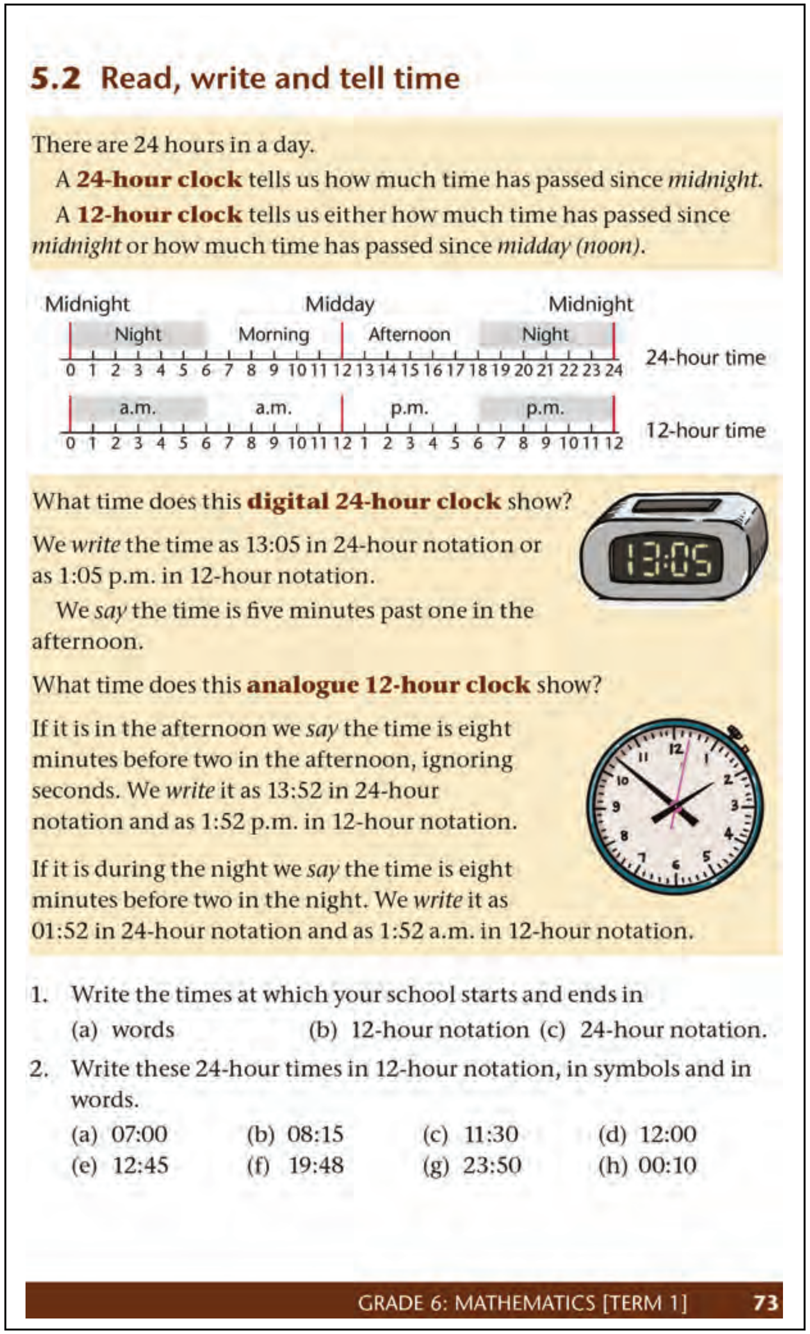
Notes on questions

This section is fairly long. Consider splitting questions up for classwork and homework. You could, for example, use questions 1, 2(a), (d), (e), (f), (h), 3(a), (c), (e) and 4(a), (b) for classwork and the rest for homework.

Answers

1. (a)–(c) Learners' answers will differ from school to school.

2.	(a)	7:00 a.m.	seven o'clock in the morning
	(b)	8:15 a.m.	quarter past eight in the morning
	(c)	11:30 a.m.	eleven thirty in the morning
	(d)	12:00 p.m.	midday, noon or 12 noon
	(e)	12:45 p.m.	quarter to one in the afternoon
	(f)	7:48 p.m.	twelve minutes to eight in the evening
	(g)	11:50 p.m.	ten minutes to twelve at night
	(h)	12:10 a.m.	ten minutes past twelve at night or ten minutes past midnight



5.2 Read, write and tell time

There are 24 hours in a day.

A **24-hour clock** tells us how much time has passed since *midnight*.

A **12-hour clock** tells us either how much time has passed since *midnight* or how much time has passed since *midday (noon)*.

What time does this **digital 24-hour clock** show?

We *write* the time as 13:05 in 24-hour notation or as 1:05 p.m. in 12-hour notation.

We *say* the time is five minutes past one in the afternoon.

What time does this **analogue 12-hour clock** show?

If it is in the afternoon we *say* the time is eight minutes before two in the afternoon, ignoring seconds. We *write* it as 13:52 in 24-hour notation and as 1:52 p.m. in 12-hour notation.

If it is during the night we *say* the time is eight minutes before two in the night. We *write* it as 01:52 in 24-hour notation and as 1:52 a.m. in 12-hour notation.

1. Write the times at which your school starts and ends in
   (a) words (b) 12-hour notation (c) 24-hour notation.
2. Write these 24-hour times in 12-hour notation, in symbols and in words.
   (a) 07:00 (b) 08:15 (c) 11:30 (d) 12:00
   (e) 12:45 (f) 19:48 (g) 23:50 (h) 00:10

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Notes on questions

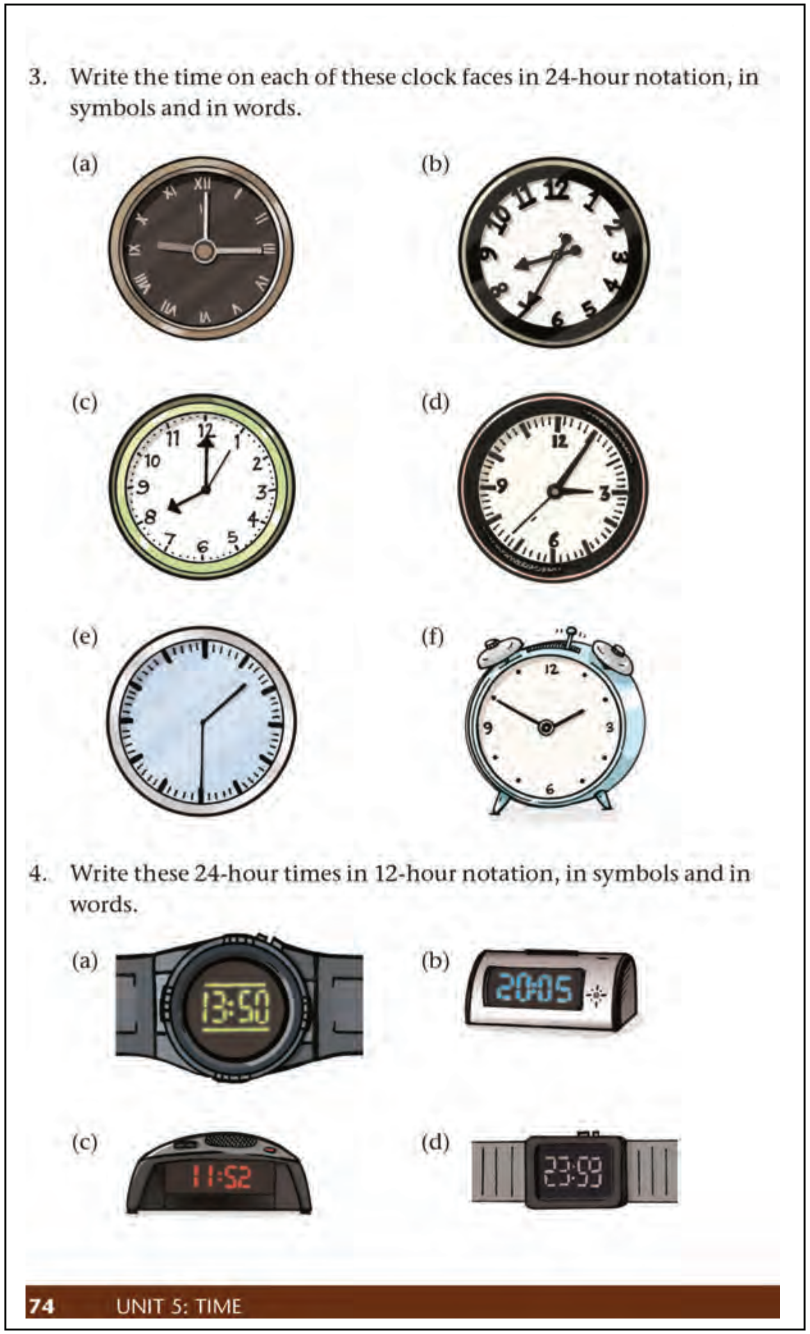
Emphasise that a 12-hour clock does not tell you whether the time is morning or afternoon. So, for example, people who work inside a mine may not be able to tell whether it is daytime or night-time outside by looking at a 12-hour clock.

If you wish, you could ask learners to assume that all times in question 3 are in the morning. In question 4, learners will practise converting afternoon 24-hour time to 12-hour time. If you want learners to have more practice converting 24-hour afternoon or evening time to 12-hour time, indicate to them which of the clocks you want to show time *after* noon (and then adjust the answers accordingly).

Answers

3.	(a)	09:15:00	or	21:15	quarter past nine exactly
	(b)	08:35	or	20:35	twenty-five minutes to nine
	(c)	08:00:05	or	20:00:05	five seconds past eight
	(d)	03:05:37	or	15:05:37	five minutes and thirty-seven seconds past three
	(e)	01:30	or	13:30	half past one or one thirty
	(f)	01:50	or	13:50	ten minutes to two

4.	(a)	1:50 p.m.	ten minutes to two in the afternoon
	(b)	8:05 p.m.	five minutes past eight in the evening
	(c)	11:52 a.m.	eight minutes to twelve noon or eight minutes before noon
	(d)	11:59 p.m.	one minute before midnight or one minute to twelve at night



3. Write the time on each of these clock faces in 24-hour notation, in symbols and in words.

(a) (b)

(c) (d)

(e) (f)

4. Write these 24-hour times in 12-hour notation, in symbols and in words.

(a) (b)

(c) (d)

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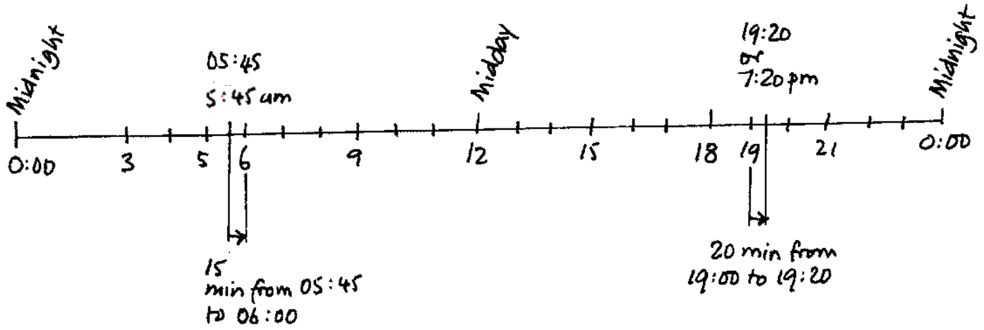
5.3 Time intervals

Teaching guidelines

Learners could do questions 1(a) and (c) in class, and 1(b) and (d) for homework.

Stress to learners that not only should they take note of the morning and afternoon time differences on these clocks; they need to also check the dates. For example, sometimes the second clock shows one or two days after the first clock.

You may draw a number line on the board. For example, for question 1(a):



Working out the times on a number line helps learners to visually break up the segments of time in a logical way.

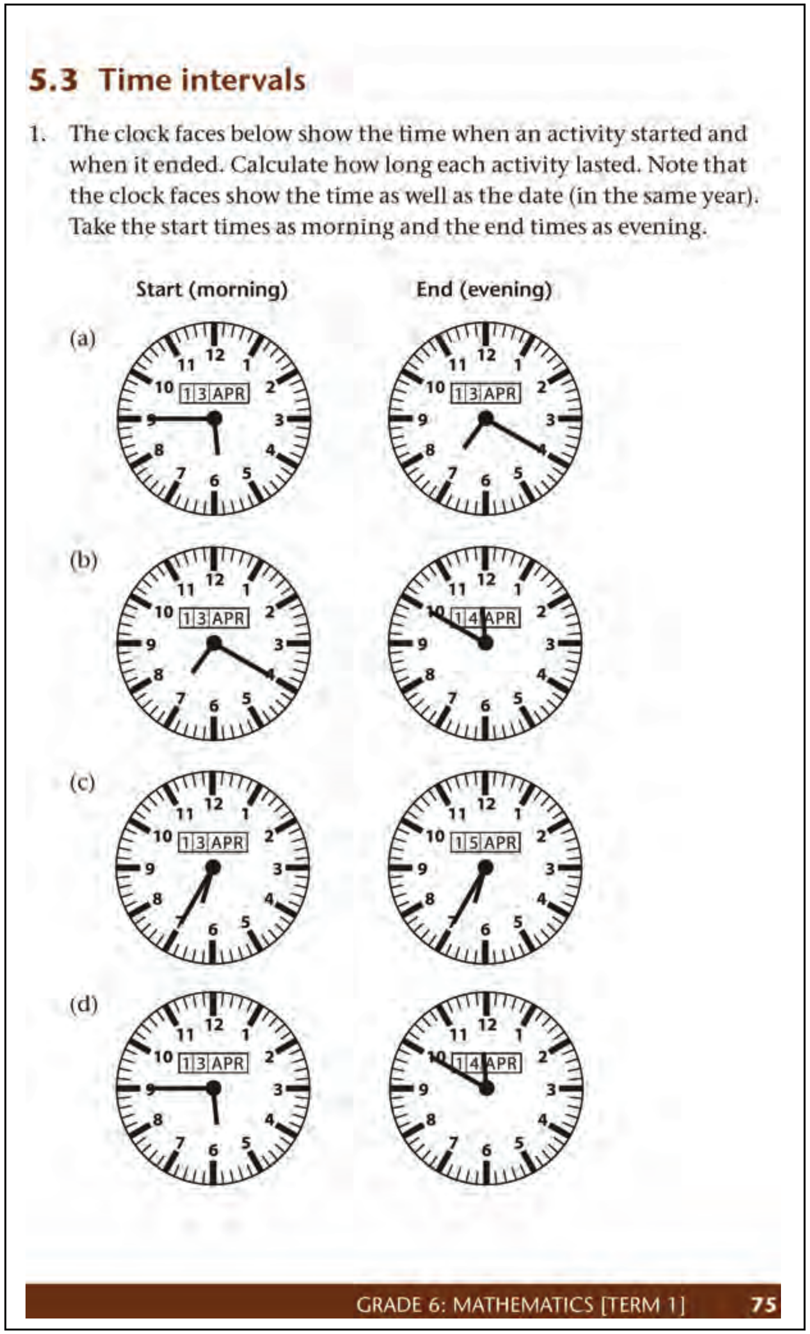
Tell learners to add the whole hours first. It adds up to 13 hours from 06:00 until 19:00.

Then tell them to work out the minutes. From 05:45 until 06:00 is one quarter of an hour, or 15 minutes. From 19:00 until 19:20 is 20 minutes. If we add the minutes together we get 35 minutes. So, the total time interval is 13 hours and 35 minutes.

(*continued on the next page*)

Answers

See next page.



5.3 Time intervals

1. The clock faces below show the time when an activity started and when it ended. Calculate how long each activity lasted. Note that the clock faces show the time as well as the date (in the same year). Take the start times as morning and the end times as evening.

Start (morning) **End (evening)**

(a)

(b)

(c)

(d)

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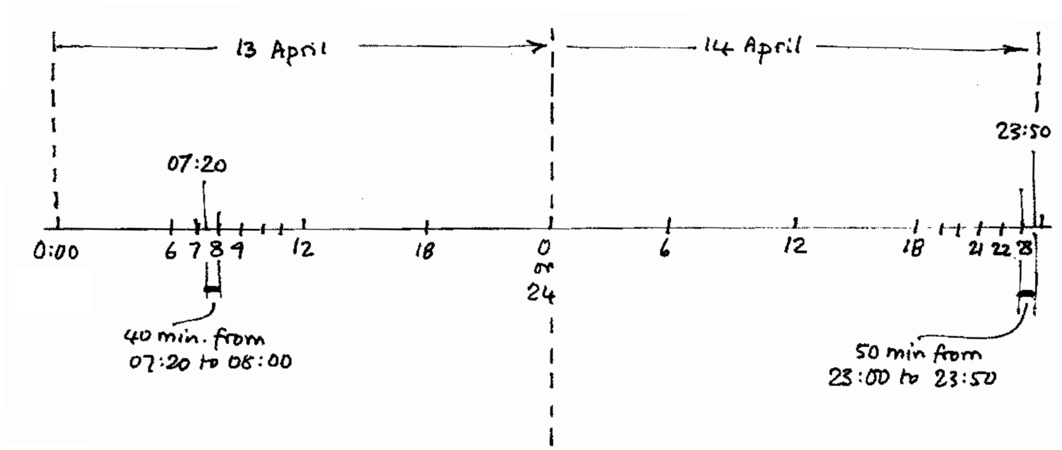
Teaching guidelines *(continued)*

Question 1(b) can be explained in a similar way. Here we must show 13 April and 14 April, as well as the hours. If we add the hours first we work it out as follows: On 13 April we have whole hours from 08:00 to 24:00 (which is the same as 00:00), i.e. 16 whole hours. From midnight or 00:00 on 14 April we have another 23 whole hours.

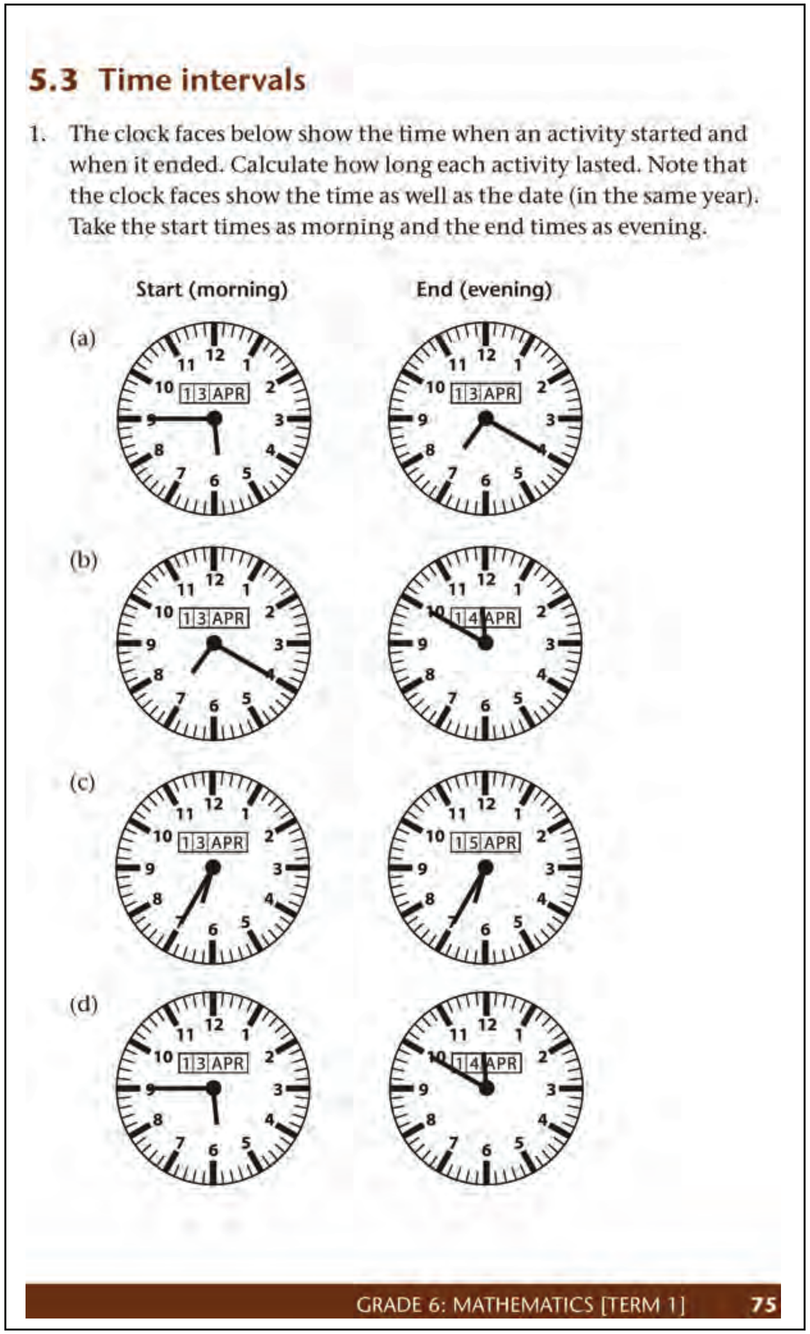


The total number of whole hours is 16 + 23, which is 39.

If we add the minutes we work it out as follows: On 13 April we have 40 minutes from 07:20 to 08:00. On 14 April we have 50 minutes from 23:00 to 23:50. So adding the minutes we get 90 minutes. 90 minutes is 1 hour and 30 minutes. So the total time interval is 39 hours + 1 hour + 30 minutes = 40 hours and 30 minutes.

Answers

1. (a) 13 hours + 15 minutes + 20 minutes
   = 13 hours and 35 minutes
   (b) 16 hours + 23 hours + 40 minutes + 50 minutes
   = 39 hours + 1 hour + 30 minutes
   = 40 hours and 30 minutes
   (c) 2 days + 12 hours
   = 48 hours + 12 hours
   = 60 hours
   (d) 24 hours + 12 hours + 6 hours + 5 minutes
   = 42 hours and 5 minutes



5.3 Time intervals

1. The clock faces below show the time when an activity started and when it ended. Calculate how long each activity lasted. Note that the clock faces show the time as well as the date (in the same year). Take the start times as morning and the end times as evening.

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Notes on questions

In question 2, learners will need to either multiply or divide by 60 to convert between hours, minutes and seconds.

Answers

2.

Hour(s)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$1\frac{1}{2}$
Minutes	15	30	45	$60 + (\frac{1}{2}$ of $60) = 90$
Seconds	900	$30 \times 60 = 1\ 800$	$45 \times 60 = 2\ 700$	$90 \times 60 = 5\ 400$

Hour(s)	2	$2\frac{1}{4}$	$150 \div 60 = 2\frac{1}{2}$	3
Minutes	$7\ 200 \div 60$ $= 120$	$120 + (\frac{1}{4}$ of $60)$ $= 120 + 15 = 135$	150	$3 \times 60 = 180$
Seconds	7 200	$135 \times 60 = 8\ 100$	$150 \times 60 = 9\ \ 000$	$180 \times 60 = 10\ 800$

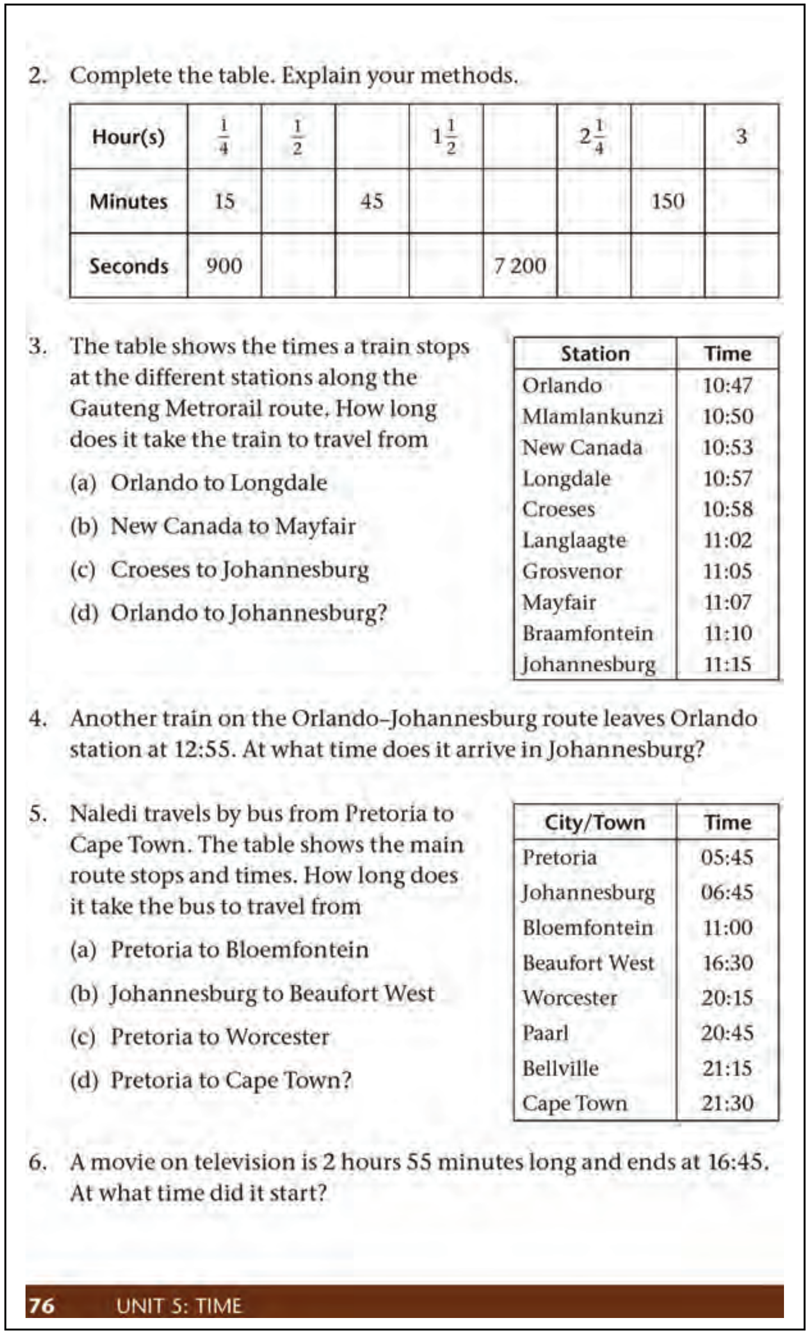
There are 60 seconds in a minute and 60 minutes in an hour. Learners must either multiply or divide by 60 to get an answer each time. Learners might also talk about how they used one answer to get another answer. For example, if you know that 120 minutes is 2 hours, and that 30 minutes is half an hour, then you also know that 150 minutes is $2\frac{1}{2}\frac{1}{2}$ hours. Learners might say (correctly) that to multiply by 60 is the same as multiplying by 6 and then multiplying by 10.

3. (a) 10 minutes (b) 14 minutes
   (c) 17 minutes (d) 18 minutes

4. 12:55 + 18 minutes = 13:13

5. (a) 5 hours and 15 minutes (b) 9 hours and 45 minutes
   (c) 14 hours and 30 minutes (d) 15 hours and 45 minutes

6. 13:50



2. Complete the table. Explain your methods.

Hour(s)	$\frac{1}{4}$	$\frac{1}{2}$		$1\frac{1}{2}$		$2\frac{1}{4}$		3
Minutes	15		45				150	
Seconds	900				7 200			

3. The table shows the times a train stops at the different stations along the Gauteng Metrorail route. How long does it take the train to travel from
   (a) Orlando to Longdale
   (b) New Canada to Mayfair
   (c) Croeses to Johannesburg
   (d) Orlando to Johannesburg?

Station	**Time**
Orlando	10:47
Mlamlankunzi	10:50
New Canada	10:53
Longdale	10:57
Croeses	10:58
Langlaagte	11:02
Grosvenor	11:05
Mayfair	11:07
Braamfontein	11:10
Johannesburg	11:15

4. Another train on the Orlando–Johannesburg route leaves Orlando station at 12:55. At what time does it arrive in Johannesburg?

5. Naledi travels by bus from Pretoria to Cape Town. The table shows the main route stops and times. How long does it take the bus to travel from
   (a) Pretoria to Bloemfontein
   (b) Johannesburg to Beaufort West
   (c) Pretoria to Worcester
   (d) Pretoria to Cape Town?

City/Town	**Time**
Pretoria	05:45
Johannesburg	06:45
Bloemfontein	11:00
Beaufort West	16:30
Worcester	20:15
Paarl	20:45
Bellville	21:15
Cape Town	21:30

6. A movie on television is 2 hours 55 minutes long and ends at 16:45. At what time did it start?

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5.4 Time intervals on the stopwatch

Teaching guidelines

Learners can either use stopwatches that occur as single instruments, or stopwatches on cell phones or wrist watches.

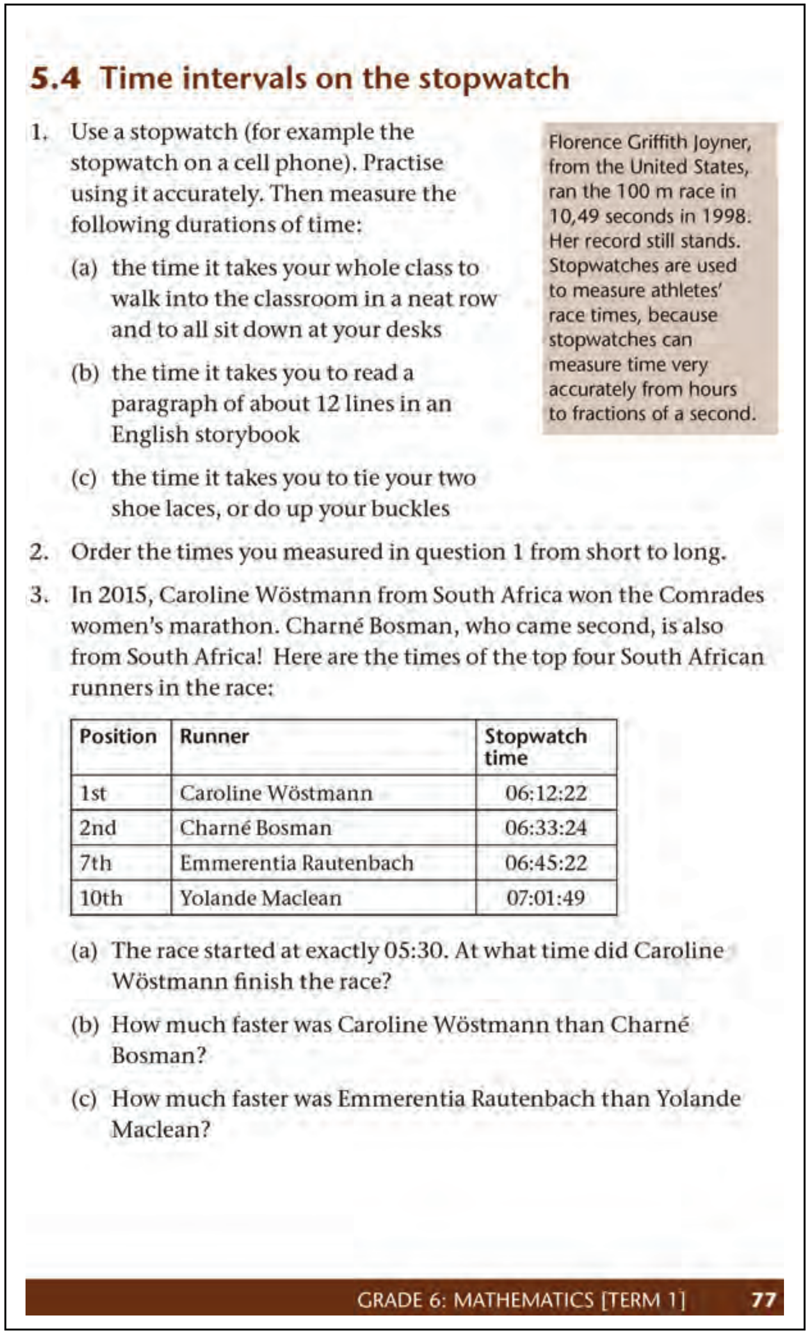
Digital stopwatches are usually easier to read than analogue stopwatches. Many cell phones have a stopwatch function, or the possibility of downloading a free stopwatch app. While learners are busy with classwork, you could work with small groups of learners to show them how a stopwatch works.

Notes on questions

You can use question 3(c) as a challenge because learners will need to convert between minutes and seconds in different ways to get the answer.

Answers

1. (a)–(c) Times will differ from class to class and from learner to learner.
2. Learners' answers will differ from class to class.
3. (a) 11:42:22
   (b) 21 minutes and 2 seconds
   (c) 16 minutes and 27 seconds



5.4 Time intervals on the stopwatch

1. Use a stopwatch (for example the stopwatch on a cell phone). Practise using it accurately. Then measure the following durations of time:
   (a) the time it takes your whole class to walk into the classroom in a neat row and to all sit down at your desks
   (b) the time it takes you to read a paragraph of about 12 lines in an English storybook
   (c) the time it takes you to tie your two shoe laces, or do up your buckles

Florence Griffith Joyner, from the United States, ran the 100 m race in 10,49 seconds in 1998. Her record still stands. Stopwatches are used to measure athletes' race times, because stopwatches can measure time very accurately from hours to fractions of a second.

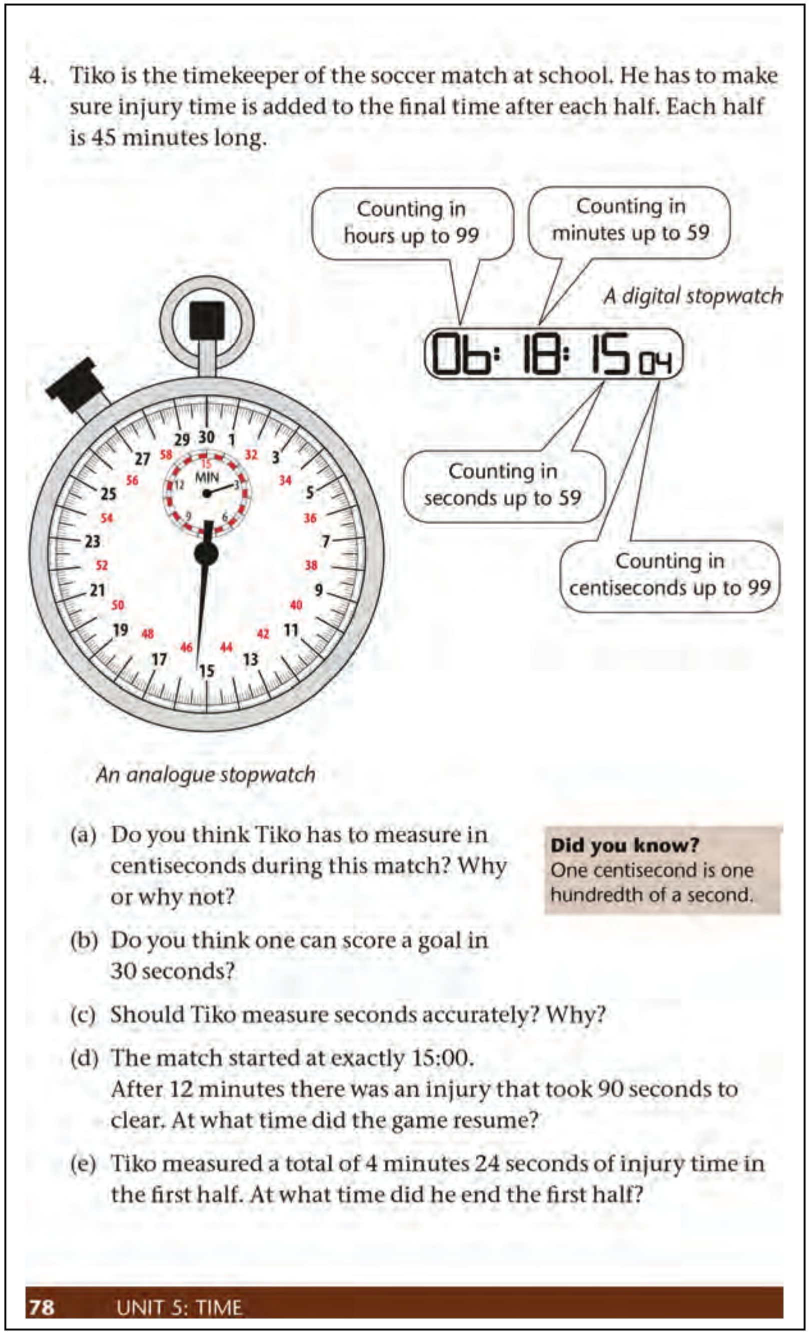
2. Order the times you measured in question 1 from short to long.
3. In 2015, Caroline Wöstmann from South Africa won the Comrades women's marathon. Charné Bosman, who came second, is also from South Africa! Here are the times of the top four South African runners in the race:

Position	Runner	Stopwatch time
1st	Caroline Wöstmann	06:12:22
2nd	Charné Bosman	06:33:24
7th	Emmerentia Rautenbach	06:45:22
10th	Yolande Maclean	07:01:49

   (a) The race started at exactly 05:30. At what time did Caroline Wöstmann finish the race?
   (b) How much faster was Caroline Wöstmann than Charné Bosman?
   (c) How much faster was Emmerentia Rautenbach than Yolande Maclean?

Answers

4. (a) No, centiseconds are $\frac{1}{100}$ or one hundredth of a second; time less than one second is too short to add to a match. Adding less than 1 second to a match will not change the score.
   (b) Yes, it is possible.
   (c) Yes, because adding the wrong number of seconds could change the outcome of the match.
   (d) 15:13:30
   (e) Total time: 45 minutes + 4 minutes and 24 seconds = 49 minutes and 24 seconds
   Final time: 15:49:24



4. Tiko is the timekeeper of the soccer match at school. He has to make sure injury time is added to the final time after each half. Each half is 45 minutes long.

A digital stopwatch

An analogue stopwatch

(a) Do you think Tiko has to measure in centiseconds during this match? Why or why not?

Did you know?
One centisecond is one hundredth of a second.

(b) Do you think one can score a goal in 30 seconds?

(c) Should Tiko measure seconds accurately? Why?

(d) The match started at exactly 15:00.
After 12 minutes there was an injury that took 90 seconds to clear. At what time did the game resume?

(e) Tiko measured a total of 4 minutes 24 seconds of injury time in the first half. At what time did he end the first half?

5.5 Years, decades and centuries

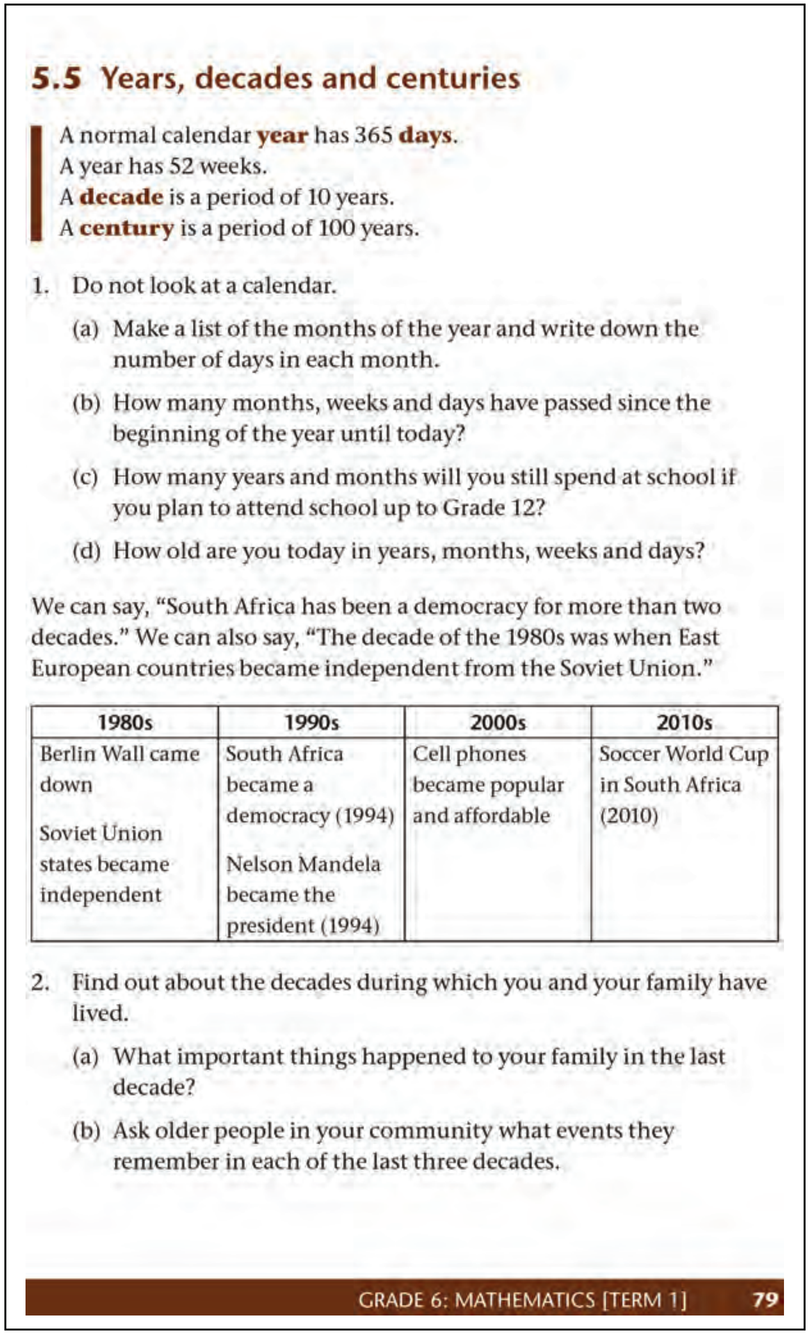
Teaching guidelines

Learners will need to do question 2 at home. You may want to give them a few days to complete this question because they need to talk to several people in the community. Another option could be to invite a few older people to come and tell the class about what they remember of the last three decades (i.e. 30 years).

Answers

1. (a) January (31), February (28 in a non-leap year; 29 in a leap year), March (31), April (30), May (31), June (30), July (31), August (31), September (30), October (31), November (30), December (31)

   (b)–(d) Answers will differ from class to class depending on the date on which learners do this section. If you are teaching this in Term 1, you might be in late February. So, if learners give an answer of 5 months, for example, ask them how they can work out whether their answer might be right or wrong.
2. (a)–(b) Learners' answers will differ.



5.5 Years, decades and centuries

A normal calendar **year** has 365 **days**.
A year has 52 weeks.
A **decade** is a period of 10 years.
A **century** is a period of 100 years.

1. Do not look at a calendar.
   - (a) Make a list of the months of the year and write down the number of days in each month.
   - (b) How many months, weeks and days have passed since the beginning of the year until today?
   - (c) How many years and months will you still spend at school if you plan to attend school up to Grade 12?
   - (d) How old are you today in years, months, weeks and days?

We can say, "South Africa has been a democracy for more than two decades." We can also say, "The decade of the 1980s was when East European countries became independent from the Soviet Union."

1980s	1990s	2000s	2010s
Berlin Wall came down Soviet Union states became independent	South Africa became a democracy (1994) Nelson Mandela became the president (1994)	Cell phones became popular and affordable	Soccer World Cup in South Africa (2010)

2. Find out about the decades during which you and your family have lived.
   - (a) What important things happened to your family in the last decade?
   - (b) Ask older people in your community what events they remember in each of the last three decades.

Teaching guidelines

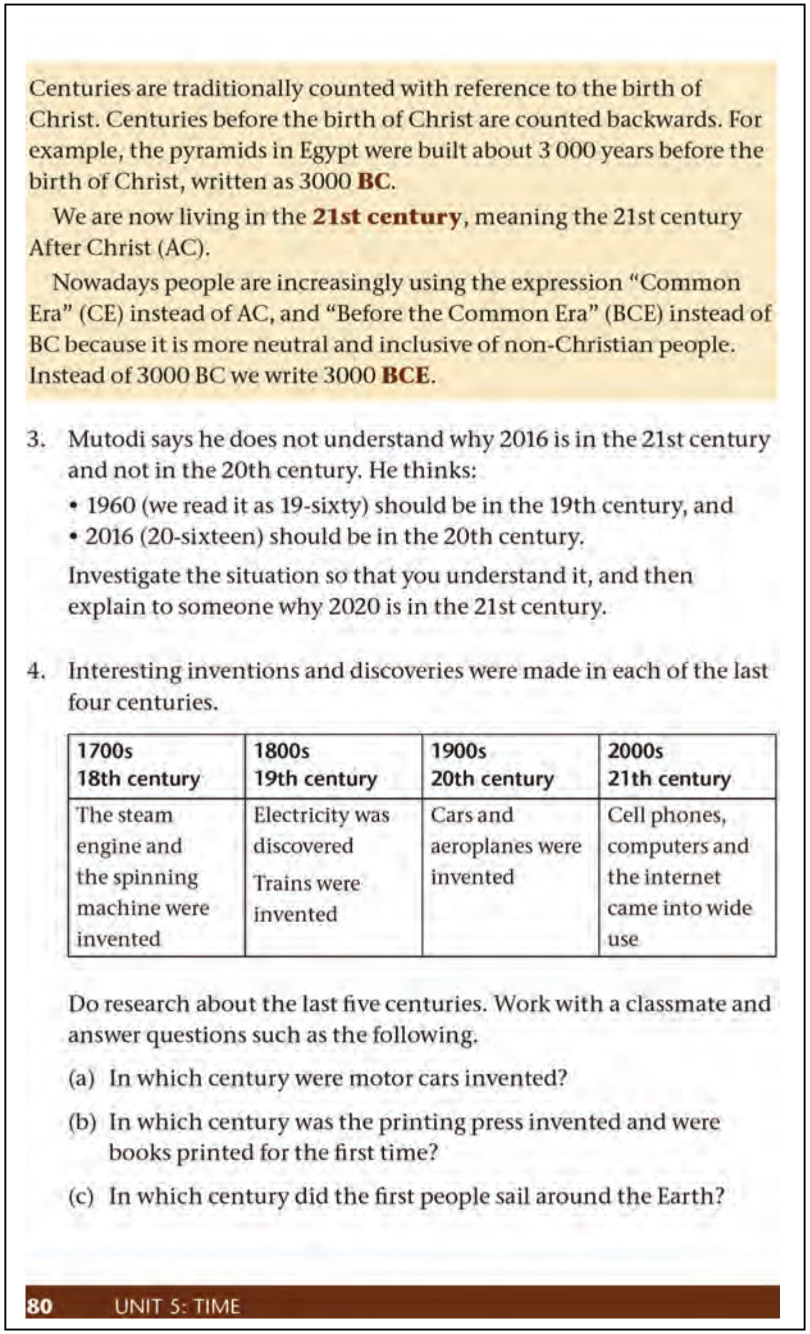
Learners will need time to do question 4. If they are not able to access a library or the internet, you may need to bring in some historical sources for them to read in order to answer question 4. They could hand in their answers to question 4 after a few days.

Possible misconceptions

It is commonly accepted that in the 1400s, Johannes Gutenberg invented the printing press. However, the first moveable type presses were invented by Bi Sheng in China during the 11th century. The first moveable type metal presses were invented by a Korean Buddhist monk named Baegun in 1377. Therefore, it is safe to say that printing presses existed before Johannes Gutenberg introduced the printing press to Europe in the 1400s. Some people say that Gutenberg changed the world by manufacturing printing presses, not by inventing the printing press.

Answers

3. Learners' answers will differ. Perhaps one way to think about it is to compare it to birthdays, for instance. We say that a baby is one year old when the baby has already lived for 12 months. Children are not considered to be one year old in their first year of life. Instead, they are considered to be one year old in the second year of their life. Similarly, the years from 1 to 99 were the first century. The years 200 to 299 were the third century, etc. So, the years 2000 to 2099 form the 21st century.
4. (a) In the 1800s, i.e. in the 19th century.
   (b) Refer to the comments under "Possible misconceptions". Learners may only mention the printing press developed by Gutenberg in the 1400s.
   (c) In the 1500s, i.e. in the sixteenth century.



Centuries are traditionally counted with reference to the birth of Christ. Centuries before the birth of Christ are counted backwards. For example, the pyramids in Egypt were built about 3 000 years before the birth of Christ, written as 3000 **BC**.

We are now living in the **21st century**, meaning the 21st century After Christ (AC).

Nowadays people are increasingly using the expression "Common Era" (CE) instead of AC, and "Before the Common Era" (BCE) instead of BC because it is more neutral and inclusive of non-Christian people. Instead of 3000 BC we write 3000 **BCE**.

3. Mutodi says he does not understand why 2016 is in the 21st century and not in the 20th century. He thinks:
   - 1960 (we read it as 19-sixty) should be in the 19th century, and
   - 2016 (20-sixteen) should be in the 20th century.

   Investigate the situation so that you understand it, and then explain to someone why 2020 is in the 21st century.

4. Interesting inventions and discoveries were made in each of the last four centuries.

1700s 18th century	1800s 19th century	1900s 20th century	2000s 21th century
The steam engine and the spinning machine were invented	Electricity was discovered Trains were invented	Cars and aeroplanes were invented	Cell phones, computers and the internet came into wide use

Do research about the last five centuries. Work with a classmate and answer questions such as the following.

(a) In which century were motor cars invented?

(b) In which century was the printing press invented and were books printed for the first time?

(c) In which century did the first people sail around the Earth?

5.6 A short history of calendars

Teaching guidelines

Read through the brief history of the different types of calendars with the learners.

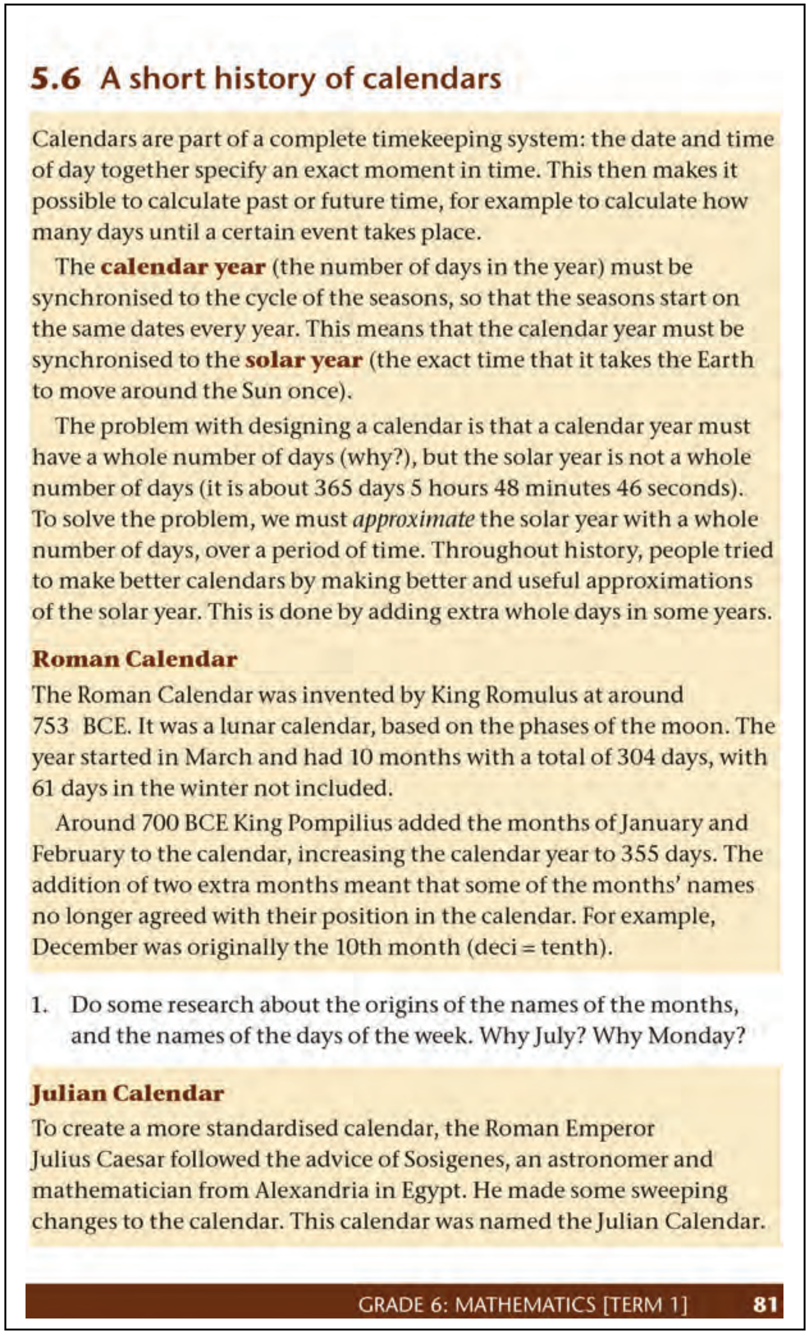
Answers

1. Origins of the names of the **months** of the year:

   January: Named after the Roman god of beginnings and endings, Janus
   February: Named after Februa, the Roman feast of purification
   March: Named after the Roman god of war, Mars
   April: Named after either *aperire*, which means "to open", or the Greek god of love, Aphrodite
   May: Named after the Roman goddess of love and honour, Maiesta
   June: Named after Juno, the queen of the gods
   July: Named after Julius Caesar
   August: Named after Augustus Caesar
   September: Named after *septem*, which means "seven" (the Roman calendar started with March as the first month, so September would have been the seventh month)
   October: Named after *octo*, which means "eight"
   November: Named after *novem*, which means "nine"
   December: Named after *decem*, which means "ten"

   Origins of the names of the **days** of the week:

   Monday: Comes from *monandaeg*, an Anglo-Saxon word that means the "moon's day"
   Tuesday: Named after a one-handed Norse god, *Týr*
   Wednesday: Named after the Germanic god, *Wõdan*
   Thursday: Named after a Norse god, *Thor*
   Friday: Named after a Norse goddess, *Frigg*
   Saturday: Named after *dies Saturni*, which means "Saturn's day"
   Sunday: Named after *dies solis*, which means "the sun's day"



5.6 A short history of calendars

Calendars are part of a complete timekeeping system: the date and time of day together specify an exact moment in time. This then makes it possible to calculate past or future time, for example to calculate how many days until a certain event takes place.

The **calendar year** (the number of days in the year) must be synchronised to the cycle of the seasons, so that the seasons start on the same dates every year. This means that the calendar year must be synchronised to the **solar year** (the exact time that it takes the Earth to move around the Sun once).

The problem with designing a calendar is that a calendar year must have a whole number of days (why?), but the solar year is not a whole number of days (it is about 365 days 5 hours 48 minutes 46 seconds). To solve the problem, we must *approximate* the solar year with a whole number of days, over a period of time. Throughout history, people tried to make better calendars by making better and useful approximations of the solar year. This is done by adding extra whole days in some years.

Roman Calendar

The Roman Calendar was invented by King Romulus at around 753 BCE. It was a lunar calendar, based on the phases of the moon. The year started in March and had 10 months with a total of 304 days, with 61 days in the winter not included.

Around 700 BCE King Pompilius added the months of January and February to the calendar, increasing the calendar year to 355 days. The addition of two extra months meant that some of the months' names no longer agreed with their position in the calendar. For example, December was originally the 10th month (deci = tenth).

1. Do some research about the origins of the names of the months, and the names of the days of the week. Why July? Why Monday?

Julian Calendar

To create a more standardised calendar, the Roman Emperor Julius Caesar followed the advice of Sosigenes, an astronomer and mathematician from Alexandria in Egypt. He made some sweeping changes to the calendar. This calendar was named the Julian Calendar.

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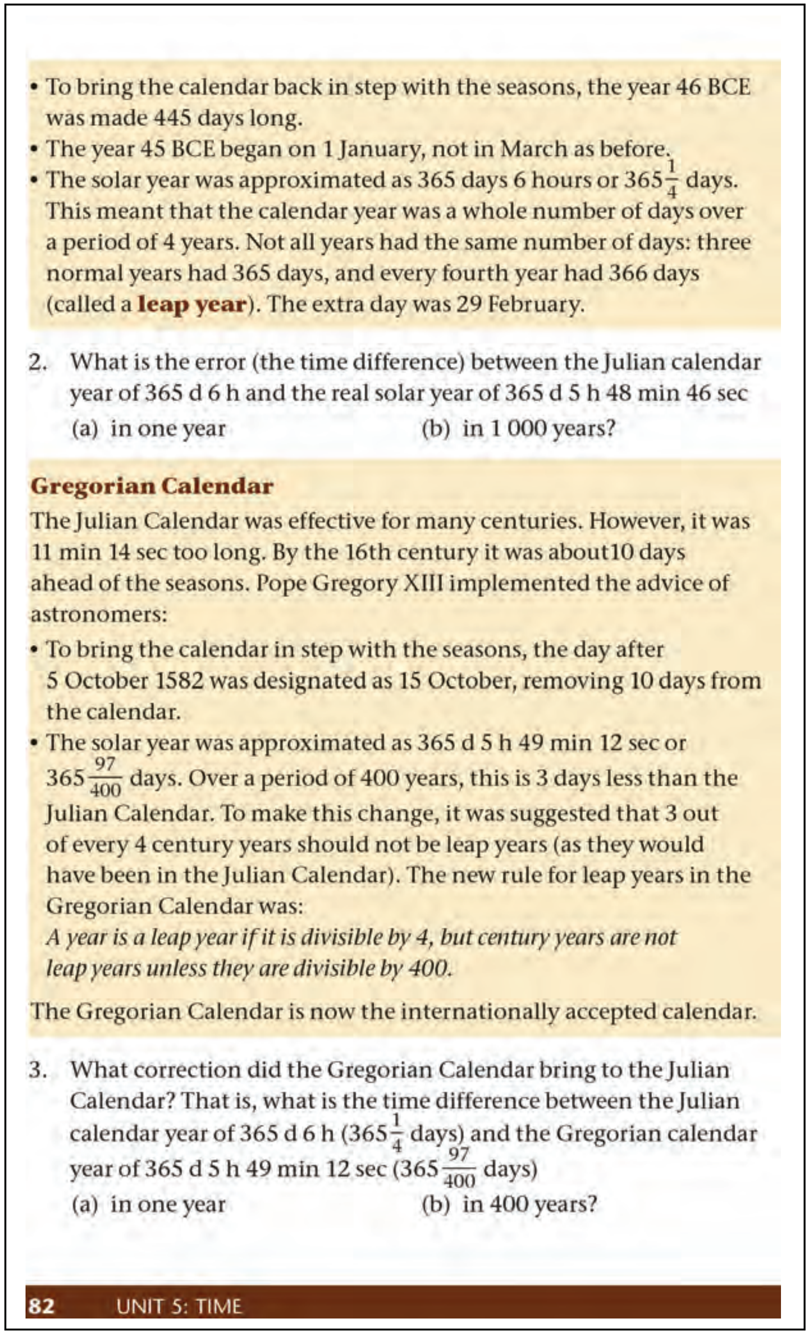
Notes on questions

You can use questions 2(b) and 3(b) as a challenge, or you could let the class work through these questions together.

Answers

2. (a) 11 min 14 sec
   (b) 187 hours 13 minutes and 20 seconds or 7 days 19 hours 13 minutes 20 seconds
3. (a) 10 min 48 sec
   (b) 3 days



- To bring the calendar back in step with the seasons, the year 46 BCE was made 445 days long.
- The year 45 BCE began on 1 January, not in March as before.
- The solar year was approximated as 365 days 6 hours or $365\frac{1}{4}$ days. This meant that the calendar year was a whole number of days over a period of 4 years. Not all years had the same number of days: three normal years had 365 days, and every fourth year had 366 days (called a **leap year**). The extra day was 29 February.

2. What is the error (the time difference) between the Julian calendar year of 365 d 6 h and the real solar year of 365 d 5 h 48 min 46 sec
   (a) in one year (b) in 1 000 years?

Gregorian Calendar

The Julian Calendar was effective for many centuries. However, it was 11 min 14 sec too long. By the 16th century it was about10 days ahead of the seasons. Pope Gregory XIII implemented the advice of astronomers:

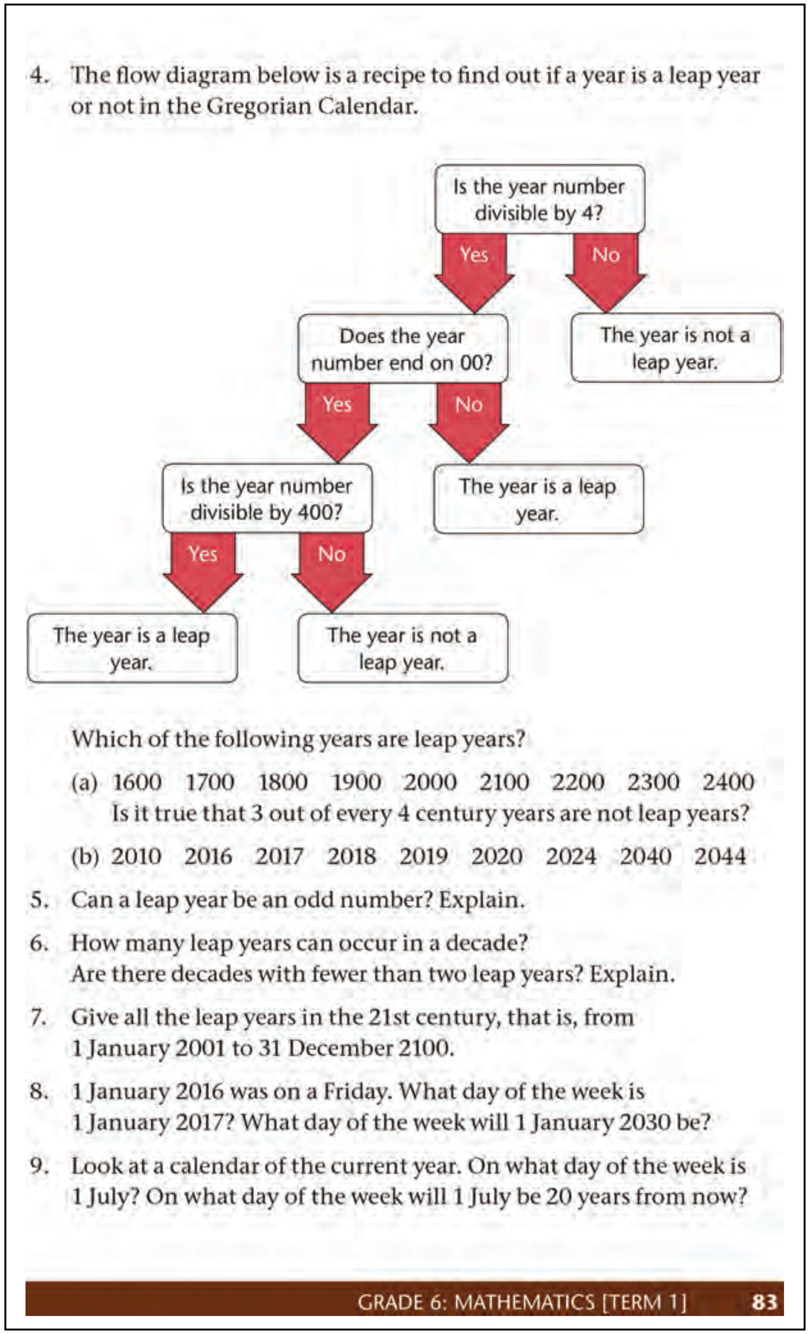
- To bring the calendar in step with the seasons, the day after 5 October 1582 was designated as 15 October, removing 10 days from the calendar.
- The solar year was approximated as 365 d 5 h 49 min 12 sec or $365\frac{97}{400}$ days. Over a period of 400 years, this is 3 days less than the Julian Calendar. To make this change, it was suggested that 3 out of every 4 century years should not be leap years (as they would have been in the Julian Calendar). The new rule for leap years in the Gregorian Calendar was:
  A year is a leap year if it is divisible by 4, but century years are not leap years unless they are divisible by 400.

The Gregorian Calendar is now the internationally accepted calendar.

3. What correction did the Gregorian Calendar bring to the Julian Calendar? That is, what is the time difference between the Julian calendar year of 365 d 6 h ($365\frac{1}{4}$ days) and the Gregorian calendar year of 365 d 5 h 49 min 12 sec ($365\frac{97}{400}$ days)
   (a) in one year (b) in 400 years?

Answers

4. (a) 1600; 2000; 2400
    Yes, three out of every four century years are not leap years.
   (b) 2016; 2020; 2024; 2040; 2044
5. No, all multiples of 4 and 400 are even numbers.
6. Two or three.
   No, the multiples of 4 follow this pattern: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, etc. There are two or three multiples for every 10 years, or one decade.
7. 2004, 2008, 2012, 2016, 2020, 2024, 2028, 2032, 2036, 2040, 2044, 2048, 2052, 2056, 2060, 2064, 2068, 2072, 2076, 2080, 2084, 2088, 2092, 2096, 2100
8. 1 January 2017 is a Sunday; 1 January 2030 is a Tuesday.
9. Answers will differ, depending on the year.



4. The flow diagram below is a recipe to find out if a year is a leap year or not in the Gregorian Calendar.

Which of the following years are leap years?

(a) 1600 1700 1800 1900 2000 2100 2200 2300 2400
Is it true that 3 out of every 4 century years are not leap years?

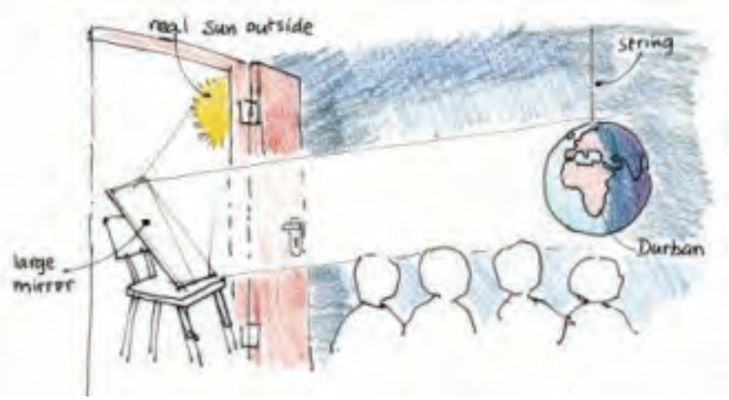
(b) 2010 2016 2017 2018 2019 2020 2024 2040 2044

5. Can a leap year be an odd number? Explain.
6. How many leap years can occur in a decade?
   Are there decades with fewer than two leap years? Explain.
7. Give all the leap years in the 21st century, that is, from 1 January 2001 to 31 December 2100.
8. 1 January 2016 was on a Friday. What day of the week is 1 January 2017? What day of the week will 1 January 2030 be?
9. Look at a calendar of the current year. On what day of the week is 1 July? On what day of the week will 1 July be 20 years from now?

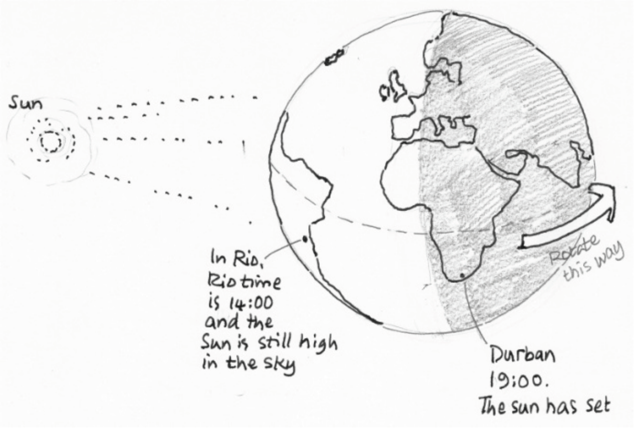
5.7 Time zones

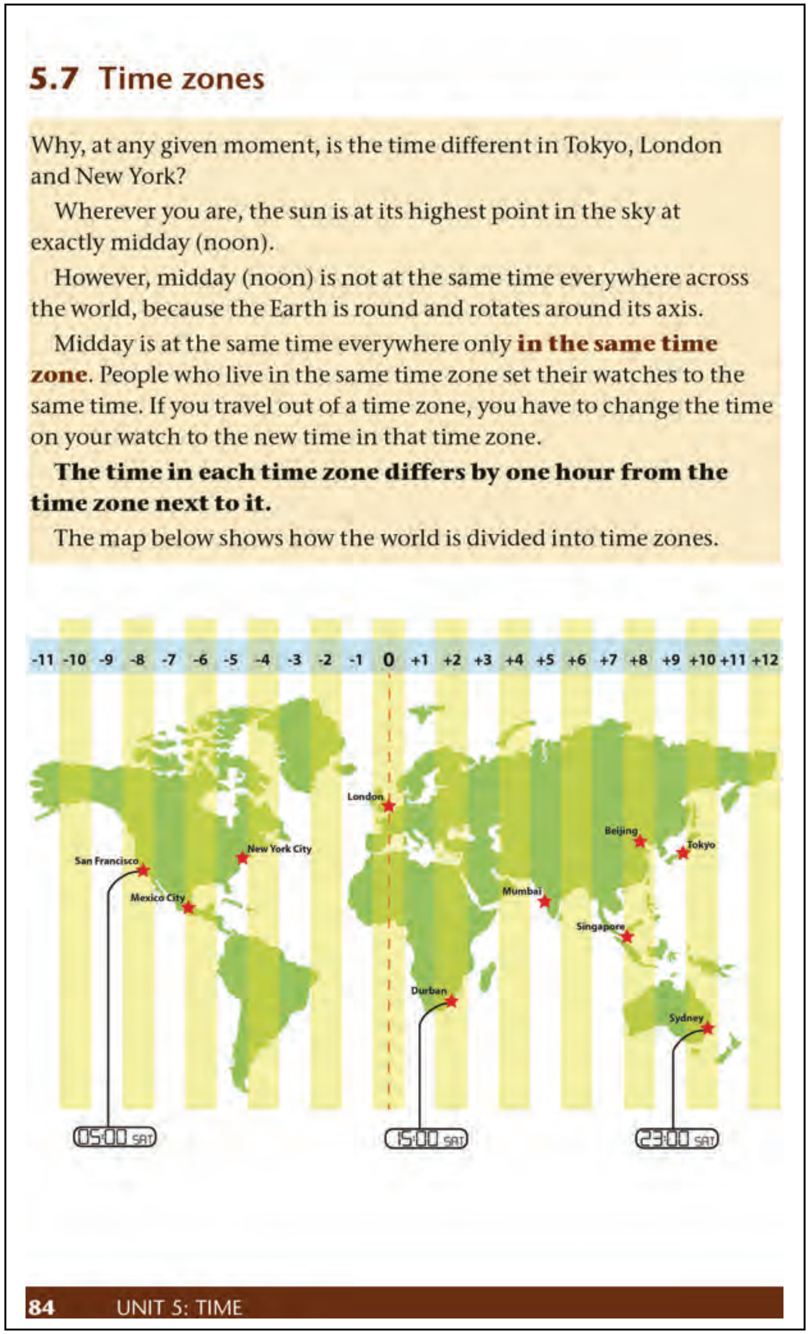
Teaching guidelines

This may be a difficult topic to teach if your learners lack clear ideas about where other places are in South Africa or the rest of the world. Go through it slowly and build up their understanding. You will need a world globe and a large mirror to reflect sunlight into the classroom. In the following picture you see a globe with real sunlight on it. Set up a large mirror (taken from a dressing-table, for example) to reflect sunlight into the classroom. Don't waste time shining a torch onto the globe as it won't be convincing enough to the learners and, of course, the torch moves around, unlike the real sun.



In the following picture you can see South Africa and part of South America. In 2016 the Olympic Games were held in Rio de Janeiro in South America. The South African Wayde van Niekerk won the 400 m race in 43:03 seconds, breaking a world record that had been unbroken for 17 years. The race was run at 20:00 Rio de Janeiro time (that is, 8:00 p.m.).





5.7 Time zones

Why, at any given moment, is the time different in Tokyo, London and New York?

Wherever you are, the sun is at its highest point in the sky at exactly midday (noon).

However, midday (noon) is not at the same time everywhere across the world, because the Earth is round and rotates around its axis.

Midday is at the same time everywhere only **in the same time zone**. People who live in the same time zone set their watches to the same time. If you travel out of a time zone, you have to change the time on your watch to the new time in that time zone.

The time in each time zone differs by one hour from the time zone next to it.

The map below shows how the world is divided into time zones.

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However, people in South Africa had to stay up until 01:00 (an hour past midnight) to watch the race live on TV. Ask learners why South Africans could not watch the race at 20:00. You can show them the answer using the globe.

Referring back to the illustration, the time in Durban is not yet 01:00, it is only 19:00 and the sun has set an hour ago. Far away in Rio, the afternoon sun is high in the sky and Rio time is 14:00.

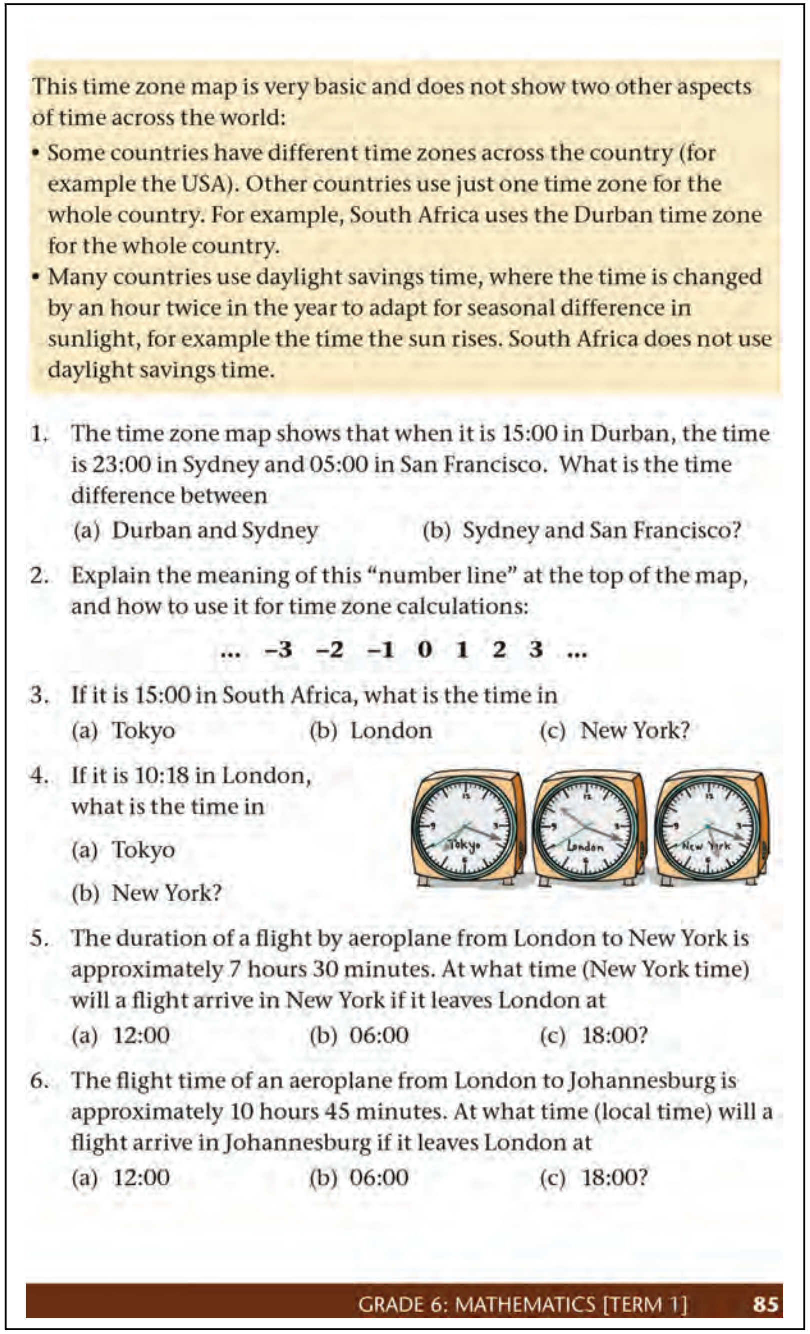
Set the globe with the shadow's edge going through Africa, and South America in full sunshine. In Durban people are finishing their supper, but in Rio people have only just finished their lunch.

Now rotate the globe slowly in the direction that the arrow shows (you rotate Durban further into the shadow and away from the sunlight). The race was run at 20:00, Rio de Janeiro time.

When the learners have answered that question, ask them how many hours Rio is behind Durban time. (The number of hours is 5.) The time zone map in the Learner Book will give them the answer, because there are five time zones between Durban and Rio. If the time in Durban is 19:00, then they can count backwards: 18:00, 17:00, 16:00, 15:00 and 14:00, which makes five time zones.

Answers

1. (a) Durban is 8 hours ahead of Sydney.
   (b) San Francisco is 18 hours behind Sydney or Sydney is 18 hours ahead of San Francisco.
2. The "number line" shows how many hours ahead or behind that time zone is of the standard Greenwich Mean Time (GMT). South African time is two hours later than GMT.
3. (a) 22:00
   (b) 13:00
   (c) 08:00
4. (a) 19:18
   (b) 05:18
5. (a) 19:30 – 5 = 14:30
   (b) 13:30 – 5 = 08:30
   (c) 20:30
6. (a) 00:45 the following morning
   (b) 18:45
   (c) 06:45 the following morning



This time zone map is very basic and does not show two other aspects of time across the world:

- Some countries have different time zones across the country (for example the USA). Other countries use just one time zone for the whole country. For example, South Africa uses the Durban time zone for the whole country.
- Many countries use daylight savings time, where the time is changed by an hour twice in the year to adapt for seasonal difference in sunlight, for example the time the sun rises. South Africa does not use daylight savings time.

1. The time zone map shows that when it is 15:00 in Durban, the time is 23:00 in Sydney and 05:00 in San Francisco. What is the time difference between
   (a) Durban and Sydney (b) Sydney and San Francisco?
2. Explain the meaning of this "number line" at the top of the map, and how to use it for time zone calculations:

   … –3 –2 –1 0 1 2 3 …
3. If it is 15:00 in South Africa, what is the time in
   (a) Tokyo (b) London (c) New York?
4. If it is 10:18 in London, what is the time in
   (a) Tokyo
   (b) New York?
5. The duration of a flight by aeroplane from London to New York is approximately 7 hours 30 minutes. At what time (New York time) will a flight arrive in New York if it leaves London at
   (a) 12:00 (b) 06:00 (c) 18:00?
6. The flight time of an aeroplane from London to Johannesburg is approximately 10 hours 45 minutes. At what time (local time) will a flight arrive in Johannesburg if it leaves London at
   (a) 12:00 (b) 06:00 (c) 18:00?

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Grade 6 Term 1 Unit 6 Properties of two-dimensional shapes

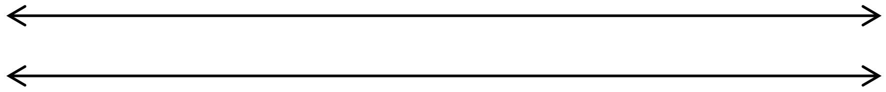
Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
6.1 Naming figures by the number of sides	Identifying and naming polygons	86
6.2 Angles	The angle concept	87 to 91
6.3 Angles of different sizes	Classifying and naming types of angles	92 to 95
6.4 Parallelograms	Identifying (seeing) angles in figures, classifying and comparing them	96

CAPS time allocation	8 hours
CAPS page references	21 to 22 and 229 to 232

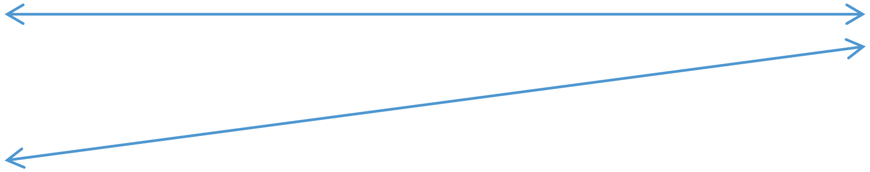
Mathematical background

Rectangles are quadrilaterals with four equal right angles *and* with opposite sides equal. Squares are special rectangles – they are rectangles with the extra condition that "*all* four sides have the same length". Parallelograms are quadrilaterals with their *opposite angles* equal and their *opposite sides* equal. This means that rectangles are special kinds of parallelograms (the extra condition being "*all* angles are equal", not just the opposite ones).

Two lines may have the same direction, like the opposite sides of a rectangle or any other parallelogram, or two lines may have different directions.



Two lines with different directions are said to be **at an angle** to each other.



The directions of the two blue lines differ more than the directions of the two red lines. The angle between the blue lines is bigger than the angle between the red lines.

Angles are not objects, like triangles, rectangles, circles or prisms are.

The idea of angle is like the idea of distance:
we refer to the **distance between two points**, or the length of a line.

Likewise, we refer to the **angle between two lines**.

The extent of the difference between the directions of two lines can be measured in terms of how much you have to turn the one line to make its direction equal to that of the other line.

Resources

Corrugated cardboard boxes, for example A4 paper boxes, scissors, loose sheets of A4 paper

6.1 Naming figures by the number of sides

Mathematical notes

The word "polygon" literally means many corners: the word stem "poly" means many and "gon" means corner/angle (not side). Any polygon has as many sides as angles.

Teaching guidelines

Ask learners to look at the variety of figures in question 1. Point out that different figures have different numbers of sides. Ask some questions, like: *"How many corners does Figure C have, and how many corners does Figure E have?"* Indicate that Figure G may be described as "a figure with seven sides" or "a figure with seven corners".

Explain the meaning of the word stems "tri", "quadri", "penta", etc. It may be useful to write it on the board:

tri means three
quadri means four
penta means five
etc.

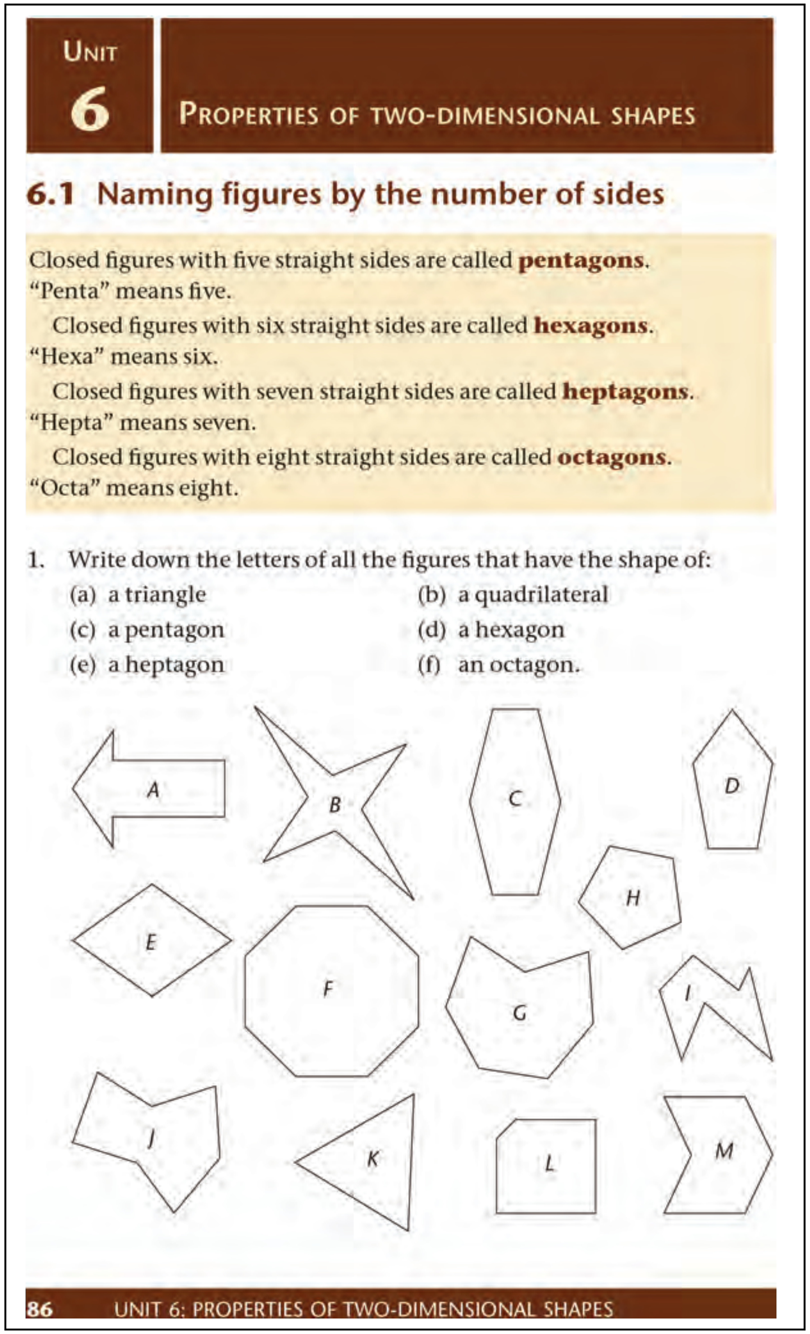
To answer question 1, learners have to count the sides (or corners) of each figure, then select the correct name.

Possible misconceptions

Some learners may have *regular* two-dimensional shapes in mind when they are asked to make decisions about the characteristics, names, etc. of figures they are shown. They may say, for instance, that an irregular heptagon is not a heptagon (e.g. if it is shaped like an arrow). In such cases remind them that **the naming is about the number of sides, not about their orientation or length.**

Answers

1. (a) Triangles: K (b) Quadrilaterals: E
   (c) Pentagons: D, L, H (d) Hexagons: C, M
   (e) Heptagons: G, I, J, A (f) Octagons: B, F



UNIT
6
PROPERTIES OF TWO-DIMENSIONAL SHAPES

6.1 Naming figures by the number of sides

Closed figures with five straight sides are called **pentagons**. "Penta" means five.

Closed figures with six straight sides are called **hexagons**. "Hexa" means six.

Closed figures with seven straight sides are called **heptagons**. "Hepta" means seven.

Closed figures with eight straight sides are called **octagons**. "Octa" means eight.

1. Write down the letters of all the figures that have the shape of:
   (a) a triangle (b) a quadrilateral
   (c) a pentagon (d) a hexagon
   (e) a heptagon (f) an octagon.

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6.2 Angles

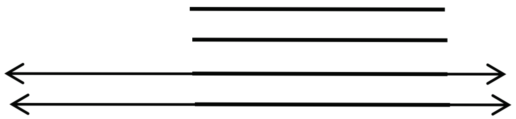
Mathematical notes

The idea of angle can only be properly understood with reference to the ideas of **direction** and **rotation (turn)**. Two lines that do not have the same direction are said to be **at an angle to each other**. The extent of the difference between the directions of two lines can be measured in terms of how much you have to turn the one line to make its direction equal to that of the other line.

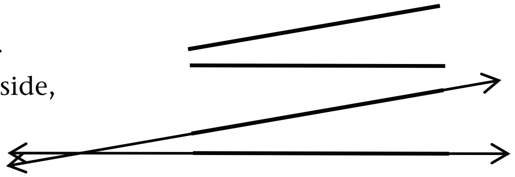
Teaching guidelines

Allow learners to do questions 1, 2, and 3 (continued on next page). When they have finished, you may consolidate by drawing pairs of lines (like below) on the board. Describing each pair in the way indicated below, may be useful for this purpose.

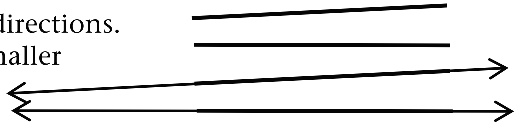
A. These two lines have the same direction. They will remain at the same distance from each other no matter how far you extend them.



B. These two lines have different directions. If they are both extended, they will go further and further away from each other on the one side, and get closer to each other on the other side and eventually meet.

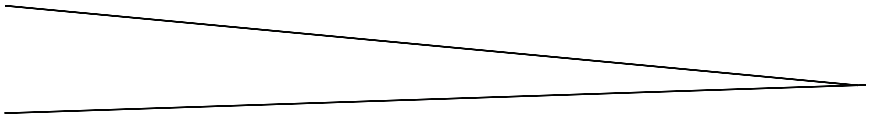


C. These two lines have only slightly different directions. We say the angle between the lines in C is smaller than the angle between the lines in B.

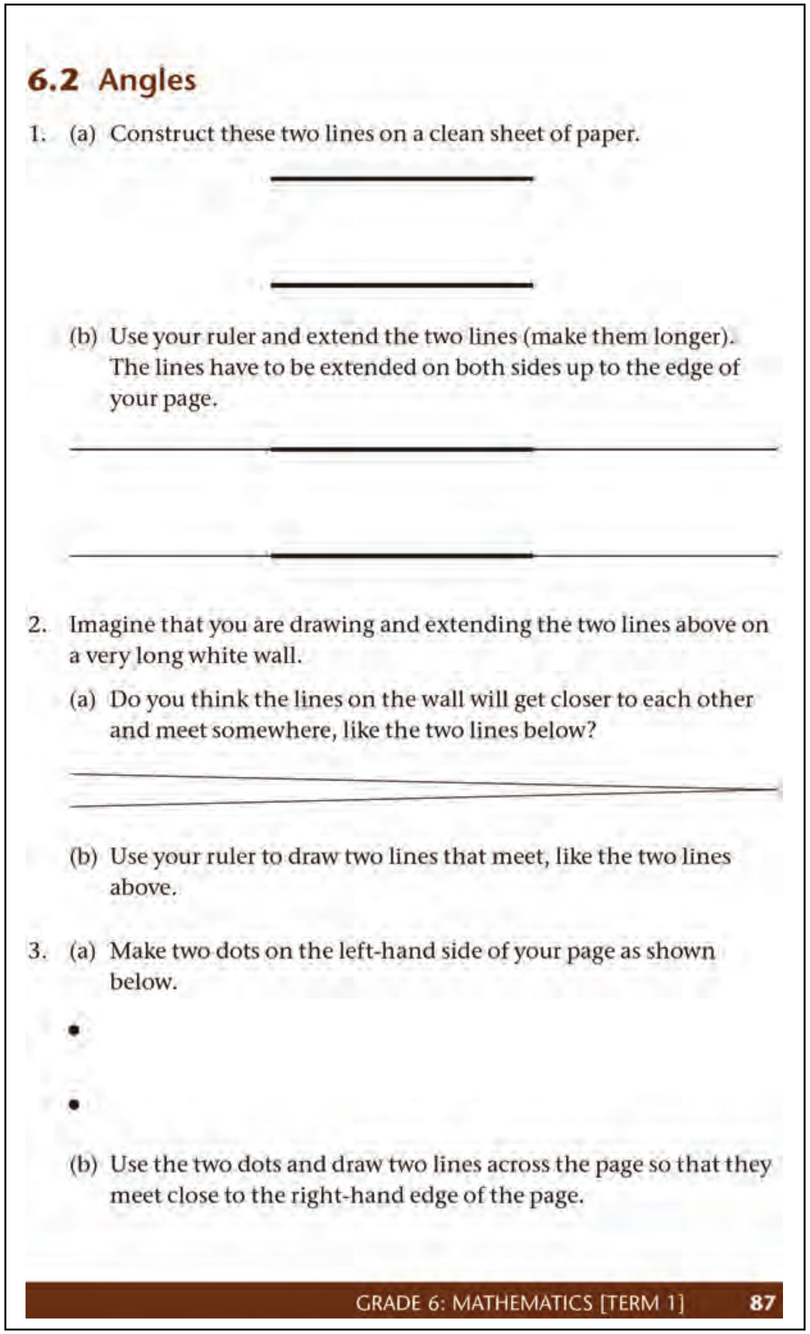


Answers

1. (a)–(b) Two lines as in the Learner Book.
2. (a) The lines will not meet.
   (b)



3. See next page.



6.2 Angles

1. (a) Construct these two lines on a clean sheet of paper.

   (b) Use your ruler and extend the two lines (make them longer). The lines have to be extended on both sides up to the edge of your page.

2. Imagine that you are drawing and extending the two lines above on a very long white wall.

   (a) Do you think the lines on the wall will get closer to each other and meet somewhere, like the two lines below?

   (b) Use your ruler to draw two lines that meet, like the two lines above.

3. (a) Make two dots on the left-hand side of your page as shown below.

   (b) Use the two dots and draw two lines across the page so that they meet close to the right-hand edge of the page.

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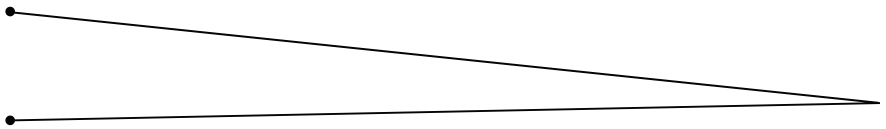
Answers *(continued)*

3. (a)–(b)



(c) Two lines as in the Learner Book.

Teaching guidelines

This page continues to develop the idea that two lines can be at an angle to each other. To stimulate discussion, you may put these questions to the class:

If two lines remain at the same distance from each other, even when they are extended, can they ever meet?

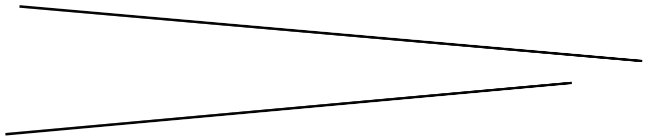
If two lines never meet, even when they are extended, are they everywhere at the same distance from each other?

Notes on questions

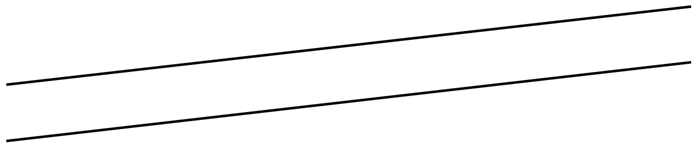
Question 4 is actually a repetition of question 3, but it is phrased differently to ensure that learners engage with the expression "at an angle to each other".

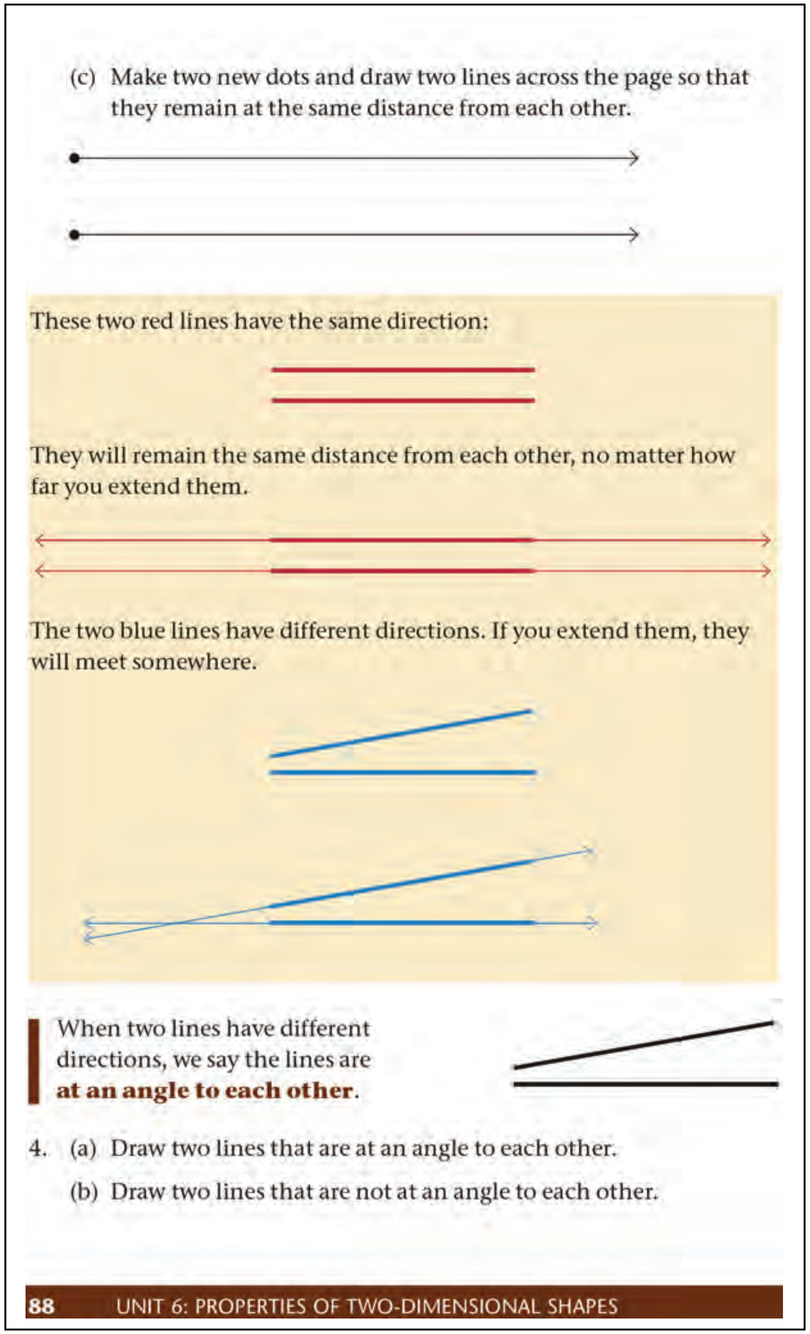
Answers

4. (a) Example:



(b) Example:





(c) Make two new dots and draw two lines across the page so that they remain at the same distance from each other.

These two red lines have the same direction:

They will remain the same distance from each other, no matter how far you extend them.

The two blue lines have different directions. If you extend them, they will meet somewhere.

When two lines have different directions, we say the lines are **at an angle to each other**.

4. (a) Draw two lines that are at an angle to each other.

(b) Draw two lines that are not at an angle to each other.

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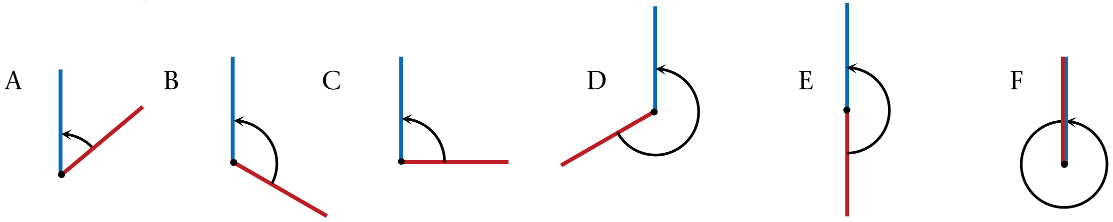
Mathematical notes

Like length, area or volume, angle is a particular type of quantity, a **magnitude**, something that can be measured. Angle can be understood as the amount by which one line needs to be turned so that it has the same direction as a reference line. In the diagrams below, the amount by which the red line needs to be turned around the black dot is indicated by a curved arrow (arc).



Teaching guidelines

The folded cardboard strip is a tool to let learners experience turning an object (one arm of the folded strip) to change the angle it forms with another object (the other arm of the folded strip). Corrugated cardboard, such as the cardboard used for some brands of A4 paper boxes, is ideal for this purpose. Cut the strips so that the corrugations run across the strips' width:

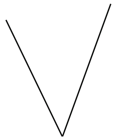
It may be best if you cut the strips beforehand to save classroom time. Cut them about 2 cm wide.

Also make a bigger strip for yourself, which you can use for demonstration purposes.

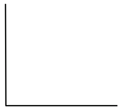
To understand angle as an amount of turn, it is important that learners use the words "smaller" and "bigger" in relation to angles.

Answers

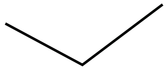
5. Practical work
6. (a) No, the arms are wider apart in Photo B.
   (b) In Photo C.
7. (a)

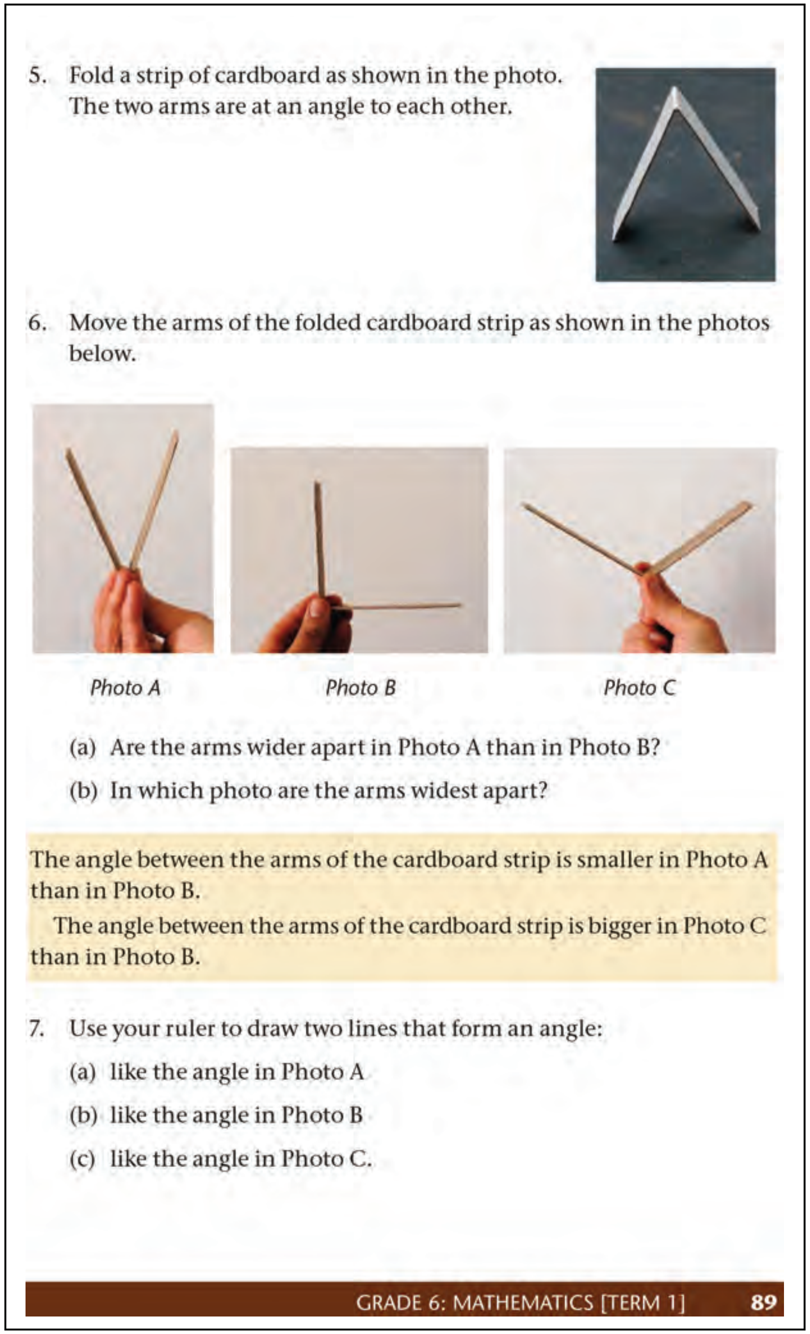


(b)



(c)





5. Fold a strip of cardboard as shown in the photo. The two arms are at an angle to each other.

6. Move the arms of the folded cardboard strip as shown in the photos below.

Photo A　Photo B　Photo C

(a) Are the arms wider apart in Photo A than in Photo B?

(b) In which photo are the arms widest apart?

The angle between the arms of the cardboard strip is smaller in Photo A than in Photo B.

The angle between the arms of the cardboard strip is bigger in Photo C than in Photo B.

7. Use your ruler to draw two lines that form an angle:

(a) like the angle in Photo A

(b) like the angle in Photo B

(c) like the angle in Photo C.

Teaching guidelines

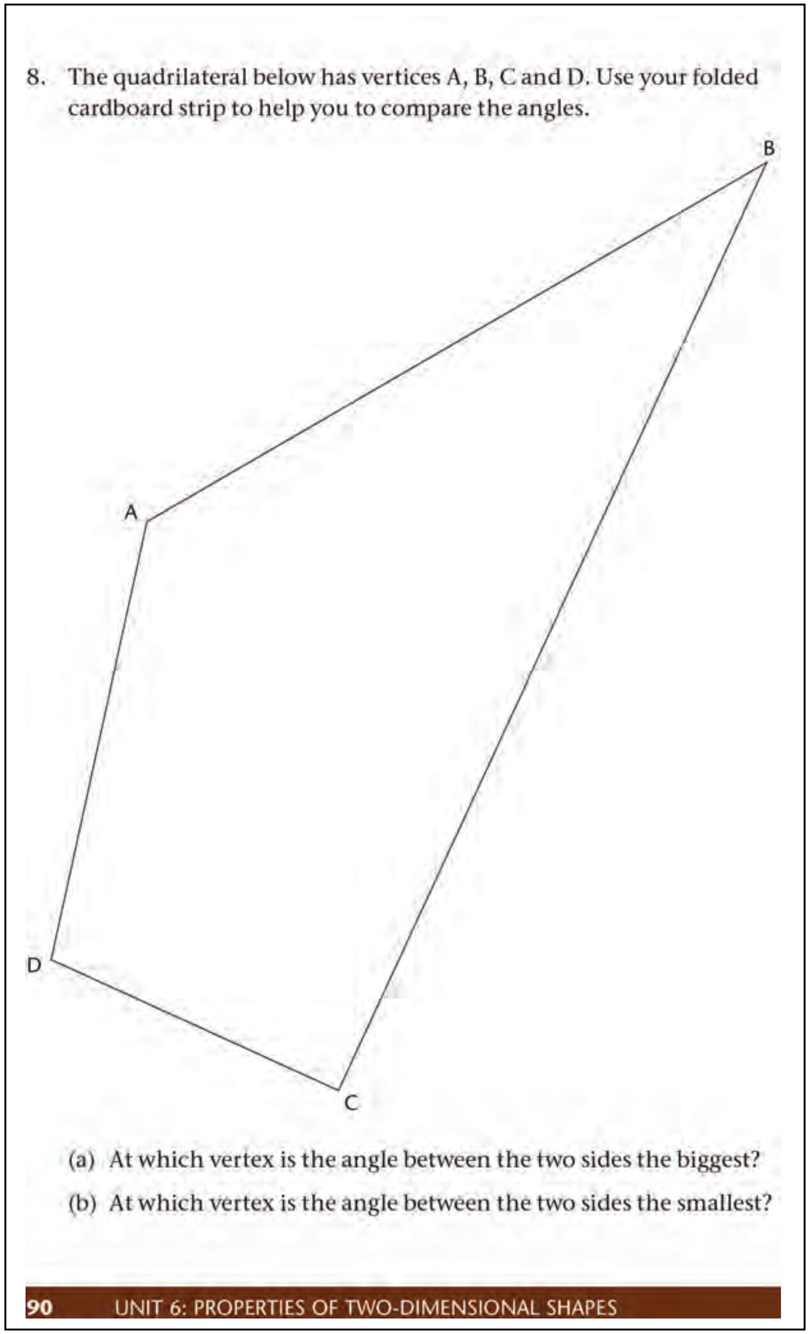
In question 6 on the previous page, learners increased or decreased the angle between the arms of the cardboard strip to develop a sense of angle as an amount of turn, or of how wide the arms are opened.

In question 8 they use the folded cardboard strip to compare the size of angles in a figure.

It will be useful if you make a large similar drawing on the board and demonstrate how the angles can be compared by using a folded cardboard strip.

Answers

8. (a) Vertex A
   (b) Vertex B



8. The quadrilateral below has vertices A, B, C and D. Use your folded cardboard strip to help you to compare the angles.

(a) At which vertex is the angle between the two sides the biggest?

(b) At which vertex is the angle between the two sides the smallest?

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Teaching guidelines

A sheet of A4 paper is a useful example of a rectangular shape. Provide each learner with an A4 sheet.

Apart from continuing the development of the concept of angle, questions 9 to 12 are intended to provide learners with opportunities to experience the similarities and differences between squares, rectangles and parallelograms. To utilise these opportunities in questions 9 to 11, you may put questions like the following to the class:

Which of figures A, B, C and D are rectangles? (B and C)

Is there a square? (Yes, B)

Which figures are not rectangles? (A and D)

Question 12 offers learners a first opportunity to investigate the properties of parallelograms. This is extended in Section 6.4.

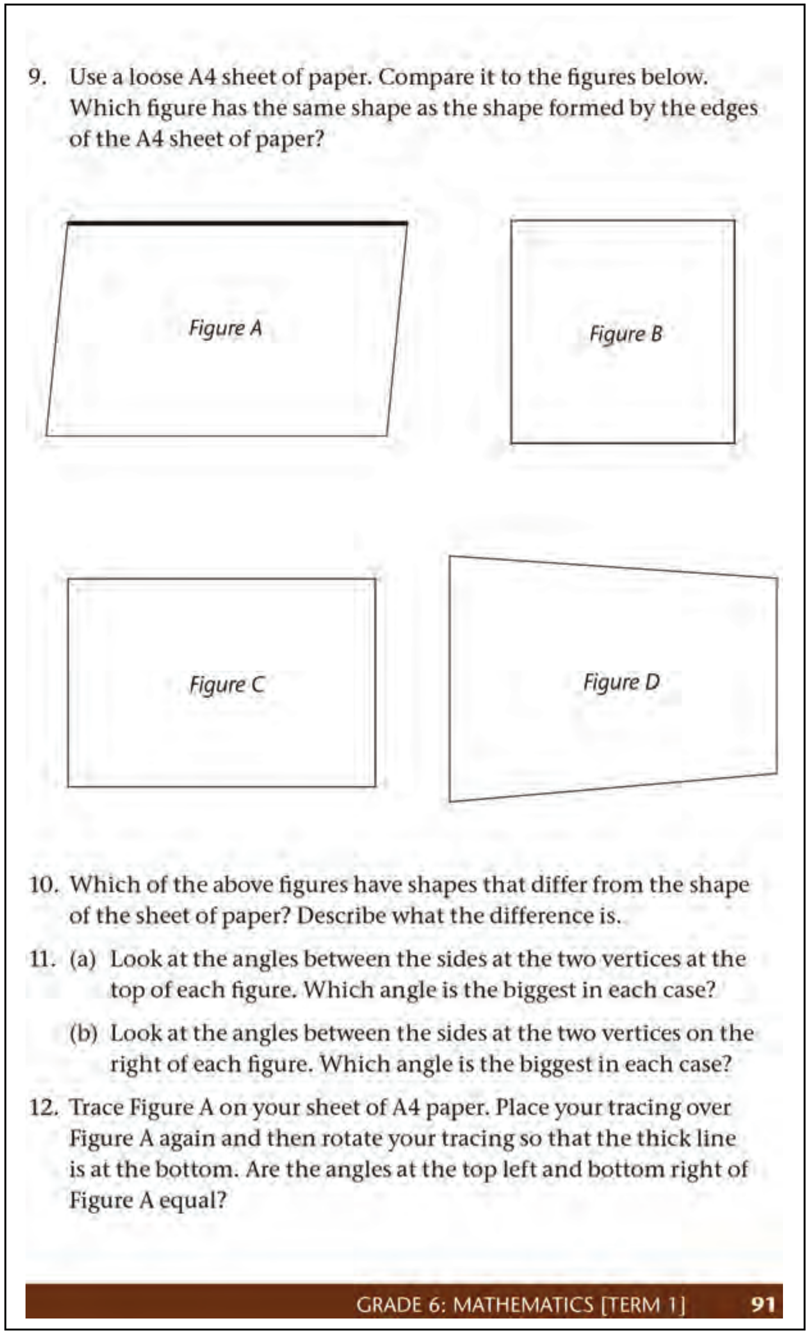
Answers

9. Figure C
10. Figures A, B and D

    Figure B has four sides of equal length; the sheet of paper only has opposite sides of equal length.

    Figures A and D do not have four equal angles like the sheet of paper.
11. (a) In Figure A, the top left angle is bigger than the top right angle. In Figures B and C, the two angles are equal. In Figure D, the top right angle is bigger than the top left angle.

    (b) In Figure A, the top right angle is smaller than the bottom right angle. In Figures B, C and D the top and bottom right angles are equal.
12. Yes



9. Use a loose A4 sheet of paper. Compare it to the figures below. Which figure has the same shape as the shape formed by the edges of the A4 sheet of paper?

10. Which of the above figures have shapes that differ from the shape of the sheet of paper? Describe what the difference is.
11. (a) Look at the angles between the sides at the two vertices at the top of each figure. Which angle is the biggest in each case?

    (b) Look at the angles between the sides at the two vertices on the right of each figure. Which angle is the biggest in each case?
12. Trace Figure A on your sheet of A4 paper. Place your tracing over Figure A again and then rotate your tracing so that the thick line is at the bottom. Are the angles at the top left and bottom right of Figure A equal?

GRADE 6: MATHEMATICS [TERM 1] 91

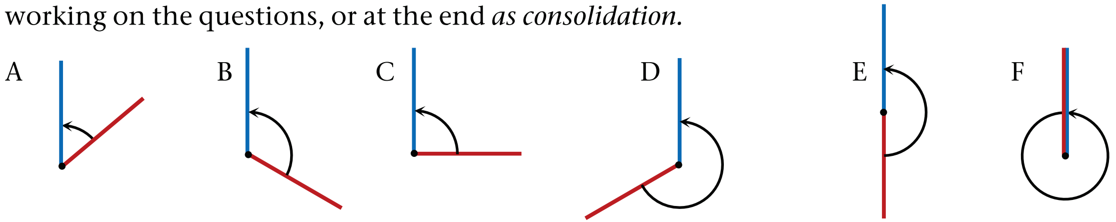
6.3 Angles of different sizes

Teaching guidelines

The idea of a right angle provides a useful reference point for developing knowledge about different angle sizes. Make sure learners keep the right-angle templates they make in question 1, for later use.

The activities in this section provide learners with opportunities to form angles of different sizes and to learn the names of the different size groups (acute, obtuse, etc.).

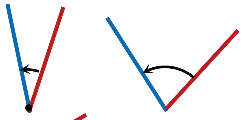
You may find it useful to draw these six angles on the board and provide explanations (along the lines of the text below) *before* learners engage with the activities, *while* they are working on the questions, or at the end *as consolidation*.



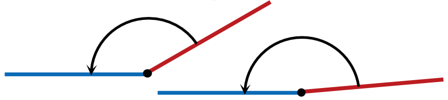
The red line in Diagram B has to be turned more than the red line in Diagram A, to coincide with the blue line (and hence have the same direction). We say **the angle between the red and blue lines in Diagram B is bigger than the angle between the red and blue lines in Diagram A**. The red line in Diagram D has to be turned even more.

The red line in Diagram E has to be turned through half a revolution to coincide with the blue line. An angle of this size is called a **straight angle**. The angle in Diagram C is half of a straight angle. An angle of this size is called a **right angle**.

Angles smaller than right angles, like the angle in Diagram A and in the two diagrams alongside, are called **acute angles**.

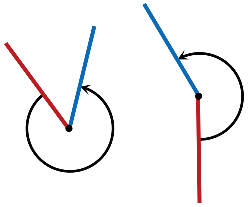


Angles bigger than right angles but smaller than straight angles, like the angle in Diagram B and in the two diagrams alongside, are called **obtuse angles**.



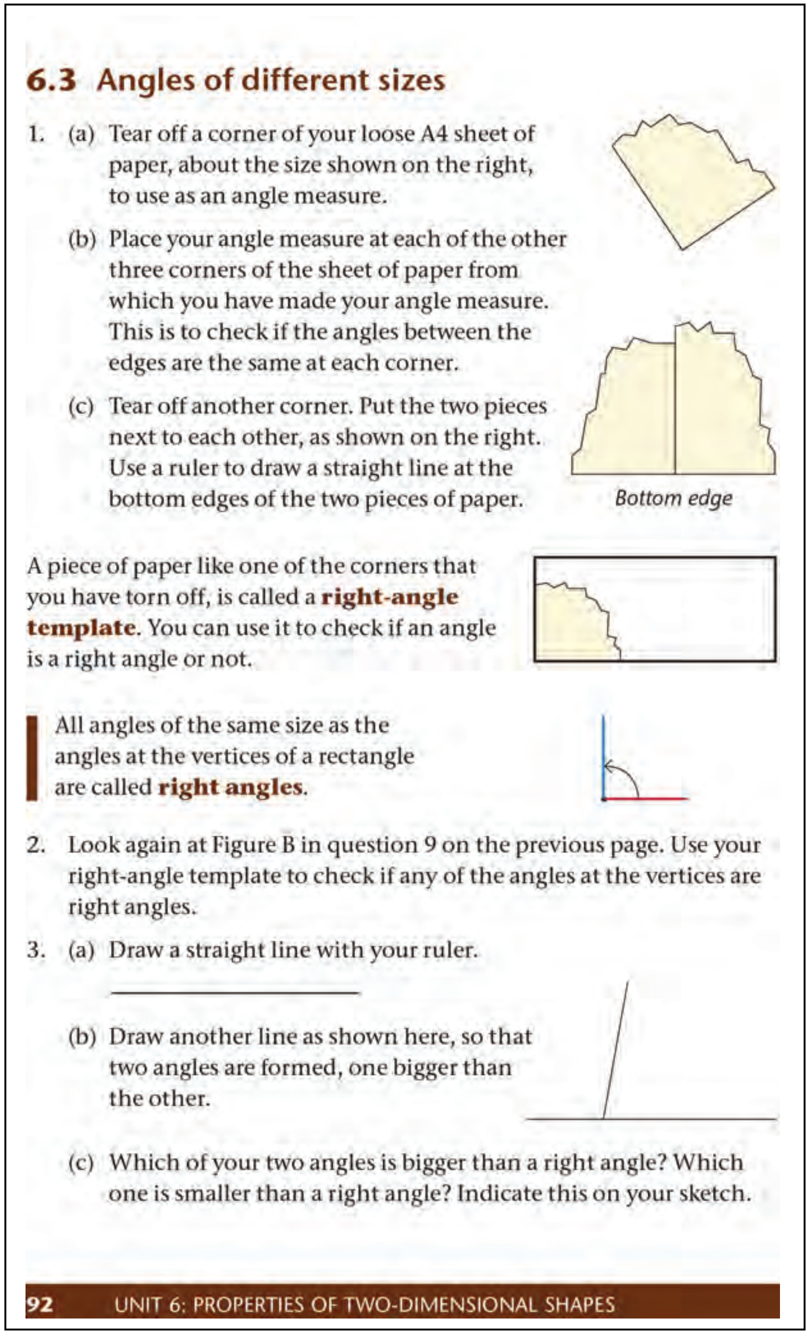
The red and blue lines in Diagram F have the same direction. If the red line is turned right around the black dot so that it returns to its original position, it has been turned through a full **revolution**.

Angles bigger than a straight angle but smaller than a revolution, like the angle in Diagram D and in the two diagrams alongside, are called **reflex angles**.



Answers

See next page.



6.3 Angles of different sizes

1. (a) Tear off a corner of your loose A4 sheet of paper, about the size shown on the right, to use as an angle measure.
   (b) Place your angle measure at each of the other three corners of the sheet of paper from which you have made your angle measure. This is to check if the angles between the edges are the same at each corner.
   (c) Tear off another corner. Put the two pieces next to each other, as shown on the right. Use a ruler to draw a straight line at the bottom edges of the two pieces of paper.

A piece of paper like one of the corners that you have torn off, is called a **right-angle template**. You can use it to check if an angle is a right angle or not.

All angles of the same size as the angles at the vertices of a rectangle are called **right angles**.

2. Look again at Figure B in question 9 on the previous page. Use your right-angle template to check if any of the angles at the vertices are right angles.
3. (a) Draw a straight line with your ruler.
   (b) Draw another line as shown here, so that two angles are formed, one bigger than the other.
   (c) Which of your two angles is bigger than a right angle? Which one is smaller than a right angle? Indicate this on your sketch.

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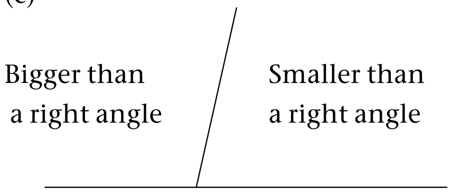
Answers

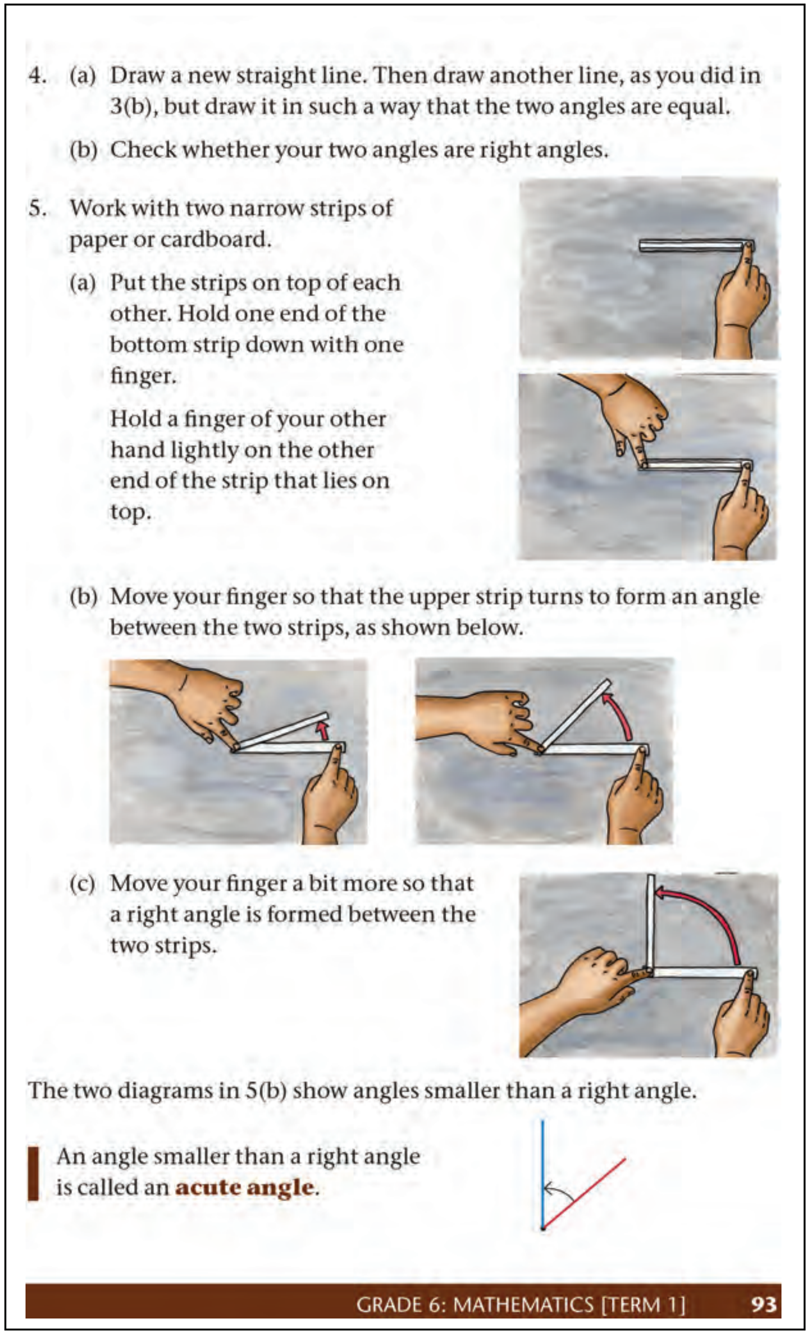
1. Practical work
2. All four angles are right angles.
3. (c)



4.–5. Practical work

Teaching guidelines

There is some danger that learners may focus just on the last bit of turning while doing the activities in questions 5 to 7. For example, in 5(c) they will experience only a small angle (small amount of turn) as the difference between the angles shown in the two diagrams. Ensure that they understand that the angle is the whole movement from the starting position where the one strip was directly on top of the other one.



4. (a) Draw a new straight line. Then draw another line, as you did in 3(b), but draw it in such a way that the two angles are equal.
   (b) Check whether your two angles are right angles.

5. Work with two narrow strips of paper or cardboard.
   (a) Put the strips on top of each other. Hold one end of the bottom strip down with one finger.

   Hold a finger of your other hand lightly on the other end of the strip that lies on top.

   (b) Move your finger so that the upper strip turns to form an angle between the two strips, as shown below.

   (c) Move your finger a bit more so that a right angle is formed between the two strips.

The two diagrams in 5(b) show angles smaller than a right angle.

An angle smaller than a right angle is called an **acute angle**.

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Teaching guidelines

The idea of a straight angle may challenge learners: they may sense that the two lines are not at an angle to each other. To assist them, emphasise that **angle is an amount of turn**, not an object formed by lines. An angle describes how much you have to turn one line so that it has the same direction as another line.

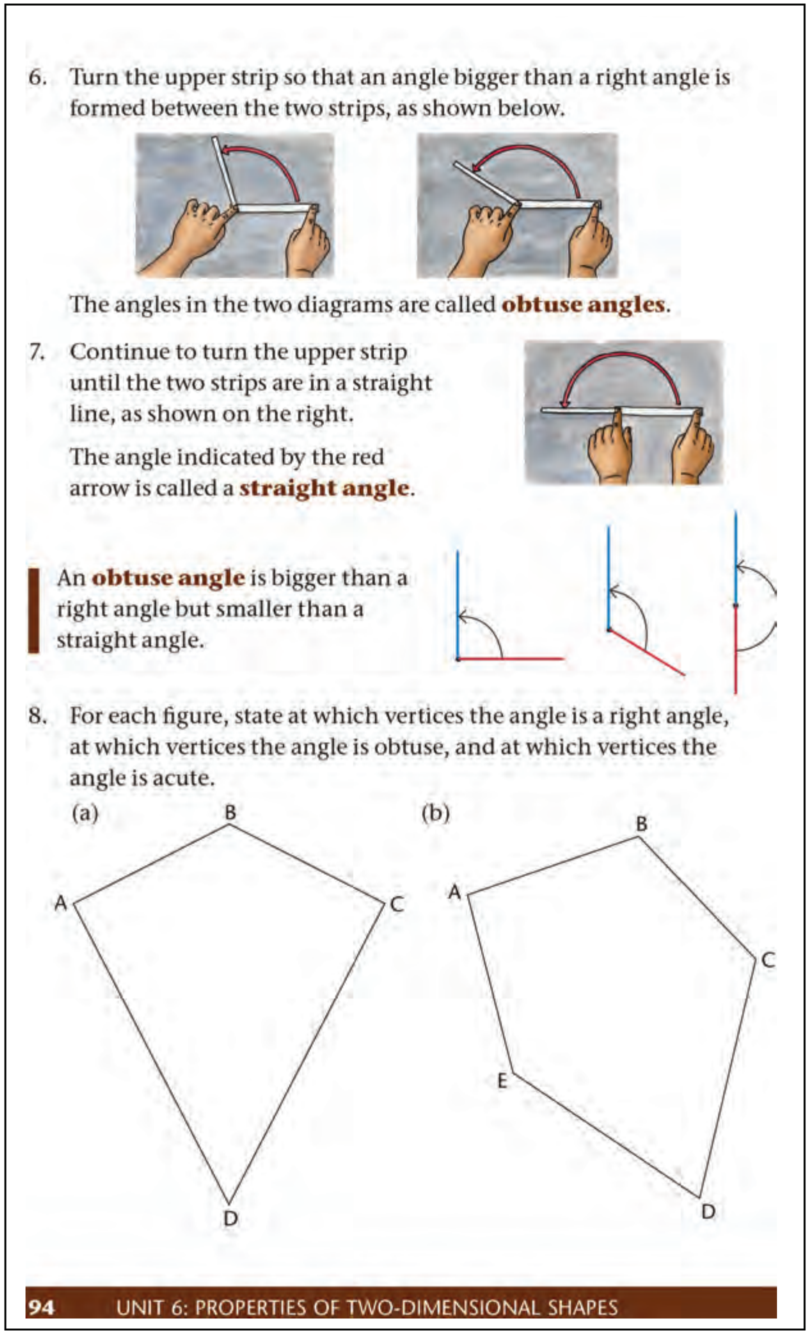
Notes on questions

In question 8, the angle at Vertex A is slightly smaller than a right angle. Remind learners that they can use a right-angle template (refer to question 1 on page 92 of the Learner Book) to check and compare the sizes of angles.

Answers

6.–7. Practical work

8. (a) The angles at A and C are right angles.
   The angle at B is obtuse, and the angle at A is acute.
   (b) There is no right angle. The angles at A, B, C and E are obtuse. The angle at D is acute.



6. Turn the upper strip so that an angle bigger than a right angle is formed between the two strips, as shown below.

The angles in the two diagrams are called **obtuse angles**.

7. Continue to turn the upper strip until the two strips are in a straight line, as shown on the right.

The angle indicated by the red arrow is called a **straight angle**.

An **obtuse angle** is bigger than a right angle but smaller than a straight angle.

8. For each figure, state at which vertices the angle is a right angle, at which vertices the angle is obtuse, and at which vertices the angle is acute.

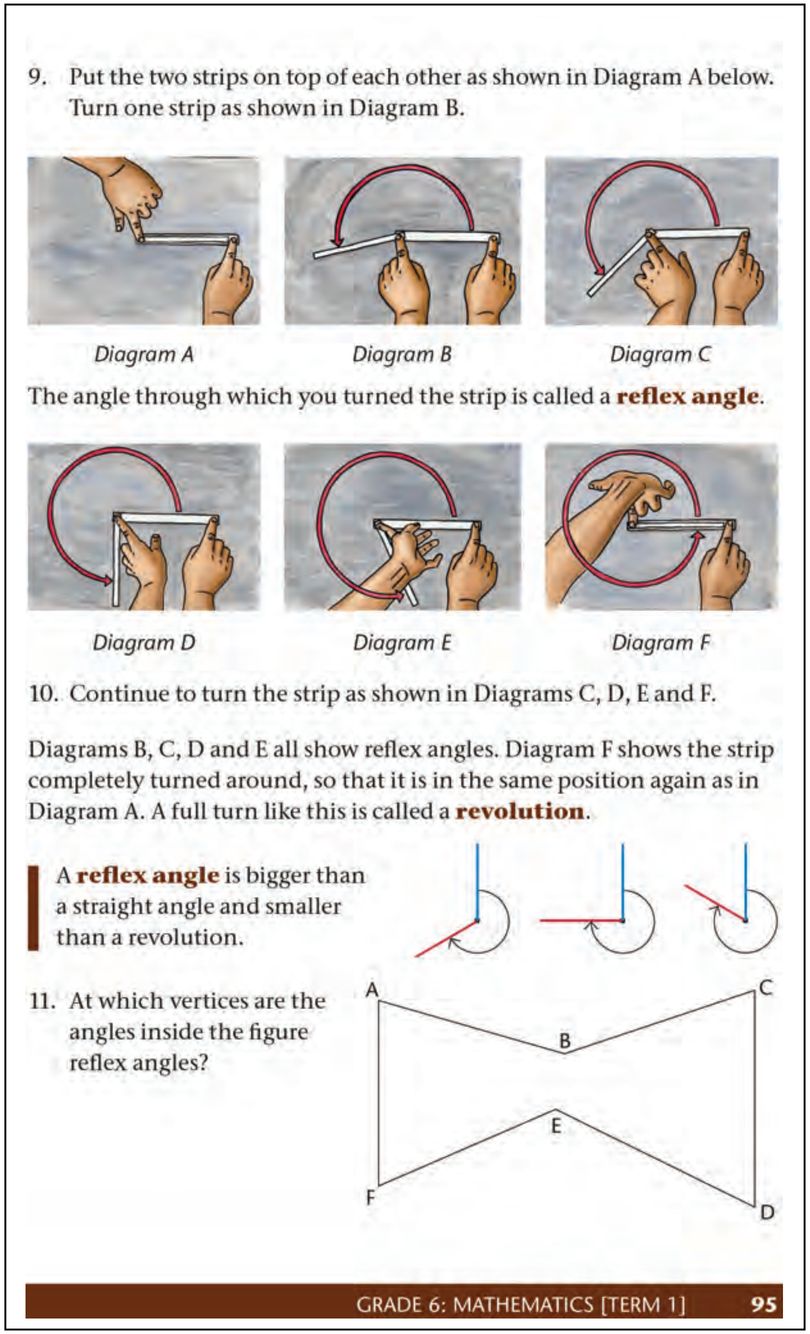
Teaching guidelines

It may be helpful to learners if you point out that an angle can also be bigger than one revolution. For example, the upper strip of paper that learners used to form angles of different sizes, can be turned twice around or three times around to make angles of two or three revolutions. An angle is an amount of turn.

In question 11 learners may tend to regard the outer angles at Vertices B and E. Point out to them that the question is about the angles inside the polygon.

Answers

11. Vertices B and E



9. Put the two strips on top of each other as shown in Diagram A below. Turn one strip as shown in Diagram B.

Diagram A *Diagram B* *Diagram C*

The angle through which you turned the strip is called a **reflex angle**.

Diagram D *Diagram E* *Diagram F*

10. Continue to turn the strip as shown in Diagrams C, D, E and F.

Diagrams B, C, D and E all show reflex angles. Diagram F shows the strip completely turned around, so that it is in the same position again as in Diagram A. A full turn like this is called a **revolution**.

> A **reflex angle** is bigger than a straight angle and smaller than a revolution.

11. At which vertices are the angles inside the figure reflex angles?

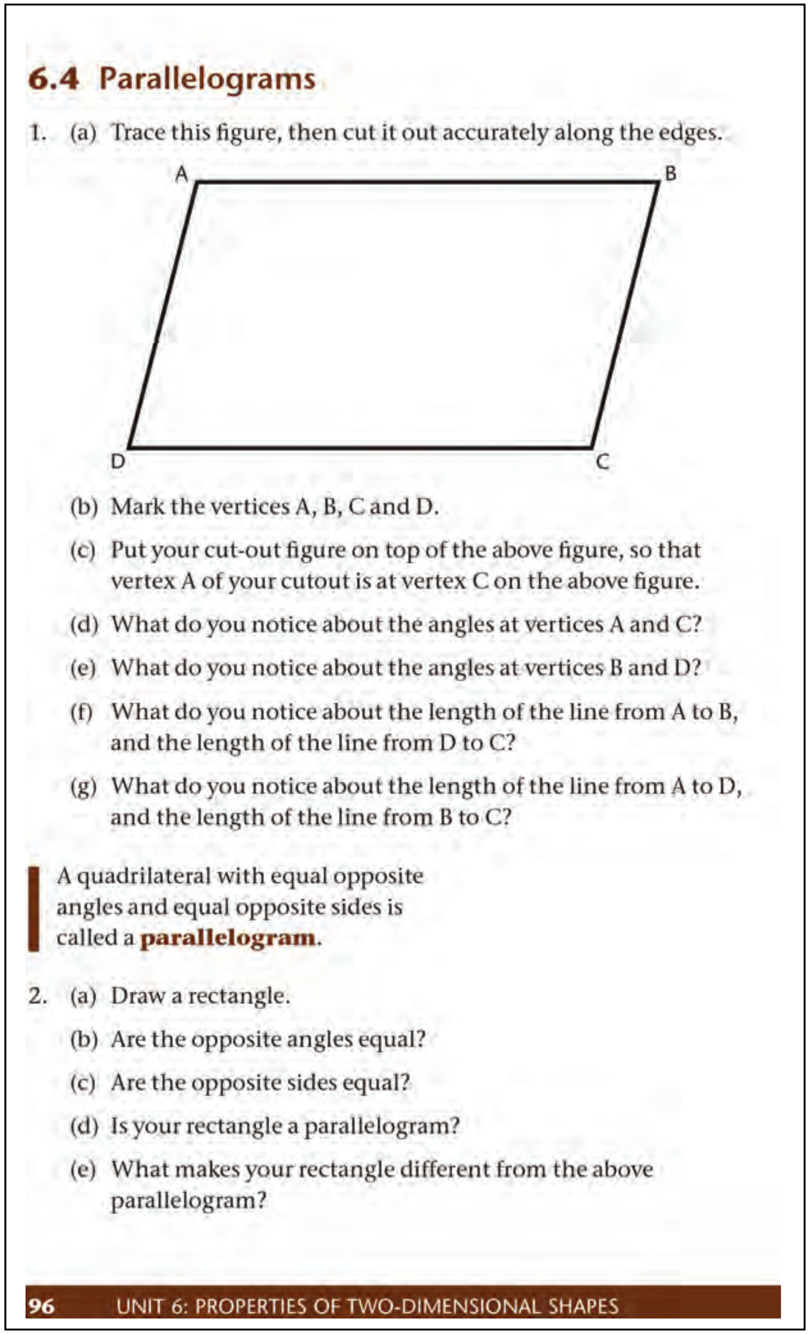
6.4 Parallelograms

Teaching guidelines

In question 1, learners can investigate the properties of parallelograms by turning the tracing through half a revolution. Alternatively, they can mark the vertices on their cutout and then tear off the corners, so that they can compare the angles with the angles in the figure in the Learner Book.

Answers

1. (a)–(c) Practical work
   - (d) The angles at A and C are equal.
   - (e) The angles at B and D are equal.
   - (f) The lengths are the same.
   - (g) The lengths are the same.
2. (a) Learners may draw a rectangle or a square (a square is a special type of rectangle).
   - (b) Yes
   - (c) Yes
   - (d) Yes
   - (e) It has right angles. So, all angles are equal, not just the opposite ones.



6.4 Parallelograms

1. (a) Trace this figure, then cut it out accurately along the edges.

   (b) Mark the vertices A, B, C and D.

   (c) Put your cut-out figure on top of the above figure, so that vertex A of your cutout is at vertex C on the above figure.

   (d) What do you notice about the angles at vertices A and C?

   (e) What do you notice about the angles at vertices B and D?

   (f) What do you notice about the length of the line from A to B, and the length of the line from D to C?

   (g) What do you notice about the length of the line from A to D, and the length of the line from B to C?

A quadrilateral with equal opposite angles and equal opposite sides is called a **parallelogram**.

2. (a) Draw a rectangle.

   (b) Are the opposite angles equal?

   (c) Are the opposite sides equal?

   (d) Is your rectangle a parallelogram?

   (e) What makes your rectangle different from the above parallelogram?

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Grade 6 Term 1 Unit 7 Data handling

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
7.1 Understanding the data context	Reading and interpreting information about citrus farming in South Africa	97 to 98
7.2 Interpreting graphs	One-to-many pictographs, pie charts, double bar graphs	99 to 101
7.3 Organising data	Making categories, organising raw data in a table, mode and median	102 to 105
7.4 Project	Design and use appropriate tally tables etc. to collect data; represent, analyse, summarise and report findings	106 to 107

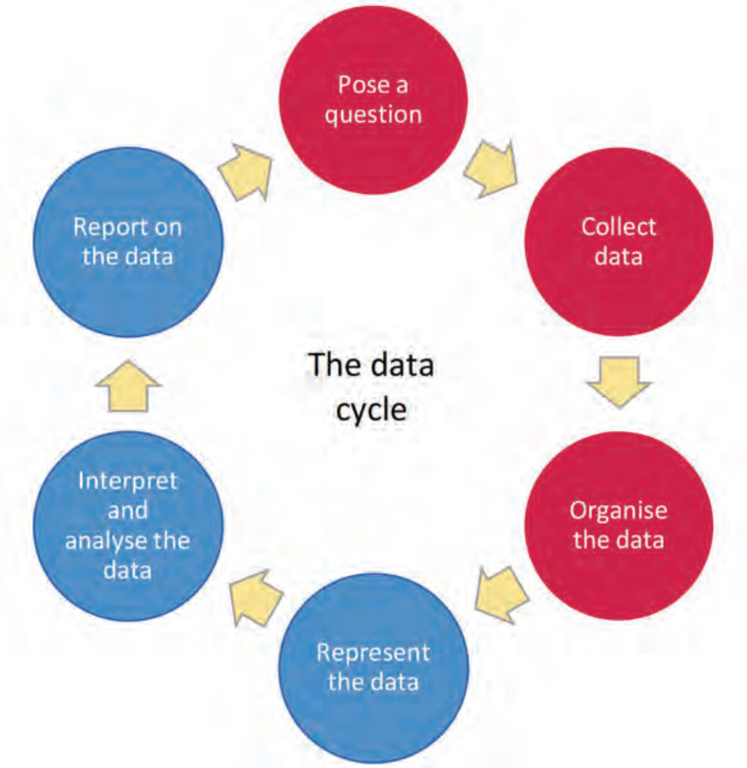
CAPS time allocation	10 hours
CAPS page references	30 to 31 and 233 to 234

Mathematical background

Politicians, business managers, school principals, mayors, municipal managers, medical doctors, employers, union leaders and many others need reliable information in order to take wise decisions. For example, a mayor may be faced with the **question** of what is more important: to build another hospital or to build a new fire station.

The origin of information is **data** (facts) that can be **collected** by making observations (e.g. of cars passing an intersection), taking measurements (e.g. of rainfall or temperature at various places at various times), making counts (e.g. of how many people have different diseases), asking questions (e.g. in questionnaires about people's favourite foods) and many other ways. Large amounts of data are often needed to make wise decisions, hence the data needs to be **organised** and **represented** in a way that makes it possible to **interpret** and **analyse** it to find out what it really tells us.

People like the ones mentioned above do not have time to gather and process data themselves. They rely on other people to do this for them, and also on such people to produce **reports** that summarise the main findings that emerge from the analysis and interpretation of the data.



Resources

Round objects (e.g. mugs or plastic cups) to draw circles, scissors, world map or globe

7.1 Understanding the data context

Teaching guidelines

Remind learners of the purpose of data handling. People collect data about a specific topic or issue in order to make a decision about the topic or issue, or to answer a question. The data can be presented in a way that makes it easy to interpret, or to make comparisons.

There may be learners in your class who are unable to distinguish colours, especially reds, greens, browns and oranges, or even blue and purple shades. About 1 out of every 8 males and 1 out of every 200 females have this problem. This can make it difficult to distinguish the colours in pie charts and in double bar graphs (Section 7.1 question 1, Section 7.2 questions 2 and 3). If learners struggle to distinguish colours, pair them with learners who do not have this difficulty.

The citrus farming data is real, although the farm in the story is not a real farm.

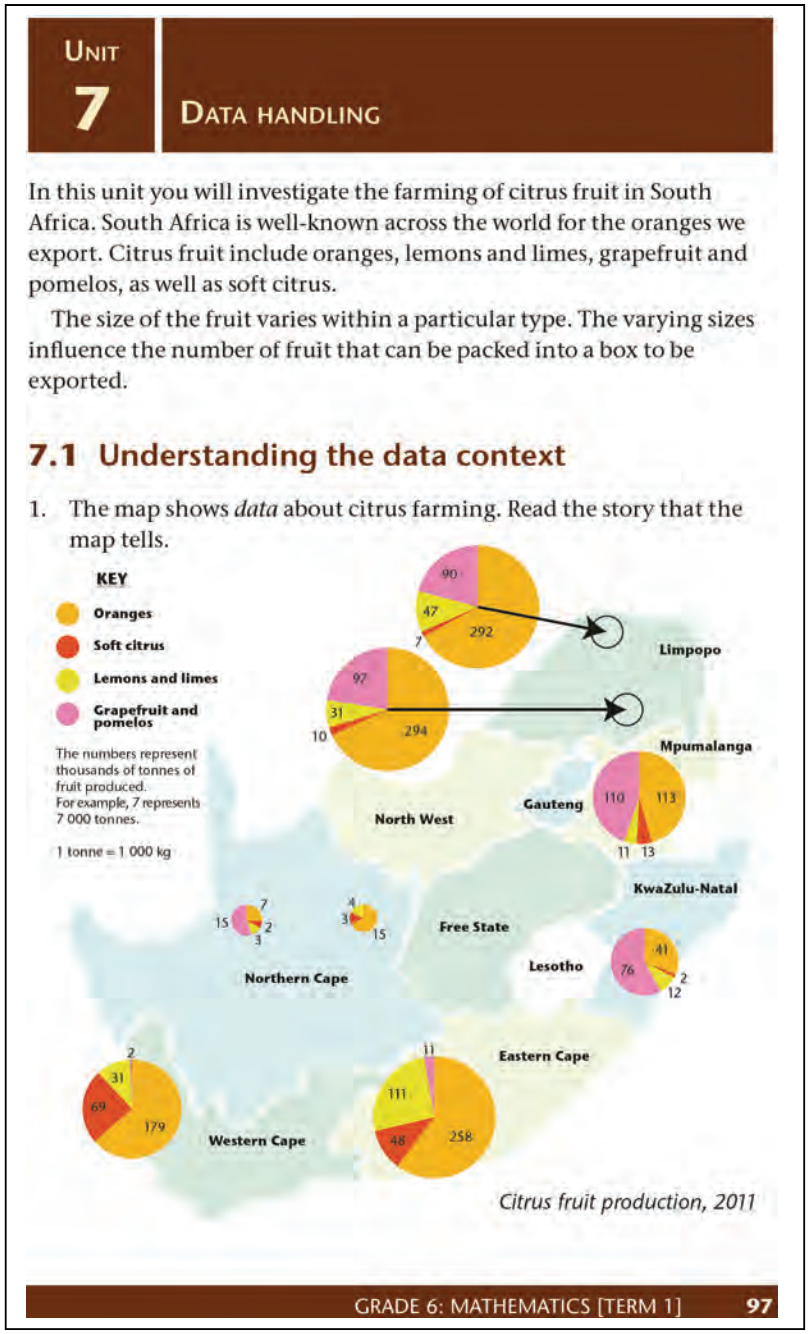
Work with the Social Sciences (Geography) teacher to explore the production of citrus in South Africa in greater depth.

Prepare the tables and graphs on the board or on posters to use in class discussions.

Before learners begin to answer the questions on page 98, ask them what they can "read" on the map on page 97.

There are two pie charts for both Limpopo and the Northern Cape. You many need to help learners to understand that this indicates that there are two major citrus producing regions in each of these provinces. Learners will need to look at both pie charts for these provinces. They will need to add the amounts (tonnes) from both pie charts. For example, the total mass of oranges produced in Limpopo is 292 tonnes + 294 tonnes.

Ask learners what information is given in the table in question 2 before they answer the questions.



UNIT 7 DATA HANDLING

In this unit you will investigate the farming of citrus fruit in South Africa. South Africa is well-known across the world for the oranges we export. Citrus fruit include oranges, lemons and limes, grapefruit and pomelos, as well as soft citrus.

The size of the fruit varies within a particular type. The varying sizes influence the number of fruit that can be packed into a box to be exported.

7.1 Understanding the data context

1. The map shows *data* about citrus farming. Read the story that the map tells.

Citrus fruit production, 2011

GRADE 6: MATHEMATICS [TERM 1] 97

Notes on questions

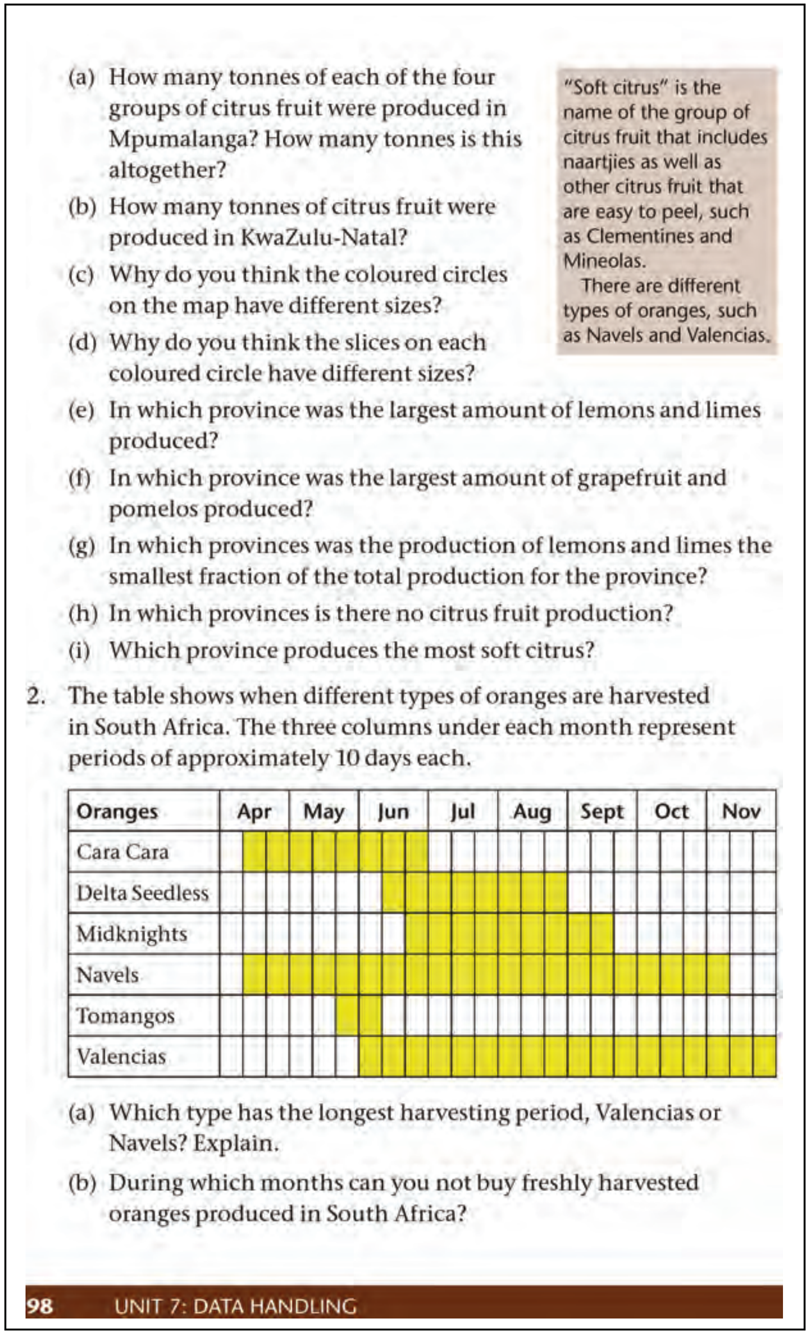
In question 1(f), learners need to add the amounts of grapefruit and pomelos produced in both citrus growing regions of Limpopo. This amounts to a “grouping of data” activity.

The answer to question 1(g) can be observed visually by the small sector of the slice representing lemons and limes. It can also be found by calculating the fraction (ratio) for each province. In the latter case, the data for the two production areas in Limpopo have to be combined, and the same goes for the data for the two production areas in the Northern Cape.

In the table in question 2, each month is divided into thirds, which is approximately 10 days. When learners answer question 2(a) it will be easy for them to see that Valencias are harvested for 6 months. When learners work out the harvest period for Navels they can see that Navels are harvested for $\frac{2}{3}$ of April, all of May, June, July, August, September and October and then $\frac{1}{3}$ of November. This makes 7 months in total. Learners may also count all the little blocks (21) and divide by 3 (each small block represents a third of the month).

Answers

1. (a) Grapefruit and pomelos: 110 000 tonnes
   Oranges: 113 000 tonnes
   Lemon and limes: 11 000 tonnes
   Soft citrus: 13 000 tonnes
   Total: 247 000 tonnes
   (b) 131 000 tonnes
   (c) The surface area of a coloured circle for a certain region (i.e. province or region in a province) represents the total amount (mass) of production of all citrus fruit in that region relative to the amounts produced in other regions.
   (d) The surface areas (and the angles) of the slices of a coloured circle represent the amount (mass) of production of a specific citrus fruit in a region relative to the total amount of citrus fruit produced in that region.
   (e) Eastern Cape (f) Limpopo
   (g) Mpumalanga (h) North West, Gauteng, Free State
   (i) Western Cape
2. (a) Navels have the longest harvesting period, namely 21 ÷ 3 = 7 months.
   Valencias are only harvested for 6 months.
   (b) From December to the beginning of April
   OR: December, January, February, March, and the first 10 days of April.



(a) How many tonnes of each of the four groups of citrus fruit were produced in Mpumalanga? How many tonnes is this altogether?

(b) How many tonnes of citrus fruit were produced in KwaZulu-Natal?

(c) Why do you think the coloured circles on the map have different sizes?

(d) Why do you think the slices on each coloured circle have different sizes?

(e) In which province was the largest amount of lemons and limes produced?

(f) In which province was the largest amount of grapefruit and pomelos produced?

(g) In which provinces was the production of lemons and limes the smallest fraction of the total production for the province?

(h) In which provinces is there no citrus fruit production?

(i) Which province produces the most soft citrus?

“Soft citrus” is the name of the group of citrus fruit that includes naartjies as well as other citrus fruit that are easy to peel, such as Clementines and Mineolas.

There are different types of oranges, such as Navels and Valencias.

2. The table shows when different types of oranges are harvested in South Africa. The three columns under each month represent periods of approximately 10 days each.

Oranges	Apr			May			Jun			Jul			Aug			Sept			Oct			Nov		
Cara Cara		🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨															
Delta Seedless								🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨									
Midknights									🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨							
Navels		🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨		
Tomangos						🟨	🟨																	
Valencias							🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨	🟨

(a) Which type has the longest harvesting period, Valencias or Navels? Explain.

(b) During which months can you not buy freshly harvested oranges produced in South Africa?

98 UNIT 7: DATA HANDLING

7.2 Interpreting graphs

Teaching guidelines

A pictograph is used in this question 1. You can make learners aware of the special features of pictographs. While it makes the data visible and easy to read or make comparisons with, the information given is not as accurate as in a table or a line graph.

Ask learners why they think a pictograph is suitable to present the data in this context.

Use a world map or globe to show learners where the countries are located.

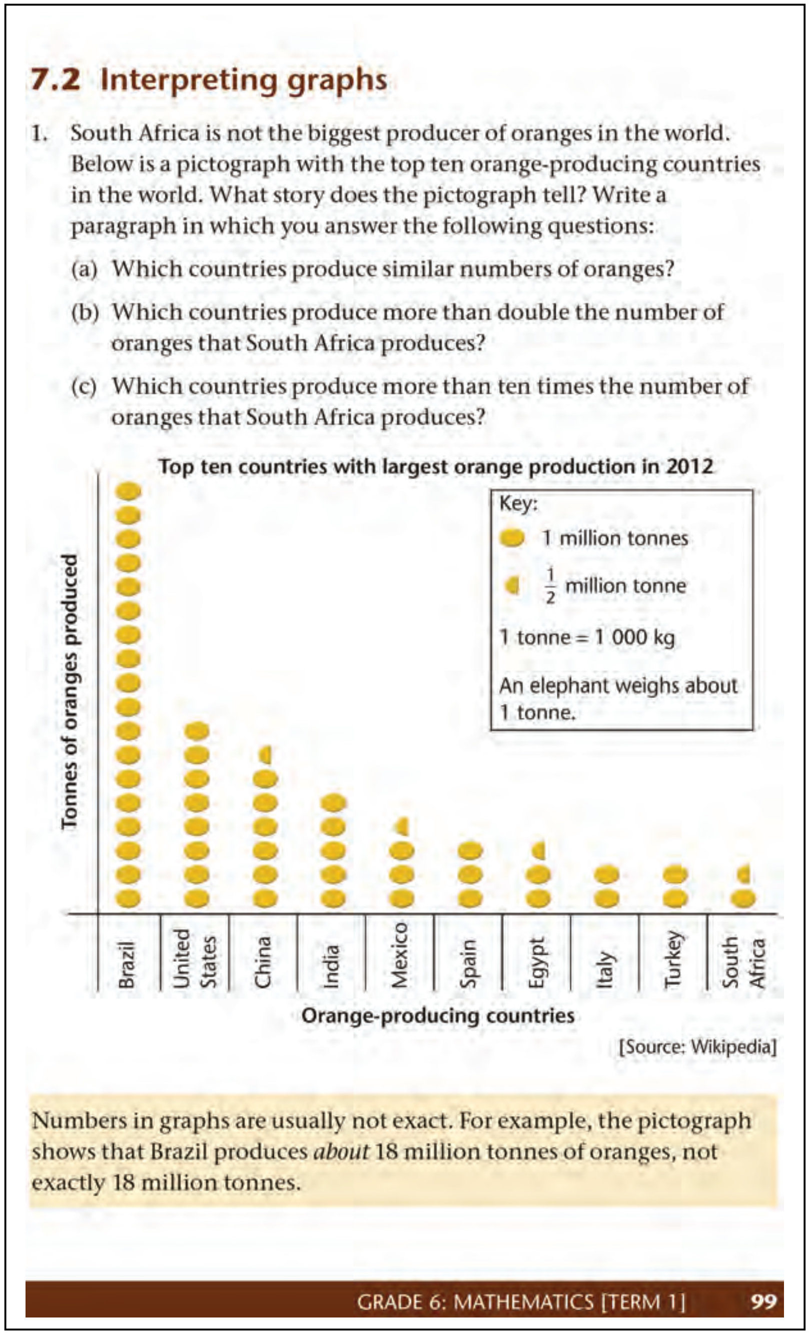
Notes on questions

Learners should write a paragraph as their answer for question 1. Although the Learner Book specifies three questions for them to address, they do not have to separate their answers into (a), (b) and (c) parts.

Answers

1. The ways in which learners structure their answers will vary, but the information is indisputable.

   South Africa produces about $1\frac{1}{2}$ million tonnes of oranges per year. Italy and Turkey both produce about 2 million tonnes of oranges per year. Spain produces about 3 million tonnes of oranges per year, which is double what South Africa produces. Egypt produces just under double what we produce annually. Mexico, India, China, the USA and Brazil all produce more than double what we produce annually. Brazil produces about 18 million tonnes per year, which is more than ten times the number of oranges that South Africa produces every year.



7.2 Interpreting graphs

1. South Africa is not the biggest producer of oranges in the world. Below is a pictograph with the top ten orange-producing countries in the world. What story does the pictograph tell? Write a paragraph in which you answer the following questions:
   (a) Which countries produce similar numbers of oranges?
   (b) Which countries produce more than double the number of oranges that South Africa produces?
   (c) Which countries produce more than ten times the number of oranges that South Africa produces?

[Source: Wikipedia]

Numbers in graphs are usually not exact. For example, the pictograph shows that Brazil produces *about* 18 million tonnes of oranges, not exactly 18 million tonnes.

Teaching guidelines

Learners should use their knowledge of fractions to say how they estimate the various fractional parts of the pie chart. Let them draw lines or fold another circle to make sectors of similar sizes as on the graphs, for example a sixth, a third, an eighth, etc.

Prepare the graphs on the board or on posters for use in class discussions.

Notes on questions

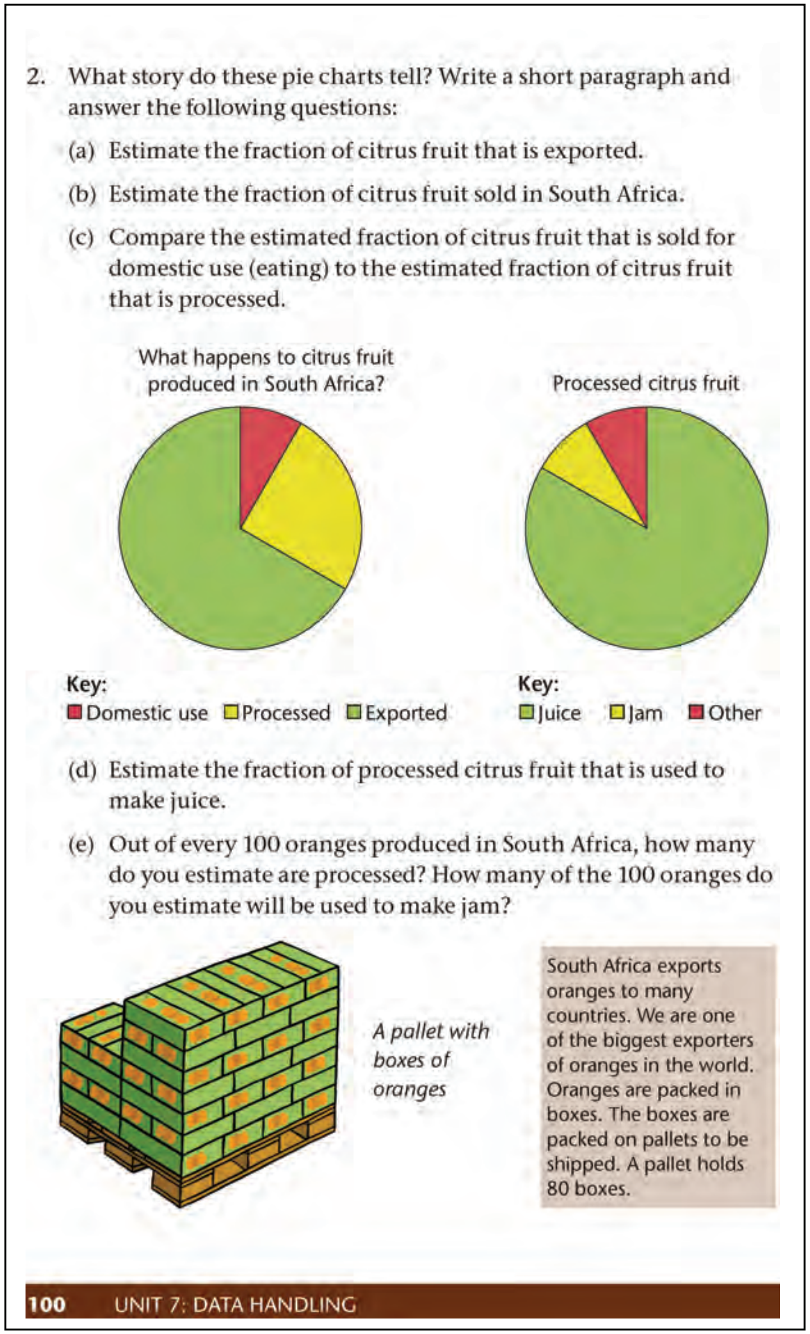
Learners' answers may differ as they are estimating. It is important that you ask learners to explain the reasons for their estimates.

In question 2(b) learners need to consider the fruit that is processed as part of the fruit that is not exported, so this also needs to be considered as "sold in South Africa".

In question 2(c) learners should use the "processed" sector (which shows $\frac{1}{4}$) as a reference size. This will allow them to see that the "domestic use" sector is less than half of a quarter i.e. less than $\frac{1}{8}$.

Answers

2. (a) About $\frac{2}{3}$ of SA's citrus fruit is exported (the green sector of the pie chart on the left).
   (b) About $\frac{1}{3}$ of SA's citrus fruit is not exported (the red and yellow sectors of the pie chart on the left).
   (c) The red sector is about one third of the yellow sector (pie chart on the left). So about three times as much citrus is processed than sold for eating. One quarter of South Africa's citrus fruit is processed. The amount of citrus for domestic use can be estimated to be about $\frac{1}{12}$ of all the citrus produced in SA. Reasonable answers could be anything from $\frac{1}{10}$ to $\frac{1}{14}$.
   (d) Estimates will differ. Ask learners to explain the basis for their estimates. More than $\frac{3}{4}$ of the processed citrus is used to make juice. $\frac{8}{10}$ or $\frac{4}{5}$ could be a good estimate here (with each of the other small sectors about $\frac{1}{10}$ each).
   (e) About $\frac{1}{4}$ of the oranges are processed, therefore 25 out of 100 fruit. About $\frac{1}{10}$ of the oranges that are processed are used to make jam ($\frac{1}{10}$ of 25), which is about 2 out of every 100 oranges.



2. What story do these pie charts tell? Write a short paragraph and answer the following questions:
   (a) Estimate the fraction of citrus fruit that is exported.
   (b) Estimate the fraction of citrus fruit sold in South Africa.
   (c) Compare the estimated fraction of citrus fruit that is sold for domestic use (eating) to the estimated fraction of citrus fruit that is processed.

What happens to citrus fruit produced in South Africa?

Key:
🟥 Domestic use 🟨 Processed 🟩 Exported

Processed citrus fruit

Key:
🟩 Juice 🟨 Jam 🟥 Other

   (d) Estimate the fraction of processed citrus fruit that is used to make juice.
   (e) Out of every 100 oranges produced in South Africa, how many do you estimate are processed? How many of the 100 oranges do you estimate will be used to make jam?

A pallet with boxes of oranges

South Africa exports oranges to many countries. We are one of the biggest exporters of oranges in the world. Oranges are packed in boxes. The boxes are packed on pallets to be shipped. A pallet holds 80 boxes.

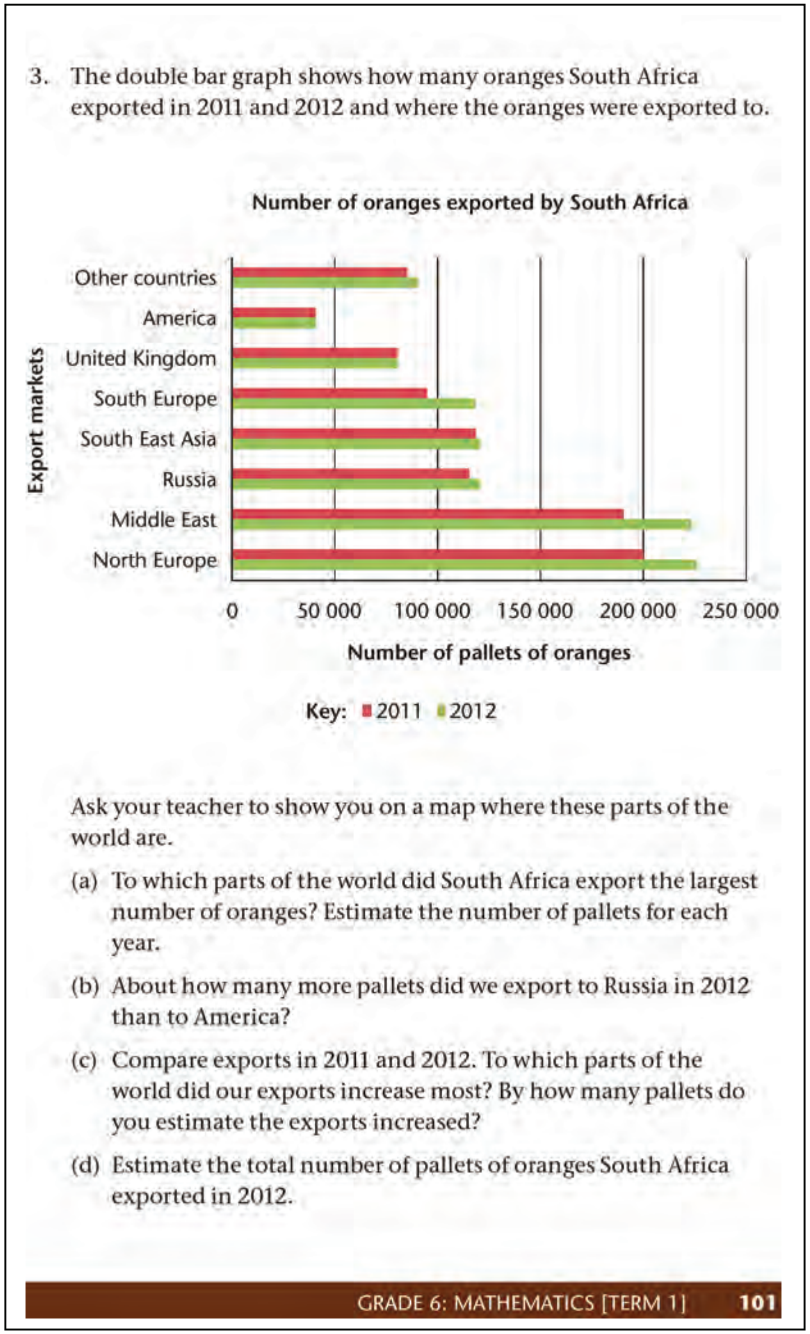
Teaching guidelines

Ask learners to explain their thinking when estimating. Their estimating will involve finding fractions of 50 000, because the horizontal axis is scaled in steps of 50 000.

Use a world map or globe to show learners where the countries are located.

Answers

3. (a) North Europe
   In 2011 about 200 000 pallets were exported to North Europe.
   In 2012 about 225 000 pallets were exported to North Europe.
   (b) Learners' answers will differ as they are estimating the differences between the lengths of the bars. Answers may be around 80 000 pallets.
   (c) The Middle East
   Learners' answers will differ as they are estimating the differences between the lengths of the bars. Answers may be around 35 000 pallets.
   (d) Learners' answers will differ as they are estimating the lengths of the bars.
   225 000 + 225 000 + 120 000 + 120 000 + 120 000 + 75 000 + 40 000 + 90 000 = 1 015 000. So most answers should be between 1 000 000 and 1 025 000 pallets in total.



3. The double bar graph shows how many oranges South Africa exported in 2011 and 2012 and where the oranges were exported to.

Ask your teacher to show you on a map where these parts of the world are.

(a) To which parts of the world did South Africa export the largest number of oranges? Estimate the number of pallets for each year.

(b) About how many more pallets did we export to Russia in 2012 than to America?

(c) Compare exports in 2011 and 2012. To which parts of the world did our exports increase most? By how many pallets do you estimate the exports increased?

(d) Estimate the total number of pallets of oranges South Africa exported in 2012.

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7.3 Organising data

Mathematical notes

Proportional reasoning is developed in this section. Learners must develop an understanding that more oranges of a smaller size will fit into a box, and fewer oranges of a larger size will fit into a box. Therefore, more boxes are needed for the same number of large oranges than small oranges.

Graphs provide a picture of data. This picture facilitates the analysis of the data. We can also analyse data by examining how spread out or clustered it is and what a typical value is. In this section learners use two measures to describe the typical value of the data: the **mode** (the data value that occurs most frequently) and the **median** (middle value of the set of data points). The median indirectly requires learners to look at the spread of the data. The median and the mode for the same data in a pictograph are compared, so that the usefulness of the median as the summary value can be demonstrated.

A lot of data handling involves reasoning in uncertain situations. This can make learners feel insecure, because there tends to be much more certainty in other areas of Mathematics where there are usually one or more definite answers. In data handling learners need to use their analysis of the data as evidence to back up an argument.

Teaching guidelines

Demonstrate how the width of an orange is determined. Prepare the tables and graphs on the board or on posters for use in class discussions. Use a world map or globe to show learners where the export markets featuring in question 1 are located.

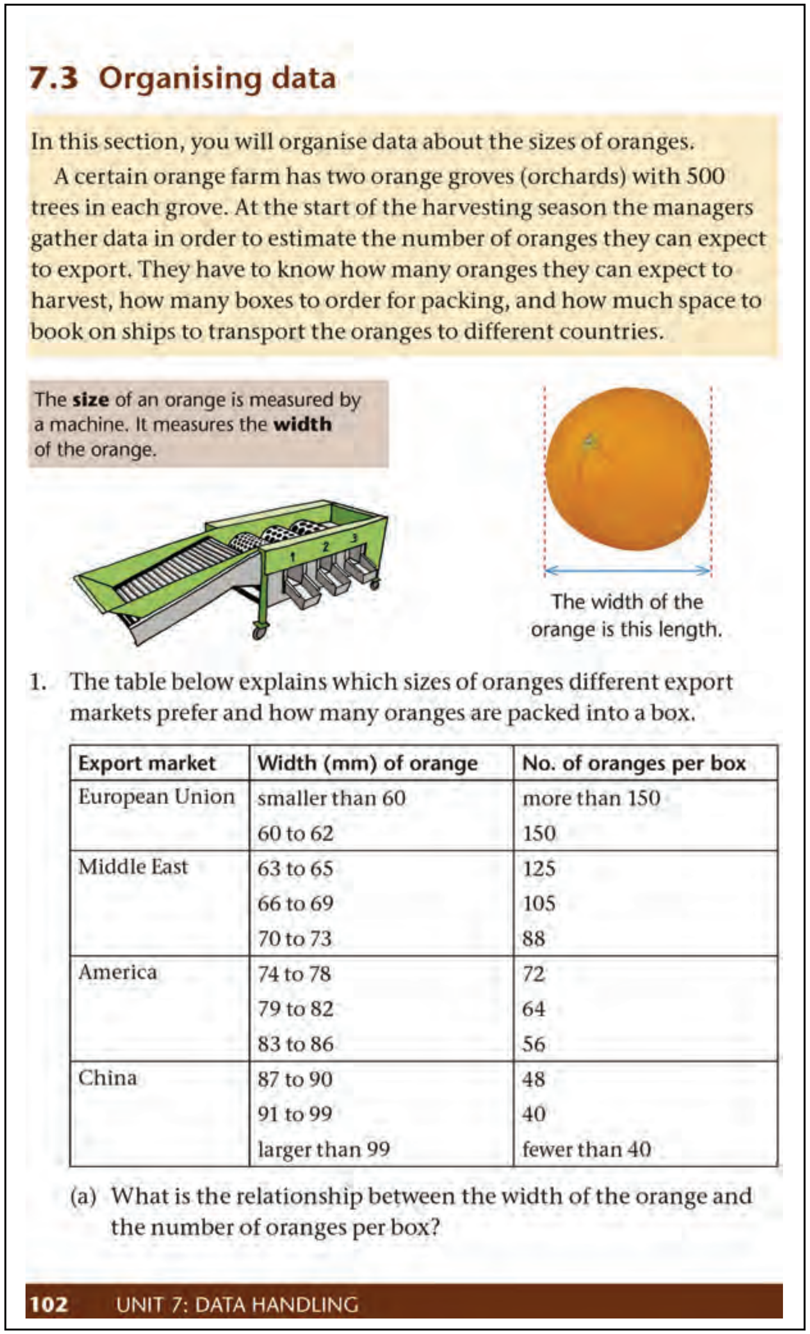
Notes on questions

Use question 1(b) to get access to learners' intuitions about using a representative tree to estimate the number of oranges on all trees.

Let learners talk about the visual picture of the graph in question 2. If they see two clumps of data they are starting to consider the graph as a distribution.

Answers

1. (a) There are two relationships here. The greater the width of the orange, the fewer fit in a box. The greater the width of an orange, the more boxes you need to pack the same number of oranges.



7.3 Organising data

In this section, you will organise data about the sizes of oranges.

A certain orange farm has two orange groves (orchards) with 500 trees in each grove. At the start of the harvesting season the managers gather data in order to estimate the number of oranges they can expect to export. They have to know how many oranges they can expect to harvest, how many boxes to order for packing, and how much space to book on ships to transport the oranges to different countries.

The **size** of an orange is measured by a machine. It measures the **width** of the orange.

The width of the orange is this length.

1. The table below explains which sizes of oranges different export markets prefer and how many oranges are packed into a box.

Export market	Width (mm) of orange	No. of oranges per box
European Union	smaller than 60	more than 150
	60 to 62	150
Middle East	63 to 65	125
	66 to 69	105
	70 to 73	88
America	74 to 78	72
	79 to 82	64
	83 to 86	56
China	87 to 90	48
	91 to 99	40
	larger than 99	fewer than 40

(a) What is the relationship between the width of the orange and the number of oranges per box?

Answers

1. (b) Learners' answers will differ. Accept all reasonable answers. One example is: The farmers can look at the data for the previous year. If they have no data, they can choose a tree in each of the two groves and count the oranges on the trees. This is to get an idea of a typical or representative tree. If they think the other trees in the groves are similar to the trees they have chosen, they can multiply the number of oranges they have counted per tree by the number of trees in the grove.

Notes on questions

In question 2 we develop learners' intuitions about the median as a representative value. Allow discussion before you tell them how to get the middlemost width of all the oranges. Question 2(c) requires that they find a middlemost value (median) for each clump of data.

Teaching guidelines

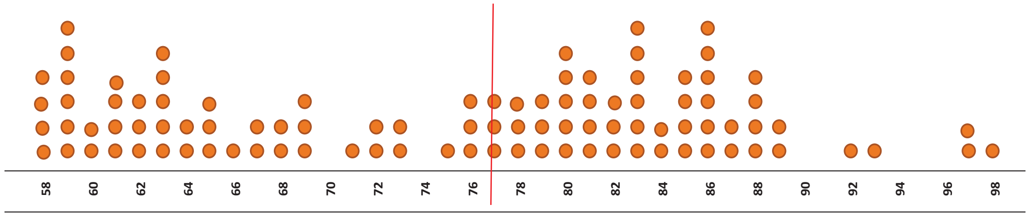
When you discuss the clusters of oranges on the graph in question 2, use a clean sheet of paper to first cover the one cluster, and then the other, to help learners "see" the clusters. Where the clusters end is a matter for discussion and agreement.

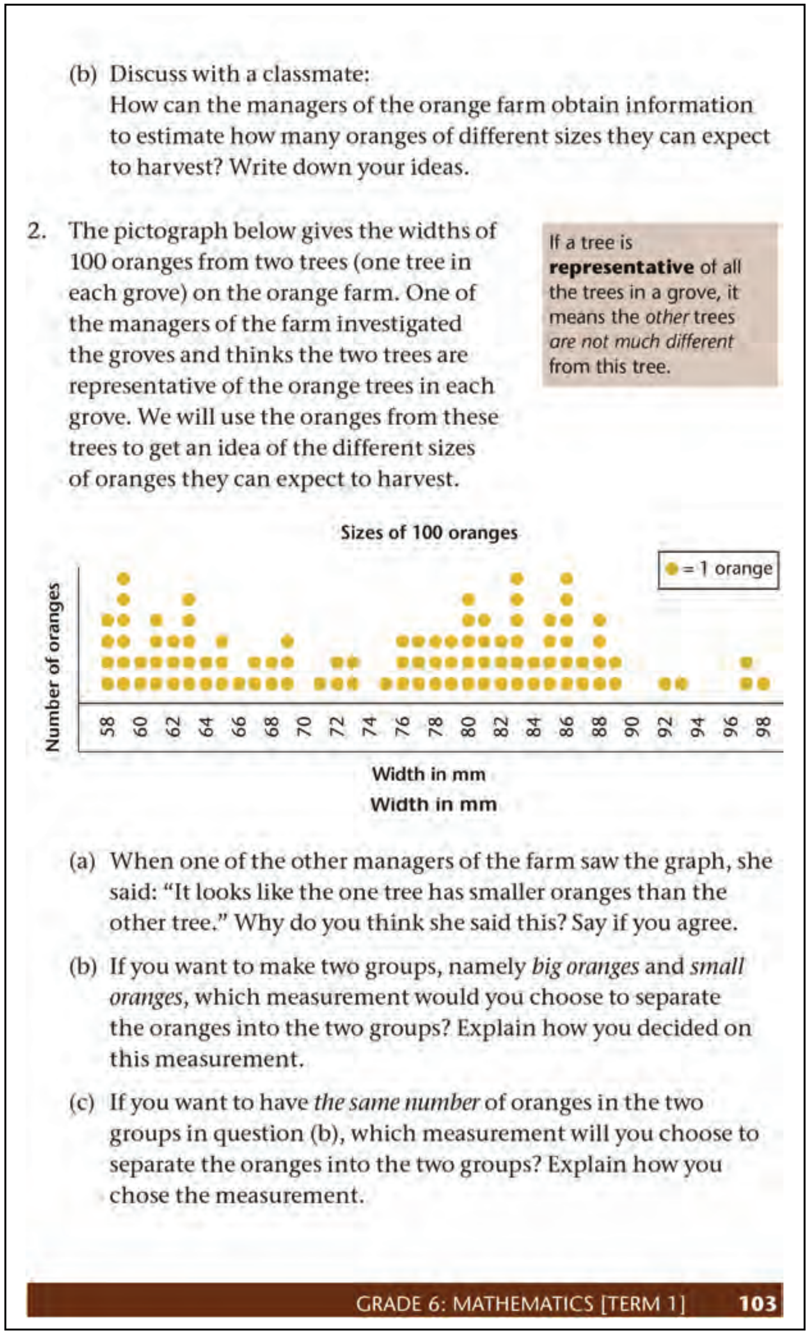
Answers

2. (a) Answers will differ. Some learners may say that it looks like there are two groups of oranges: one group have widths between 58 mm and about 70 mm, and the other group have widths between 76 mm and about 90 mm. Some oranges are just a little smaller or bigger than those in the two groups. These two groups could also represent the range of orange sizes on each of the trees.

   (b) Answers may differ. Accept all reasonable answers that separate the two clumps of data. Example: Oranges that are smaller than 74 cm in width are small, oranges with widths bigger than 74 cm are large. This range corresponds with the size of the oranges in the four markets that we saw in question 1 (Europe and the Middle East less than 74 mm and America and China bigger than 74 mm).

   (c) Count all the dots (data points) and divide by two: 98 ÷ 2 = 49. Count off the first 49 dots, and mark the width of the 49th orange. The middlemost width is about 77 mm.





(b) Discuss with a classmate:
How can the managers of the orange farm obtain information to estimate how many oranges of different sizes they can expect to harvest? Write down your ideas.

2. The pictograph below gives the widths of 100 oranges from two trees (one tree in each grove) on the orange farm. One of the managers of the farm investigated the groves and thinks the two trees are representative of the orange trees in each grove. We will use the oranges from these trees to get an idea of the different sizes of oranges they can expect to harvest.

If a tree is **representative** of all the trees in a grove, it means the *other* trees *are not much different* from this tree.

(a) When one of the other managers of the farm saw the graph, she said: "It looks like the one tree has smaller oranges than the other tree." Why do you think she said this? Say if you agree.

(b) If you want to make two groups, namely *big oranges* and *small oranges*, which measurement would you choose to separate the oranges into the two groups? Explain how you decided on this measurement.

(c) If you want to have *the same number* of oranges in the two groups in question (b), which measurement will you choose to separate the oranges into the two groups? Explain how you chose the measurement.

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Teaching guidelines

To explain what the median is, you can write the following numbers on the board and ask learners to rewrite the numbers from smallest to biggest:
23 62 35 44 56 32 28 55 61 43 51

- Arrange data in ascending order: 23 28 32 35 43 44 51 55 56 61 62
- The median is the number in the middle: 44.

In the case of an even number of data items, the median is the sum of the data items in the middle, divided by two: 23 62 35 44 56 32 28 55 61 43

- Arrange data in ascending order: 23 28 32 35 43 44 55 56 61 62
- The median is the number in the middle: (43 + 44) ÷ 2 = 43,5.

Notes on questions

The mode is the data value that appears most frequently. On the pictograph it is the data values with the highest stacks of dots. Some data sets have more than one mode: bimodal sets have two modes and multimodal sets have more than two modes. In this set of 100 oranges, there are 6 oranges with widths of 59 mm, 6 oranges with widths of 83 mm and 6 oranges with widths of 86 mm.

Answers

2. (d) Answers may differ. Some learners may choose the width with the highest stack of dots (mode) and say 59 mm. Other learners may say 59 mm is too small and choose a width that is closer to the middle of the clump of small oranges. These learners show an intuitive understanding of median. The median of the "small" oranges is about 64 mm. (There are 49 oranges in each group. Half of 49 is 24,5. If you count the first 25 oranges you get to the width of 64 mm.)

   (e) Similar to question (d). The median of the "large" oranges is 84 mm. There are 49 oranges in each group. Half of 49 is 24,5. If you count back 25 oranges from the largest orange you get to the width of 84 mm.

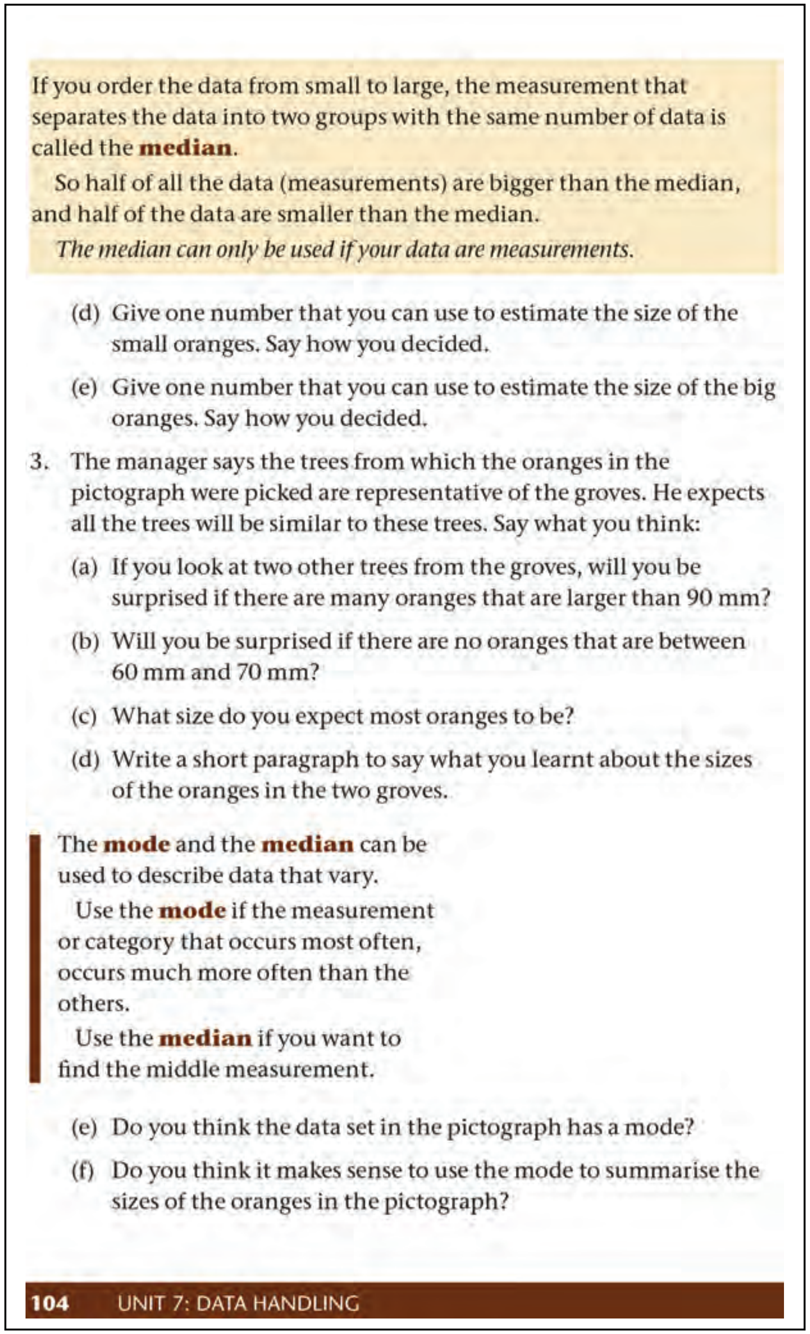
3. (a) Yes, because so few of these oranges (5 out of 98) are larger than 90 mm wide.

   (b) Yes, because more than half of these oranges are between 60 mm and 70 mm wide.

   (c) Between 58 mm and 88 mm

   (d) Answers may differ. Example: The width of all the oranges was less than 100 mm and more than 57 mm. One group of oranges (possibly from one grove) tended to be from 58 mm to 73 mm wide. Another group of oranges (possibly from another grove) tended to be from 71 mm to 98 mm wide. So it seems as if the farm has one grove that produces "small" oranges and another grove that produces "large" oranges.

   (e)–(f) See next page.



If you order the data from small to large, the measurement that separates the data into two groups with the same number of data is called the **median**.

So half of all the data (measurements) are bigger than the median, and half of the data are smaller than the median.

The median can only be used if your data are measurements.

(d) Give one number that you can use to estimate the size of the small oranges. Say how you decided.

(e) Give one number that you can use to estimate the size of the big oranges. Say how you decided.

3. The manager says the trees from which the oranges in the pictograph were picked are representative of the groves. He expects all the trees will be similar to these trees. Say what you think:

   (a) If you look at two other trees from the groves, will you be surprised if there are many oranges that are larger than 90 mm?

   (b) Will you be surprised if there are no oranges that are between 60 mm and 70 mm?

   (c) What size do you expect most oranges to be?

   (d) Write a short paragraph to say what you learnt about the sizes of the oranges in the two groves.

The **mode** and the **median** can be used to describe data that vary.

Use the **mode** if the measurement or category that occurs most often, occurs much more often than the others.

Use the **median** if you want to find the middle measurement.

   (e) Do you think the data set in the pictograph has a mode?

   (f) Do you think it makes sense to use the mode to summarise the sizes of the oranges in the pictograph?

Answers *(continued)*

3. (e) There are three modes: oranges with widths of 59 mm, oranges with widths of 83 mm and oranges with widths of 86 mm.
   (f) It only helps us with part of the analysis. The mode for the small oranges seems too small to tell the story of what a typical small orange is. The mode isn't helpful for deciding what a typical large orange is. So the median is a better summary for the whole graph, and for the two groups of data.

Teaching guidelines

Prepare the tally table on the board or on a poster for use in class discussions.

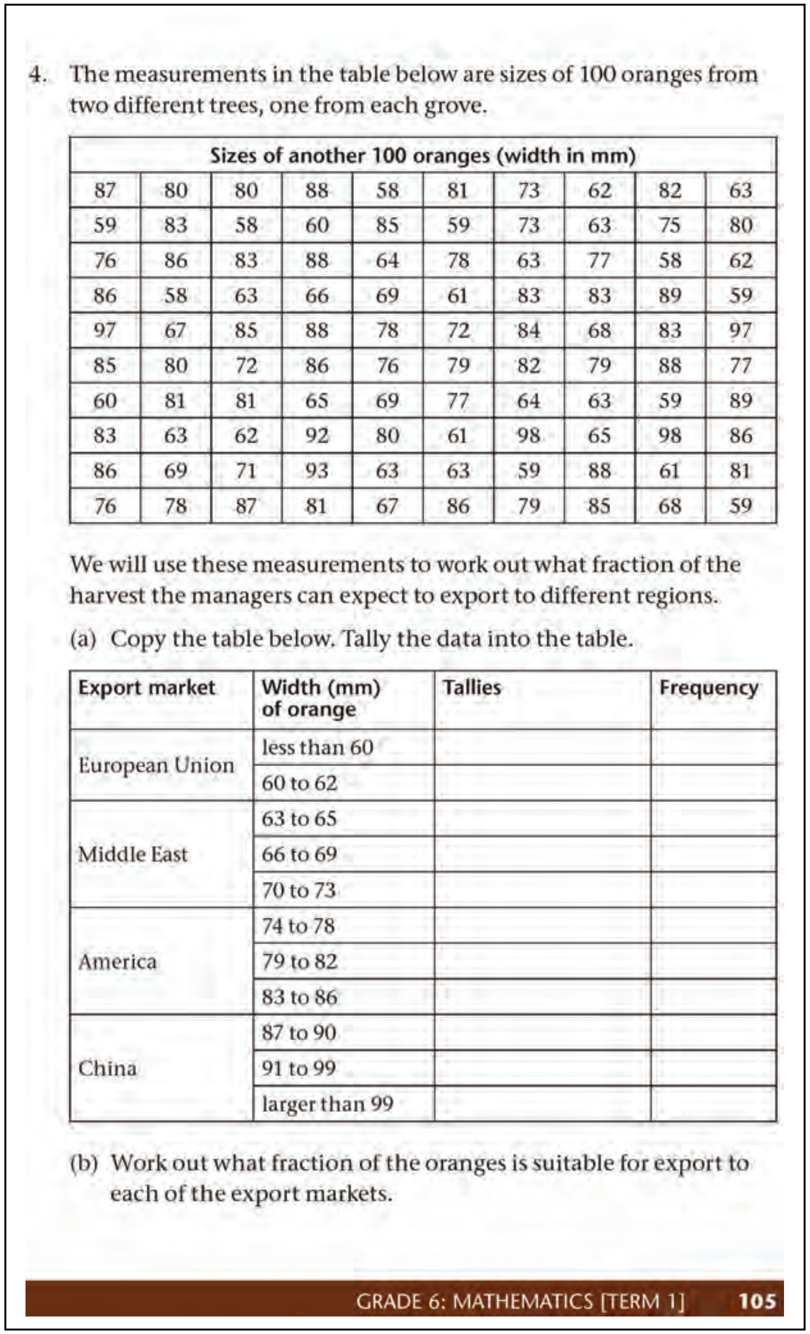
Answers

4. (a)

Export market	**Width (mm) of orange**	**Tallies**	**Frequency**
European Union	less than 60	𝍸 𝍸	10
	60 to 62	𝍸 ///	8
Middle East	63 to 65	𝍸 𝍸 //	12
	66 to 69	𝍸 ///	8
	70 to 73	𝍸	5
America	74 to 78	𝍸 𝍸	10
	79 to 82	𝍸 𝍸 𝍸	15
	83 to 86	𝍸 𝍸 𝍸 //	17
China	87 to 90	𝍸 ////	9
	91 to 99	𝍸 /	6
	larger than 99		0
		Total	100

(b) Fraction of export market:

European Union: $\frac{18}{100}$ Middle East: $\frac{25}{100}$ America: $\frac{42}{100}$ China: $\frac{15}{100}$



4. The measurements in the table below are sizes of 100 oranges from two different trees, one from each grove.

Sizes of another 100 oranges (width in mm)									
87	80	80	88	58	81	73	62	82	63
59	83	58	60	85	59	73	63	75	80
76	86	83	88	64	78	63	77	58	62
86	58	63	66	69	61	83	83	89	59
97	67	85	88	78	72	84	68	83	97
85	80	72	86	76	79	82	79	88	77
60	81	81	65	69	77	64	63	59	89
83	63	62	92	80	61	98	65	98	86
86	69	71	93	63	63	59	88	61	81
76	78	87	81	67	86	79	85	68	59

We will use these measurements to work out what fraction of the harvest the managers can expect to export to different regions.

(a) Copy the table below. Tally the data into the table.

Export market	Width (mm) of orange	Tallies	Frequency
European Union	less than 60		
	60 to 62		
Middle East	63 to 65		
	66 to 69		
	70 to 73		
America	74 to 78		
	79 to 82		
	83 to 86		
China	87 to 90		
	91 to 99		
	larger than 99		

(b) Work out what fraction of the oranges is suitable for export to each of the export markets.

7.4 Project

Teaching guidelines

This project will take about three weeks to complete. Learners must plan the project in the first week. During the second week they must gather and record the data. During the third week they must represent, analyse and interpret the data, and write a report.

It is important that you dedicate weekly time to monitor learners' progress and to support them.

Help learners to decide on headings for the different parts of their reports. They may use headings such as:

- **The questions we want to answer**
- **Data gathering** (Here they tell how and where they gathered data.)
- **Representation of the data** (Here they provide their tables and graphs.)
- **Analysis and summary of the data** (Here they provide calculations and summary values that are relevant to the questions.)
- **Interpretation of the data** (Here they write a paragraph to interpret the data and summaries.)

Provide learners with paper to draw the graphs.

Week 1

Help learners with the following preparations:

- Forming groups and deciding how they will share the work to gather relevant data to answer the two given questions, as well as their own question.
- Deciding what data to gather to answer each question.
- Preparing appropriate tally tables to complete when collecting the data.

If necessary, provide learners with a letter from the school requesting shop owners and vendors to allow them to count the number of oranges (or other citrus fruit) in a bag.

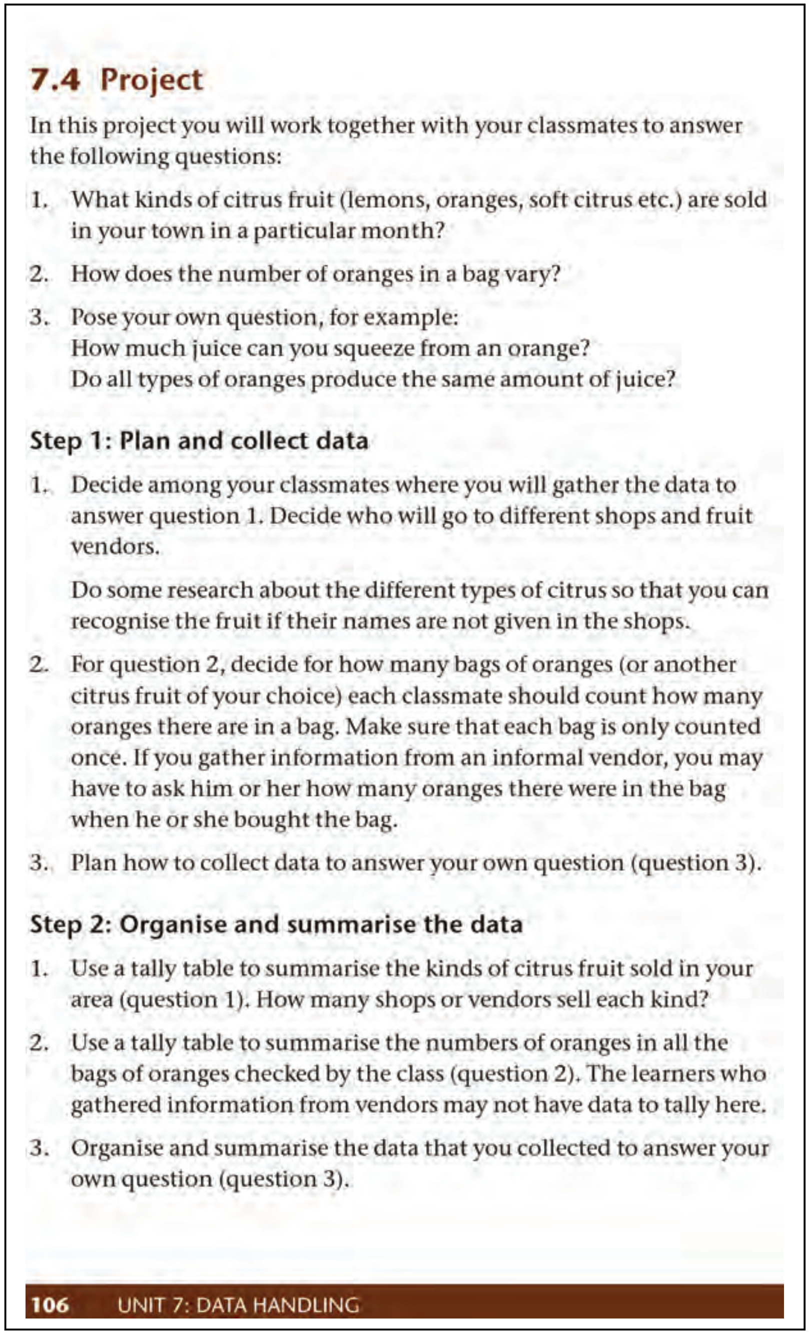
To answer question 1, learners will have to enquire at many shops and vendors. For question 2, they must count the fruit in at least three bags.

Week 2

Learners gather data as planned.

Week 3

Share your assessment rubric with learners. An example is provided on the next page.



7.4 Project

In this project you will work together with your classmates to answer the following questions:

1. What kinds of citrus fruit (lemons, oranges, soft citrus etc.) are sold in your town in a particular month?
2. How does the number of oranges in a bag vary?
3. Pose your own question, for example:
   How much juice can you squeeze from an orange?
   Do all types of oranges produce the same amount of juice?

Step 1: Plan and collect data

1. Decide among your classmates where you will gather the data to answer question 1. Decide who will go to different shops and fruit vendors.

   Do some research about the different types of citrus so that you can recognise the fruit if their names are not given in the shops.
2. For question 2, decide for how many bags of oranges (or another citrus fruit of your choice) each classmate should count how many oranges there are in a bag. Make sure that each bag is only counted once. If you gather information from an informal vendor, you may have to ask him or her how many oranges there were in the bag when he or she bought the bag.
3. Plan how to collect data to answer your own question (question 3).

Step 2: Organise and summarise the data

1. Use a tally table to summarise the kinds of citrus fruit sold in your area (question 1). How many shops or vendors sell each kind?
2. Use a tally table to summarise the numbers of oranges in all the bags of oranges checked by the class (question 2). The learners who gathered information from vendors may not have data to tally here.
3. Organise and summarise the data that you collected to answer your own question (question 3).

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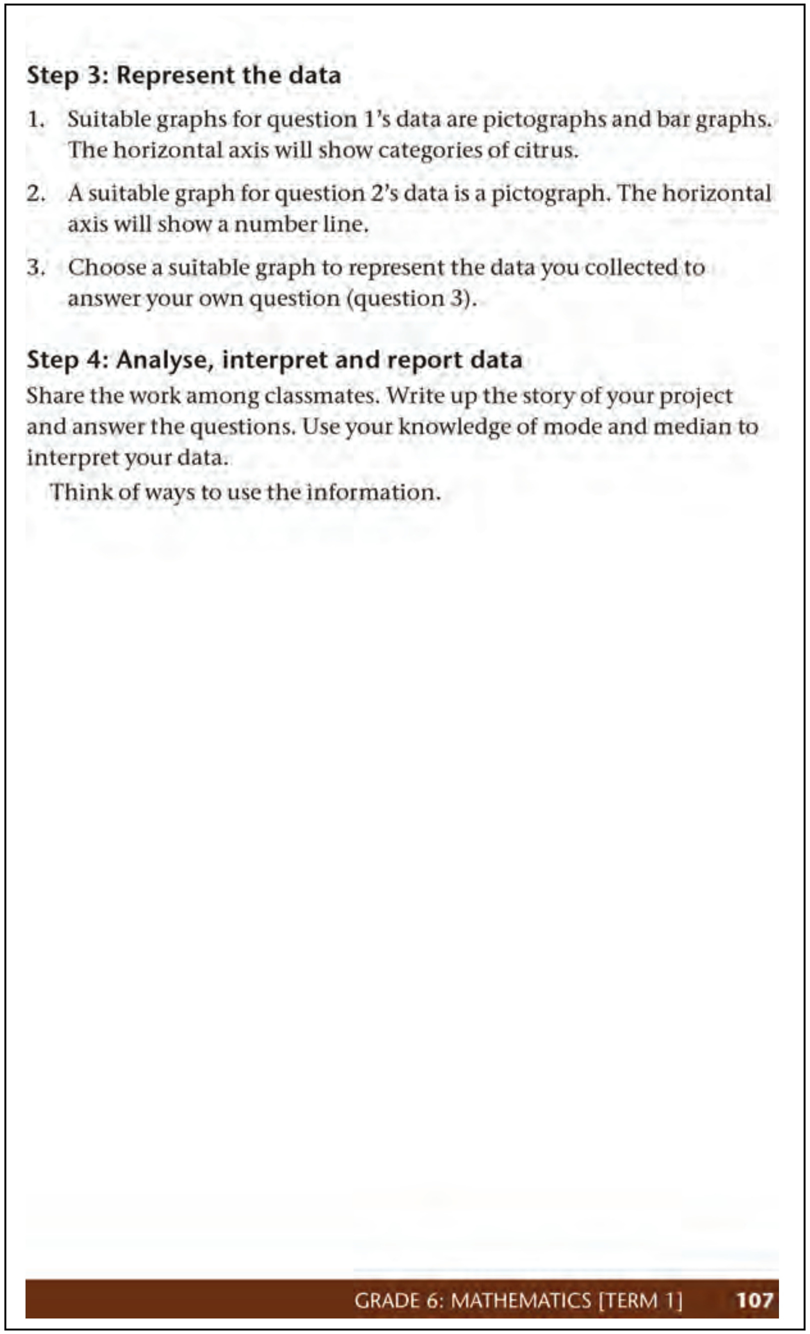
Possible assessment rubric		
Data gathering	7 marks	Suitable tally tables to record the data gathered for each question (1 mark for each of the three questions: 1 × 3) Three or more shops/outlets/vendor stands sampled (1 mark) Three or more bags sampled (1 mark) Appropriate question posed for question 3 (1 mark) Suitable categories chosen for question 3 (1 mark)
Data representation	21 marks	Appropriate headings for each graph (1 mark for each of the three graphs: 1 × 3) Axes or key correctly labelled (1 mark for each of the three graphs: 1 × 3) Suitable scales (in bar graphs) or categories/ranges (in pictographs) used (1 mark for each of the three graphs: 1 × 3) Bars or icons accurately drawn/placed (3 marks for each of the three graphs: 3 × 3) Graphs neatly drawn (1 mark for each of the three graphs: 1 × 3)
Data interpretation and reporting	18 marks	Description of how and where data was gathered (1 mark for each of the three questions: 1 × 3) Calculations and summary values shown for each question (2 marks per question: 2 × 3) Findings described in written paragraph (3 marks per question: 3 × 3)
Presentation	4 marks	Up to 5 marks for presentation (cover page with name, grade, class, title of project; headings of sections: see the bulleted list under "Teaching guidelines" on the previous page)
TOTAL	50 marks	



Step 3: Represent the data

1. Suitable graphs for question 1's data are pictographs and bar graphs. The horizontal axis will show categories of citrus.
2. A suitable graph for question 2's data is a pictograph. The horizontal axis will show a number line.
3. Choose a suitable graph to represent the data you collected to answer your own question (question 3).

Step 4: Analyse, interpret and report data

Share the work among classmates. Write up the story of your project and answer the questions. Use your knowledge of mode and median to interpret your data.

Think of ways to use the information.

GRADE 6: MATHEMATICS [TERM 1] **107**

Grade 6 Term 1 Unit 8 Numeric patterns

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
	Introduction	108
8.1 Revising sequences of multiples	Consolidating sequences of multiples	109
8.2 Non-multiple sequences	Finding rules for families of sequences with a constant difference	110 to 112
8.3 Flow diagrams and rules	Consolidating completing flow diagrams	113 to 114
8.4 Tables and rules	Consolidating derived rules for families of sequences	115 to 116

CAPS time allocation	4 hours
CAPS page references	18 to 19 and 235 to 238

Mathematical background

Numeric patterns, as part of the Content Area "Patterns, Functions and Algebra", should serve as building blocks to develop the basic concepts of algebra in the Senior and FET phases. The study of numeric patterns should develop the idea of a relationship between two variable quantities, for example:

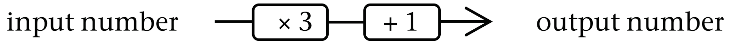
One variable quantity (the "input numbers")	1	2	3	4	5	6	7	8	9	10	11
Another variable quantity (the "output numbers")	4	7	10	13	16	19	22	25	28	31	34

The word **pattern** means that something is repeated. In the above case, the sequence 4, 7, 10, 13, 16, ... can be formed by repeatedly adding 3. This pattern in the sequence can be performed by performing the same calculation each time to move from one number to the next. Such a pattern is called a **recursive pattern**. The word "recur" means "repeat".

The above sequence of output numbers can also be formed by multiplying each input number by 3 and adding 1:

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
3 × **1** + 1	3 × **2** + 1	3 × **3** + 1	3 × **4** + 1	3 × **5** + 1	3 × **6** + 1	3 × **7** + 1	3 × **8** + 1	3 × **9** + 1	3 × **10** + 1	3 × **11** + 1
4	**7**	**10**	**13**	**16**	**19**	**22**	**25**	**28**	**31**	**34**

A relationship between two variable quantities, in which each value of the second quantity is uniquely determined by the corresponding value of the first quantity, is called a **function** – the middle word in the CAPS title for this Content Area. In the above case, the link between the input and output numbers (also called the independent and dependent variables) is given by the calculation plan (rule) "multiply the input number by 3 and add 1", which can also be represented as $3 \times \square + 1$, or with this flow diagram:



Resources

Calculators

Overview of the approach to Numeric Patterns

The work on numeric patterns was designed along the following principles and guides:

Sequences of multiples

The sequences of multiples (the "tables") are first thoroughly developed and reinforced with the intention that they will become easy for learners and a building block from which other sequences can be studied.

It is established that all the sequences of multiples are of the same type:

- The multiples of *k* have a constant difference of +*k* between consecutive numbers (the "horizontal" pattern). For example, for the multiples of 6 the constant difference is 6.
- The multiples of *k* have a rule of the form ×*k* (the "vertical" pattern). For example, for the multiples of 6 the rule is *Multiple no. = 6 × Position no.*

Families of sequences

Next it is established that sequences that are obviously different, can be the same in some respects. For example the sequences in the series of sequences below are clearly different, but are nevertheless the same in that they share the property that they have a constant difference of 4:

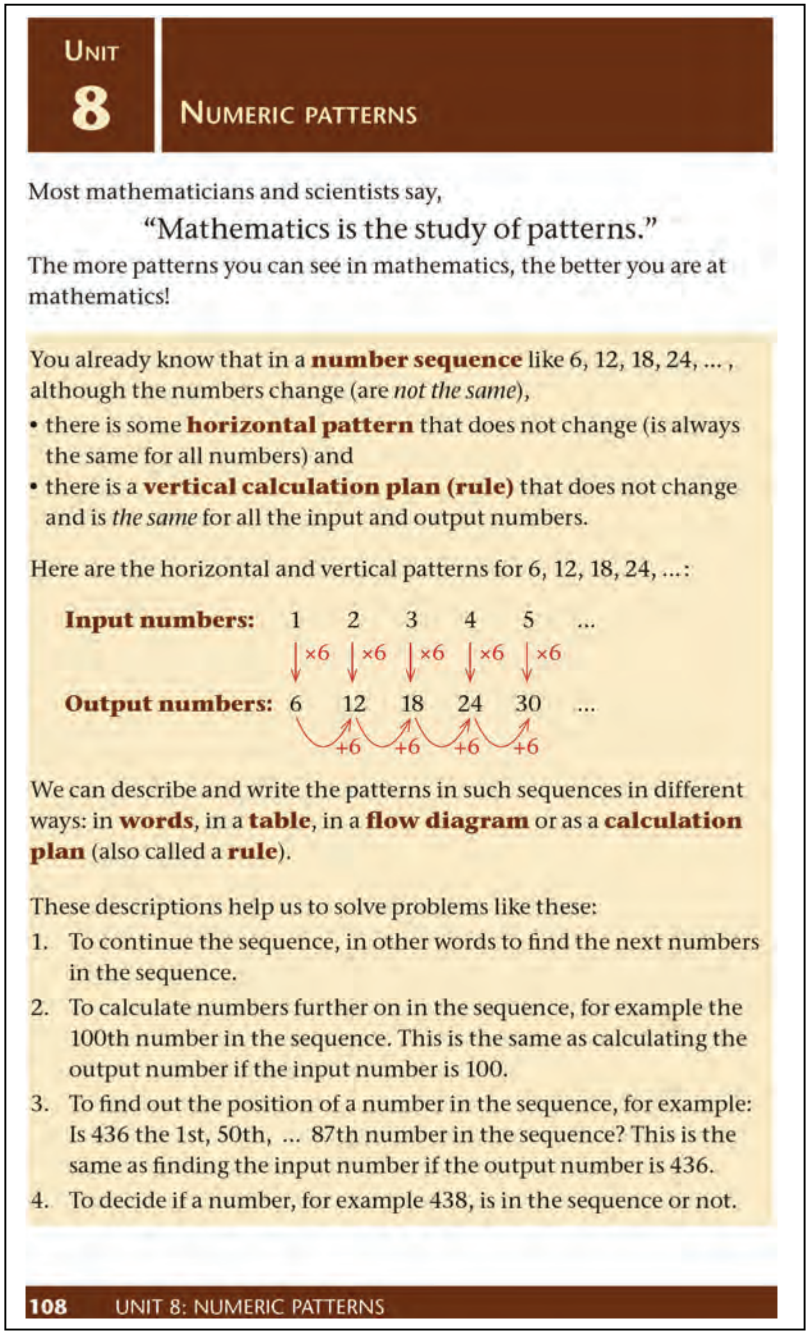
3, 7, 11, 15, 19, 23, 27, ...
4, 8, 12, 16, 20, 24, 28, ...
5, 9, 13, 17, 21, 25, 29, ...
6, 10, 14, 18, 22, 26, 30, ...

We call them "a family of sequences".

By comparing flow diagrams, tables and rules with a focus on the *relationship between these sequences*, a relationship between the calculation rules for these families of sequences can be identified, like this:

	Sequence	Description in words	Flow diagram/ Rule	
	3, 7, 11, 15, 19, 23, 27, ...	one less than multiples of 4	$\rightarrow\boxed{\times 4}\rightarrow\boxed{-1}\rightarrow$	
−1				−1
	4, 8, 12, 16, 20, 24, 28, ...	**multiples of 4** *Easy! Start here!*	$\rightarrow\boxed{\times 4}\rightarrow$	
+2				+2
	6, 10, 14, 18, 22, 26, 30,...	two more than multiples of 4	$\rightarrow\boxed{\times 4}\rightarrow\boxed{+2}\rightarrow$	

Typical of real understanding, there is more to understand than in rote learning. But once understood it is tremendously empowering: it offers deeper insights, is transferable to other contexts, is easier to remember and apply, and it makes more learning possible.



UNIT
8
NUMERIC PATTERNS

Most mathematicians and scientists say,

"Mathematics is the study of patterns."

The more patterns you can see in mathematics, the better you are at mathematics!

You already know that in a **number sequence** like 6, 12, 18, 24, ... , although the numbers change (are *not the same*),

- there is some **horizontal pattern** that does not change (is always the same for all numbers) and
- there is a **vertical calculation plan (rule)** that does not change and is *the same* for all the input and output numbers.

Here are the horizontal and vertical patterns for 6, 12, 18, 24, ...:

Input numbers:	1	2	3	4	5	...
	↓×6	↓×6	↓×6	↓×6	↓×6	
Output numbers:	6	12	18	24	30	...
		+6	+6	+6	+6	

We can describe and write the patterns in such sequences in different ways: in **words**, in a **table**, in a **flow diagram** or as a **calculation plan** (also called a **rule**).

These descriptions help us to solve problems like these:

1. To continue the sequence, in other words to find the next numbers in the sequence.
2. To calculate numbers further on in the sequence, for example the 100th number in the sequence. This is the same as calculating the output number if the input number is 100.
3. To find out the position of a number in the sequence, for example: Is 436 the 1st, 50th, ... 87th number in the sequence? This is the same as finding the input number if the output number is 436.
4. To decide if a number, for example 438, is in the sequence or not.

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8.1 Revising sequences of multiples

Teaching guidelines

We need to thoroughly reinforce sequences of multiples ("the times table") so that they will become easy for learners and a building block for non-multiple sequences.

Critical knowledge

All learners should understand, know and be able to apply the knowledge common to all multiple sequences: The multiples of k have (1) a constant difference of $+k$ and (2) a rule of the form $\times k$, for example the rule for multiples of 3 is *Multiple no.* = 3 × *Position no.*

Notes on questions

Problem solving is all about asking yourself the right questions, by reformulating the given question(s) from new information you have. So for question 1(e), in terms of Sequence A, the original question is: "Is 465 a number in the sequence? How do you know?" After recognising Sequence A as multiples of 3, the question should be reformulated to: "Is 465 a multiple of 3?" followed by: "How do I find out or know that it is a multiple of 3?"

Then you answer your own question: "If 465 divided by 3 has no remainder."

Then you do it (*with a calculator*): 465 ÷ 3 = 155.

There is no remainder, so 465 is a multiple of 3. So 465 is in the sequence 3, 6, 9, 12, ...

Answers

1. A: (a)(b) ..., 21, 24, 27, 30, 33, ... 100×3 = 300
(c)(d) 360 is in the sequence – it is a multiple of 3 (multiple no. 120)
(e) 465 is in the sequence – 465 is a multiple of 3

B: (a)(b) ..., 28, 32, 36, 40, 44, ... 100×4 = 400
(c)(d) 360 is in the sequence – it is a multiple of 4 (multiple no. 90)
(e) 465 is not in the sequence – 465 is not a multiple of 4

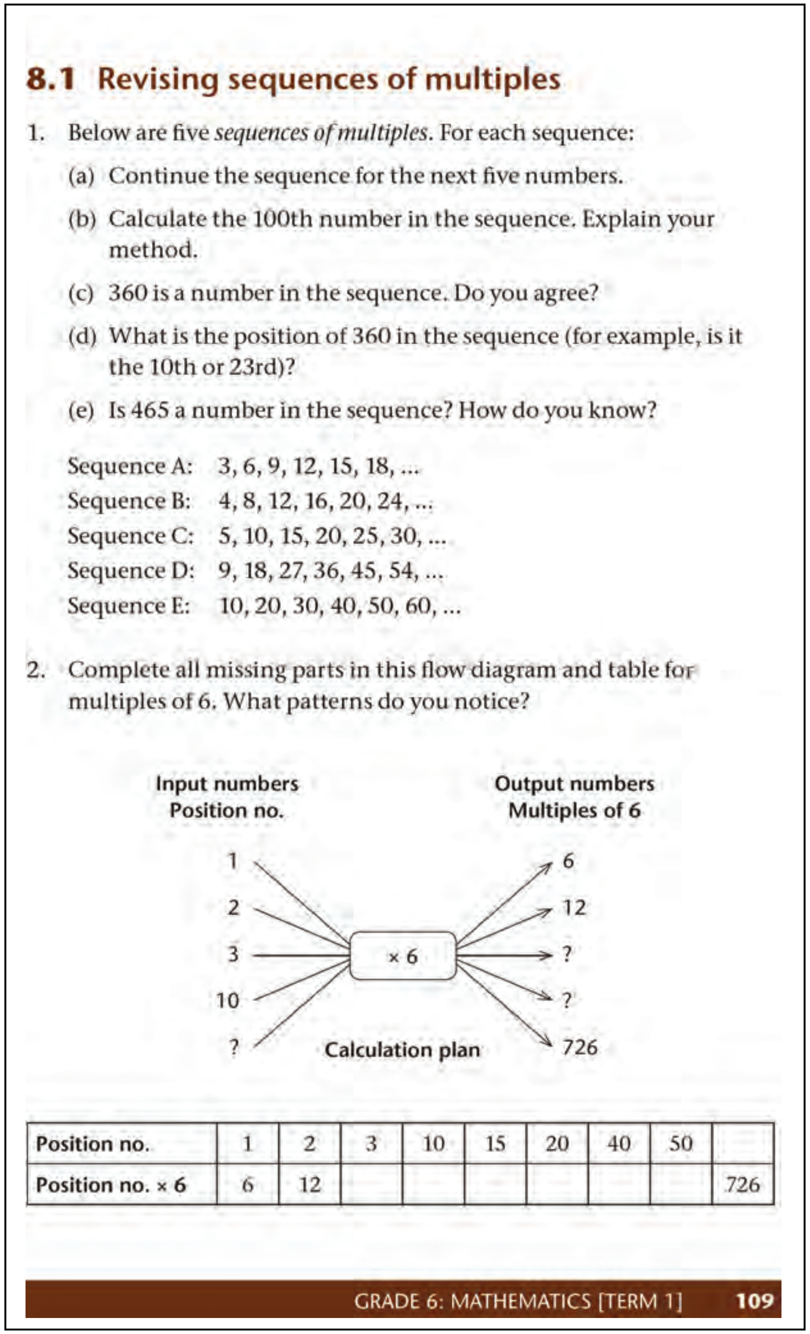
C: (a)(b) ..., 35, 40, 45, 50, 55, ... 100×5 = 500
(c)(d) 360 is in the sequence – it is a multiple of 5 (multiple no. 72)
(e) 465 is in the sequence – 465 is a multiple of 5

D: (a)(b) ..., 63, 72, 81, 90, 99, ... 100×9 = 900
(c)(d) 360 is in the sequence – it is a multiple of 9 (multiple no. 40)
(e) 465 is not in the sequence – 465 is not a multiple of 9

E: (a)(b) ..., 70, 80, 90, 100, 110, ... 100×10 = 1 000
(c)(d) 360 is in the sequence – it is a multiple of 9 (multiple no. 40)
(e) 465 is not in the sequence – 465 is not a multiple of 10

2. Output values are multiples of 6; so, for example, input for 726 is 726 ÷ 6 = 121.

Position no.	1	2	3	10	15	20	40	50	121
Position no. × 6	6	12	18	60	90	120	240	300	726



8.1 Revising sequences of multiples

1. Below are five *sequences of multiples*. For each sequence:
(a) Continue the sequence for the next five numbers.
(b) Calculate the 100th number in the sequence. Explain your method.
(c) 360 is a number in the sequence. Do you agree?
(d) What is the position of 360 in the sequence (for example, is it the 10th or 23rd)?
(e) Is 465 a number in the sequence? How do you know?

Sequence A: 3, 6, 9, 12, 15, 18, ...
Sequence B: 4, 8, 12, 16, 20, 24, ...
Sequence C: 5, 10, 15, 20, 25, 30, ...
Sequence D: 9, 18, 27, 36, 45, 54, ...
Sequence E: 10, 20, 30, 40, 50, 60, ...

2. Complete all missing parts in this flow diagram and table for multiples of 6. What patterns do you notice?

Position no.	1	2	3	10	15	20	40	50	
Position no. × 6	6	12							726

8.2 Non-multiple sequences

Teaching guidelines

The purpose of this section is to learn how to find the calculation plan (rule) for non-multiple sequences with a constant difference, in order to make the following two problem types easier:

- to find the 100th number in the sequence (finding output numbers)
- to find out whether a certain number, for example 465, is in the sequence and in what position it is (finding input numbers).

This section is a tutorial activity where learners learn by doing the activities, with meta-support about thinking strategies from you.

The different questions approach the concepts from different perspectives, for example from tables to rules (question 1), from flow diagrams to rules (question 2), and from sequences to rules (questions 3 to 5). You should help learners to see that these different representations are all equivalent.

To lay a sound foundation, it is important that all learners should do all the questions. It is important that learners do not have the mindset of answering each question as a standalone, isolated question. Rather, the learning vehicle is that learners will see the *relationship between the sequences* in the designed learning activities and also the relationship between the flow diagrams and the rules. If they do, they will have developed a very important and useful problem-solving tool, and it will make the work easy and they can finish quickly.

Answers

1. (a) The sequences all have a constant difference of 5, but different first numbers.

   (b) The vertical difference between the sequences is 1.

   (c)

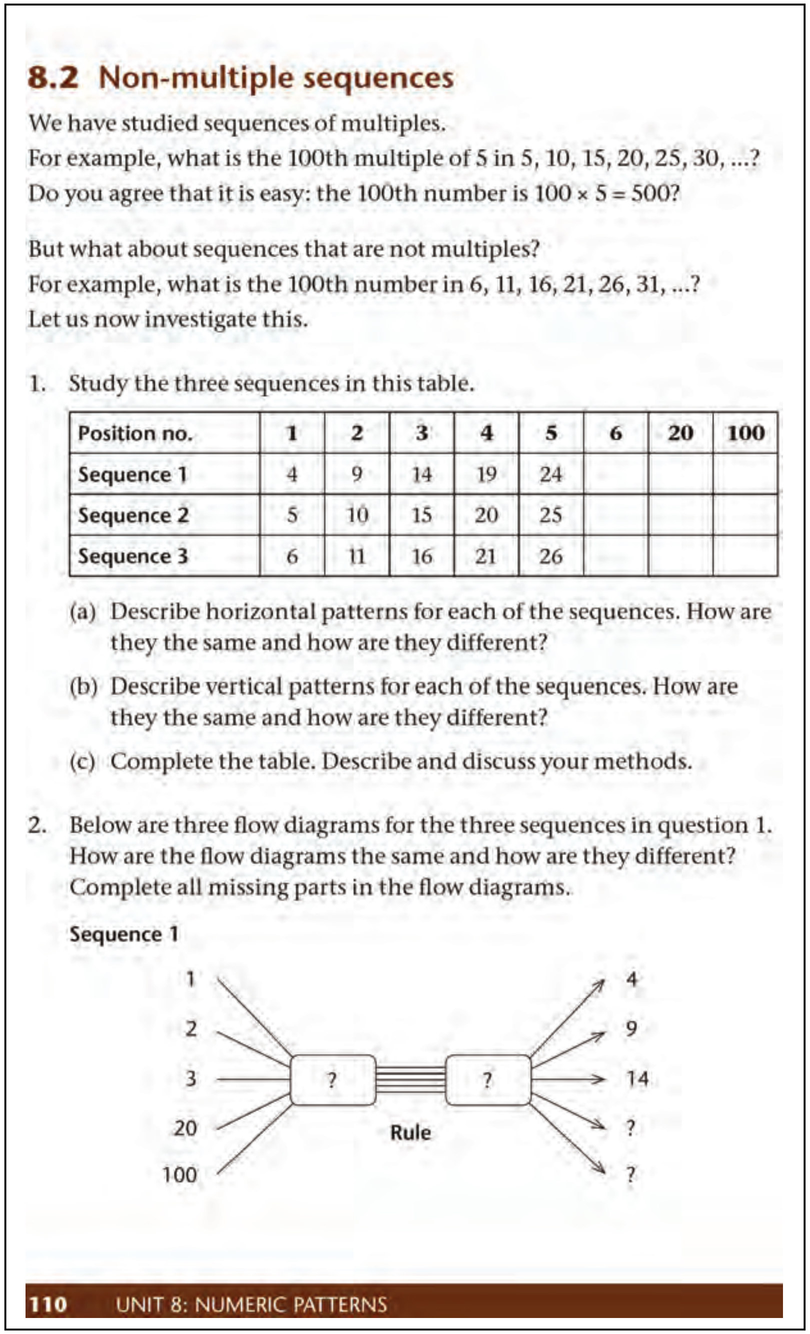
Position no.	**1**	**2**	**3**	**4**	**5**	**6**	**20**	**100**
Sequence 1	4	9	14	19	24	29	99	499
Sequence 2	5	10	15	20	25	30	100	500
Sequence 3	6	11	16	21	26	31	101	501

2. Sequence 1: 20, 100 –$\boxed{\times 5}$–$\boxed{-1}$→ 99, 499

   Sequence 2: 20, 100 –$\boxed{\times 5}$–$\boxed{+0}$→ 100, 500

   Sequence 3: 20, 100 –$\boxed{\times 5}$–$\boxed{+1}$→ 101, 501

Note: The alternative one-line flow diagram notation used here is for teachers only, NOT for learners.



8.2 Non-multiple sequences

We have studied sequences of multiples.

For example, what is the 100th multiple of 5 in 5, 10, 15, 20, 25, 30, ...?

Do you agree that it is easy: the 100th number is 100 × 5 = 500?

But what about sequences that are not multiples?

For example, what is the 100th number in 6, 11, 16, 21, 26, 31, ...?

Let us now investigate this.

1. Study the three sequences in this table.

Position no.	**1**	**2**	**3**	**4**	**5**	**6**	**20**	**100**
Sequence 1	4	9	14	19	24			
Sequence 2	5	10	15	20	25			
Sequence 3	6	11	16	21	26			

   (a) Describe horizontal patterns for each of the sequences. How are they the same and how are they different?

   (b) Describe vertical patterns for each of the sequences. How are they the same and how are they different?

   (c) Complete the table. Describe and discuss your methods.

2. Below are three flow diagrams for the three sequences in question 1. How are the flow diagrams the same and how are they different? Complete all missing parts in the flow diagrams.

Sequence 1

A common error

You should note a very common error, as shown in the table:

×20

Position no.	1	2	3	4	5	100
Sequence no.	7	13	19	25	31	620 ✗

×20

Here learners see and use the relationship 100 = 5×20, which is of course correct. The error is to assume that the same ×20 relationship holds between the corresponding output numbers. Learners then get *Output no.* 100 = 31×20 = 620, which is wrong.

What should you do?

Firstly, you have to get learners to realise that the answer is wrong. This can be done by comparing it to answers obtained by other learners and discussing the correctness of their methods, and therefore of their answers. Two other, correct calculation plans are:

Output no. = 6 × *Input no.* + 1, so *Output no.* 100 = 6 × 100 + 1 = 601

Output no. 100 = 31 + 95×6 = 601, as illustrated below:

+1 +1 +1 +1 +95

Position no.	1	2	3	4	5	100
Sequence no.	7	13	19	25	31	

+6 +6 +6 +6 +95×6

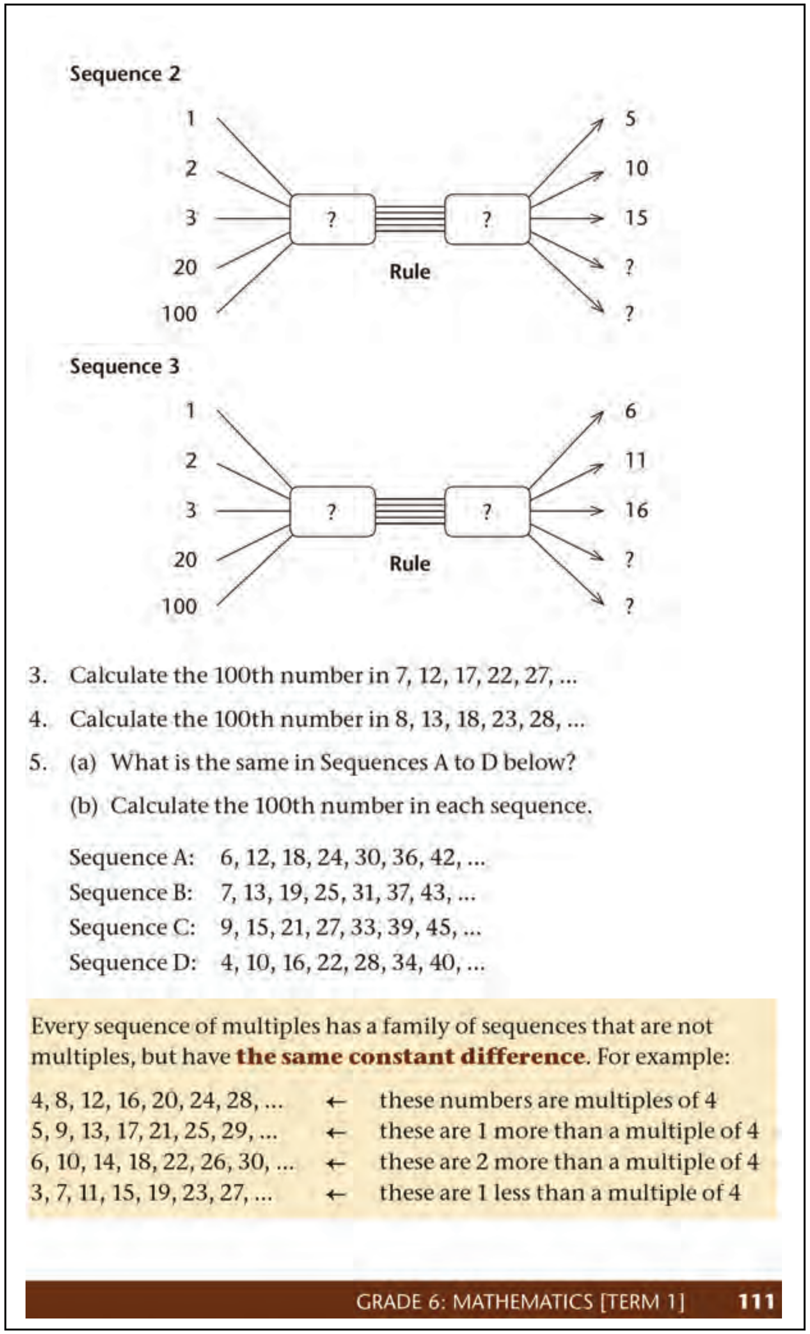
Secondly, learners have to understand that this multiplication strategy is not a property of non-multiple sequences, as illustrated here. If the property did hold, Output number 2 would be 7×2 = 14 and Output number 6 would be 13×3 = 39. But they are not.

×2 ×3

Position no.	1	2	3	4	5	6	100
Sequence no.	7	13	19	25	31	37	

Answers

3. 100×5 + 2 = 502
4. 100×5 + 3 = 503
5. (a) They all have a constant difference of 6.
   (b) A: 100×6 + 0 = 600
   B: 100×6 + 1 = 601
   C: 100×6 + 3 = 603
   D: 100×6 – 2 = 598



Sequence 2

Sequence 3

3. Calculate the 100th number in 7, 12, 17, 22, 27, ...
4. Calculate the 100th number in 8, 13, 18, 23, 28, ...
5. (a) What is the same in Sequences A to D below?
   (b) Calculate the 100th number in each sequence.

Sequence A: 6, 12, 18, 24, 30, 36, 42, ...
Sequence B: 7, 13, 19, 25, 31, 37, 43, ...
Sequence C: 9, 15, 21, 27, 33, 39, 45, ...
Sequence D: 4, 10, 16, 22, 28, 34, 40, ...

Every sequence of multiples has a family of sequences that are not multiples, but have **the same constant difference**. For example:

4, 8, 12, 16, 20, 24, 28, ...	←	these numbers are multiples of 4
5, 9, 13, 17, 21, 25, 29, ...	←	these are 1 more than a multiple of 4
6, 10, 14, 18, 22, 26, 30, ...	←	these are 2 more than a multiple of 4
3, 7, 11, 15, 19, 23, 27, ...	←	these are 1 less than a multiple of 4

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Teaching guidelines

It would be good if you were to do an interactive presentation along the lines of the shaded paragraph (Zukele's plan), maybe with other examples.

Some learners may find it difficult to immediately find the vertical pattern or rule needed to find the output for 100:

Position no.	**1**	**2**	**3**	**4**	**5**	**100**
	↓?	↓?	↓?	↓?	↓?	↓?
Sequence 2	10	14	18	22	26	?

You can help learners to change the one-step approach to two steps by using its relationship to the known sequence of multiples of 4 as a stepping stone (an intermediate or help sequence):

Position no.	**1**	**2**	**3**	**4**	**5**	**100**
	↓?	↓?	↓?	↓?	↓?	↓?
Sequence 1	4	8	12	16	20	?
	↓?	↓?	↓?	↓?	↓?	↓?
Sequence 2	10	14	18	22	26	?

Learners should find this two-step approach relatively easy. The knowledge required is:

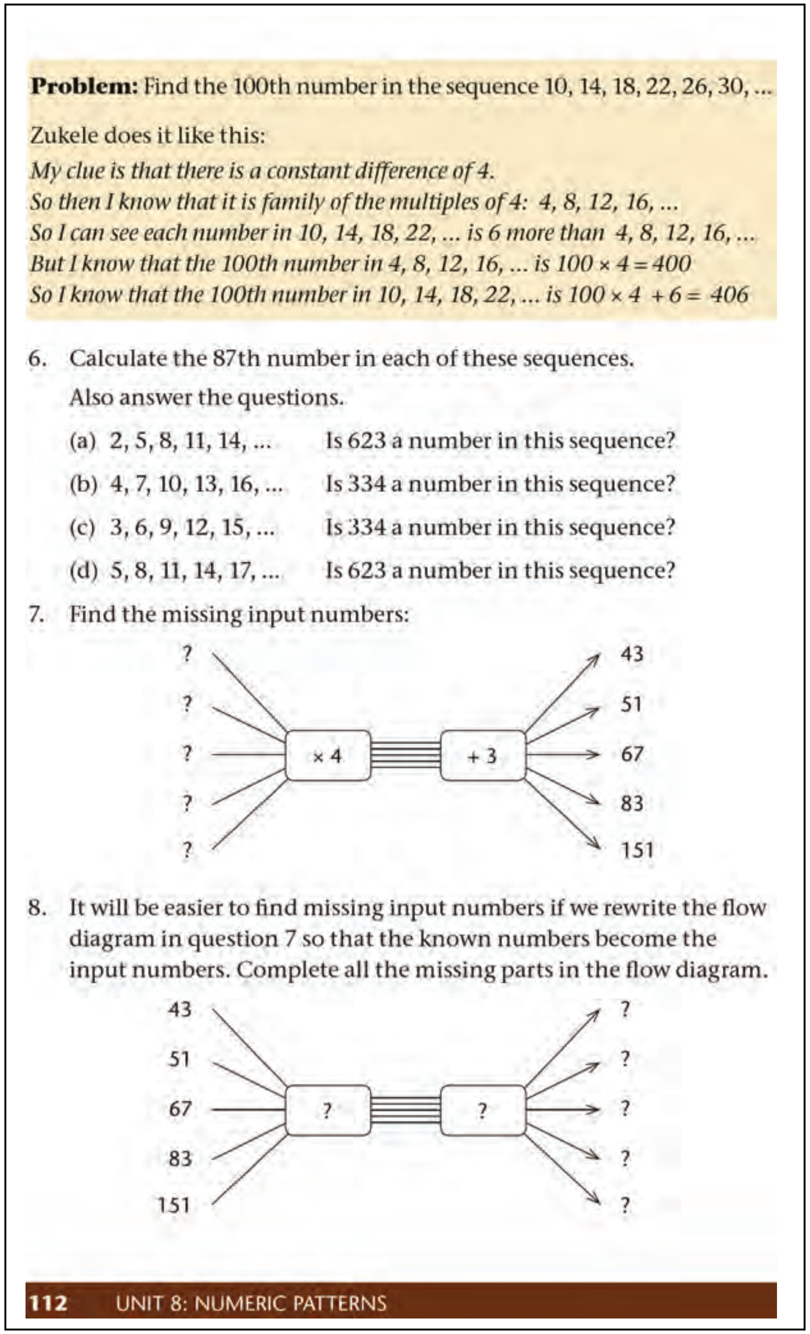
- The recognition that Sequence 1 and Sequence 2 are the same in the sense that they both have a constant difference of 4.
- The recognition of Sequence 1 (multiples of 4) as easy and do-able.
- Finding the relationship between Sequence 1 and Sequence 2, which in this case is +6.

Answers

6. (a) 3×87 – 1 = 260 623 ÷ 3 = 207 rem 2, so 623 is in the sequence (multiple no. 208).
   (b) 3×87 + 1 = 262 334 ÷ 3 = 111 rem 1, so 334 is in the sequence (multiple no. 111).
   (c) 3×87 + 0 = 261 334 ÷ 3 = 111 rem 1, so 334 is not in the sequence.
   (d) 3×87 + 2 = 263 Yes (multiple no. 87).

7. 10, 12, 16, 20, 37 $\rightarrow \boxed{\times 4} \rightarrow \boxed{+3} \rightarrow$ 43, 51, 67, 83, 151

8. 43, 51, 67, 83, 151 $\rightarrow \boxed{-3} \rightarrow \boxed{\div 4} \rightarrow$ 10, 12, 16, 20, 37



Problem: Find the 100th number in the sequence 10, 14, 18, 22, 26, 30, ...

Zukele does it like this:

My clue is that there is a constant difference of 4.
So then I know that it is family of the multiples of 4: 4, 8, 12, 16, ...
So I can see each number in 10, 14, 18, 22, ... is 6 more than 4, 8, 12, 16, ...
But I know that the 100th number in 4, 8, 12, 16, ... is 100 × 4 = 400
So I know that the 100th number in 10, 14, 18, 22, ... is 100 × 4 + 6 = 406

6. Calculate the 87th number in each of these sequences.
   Also answer the questions.

(a) 2, 5, 8, 11, 14, ...	Is 623 a number in this sequence?
(b) 4, 7, 10, 13, 16, ...	Is 334 a number in this sequence?
(c) 3, 6, 9, 12, 15, ...	Is 334 a number in this sequence?
(d) 5, 8, 11, 14, 17, ...	Is 623 a number in this sequence?

7. Find the missing input numbers:

8. It will be easier to find missing input numbers if we rewrite the flow diagram in question 7 so that the known numbers become the input numbers. Complete all the missing parts in the flow diagram.

8.3 Flow diagrams and rules

Teaching guidelines

For learners who have grasped the approach, this is a quick consolidation or reinforcement exercise, maybe leading to new insights. For learners who have not yet grasped the necessary concepts, it offers another opportunity to do so.

Again, if learners do not handle each new question as a stand-alone, but notice the relationship between (a), (b), (c)... they will find the work easy and can work quickly.

For example, once they recognise (a) as the multiples of 4 (the 4-times table), it should be clear that for (b) the Output numbers are 1 more, and in (c) another 1 more, etc.

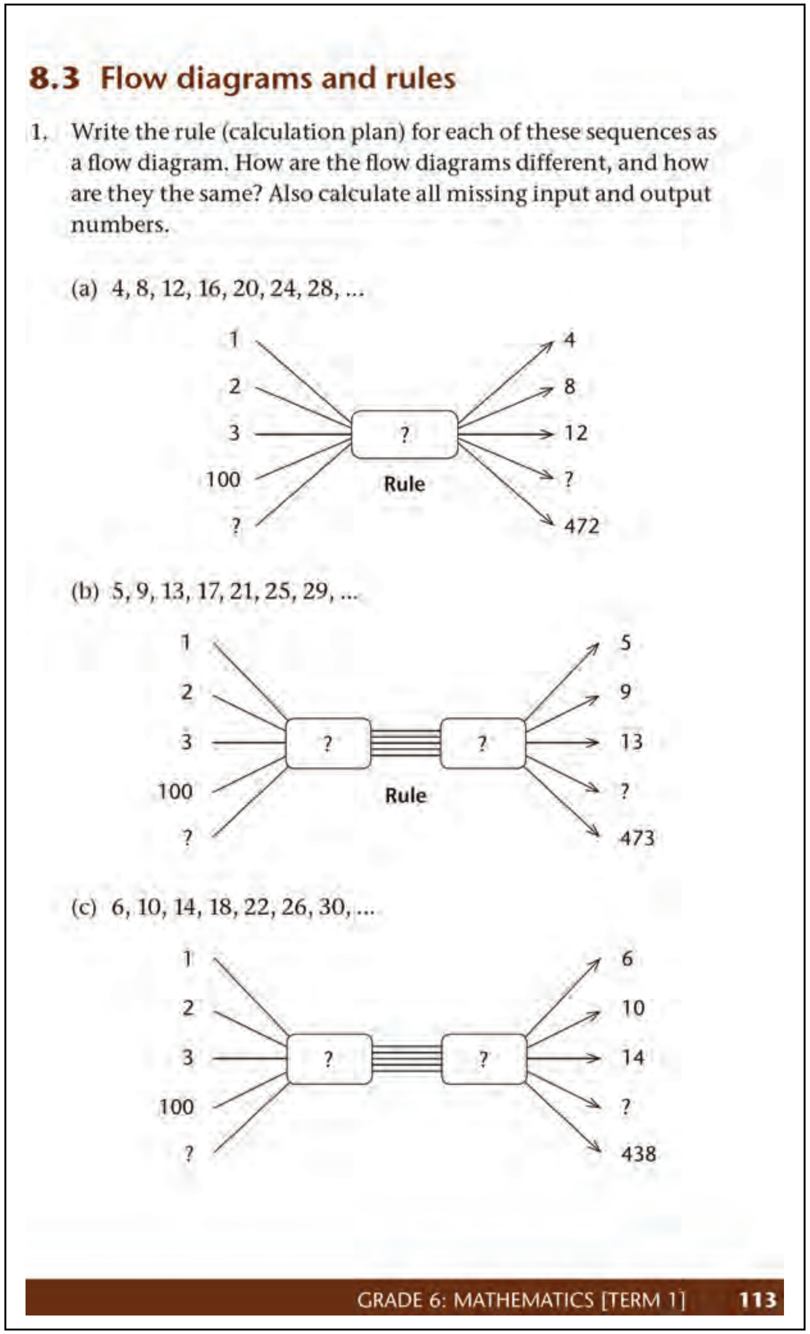
Because all the sequences have a constant difference of 4, all their flow diagrams have a $-\boxed{\times 4}-$ operator. However, the first number in each sequence is different, and so the flow diagrams have different addition or subtraction operators.

Answers

1. (a) 1, 2, 3, 100, 118 $-\boxed{\times 4}\rightarrow$ 4, 8, 12, 400, 472
   (b) 1, 2, 3, 100, 118 $-\boxed{\times 4}-\boxed{+1}\rightarrow$ 5, 9, 13, 401, 473
   (c) 1, 2, 3, 100, 109 $-\boxed{\times 4}-\boxed{+2}\rightarrow$ 6, 10, 14, 402, 438

   Questions (d) and (e) are on the next page in the Learner Book.

   (d) 1, 2, 3, 100, 209 $-\boxed{\times 4}-\boxed{-1}\rightarrow$ 3, 7, 11, 399, 835
   (e) 1, 2, 3, 100, 161 $-\boxed{\times 4}-\boxed{-3}\rightarrow$ 1, 5, 9, 397, 641



8.3 Flow diagrams and rules

1. Write the rule (calculation plan) for each of these sequences as a flow diagram. How are the flow diagrams different, and how are they the same? Also calculate all missing input and output numbers.

(a) 4, 8, 12, 16, 20, 24, 28, ...

(b) 5, 9, 13, 17, 21, 25, 29, ...

(c) 6, 10, 14, 18, 22, 26, 30, ...

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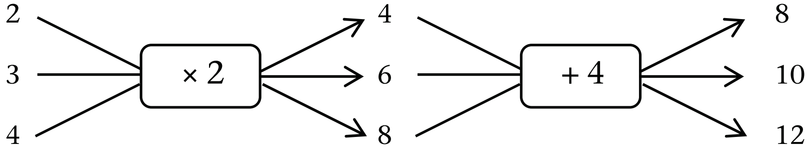
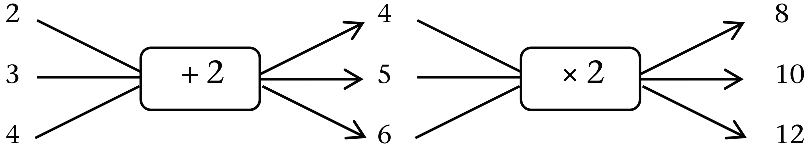
Note on order of operations and flow diagrams

You should make sure that leaners understand flow diagrams with two operators.

The flow diagram representation carries an intuitive left-to-right procedure: the first input produces the first output, the second input produces the second output, for example:



The left-to-right convention means that there is no need to learn rules such as BODMAS for the order of operations (first multiply before you add). BODMAS does not apply in the diagrams. For example, the following flow diagram is equivalent to the above.



The flow diagram's left-to-right procedure plays the same role as brackets in numerical expressions. For example, to calculate the output value for the input 3, the first diagram uses the arithmetic expression (3×2) + 4, and the second diagram uses (3+2) × 2, and of course (3×2) + 4 = (3+2) × 2.

Answers

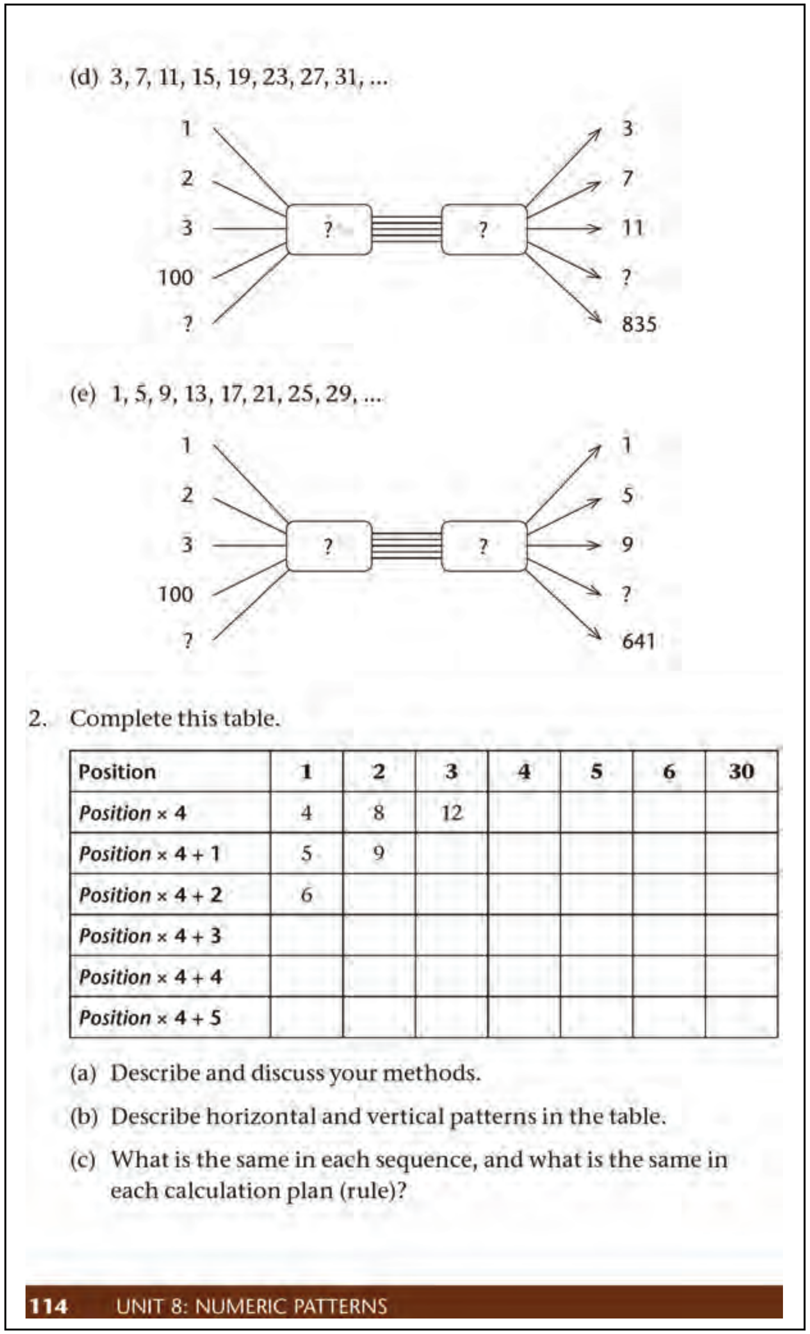
2.

Position	**1**	**2**	**3**	**4**	**5**	**6**	**30**
***Position* × 4 + 0**	4	8	12	16	20	24	120
***Position* × 4 + 1**	5	9	13	17	21	25	121
***Position* × 4 + 2**	6	10	14	18	22	26	122
***Position* × 4 + 3**	7	11	15	19	23	27	123
***Position* × 4 + 4**	8	12	16	20	24	28	124
***Position* × 4 + 5**	9	13	17	21	25	29	125

All sequences have a constant horizontal difference of 4.

All calculations plans have a –$\boxed{\times 4}$– operator.

(a) to (c): Consider learners' answers.



(d) 3, 7, 11, 15, 19, 23, 27, 31, ...

(e) 1, 5, 9, 13, 17, 21, 25, 29, ...

2. Complete this table.

Position	1	2	3	4	5	6	30
Position × 4	4	8	12				
Position × 4 + 1	5	9					
Position × 4 + 2	6						
Position × 4 + 3							
Position × 4 + 4							
Position × 4 + 5							

(a) Describe and discuss your methods.

(b) Describe horizontal and vertical patterns in the table.

(c) What is the same in each sequence, and what is the same in each calculation plan (rule)?

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8.4 Tables and rules

Teaching guidelines

For learners who have grasped the approach, this is a consolidation or reinforcement exercise, maybe leading to new insights by now working via given rules and tables. For learners who have not yet grasped the necessary concepts, it offers another opportunity to do so.

Questions 3 and 4 may serve as a good diagnostic assessment to you as well as your learners about their understanding of the core concepts and skills in the unit.

Notes on questions

Learners may approach question 1(a) differently, depending on their knowledge. A direct approach may be to simply calculate the output values by substitution of some of the given input values for each of the given rules, to check which rule gives the same values as those in the table.

Another approach would be to use our knowledge of the relationship between the constant differences in a sequence and the form of the rule. Here there is a constant horizontal difference of 5 in the sequence 2, 7, 12, 17, ... so there should be a "×5" in the rule, so there are only two possibilities, and these can easily be checked by applying the two rules to the given input numbers:

{0, 1, 2, ... } → *Input number* × 5 + 2 → {2, 7, 12, ...} Correct.

{0, 1, 2, ... } → (*Input number* + 2) × 5 → {10, 15, 20, ...} Not correct.

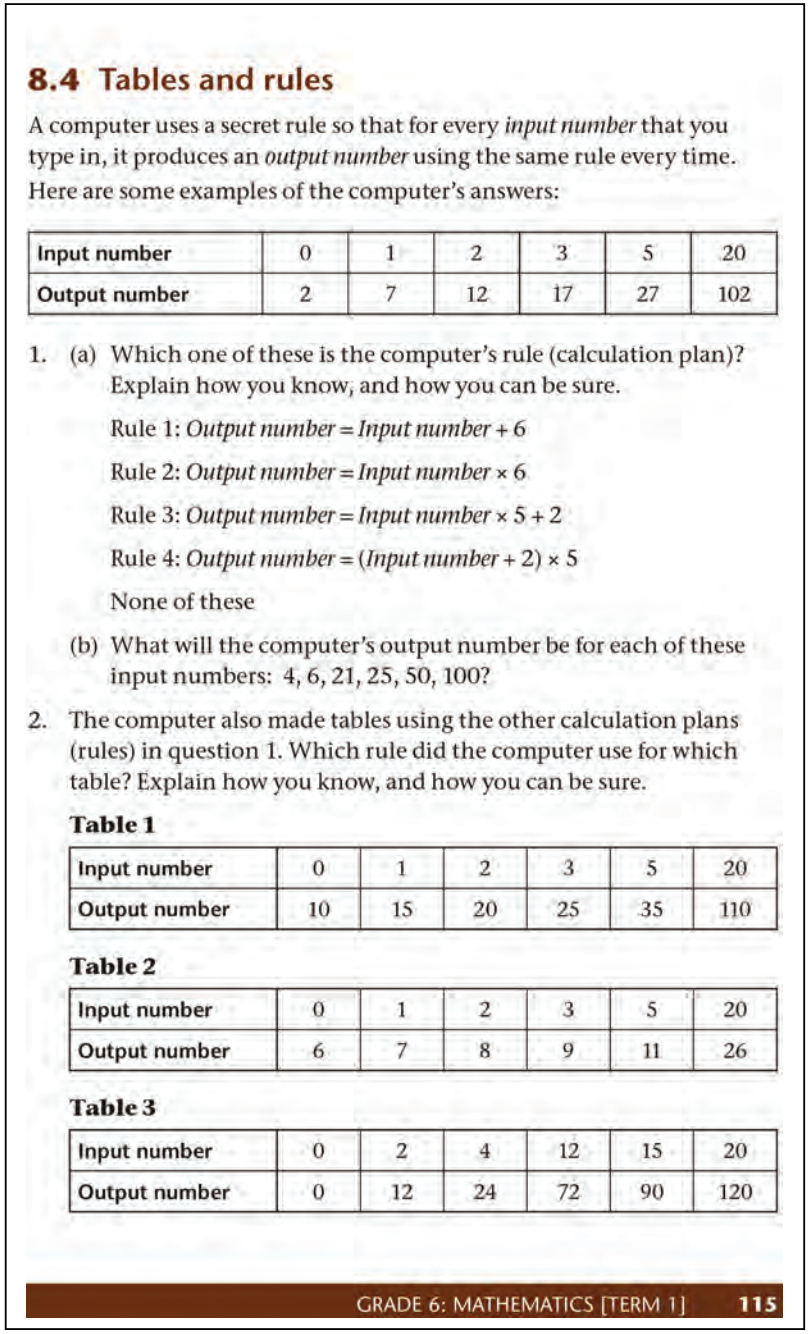
Answers

1. (a) Rule 3, i.e. *Output number* = *Input number* × 5 + 2.
   There is a constant horizontal difference of 5 between the output numbers.
   (b) 4, 6, 21, 25, 50, 100 $\rightarrow\boxed{\times 5}\rightarrow\boxed{+2}\rightarrow$ 22, 32, 107, 127, 252, 502

2. Table 1: Rule 4, i.e. *Output number* = (*Input number* + 2) × 5

   Table 2: Rule 1, i.e. *Output number* = *Input number* + 6

   Table 3: Rule 2, i.e. *Output number* = *Input number* × 6



8.4 Tables and rules

A computer uses a secret rule so that for every *input number* that you type in, it produces an *output number* using the same rule every time. Here are some examples of the computer's answers:

Input number	0	1	2	3	5	20
Output number	2	7	12	17	27	102

1. (a) Which one of these is the computer's rule (calculation plan)? Explain how you know, and how you can be sure.

   Rule 1: *Output number* = *Input number* + 6

   Rule 2: *Output number* = *Input number* × 6

   Rule 3: *Output number* = *Input number* × 5 + 2

   Rule 4: *Output number* = (*Input number* + 2) × 5

   None of these

   (b) What will the computer's output number be for each of these input numbers: 4, 6, 21, 25, 50, 100?

2. The computer also made tables using the other calculation plans (rules) in question 1. Which rule did the computer use for which table? Explain how you know, and how you can be sure.

Table 1

Input number	0	1	2	3	5	20
Output number	10	15	20	25	35	110

Table 2

Input number	0	1	2	3	5	20
Output number	6	7	8	9	11	26

Table 3

Input number	0	2	4	12	15	20
Output number	0	12	24	72	90	120

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Notes on the questions

Once learners recognise the sequence of Output numbers in Table 4 as the multiples of 12 (the 12-times table), they can easily find all missing Output numbers using the rule

Output number = *Input number* × 12

If learners do not handle Table 5 as a completely new question, but recognise that the Output numbers in Table 5 are all 2 more than the corresponding Output numbers in Table 4, they can easily find all missing Output numbers in Table 5 by simply adding 2 to the values in Table 4, or by using the rule

Output number = *Input number* × 12 + 2

Answers

3. (a) **Table 4**

Input number	1	2	3	4	5	6	17	60
Output number	12	24	36	48	60	72	204	720

Table 5

Input number	1	2	3	4	5	6	17	60
Output number	14	26	38	50	62	74	206	722

(b) Table 4: $17, 60 \rightarrow \boxed{\times 12} \rightarrow \boxed{+0} \rightarrow 204, 720$

Table 5: $17, 60 \rightarrow \boxed{\times 12} \rightarrow \boxed{+2} \rightarrow 206, 722$

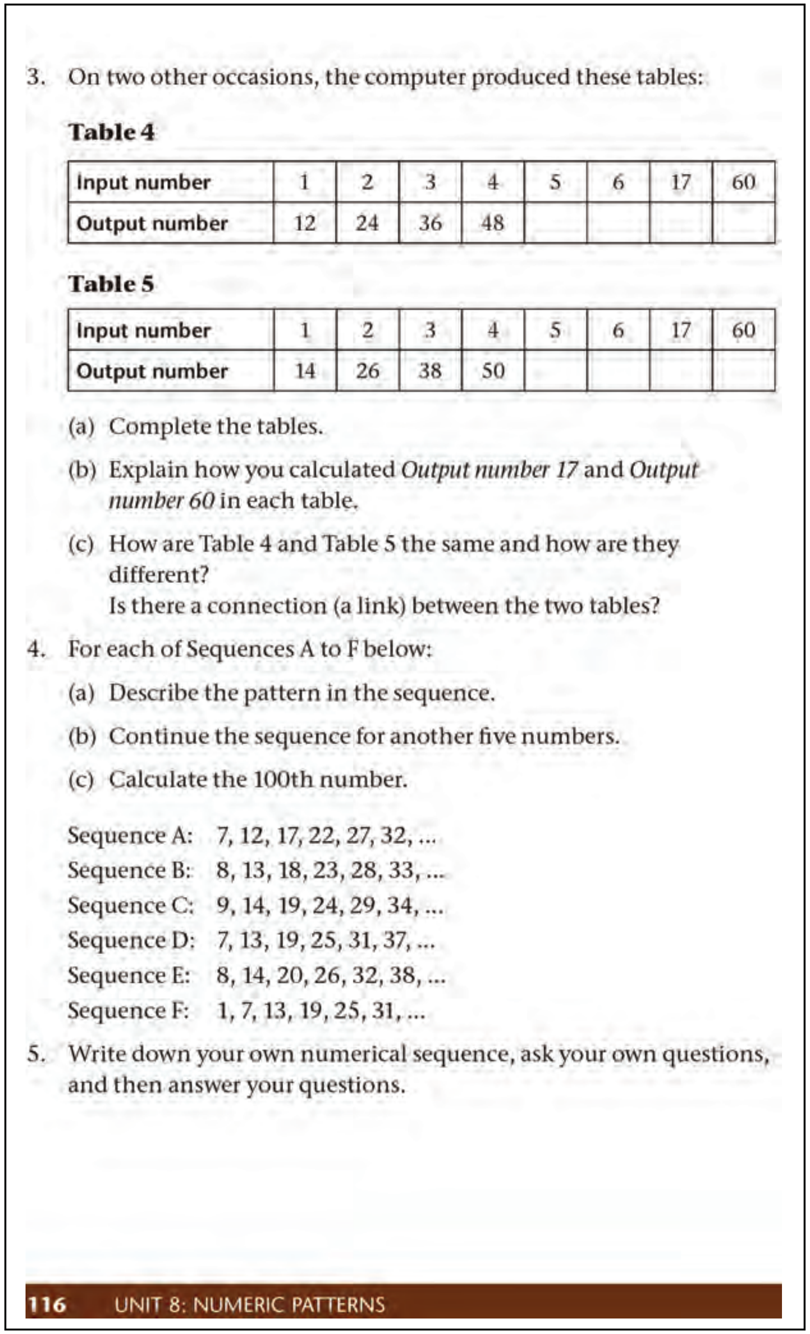
(c) Tables 4 and 5 both have a constant difference of 12.

The values in Table 5 are 2 more than those in Table 4.

4.

A:	(a) 2 more than multiples of 5	(b) 37, 42, 47, 52, 57	(c) 502
B:	(a) 3 more than multiples of 5	(b) 38, 43, 48, 53, 58	(c) 503
C:	(a) 4 more than multiples of 5	(b) 39, 44, 49, 54, 59	(c) 504
D:	(a) 1 more than multiples of 6	(b) 43, 49, 55, 61, 67	(c) 601
E:	(a) 2 more than multiples of 6	(b) 44, 50, 56, 62, 68	(c) 602
F:	(a) 5 less than multiples of 6	(b) 37, 43, 49, 55, 61	(c) 595

5. Individual work. It may be a good idea to let learners exchange their work and check each other's work. (Be open to sequences that may not be linear and similar to the ones used. Learners may come up with Fibonacci sequences, for example.)



3. On two other occasions, the computer produced these tables:

Table 4

Input number	1	2	3	4	5	6	17	60
Output number	12	24	36	48				

Table 5

Input number	1	2	3	4	5	6	17	60
Output number	14	26	38	50				

(a) Complete the tables.

(b) Explain how you calculated *Output number 17* and *Output number 60* in each table.

(c) How are Table 4 and Table 5 the same and how are they different?
Is there a connection (a link) between the two tables?

4. For each of Sequences A to F below:

(a) Describe the pattern in the sequence.

(b) Continue the sequence for another five numbers.

(c) Calculate the 100th number.

Sequence A:	7, 12, 17, 22, 27, 32, ...
Sequence B:	8, 13, 18, 23, 28, 33, ...
Sequence C:	9, 14, 19, 24, 29, 34, ...
Sequence D:	7, 13, 19, 25, 31, 37, ...
Sequence E:	8, 14, 20, 26, 32, 38, ...
Sequence F:	1, 7, 13, 19, 25, 31, ...

5. Write down your own numerical sequence, ask your own questions, and then answer your questions.

116 UNIT 8: NUMERIC PATTERNS

Term 2

Grade 6 Term 2 Unit 1 Whole numbers

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
1.1 Numbers bigger than a million	Developing a sense of the magnitude of larger numbers	119 to 121
1.2 Count beyond 1 million	Counting activities to strengthen number concept	122
1.3 Represent and order 9-digit numbers	Naming and comparing larger numbers, expanded notation	123 to 124

CAPS time allocation	1 hour
CAPS page references	13 to 15 and 240

Mathematical background

While the number names up to 1 000 are not difficult to grasp, the number names for larger numbers can become a challenge for learners, and it is important that they understand how the **number naming system** works.

The number names for all whole numbers up to 9-digit numbers are formed by combining the following thirty words:

A. The names for numbers up to ten: *one, two, three, four, five, six, seven, eight, nine, ten*
B. The names for numbers between ten and twenty: *eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen*
C. The names for multiples of ten up to ninety: *twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety*
D. The names for the second, third and sixth powers of ten: *hundred, thousand, million.*

The number names for numbers between hundred and thousand are built up with the words in Lists A, B and C above, and the word *hundred*.
For example, the number name for 574 is *five hundred and seventy-four*.

The number names for numbers between thousand and million are also built up with the words in Lists A, B and C above, and the word *hundred,* **as well as the word *thousand***. For example, the number name for 574 000 is *five hundred and seventy-four* ***thousand***.

The number names for numbers between million and 1 000 million are also built up with the words in Lists A, B and C above, and the word *hundred,* **as well as the word *million***. For example, the number name for 574 000 000 is *five hundred and seventy-four* ***million***.

1.1 Numbers bigger than a million

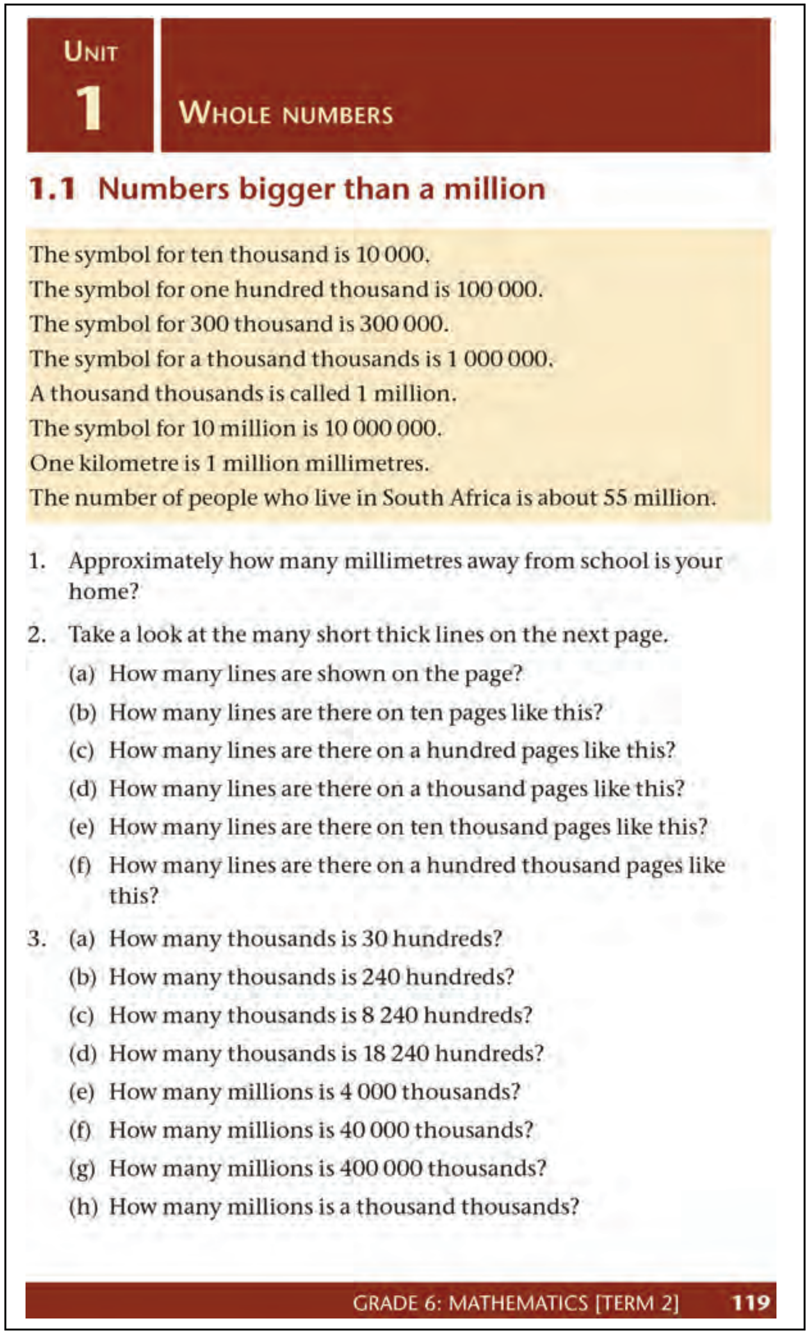
Teaching guidelines

The three questions are intended to help learners to form a sense of the magnitudes represented by large numbers, since they often do not have real-life experiences of large collections of objects. Tell them that this is the purpose of the questions.

Question 3 can be given for homework.

Answers

1. Learners' answers will differ.
2. (a) 10 000 (b) 100 000
   (c) 1 000 000 (d) 10 000 000
   (e) 100 000 000 (f) 1 000 000 000
3. (a) 3 thousands (b) 24 thousands
   (c) 824 thousands (d) 1 824 thousands
   (e) 4 millions (f) 40 millions
   (g) 400 millions (h) 1 million



UNIT 1

WHOLE NUMBERS

1.1 Numbers bigger than a million

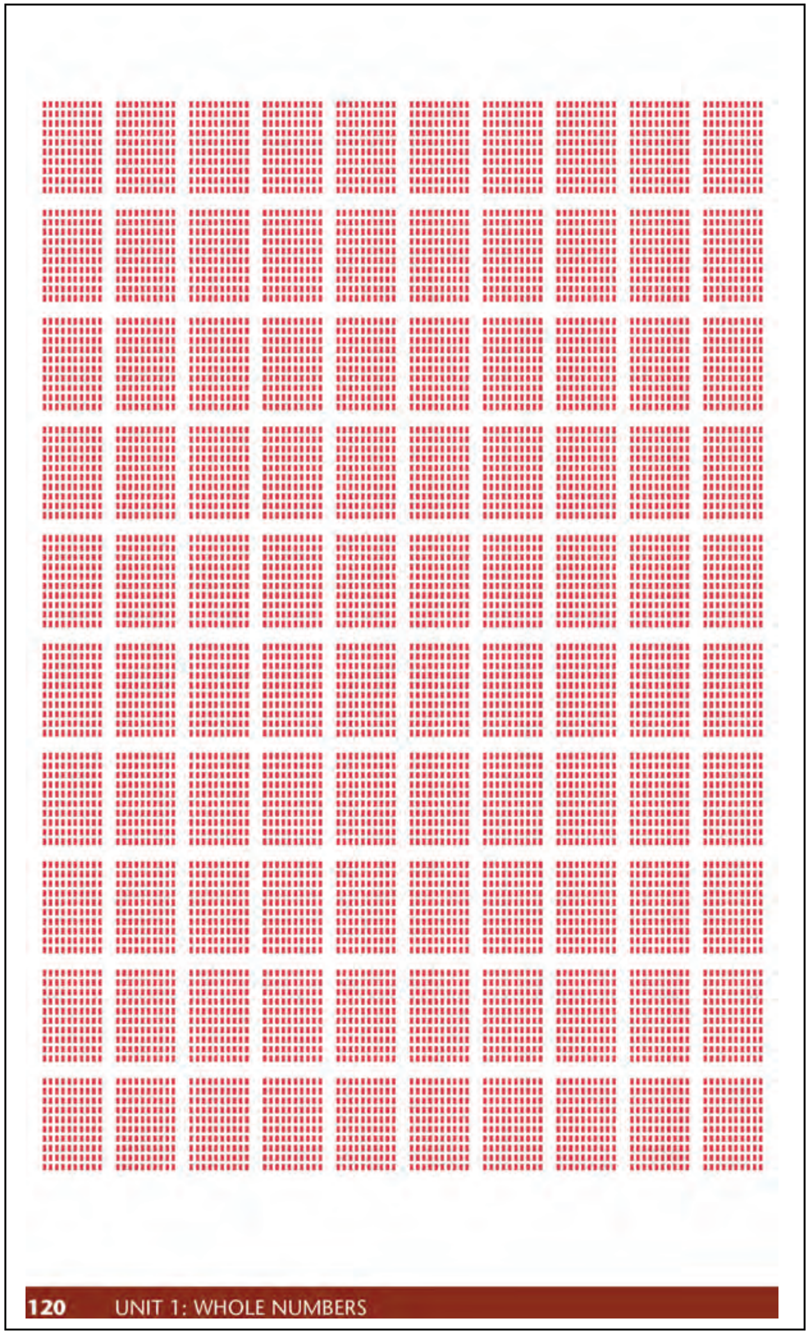
The symbol for ten thousand is 10 000.
The symbol for one hundred thousand is 100 000.
The symbol for 300 thousand is 300 000.
The symbol for a thousand thousands is 1 000 000.
A thousand thousands is called 1 million.
The symbol for 10 million is 10 000 000.
One kilometre is 1 million millimetres.
The number of people who live in South Africa is about 55 million.

1. Approximately how many millimetres away from school is your home?
2. Take a look at the many short thick lines on the next page.
   (a) How many lines are shown on the page?
   (b) How many lines are there on ten pages like this?
   (c) How many lines are there on a hundred pages like this?
   (d) How many lines are there on a thousand pages like this?
   (e) How many lines are there on ten thousand pages like this?
   (f) How many lines are there on a hundred thousand pages like this?
3. (a) How many thousands is 30 hundreds?
   (b) How many thousands is 240 hundreds?
   (c) How many thousands is 8 240 hundreds?
   (d) How many thousands is 18 240 hundreds?
   (e) How many millions is 4 000 thousands?
   (f) How many millions is 40 000 thousands?
   (g) How many millions is 400 000 thousands?
   (h) How many millions is a thousand thousands?

Teaching guidelines

If time permits, you may ask a variety of other questions with reference to this array of 10 000 lines. Some examples are given below:

1. How many pages like this are needed to make up one million lines?
2. How many pages like this are needed to make up 10 million lines?
3. How many blocks of 100 lines each are shown on this page?
4. If 15 lines are removed from each block, how many lines will remain on the page?
5. If 38 blocks are coloured blue and the other blocks remain red, how many lines will be red?
6. If 2 367 of the lines are removed, how many lines will remain?
7. If the array of lines is divided into four equal parts, how many lines will there be in each part?



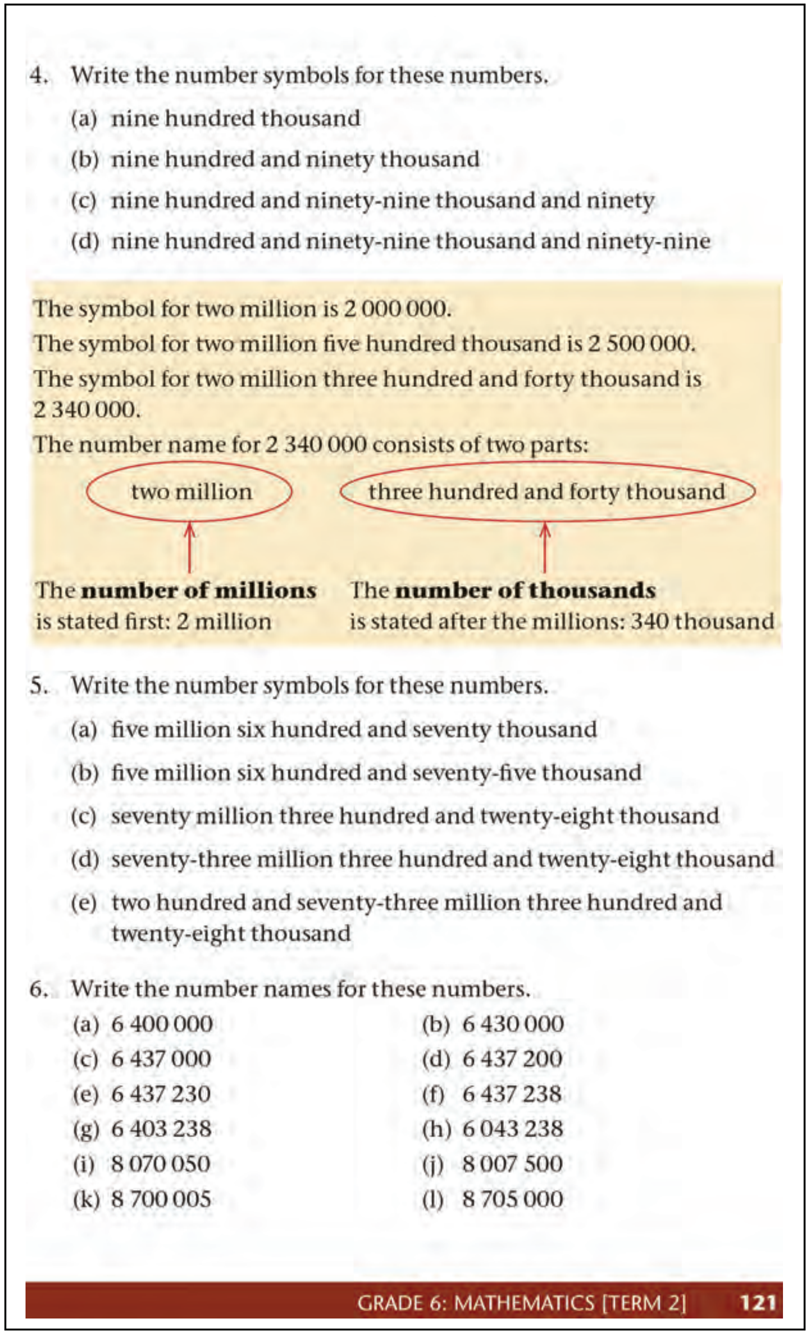
120 UNIT 1: WHOLE NUMBERS

Teaching guidelines

The naming system for larger numbers can be explained with reference to the shaded passage. The key aspect of naming larger numbers (beyond a million) is that the number is thought of as consisting of a millions part, a thousands part and a units part, and these three parts form the structure of the number name, for example 524 674 839 is named as "524 million 674 thousand and 839", in other words as *five hundred and twenty-four* **million** *six hundred and seventy-four* **thousand** *eight hundred and thirty-nine*.

Answers

4. (a) 900 000 (b) 990 000
   (c) 999 090 (d) 999 099
5. (a) 5 670 000 (b) 5 675 000
   (c) 70 328 000 (d) 73 328 000
   (e) 273 328 000
6. (a) six million four hundred thousand
   (b) six million four hundred and thirty thousand
   (c) six million four hundred and thirty-seven thousand
   (d) six million four hundred and thirty-seven thousand two hundred
   (e) six million four hundred and thirty-seven thousand two hundred and thirty
   (f) six million four hundred and thirty-seven thousand two hundred and thirty-eight
   (g) six million four hundred and three thousand two hundred and thirty-eight
   (h) six million forty-three thousand two hundred and thirty-eight
   (i) eight million seventy thousand and fifty
   (j) eight million seven thousand five hundred
   (k) eight million seven hundred thousand and five
   (l) eight million seven hundred and five thousand



4. Write the number symbols for these numbers.
   (a) nine hundred thousand
   (b) nine hundred and ninety thousand
   (c) nine hundred and ninety-nine thousand and ninety
   (d) nine hundred and ninety-nine thousand and ninety-nine

The symbol for two million is 2 000 000.
The symbol for two million five hundred thousand is 2 500 000.
The symbol for two million three hundred and forty thousand is 2 340 000.
The number name for 2 340 000 consists of two parts:

two million | three hundred and forty thousand

The **number of millions** is stated first: 2 million

The **number of thousands** is stated after the millions: 340 thousand

5. Write the number symbols for these numbers.
   (a) five million six hundred and seventy thousand
   (b) five million six hundred and seventy-five thousand
   (c) seventy million three hundred and twenty-eight thousand
   (d) seventy-three million three hundred and twenty-eight thousand
   (e) two hundred and seventy-three million three hundred and twenty-eight thousand

6. Write the number names for these numbers.
   (a) 6 400 000 (b) 6 430 000
   (c) 6 437 000 (d) 6 437 200
   (e) 6 437 230 (f) 6 437 238
   (g) 6 403 238 (h) 6 043 238
   (i) 8 070 050 (j) 8 007 500
   (k) 8 700 005 (l) 8 705 000

GRADE 6: MATHEMATICS [TERM 2] 121

1.2 Count beyond 1 million

Teaching guidelines

Counting is the way in which learners form a sense of the sizes of smaller numbers.

Ask them how long they think they would take to count to 1 000, to 10 000, to 1 000 000.

Counting in large intervals as required by questions 1 to 4 may help them to form a sense of larger numbers.

Answers

1\. (a) 800 000 850 000 900 000 950 000 1 000 000
1 050 000 1 100 000 1 150 000 1 200 000

(b) 990 000 992 000 994 000 996 000 998 000
1 000 000 1 002 000 1 004 000 1 006 000 1 008 000
1 010 000 1 012 000

(c) 100 000 600 000 1 100 000 1 600 000 2 100 000
2 600 000 3 100 000 3 600 000

(d) 4 000 000 4 250 000 4 500 000 4 750 000 5 000 000
5 250 000 5 500 000 5 750 000 6 000 000

(e) 41 000 000 41 250 000 41 500 000 41 750 000 42 000 000
42 250 000 42 500 000 42 750 000 43 000 000

(f) 423 000 000 423 250 000 423 500 000 423 750 000 424 000 000
424 250 000 424 500 000 424 750 000 425 000 000

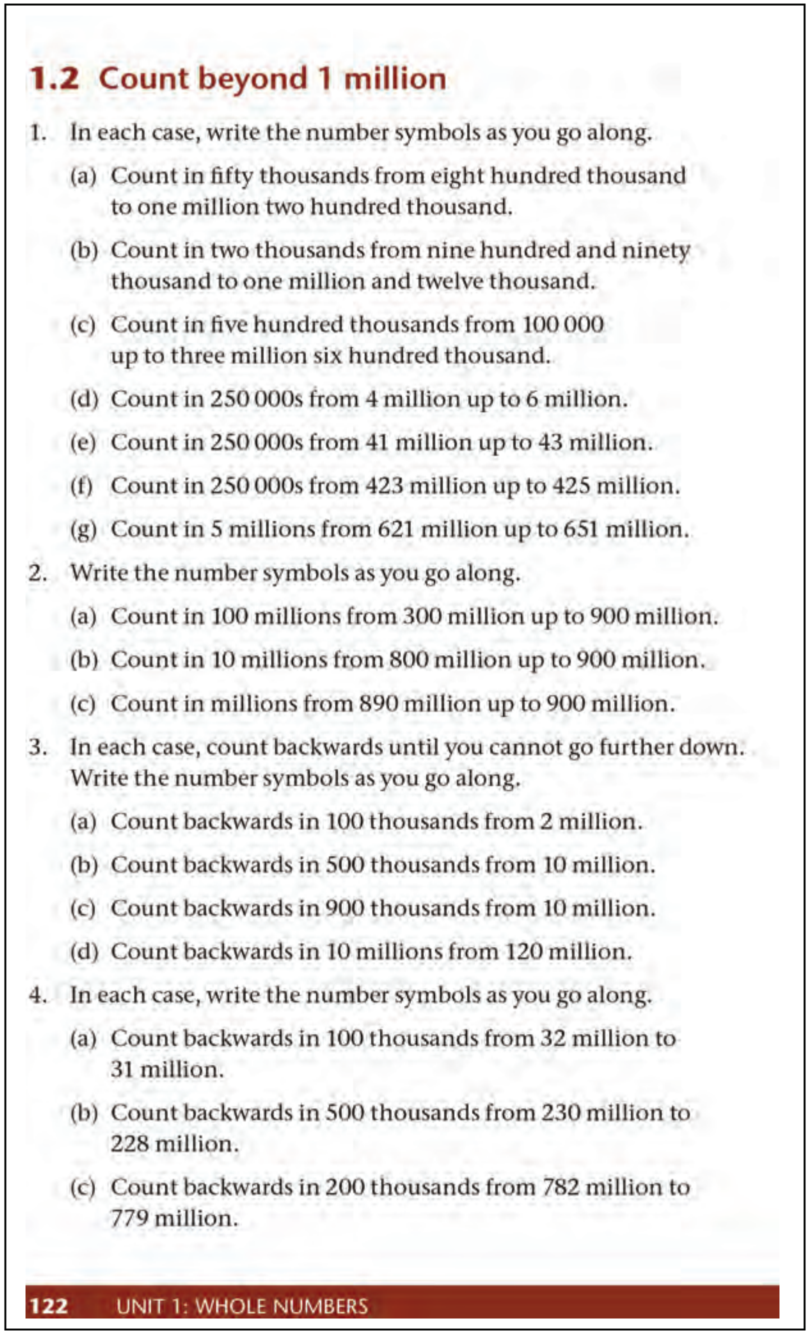
(g) 621 000 000 626 000 000 631 000 000 636 000 000 641 000 000
646 000 000 651 000 000

2\. (a) 300 000 000 400 000 000 500 000 000 600 000 000
700 000 000 800 000 000 900 000 000

(b) 800 000 000 810 000 000 820 000 000 830 000 000
840 000 000 850 000 000 860 000 000 870 000 000
880 000 000 890 000 000 900 000 000

(c) 890 000 000 891 000 000 892 000 000 893 000 000
894 000 000 895 000 000 896 000 000 897 000 000
898 000 000 899 000 000 900 000 000

3.–4. See next page.



1.2 Count beyond 1 million

1. In each case, write the number symbols as you go along.
   - (a) Count in fifty thousands from eight hundred thousand to one million two hundred thousand.
   - (b) Count in two thousands from nine hundred and ninety thousand to one million and twelve thousand.
   - (c) Count in five hundred thousands from 100 000 up to three million six hundred thousand.
   - (d) Count in 250 000s from 4 million up to 6 million.
   - (e) Count in 250 000s from 41 million up to 43 million.
   - (f) Count in 250 000s from 423 million up to 425 million.
   - (g) Count in 5 millions from 621 million up to 651 million.
2. Write the number symbols as you go along.
   - (a) Count in 100 millions from 300 million up to 900 million.
   - (b) Count in 10 millions from 800 million up to 900 million.
   - (c) Count in millions from 890 million up to 900 million.
3. In each case, count backwards until you cannot go further down. Write the number symbols as you go along.
   - (a) Count backwards in 100 thousands from 2 million.
   - (b) Count backwards in 500 thousands from 10 million.
   - (c) Count backwards in 900 thousands from 10 million.
   - (d) Count backwards in 10 millions from 120 million.
4. In each case, write the number symbols as you go along.
   - (a) Count backwards in 100 thousands from 32 million to 31 million.
   - (b) Count backwards in 500 thousands from 230 million to 228 million.
   - (c) Count backwards in 200 thousands from 782 million to 779 million.

122 UNIT 1: WHOLE NUMBERS

Answers *(continued)*

3. (a) 2 000 000 1 900 000 1 800 000 1 700 000 1 600 000 1 500 000
1 400 000 1 300 000 1 200 000 1 100 000 1 000 000 900 000
800 000 700 000 600 000 500 000 400 000 300 000
200 000 100 000 0

(b) 10 000 000 9 500 000 9 000 000 8 500 000 8 000 000 7 500 000
7 000 000 6 500 000 6 000 000 5 500 000 5 000 000 4 500 000
4 000 000 3 500 000 3 000 000 2 500 000 2 000 000 1 500 00
1 000 000 500 000 0

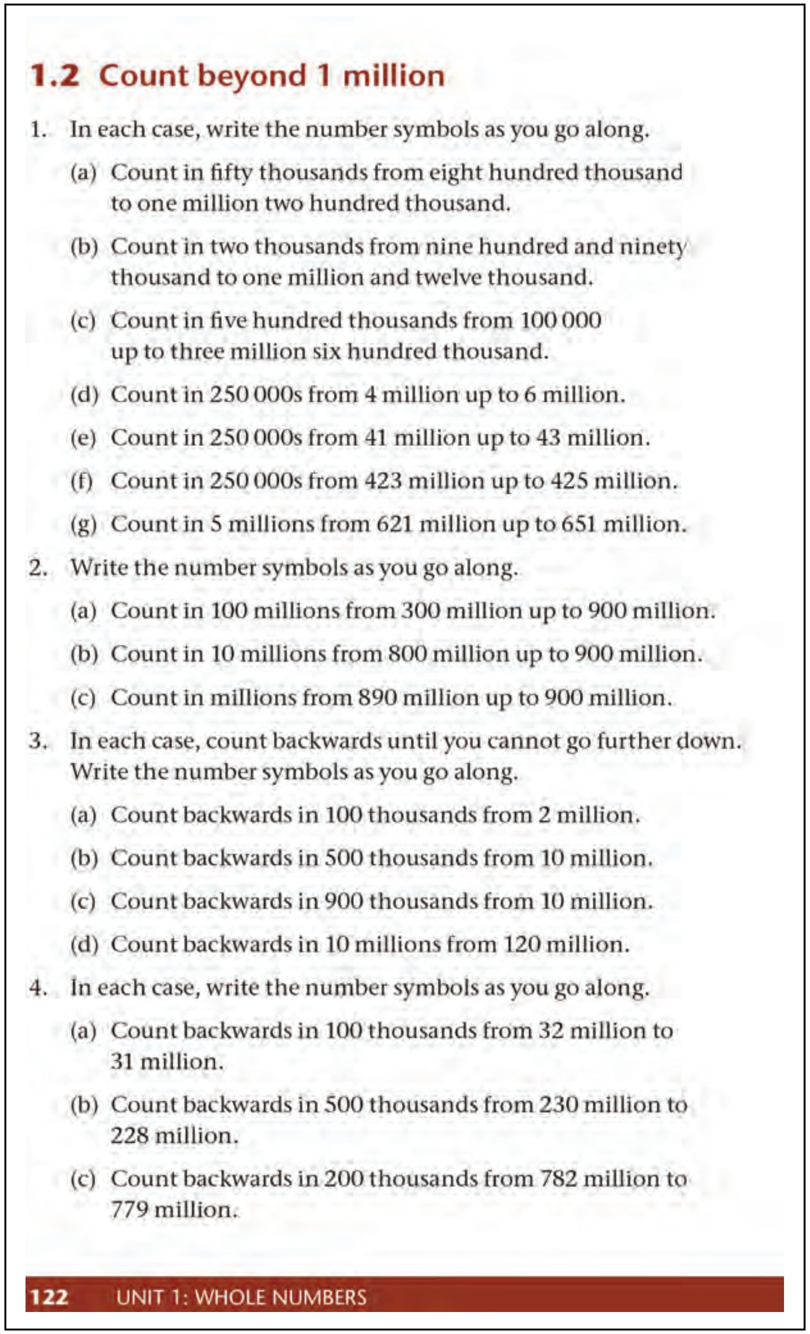
(c) 10 000 000 9 100 000 8 200 000 7 300 000 6 400 000 5 500 000
4 600 000 3 700 000 2 800 000 1 900 000 1 000 000 100 000

(d) 120 000 000 110 000 000 100 000 000 90 000 000 80 000 000
70 000 000 60 000 000 50 000 000 40 000 000 30 000 000
20 000 000 10 000 000 0

4. (a) 32 000 000 31 900 000 31 800 000 31 700 000 31 600 000
31 500 000 31 400 000 31 300 000 31 200 000 31 100 000
31 000 000

(b) 230 000 000 229 500 000 229 000 000 228 500 000 228 000 000

(c) 782 000 000 781 800 000 781 600 000 781 400 000 781 200 000
781 000 000 780 800 000 780 600 000 780 400 000 780 200 000
780 000 000 779 800 000 779 600 000 779 400 000 779 200 000
779 000 000



1.2 Count beyond 1 million

1. In each case, write the number symbols as you go along.
   (a) Count in fifty thousands from eight hundred thousand to one million two hundred thousand.
   (b) Count in two thousands from nine hundred and ninety thousand to one million and twelve thousand.
   (c) Count in five hundred thousands from 100 000 up to three million six hundred thousand.
   (d) Count in 250 000s from 4 million up to 6 million.
   (e) Count in 250 000s from 41 million up to 43 million.
   (f) Count in 250 000s from 423 million up to 425 million.
   (g) Count in 5 millions from 621 million up to 651 million.
2. Write the number symbols as you go along.
   (a) Count in 100 millions from 300 million up to 900 million.
   (b) Count in 10 millions from 800 million up to 900 million.
   (c) Count in millions from 890 million up to 900 million.
3. In each case, count backwards until you cannot go further down. Write the number symbols as you go along.
   (a) Count backwards in 100 thousands from 2 million.
   (b) Count backwards in 500 thousands from 10 million.
   (c) Count backwards in 900 thousands from 10 million.
   (d) Count backwards in 10 millions from 120 million.
4. In each case, write the number symbols as you go along.
   (a) Count backwards in 100 thousands from 32 million to 31 million.
   (b) Count backwards in 500 thousands from 230 million to 228 million.
   (c) Count backwards in 200 thousands from 782 million to 779 million.

122 UNIT 1: WHOLE NUMBERS

1.3 Represent and order 9-digit numbers

Teaching guidelines

You may assess learners' ability to name large numbers by writing some 9-digit number symbols on the board and asking learners to read them softly to themselves, then to write the number names on loose sheets of paper that you can take in and mark, for example:

273 437 628

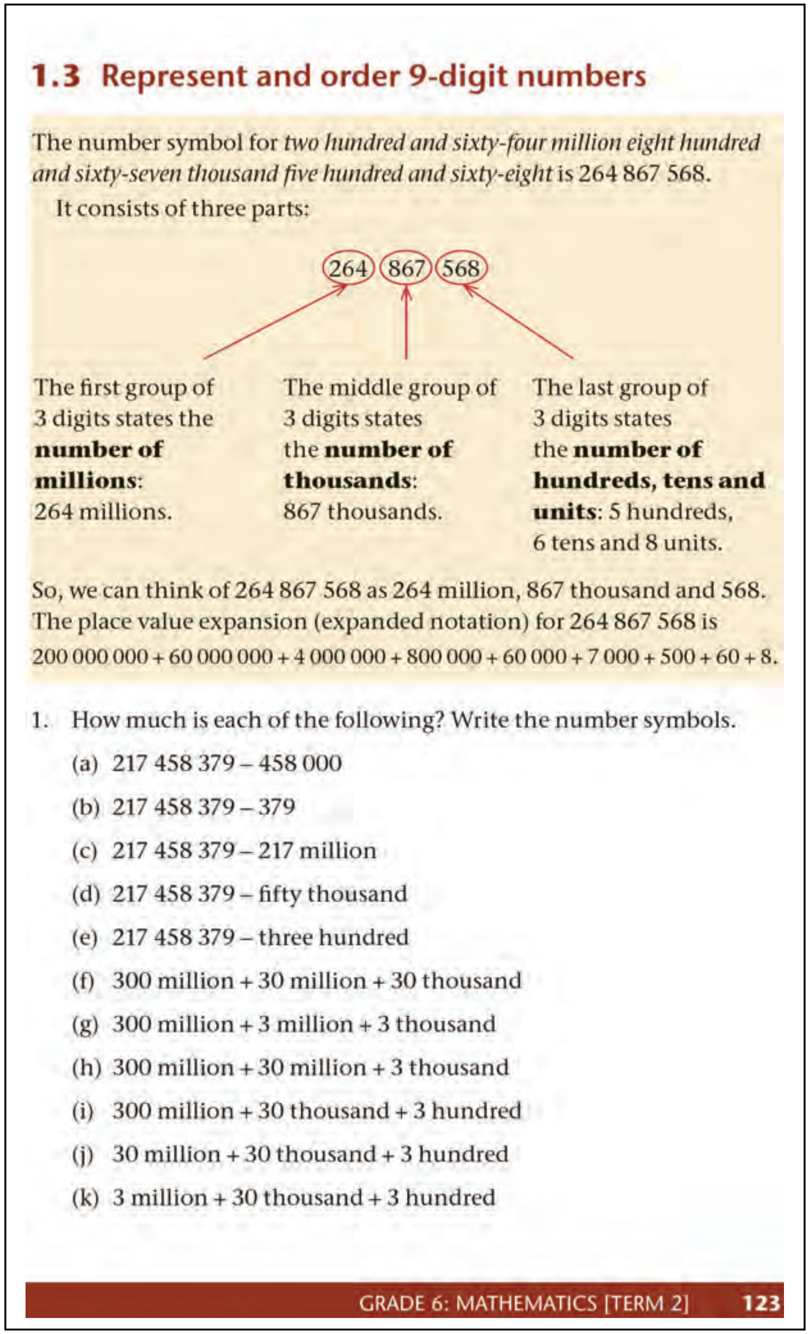
728 554 193

This assessment (with different numbers) may be repeated at the end of the section to monitor possible progress.

After you have taken in the answer sheets, you may use the notes in the shaded passage on the board to again explain how the number naming system works:
the millions (1 to 999 million), thousands (1 to 999 thousand) and units (1 to 999) parts are mentioned separately when we say a number.

Answers

1. (a) 217 000 379
   (b) 217 458 000
   (c) 458 379
   (d) 217 408 379
   (e) 217 458 079
   (f) 330 030 000
   (g) 303 003 000
   (h) 330 003 000
   (i) 300 030 300
   (j) 30 030 300
   (k) 3 030 300



1.3 Represent and order 9-digit numbers

The number symbol for *two hundred and sixty-four million eight hundred and sixty-seven thousand five hundred and sixty-eight* is 264 867 568.

It consists of three parts:

264 867 568

The first group of 3 digits states the **number of millions**: 264 millions.

The middle group of 3 digits states the **number of thousands**: 867 thousands.

The last group of 3 digits states the **number of hundreds, tens and units**: 5 hundreds, 6 tens and 8 units.

So, we can think of 264 867 568 as 264 million, 867 thousand and 568.
The place value expansion (expanded notation) for 264 867 568 is
200 000 000 + 60 000 000 + 4 000 000 + 800 000 + 60 000 + 7 000 + 500 + 60 + 8.

1. How much is each of the following? Write the number symbols.
   (a) 217 458 379 – 458 000
   (b) 217 458 379 – 379
   (c) 217 458 379 – 217 million
   (d) 217 458 379 – fifty thousand
   (e) 217 458 379 – three hundred
   (f) 300 million + 30 million + 30 thousand
   (g) 300 million + 3 million + 3 thousand
   (h) 300 million + 30 million + 3 thousand
   (i) 300 million + 30 thousand + 3 hundred
   (j) 30 million + 30 thousand + 3 hundred
   (k) 3 million + 30 thousand + 3 hundred

GRADE 6: MATHEMATICS [TERM 2] 123

Answers

2. (a) 5 850 456 (b) 101 054 348 (c) 32 040 375
(d) 784 618 013 (e) 7 190 003 (f) 960 864 010
(g) 110 101 100

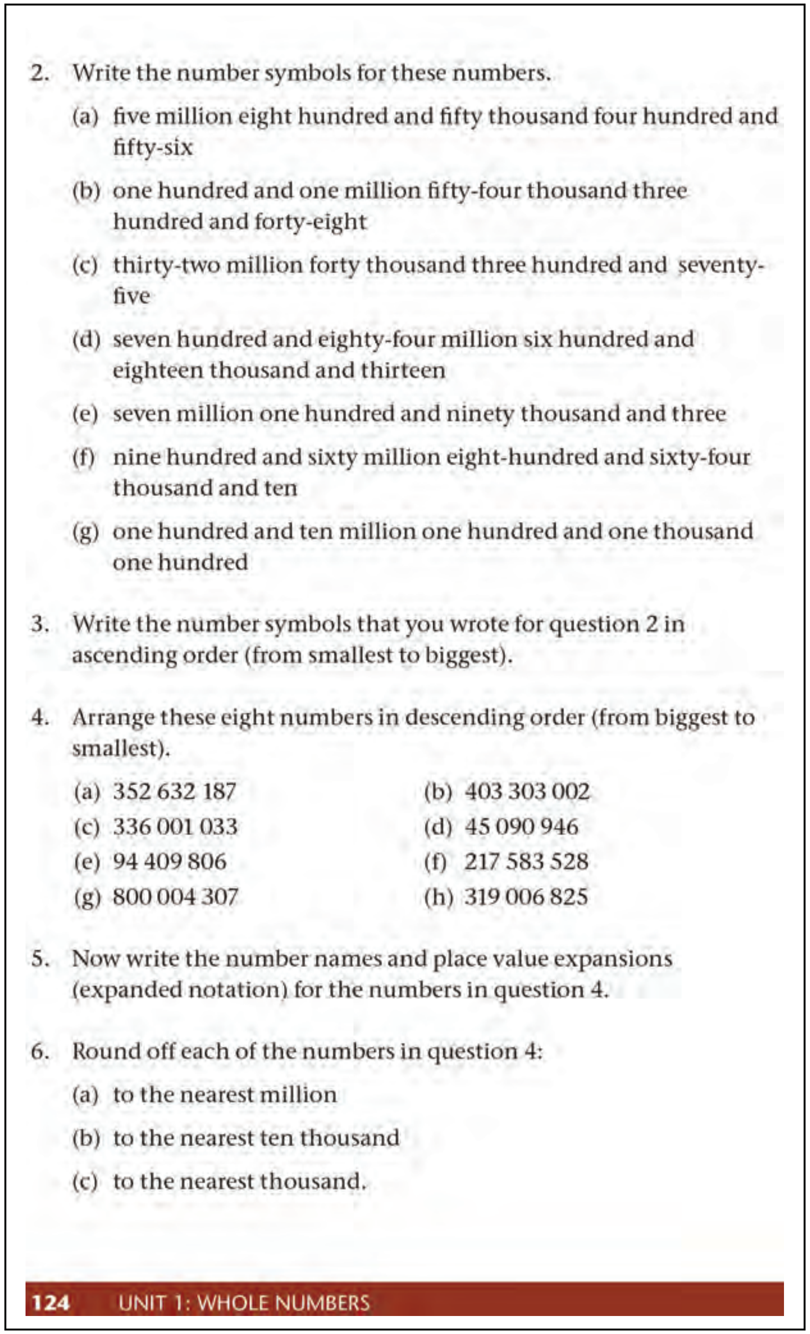
3. 5 850 456 7 190 003 32 040 375 101 054 348
110 101 100 784 618 013 960 864 010

4. 800 000 000 403 303 002 352 632 187 336 001 033
319 006 825 217 583 528 94 409 806 45 090 946

5. (a) three hundred and fifty-two million six hundred and thirty-two thousand one hundred and eighty-seven
300 000 000 + 50 000 000 + 2 000 000 + 600 000 + 30 000 + 2 000 + 100 + 80 + 7
(b) four hundred and three million three hundred and three thousand and two
400 000 000 + 3 000 000 + 300 000 + 3 000 + 2
(c) three hundred and thirty-six million one thousand and thirty-three
300 000 000 + 30 000 000 + 6 000 000 + 1 000 + 30 + 3
(d) forty-five million ninety thousand nine hundred and forty-six
40 000 000 + 5 000 000 + 90 000 + 900 + 40 + 6
(e) ninety-four million four hundred and nine thousand eight hundred and six
90 000 000 + 4 000 000 + 400 000 + 9 000 + 800 + 6
(f) two hundred and seventeen million five hundred and eighty-three thousand five hundred and twenty-eight
200 000 000 + 10 000 000 + 7 000 000 + 500 000 + 80 000 + 3 000 + 500 + 20 + 8
(g) eight hundred million four thousand three hundred and seven
800 000 000 + 4 000 + 300 + 7
(h) three hundred and nineteen million six thousand eight hundred and twenty-five
300 000 000 + 10 000 000 + 9 000 000 + 6 000 + 800 + 20 + 5

6.

Rounded off to the nearest…	**(a) million**	**(b) ten thousand**	**(c) thousand**
352 632 187	353 000 000	352 630 000	352 632 000
403 303 002	403 000 000	403 300 000	403 303 000
336 001 033	336 000 000	336 000 000	336 001 000
45 090 946	45 000 000	45 090 000	45 091 000
94 409 806	94 000 000	94 410 000	94 410 000
217 583 528	218 000 000	217 580 000	217 584 000
800 004 307	800 000 000	800 000 000	800 004 000
319 006 825	319 000 000	319 010 000	319 007 000



2. Write the number symbols for these numbers.
   (a) five million eight hundred and fifty thousand four hundred and fifty-six
   (b) one hundred and one million fifty-four thousand three hundred and forty-eight
   (c) thirty-two million forty thousand three hundred and seventy-five
   (d) seven hundred and eighty-four million six hundred and eighteen thousand and thirteen
   (e) seven million one hundred and ninety thousand and three
   (f) nine hundred and sixty million eight-hundred and sixty-four thousand and ten
   (g) one hundred and ten million one hundred and one thousand one hundred

3. Write the number symbols that you wrote for question 2 in ascending order (from smallest to biggest).

4. Arrange these eight numbers in descending order (from biggest to smallest).
   (a) 352 632 187
   (b) 403 303 002
   (c) 336 001 033
   (d) 45 090 946
   (e) 94 409 806
   (f) 217 583 528
   (g) 800 004 307
   (h) 319 006 825

5. Now write the number names and place value expansions (expanded notation) for the numbers in question 4.

6. Round off each of the numbers in question 4:
   (a) to the nearest million
   (b) to the nearest ten thousand
   (c) to the nearest thousand.

124 UNIT 1: WHOLE NUMBERS

Grade 6 Term 2 Unit 2 Whole numbers: Multiplication

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
2.1 Extending multiplication facts	Doing basic mental mathematics	125 to 126
2.2 Summarise and practise multiplication facts	Doing basic mental mathematics	127 to 128
2.3 Products and factors	Developing the concepts of factors and products	128 to 131
2.4 Multiplying with factors	Breaking down a number into factors to simplify the multiplication	132
2.5 Different ways of recording multiplication	Practising multiplication skills	133 to 135
2.6 Apply your multiplication skills	Recording work in column format	136
2.7 Mental calculation versus the calculator!	Knowing when and when not to use the calculator	137
2.8 Use estimation to check the calculator	Estimating	138
2.9 Use equivalence to check the calculator	Using equivalence when doing multiplication	139
2.10 Use inverses to check the calculator	Using multiplication and division as inverse operations	140 to 141

CAPS time allocation	5 hours
CAPS page references	13 to 15 and 241 to 243

Mathematical background

When learners multiply whole numbers, they need to follow these steps: **Example**

Step 1: Break down the numbers into place value parts: 36 × 47 = (30 + 6) × (40 + 7)

Step 2: Distribute multiplication over addition: = (30 + 6) × 40 + (30 + 6) × 7, and again:

= 30 × 40 + 6 × 40 + 30 × 7 + 6 × 7

Step 3: Calculate the small products by using known facts: = 1 200 + 240 + 210 + 42

Step 4: Add up the parts: = 1 692

Exactly the same thinking can also be recorded, in slightly less detail, in column format as shown on the right (see the notes in this regard in Section 2.5).

The two expositions given here (with number sentences and in column format respectively) do not reflect different methods of multiplication. The two expositions only reflect different ways of representing the same method.

A different method of multiplication is described on pages 132 of the Learner Book. Learners should know that they have the freedom to use any method of calculation, provided they use it correctly and can explain it.

$$\begin{array}{rl} 36 & \\ \times\ \underline{47} & \\ 42 & \ldots\ 6 \times 7 \\ 210 & \ldots\ 30 \times 7 \\ 240 & \ldots\ 6 \times 40 \\ \underline{1\ 200} & \ldots\ 30 \times 40 \\ 1\ 692 & \end{array}$$

2.1 Extending multiplication facts

Teaching guidelines

Begin this unit by improving on learners' basic multiplication facts. Write 254 × 78 on the board. After breaking up the numbers into their place value parts, ask learners to provide the answers to the simpler products, such as 4 × 8 and 200 × 70.

Mathematical notes

The two principal ways of producing the answers for simple multiplication tasks, such as 30 × 60, are:

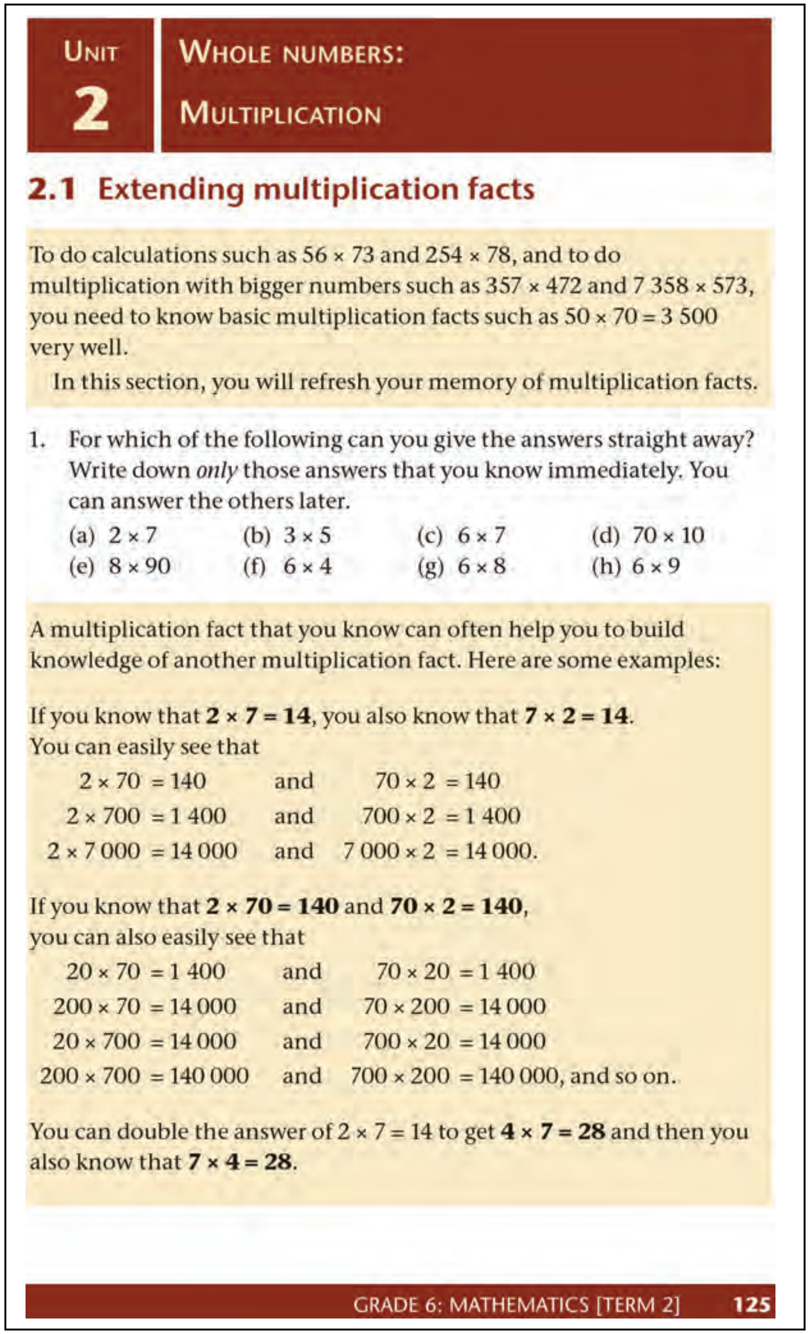
- recalling known facts, for example 3 × 6 = 18
- producing the required fact from a known fact, for example 30 × 60 = 3 × 6 × 10 × 10 = 18 × 100 = 1 800.

Three ways of producing facts from known facts are described in the second shaded passage:

- **Using the commutative property of multiplication**
  For example, a learner may recall that 3 × 7 = 21 but may have difficulty in recalling that 7 × 3 = 21. Understanding the commutative property of multiplication helps learners to identify that 7 × 3 = 21 and 3 × 7 = 21: it merely requires swapping the digits around.
- **Multiplying by ten**
  For example, if learners know that 6 × 8 = 48, then they will also know that 6 × 8 tens = 48 tens, i.e. 6 × 80 = 480.
- **Doubling**
  For example, if learners know that 7 × 5 = 35, they will also know that 14 × 5 = double 35, which is 70.

Answers

1. (a) 14 (b) 15 (c) 42 (d) 700
(e) 720 (f) 24 (g) 48 (h) 54



UNIT 2

WHOLE NUMBERS:
MULTIPLICATION

2.1 Extending multiplication facts

To do calculations such as 56 × 73 and 254 × 78, and to do multiplication with bigger numbers such as 357 × 472 and 7 358 × 573, you need to know basic multiplication facts such as 50 × 70 = 3 500 very well.

In this section, you will refresh your memory of multiplication facts.

1. For which of the following can you give the answers straight away? Write down *only* those answers that you know immediately. You can answer the others later.
(a) 2 × 7 (b) 3 × 5 (c) 6 × 7 (d) 70 × 10
(e) 8 × 90 (f) 6 × 4 (g) 6 × 8 (h) 6 × 9

A multiplication fact that you know can often help you to build knowledge of another multiplication fact. Here are some examples:

If you know that **2 × 7 = 14**, you also know that **7 × 2 = 14**.
You can easily see that
2 × 70 = 140 and 70 × 2 = 140
2 × 700 = 1 400 and 700 × 2 = 1 400
2 × 7 000 = 14 000 and 7 000 × 2 = 14 000.

If you know that **2 × 70 = 140** and **70 × 2 = 140**,
you can also easily see that
20 × 70 = 1 400 and 70 × 20 = 1 400
200 × 70 = 14 000 and 70 × 200 = 14 000
20 × 700 = 14 000 and 700 × 20 = 14 000
200 × 700 = 140 000 and 700 × 200 = 140 000, and so on.

You can double the answer of 2 × 7 = 14 to get **4 × 7 = 28** and then you also know that **7 × 4 = 28**.

Teaching guidelines

Once learners have completed question 4, use the shaded passage to explain and demonstrate a fourth way of producing facts from known facts. Here is another example: 8 × 6 means 8 sixes together, which is 48. Therefore, 1 six more is 48 + 6 = 54. So, 9 × 6 = 54.

Answers

2.

Facts that can be easily formed by building on 2 × 7 = 14				
×	7	70	700	7 000
2	14	140	1 400	14 000
20	140	1 400	14 000	140 000
200	1 400	14 000	140 000	1 400 000
2 000	14 000	140 000	1 400 000	14 000 000
4	28	280	2 800	28 000
40	280	2 800	28 000	280 000
400	2 800	28 000	280 000	2 800 000
4 000	28 000	280 000	2 800 000	28 000 000

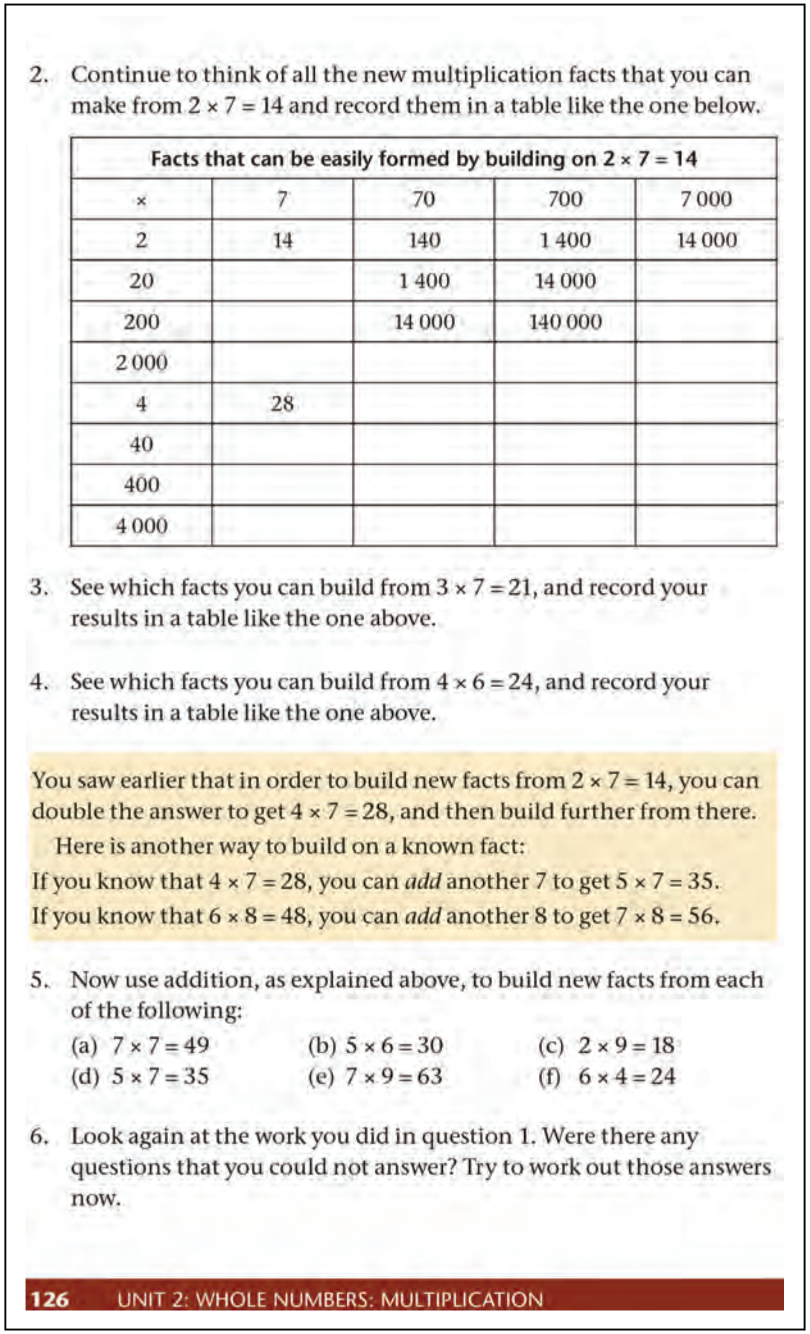
3.

Facts that can be easily formed by building on 3 × 7 = 21				
×	7	70	700	7 000
3	21	210	2 100	21 000
30	210	2 100	21 000	210 000
300	2 100	21 000	210 000	2 100 000
3 000	21 000	210 000	2 100 000	21 000 000
6	42	420	4 200	42 000
60	420	4 200	42 000	420 000
600	4 200	42 000	420 000	4 200 000
6 000	42 000	420 000	4 200 000	42 000 000

4.

Facts that can be easily formed by building on 4 × 6 = 24				
×	6	60	600	6 000
4	24	240	2 400	24 000
40	240	2 400	24 000	240 000
400	2 400	24 000	240 000	2 400 000
4 000	24 000	240 000	2 400 000	24 000 000
8	48	480	4 800	48 000
80	480	4 800	48 000	480 000
800	4 800	48 000	480 000	4 800 000
8 000	48 000	480 000	4 800 000	48 000 000

5. (a) 8 × 7 = 56; 7 × 8 = 56 (b) 6 × 6 = 36; 5 × 7 = 35 (c) 3 × 9 = 27; 2 × 8 = 16
(d) 6 × 7 = 42; 5 × 8 = 40 (e) 8 × 9 = 72; 7 × 8 = 56 (f) 7 × 4 = 28; 6 × 5 = 30



2. Continue to think of all the new multiplication facts that you can make from 2 × 7 = 14 and record them in a table like the one below.

Facts that can be easily formed by building on 2 × 7 = 14				
×	7	70	700	7 000
2	14	140	1 400	14 000
20		1 400	14 000	
200		14 000	140 000	
2 000				
4	28			
40				
400				
4 000				

3. See which facts you can build from 3 × 7 = 21, and record your results in a table like the one above.

4. See which facts you can build from 4 × 6 = 24, and record your results in a table like the one above.

You saw earlier that in order to build new facts from 2 × 7 = 14, you can double the answer to get 4 × 7 = 28, and then build further from there.

Here is another way to build on a known fact:

If you know that 4 × 7 = 28, you can *add* another 7 to get 5 × 7 = 35.

If you know that 6 × 8 = 48, you can *add* another 8 to get 7 × 8 = 56.

5. Now use addition, as explained above, to build new facts from each of the following:
(a) 7 × 7 = 49 (b) 5 × 6 = 30 (c) 2 × 9 = 18
(d) 5 × 7 = 35 (e) 7 × 9 = 63 (f) 6 × 4 = 24

6. Look again at the work you did in question 1. Were there any questions that you could not answer? Try to work out those answers now.

2.2 Summarise and practise multiplication facts

Teaching guidelines

Question 1 will help learners to recall some facts they already know and produce some facts they do not know. Remind learners that in order to produce a fact it often helps to start with a known fact. For example, if learners do not know how much 7 × 8 is, they then need to think about other multiples of 7 or 8 that they do know.

- If learners know that 7 × 7 = 49, they can easily find 7 × 8 by adding 7, i.e. 7 × 8 = 7 × 7 + 7 = 49 + 7 = 56.
- If learners know that 2 × 7 = 14, they can easily find 4 × 7 by doubling, i.e. 14 + 14 = 28, and then they can find 8 × 7 by doubling again, i.e. 8 × 7 = 28 + 28.

Write 8 × 9 on the board and ask learners to suggest another fact from which the answer for 8 × 9 can possibly be derived.

Learners may use the completed table for question 1 to access information, and then multiply by 10. For example, the answer for 40 × 90 can be found by starting with 4 × 9.

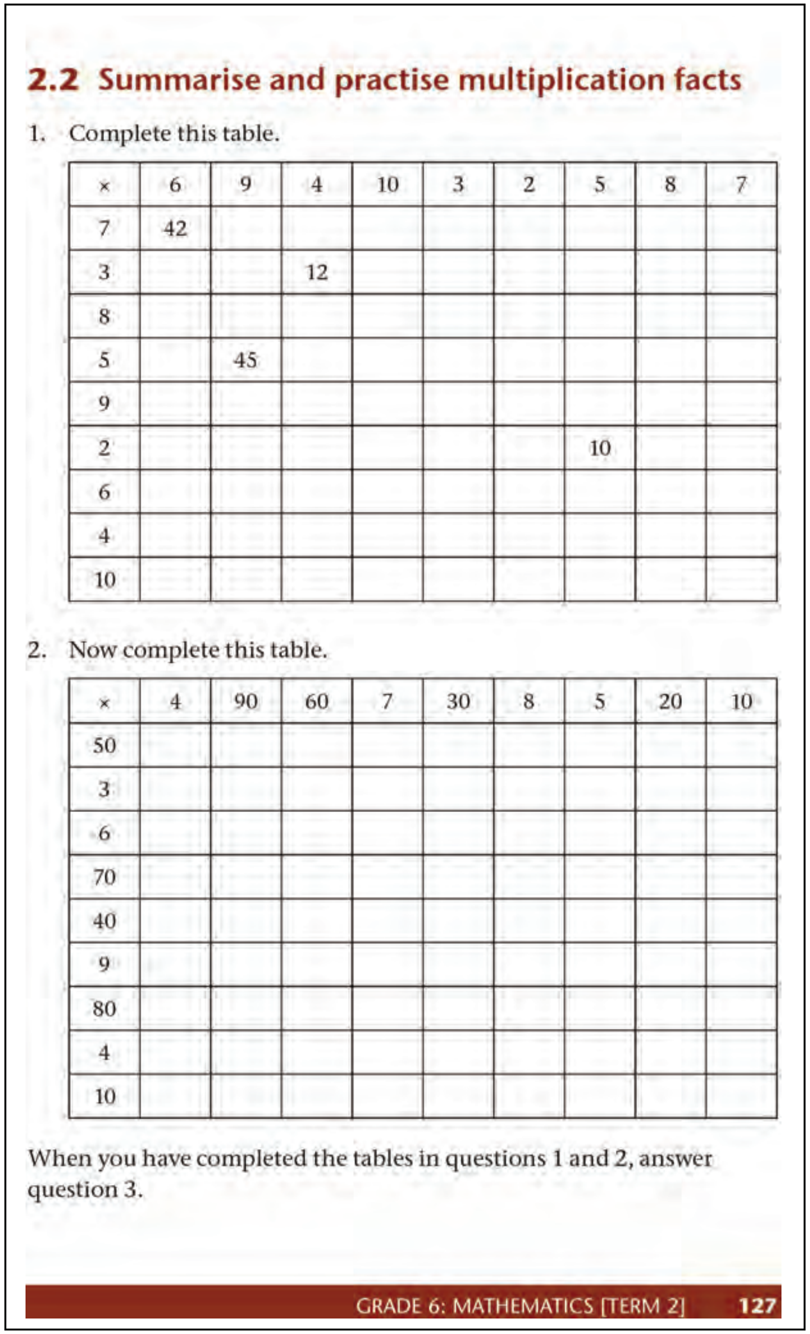
Answers

1.

×	6	9	4	10	3	2	5	8	7
7	42	63	28	70	21	14	35	56	49
3	18	27	12	30	9	6	15	24	21
8	48	72	32	80	24	16	40	64	56
5	30	45	20	50	15	10	25	40	35
9	54	81	36	90	27	18	45	72	63
2	12	18	8	20	6	4	10	16	14
6	36	54	24	60	18	12	30	48	42
4	24	36	16	40	12	8	20	32	28
10	60	90	40	100	30	20	50	80	70

2.

×	4	90	60	7	30	8	5	20	10
50	200	4 500	3 000	350	1 500	400	250	1 000	500
3	12	360	180	21	90	24	15	60	30
6	24	540	360	42	180	48	30	120	60
70	280	6 300	4 200	490	2 100	560	350	140	700
40	160	3 600	2 400	280	1 200	320	200	800	400
9	36	810	540	63	270	72	45	180	90
80	320	7 200	4 800	560	2 400	640	400	1 600	800
4	16	360	240	28	120	32	20	80	40
10	40	900	600	70	300	80	50	200	100



2.2 Summarise and practise multiplication facts

1. Complete this table.

×	6	9	4	10	3	2	5	8	7
7	42								
3			12						
8									
5		45							
9									
2							10		
6									
4									
10									

2. Now complete this table.

×	4	90	60	7	30	8	5	20	10
50									
3									
6									
70									
40									
9									
80									
4									
10									

When you have completed the tables in questions 1 and 2, answer question 3.

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Notes on questions

Learners do not have to consult the tables when they do question 3, but they may do so if needed. For example, if learners do not immediately know that 40 × 7 = 280 when they do question 3(a), they can look up the answer for 40 × 7 in question 2.

Similarly, if learners do not recall the answers for 90 × 80 and 7 × 80 when they do question 3(d), they can look it up in the table they completed in question 2.

The same applies to question 5. For example, when learners do question 5(b), they may look up 900 × 70 and 40 × 70 in the completed table for question 4.

Questions 3 and 5 are intended to promote learners' awareness of the use of known or easy-to-produce facts in multiplication with multi-digit numbers, and hence to make them realise how important it is that they are fluent with respect to basic multiplication facts.

You may extend question 5 to 476 × 87.

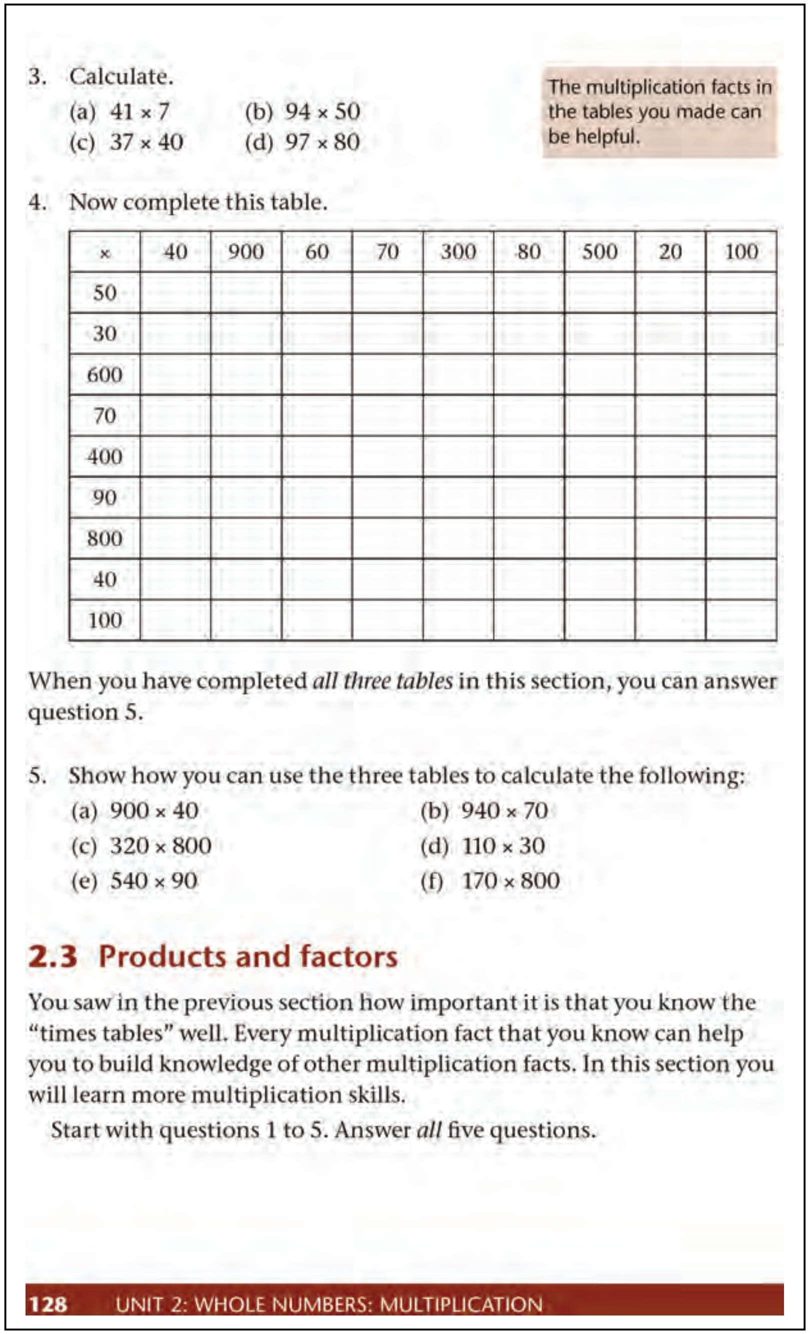
Answers

3. (a) 287 (b) 4 700 (c) 1 480 (d) 7 760

4.

×	40	900	60	70	300	80	500	20	100
50	2 000	45 000	3 000	3 500	15 000	4 000	25 000	1 000	5 000
30	1 200	27 000	1 800	2 100	9 000	2 400	15 000	600	3 000
600	24 000	540 000	36 000	42 000	180 000	48 000	300 000	12 000	60 000
70	2 800	63 000	4 200	4 900	21 000	5 600	35 000	1 400	7 000
400	16 000	360 000	24 000	28 000	120 000	32 000	200 000	8 000	40 000
90	3 600	81 000	5 400	6 300	27 000	7 200	45 000	1 800	9 000
800	32 000	720 000	48 000	56 000	240 000	64 000	400 000	16 000	80 000
40	1 600	36 000	2 400	2 800	12 000	3 200	20 000	800	4 000
100	4 000	90 000	6 000	7 000	30 000	8 000	50 000	2 000	10 000

5. (a) 900 × 40 = 36 000 (from the 3rd table, i.e. the table in question 4)
   (b) 940 × 70 = 900 × 70 + 40 × 70 = 63 000 + 2 800 = 65 800 (from the 3rd table)
   (c) 320 × 800 = 300 × 800 + 20 × 800 = 240 000 + 16 000 = 256 000 (from the 3rd table)
   (d) 110 × 30 = 100 × 30 + 10 × 30 = 3 000 + 300 = 3 300 (from the 3rd and 2nd tables)
   (e) 540 × 90 = 500 × 90 + 40 × 90 = 45 000 + 3 600 = 48 600 (from the 3rd table)
   (f) 170 × 800 = 100 × 800 + 70 × 800 = 80 000 + 56 000 = 136 000 (from the 3rd table)



3. Calculate.
   (a) 41 × 7 (b) 94 × 50
   (c) 37 × 40 (d) 97 × 80

The multiplication facts in the tables you made can be helpful.

4. Now complete this table.

×	40	900	60	70	300	80	500	20	100
50									
30									
600									
70									
400									
90									
800									
40									
100									

When you have completed *all three tables* in this section, you can answer question 5.

5. Show how you can use the three tables to calculate the following:
   (a) 900 × 40 (b) 940 × 70
   (c) 320 × 800 (d) 110 × 30
   (e) 540 × 90 (f) 170 × 800

2.3 Products and factors

You saw in the previous section how important it is that you know the "times tables" well. Every multiplication fact that you know can help you to build knowledge of other multiplication facts. In this section you will learn more multiplication skills.

Start with questions 1 to 5. Answer *all* five questions.

2.3 Products and factors

Mathematical notes

When three or more numbers are to be multiplied, this can be done in any order. This is the so-called associative property of multiplication (learners need *not* be burdened with the term “associative”). For example, all of the following calculation plans will produce the same result, namely 60:

$$(5 \times 4) \times 3 \qquad (5 \times 3) \times 4 \qquad (3 \times 4) \times 5$$

Many people realise this intuitively. Note that the above are three distinctly different calculation plans, with 20 × 3, 15 × 4 and 12 × 5 as second steps in the three cases respectively. But (5 × 4) × 3, (4 × 5) × 3, 3 × (5 × 4) and 3 × (4 × 5) are all different representations of exactly the same plan: they all specify exactly the same sequence of actions.

The associative property of multiplication (and of addition) relates to the order in which **operations** (multiplications or additions) **are executed** when a calculation plan with three or more multiplications (additions) is implemented. The commutative property relates to the order in which **numbers are considered** when a single multiplication or addition is performed.

Teaching guidelines

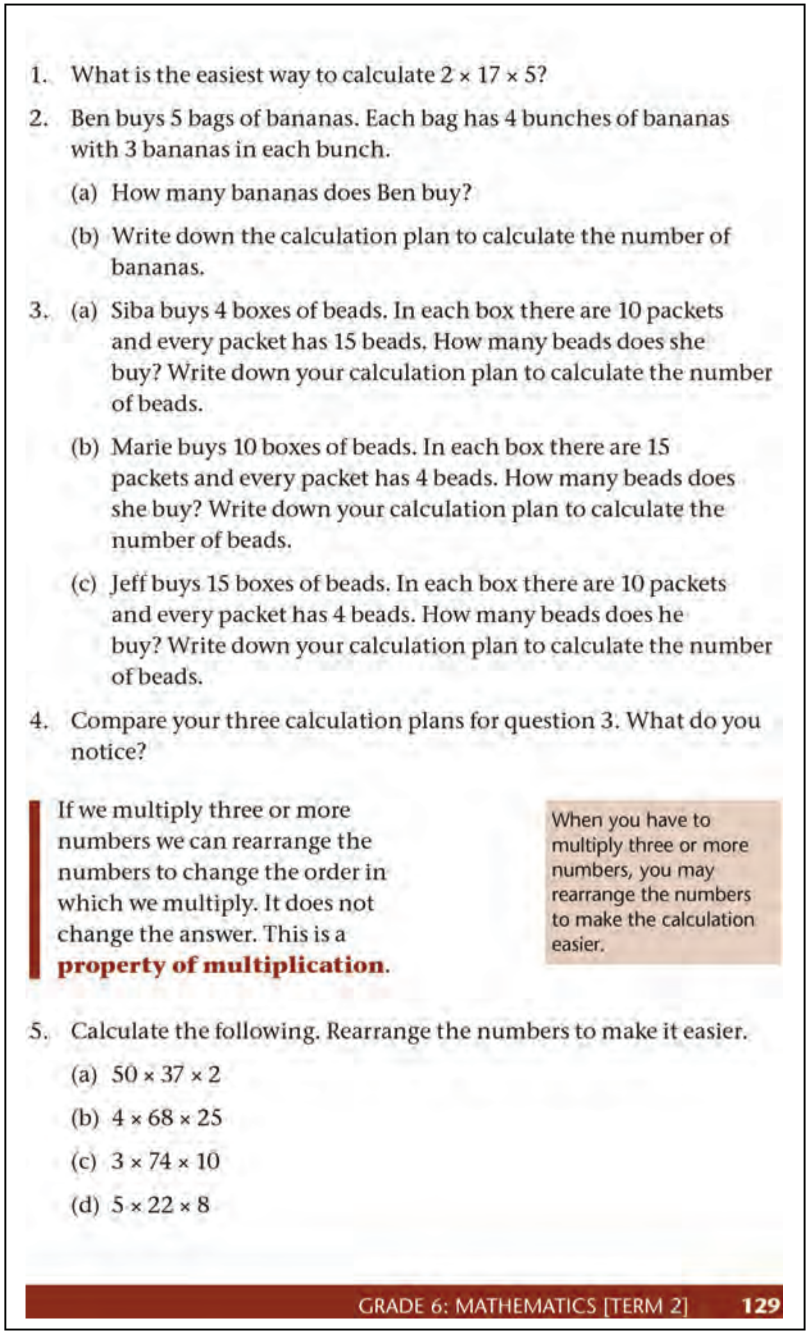
Questions 1 to 5 are intended to promote:

- awareness of the fact that when three or more numbers are to be multiplied, this can be done in any order, and
- the use of this property of multiplication to simplify calculations.

Learners’ responses to question 1 will be very informative.

Learners who state 2 × 5 × 17 or 2 × 5 = 10 followed by 10 × 17 clearly already have an intuitive awareness of the associativity of multiplication. They exhibit the confidence to rearrange the order in which multiplications are performed when evaluating a product with three or more factors. While these learners will not necessarily learn anything new by doing questions 2 to 5, the work will provide them with useful practice.

Some of the learners who do 2 × 17 = 34, then 34 × 5 = 170 for question 1 may also be aware of the associativity of multiplication, but may not think of utilising it here because they find the calculations in the given order quite easy anyway (doubling, then multiplication by 5). However, other learners who do 2 × 17 = 34, then 34 × 5 = 170 may be unaware of associativity and hence the possibility of replacing 2 × 17 × 5 with the equivalent calculation plan 2 × 5 × 17. For these learners, questions 2 to 5 may be an important opportunity to become aware of the associativity of multiplication and how it may be utilised to simplify calculations.



1. What is the easiest way to calculate 2 × 17 × 5?
2. Ben buys 5 bags of bananas. Each bag has 4 bunches of bananas with 3 bananas in each bunch.
   (a) How many bananas does Ben buy?
   (b) Write down the calculation plan to calculate the number of bananas.
3. (a) Siba buys 4 boxes of beads. In each box there are 10 packets and every packet has 15 beads. How many beads does she buy? Write down your calculation plan to calculate the number of beads.
   (b) Marie buys 10 boxes of beads. In each box there are 15 packets and every packet has 4 beads. How many beads does she buy? Write down your calculation plan to calculate the number of beads.
   (c) Jeff buys 15 boxes of beads. In each box there are 10 packets and every packet has 4 beads. How many beads does he buy? Write down your calculation plan to calculate the number of beads.
4. Compare your three calculation plans for question 3. What do you notice?

If we multiply three or more numbers we can rearrange the numbers to change the order in which we multiply. It does not change the answer. This is a **property of multiplication**.

When you have to multiply three or more numbers, you may rearrange the numbers to make the calculation easier.

5. Calculate the following. Rearrange the numbers to make it easier.
   (a) 50 × 37 × 2
   (b) 4 × 68 × 25
   (c) 3 × 74 × 10
   (d) 5 × 22 × 8

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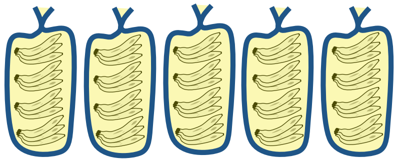
Teaching guidelines *(continued)*

The four contextual questions serve to promote awareness of the associative property of multiplication, i. e. the fact that it is immaterial in which order multiplications are executed in a string of three or more multiplications. In none of the four scenarios does the context indicate which multiplication should be done first.

In question 2 for instance, it is clearly immaterial whether you

- first calculate how many bunches of bananas there are (5 × 4 = 20) and then multiply by 3, or
- first calculate how many bananas there are in each bag (3 × 4 = 12) and then multiply by 5.



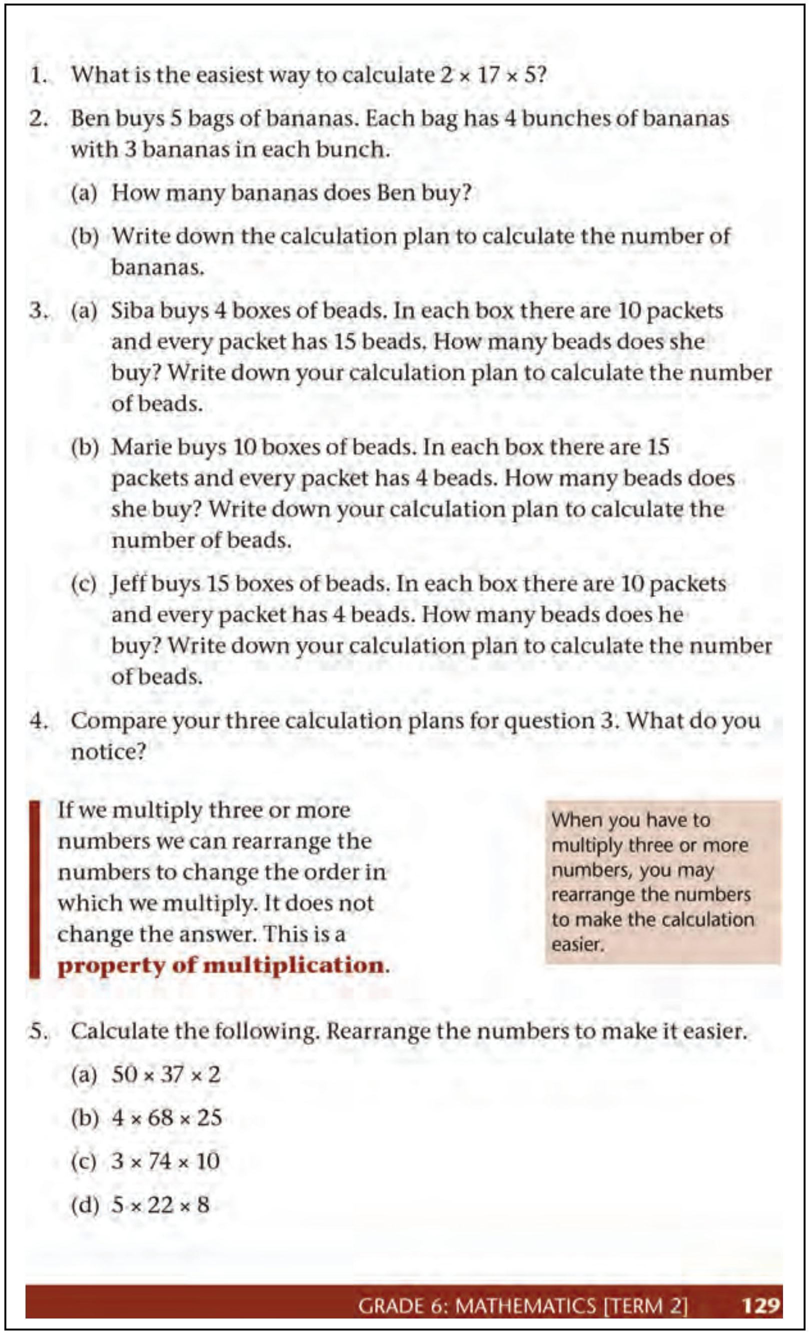
Hence the different calculations plans 5 × 4 × 3 and 3 × 4 × 5 are equally appropriate representations of the numerical aspects of the situation. To promote awareness of associativity, you may write the above two alternative ways of finding the number of bananas on the board once learners have completed the question. To further promote awareness of associativity, you may ask learners whether 3 × 5 × 4, i.e. 3 × 5 = 15 followed by 15 × 4 will produce the same answer as 5 × 4 × 3 and 3 × 4 × 5.

Note that questions 3(a), (b) and (c) require learners to write down their calculation plans. You may suggest that learners write their plans for all three questions before they do the calculations. The descriptions of the contexts may lead many learners to write 4 × 15 × 10, 10 × 15 × 4 and 15 × 10 × 4 for (a), (b) and (c) respectively, though some learners may write 15 × 10 × 4, 4 × 15 × 10 and 4 × 10 × 15. Some learners may recognise that the same calculation plan can work for all three scenarios, and write only one plan. Write all these calculation plans on the board (some are actually the same) and ask learners to predict whether the answers for the different plans will be the same or different.

In question 5 you may ask learners why they choose a specific arrangement in each case.

Answers

1. 2 × 5 = 10 and 10 × 17 = 170
2. (a) 60 bananas (b) 5 × 4 × 3; or 3 × 5 × 4; or 3 × 4 × 5
3. (a) 4 × 15 × 10 = 60 × 10 = 600 beads (b) 10 × 15 × 4 = 150 × 4 = 600 beads
   (c) 15 × 10 × 4 = 150 × 4 = 600 beads
4. The answers are all the same. It does not matter in what order you multiply the numbers as the same numbers are being multiplied.
5. (a) 50 × 2 × 37 = 100 × 37 = 3 700 (b) 4 × 25 × 68 = 100 × 68 = 6 800
   (c) 3 × 74 × 10 = 222 × 10 = 2 220 (d) 5 × 8 × 22 = 40 × 22 = 880



1. What is the easiest way to calculate 2 × 17 × 5?
2. Ben buys 5 bags of bananas. Each bag has 4 bunches of bananas with 3 bananas in each bunch.
   (a) How many bananas does Ben buy?
   (b) Write down the calculation plan to calculate the number of bananas.
3. (a) Siba buys 4 boxes of beads. In each box there are 10 packets and every packet has 15 beads. How many beads does she buy? Write down your calculation plan to calculate the number of beads.
   (b) Marie buys 10 boxes of beads. In each box there are 15 packets and every packet has 4 beads. How many beads does she buy? Write down your calculation plan to calculate the number of beads.
   (c) Jeff buys 15 boxes of beads. In each box there are 10 packets and every packet has 4 beads. How many beads does he buy? Write down your calculation plan to calculate the number of beads.
4. Compare your three calculation plans for question 3. What do you notice?

If we multiply three or more numbers we can rearrange the numbers to change the order in which we multiply. It does not change the answer. This is a **property of multiplication**.

When you have to multiply three or more numbers, you may rearrange the numbers to make the calculation easier.

5. Calculate the following. Rearrange the numbers to make it easier.
   (a) 50 × 37 × 2
   (b) 4 × 68 × 25
   (c) 3 × 74 × 10
   (d) 5 × 22 × 8

Teaching guidelines

Questions 6 to 11 serve two purposes:

- to consolidate and extend knowledge of factors, and
- to provide practise in Mental Mathematics (basic multiplication facts).

Question 10 is intended as a challenge for learners who work much faster than the majority. It is quite demanding.

Answers

6. No, they are not. Many other numbers are factors of 900, for example: 20, 30, 45, 60, 90, 300, 450.

7.–8. There are many possibilities, for example:

900 = 2 × 10 × 45; 900 = 6 × 30 × 5; 900 = 9 × 4 × 25; 900 = 10 × 18 × 5; 900 = 3 × 20 × 15;
900 = 50 × 2 × 9; 900 = 4 × 3 × 75

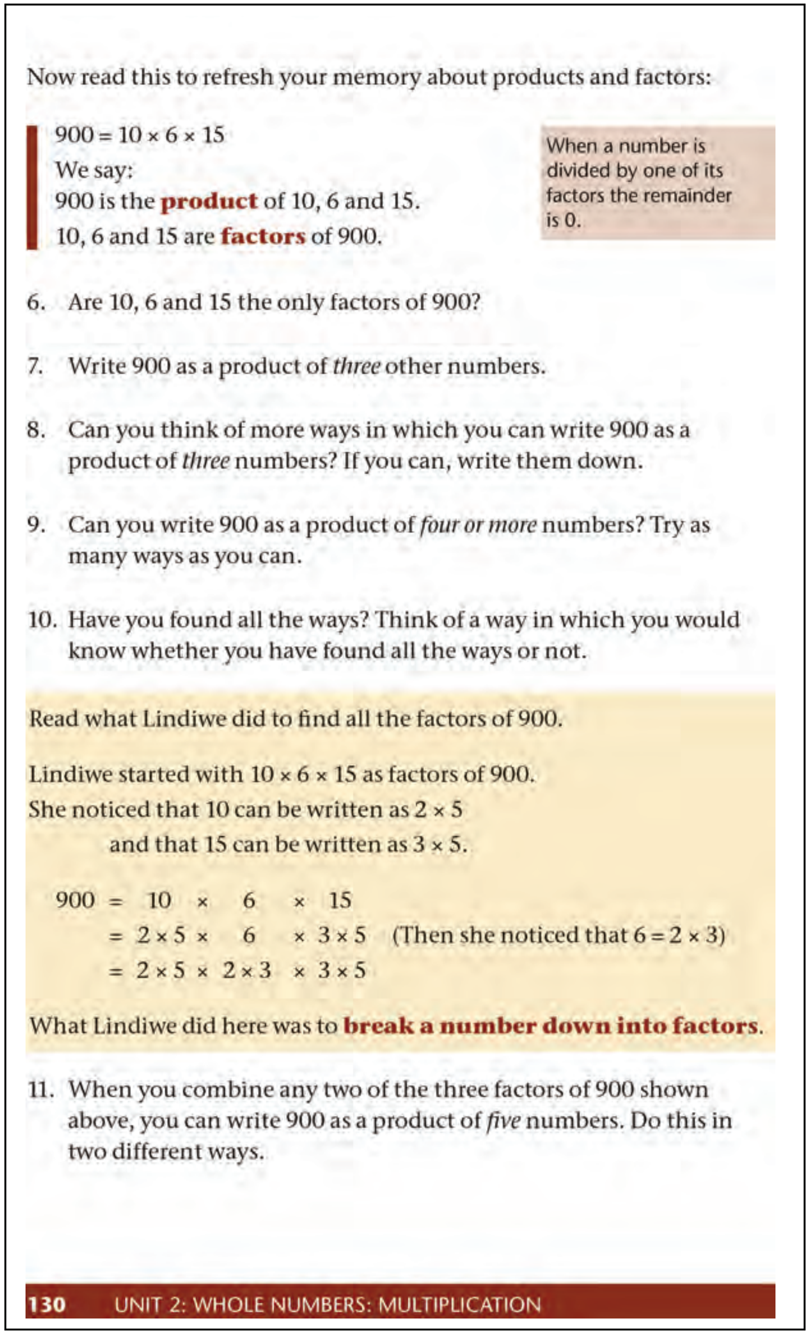
9. There are many possibilities, for example:

900 = 2 × 3 × 10 × 15; 900 = 4 × 3 × 5 × 15; 900 = 25 × 3 × 2 × 6; 900 = 2 × 9 × 10 × 5

10. See next page.

11. Consider learners' answers, for example:

900 = 2 × 3 × 2 × 5 × 15; 900 = 2 × 2 × 3 × 5 × 15; 900 = 5 × 5 × 3 × 2 × 6;
900 = 2 × 2 × 5 × 5 × 9



Now read this to refresh your memory about products and factors:

900 = 10 × 6 × 15
We say:
900 is the **product** of 10, 6 and 15.
10, 6 and 15 are **factors** of 900.

When a number is divided by one of its factors the remainder is 0.

6. Are 10, 6 and 15 the only factors of 900?

7. Write 900 as a product of *three* other numbers.

8. Can you think of more ways in which you can write 900 as a product of *three* numbers? If you can, write them down.

9. Can you write 900 as a product of *four or more* numbers? Try as many ways as you can.

10. Have you found all the ways? Think of a way in which you would know whether you have found all the ways or not.

Read what Lindiwe did to find all the factors of 900.

Lindiwe started with 10 × 6 × 15 as factors of 900.
She noticed that 10 can be written as 2 × 5
and that 15 can be written as 3 × 5.

$$\begin{aligned} 900 &= 10 \times 6 \times 15 \\ &= 2 \times 5 \times 6 \times 3 \times 5 \quad \text{(Then she noticed that } 6 = 2 \times 3\text{)} \\ &= 2 \times 5 \times 2 \times 3 \times 3 \times 5 \end{aligned}$$

What Lindiwe did here was to **break a number down into factors**.

11. When you combine any two of the three factors of 900 shown above, you can write 900 as a product of *five* numbers. Do this in two different ways.

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Answer for question 10

There is only one way in which 900 can be expressed as a product of 6 numbers: 900 = 2 × 2 × 3 × 3 × 5 × 5 900 can be expressed as a product of 5 numbers in these 6 different ways only: A. (2 × 2) × 3 × 3 × 5 × 5 = 4 × 3 × 3 × 5 × 5 B. (2 × 3) × 2 × 3 × 5 × 5 = 6 × 2 × 3 × 5 × 5 C. (2 × 5) × 2 × 3 × 3 × 5 = 10 × 2 × 3 × 3 × 5 D. (3 × 3) × 2 × 2 × 5 × 5 = 9 × 2 × 2 × 5 × 5 E. (3 × 5) × 2 × 2 × 3 × 5 = 15 × 2 × 2 × 3 × 5 F. (5 × 5) × 2 × 2 × 3 × 3 = 25 × 2 × 2 × 3 × 3	Any two of the factors of the product in A alongside can be combined to express 900 as a product of 4 numbers: (4 × 3) × 3 × 5 × 5 = 12 × 3 × 5 × 5 (4 × 5) × 3 × 3 × 5 = 20 × 3 × 3 × 5 (3 × 5) × 4 × 3 × 5 = 15 × 4 × 3 × 5 (3 × 3) × 5 × 5 × 4 = 9 × 5 × 5 × 4 (5 × 5) × 3 × 3 × 4 = 25 × 3 × 3 × 4 Similarly, the products D and F (each with two repeated factors like A) provide five different ways of expressing 900 as a product of 4 numbers. Hence the products A, D and F account for 15 different ways of expressing 900 as a product of 4 numbers.
The products B, C and E have the same structure in the sense that they have only one repeated factor each. It is hence logical to assume that each of these products will provide the same number of different ways to express 900 as a product of four different numbers. Any two of the factors of the product in B can be combined to express 900 as a product of 4 numbers: (6 × 2) × 3 × 5 × 5 = 12 × 3 × 5 × 5 (6 × 3) × 2 × 5 × 5 = 18 × 2 × 5 × 5 (6 × 5) × 3 × 2 × 5 = 30 × 3 × 2 × 5 (2 × 3) × 5 × 5 × 6 = 6 × 5 × 5 × 6 (2 × 5) × 5 × 3 × 6 = 10 × 5 × 3 × 6 (3 × 5) × 5 × 2 × 6 = 15 × 5 × 2 × 6 (5 × 5) × 3 × 2 × 6 = 25 × 3 × 2 × 6 Similarly, the products C and E will each provide seven different ways of expressing 900 as a product of 4 numbers.	**In summary:** 900 can be expressed in **one** way only as a product of 6 numbers: 2 × 2 × 3 × 3 × 5 × 5 (3 factors, each repeated twice). 900 can be expressed in **six** ways as a product of 5 numbers (three of these ways have two repeated factors; the other three have one repeated factor only). Each product of 5 numbers with two repeated factors that yield 900 produces five ways of expressing 900 as a product of 4 numbers, hence **15** ways in total. Each product of 5 numbers with one repeated factor that yield 900 produces seven ways of expressing 900 as a product of 4 numbers, hence **21** ways in total. In total there are 1 + 6 + 15 + 21 = 43 ways in which 900 can be expressed as a product of four or more numbers.

Possible further questions

1. In how many different ways can 11 025 (3 × 3 × 5 × 5 × 7 × 7) be expressed as a product of four or more different numbers?
2. Which other numbers can be expressed in the same number of ways as a product of four or more different numbers?
3. In how many different ways can 2 × 3 × 5 × 7 × 11 × 13 be expressed as a product of four or more different numbers?
4. In how many different ways can 2 × 3 × 5 × 7 be expressed as a product of two or more different numbers?
5. In how many different ways can 2 × 2 × 3 × 3 be expressed as a product of two or more different numbers?

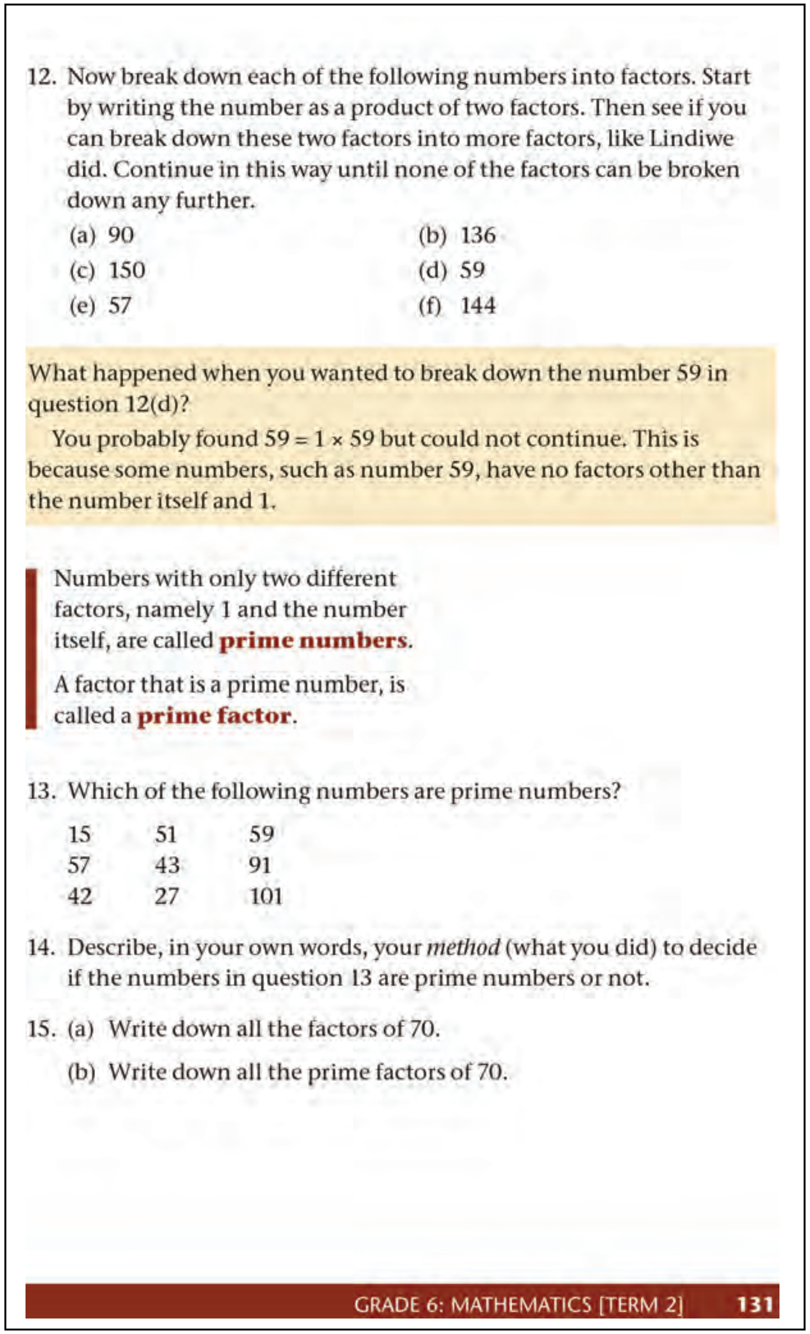
Answers

12. (a) $90 = 2 \times 3 \times 3 \times 5$ (b) $136 = 2 \times 2 \times 2 \times 17$
    (c) $150 = 2 \times 3 \times 5 \times 5$ (d) $59 = 1 \times 59$
    (e) $57 = 3 \times 19$ (f) $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
13. 59; 43; 101
14. Learners describe their methods in their own words.
15. (a) 1; 2; 5; 7; 10; 14; 35; 70
    (b) 2; 5; 7

Mathematical notes

Note that there are also other ways of evaluating a product such as 36 × 47, for example:

- $$\begin{aligned} 36 \times 47 = 10 \times 47 + 10 \times 47 + 10 \times 47 + 6 \times 47 &= 470 + 470 + 470 + 6 \times 50 - 18 \\ &= 3 \times 500 - 3 \times 30 + 6 \times 50 - 6 \times 3 \\ &= 1\,500 - 90 + 300 - 18 \\ &= 1\,410 + 282 \\ &= 1\,692 \end{aligned}$$
- $36 \times 47 = 2 \times 2 \times 3 \times 3 \times 47 = 2 \times 2 \times 3 \times 141 = 2 \times 2 \times 423 = 2 \times 846 = 1\,692$
- $36 \times 47 = 36 \times 50 - 36 \times 3 = (36 \times 100) \div 2 - 36 \times 3 = 1\,800 - 108 = 1\,692$



12. Now break down each of the following numbers into factors. Start by writing the number as a product of two factors. Then see if you can break down these two factors into more factors, like Lindiwe did. Continue in this way until none of the factors can be broken down any further.
    (a) 90 (b) 136
    (c) 150 (d) 59
    (e) 57 (f) 144

What happened when you wanted to break down the number 59 in question 12(d)?

You probably found 59 = 1 × 59 but could not continue. This is because some numbers, such as number 59, have no factors other than the number itself and 1.

Numbers with only two different factors, namely 1 and the number itself, are called **prime numbers**.

A factor that is a prime number, is called a **prime factor**.

13. Which of the following numbers are prime numbers?

15 51 59
57 43 91
42 27 101

14. Describe, in your own words, your *method* (what you did) to decide if the numbers in question 13 are prime numbers or not.
15. (a) Write down all the factors of 70.
    (b) Write down all the prime factors of 70.

2.4 Multiplying with factors

Mathematical notes

In the breaking-down and building-up method of multiplication, **both** numbers are broken down into place value parts to replace the product of two numbers with a sum of simple products, for example:

678×42 is replaced by $600 \times 40 + 600 \times 2 + 70 \times 40 + 70 \times 2 + 8 \times 40 + 8 \times 2$.

In the method described in the shaded passage, **one** of the numbers is broken down into prime factors, for example:

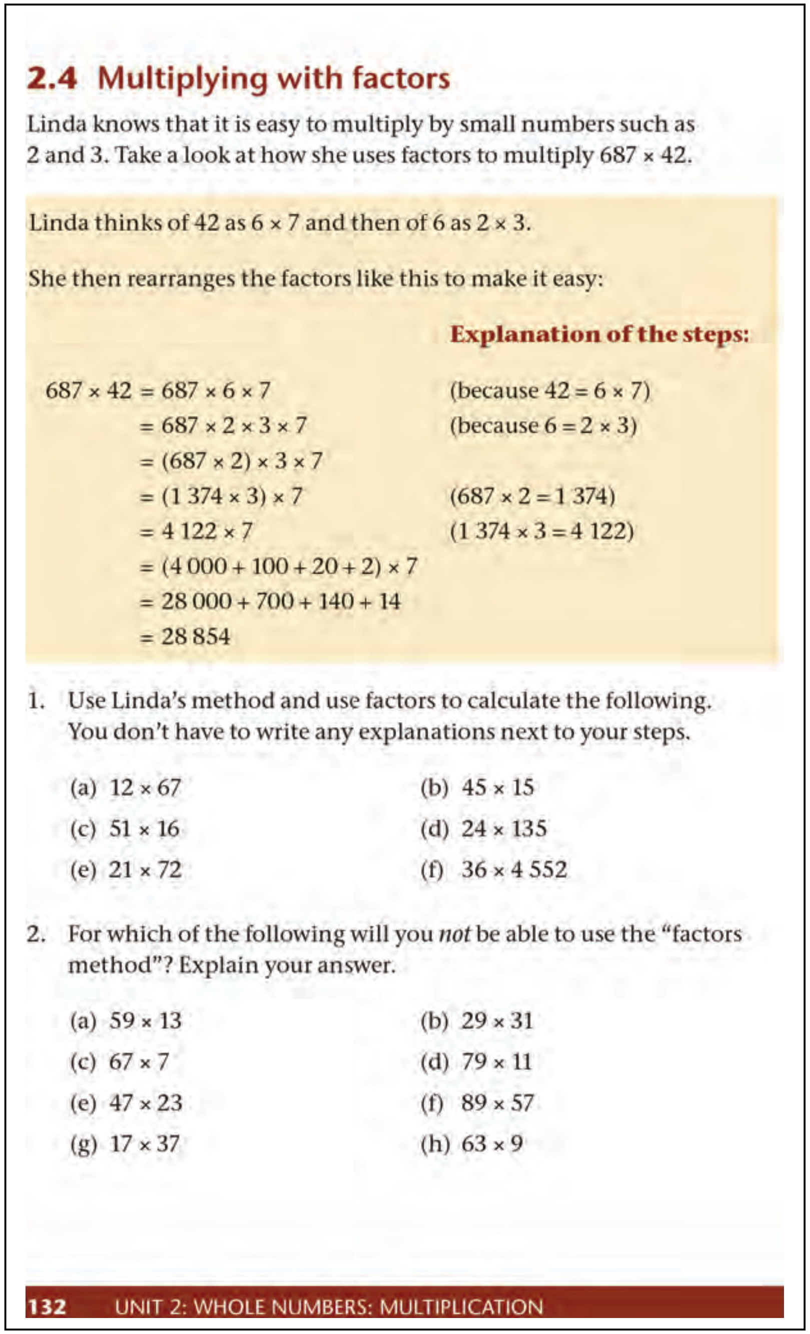
678×42 is replaced by $687 \times (2 \times 3 \times 7) = 687 \times 2 + 687 \times 3 + 687 \times 7$

Answers

1. (a) $67 \times 2 \times 2 \times 3 = 804$
   (b) $45 \times 15 = 45 \times 5 \times 3 = 675$
   (c) $51 \times 16 = 51 \times 2 \times 2 \times 2 \times 2 = 816$
   (d) $24 \times 135 = 135 \times 3 \times 2 \times 2 \times 2 = 3\,240$
   (e) $21 \times 72 = 21 \times 3 \times 3 \times 2 \times 2 \times 2 = 1\,512$
   (f) $36 \times 4\,552 = 4\,552 \times 3 \times 3 \times 2 \times 2 = 163\,872$

   Note: There are different possible ways, depending on which factors are chosen for the first step.

2. (a) 59×13 (b) 29×31 (c) 67×7
   (d) 79×11 (e) 47×23 (g) 17×37
   In each case, both numbers are prime factors.



2.4 Multiplying with factors

Linda knows that it is easy to multiply by small numbers such as 2 and 3. Take a look at how she uses factors to multiply 687×42.

Linda thinks of 42 as 6×7 and then of 6 as 2×3.

She then rearranges the factors like this to make it easy:

	Explanation of the steps:
$687 \times 42 = 687 \times 6 \times 7$	(because $42 = 6 \times 7$)
$= 687 \times 2 \times 3 \times 7$	(because $6 = 2 \times 3$)
$= (687 \times 2) \times 3 \times 7$	
$= (1\,374 \times 3) \times 7$	($687 \times 2 = 1\,374$)
$= 4\,122 \times 7$	($1\,374 \times 3 = 4\,122$)
$= (4\,000 + 100 + 20 + 2) \times 7$	
$= 28\,000 + 700 + 140 + 14$	
$= 28\,854$	

1. Use Linda's method and use factors to calculate the following. You don't have to write any explanations next to your steps.
   (a) 12×67 (b) 45×15
   (c) 51×16 (d) 24×135
   (e) 21×72 (f) $36 \times 4\,552$

2. For which of the following will you *not* be able to use the "factors method"? Explain your answer.
   (a) 59×13 (b) 29×31
   (c) 67×7 (d) 79×11
   (e) 47×23 (f) 89×57
   (g) 17×37 (h) 63×9

2.5 Different ways of recording multiplication

Mathematical background

While the column format is highly useful in terms of the amount of writing to be done, it has the disadvantage that the actual thinking steps are not recorded. For example:

Recording in detail

$$\begin{aligned}34 \times 63 &= (30 + 4) \times (60 + 3)\\ &= 30 \times (60 + 3) + 4 \times (60 + 3)\\ &= 30 \times 60 + 30 \times 3 + 4 \times 60 + 4 \times 3\\ &= 1\ 800 + 90 + 240 + 12\\ &= 2\ 142\end{aligned}$$

Recording in columns

$$\begin{array}{r} 34\\ \underline{\times\ 63}\\ 12\\ 90\\ 240\\ +\ \underline{1\ 800}\\ 2\ 142\end{array} \qquad \begin{array}{r} 34\\ \underline{\times\ 63}\\ 102\\ \underline{+\ 2\ 040}\\ 2\ 142\end{array}$$

The detailed recording on the left-hand side shows the place value parts into which the two numbers have been broken down, as well as which numbers were multiplied to obtain the parts 12, 90, 240 and 1 800 of the answer. In the column format, the place value parts and the numbers that were multiplied to form the part answers are not shown.

The detailed recording shows how the distributive property is applied. This is not shown in the column format.

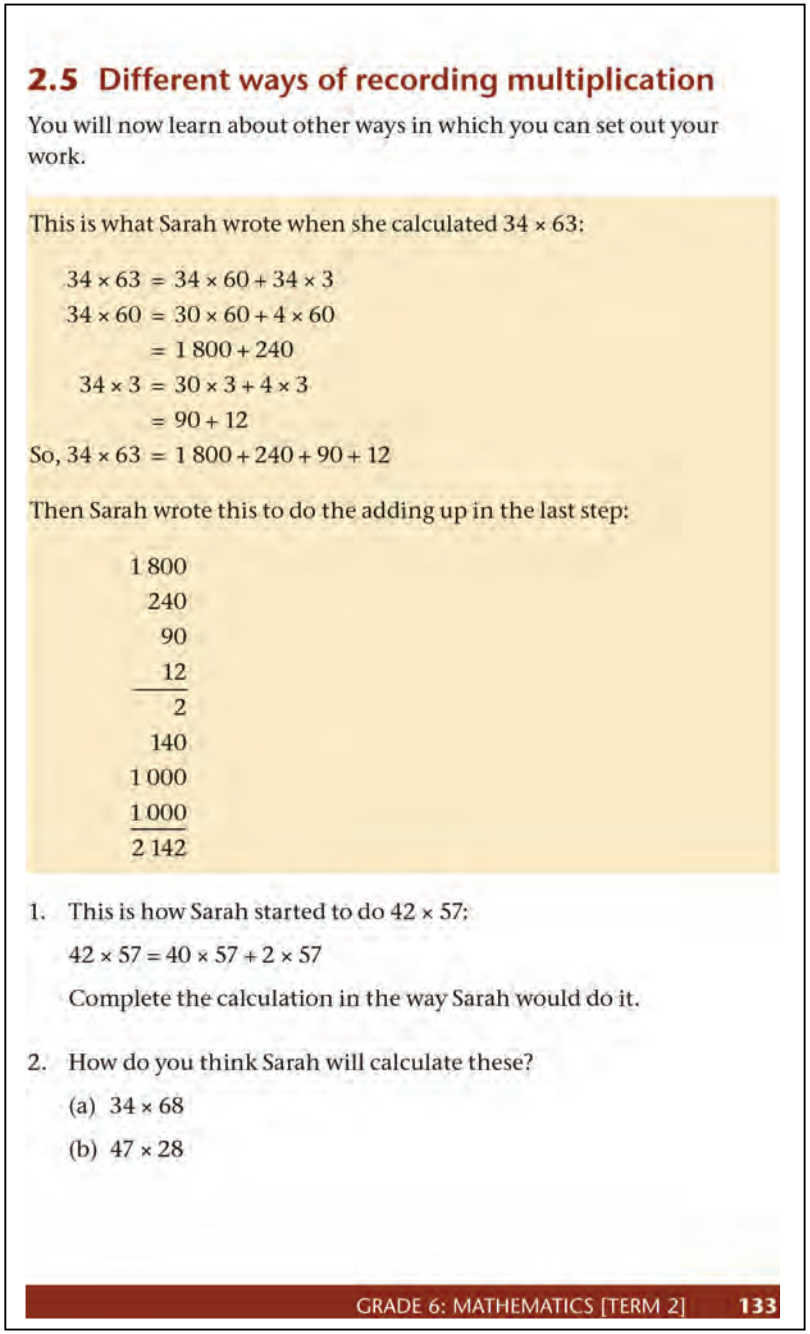
Taken as a whole, the **logic** of the procedure is explicitly shown in the detailed recording, but is hidden in the column format.

One of the major reasons why learners are required to provide a detailed recording of their multiplication work up to the first half of Grade 6, is to promote understanding of the logic of the process. Understanding the logic of calculation methods provides a powerful basis for making sense of algebra in the Senior Phase. It promotes a view of mathematics as an understandable logic-based activity, and provides learners with an appreciation of their own logical and mathematical abilities.

A few decades ago, when cheap hand-held calculators were not available, it was deemed necessary to introduce learners to the column format so that they could become proficient in doing multiplication with a minimum of writing (and thinking). Today, however, multiplication is normally done with calculators – that is, in life out of school – and the need for proficiency in doing multiplication "by hand" has largely disappeared.

Answers

See next page.



2.5 Different ways of recording multiplication

You will now learn about other ways in which you can set out your work.

This is what Sarah wrote when she calculated 34 × 63:

$$\begin{aligned}34 \times 63 &= 34 \times 60 + 34 \times 3\\ 34 \times 60 &= 30 \times 60 + 4 \times 60\\ &= 1\ 800 + 240\\ 34 \times 3 &= 30 \times 3 + 4 \times 3\\ &= 90 + 12\\ \text{So, } 34 \times 63 &= 1\ 800 + 240 + 90 + 12\end{aligned}$$

Then Sarah wrote this to do the adding up in the last step:

$$\begin{array}{r} 1\ 800\\ 240\\ 90\\ \underline{12}\\ 2\\ 140\\ 1\ 000\\ \underline{1\ 000}\\ 2\ 142\end{array}$$

1. This is how Sarah started to do 42 × 57:

   42 × 57 = 40 × 57 + 2 × 57

   Complete the calculation in the way Sarah would do it.

2. How do you think Sarah will calculate these?

   (a) 34 × 68

   (b) 47 × 28

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Answers

1. 42 × 57 = 40 × 57 + 2 × 57

   40 × 57 = 40 × 50 + 40 × 7 and 2 × 57 = 2 × 50 + 2 × 7
   = 2 000 + 280 = 100 + 14

   So, 42 × 57 = 2 000 + 280 + 100 + 14

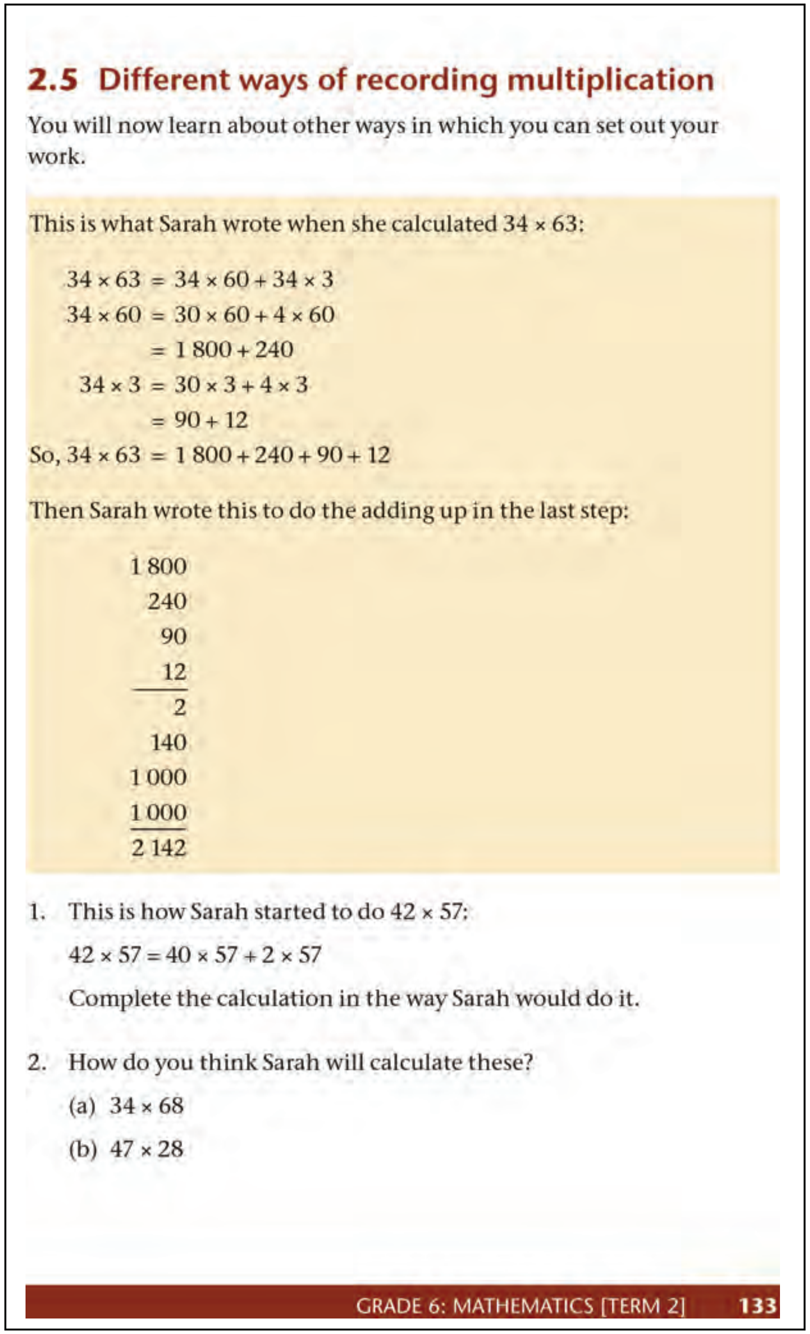
   2 000
   280
   100
   14
   4
   90
   300
   2 000
   2 394

2. (a) 30 × 68 + 4 × 68 and break it up to 30 × 60 + 30 × 8 + 4 × 60 + 4 × 8

   1 800
   240
   240
   32
   2
   110
   1 200
   1 000
   2 312

   So, 34 × 68 = 2 312

   (b) Similarly, 47 × 28 = 1 316



2.5 Different ways of recording multiplication

You will now learn about other ways in which you can set out your work.

This is what Sarah wrote when she calculated 34 × 63:

34 × 63 = 34 × 60 + 34 × 3
34 × 60 = 30 × 60 + 4 × 60
= 1 800 + 240
34 × 3 = 30 × 3 + 4 × 3
= 90 + 12
So, 34 × 63 = 1 800 + 240 + 90 + 12

Then Sarah wrote this to do the adding up in the last step:

1 800
240
90
12
2
140
1 000
1 000
2 142

1. This is how Sarah started to do 42 × 57:

   42 × 57 = 40 × 57 + 2 × 57

   Complete the calculation in the way Sarah would do it.

2. How do you think Sarah will calculate these?
   (a) 34 × 68
   (b) 47 × 28

GRADE 6: MATHEMATICS [TERM 2] 133

Possible misconceptions

There are two distinct challenges when introducing learners to the column format of recording the breaking-down and building-up method of multiplication:

- Learners may lose their understanding of the logic of the actual mathematical thinking involved in the process, namely to break down into place value parts, distributing and evaluating the set of simple products.
- Learners may in fact come to understand working in columns as a new method of multiplication, different to the breaking down and building up method based on the distributive property.

A way to protect learners against such breakdowns in their understanding is to get them to write notes next to the column exposition, stating the products that were evaluated to produce the part answers. This is clearly shown in the shaded passage.

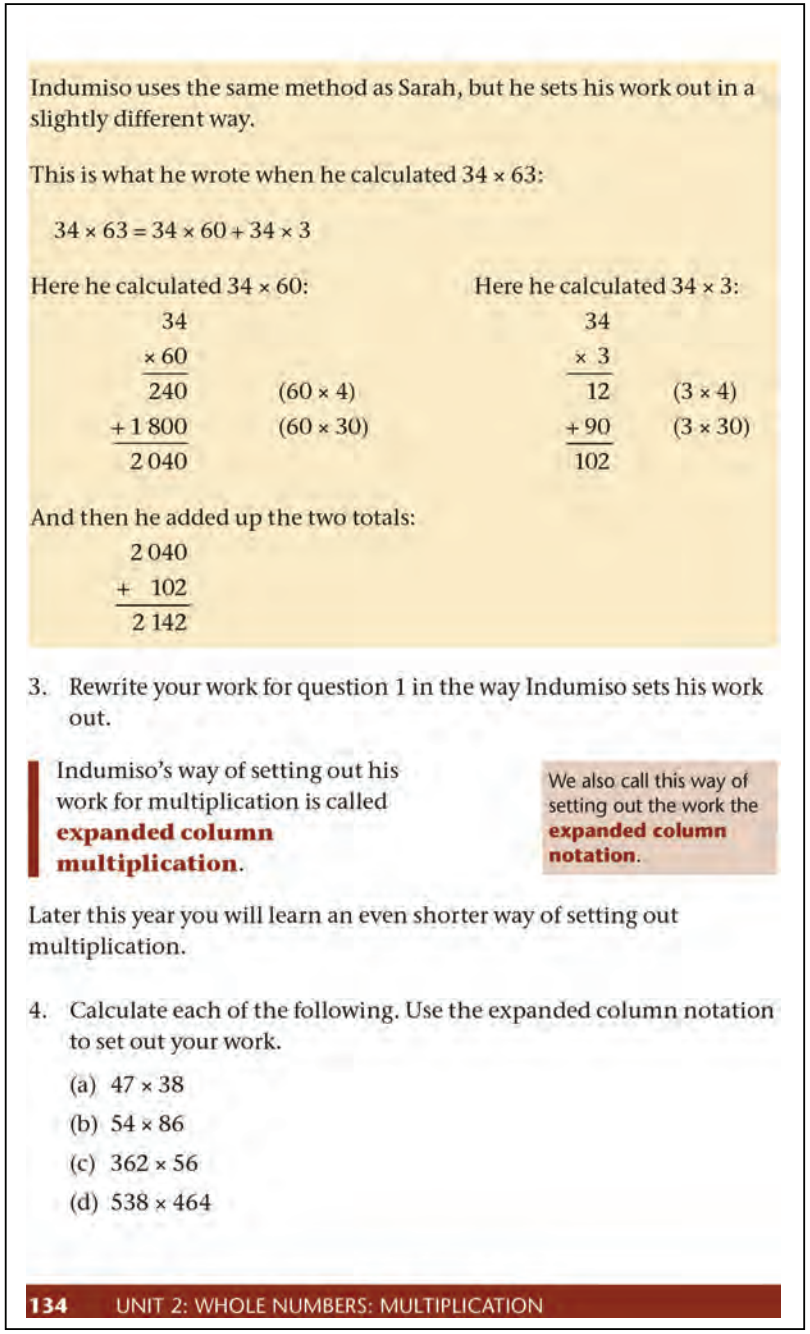
Answers

3. 42 × 57 = 42 × 50 + 42 × 7

42		42	
× 50		× 7	
100	(50 × 2)	14	(7 × 2)
+ 2 000	(50 × 40)	+ 280	(7 × 40)
2 100		294	

2 100
\+ 294
2 394

4. Learners should use the expanded column notation to set out their work.
(a) 1 786 (b) 4 644 (c) 20 272 (d) 249 632



Indumiso uses the same method as Sarah, but he sets his work out in a slightly different way.

This is what he wrote when he calculated 34 × 63:

34 × 63 = 34 × 60 + 34 × 3

Here he calculated 34 × 60:		Here he calculated 34 × 3:	
34		34	
× 60		× 3	
240	(60 × 4)	12	(3 × 4)
+ 1 800	(60 × 30)	+ 90	(3 × 30)
2 040		102	

And then he added up the two totals:

2 040
\+ 102
2 142

3. Rewrite your work for question 1 in the way Indumiso sets his work out.

Indumiso's way of setting out his work for multiplication is called **expanded column multiplication**.

We also call this way of setting out the work the **expanded column notation**.

Later this year you will learn an even shorter way of setting out multiplication.

4. Calculate each of the following. Use the expanded column notation to set out your work.
(a) 47 × 38
(b) 54 × 86
(c) 362 × 56
(d) 538 × 464

Notes on questions

Questions 5, 7 and 9 address the misconceptions described on the previous page.

Answers

5. (a) 400 × 7 (b) 400 × 80 (c) 400 × 500
(d) 70 × 7 (e) 70 × 80 (f) 70 × 500
(g) 3 × 7 (h) 3 × 80 (i) 3 × 500

6. 183 791

7. 765 = 700 + 60 + 5

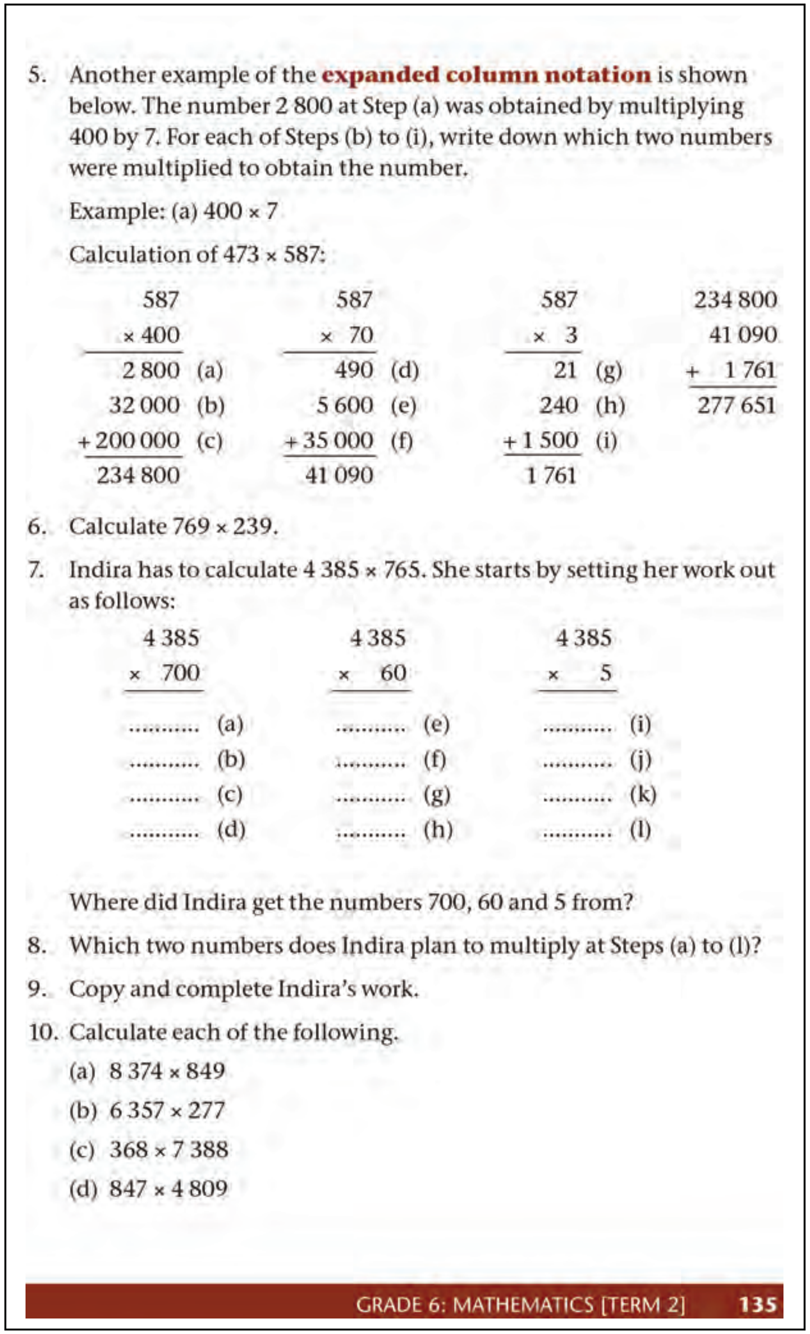
8. (a) 700 × 5 (b) 700 × 80 (c) 700 × 300 (d) 700 × 4 000
(e) 60 × 5 (f) 60 × 80 (g) 60 × 300 (h) 60 × 4 000
(i) 5 × 5 (j) 5 × 80 (k) 5 × 300 (l) 5 × 4 000

9.

4 385		4 385		4 385	
× 700		× 60		× 5	
3 500	(a)	300	(e)	25	(i)
56 000	(b)	4 800	(f)	400	(j)
210 000	(c)	18 000	(g)	1 500	(k)
2 800 000	(d)	240 000	(h)	20 000	(l)
3 069 500		263 100		21 925	

Therefore, 4 385 × 765 = 3 069 500 + 263 100 + 21 925 = 3 354 525

10. (a) 7 109 526 (b) 1 760 889 (c) 2 718 784 (d) 4 073 223



5. Another example of the **expanded column notation** is shown below. The number 2 800 at Step (a) was obtained by multiplying 400 by 7. For each of Steps (b) to (i), write down which two numbers were multiplied to obtain the number.

Example: (a) 400 × 7

Calculation of 473 × 587:

587		587		587		234 800
× 400		× 70		× 3		41 090
2 800	(a)	490	(d)	21	(g)	+ 1 761
32 000	(b)	5 600	(e)	240	(h)	277 651
+ 200 000	(c)	+ 35 000	(f)	+ 1 500	(i)	
234 800		41 090		1 761		

6. Calculate 769 × 239.

7. Indira has to calculate 4 385 × 765. She starts by setting her work out as follows:

4 385		4 385		4 385	
× 700		× 60		× 5	
....	(a)	....	(e)	....	(i)
....	(b)	....	(f)	....	(j)
....	(c)	....	(g)	....	(k)
....	(d)	....	(h)	....	(l)

Where did Indira get the numbers 700, 60 and 5 from?

8. Which two numbers does Indira plan to multiply at Steps (a) to (l)?

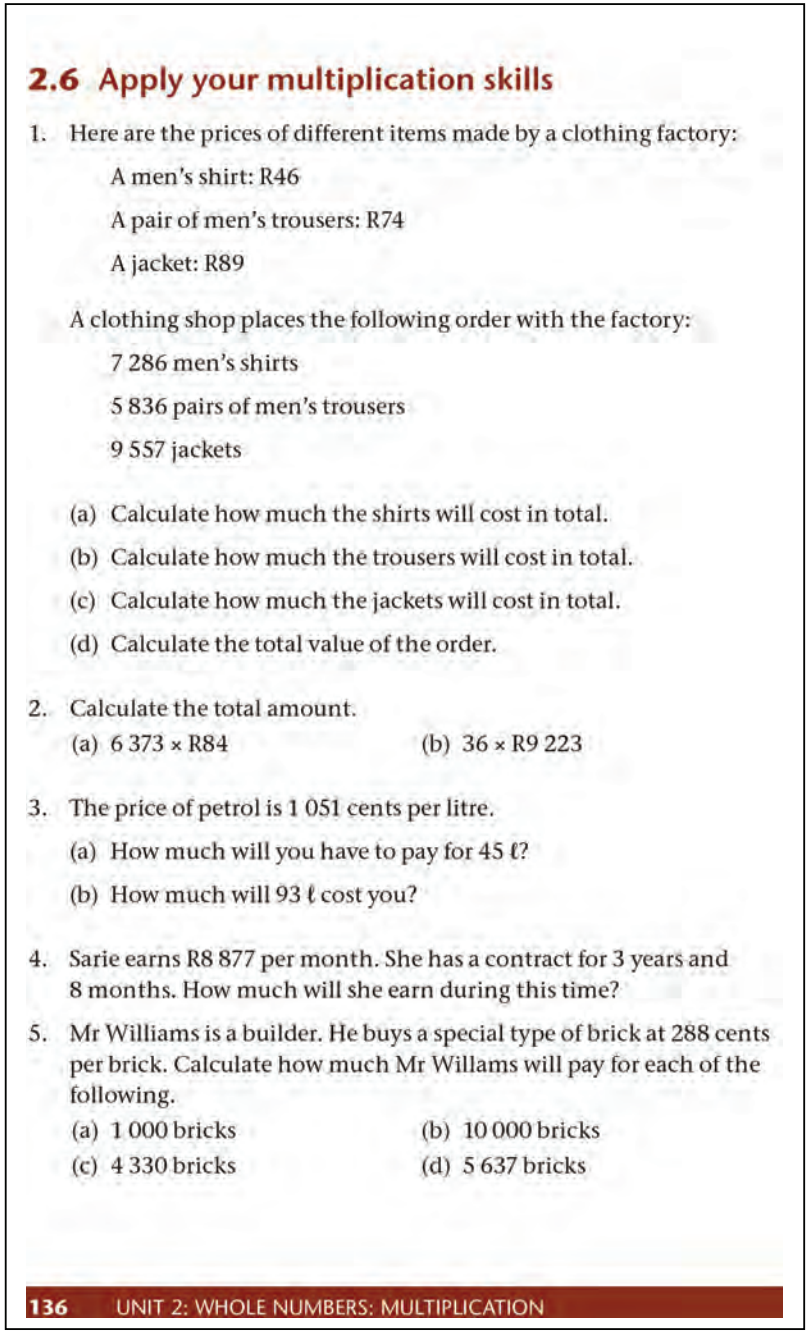
9. Copy and complete Indira's work.

10. Calculate each of the following.
    (a) 8 374 × 849
    (b) 6 357 × 277
    (c) 368 × 7 388
    (d) 847 × 4 809

2.6 Apply your multiplication skills

Answers

1. (a) 7 286 × 46 = R335 156
   (b) 5 836 × 74 = R431 864
   (c) 9 557 × 89 = R850 573
   (d) R335 156 + R431 864 + R850 573 = R1 617 593
2. (a) R535 332
   (b) R332 028
3. (a) 1 051 × 45 = 47 295 cents = R472,95
   (b) 1 051 × 93 = 97 743 cents = R977,43
4. Three years and eight months = 3 × 12 + 8 = 44 months
   Sarie will earn R8 877 × 44 = R390 588
5. (a) 1 000 × 288 = 288 000 cents = R2 880,00
   (b) 10 000 × 288 = 2 880 000 cents = R28 800,00
   (c) 4 330 × 288 = 1 247 040 cents = R12 470,40
   (d) 5 637 × 288 = 1 623 456 cents = R16 234,56



2.6 Apply your multiplication skills

1. Here are the prices of different items made by a clothing factory:
   A men's shirt: R46
   A pair of men's trousers: R74
   A jacket: R89

   A clothing shop places the following order with the factory:
   7 286 men's shirts
   5 836 pairs of men's trousers
   9 557 jackets

   (a) Calculate how much the shirts will cost in total.
   (b) Calculate how much the trousers will cost in total.
   (c) Calculate how much the jackets will cost in total.
   (d) Calculate the total value of the order.
2. Calculate the total amount.
   (a) 6 373 × R84 (b) 36 × R9 223
3. The price of petrol is 1 051 cents per litre.
   (a) How much will you have to pay for 45 ℓ?
   (b) How much will 93 ℓ cost you?
4. Sarie earns R8 877 per month. She has a contract for 3 years and 8 months. How much will she earn during this time?
5. Mr Williams is a builder. He buys a special type of brick at 288 cents per brick. Calculate how much Mr Willams will pay for each of the following.
   (a) 1 000 bricks (b) 10 000 bricks
   (c) 4 330 bricks (d) 5 637 bricks

2.7 Mental calculation versus the calculator!

Possible misconceptions

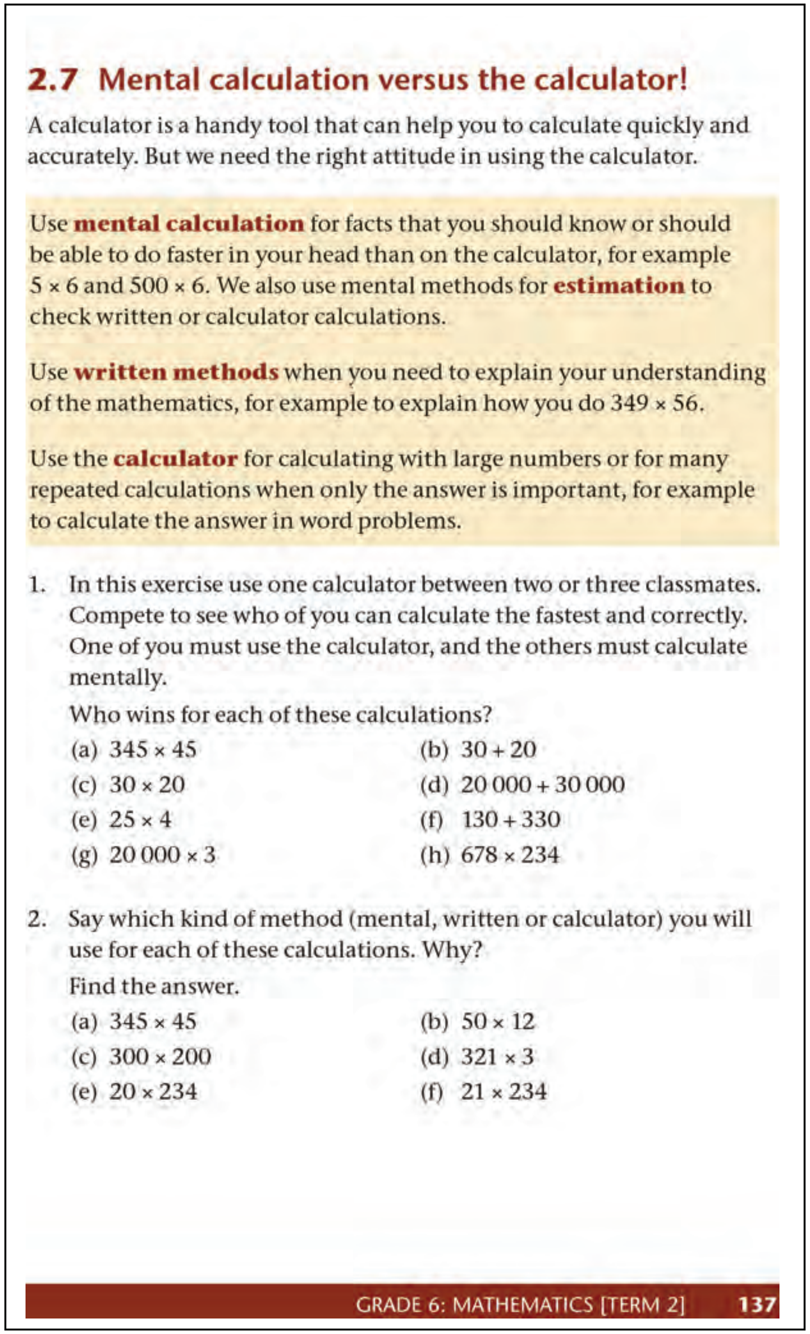
While the calculator is a powerful mathematical tool and developing competence in using it is critically important, the danger exists that learners will adopt the attitude that they cannot and need not be able to do calculations, especially multiplication, without the calculator.

It is therefore important that learners develop the perspectives described in the shaded passage and adopt a responsible attitude towards using the calculator. The learning activities in Sections 2.7 to 2.10 are specifically designed to promote the perspectives described in the shaded passage. Explain this thoroughly to learners and revisit these perspectives from time to time.

Answers

1. (a) 15 525 (b) 50 (c) 600 (d) 50 000
(e) 100 (f) 460 (g) 60 000 (h) 158 652

2. (a) 345 × 45 = 15 525 calculator
(b) 50 × 12 = 600 mentally
(c) 300 × 200 = 60 000 mentally
(d) 321 × 3 = 963 mentally/written
(e) 20 × 234 = 4 680 mentally; double 234 and × 10
(f) 21 × 234 = 4 914 written; one 234 more than (e)



2.7 Mental calculation versus the calculator!

A calculator is a handy tool that can help you to calculate quickly and accurately. But we need the right attitude in using the calculator.

Use **mental calculation** for facts that you should know or should be able to do faster in your head than on the calculator, for example 5 × 6 and 500 × 6. We also use mental methods for **estimation** to check written or calculator calculations.

Use **written methods** when you need to explain your understanding of the mathematics, for example to explain how you do 349 × 56.

Use the **calculator** for calculating with large numbers or for many repeated calculations when only the answer is important, for example to calculate the answer in word problems.

1. In this exercise use one calculator between two or three classmates. Compete to see who of you can calculate the fastest and correctly. One of you must use the calculator, and the others must calculate mentally.
Who wins for each of these calculations?
(a) 345 × 45 (b) 30 + 20
(c) 30 × 20 (d) 20 000 + 30 000
(e) 25 × 4 (f) 130 + 330
(g) 20 000 × 3 (h) 678 × 234

2. Say which kind of method (mental, written or calculator) you will use for each of these calculations. Why?
Find the answer.
(a) 345 × 45 (b) 50 × 12
(c) 300 × 200 (d) 321 × 3
(e) 20 × 234 (f) 21 × 234

2.8 Use estimation to check the calculator

Mathematical notes

The idea of “sandwiching” the answer to a mathematical question between upper and lower limits – as described in Cyndi’s way of estimating answers – forms the basic strategy of some important modern mathematical practices. Learning to think this way not only promotes learners’ proficiency at doing calculations by hand and by using the calculator; it also develops a thinking strategy that may support further learning in Mathematics, even at very high levels. However, while the strategy may seem obvious to you, it may not be transparent to learners and some careful teaching is required here.

Teaching guidelines

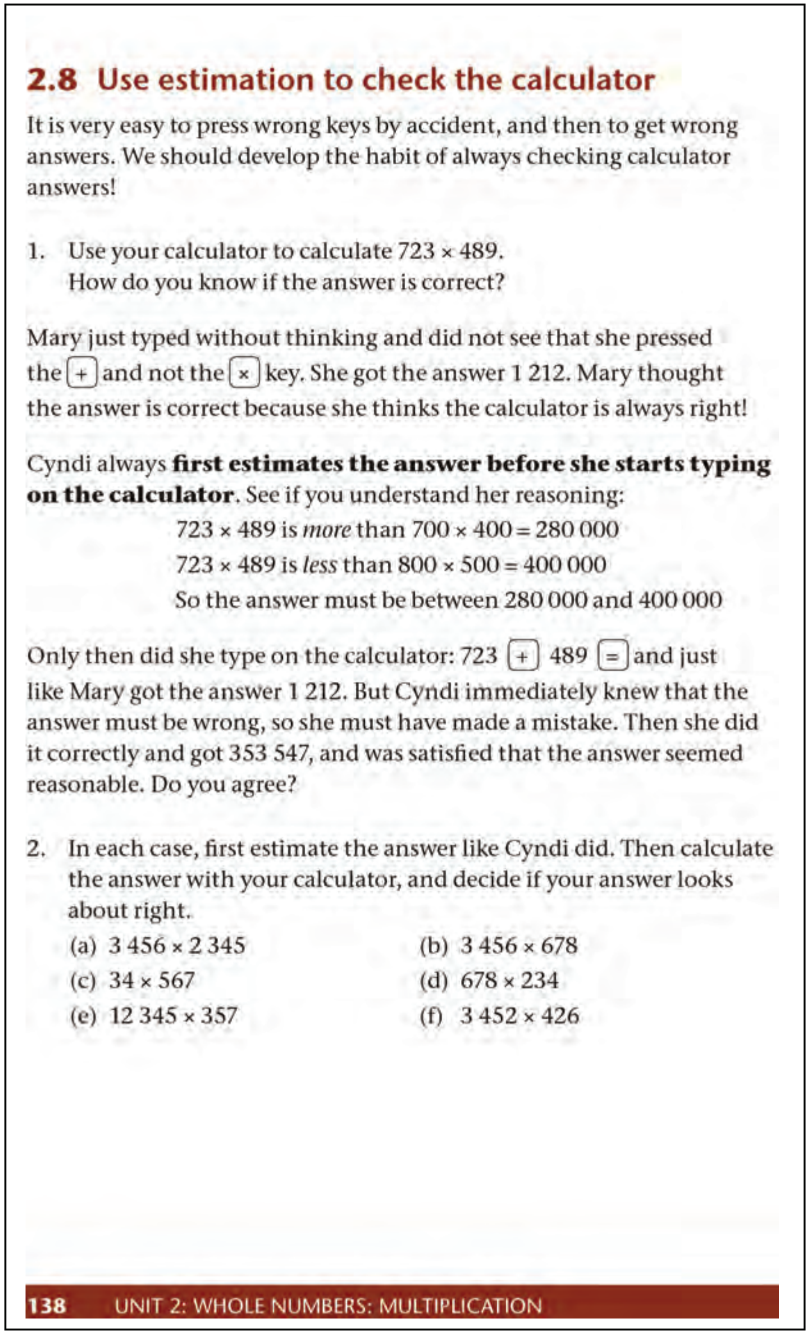
One approach is to act out the third paragraph on page 138 in class, i.e. take the calculator in your hand, type on it, and write 723 × 489 = 1 212 on the board. Ask learners if they think this answer is correct. Also ask learners to make suggestions on how the answer can be checked. It is important that they focus on the question of **how** the answer can be checked before moving on with the lesson.

Then ask learners whether they would expect the answer for 723 × 489 to be **less than** 700 × 500, **between** 700 × 500 and 800 × 500, or **bigger than** 800 × 500.

?
700 × 500
?
800 × 500
?

Answers

1. 723 × 489 = 353 547
   Consider learners’ suggestions, for example:
   Estimate the answer before you calculate; repeat the calculation to check the answer; judge the reasonableness.
2. (a) Answer must be between 6 000 000 and 12 000 000; 3 456 × 2 345 = 8 104 320
   (b) Answer must be between 1 800 000 and 2 800 000; 3 456 × 678 = 2 343 168
   (c) Answer must be between 15 000 and 24 000; 34 × 567 = 19 278
   (d) Answer must be between 120 000 and 210 000; 678 × 234 = 158 652
   (e) Answer must be between 3 600 000 and 5 200 000; 12 345 × 357 = 4 407 165
   (f) Answer must be between 1 200 000 and 2 000 000; 3 452 × 426 = 1 470 552



2.8 Use estimation to check the calculator

It is very easy to press wrong keys by accident, and then to get wrong answers. We should develop the habit of always checking calculator answers!

1. Use your calculator to calculate 723 × 489.
   How do you know if the answer is correct?

Mary just typed without thinking and did not see that she pressed the $\boxed{+}$ and not the $\boxed{\times}$ key. She got the answer 1 212. Mary thought the answer is correct because she thinks the calculator is always right!

Cyndi always **first estimates the answer before she starts typing on the calculator**. See if you understand her reasoning:

723 × 489 is *more* than 700 × 400 = 280 000
723 × 489 is *less* than 800 × 500 = 400 000
So the answer must be between 280 000 and 400 000

Only then did she type on the calculator: 723 $\boxed{+}$ 489 $\boxed{=}$ and just like Mary got the answer 1 212. But Cyndi immediately knew that the answer must be wrong, so she must have made a mistake. Then she did it correctly and got 353 547, and was satisfied that the answer seemed reasonable. Do you agree?

2. In each case, first estimate the answer like Cyndi did. Then calculate the answer with your calculator, and decide if your answer looks about right.
   (a) 3 456 × 2 345
   (b) 3 456 × 678
   (c) 34 × 567
   (d) 678 × 234
   (e) 12 345 × 357
   (f) 3 452 × 426

138 UNIT 2: WHOLE NUMBERS: MULTIPLICATION

2.9 Use equivalence to check the calculator

Teaching guidelines

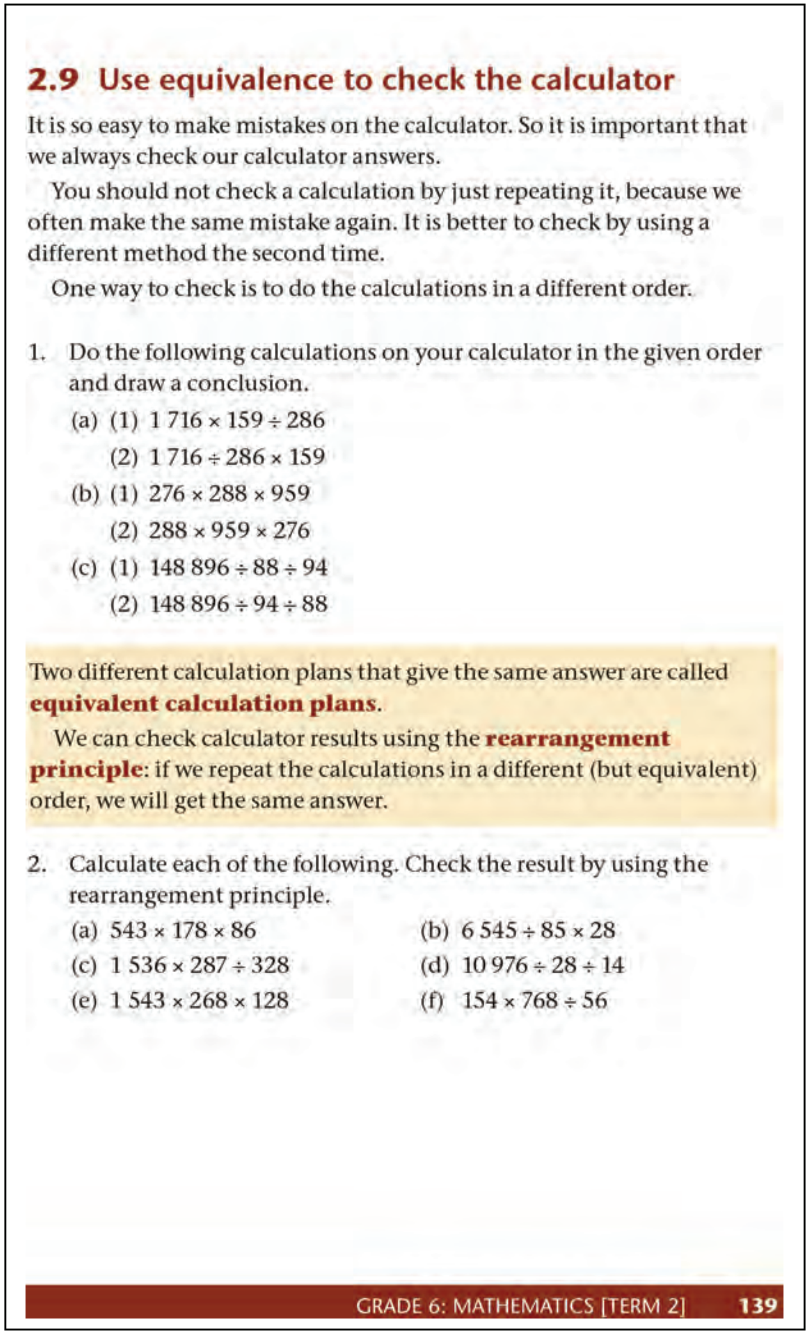
While equivalent calculation plans do provide a way of checking answers, the real value of this work is that it promotes understanding of equivalence and the properties of operations.

Answers

1. (a) (1) 954 (2) 954
   (b) (1) 76 228 992 (2) 76 228 992
   (c) (1) 18 (2) 18

   One way of checking calculator results is doing the calculations in a different but equivalent order.

2. (a) 8 312 244 (b) 2 156
   (c) 1 344 (d) 28
   (e) 52 931 072 (f) 2 112



2.9 Use equivalence to check the calculator

It is so easy to make mistakes on the calculator. So it is important that we always check our calculator answers.

You should not check a calculation by just repeating it, because we often make the same mistake again. It is better to check by using a different method the second time.

One way to check is to do the calculations in a different order.

1. Do the following calculations on your calculator in the given order and draw a conclusion.
   (a) (1) 1 716 × 159 ÷ 286
   (2) 1 716 ÷ 286 × 159
   (b) (1) 276 × 288 × 959
   (2) 288 × 959 × 276
   (c) (1) 148 896 ÷ 88 ÷ 94
   (2) 148 896 ÷ 94 ÷ 88

Two different calculation plans that give the same answer are called **equivalent calculation plans**.

We can check calculator results using the **rearrangement principle**: if we repeat the calculations in a different (but equivalent) order, we will get the same answer.

2. Calculate each of the following. Check the result by using the rearrangement principle.
   (a) 543 × 178 × 86 (b) 6 545 ÷ 85 × 28
   (c) 1 536 × 287 ÷ 328 (d) 10 976 ÷ 28 ÷ 14
   (e) 1 543 × 268 × 128 (f) 154 × 768 ÷ 56

GRADE 6: MATHEMATICS [TERM 2] 139

2.10 Use inverses to check the calculator

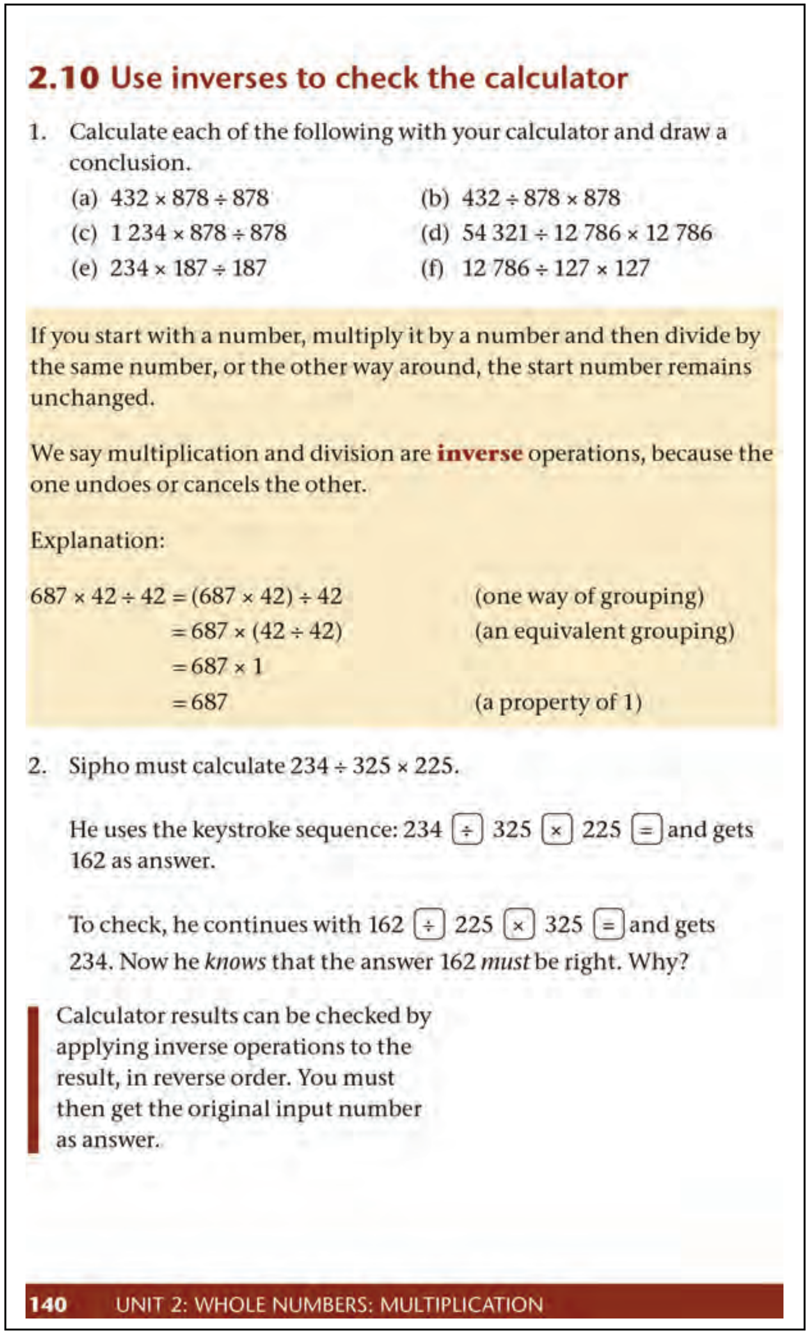
Teaching guidelines

This is possibly the easiest way to check answers obtained by using a calculator. Note that the value of spending some class time on this is not just that learners will acquire a method to check their calculator work, but that this may reinforce their awareness and understanding of inverse operations.

Answers

1. (a) 432 (b) 432 (c) 1 234
   (d) 54 321 (e) 234 (f) 12 786

   If you multiply and then divide by the same number (or the other way round) you get the original starting number as an answer. So, applying the inverse operation to the calculator answer in reverse order is a way to check whether the calculator answer is correct.
2. Because he ended with the starting number after applying the inverse operations in reverse order to the calculator answer.



2.10 Use inverses to check the calculator

1. Calculate each of the following with your calculator and draw a conclusion.
   (a) 432 × 878 ÷ 878 (b) 432 ÷ 878 × 878
   (c) 1 234 × 878 ÷ 878 (d) 54 321 ÷ 12 786 × 12 786
   (e) 234 × 187 ÷ 187 (f) 12 786 ÷ 127 × 127

If you start with a number, multiply it by a number and then divide by the same number, or the other way around, the start number remains unchanged.

We say multiplication and division are **inverse** operations, because the one undoes or cancels the other.

Explanation:

687 × 42 ÷ 42 = (687 × 42) ÷ 42 (one way of grouping)
= 687 × (42 ÷ 42) (an equivalent grouping)
= 687 × 1
= 687 (a property of 1)

2. Sipho must calculate 234 ÷ 325 × 225.

   He uses the keystroke sequence: 234 $\boxed{\div}$ 325 $\boxed{\times}$ 225 $\boxed{=}$ and gets 162 as answer.

   To check, he continues with 162 $\boxed{\div}$ 225 $\boxed{\times}$ 325 $\boxed{=}$ and gets 234. Now he *knows* that the answer 162 *must* be right. Why?

Calculator results can be checked by applying inverse operations to the result, in reverse order. You must then get the original input number as answer.

Notes on questions

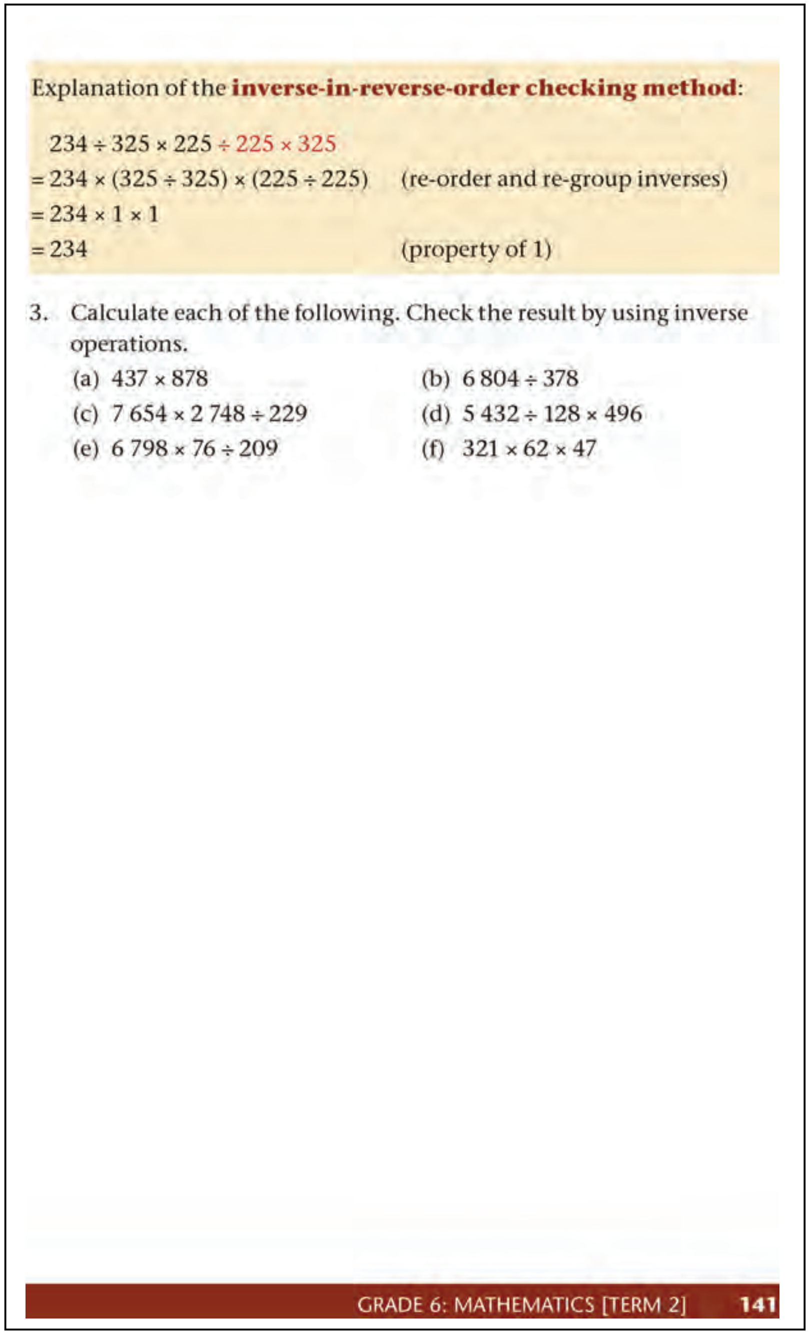
There is another way of checking answers that does not involve using the calculator. To check, for example, if 723 × 489 = 1 212 is correct, learners may start to do the calculation accurately without the calculator and observe the last digit of the answer. Since 3 × 9 = 27 the last digit is clearly 7, not 2 as in 1 212.

Answers

3. Learners calculate and check their answers by applying inverse operations.

   (a) 383 686 (b) 18 (c) 91 848

   (d) 21 049 (e) 2 472 (f) 935 394



Explanation of the **inverse-in-reverse-order checking method**:

234 ÷ 325 × 225 ÷ 225 × 325
= 234 × (325 ÷ 325) × (225 ÷ 225) (re-order and re-group inverses)
= 234 × 1 × 1
= 234 (property of 1)

3. Calculate each of the following. Check the result by using inverse operations.
   (a) 437 × 878
   (b) 6 804 ÷ 378
   (c) 7 654 × 2 748 ÷ 229
   (d) 5 432 ÷ 128 × 496
   (e) 6 798 × 76 ÷ 209
   (f) 321 × 62 × 47

GRADE 6: MATHEMATICS [TERM 2] 141

Grade 6 Term 2 Unit 3 Properties of three-dimensional objects

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
3.1 Prisms	Prisms with a variety of end faces	142 to 144
3.2 Faces, edges and vertices of prisms	Establishing standard terms for parts of prisms	145
3.3 Pyramids	Pyramids with a variety of bases	146 to 147
3.4 Build 3-D objects with straws or sticks	Using straws/sticks for the edges of 3-D objects	148
3.5 Nets of prisms and pyramids	Drawing nets for objects, and folding nets to make objects	149 to 151

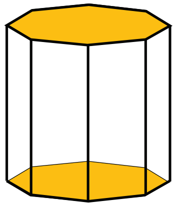
CAPS time allocation	5 hours
CAPS page references	22 and 244 to 246

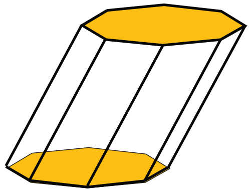
Mathematical background

Prisms are 3-D objects with two identical and identically orientated polygonal faces on opposite sides. These two faces are connected by faces in the form of parallelograms that all have the same length.

If the connecting faces are rectangles, the prism is called a **right prism**.
If the connecting faces are not rectangles, the prism is called a **skew** or **oblique prism**.

A prism with six faces that are all rectangles is called a **rectangular prism**.


A right prism


A skew (oblique) prism

Pyramids are 3-D objects that have a polygonal base face with triangular faces attached to the base. The triangular faces meet at a point opposite the base face.

We call the 2-D shapes that make up the **surface** of the prism or pyramid the **faces** of the object. We call the lines where faces meet the **edges** of the 3-D object. We call the points or corners where the faces meet the **vertices** (singular: vertex) of the 3-D object.

A **cross-section** of a 3-D object is any cut through the object that produces two identical, flat surfaces on the two parts that result.

The **net** of a prism or pyramid is all of its 2-D faces laid out flat, connected to each other along some edges.

Apart from learning about the properties of rectangular prisms and various kinds of pyramids, it is critically important that learners learn to look closely at drawings and pictures of 3-D objects, and notice detail such as the faces, edges and vertices. It is also critically important that learners learn to describe objects in words with reference to faces, edges and vertices, and to read and interpret descriptions of objects. To promote these skills, discussions in which all learners participate are very important.

Resources

Clay or sticky putty, drinking straws or sticks, paper/cardboard, scissors, models of the prisms and pyramids dealt with in this unit

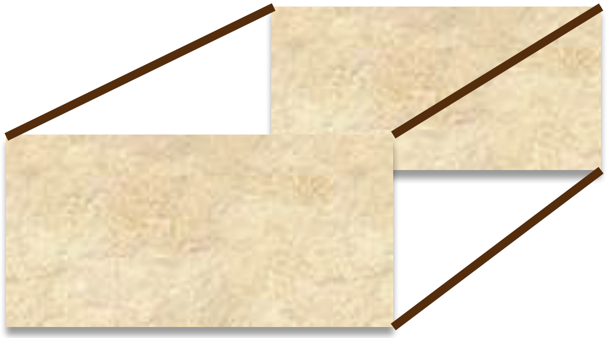
3.1 Prisms

Teaching guidelines

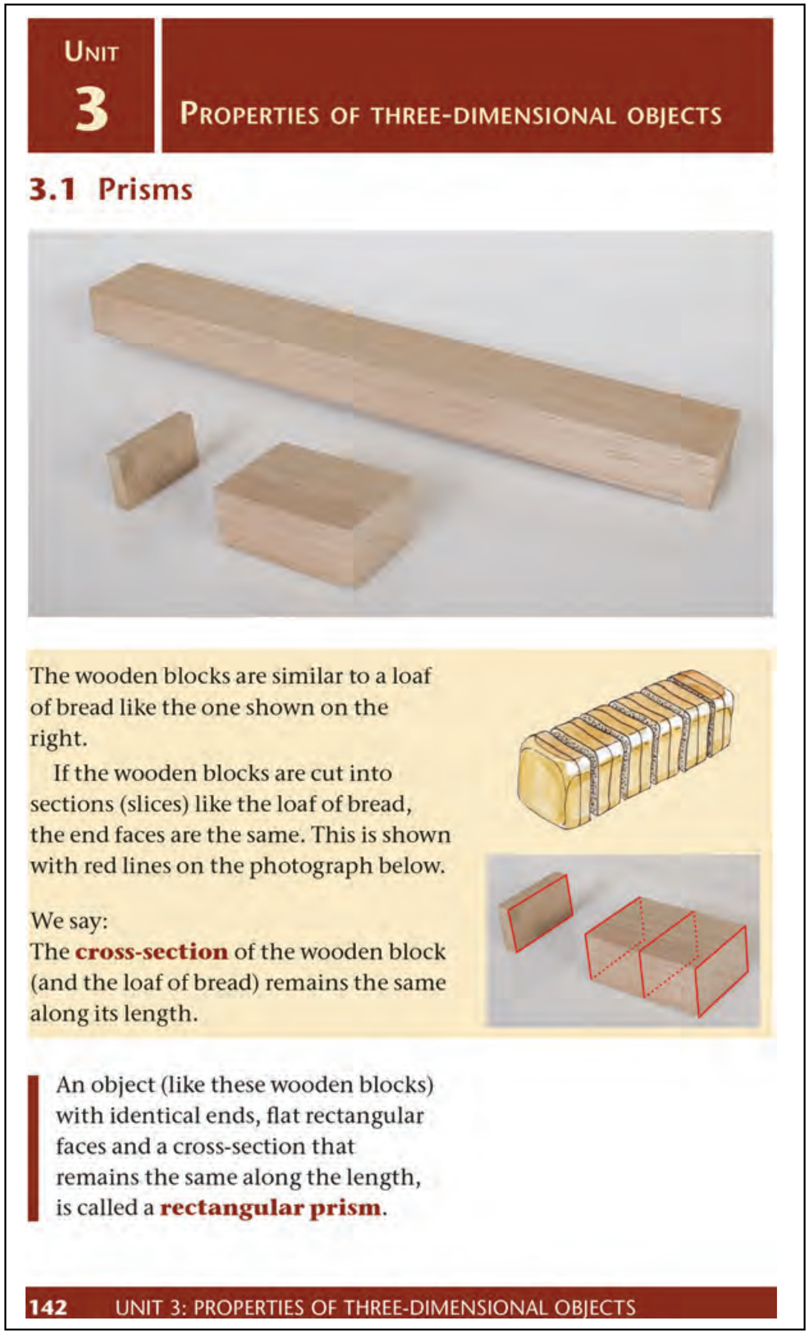
It is quite critical that you show learners some prism-shaped objects, for example a brick, a carton, a loaf of bread, a box of matches and a piece of plank. The vast majority of classrooms are also in the shape of prisms: when they are in class, learners are actually inside a prism!

Although an uncut loaf of bread is only approximately a prism, it is a useful example in class because you can easily slice it in such a way that two faces identical to the ends are exposed. This shows a key characteristic of prisms: they have what is called a **uniform cross-section** identical to the two equal opposite faces.

It is also useful to have a prism made of two flat pieces of cardboard as the identical opposite faces, and drinking straws or thin wooden sticks that correspond to the edges of the other faces.



Learners should be empowered to engage with drawings/pictures of 3-D objects as well as with verbal descriptions of objects. Hence they need to acquire the vocabulary and forms of expression required to communicate effectively about 3-D objects. Revise the terms face, edge and vertex while demonstrating.



UNIT 3 PROPERTIES OF THREE-DIMENSIONAL OBJECTS

3.1 Prisms

The wooden blocks are similar to a loaf of bread like the one shown on the right.

If the wooden blocks are cut into sections (slices) like the loaf of bread, the end faces are the same. This is shown with red lines on the photograph below.

We say:
The **cross-section** of the wooden block (and the loaf of bread) remains the same along its length.

An object (like these wooden blocks) with identical ends, flat rectangular faces and a cross-section that remains the same along the length, is called a **rectangular prism**.

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Possible misconceptions

Learners may easily form the idea that all prisms are rectangular prisms. Let them look at all the pictures in the top row of the shaded passage.

- *Ask whether all four objects are prisms (they are).*
- *Ask whether they are all rectangular prisms (they are not).*
- *Ask whether an object can be a prism but not a rectangular prism (it can).*

Asking these questions and taking answers from learners will also provide learners with experience of using language to describe 3-D objects, and in this way prepare them for question 1.

Learners also easily form the misconception that a cube is not a prism. A cube is a prism; indeed, it is a special kind of prism of which all the faces are squares. Ask learners: *"Can all the faces of any prism that is not a cube be identical?"*

Teaching guidelines

Question 1 provides learners with an opportunity to interpret verbal descriptions and 2-D drawings and pictures of 3-D objects.

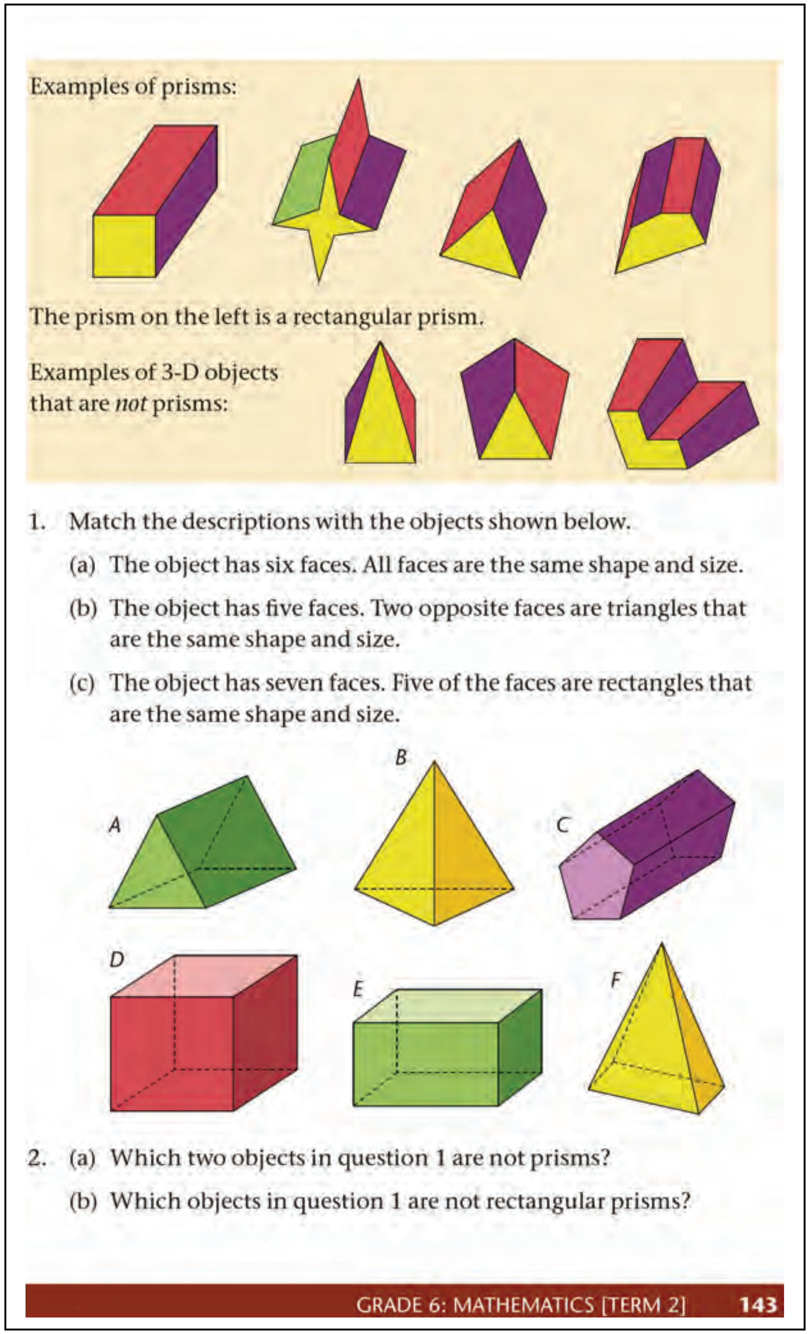
If learners seem to need support, you may ask questions like the following, and suggest to them that they put similar questions to themselves about each of Objects A to F:

- *How many faces does the 3-D object have?*
- *What are the shapes of its faces?*
- *How many faces are mentioned in each of questions (a), (b) and (c)?*

Ask learners to write explanations for their answers to question 2, and take this in for thorough marking. This will provide you with good information about the level of learners' understanding and language usage about 3-D objects.

Answers

1. (a) Object D
   (b) Object A
   (c) Object C
2. (a) Objects B and F
   (b) Objects A, B, C and F



Examples of prisms:

The prism on the left is a rectangular prism.

Examples of 3-D objects that are *not* prisms:

1. Match the descriptions with the objects shown below.
   (a) The object has six faces. All faces are the same shape and size.
   (b) The object has five faces. Two opposite faces are triangles that are the same shape and size.
   (c) The object has seven faces. Five of the faces are rectangles that are the same shape and size.

2. (a) Which two objects in question 1 are not prisms?
   (b) Which objects in question 1 are not rectangular prisms?

GRADE 6: MATHEMATICS [TERM 2] **143**

Teaching guidelines

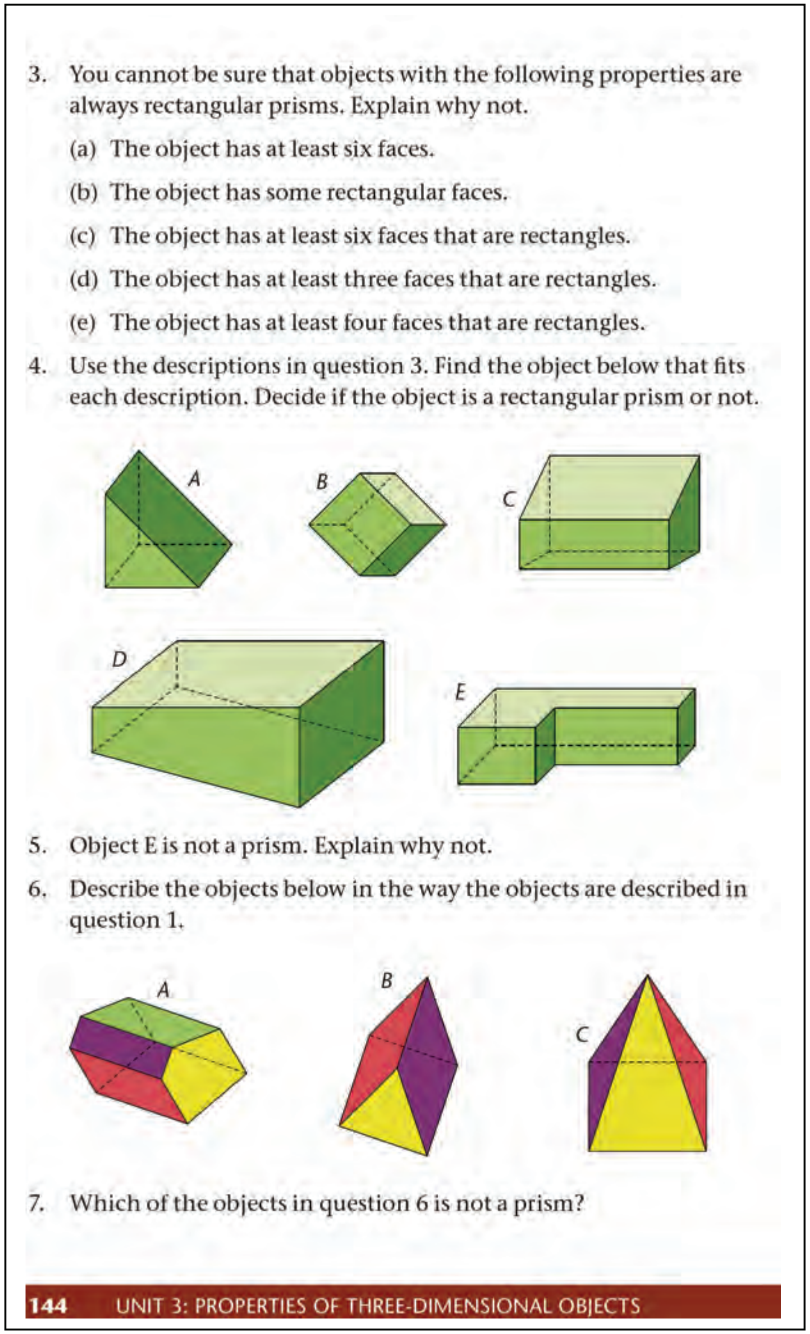
Question 3 requires that learners read carefully, and take note of what is *not* being said. For example, in 3(a) it is not said that the faces are rectangular, hence once cannot be sure that the object is a rectangular prism. In 3(d) nothing is said about the other faces, hence there may be faces that are not rectangles.

Some questions may be answered effectively by citing a 3-D object that has the stated properties, but is not a rectangular prism. For example, 3(c) can be answered by pointing out that a hexagonal prism (e.g. as on page 145 of the Learner Book) has six faces that are rectangles.

Allow learners to engage with question 3 individually for some time, then conduct a class discussion.

Answers

3. (a) All six faces must be rectangular.
   (b) All faces need to be rectangular.
   (c) A hexagonal prism, for example, has six faces that are rectangles.
   (d) The object could be a triangular prism.
   (e) The remaining two faces are not necessarily rectangular.
4. (a) Objects B, C and D, but only Object B is a rectangular prism.
   (b) Objects A, B, C, D and E, but only Object B is a rectangular prism.
   (c) Only Object B and it is a rectangular prism.
   (d) All the objects, but only B is a rectangular prism.
   (e) Objects C and D, but they are not rectangular prisms.
5. The end faces of the object are not identical.
6. A. The object has 5 rectangular faces. Two opposite ends are pentagons that have the same shape and size.
   B. The object has 3 rectangular faces. Two opposite ends are triangles that have the same shape and size.
   C. The object has 5 faces: four triangular faces and one quadrilateral face.
7. Object C



3. You cannot be sure that objects with the following properties are always rectangular prisms. Explain why not.
   (a) The object has at least six faces.
   (b) The object has some rectangular faces.
   (c) The object has at least six faces that are rectangles.
   (d) The object has at least three faces that are rectangles.
   (e) The object has at least four faces that are rectangles.
4. Use the descriptions in question 3. Find the object below that fits each description. Decide if the object is a rectangular prism or not.

5. Object E is not a prism. Explain why not.
6. Describe the objects below in the way the objects are described in question 1.

7. Which of the objects in question 6 is not a prism?

144 UNIT 3: PROPERTIES OF THREE-DIMENSIONAL OBJECTS

3.2 Faces, edges and vertices of prisms

Teaching guidelines

If possible, have various models handy to help learners to “see” what is represented by the diagrams given here.

Possible misconceptions

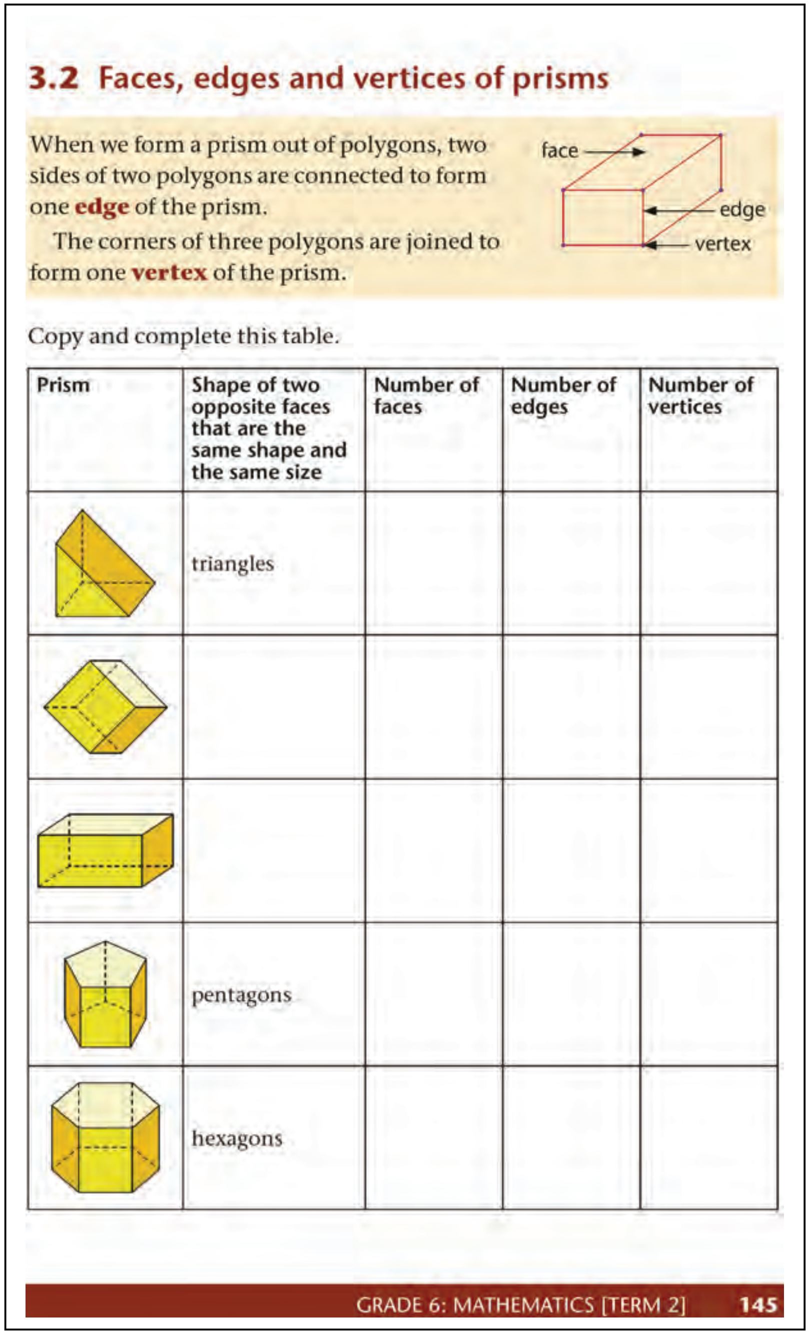
Note that only regular polygonal prisms are shown. Learners may form restricted understandings of prisms as a result. Disrupt this line of thinking by providing additional examples where the end faces are not regular polygons (all the table entries will be the same for the irregular ones, a very useful enrichment).

Notes on questions

The table provided is a tool to assist learners in keeping track/comparing the properties of the various objects. Be sure to remind them that completing the table is not the primary objective, but rather comparing the properties. The purpose of this section is, after all, to develop a more detailed grasp of prisms. Learners need not copy the diagrams when they copy the table. They can write the names of the prisms in the first column instead.

Answers

Prism	Shape of two opposite faces that are the same shape and the same size	Number of faces	Number of edges	Number of vertices
Triangular prism	triangles	5	9	6
Cube	squares	6	12	8
Rectangular prism	rectangles	6	12	8
Pentagonal prism	pentagons	7	15	10
Hexagonal prism	hexagons	8	18	12



3.2 Faces, edges and vertices of prisms

When we form a prism out of polygons, two sides of two polygons are connected to form one **edge** of the prism.

The corners of three polygons are joined to form one **vertex** of the prism.

Copy and complete this table.

Prism	Shape of two opposite faces that are the same shape and the same size	Number of faces	Number of edges	Number of vertices
	triangles			
	pentagons			
	hexagons			

3.3 Pyramids

Mathematical notes

Pyramids are 3-D objects that have a polygonal base face, with triangles attached to each side. The triangles come together at a point opposite the base.

Teaching guidelines

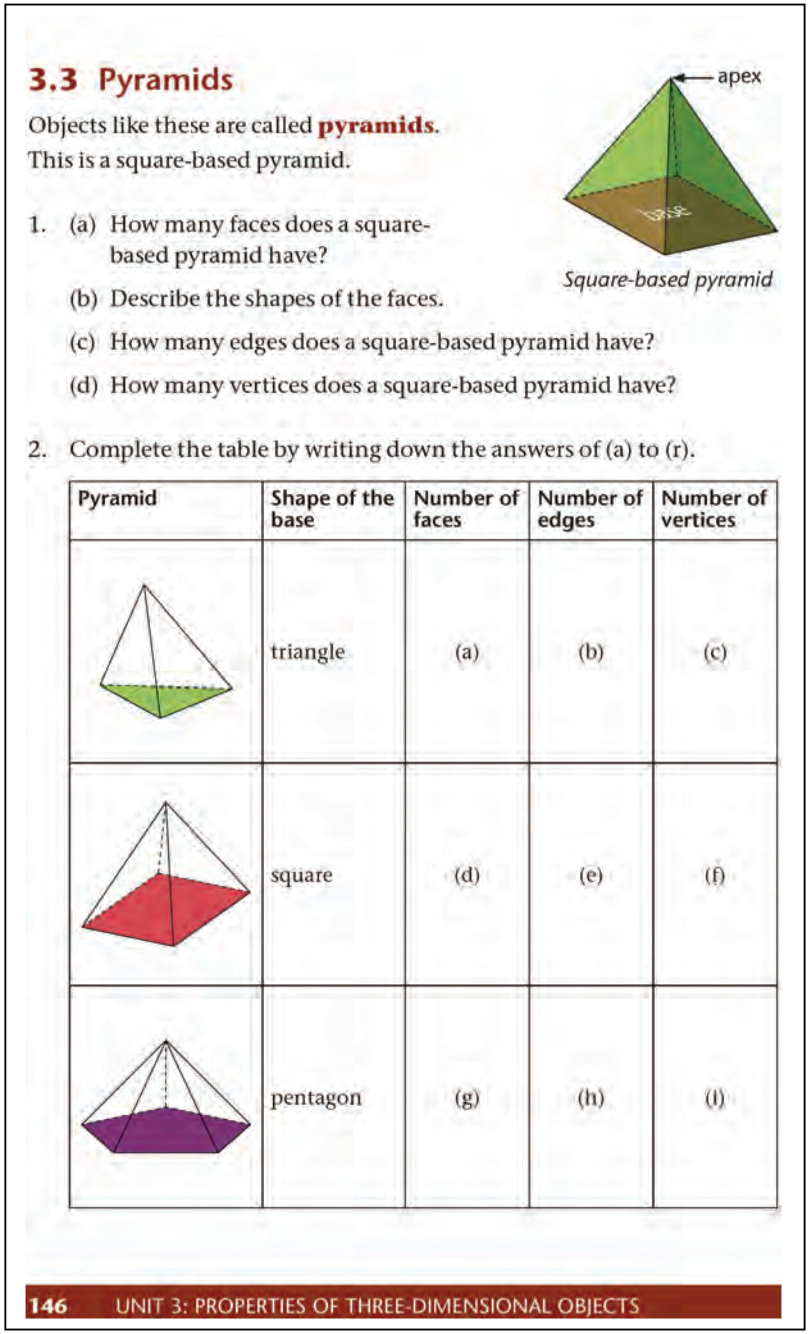
If possible, have various models handy to help your learners "see" what is represented by the diagrams given here.

Notes on questions

Emphasise that the focus is on the objects, not on completing the table for its own sake.

Answers

1. (a) 5 faces
   (b) 4 triangles, 1 square
   (c) 8 edges
   (d) 5 vertices
2. (a) 4 faces (b) 6 edges (c) 4 vertices
   (d) 5 faces (e) 8 edges (f) 5 vertices
   (g) 6 faces (h) 10 edges (i) 6 vertices



3.3 Pyramids

Objects like these are called **pyramids**.
This is a square-based pyramid.

Square-based pyramid

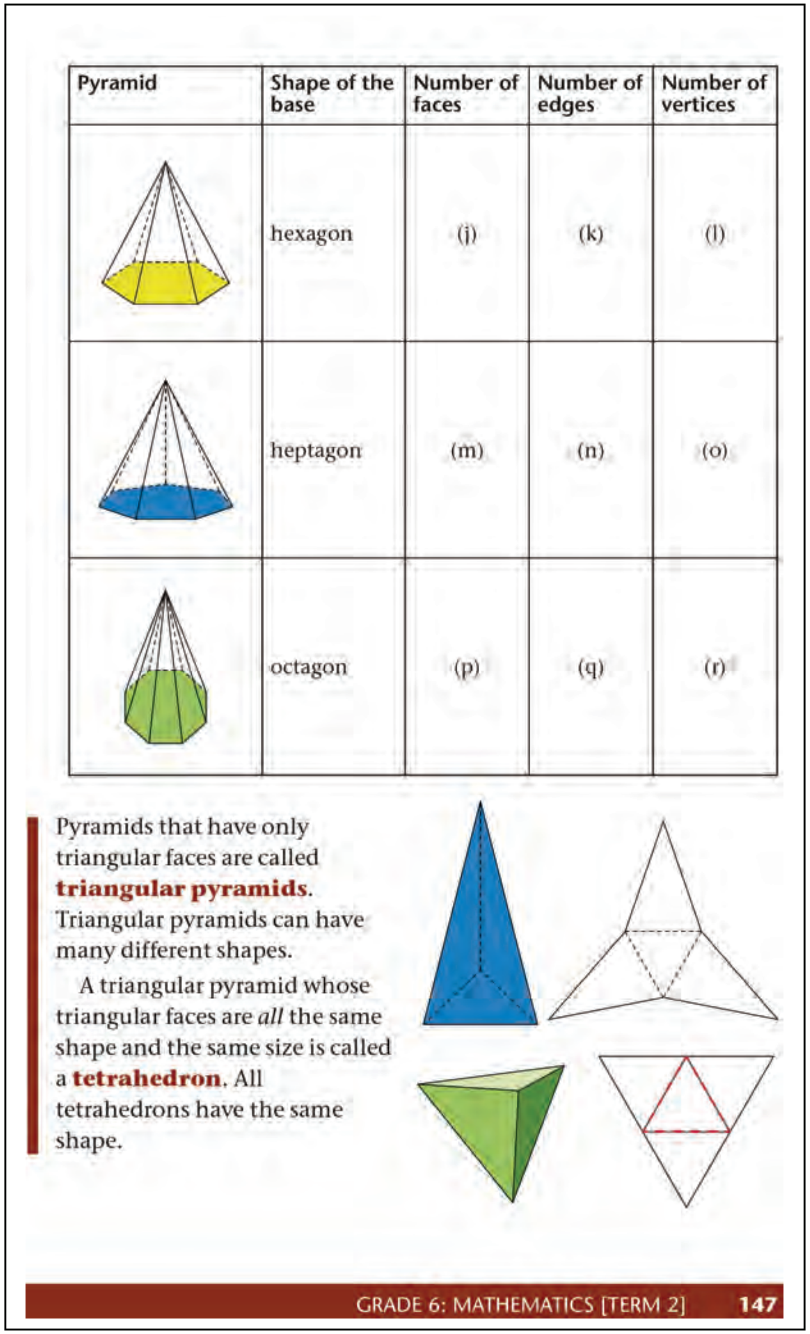
1. (a) How many faces does a square-based pyramid have?
   (b) Describe the shapes of the faces.
   (c) How many edges does a square-based pyramid have?
   (d) How many vertices does a square-based pyramid have?

2. Complete the table by writing down the answers of (a) to (r).

Pyramid	Shape of the base	Number of faces	Number of edges	Number of vertices
	triangle	(a)	(b)	(c)
	square	(d)	(e)	(f)
	pentagon	(g)	(h)	(i)

Answers (*continued*)

2. (j) 7 faces (k) 12 edges (l) 7 vertices
(m) 8 faces (n) 14 edges (o) 8 vertices
(p) 9 faces (q) 16 edges (r) 9 vertices



Pyramid	Shape of the base	Number of faces	Number of edges	Number of vertices
	hexagon	(j)	(k)	(l)
	heptagon	(m)	(n)	(o)
	octagon	(p)	(q)	(r)

Pyramids that have only triangular faces are called **triangular pyramids**. Triangular pyramids can have many different shapes.

A triangular pyramid whose triangular faces are *all* the same shape and the same size is called a **tetrahedron**. All tetrahedrons have the same shape.

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3.4 Build 3-D objects with straws or sticks

Mathematical notes

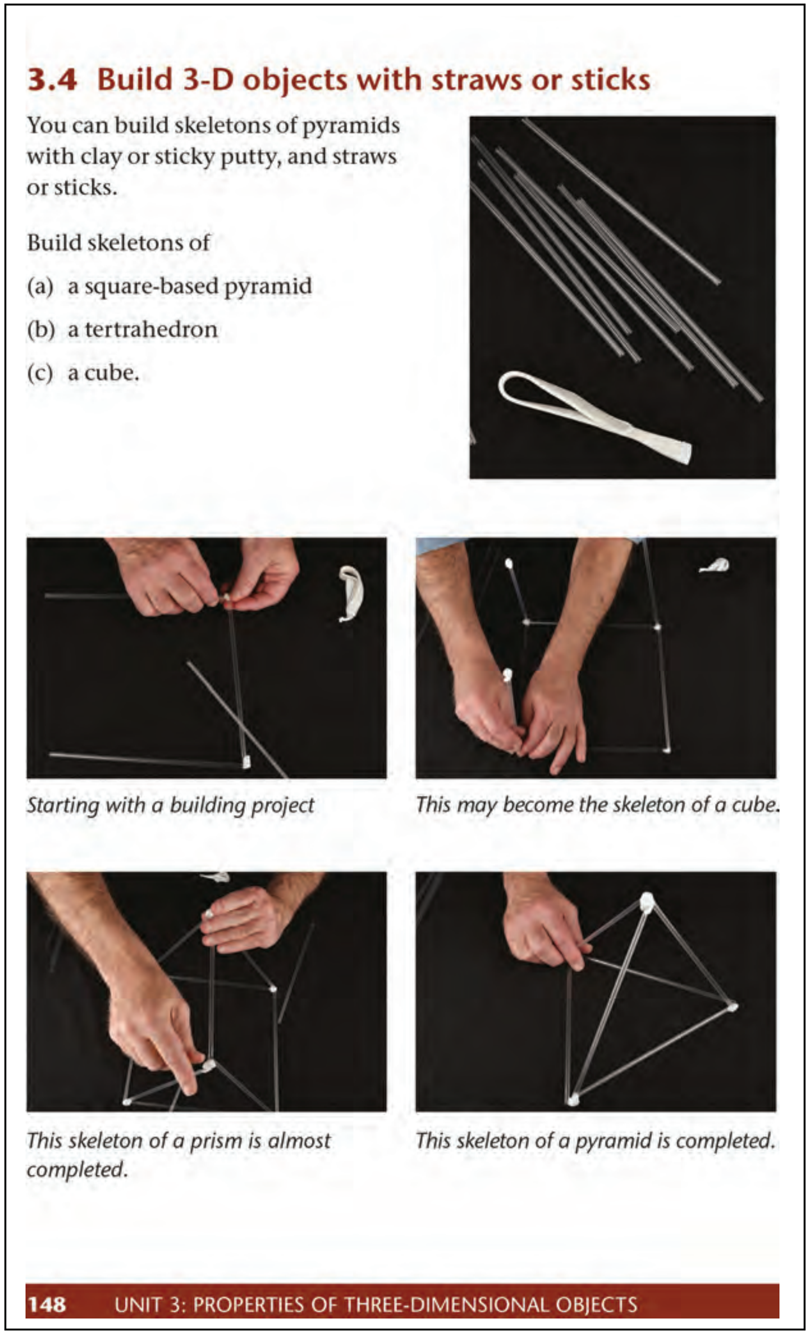
One way to build models of 3-D objects that have only polygonal faces is to use straight sticks or straws. The resulting models show the edges and vertices of the objects directly. The faces are the imaginary flat surfaces on the outside of the model that are enclosed by the sticks.

Teaching guidelines

This can be a time-consuming activity, but it is an extremely rewarding one for your learners. It serves a number of purposes: primarily developing hands-on model building skills, but also drawing learners into thinking hard about the spatial arrangement of the faces, edges and vertices of the object.

If possible, give learners additional building tasks to allow them to further explore the spatial arrangements of faces, edges and vertices in 3-D objects. Many may decide to pursue this on their own anyway and will want to "show and tell" if given the opportunity, something that should be encouraged!

For enrichment, allow learners to attempt to build objects with sticks or straws of different lengths to see what will and will not work.



3.4 Build 3-D objects with straws or sticks

You can build skeletons of pyramids with clay or sticky putty, and straws or sticks.

Build skeletons of

(a) a square-based pyramid

(b) a tertrahedron

(c) a cube.

Starting with a building project

This may become the skeleton of a cube.

This skeleton of a prism is almost completed.

This skeleton of a pyramid is completed.

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3.5 Nets of prisms and pyramids

Mathematical notes

The net of a 3-D object has all of its faces laid out flat but connected in some way. It is important to be able to see which sides and faces in a net are connected along their edges in the 3-D object, which faces may be opposite each other, and which corners of the faces come together at the vertices of the object. Understanding how the net of the object relates to the object itself is very important in developing a fuller grasp of the spatial arrangement of the sides and faces of the object. Comparing the net to the models built in the previous section provides further opportunity for deepening spatial understanding.

Teaching guidelines

Working with nets is another way of making 3-D objects. More importantly, however, is that working with nets provides learners with opportunities to analyse the elements of 3-D objects and strengthen their language skills with respect to 3-D objects. Question 1 is as much about learning to read and interpret text relating to 3-D objects as it is about relating the elements of the net to the elements of the prism.

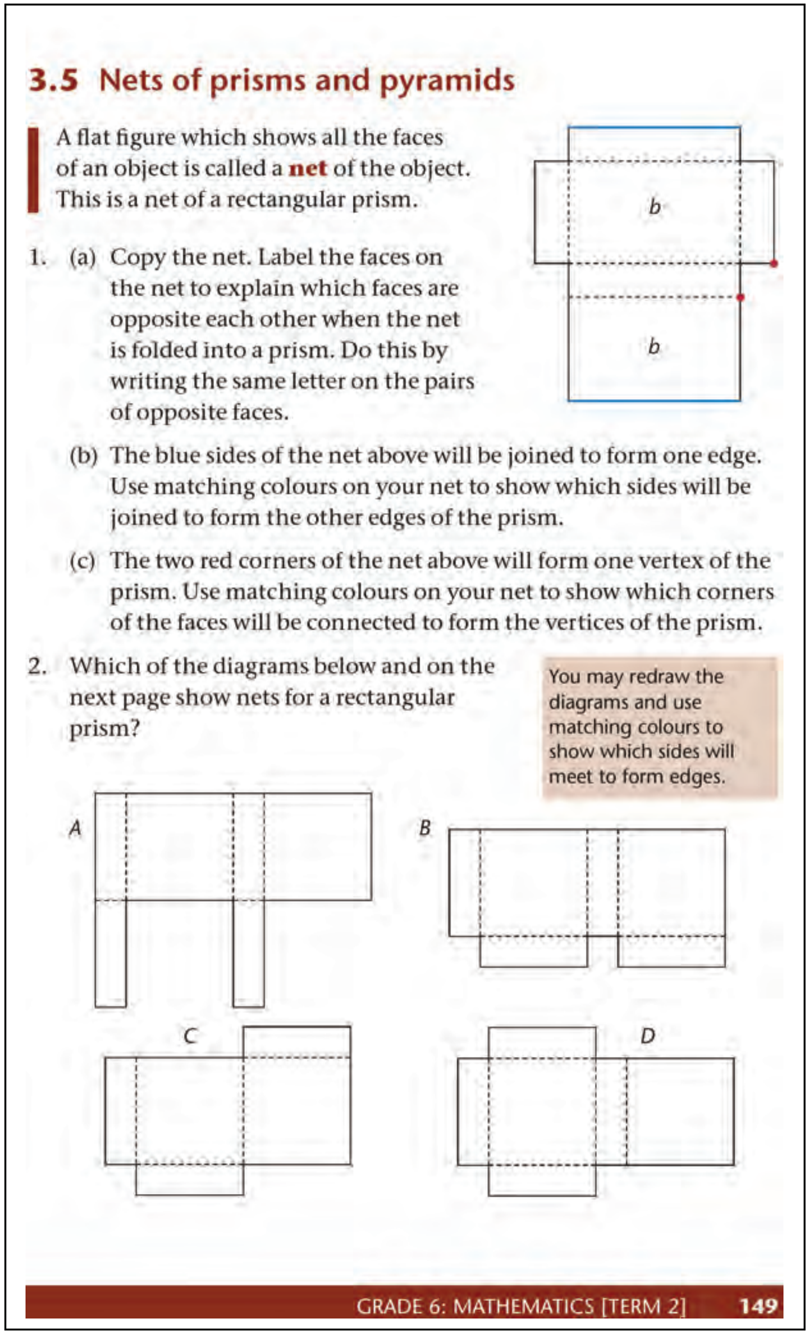
Many of your learners have probably seen an "exploded" (opened and flattened) cardboard box. This is a good way to introduce the idea of a net. Allow your learners to investigate which faces are connected along their edges and which faces are opposite each other (each face is connected to four others and there are six in total – three pairs of faces that are identical and opposite each other). Do this before moving on to other prisms, and pyramids.

As far as possible, provide learners with actual nets to cut out and fold. Encourage them to investigate how the edges and corners come together. Many repetitions of folding and unfolding may be necessary before they begin to develop a "mental map" of the relationships. Allow them to compare the net of each object with a model of the object built in the previous section.

Answers

1. (a)–(c) Learners' own work
2. Only Diagram D is a net for a rectangular prism.

   In Diagrams A and B two sides will overlap and one face will be open/missing. In Diagrams C and E a face is missing. In Diagram F one of the faces is in the wrong position.



3.5 Nets of prisms and pyramids

A flat figure which shows all the faces of an object is called a **net** of the object. This is a net of a rectangular prism.

1. (a) Copy the net. Label the faces on the net to explain which faces are opposite each other when the net is folded into a prism. Do this by writing the same letter on the pairs of opposite faces.

   (b) The blue sides of the net above will be joined to form one edge. Use matching colours on your net to show which sides will be joined to form the other edges of the prism.

   (c) The two red corners of the net above will form one vertex of the prism. Use matching colours on your net to show which corners of the faces will be connected to form the vertices of the prism.

2. Which of the diagrams below and on the next page show nets for a rectangular prism?

You may redraw the diagrams and use matching colours to show which sides will meet to form edges.

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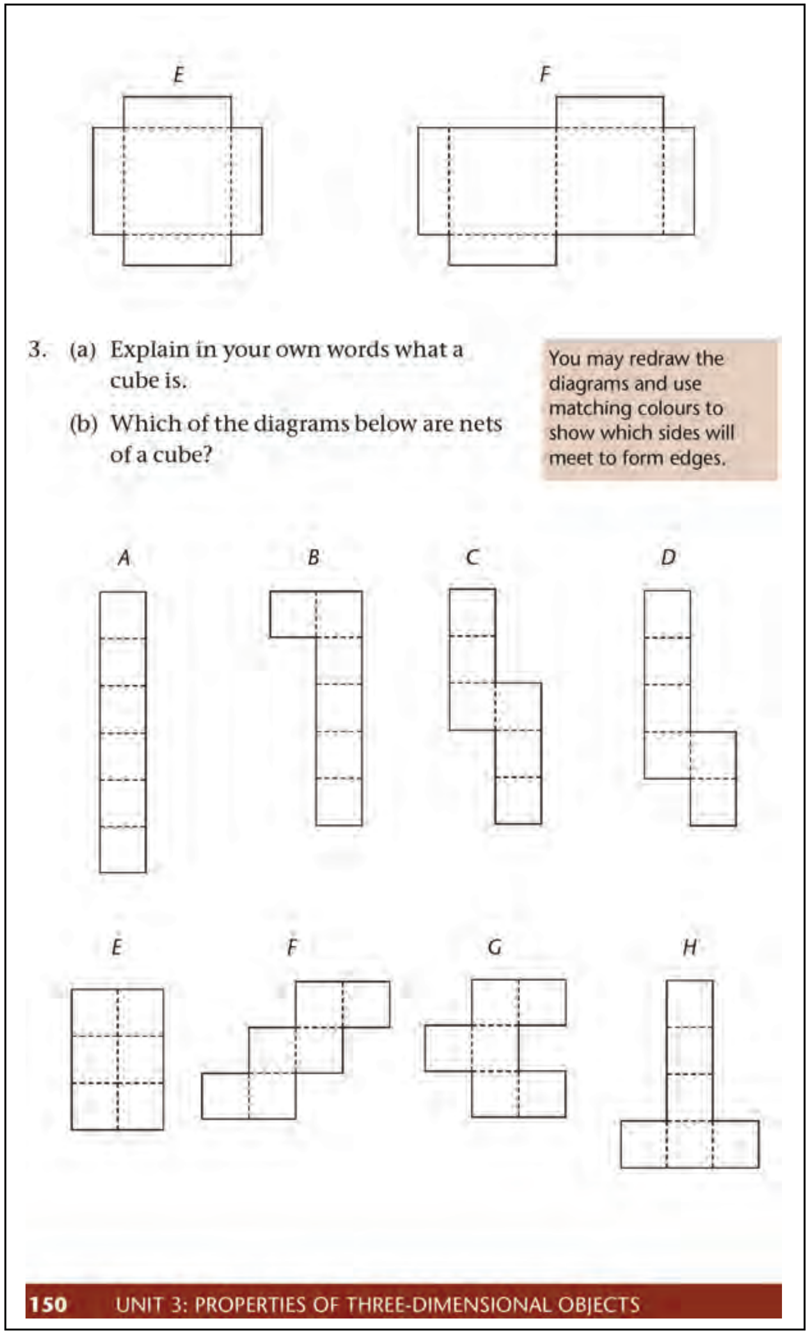
Possible misconceptions

The spatial arrangement of the faces of a rectangular prism can be very challenging for young learners. If they struggle to "see" how the faces relate, especially with a cube where all six faces are identical squares, give them some cut-outs of nets and let them fold them into the prism, and unfold them to investigate which edges meet and which faces are opposite each other. Insufficient experience viewing 3-D objects, and folding and unfolding their nets to see how the parts fit, will result in learners having a great deal of trouble identifying relationships between faces and edges.

Answers

3. (a) A cube is an object with six identical/equal square faces.
   (b) Cubes: C, F and H



3. (a) Explain in your own words what a cube is.
   (b) Which of the diagrams below are nets of a cube?

You may redraw the diagrams and use matching colours to show which sides will meet to form edges.

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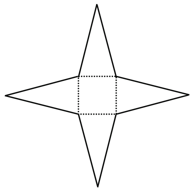
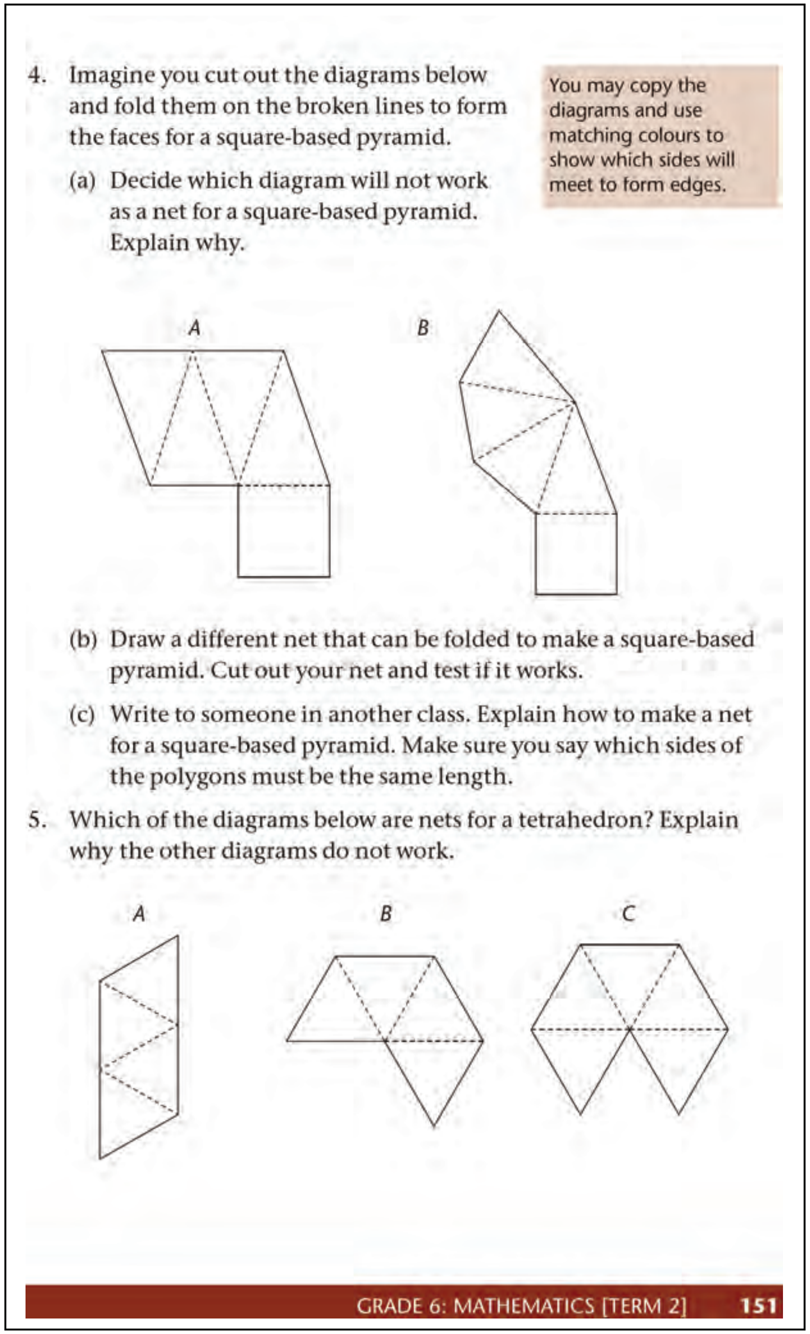
Answers

4. (a) Diagram B: the base edges of the four triangular faces will meet the four edges of the square base.
   (b) Example:



   (c) Learners' own work
   Example: Draw a square. At each side of the square draw identical triangles.

5. Diagram A: Yes
   Diagram B: Yes
   Diagram C: No; only four faces are needed and the net has five faces.



4. Imagine you cut out the diagrams below and fold them on the broken lines to form the faces for a square-based pyramid.
   (a) Decide which diagram will not work as a net for a square-based pyramid. Explain why.

You may copy the diagrams and use matching colours to show which sides will meet to form edges.

   (b) Draw a different net that can be folded to make a square-based pyramid. Cut out your net and test if it works.
   (c) Write to someone in another class. Explain how to make a net for a square-based pyramid. Make sure you say which sides of the polygons must be the same length.

5. Which of the diagrams below are nets for a tetrahedron? Explain why the other diagrams do not work.

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Grade 6 Term 2 Unit 4 Geometric patterns

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
4.1 Making beautiful patterns	Connecting geometric and numerical properties	152 to 153
4.2 Writing calculation plans	Learning to use the geometric structures to deduce rules	154 to 155
4.3 Describing patterns	Applying and practising structural thinking to deduce rules	156 to 157
4.4 From pictures to tables	More application and practice	158
4.5 More pictures and tables	More application and practice	159 to 160

CAPS time allocation	6 hours
CAPS page references	19 and 247 to 249

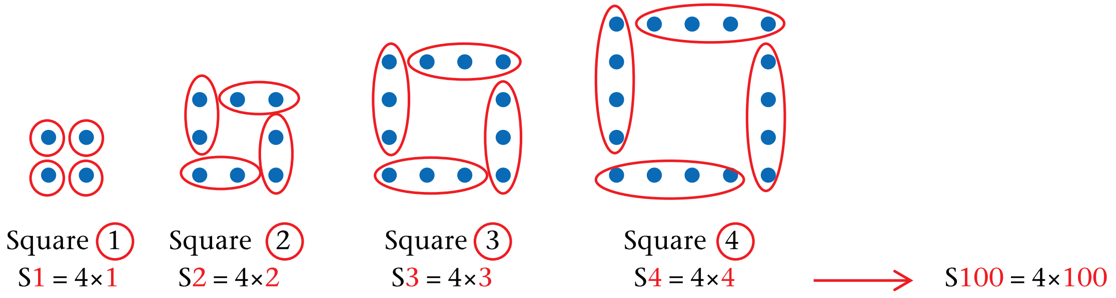
While providing opportunities to develop understanding of patterns, continuing sequences or completing tables according to a pattern also contributes to the development of the **Mental Mathematics** section of the CAPS.

In terms of developing the concept of geometric patterns, Sections 4.2 and 4.3 are **critically important**. Section 4.5 is not essential and may be used for enrichment and consolidation, as necessary.

Mathematical background

The approach in this unit is not to reduce the work on geometric patterns to numeric patterns in tables – that too – but to capitalise on the *visual* aspects of geometric representations as a method to find **rules** based on the *structure* of the geometric figures. This implies that you should help learners to not simply *count* the number of dots in a figure (counting in ones), but to use "clever counting" by identifying appropriate larger units. Then they should not actually *count* the larger units, but rather write down a **numerical expression** (calculation plan or **rule**) describing the number of dots. It is very important that learners learn to withhold immediate calculation of a numerical expression; they should first study the structure of the expression as an object.

To find a general rule for the pattern requires a second level of pattern recognition, namely recognising the structure in a series of numerical expressions – what is unchanged (is **constant**) and what changes (is **variable**). This process is illustrated below.



4.1 Making beautiful patterns

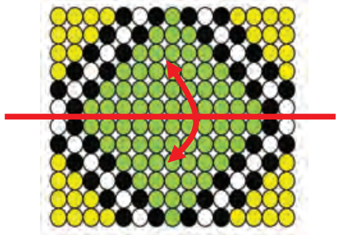
Teaching guidelines

This is a very rich mathematical activity – rich in content and processes, with rich learning and mathematical rewards.

Although it may look as if the counting of the beads will be laborious and time-consuming, with the right mathematical mindset it should not be. And that is what mathematics is all about.

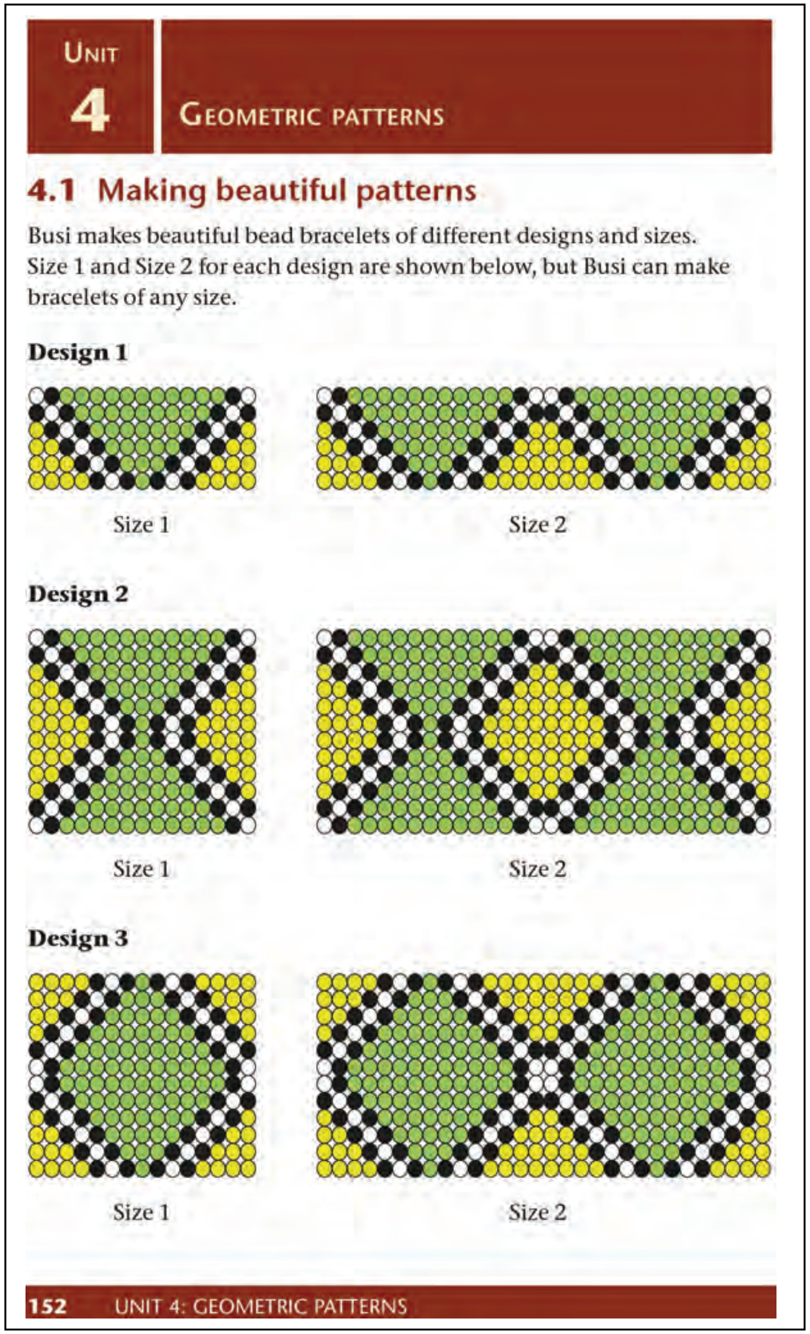
We therefore strongly advise that learners do all the activities. There are at least two reasons why it should not be time-consuming. Firstly, to enter the fray of mathematical reasoning, learners should not count in ones, but should use "clever counting", i.e. identify larger repeating units and analyse the *structure* of the situation. For example, in Design 1, Size 2, there are 4 strips of white beads with 6 beads in each, so we know immediately that there are 4×6 = 24 white beads.

Secondly, we can use the properties of the geometric figures in the design; these figures already are larger units of triangles, squares and rectangles, not beads that must be counted one by one. For example, take the green triangle in Design 1: if we reflect the triangle, we get a 6×6 square – see the square in Design 3. Also see the Challenge on page 153 of the Learner Book.



We should also realise that it is not necessary to analyse Design 2 and Design 3 in detail – Design 2 is merely double Design 1 through reflection! And Design 3 has exactly the same number of beads as Design 2 through transformation of the design figures.

You should not tell the learners this and deprive them of a surprise and a personal learning experience.



UNIT 4 GEOMETRIC PATTERNS

4.1 Making beautiful patterns

Busi makes beautiful bead bracelets of different designs and sizes. Size 1 and Size 2 for each design are shown below, but Busi can make bracelets of any size.

Design 1

Size 1 Size 2

Design 2

Size 1 Size 2

Design 3

Size 1 Size 2

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Answers

1. (a) Learners describe the pattern in their own words.

(b)

Size	**1**	**2**	**3**	**4**	**5**	**30**
No. of white beads	12	24	36	48	60	360
No. of black beads	22	44	66	88	110	660
No. of yellow beads	20	40	60	80	100	600
No. of green beads	36	72	108	144	180	1 080
Total no. of beads	90	180	270	360	450	2 700

(c)–(e): White: 10, 20, 100 $\xrightarrow{\boxed{\times 12}}$ 120, 240, 1 200

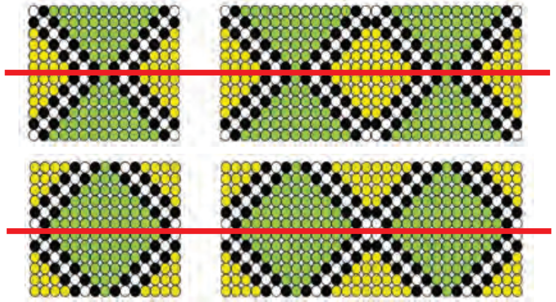
Black: 10, 20, 100 $\xrightarrow{\boxed{\times 22}}$ 220, 440, 2 200

Yellow: 10, 20, 100 $\xrightarrow{\boxed{\times 20}}$ 200, 400, 2 000

Green: 10, 20, 100 $\xrightarrow{\boxed{\times 36}}$ 360, 720, 3 600

Total: 10, 20, 100 $\xrightarrow{\boxed{\times 90}}$ 900, 1 800, 9 000

2. There is no need to do Design 2 – it is simply double Design 1, as shown here.
3. There is no need to do Design 3 – it is simply Design 2 rearranged, as shown here:



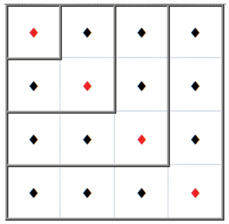
Note on the Challenge

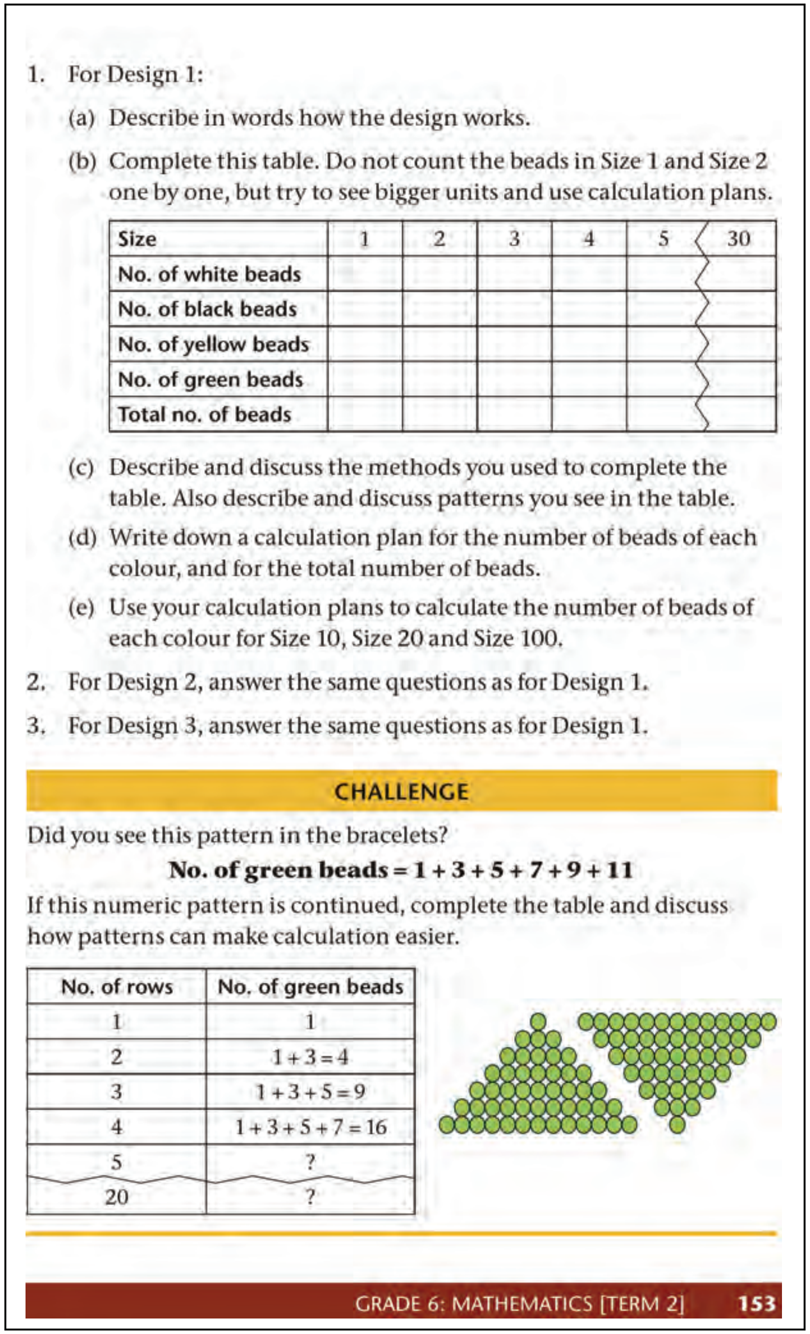
The sum of the first n odd numbers is equal to $n \times n$. Can you explain why?

So 1 + 3 + 5 + 7 ... to 20 numbers = 20 × 20 = 400

Here is a visual explanation that you may find useful:

1 + 3 + 5 + 7 + 9 + ... = ?





1. For Design 1:
   (a) Describe in words how the design works.
   (b) Complete this table. Do not count the beads in Size 1 and Size 2 one by one, but try to see bigger units and use calculation plans.

Size	1	2	3	4	5	30
No. of white beads						
No. of black beads						
No. of yellow beads						
No. of green beads						
Total no. of beads						

   (c) Describe and discuss the methods you used to complete the table. Also describe and discuss patterns you see in the table.
   (d) Write down a calculation plan for the number of beads of each colour, and for the total number of beads.
   (e) Use your calculation plans to calculate the number of beads of each colour for Size 10, Size 20 and Size 100.
2. For Design 2, answer the same questions as for Design 1.
3. For Design 3, answer the same questions as for Design 1.

CHALLENGE

Did you see this pattern in the bracelets?

No. of green beads = 1 + 3 + 5 + 7 + 9 + 11

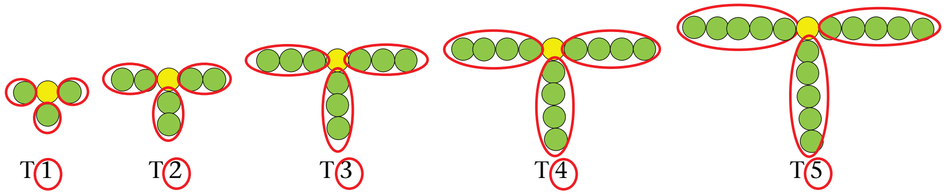
If this numeric pattern is continued, complete the table and discuss how patterns can make calculation easier.

No. of rows	No. of green beads
1	1
2	1 + 3 = 4
3	1 + 3 + 5 = 9
4	1 + 3 + 5 + 7 = 16
5	?
20	?

4.2 Writing calculation plans

Teaching guidelines

All the geometric patterns in this section can be transformed to numeric patterns by completing a table. However, the focus in this unit should be on solving the problems directly in the geometric context by "seeing" the structure in the geometric representations. This can be achieved by continuing the mindset of "clever counting". The way to "see" structure is to understand that in Figure 4 we try to see a unit of 4, in Figure 3 we try to see a unit of 3 in the same way, and so on. For example, for the T-shape in question 4:



← ←
T2 = 3×2 + 1

← ←
T3 = 3×3 + 1

← ←
Three fours plus 1
T4 = 3×4 + 1

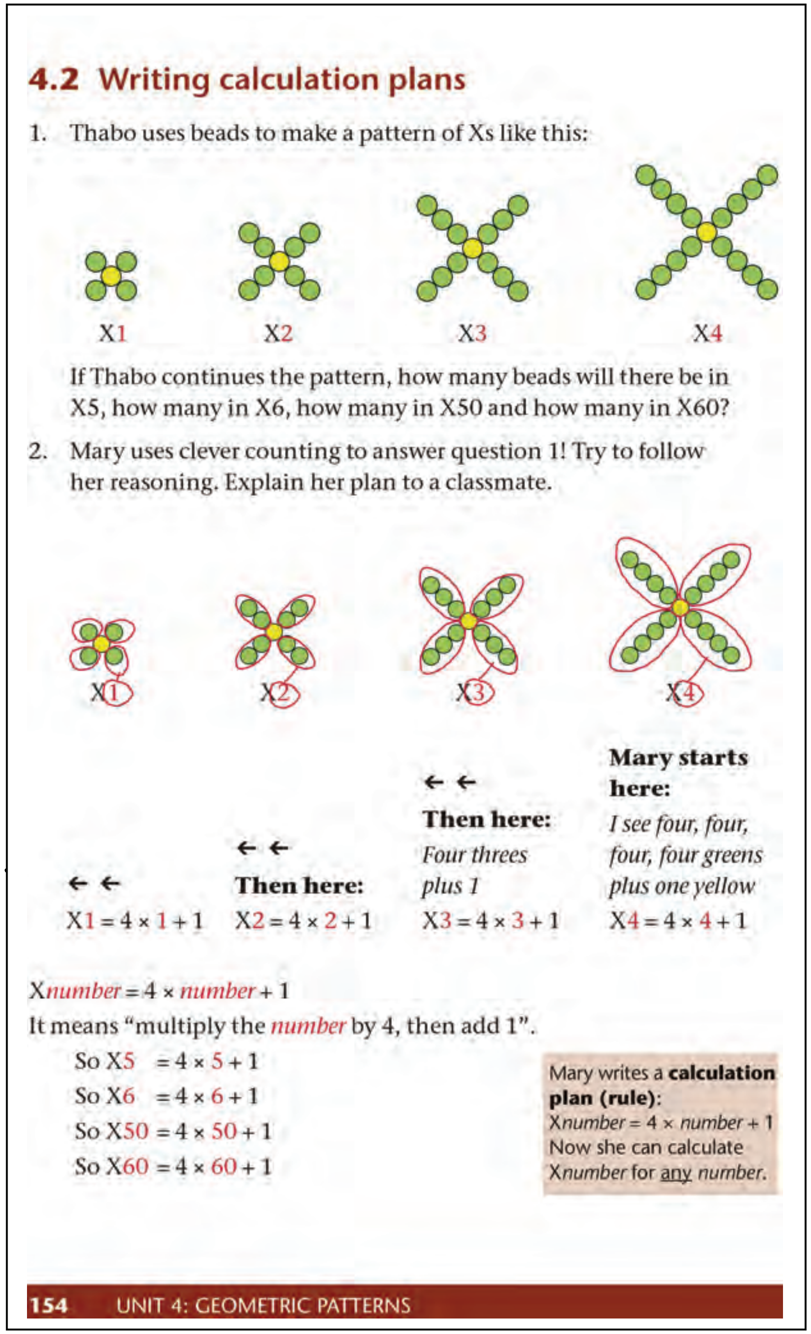
Five, five, five greens plus one yellow
T5 = 3×5 + 1

Pattern recognition now proceeds at a different level: we need to see the *structure,* not in a number sequence or in a figure, but in a series of related numerical expressions. This requires us to be very clear about what is changing (the **variable**) and what remains unchanged (**constant**). If you present it vertically it may help learners to see the pattern:

$$T1 = 3×1 + 1$$
$$T2 = 3×2 + 1$$
$$T3 = 3×3 + 1$$
$$T4 = 3×4 + 1$$
So, $T30 = 3×30 + 1$

Answers

1. X5: 21; X6: 25; X50: 201; X60: 241
2. Mary describes the structure of the pattern with a calculation plan: X*number* = 4 × *number* + 1. This way, she can calculate the number of beads in any design number in the pattern, such as X5, X6, X50, X60, etc.



4.2 Writing calculation plans

1. Thabo uses beads to make a pattern of Xs like this:

If Thabo continues the pattern, how many beads will there be in X5, how many in X6, how many in X50 and how many in X60?

2. Mary uses clever counting to answer question 1! Try to follow her reasoning. Explain her plan to a classmate.

← ←
X1 = 4 × 1 + 1

← ←
Then here:
X2 = 4 × 2 + 1

← ←
Then here:
Four threes plus 1
X3 = 4 × 3 + 1

Mary starts here:
I see four, four, four, four greens plus one yellow
X4 = 4 × 4 + 1

X*number* = 4 × *number* + 1
It means "multiply the *number* by 4, then add 1".

So X5 = 4 × 5 + 1
So X6 = 4 × 6 + 1
So X50 = 4 × 50 + 1
So X60 = 4 × 60 + 1

Mary writes a **calculation plan (rule):**
X*number* = 4 × *number* + 1
Now she can calculate X*number* for any *number*.

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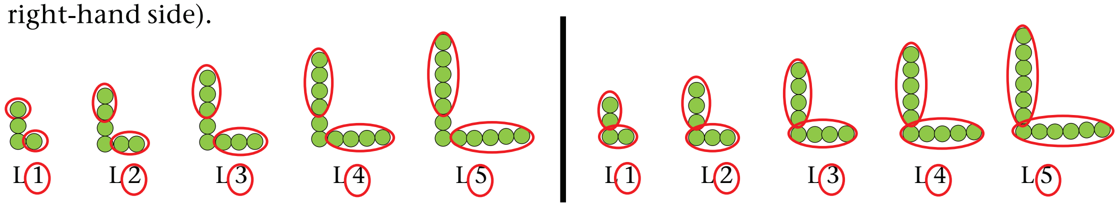
Note that if we choose different units, it will lead to **equivalent** calculation plans (different calculations leading to the same answer). For example, the work in these two diagrams leads to L87 = 2×87 + 2 = 176 (on the left-hand side) and L87 = 2×88 = 176 (on the right-hand side).



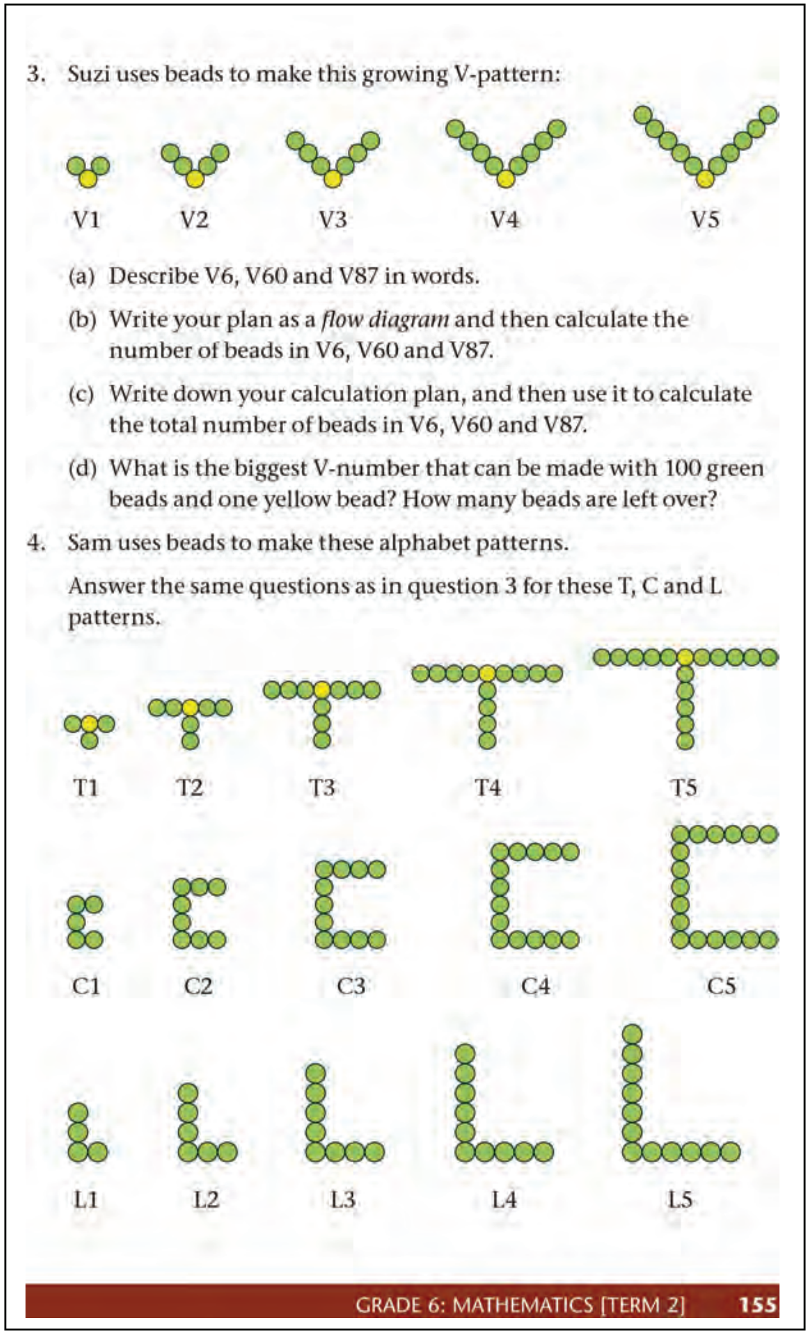
Answers

3. (a) V6: 2 arms of 6 green beads each and 1 yellow bead
V60: 2 arms of 60 green beads each and 1 yellow bead
V87: 2 arms of 87 green beads each and 1 yellow bead
(b) $6, 60, 87 \rightarrow \boxed{\times 2} \rightarrow \boxed{+ 1} \rightarrow 13, 121, 175$
(c) Multiply V-number by 2 and add 1
(d) V49; 1 green bead left over

4. T: (a) T6: 3 arms of 6 green beads each and 1 yellow bead
T60: 3 arms of 60 green beads each and 1 yellow bead
T87: 3 arms of 87 green beads each and 1 yellow bead
(b) $6, 60, 87 \rightarrow \boxed{\times 3} \rightarrow \boxed{+ 1} \rightarrow 19, 181, 262$
(c) Multiply T-number by 3 and add 1
(d) T33; one green bead left over

C: (a) C6: 3 groups of 6 beads plus 2 more
C60: 3 groups of 60 beads plus 2 more
C87: 3 groups of 87 beads plus 2 more
(b) $6, 60, 87 \rightarrow \boxed{\times 3} \rightarrow \boxed{+ 2} \rightarrow 20, 182, 263$
(c) Multiply C-number by 3 and add 2
(d) C32; two beads left over

L: (a) L6: 2 groups of 6 beads plus 2 more or 2 groups of 7
L60: 2 groups of 60 beads plus 2 more or 2 groups of 61
L87: 2 groups of 87 beads plus 2 more or 2 groups of 88
(b) $6, 60, 87 \rightarrow \boxed{\times 2} \rightarrow \boxed{+ 2} \rightarrow 14, 122, 176$ or $6, 60, 87 \rightarrow \boxed{+1} \rightarrow \boxed{\times 2} \rightarrow 14, 122, 176$
(c) Multiply L-number by 2 and add 2 or add 1 to L-number and double
(d) L49; nothing left over



3. Suzi uses beads to make this growing V-pattern:

(a) Describe V6, V60 and V87 in words.
(b) Write your plan as a *flow diagram* and then calculate the number of beads in V6, V60 and V87.
(c) Write down your calculation plan, and then use it to calculate the total number of beads in V6, V60 and V87.
(d) What is the biggest V-number that can be made with 100 green beads and one yellow bead? How many beads are left over?

4. Sam uses beads to make these alphabet patterns.

Answer the same questions as in question 3 for these T, C and L patterns.

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4.3 Describing patterns

Teaching guidelines

We emphasise again that learners should try to use the *structure* in the pictures to logically deduce calculation plans through generalisation. We illustrate with another example:

Size 1	Size 2	Size 3	Size 4
S1 = 2×1 + 0	S2 = 2×2 + 1	S3 = 2×3 + 2	S4 = 2×4 + 3

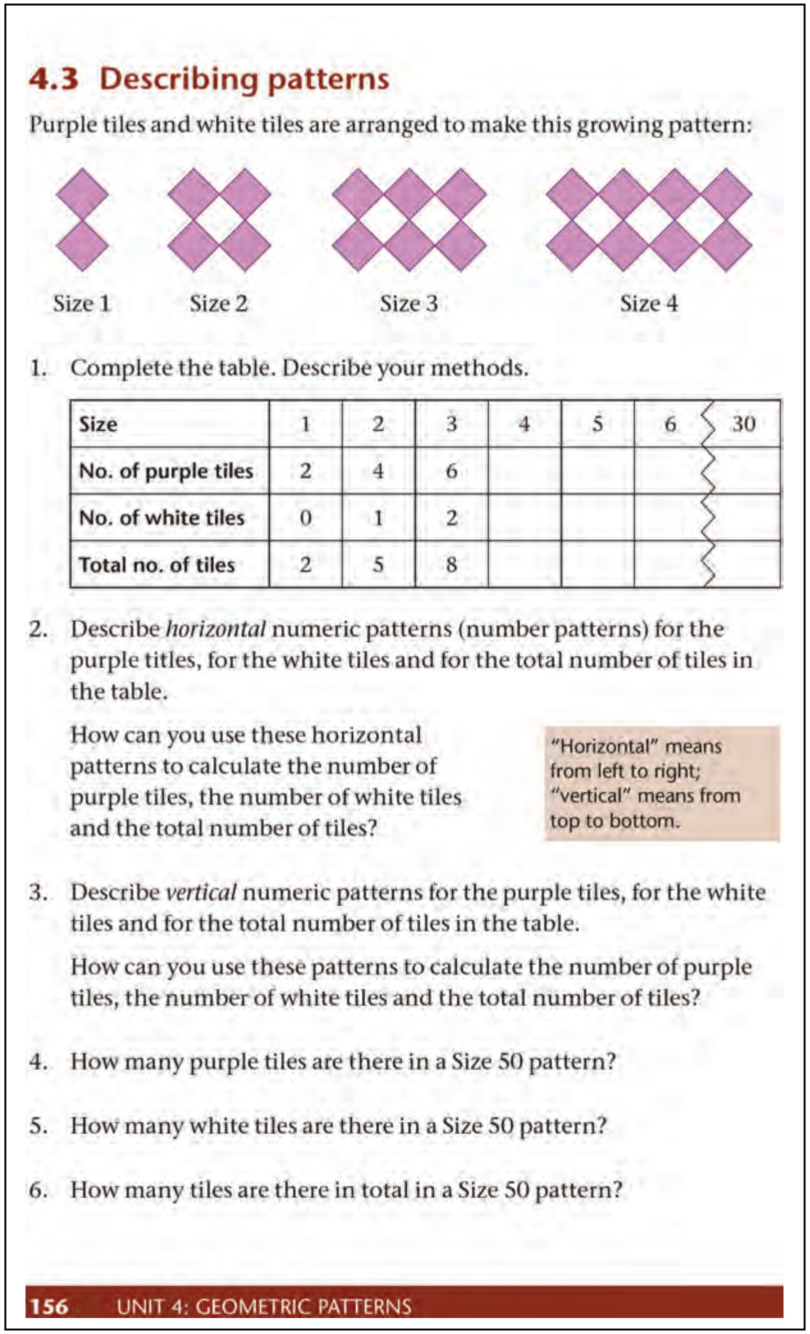
S1 = 2×1 + 0
S2 = 2×2 + 1
S3 = 2×3 + 2
S4 = 2×4 + 3
So, S30 = 2×30 + 29

Answers

1.

Size	1	2	3	4	5	6	30
No. of purple tiles	2	4	6	8	10	12	60
No. of white tiles	0	1	2	3	4	5	29
Total no. of tiles	2	5	8	11	14	17	89

2. Purple: Add 2 to every number to get the next number, starting with 2.
   White: Add 1 to every number to get the next number, starting with 0.
   Total: Add 3 to every number to get the next number, starting with 2.
3. Purple: Multiply size number by 2.
   White: Size number – 1.
   Total: Multiply size number by 2 and add 1.
   Or: Multiply size number by 3 and subtract 1.
4. 50 × 2 = 100
5. 50 – 1 = 49
6. 2 × 50 + 49 = 149



4.3 Describing patterns

Purple tiles and white tiles are arranged to make this growing pattern:

Size 1 Size 2 Size 3 Size 4

1. Complete the table. Describe your methods.

Size	1	2	3	4	5	6	30
No. of purple tiles	2	4	6				
No. of white tiles	0	1	2				
Total no. of tiles	2	5	8				

2. Describe *horizontal* numeric patterns (number patterns) for the purple titles, for the white tiles and for the total number of tiles in the table.

   How can you use these horizontal patterns to calculate the number of purple tiles, the number of white tiles and the total number of tiles?

   "Horizontal" means from left to right; "vertical" means from top to bottom.

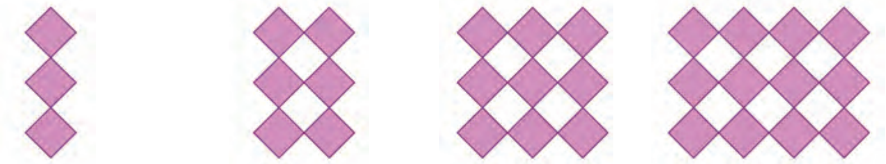
3. Describe *vertical* numeric patterns for the purple tiles, for the white tiles and for the total number of tiles in the table.

   How can you use these patterns to calculate the number of purple tiles, the number of white tiles and the total number of tiles?

4. How many purple tiles are there in a Size 50 pattern?
5. How many white tiles are there in a Size 50 pattern?
6. How many tiles are there in total in a Size 50 pattern?

Notes on questions

Note that it is sometimes useful to not focus on the size of the units, but rather on how many times a unit repeats. See if you and your learners can understand this change of thinking as illustrated for Pattern X: in Size 4 the unit 3 is repeated 4 times, and in Size 3 the unit 3 is repeated 3 times ...



Size 1 Size 2 Size 3 Size 4

X1=1×3 + 0×2 X2 = 2×3 + 1×2 X3 = 3×3 + 2×2 X4 = 4×3 + 3×2

Answers to question 7

Pattern X

1.

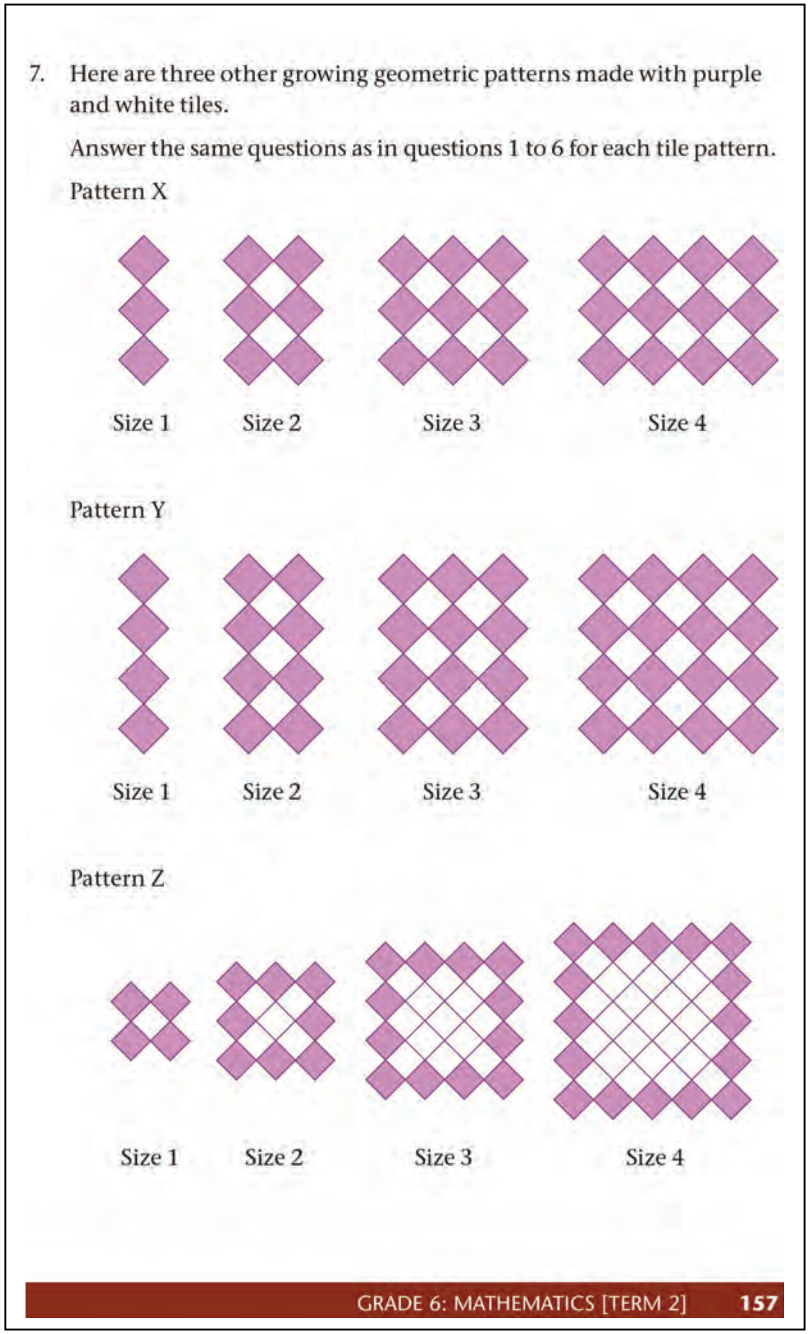
Size	1	2	3	4	5	6	30
No. of purple tiles	3	6	9	12	15	18	90
No. of white tiles	0	2	4	6	8	10	58
Total no. of tiles	3	8	13	18	23	28	148

2. Purple tiles: Add 3 to every number to get the next number, starting with 3.
   White tiles: Add 2 to every number to get the next number, starting with 0.
   Total number of tiles: Add 5 to every number to get the next one, starting with 3.
3. Purple tiles: Multiply size number by 3.
   White tiles: Multiply 1 less than size number by 2, i.e. 2×(Size number – 1).
   Total number of tiles: Multiply size number by 5 and subtract 2.
4. 50 × 3 = 150
5. 2×(50 – 1) = 2 × 49 = 98
6. 5 × 50 – 2 = 248

Pattern Y *(Answers continued on next page)*

1.

Size	1	2	3	4	5	6	30
No. of purple tiles	4	8	12	16	20	24	120
No. of white tiles	0	3	6	9	12	15	87
Total no. of tiles	4	11	18	25	32	39	207



7. Here are three other growing geometric patterns made with purple and white tiles.

   Answer the same questions as in questions 1 to 6 for each tile pattern.

   Pattern X

   Size 1 Size 2 Size 3 Size 4

   Pattern Y

   Size 1 Size 2 Size 3 Size 4

   Pattern Z

   Size 1 Size 2 Size 3 Size 4

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Answers to question 7 *(continued)*

2. Purple tiles: Add 4 to every number to get the next number, starting with 4.
   White tiles: Add 3 to every number to get the next number, starting with 0.
   Total number of tiles: Add 7 to every number to get the next one, starting with 4.
3. Purple tiles: Multiply size number by 4.
   White tiles: Multiply 1 less than size number by 3, i.e. 3×(Size number – 1).
   Total number of tiles: Multiply size number by 7 and subtract 3.
4. 50 × 4 = 200
5. 3×(50 – 1) = 3 × 49 = 147
6. 7 × 50 – 3 = 347

Pattern Z

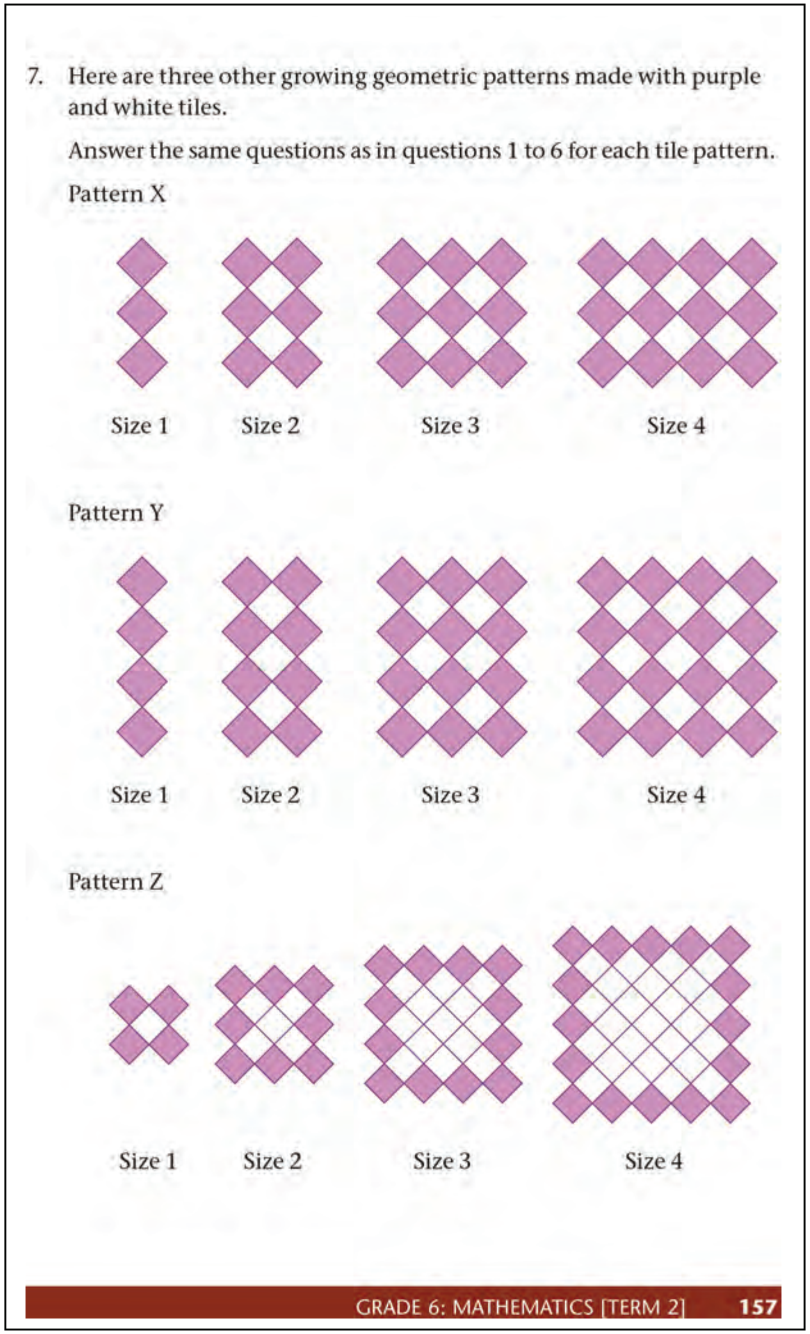
Note that Pattern Z is best left for enrichment.

Also note that in this case it is probably easier to find the total number of tiles by simply adding the number of purple and white tiles instead of trying to find a rule.

1.

Size	1	2	3	4	5	6	30
No. of purple tiles	4	8	12	16	20	24	120
No. of white tiles	1	5	13	25	41	61	1 741
Total no. of tiles	5	13	25	41	61	85	1 861

2. Purple tiles: Add 4 to every number to get the next number, starting with 4.
   White tiles: Add the number of purple tiles to every number to get the next number.
   Total no. of titles: Starting with 5, add consecutive multiples of 4, starting with 2 × 4 to each number to get the next number (i.e. 5 + 2×4 = 13; 13 + 3×4 = 25, etc.).
3. Purple tiles: Multiply size number by 4.
4. 50 × 4 = 200
5. $(50 - 1)^2 + 50^2 = 49 \times 49 + 50 \times 50 = 2\,401 + 2\,500 = 4\,901$
6. 50 × 50 + 51 × 51 = 2 500 + 2 601 = 5 101 or simply 200 + 4 901 = 5 101



7. Here are three other growing geometric patterns made with purple and white tiles.

   Answer the same questions as in questions 1 to 6 for each tile pattern.

Pattern X

Size 1 Size 2 Size 3 Size 4

Pattern Y

Size 1 Size 2 Size 3 Size 4

Pattern Z

Size 1 Size 2 Size 3 Size 4

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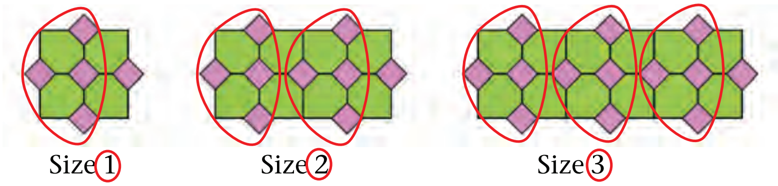
4.4 From pictures to tables

Teaching guidelines

This is an interesting geometric "family of sequences". If learners decide to make a table of values, they can use the knowledge of family of sequences we developed in the first term:

- The sequence of green tiles is 4, 8, 12, ... The multiples of 4 ...
- The sequence of purple tiles is 5, 9, 13, ... One more than multiples of 4 ...

If learners choose to follow a geometric visual approach, they can reason as follows:



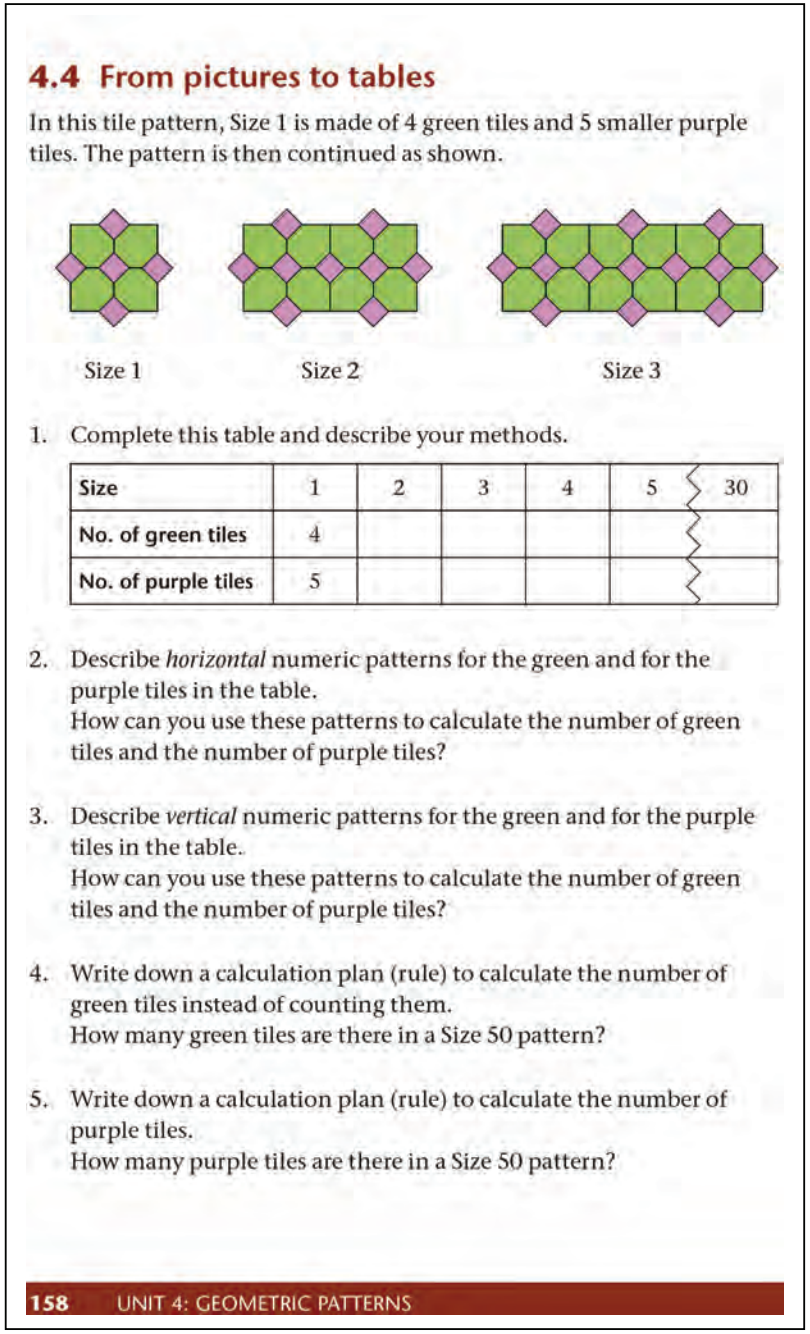
Green 1 = 1×4	Green 2 = 2×4	Green 3 = 3×4	Green 30 = 30×4
Purple 1 = 1×4 + 1	Purple 2 = 2×4 + 1	Purple 3 = 3×4 + 1	Purple 30 = 30×4 + 1

Answers

1.

Size	1	2	3	4	5	30
No. of green tiles	4	8	12	16	20	120
No. of purple tiles	5	9	13	17	21	121

2. Green tiles: Add 4 to every number to get the next number, starting with 4.
   Purple tiles: Add 4 to every number to get the next number, starting with 5.
   Green tile numbers are multiples of 4 and purple tile numbers are one more than a multiple of 4.
3. Green tiles: Multiply size number by 4.
   Purple tiles: Multiply size number by 4 and add 1.
4. Multiply size number by 4.
   Size 50 has 200 green tiles.
5. Multiply size number by 4 and add 1.
   Size 50 has 201 purple tiles.



4.4 From pictures to tables

In this tile pattern, Size 1 is made of 4 green tiles and 5 smaller purple tiles. The pattern is then continued as shown.

Size 1 Size 2 Size 3

1. Complete this table and describe your methods.

Size	1	2	3	4	5	30
No. of green tiles	4					
No. of purple tiles	5					

2. Describe *horizontal* numeric patterns for the green and for the purple tiles in the table.
   How can you use these patterns to calculate the number of green tiles and the number of purple tiles?
3. Describe *vertical* numeric patterns for the green and for the purple tiles in the table.
   How can you use these patterns to calculate the number of green tiles and the number of purple tiles?
4. Write down a calculation plan (rule) to calculate the number of green tiles instead of counting them.
   How many green tiles are there in a Size 50 pattern?
5. Write down a calculation plan (rule) to calculate the number of purple tiles.
   How many purple tiles are there in a Size 50 pattern?

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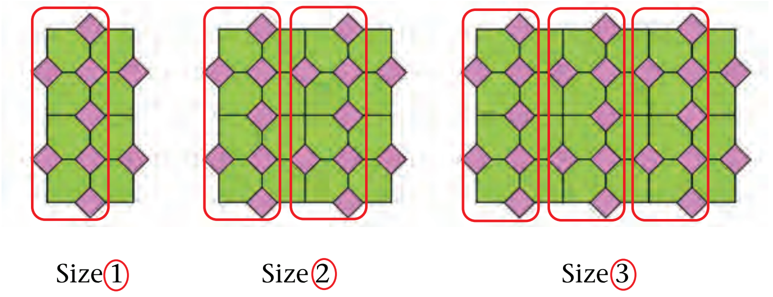
4.5 More pictures and tables

Teaching guidelines

This section is intended as an "extra", to be used for enrichment or consolidation as you may need it. It uses the same thinking as in the previous section:



Green 1 = 1×8
Purple 1 = 1×7 + 2

Green 2 = 2×8
Purple 2 = 2×7 + 2

Green 3 = 3×8
Purple 3 = 3×7 + 2 ....

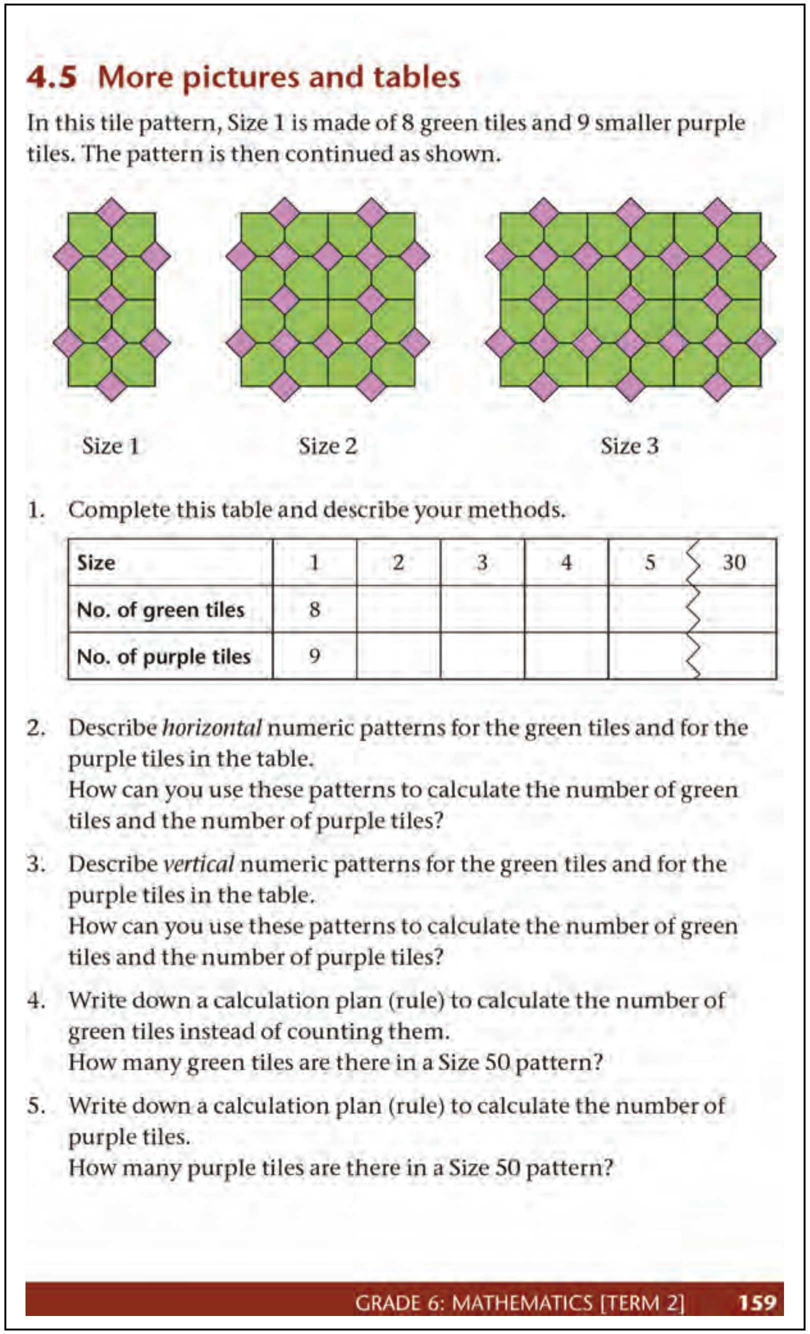
Green 30 = 30×8
Purple 30 = 30×7 + 2

Answers

1.

Size	1	2	3	4	5	30
No. of green tiles	8	16	24	32	40	200
No. of purple tiles	9	16	23	30	37	212

2. Green tiles: Starting with 8, add 8 to each number to get the next number.
   Purple tiles: Starting with 9, add 7 to each number to get the next number.
   The number of green tiles is a multiple of 8 and the number of purple tiles is two more than a multiple of 7.
3. Green tiles: Multiply size number by 8.
   Purple tiles: Multiply size number by 7 and add 2; (7 × size number + 2).
4. Green tiles = Size number × 8.
   50 × 8 = 400 green tiles in Size 50 pattern.
5. Purple tiles: Multiply size number by 7 and add 2; (7 × size number + 2).
   50 × 7 + 2 = 352 purple tiles in Size 50 pattern.



4.5 More pictures and tables

In this tile pattern, Size 1 is made of 8 green tiles and 9 smaller purple tiles. The pattern is then continued as shown.

Size 1 Size 2 Size 3

1. Complete this table and describe your methods.

Size	1	2	3	4	5	30
No. of green tiles	8					
No. of purple tiles	9					

2. Describe *horizontal* numeric patterns for the green tiles and for the purple tiles in the table.
   How can you use these patterns to calculate the number of green tiles and the number of purple tiles?
3. Describe *vertical* numeric patterns for the green tiles and for the purple tiles in the table.
   How can you use these patterns to calculate the number of green tiles and the number of purple tiles?
4. Write down a calculation plan (rule) to calculate the number of green tiles instead of counting them.
   How many green tiles are there in a Size 50 pattern?
5. Write down a calculation plan (rule) to calculate the number of purple tiles.
   How many purple tiles are there in a Size 50 pattern?

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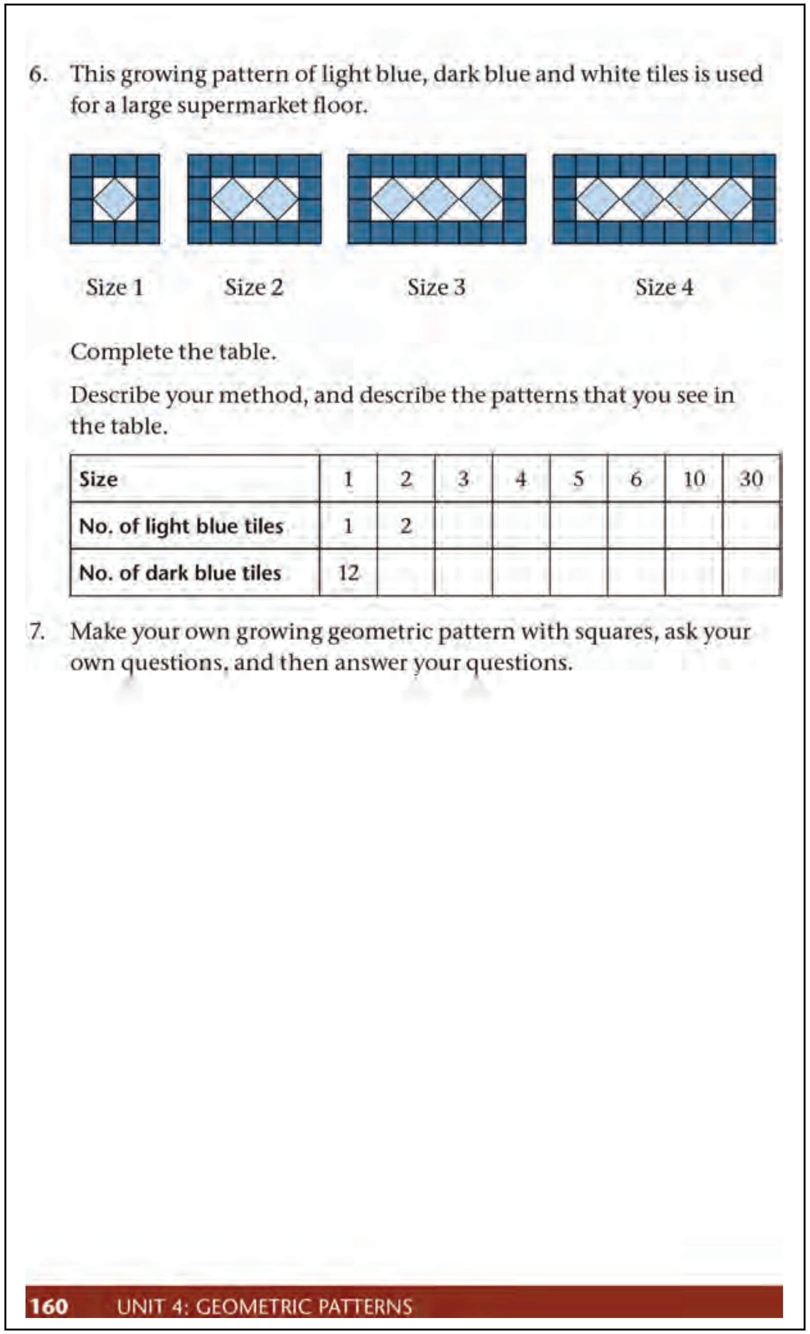
Answers

6.

Size	1	2	3	4	5	6	10	30
No. of light blue tiles	1	2	3	4	5	6	10	30
No. of dark blue tiles	12	16	20	24	28	32	48	128

Number of light blue tiles is the same as size number – add 1 to get the next number.
Dark blue tiles: starting with 12, add 4 to each number to get the next number.
Number of dark blue tiles is 4 times size number + 8.

7. Learners' own work



6. This growing pattern of light blue, dark blue and white tiles is used for a large supermarket floor.

Size 1 Size 2 Size 3 Size 4

Complete the table.

Describe your method, and describe the patterns that you see in the table.

Size	1	2	3	4	5	6	10	30
No. of light blue tiles	1	2						
No. of dark blue tiles	12							

7. Make your own growing geometric pattern with squares, ask your own questions, and then answer your questions.

160 UNIT 4: GEOMETRIC PATTERNS

Grade 6 Term 2 Unit 5 Symmetry

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
5.1 Lines of symmetry	Symmetry in polygons; identifying lines of symmetry in polygons	161
5.2 Many lines of symmetry	Polygons and other shapes with more than one line of symmetry	162 to 164

CAPS time allocation	2 hours
CAPS page references	23 and 249

Mathematical background

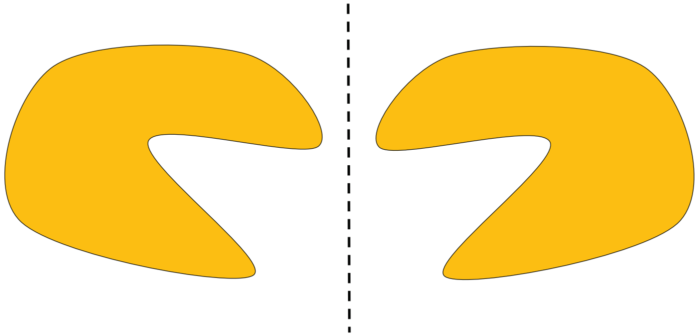
Symmetry occurs when a shape or design can be imagined as consisting of two "mirror halves". Stated differently, for any symmetrical shape we can imagine a line, called the line of symmetry, passing through the shape in such a way that if we fold along the line, every single line and point on one side of the line of symmetry lies on top of its twin on the other side of the line of symmetry – without exceptions.

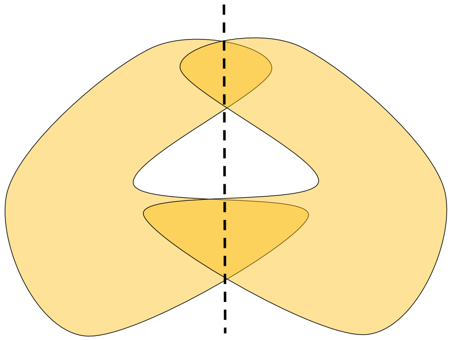
In reality, many shapes and objects that seem symmetrical aren't perfectly symmetrical. Also, some parts of a shape may have symmetry while the remaining parts do not. Some shapes may have two or more lines of symmetry. Whenever we talk of symmetry we also have to talk of a line of symmetry. If you have one, then you will always have the other.

A symmetrical design is formed whenever a shape is reflected along a line (see the units on Transformations in the Learner Book: Term 3 Unit 6, p. 247 and Term 4 Unit 9, p. 350).

"Line of symmetry" and "axis of reflection" are different names for the same thing.

In reflection we also have a line of symmetry. For example:





A shape can have more than one line of symmetry.

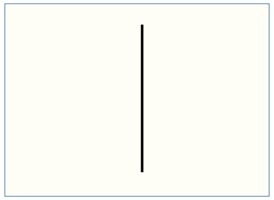
5.1 Lines of symmetry

Teaching guidelines

Learners should be familiar with symmetry by now, having studied it since Grade 4. The questions on this page provide learners with opportunities to refresh, consolidate and sharpen their understanding of symmetry. However, it may be useful to start the work by letting learners make a symmetrical figure. By drawing a symmetrical design in the way described below (which you may demonstrate on the board) learners may strengthen their understanding of symmetry substantially.

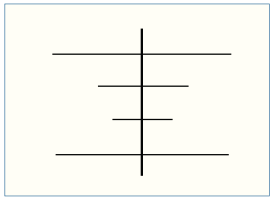
Step 1

Use a ruler and draw a vertical line on a clean sheet of paper, more or less in the middle.



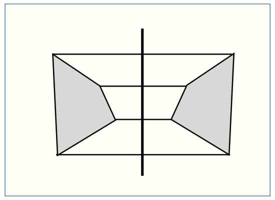
Step 2

Use the ruler to draw four lines of different lengths across, with their midpoints on the vertical line.



Step 3

Join the endpoints of the four horizontal lines on each side of the vertical line.



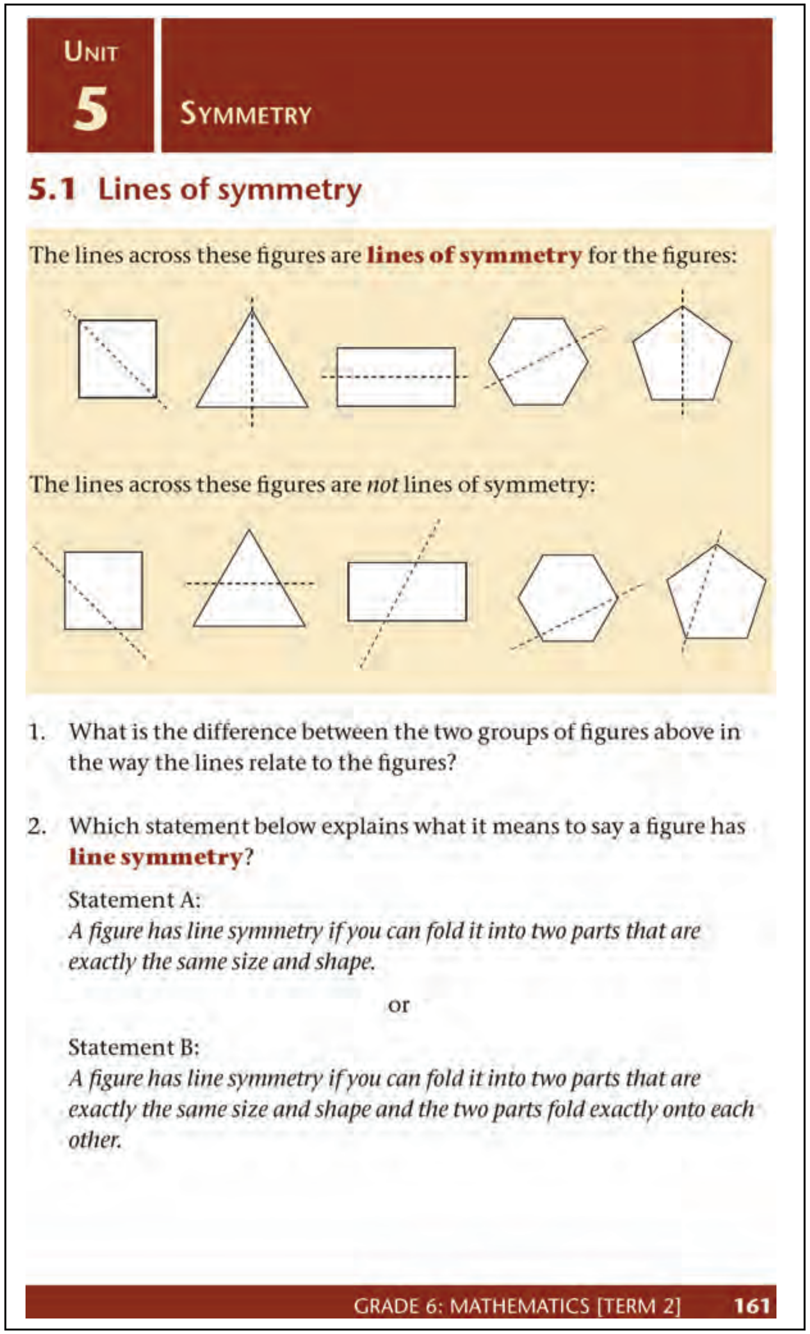
It is critical that learners try hard to produce answers for question 1, but it is not critical that they produce good answers. Many learners may not think of articulating an answer in terms of what happens if the figure is folded along the broken line, and this may make it almost impossible for them to produce a good answer to the question.

Another method is to draw a figure on one side of a sheet of paper, using thick pencil lines. Fold the sheet so that the drawn figure is on the inside, on one side, and rub hard where the figure was drawn. When you open the folded sheet, you will have a faint but visible mirror image of the original figure, and hence symmetry.

Note that the broken line through the rectangle in the middle of the bottom row in the shaded passage is not a line of symmetry.

Answers

1. The figures in the first set will form two parts that fit perfectly onto each other when folded on the broken lines. For the second set this is not true.
2. Statement B



UNIT 5 SYMMETRY

5.1 Lines of symmetry

The lines across these figures are **lines of symmetry** for the figures:

The lines across these figures are *not* lines of symmetry:

1. What is the difference between the two groups of figures above in the way the lines relate to the figures?
2. Which statement below explains what it means to say a figure has **line symmetry**?

   Statement A:
   A figure has line symmetry if you can fold it into two parts that are exactly the same size and shape.

   or

   Statement B:
   A figure has line symmetry if you can fold it into two parts that are exactly the same size and shape and the two parts fold exactly onto each other.

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5.2 Many lines of symmetry

Mathematical notes

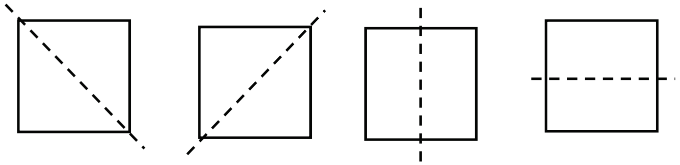
Some shapes have no symmetry (these are called asymmetrical shapes), some have one line of symmetry only, some have many but we can find them all, while others have too many to count (circular shapes are the only examples of objects with infinitely many lines of symmetry).

Notes on questions

In question 3 some learners may intuitively arrive at the incorrect decision that the parallelogram is symmetrical. Ignoring a momentary lapse in concentration, this is probably a sure sign that these learners do not fully understand symmetry. Learners who do this must be encouraged to think the folding process through (and if that fails, fold a given cut-out of the shape).

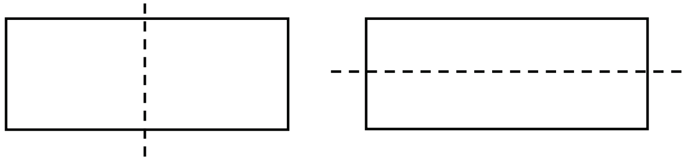
Answers

1. (a)



   (b) You can imagine folding the shape on the line or you can cut it out and fold it to test. If the parts fit exactly onto each other, your lines of symmetry are correctly drawn.

2. (a)



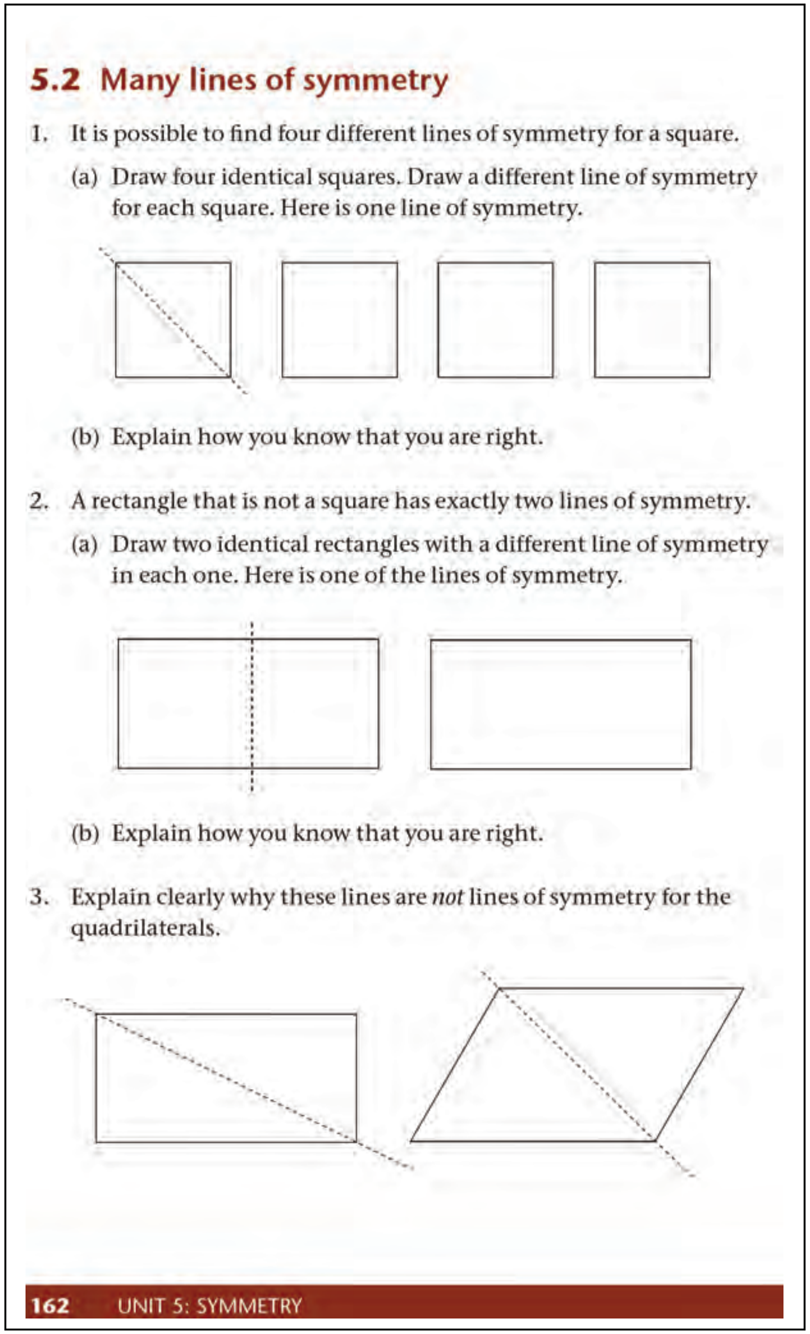
   (b) You can imagine folding the shape on the line or you can cut it out and fold it to test. If the parts fit exactly onto each other, your lines of symmetry are correctly drawn.

3. If the quadrilaterals are folded on the broken lines, the two parts will not fit exactly onto each other.

Questions for extension

A square has four equal sides and four lines of symmetry. Does any figure with four equal sides have four lines of symmetry?

Is a straight line a symmetrical figure?



5.2 Many lines of symmetry

1. It is possible to find four different lines of symmetry for a square.
   (a) Draw four identical squares. Draw a different line of symmetry for each square. Here is one line of symmetry.
   (b) Explain how you know that you are right.
2. A rectangle that is not a square has exactly two lines of symmetry.
   (a) Draw two identical rectangles with a different line of symmetry in each one. Here is one of the lines of symmetry.
   (b) Explain how you know that you are right.
3. Explain clearly why these lines are *not* lines of symmetry for the quadrilaterals.

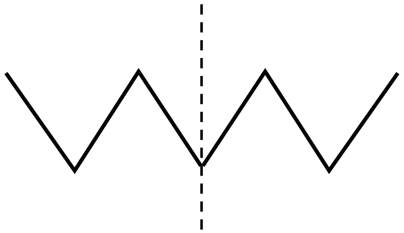
162 UNIT 5: SYMMETRY

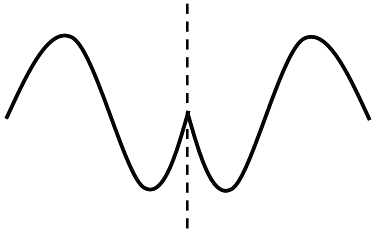
Answers

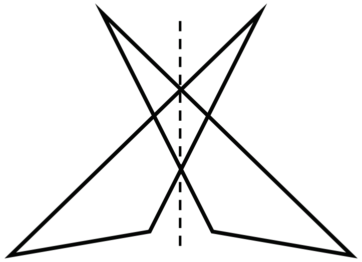
4. (a) 1 (b) 0 (c) 4
(d) 2 (e) 1 (f) 1

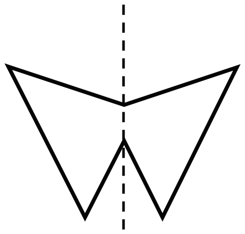
Possible misconceptions

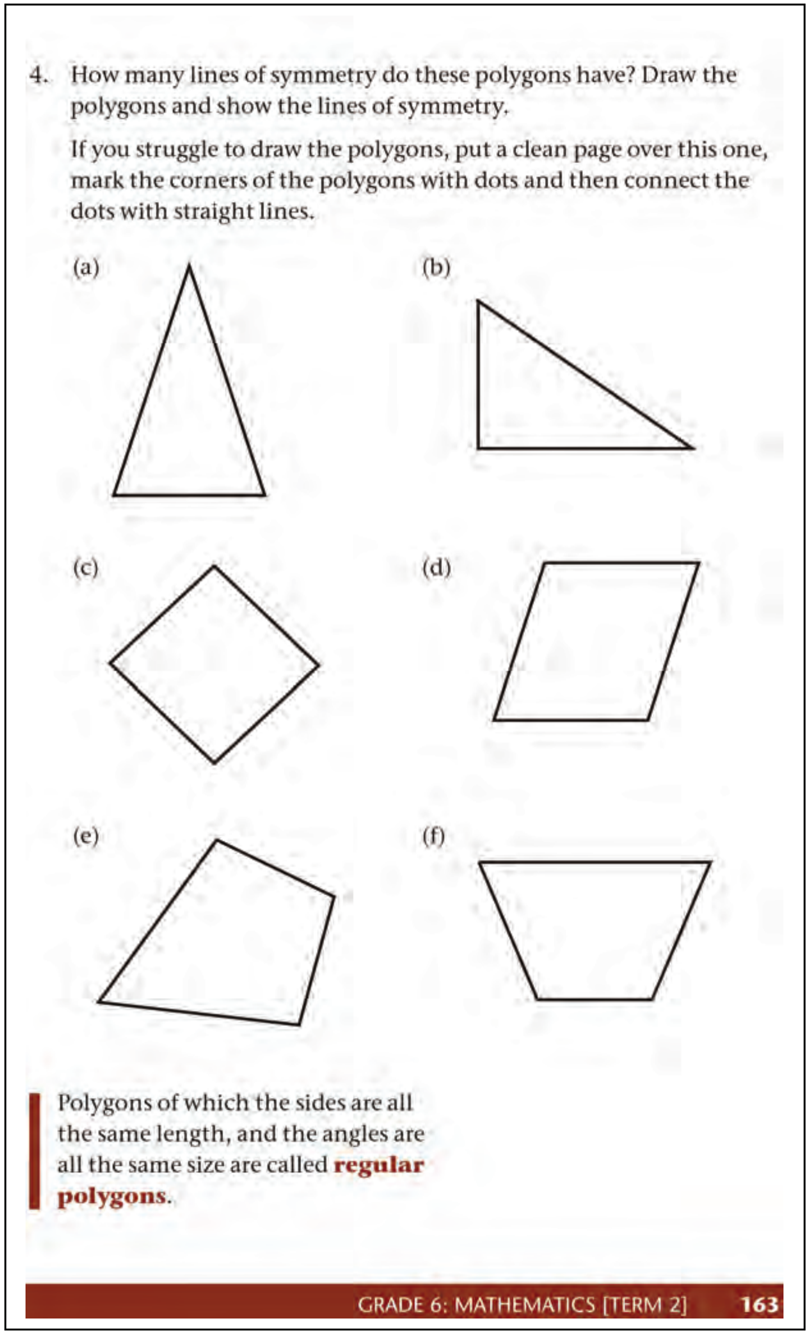
Learners may easily form the disempowering misconception that the idea of symmetry applies to (convex) polygons only. To protect them against this misconception, you may draw some examples of other symmetrical figures and configurations on the board, for example:











4. How many lines of symmetry do these polygons have? Draw the polygons and show the lines of symmetry.

If you struggle to draw the polygons, put a clean page over this one, mark the corners of the polygons with dots and then connect the dots with straight lines.

(a)

(b)

(c)

(d)

(e)

(f)

Polygons of which the sides are all the same length, and the angles are all the same size are called **regular polygons**.

Answers

5. (a) 3 sides, 3 lines of symmetry
   (b) 4 sides, 4 lines of symmetry
   (c) 5 sides, 5 lines of symmetry
   (d) 6 sides, 6 lines of symmetry
   (e) 7 sides, 7 lines of symmetry
   (f) 8 sides, 8 lines of symmetry
   (g) 10 sides, 10 lines of symmetry

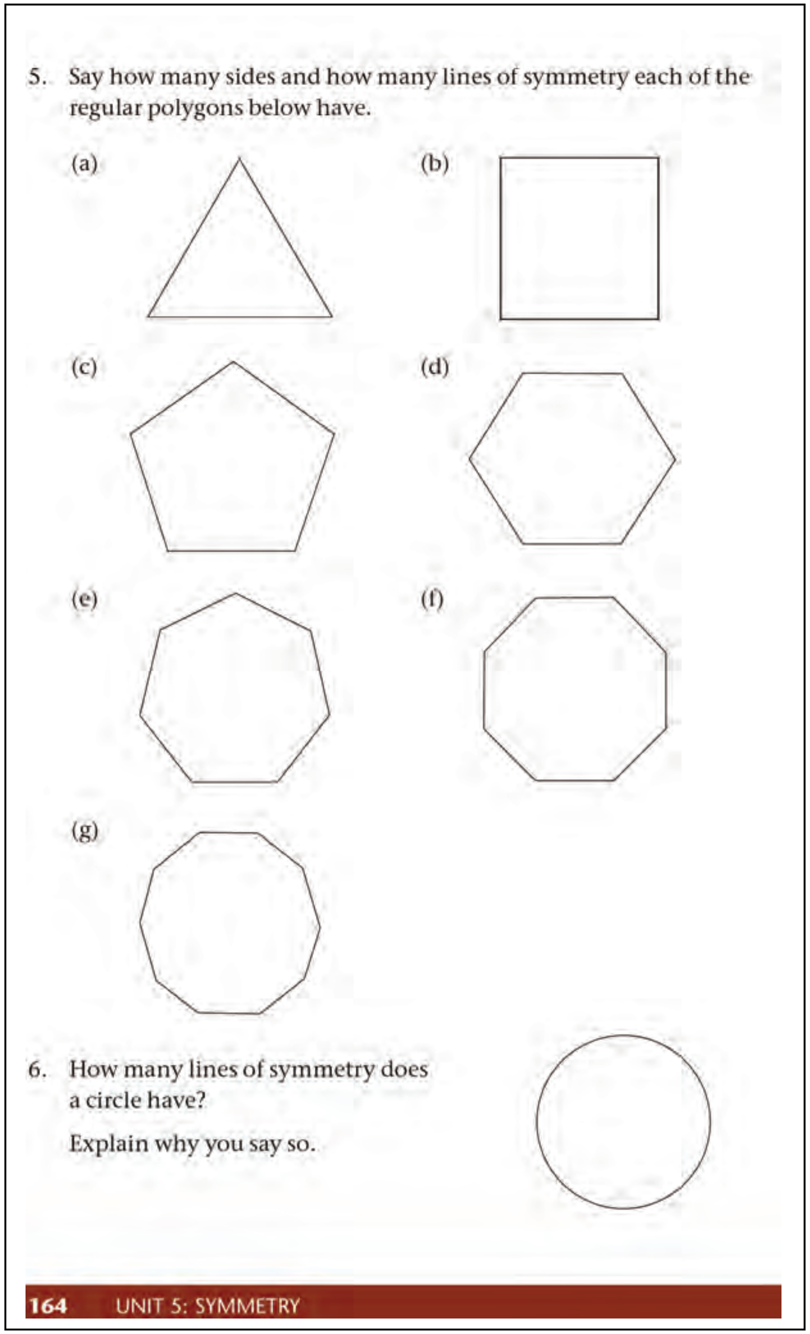
6. A circle has an infinite number of lines of symmetry.

   Any line crossing a circle, going through the centre of the circle, is a line of symmetry. There are an infinite number of lines going through the centre point.

Enrichment questions

1. Draw, freehand without a ruler, a closed shape with straight sides that has:
   (a) only one line of symmetry
   (b) only two lines of symmetry
   (c) only three lines of symmetry
   (d) only four lines of symmetry.

2. Investigate which of these figures are possible, and which are impossible:
   (a) a triangle with only one line of symmetry
   (b) a triangle with only two lines of symmetry
   (c) a triangle with only three lines of symmetry
   (d) a triangle with four lines of symmetry
   (e) a quadrilateral with only one line of symmetry
   (f) a quadrilateral with only two lines of symmetry
   (g) a quadrilateral with only three lines of symmetry
   (h) a quadrilateral with four lines of symmetry
   (i) a quadrilateral with five lines of symmetry.



5. Say how many sides and how many lines of symmetry each of the regular polygons below have.

6. How many lines of symmetry does a circle have?

   Explain why you say so.

164 UNIT 5: SYMMETRY

Grade 6 Term 2 Unit 6 Whole numbers: Division

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
6.1 What is division?	Meanings of division: examples of grouping, sharing and ratio situations in which division is required	165 to 167
6.2 Dividing by building up	Dividing by adding up	167 to 170
6.3 Practice	Solving word problems by dividing	171
6.4 The long division method	Dividing by subtracting	171 to 173
6.5 Practice	Solving word problems by dividing	174
6.6 Dividing with the calculator	Using the calculator, and estimation	175
6.7 Broken keys: estimate and improve	Using the calculator to strengthen understanding of division	176

CAPS time allocation	8 hours
CAPS page references	13 to 15 and 250 to 251

Mathematical background (also see the table in Section 6.4)

In 5 673 ÷ 147, the number 5 673 can be referred to as the **dividend**, and the number 147 as the **divisor**. While this terminology is useful for communication between mathematicians and educators, it is not advisable to burden Intermediate Phase learners with these abstract terms.

Irrespective of how it is recorded, the first step in "long division" with multi-digit numbers, for example 5 673 ÷ 147, is to identify a multiple of the divisor (in this case 30 × 147) that substantially approaches the dividend (5 673 in this case) from below.

Stated differently, the first step is to make a lower approximation of the dividend, for example 4 410 as an estimate of 5 673 in this case. The estimate is then refined by adding smaller multiples of the divisor.

The process can be executed as repeated addition (as on the immediate right) or as repeated subtraction, as in the traditional method of long division.

The key to effective thinking about how to perform division is to ask: "*With what do I have to multiply the divisor in order to get close to the dividend (the number that is divided)?*"

$$\begin{array}{rr} 30 \times 147 = & 4410 \\ 5 \times 147 = & \underline{+\ 735} \\ & 5145 \\ 3 \times 147 = & \underline{+\ 441} \\ 38 \times 147 & 5586 \\ & +\ \underline{87} \\ & 5673 \end{array}$$

$$5673 = 38 \times 147 + 87$$

$$\begin{array}{r} 3 \\ 5 \\ 30 \\ 147\overline{)5673} \\ -\ \underline{4410} \\ 1263 \\ -\ \underline{735} \\ 528 \\ -\ \underline{441} \\ 87 \end{array}$$

Division is applicable to three kinds of situations, sometimes referred to as grouping, sharing and scaling (ratio) situations. These are briefly described on the next page and form the focus of Section 6.1. The process of division becomes the focus in Section 6.2.

Resources

Calculators

6.1 What is division?

Mathematical notes

Different situations in which division is applicable are described in the shaded passage:

- A situation in which the number of parts is unknown, i.e. when the situation can be described by a number sentence of the form ? × size of each part (or rate) = total quantity, is called a **grouping** situation. To find the unknown number of parts, you have to divide. (Situation A)
- A situation in which the size of each part is unknown, i.e. when the situation can be described by a number sentence of the form number of parts × ? = total quantity, is called a **sharing** situation. To find the unknown rate or part size, you have to divide. (Situation B).
- A **scaling (ratio)** situation, in which two quantities are compared by using multiplication or division, not by stating the difference between the two quantities. For example, if you stick to a cake recipe that states 2 parts of sugar for 6 parts of cake flour, your amount of cake flour will always be 3 × your amount of sugar, and your amount of sugar will always be your amount of cake flour ÷ 3. (Situations C and D).

The five questions A, B, C, D and E in the shaded passage are given with two purposes:

- to develop awareness of the different kinds of situations in which division is applicable (meanings of division)
- to provide learners (through questions 1 to 5) with some experience of taking the first step of forming an appropriate multiple of the divisor.

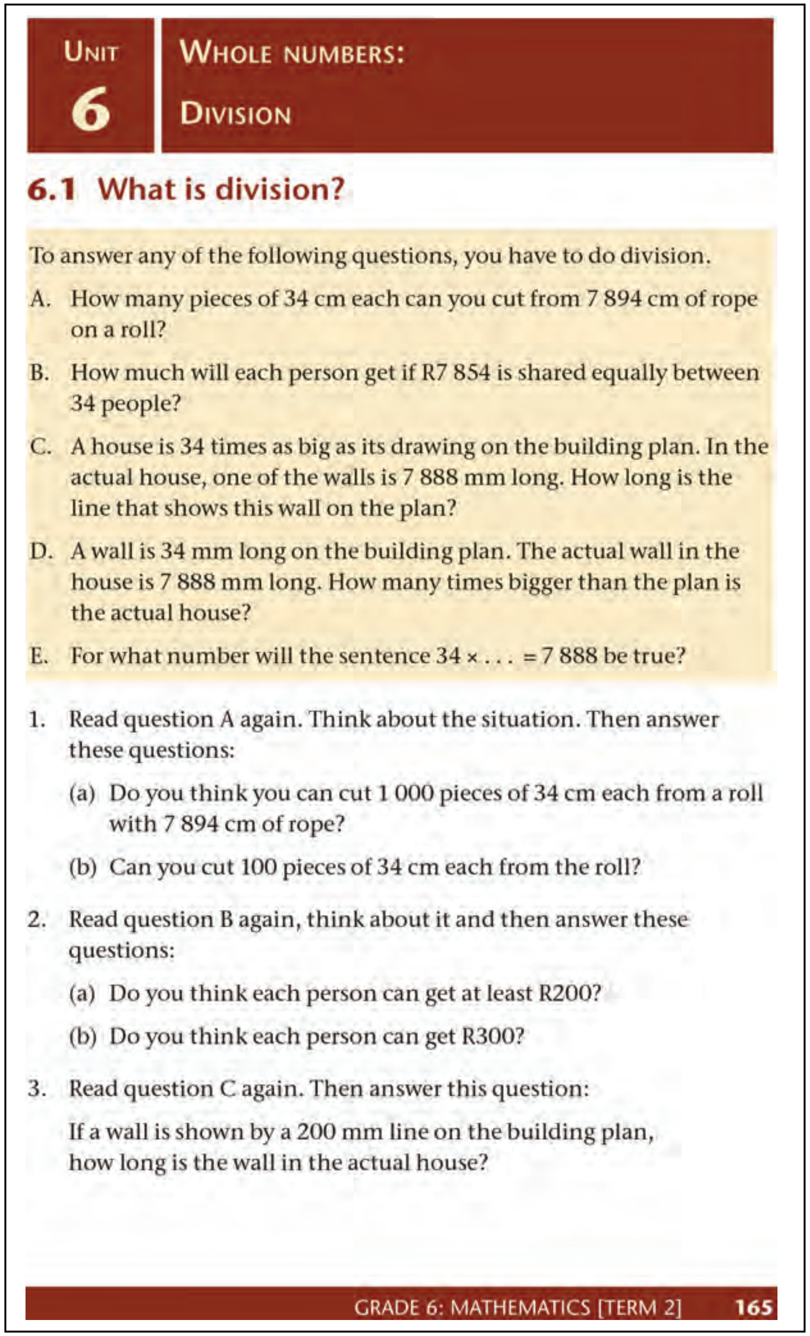
Teaching guidelines

You may start the lesson by asking learners to read questions A to E and to write roughly estimated answers. Having to estimate answers will help learners to apply their minds to understanding the given situations – which is an essential element of problem solving.

Let them then tackle questions 1 to 6. Once learners have started on these questions, you may suggest that they revise their estimates for A to E as they progress. Learners who wish to work further and find the exact answers for questions A to E while they are doing questions 1 to 5 may do so.

Answers

1. (a) No (b) Yes
2. (a) Yes (b) No
3. 6 800 mm



UNIT 6

WHOLE NUMBERS: DIVISION

6.1 What is division?

To answer any of the following questions, you have to do division.

A. How many pieces of 34 cm each can you cut from 7 894 cm of rope on a roll?

B. How much will each person get if R7 854 is shared equally between 34 people?

C. A house is 34 times as big as its drawing on the building plan. In the actual house, one of the walls is 7 888 mm long. How long is the line that shows this wall on the plan?

D. A wall is 34 mm long on the building plan. The actual wall in the house is 7 888 mm long. How many times bigger than the plan is the actual house?

E. For what number will the sentence 34 × . . . = 7 888 be true?

1. Read question A again. Think about the situation. Then answer these questions:
   (a) Do you think you can cut 1 000 pieces of 34 cm each from a roll with 7 894 cm of rope?
   (b) Can you cut 100 pieces of 34 cm each from the roll?
2. Read question B again, think about it and then answer these questions:
   (a) Do you think each person can get at least R200?
   (b) Do you think each person can get R300?
3. Read question C again. Then answer this question:
   If a wall is shown by a 200 mm line on the building plan, how long is the wall in the actual house?

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Teaching guidelines for question 6

Question 6 is intended to consolidate understanding of division and multiplication as inverse operations. The question is stated in terms of multiplication, but the answer is actually the answer for 4 731 ÷ 57. Stated differently:

When you find out with what number you have to multiply 57 to get 4 731,
you actually calculate 4 731 ÷ 57,

or

To calculate 4 731 ÷ 57, you have to find out with what number you have
to multiply 57 to get 4 731.

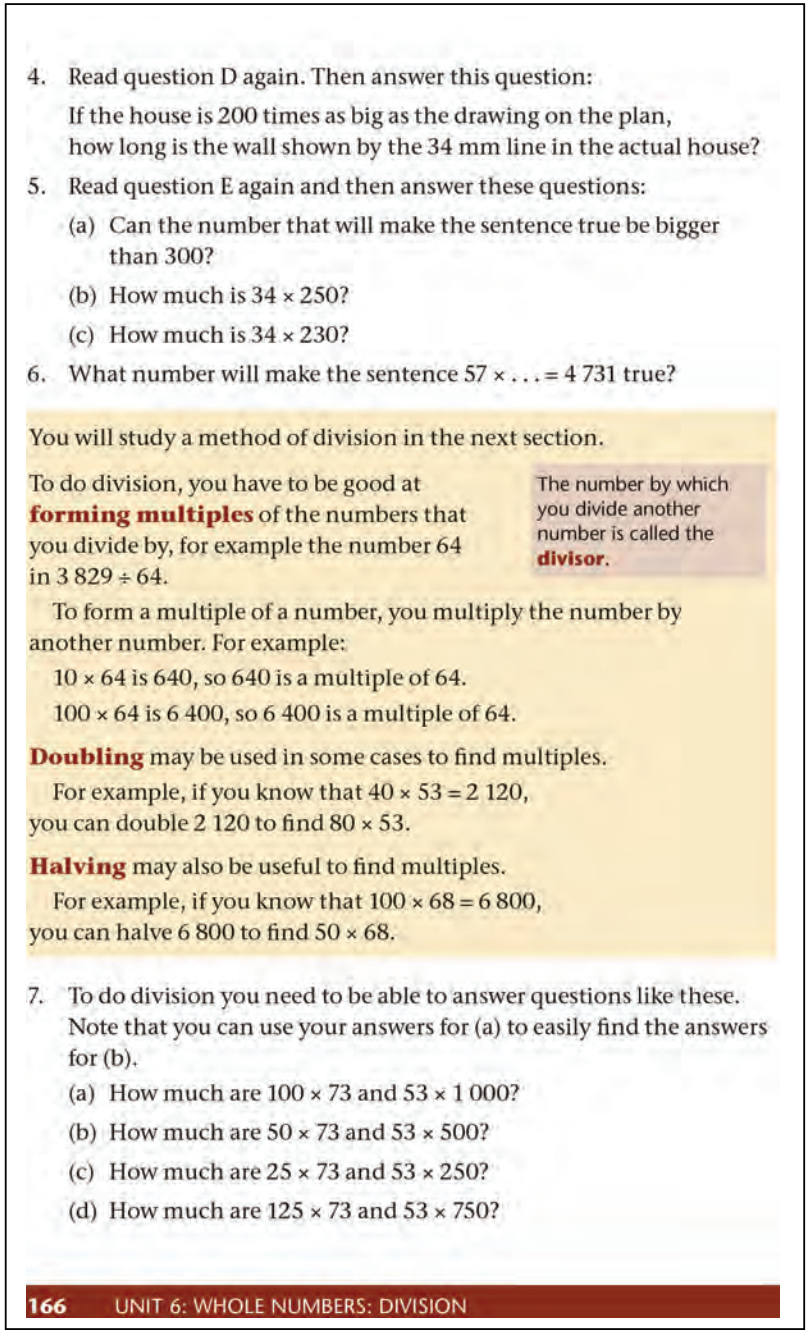
The phrase "*what do I have to multiply with ...*" is very useful in talking and thinking about division. It captures both the key logic of the division process and the mathematical idea that multiplication and division are inverse operations.

Teaching guidelines for the shaded passage and questions 7 and 8

Refer to learners' experiences in doing questions 1 to 6 to motivate them to practise multiplication with multiples of 10 and 100. You may demonstrate the techniques of doubling and halving with some examples before they engage with questions 7 and 8.

Answers

4. 6 800 mm
5. (a) No
   (b) 8 500
   (c) 7 820
6. 83
7. (a) 7 300 and 53 000
   (b) 3 650 and 26 500
   (c) 1 825 and 13 250
   (d) 9 125 and 39 750



4. Read question D again. Then answer this question:
   If the house is 200 times as big as the drawing on the plan,
   how long is the wall shown by the 34 mm line in the actual house?
5. Read question E again and then answer these questions:
   (a) Can the number that will make the sentence true be bigger than 300?
   (b) How much is 34 × 250?
   (c) How much is 34 × 230?
6. What number will make the sentence 57 × . . . = 4 731 true?

You will study a method of division in the next section.

To do division, you have to be good at **forming multiples** of the numbers that you divide by, for example the number 64 in 3 829 ÷ 64.

The number by which you divide another number is called the **divisor**.

To form a multiple of a number, you multiply the number by another number. For example:

10 × 64 is 640, so 640 is a multiple of 64.

100 × 64 is 6 400, so 6 400 is a multiple of 64.

Doubling may be used in some cases to find multiples.

For example, if you know that 40 × 53 = 2 120, you can double 2 120 to find 80 × 53.

Halving may also be useful to find multiples.

For example, if you know that 100 × 68 = 6 800, you can halve 6 800 to find 50 × 68.

7. To do division you need to be able to answer questions like these. Note that you can use your answers for (a) to easily find the answers for (b).
   (a) How much are 100 × 73 and 53 × 1 000?
   (b) How much are 50 × 73 and 53 × 500?
   (c) How much are 25 × 73 and 53 × 250?
   (d) How much are 125 × 73 and 53 × 750?

Answers

8.		× 10	× 100	× 50	× 30	× 40	× 60	× 70	× 80	× 90
(a)	37	370	3 700	1 850	1 110	1 480	2 220	2 590	2 960	3 330
(b)	76	760	7 600	3 800	2 280	3 040	4 560	5 320	6 080	6 840
(c)	98	980	9 800	4 900	2 940	3 920	5 880	6 860	7 840	8 820
(d)	43	430	4 300	2 150	1 290	1 720	2 580	3 010	3 440	3 870
(e)	38	380	3 800	1 900	1 140	1 520	2 280	2 660	3 040	3 420
(f)	55	550	5 500	2 750	1 650	2 200	3 300	3 850	4 400	4 950

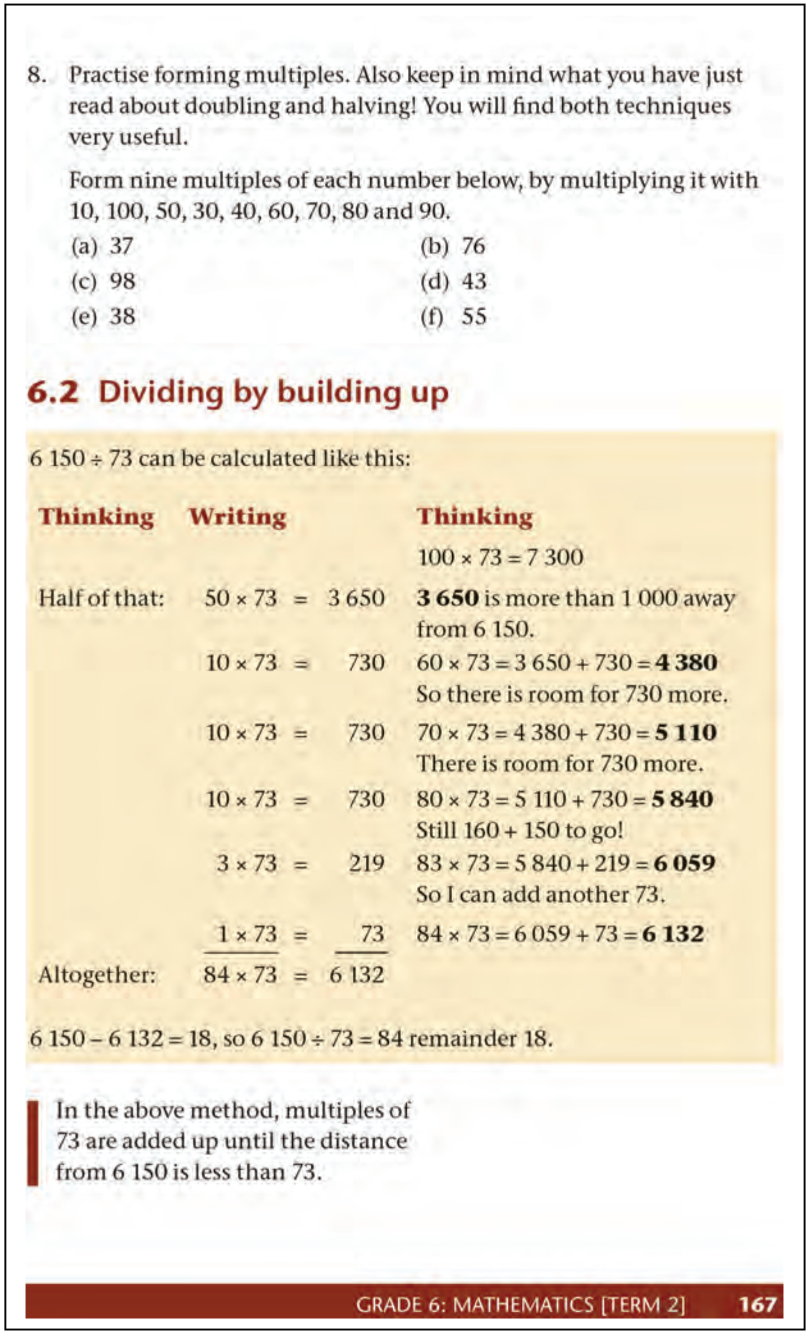
6.2 Dividing by building up

Mathematical notes

When people are thinking about a situation, they sometimes come up with more than one way to solve a problem or to find an answer to a question. We therefore use alternative methods to solve problems.

Teaching guidelines

Do an example of division by building up the answer. Unless learners have already solved all of problems A, B, C and D on page 165 of the Learner Book, it may be wise to use one of these problems as an example, rather than the example given in the shaded passage.



8. Practise forming multiples. Also keep in mind what you have just read about doubling and halving! You will find both techniques very useful.

Form nine multiples of each number below, by multiplying it with 10, 100, 50, 30, 40, 60, 70, 80 and 90.

(a) 37 (b) 76
(c) 98 (d) 43
(e) 38 (f) 55

6.2 Dividing by building up

6 150 ÷ 73 can be calculated like this:

Thinking	**Writing**	**Thinking**
		100 × 73 = 7 300
Half of that:	50 × 73 = 3 650	**3 650** is more than 1 000 away from 6 150.
	10 × 73 = 730	60 × 73 = 3 650 + 730 = **4 380** So there is room for 730 more.
	10 × 73 = 730	70 × 73 = 4 380 + 730 = **5 110** There is room for 730 more.
	10 × 73 = 730	80 × 73 = 5 110 + 730 = **5 840** Still 160 + 150 to go!
	3 × 73 = 219	83 × 73 = 5 840 + 219 = **6 059** So I can add another 73.
	1 × 73 = 73	84 × 73 = 6 059 + 73 = **6 132**
Altogether:	84 × 73 = 6 132	

6 150 – 6 132 = 18, so 6 150 ÷ 73 = 84 remainder 18.

In the above method, multiples of 73 are added up until the distance from 6 150 is less than 73.

Teaching guidelines

It will be useful to write the content of the shaded passage on the board. Inform learners that the steps in the shaded passage can be explained in at least three different ways, and then explain the steps:

- as prompted by a sharing situation (see example below)
- as prompted by a grouping situation (see example below)
- as prompted by abstract thinking about computation (see the shaded passage on page 167 of the Learner Book).

Computational actions prompted by thinking about a sharing situation

How much will each person get if 6 150 is fairly shared between 73 people?

If each person gets R50, that is R50 × 73 = R3 650

If each person gets another R10, that is R10 × 73 = R730

If each person gets another R10, that is R10 × 73 = R730

If each person gets another R10, that is R10 × 73 = R730

If each person gets another R3, that is R3 × 73 = R219

If each person gets another R1, that is R1 × 73 = R73

If each person gets R84, that is R84 × 73 = R6 132

Each person gets R84, and there is R18 left, which can also be shared into amounts smaller than R1.

Computational actions prompted by thinking about a grouping situation

How many chickens at R73 each can you buy if you have R6 132?

50 chickens cost 50 × R73 = R3 650

10 chickens cost 10 × R73 = R730

10 chickens cost 10 × R73 = R730

10 chickens cost 10 × R73 = R730

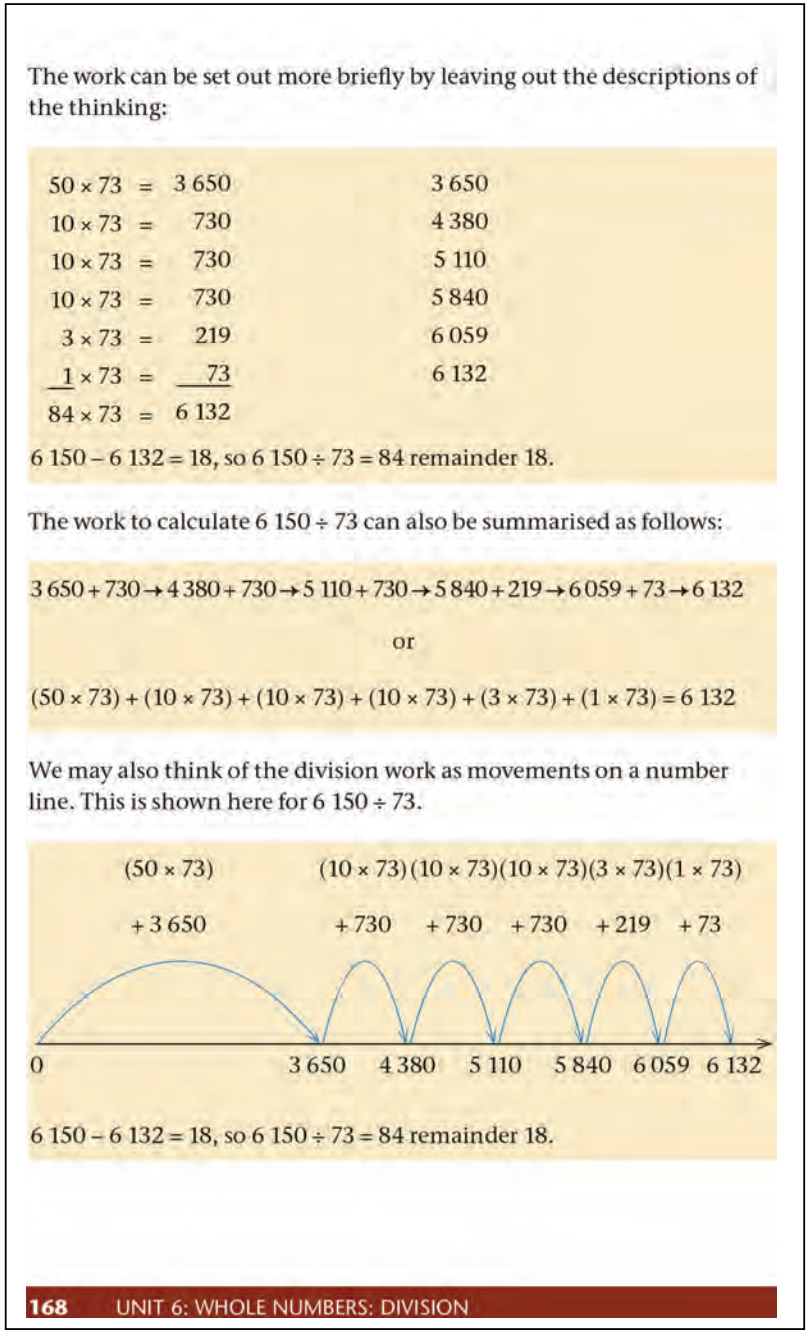
3 chickens cost 3 × R73 = R219

1 chicken costs 1 × R73 = R73

84 chickens cost 84 × R73 = R6 132

84 chickens can be bought, and there is R18 left.

Suggest to learners that they read the rest of page 168 (other ways to represent the division process) in their own time. In class it is more important to now proceed to division by using fewer steps, as shown at the top of the shaded passage on page 169.



The work can be set out more briefly by leaving out the descriptions of the thinking:

50 × 73 =	3 650	3 650
10 × 73 =	730	4 380
10 × 73 =	730	5 110
10 × 73 =	730	5 840
3 × 73 =	219	6 059
1 × 73 =	73	6 132
84 × 73 =	6 132	

6 150 – 6 132 = 18, so 6 150 ÷ 73 = 84 remainder 18.

The work to calculate 6 150 ÷ 73 can also be summarised as follows:

3 650 + 730 → 4 380 + 730 → 5 110 + 730 → 5 840 + 219 → 6 059 + 73 → 6 132

or

(50 × 73) + (10 × 73) + (10 × 73) + (10 × 73) + (3 × 73) + (1 × 73) = 6 132

We may also think of the division work as movements on a number line. This is shown here for 6 150 ÷ 73.

6 150 – 6 132 = 18, so 6 150 ÷ 73 = 84 remainder 18.

168 UNIT 6: WHOLE NUMBERS: DIVISION

Teaching guidelines

Show on the board that 6 150 ÷ 73 can be calculated in three steps only, as shown in the shaded passage, but tell learners that when they do division they may use more steps if they need to. They should, however, try to use as few steps as possible.

Let learners then do questions 1 to 6. Learners who struggle to get started with question 1 may be helped by suggesting that they think of sharing R950 fairly between 64 people, or that they ask themselves how many chickens at R64 each can be bought with R950. Thinking about a real situation (grouping or sharing) may help them to form ideas of what computational steps they may take.

Learners may sometimes "overshoot", i.e. choose a number that is too big for their answer.

For example, when calculating 950 ÷ 64 a learner may add 5 × 64 = 320 in the second step. **It is very important that they do not experience this as failure.**

Reassure learners that it is not a crisis at all when they overshoot, and suggest that they simply cross the step out neatly (or erase it if they work with pencils and have erasers), and take a smaller multiple of the number they divide with.

Calculating 950 ÷ 64:

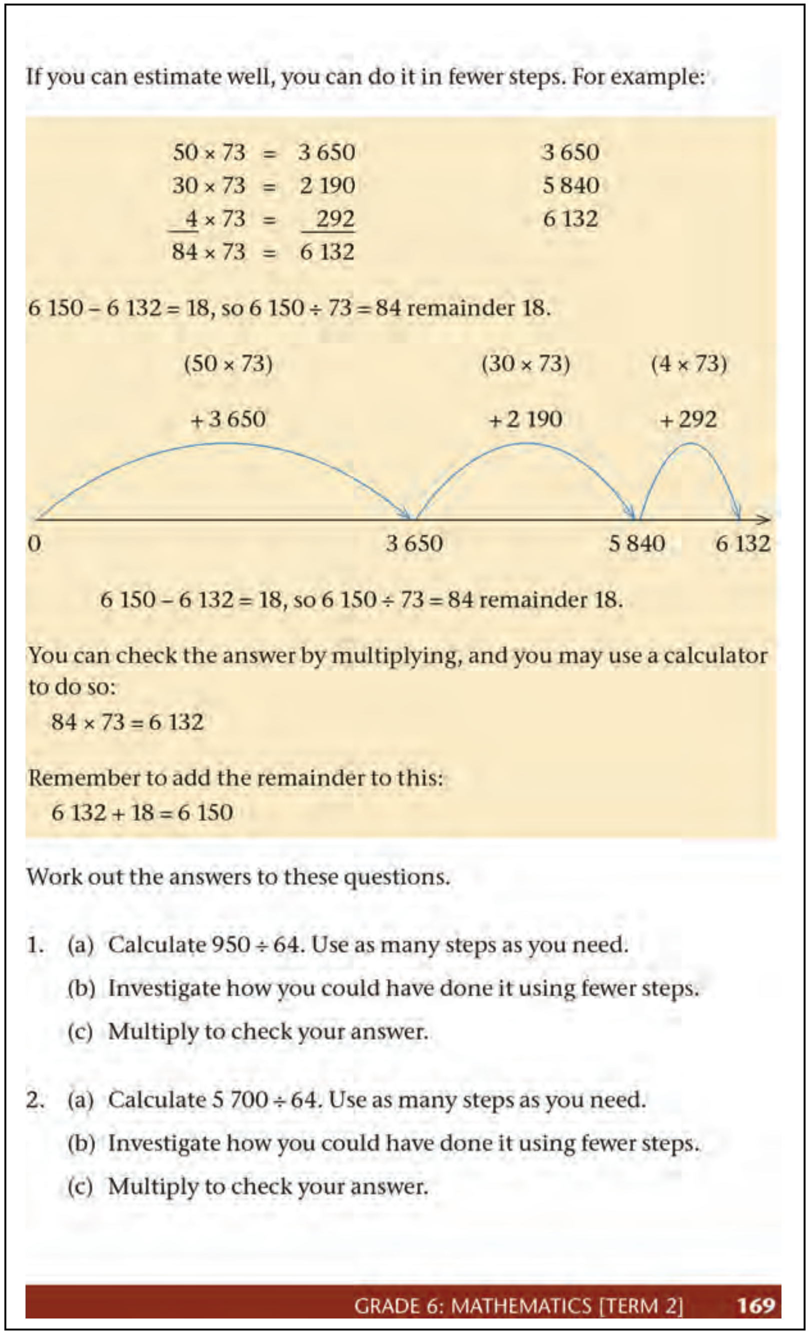
10 × 64 = 640
5 × 64 = 320   960

Calculating 950 ÷ 64:

10 × 64 = 640
~~5 × 64 = 320   960~~
4 × 64 = 256   896

Answers

1. (a) 14 remainder 54
   (b) Learners investigate their steps and eliminate some, if possible.
   (c) 64 × 14 + 54 = 950
2. (a) 89 remainder 4
   (b) Learners investigate their steps and eliminate some, if possible.
   (c) 64 × 89 + 4 = 5 700



If you can estimate well, you can do it in fewer steps. For example:

50 × 73	=	3 650	3 650
30 × 73	=	2 190	5 840
4 × 73	=	292	6 132
84 × 73	=	6 132	

6 150 – 6 132 = 18, so 6 150 ÷ 73 = 84 remainder 18.

(50 × 73) (30 × 73) (4 × 73)
+3 650 +2 190 +292
0 3 650 5 840 6 132

6 150 – 6 132 = 18, so 6 150 ÷ 73 = 84 remainder 18.

You can check the answer by multiplying, and you may use a calculator to do so:

84 × 73 = 6 132

Remember to add the remainder to this:

6 132 + 18 = 6 150

Work out the answers to these questions.

1. (a) Calculate 950 ÷ 64. Use as many steps as you need.
   (b) Investigate how you could have done it using fewer steps.
   (c) Multiply to check your answer.
2. (a) Calculate 5 700 ÷ 64. Use as many steps as you need.
   (b) Investigate how you could have done it using fewer steps.
   (c) Multiply to check your answer.

Teaching guidelines

When engaging with word problems, it is critical that learners read the question carefully and try to imagine the described situation in their minds before they decide on an operation. A good way to nudge learners towards reading and interpreting the given problem is to encourage them to produce an estimated answer first, before they start doing accurate calculations or even decide on what calculations they will do.

Learners' efforts should be directed at understanding and solving the stated problem, not at trying to identify the correct operation as quickly as possible and applying a recipe to execute it. In the case of problems like those in questions 4, 5 and 6, learners may even solve the problems successfully without consciously thinking of division.

When learners try to divide by adding up multiples of the divisor, as in the example on the right, they are sometimes hesitant to take a next step because they are afraid they may "overshoot" (see notes on the previous page).

For example, after the first step of calculating 30 × 93, learners may be afraid that they will "overshoot" beyond 3 450 in the next step, and hence stall.

Calculating 3 450 ÷ 93:

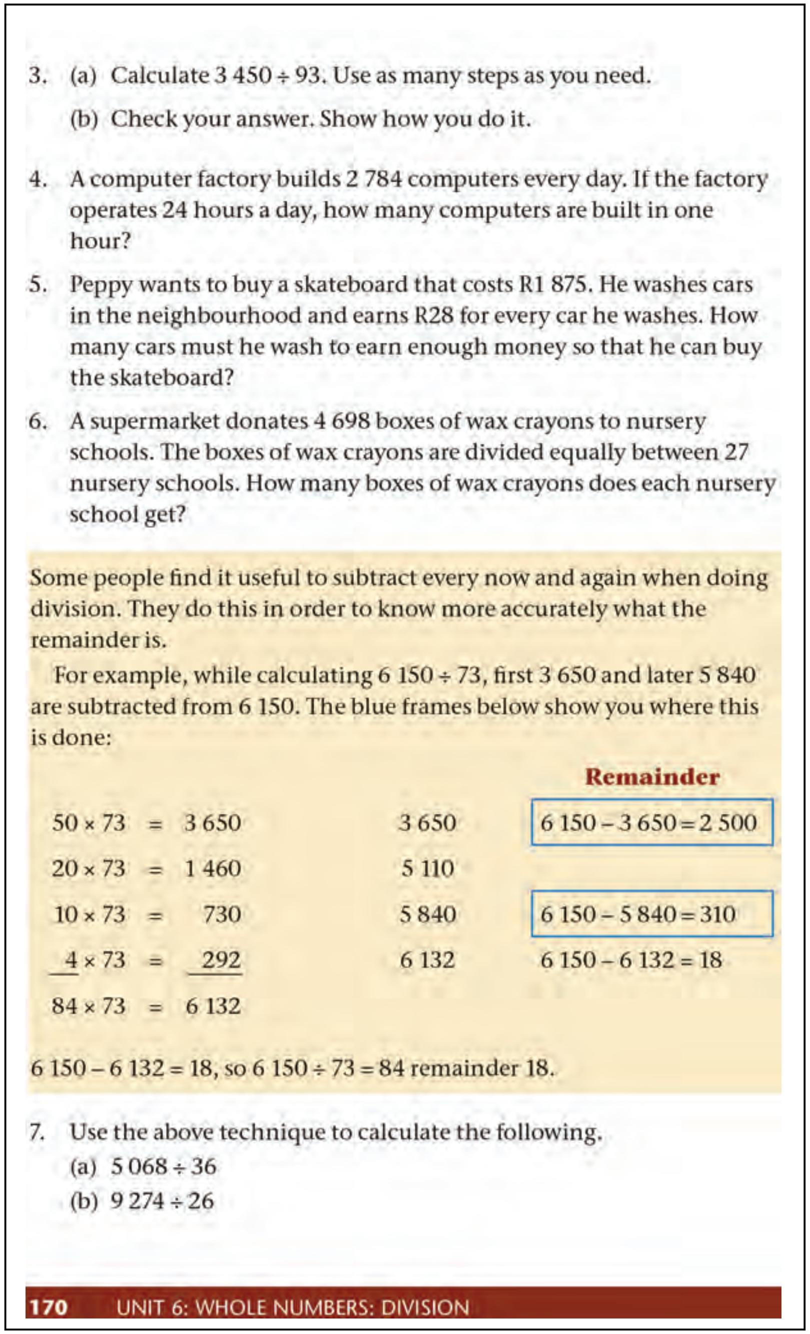
30 × 93 = 2 790
? × 93 =

To overcome this problem, one may perform subtraction as demonstrated in the shaded passage to see how much of the number that is being divided still remains. (In the traditional long division method, the remainder is actually calculated after each addition step. This practice is introduced in Section 6.4.)

You may use any of questions 3 to 6, or a different question altogether, as an example to explain the strategy of subtracting to establish the remainder after an addition step. Alternatively, you may use the example in the shaded passage.

Answers

3. (a) 37 remainder 9. Use as many steps as you need.
   (b) 37 × 93 + 9 = 3 450
4. 2 784 ÷ 24 = 116 computers are built in one hour.
5. 1 875 ÷ 28 = 66 remainder 27.
   Peppy must wash at least 67 cars to have enough money for the skateboard.
6. 4 698 ÷ 27 = 174 boxes for each nursery school.
7. Learners use the prescribed technique to calculate. The answers are:
   (a) 140 remainder 28 (b) 356 remainder 18



3. (a) Calculate 3 450 ÷ 93. Use as many steps as you need.
   (b) Check your answer. Show how you do it.
4. A computer factory builds 2 784 computers every day. If the factory operates 24 hours a day, how many computers are built in one hour?
5. Peppy wants to buy a skateboard that costs R1 875. He washes cars in the neighbourhood and earns R28 for every car he washes. How many cars must he wash to earn enough money so that he can buy the skateboard?
6. A supermarket donates 4 698 boxes of wax crayons to nursery schools. The boxes of wax crayons are divided equally between 27 nursery schools. How many boxes of wax crayons does each nursery school get?

Some people find it useful to subtract every now and again when doing division. They do this in order to know more accurately what the remainder is.

For example, while calculating 6 150 ÷ 73, first 3 650 and later 5 840 are subtracted from 6 150. The blue frames below show you where this is done:

				Remainder
50 × 73	=	3 650	3 650	6 150 – 3 650 = 2 500
20 × 73	=	1 460	5 110	
10 × 73	=	730	5 840	6 150 – 5 840 = 310
4 × 73	=	292	6 132	6 150 – 6 132 = 18
84 × 73	=	6 132		

6 150 – 6 132 = 18, so 6 150 ÷ 73 = 84 remainder 18.

7. Use the above technique to calculate the following.
   (a) 5 068 ÷ 36
   (b) 9 274 ÷ 26

6.3 Practice

Teaching guidelines

Encourage learners to estimate the answers before they start to do exact calculations, and to think logically before giving the final answer. Encourage them to check their estimates by using multiplication before proceeding to exact calculations.

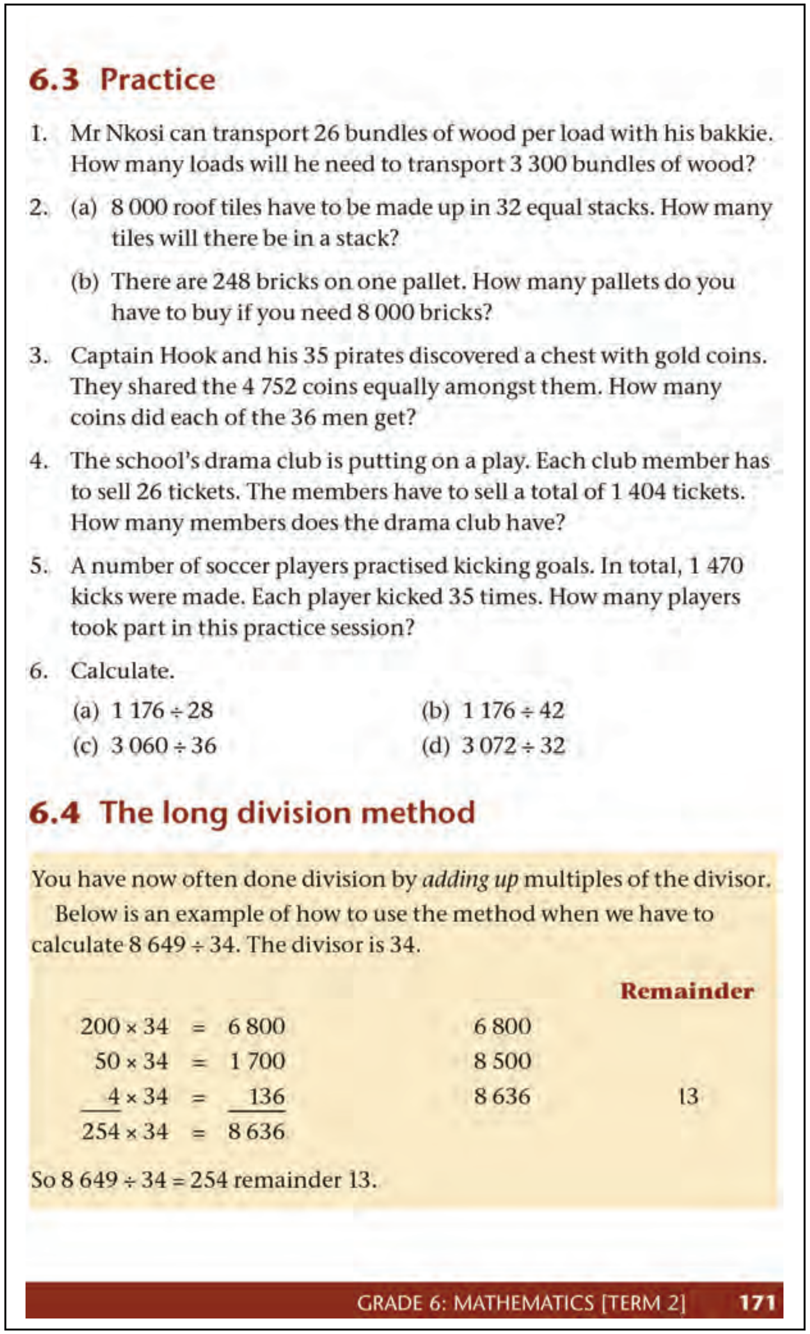
Answers

1. 3 300 ÷ 26 = 126 remainder 24, i.e. 127 loads
2. (a) 250 tiles
   (b) 8 000 ÷ 248 = 32 remainder 64, so you will need to buy 33 pallets.
3. 4 752 ÷ 36 = 132 coins
4. 54 members
5. 42 players
6. (a) 42 (b) 28 (c) 85 (d) 96

6.4 The long division method

In the method that was traditionally called long division, the first step was to take a multiple of the divisor and **subtract** it from the dividend, followed by another multiple of the divisor subtracted from the first remainder. Multiples of the divisor are subtracted from the remainders of these subtractions until a remainder equal to 0 or a remainder less than the divisor is reached. This is shown in the top part of the shaded passage on page 172. This is in contrast to the method described up to now in this unit, namely to **add** multiples of the divisor until the dividend is reached.

Demonstrate the two different methods on the board. Use 8 649 ÷ 34 as in the shaded passages, or (preferably) use a different example.



6.3 Practice

1. Mr Nkosi can transport 26 bundles of wood per load with his bakkie. How many loads will he need to transport 3 300 bundles of wood?
2. (a) 8 000 roof tiles have to be made up in 32 equal stacks. How many tiles will there be in a stack?
   (b) There are 248 bricks on one pallet. How many pallets do you have to buy if you need 8 000 bricks?
3. Captain Hook and his 35 pirates discovered a chest with gold coins. They shared the 4 752 coins equally amongst them. How many coins did each of the 36 men get?
4. The school's drama club is putting on a play. Each club member has to sell 26 tickets. The members have to sell a total of 1 404 tickets. How many members does the drama club have?
5. A number of soccer players practised kicking goals. In total, 1 470 kicks were made. Each player kicked 35 times. How many players took part in this practice session?
6. Calculate.
   (a) 1 176 ÷ 28
   (b) 1 176 ÷ 42
   (c) 3 060 ÷ 36
   (d) 3 072 ÷ 32

6.4 The long division method

You have now often done division by *adding up* multiples of the divisor.

Below is an example of how to use the method when we have to calculate 8 649 ÷ 34. The divisor is 34.

				Remainder
200 × 34	=	6 800	6 800	
50 × 34	=	1 700	8 500	
4 × 34	=	136	8 636	13
254 × 34	=	8 636		

So 8 649 ÷ 34 = 254 remainder 13.

Teaching guidelines

The first exposition in the shaded passage shows how the different methods of adding multiples and subtracting multiples of the divisor are related.

When learners do questions 1 and 3 they should use the "shorter way of recording" demonstrated in the shaded passage.

Answers

1. (a)

7 814	
– 4 200	100 × 42
3 614	
– 2 100	50 × 42
1 514	
– 1 050	25 × 42
464	
– 420	10 × 42
44	
– 42	1 × 42
2	186

So 7 814 ÷ 42
= 186 remainder 2

(b)

9 638	
– 8 400	300 × 28
1 238	
– 1 120	40 × 28
118	
– 112	4 × 28
6	344

So, 9 638 ÷ 28
= 344 remainder 6

2. (a)

100 × 42 = 4 200	4 200	
50 × 42 = 2 100	6 300	
25 × 42 = 1 050	7 350	
10 × 42 = 420	7 770	
1 × 42 = 42	7 812	7 814 – 7 812 = 2
186 × 42 = 7 812		

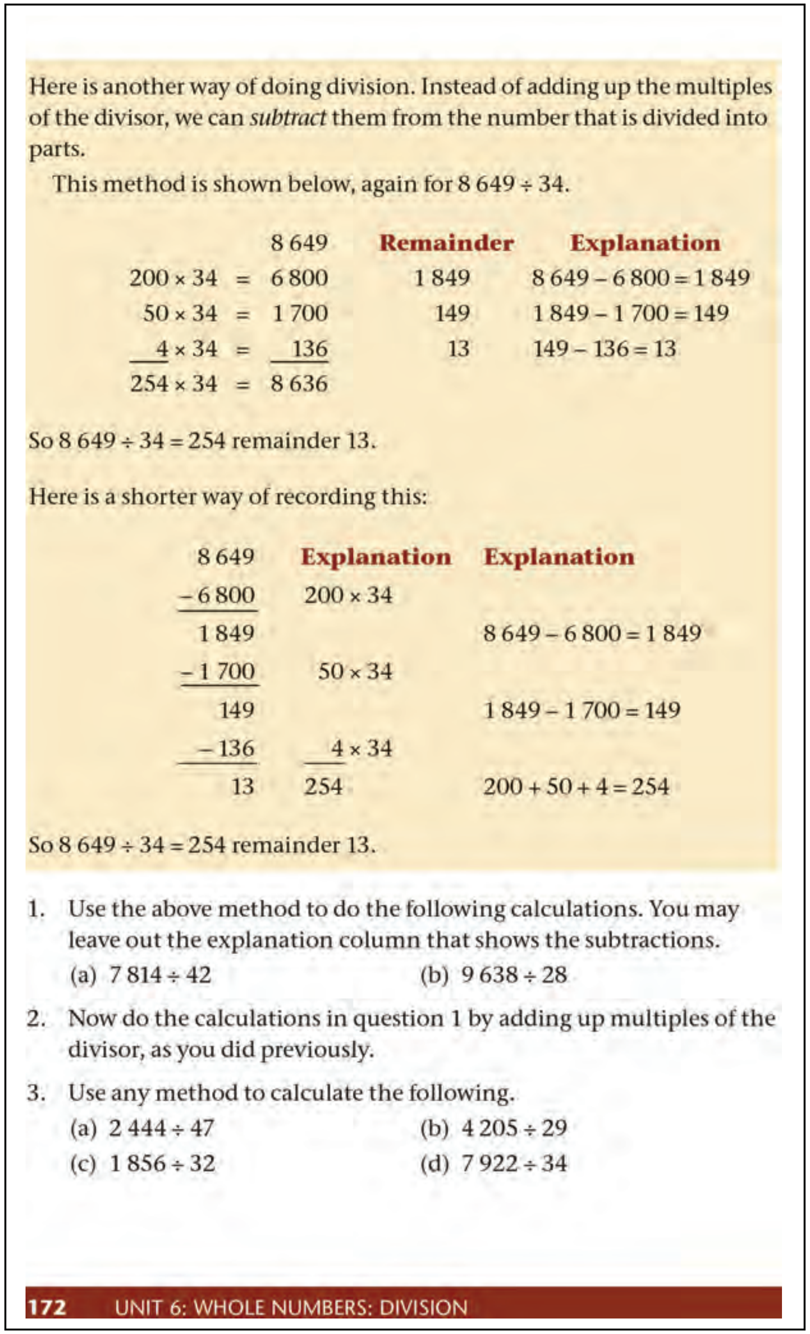
So, 7 814 ÷ 42 = 186 remainder 2

(b)

300 × 28 = 8 400	8 400	
40 × 28 = 1 120	9 520	
4 × 28 = 112	9 632	9 638 – 9 632 = 6
344 × 28 = 9 632		

So, 9 638 ÷ 28 = 344 remainder 6

3. (a) 52 (b) 145 (c) 58 (d) 233



Here is another way of doing division. Instead of adding up the multiples of the divisor, we can *subtract* them from the number that is divided into parts.

This method is shown below, again for 8 649 ÷ 34.

			Remainder	**Explanation**
		8 649		
200 × 34	=	6 800	1 849	8 649 – 6 800 = 1 849
50 × 34	=	1 700	149	1 849 – 1 700 = 149
4 × 34	=	136	13	149 – 136 = 13
254 × 34	=	8 636		

So 8 649 ÷ 34 = 254 remainder 13.

Here is a shorter way of recording this:

8 649	**Explanation**	**Explanation**
– 6 800	200 × 34	
1 849		8 649 – 6 800 = 1 849
– 1 700	50 × 34	
149		1 849 – 1 700 = 149
– 136	4 × 34	
13	254	200 + 50 + 4 = 254

So 8 649 ÷ 34 = 254 remainder 13.

1. Use the above method to do the following calculations. You may leave out the explanation column that shows the subtractions.
   (a) 7 814 ÷ 42 (b) 9 638 ÷ 28
2. Now do the calculations in question 1 by adding up multiples of the divisor, as you did previously.
3. Use any method to calculate the following.
   (a) 2 444 ÷ 47 (b) 4 205 ÷ 29
   (c) 1 856 ÷ 32 (d) 7 922 ÷ 34

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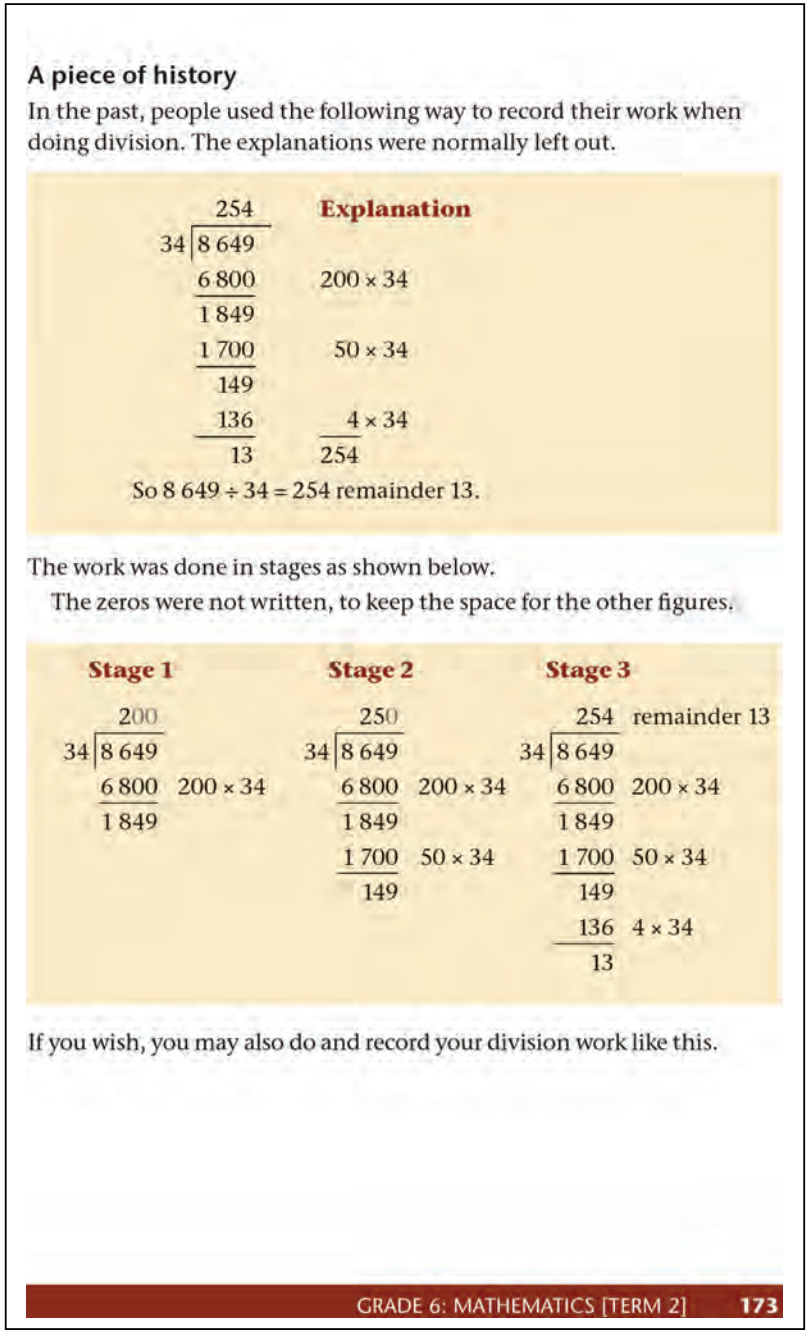
Mathematical notes

This traditional format for recording division has practically disappeared from computational practice all over the world. If learners do use it, encourage them to fill in the explanations too, with a view to sustain understanding.

Mathematical background

Multiplication and division are applicable in the following two kinds of situations:

	Examples of questions
Additive situations, in which a whole quantity can be considered to be made up of equal parts. *Example:* A consignment of sugar is packaged into a number of packets of equal mass. Situations like this can be described with a number sentence of the form: number of parts × size of each part = total quantity, or number of parts × value of each part = total value. The "value of each part" is normally called the **rate**. The number of parts can be a whole number or a fraction.	430 packets of sugar weigh 400 g each. How much sugar is this in total? (430 × 400) 1 200 kg sugar is packaged in packets of 400 g each. How many packets are there? (1 200 ÷ 400, grouping) 1 200 kg of sugar is packed into 400 equal packets. How much sugar is in each packet? (1 200 ÷ 400, sharing)
Multiplicative situations, in which one quantity can be considered as an enlargement ("stretching") or reduction ("shrinking") of another situation. *Example:* a scale drawing of a building. Situations like this can be described with a number sentence of the form: size of one object × scale factor (**ratio**) = size of another object	A house is 20 times as high as the drawing of the house on a building plan. How high is the house if the drawing is 9 cm high? (20 × 9) How high is the drawing if the house is 240 cm high? (240 ÷ 20) The height of a drawing of a house is 15 cm and the actual house is 240 cm high. What is the scale factor of the drawing? (240 ÷ 15)



A piece of history

In the past, people used the following way to record their work when doing division. The explanations were normally left out.

254	**Explanation**
34 ⟌8 649	
6 800	200 × 34
1 849	
1 700	50 × 34
149	
136	4 × 34
13	254

So 8 649 ÷ 34 = 254 remainder 13.

The work was done in stages as shown below.

The zeros were not written, to keep the space for the other figures.

Stage 1		**Stage 2**		**Stage 3**	
200		250		254	remainder 13
34 ⟌8 649		34 ⟌8 649		34 ⟌8 649	
6 800	200 × 34	6 800	200 × 34	6 800	200 × 34
1 849		1 849		1 849	
		1 700	50 × 34	1 700	50 × 34
		149		149	
				136	4 × 34
				13	

If you wish, you may also do and record your division work like this.

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6.5 Practice

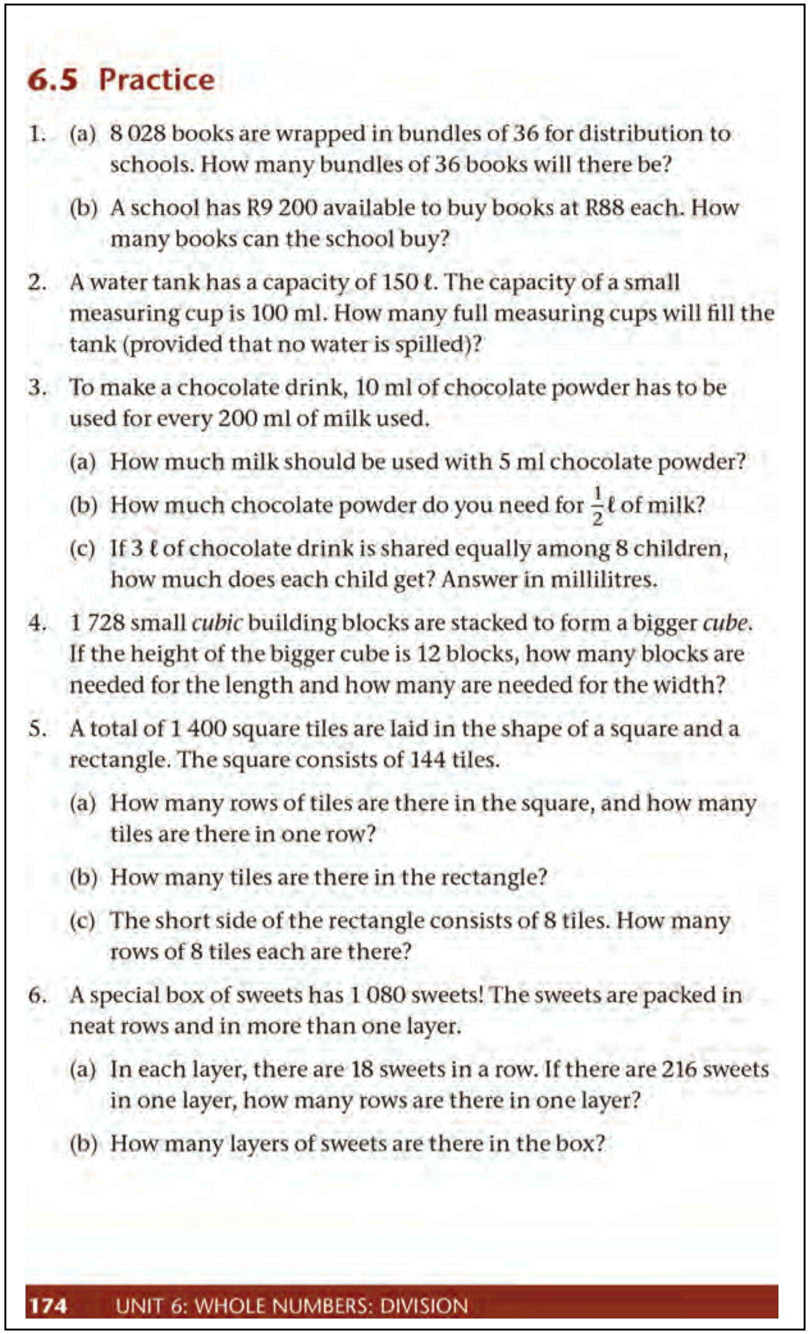
Teaching guidelines

Encourage learners to estimate the answers before they start exact calculations for questions 1 to 6. This will help them to read the questions properly before they start to do calculations.

Encourage learners to check their estimates by using multiplication before proceeding to exact calculations.

Answers

1. (a) 223 bundles
   (b) 104 books
2. 150 000 ÷ 100 = 1 500 cups
3. (a) 100 ml milk
   (b) 25 ml chocolate powder
   (c) 375 ml
4. 12 blocks for the length and 12 blocks for the width
5. (a) 12 rows and 12 tiles in one row
   (b) 1 400 – 144 = 1 256 tiles
   (c) 157 rows
6. (a) 12 rows
   (b) 5 layers



6.5 Practice

1. (a) 8 028 books are wrapped in bundles of 36 for distribution to schools. How many bundles of 36 books will there be?
   (b) A school has R9 200 available to buy books at R88 each. How many books can the school buy?
2. A water tank has a capacity of 150 ℓ. The capacity of a small measuring cup is 100 ml. How many full measuring cups will fill the tank (provided that no water is spilled)?
3. To make a chocolate drink, 10 ml of chocolate powder has to be used for every 200 ml of milk used.
   (a) How much milk should be used with 5 ml chocolate powder?
   (b) How much chocolate powder do you need for $\frac{1}{2}$ ℓ of milk?
   (c) If 3 ℓ of chocolate drink is shared equally among 8 children, how much does each child get? Answer in millilitres.
4. 1 728 small *cubic* building blocks are stacked to form a bigger *cube*. If the height of the bigger cube is 12 blocks, how many blocks are needed for the length and how many are needed for the width?
5. A total of 1 400 square tiles are laid in the shape of a square and a rectangle. The square consists of 144 tiles.
   (a) How many rows of tiles are there in the square, and how many tiles are there in one row?
   (b) How many tiles are there in the rectangle?
   (c) The short side of the rectangle consists of 8 tiles. How many rows of 8 tiles each are there?
6. A special box of sweets has 1 080 sweets! The sweets are packed in neat rows and in more than one layer.
   (a) In each layer, there are 18 sweets in a row. If there are 216 sweets in one layer, how many rows are there in one layer?
   (b) How many layers of sweets are there in the box?

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6.6 Dividing with the calculator

Teaching guidelines

It is critical that learners accept personal responsibility for the correctness of answers, even when they use a calculator. In fact, it is critical that they take note of the numbers involved in calculations, and do not just type in digits and operation signs.

Let them do the calculations for question 1; then spend time with the whole class on discussing ways in which the answers can be checked. One obvious way is to multiply the answer with the divisor (using the calculator), and to check whether the original number is obtained. For example, for question 1(a) they may obtain the answer 21. To check whether this is correct, they can calculate 21 × 324.

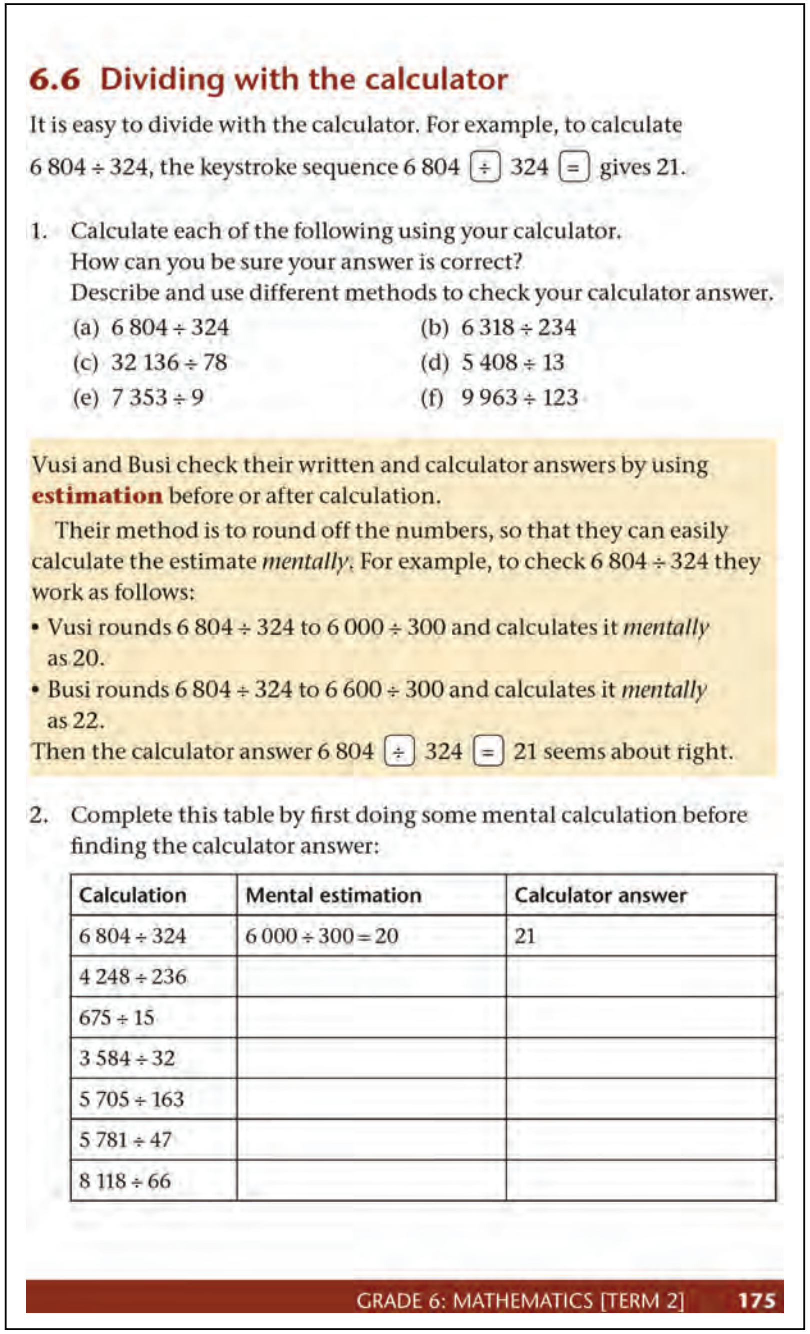
Demonstrate the estimation of answers (as described in the shaded passage) with some of the calculations in question 1.

Answers

1. (a) 21 (b) 27 (c) 412
   (d) 416 (e) 817 (f) 81

2. Estimates will differ. Example:

Calculation	**Mental estimation**	**Calculator answer**
6 804 ÷ 324	6 000 ÷ 300 = 20	21
4 248 ÷ 236	4 000 ÷ 200 = 20	18
675 ÷ 15	700 ÷ 20 = 35	45
3 584 ÷ 32	3 500 ÷ 35 = 100	112
5 705 ÷ 163	5 000 ÷ 200 = 25	35
5 781 ÷ 47	6 000 ÷ 50 = 120	123
8 118 ÷ 66	8 000 ÷ 50 = 160	123



6.6 Dividing with the calculator

It is easy to divide with the calculator. For example, to calculate 6 804 ÷ 324, the keystroke sequence 6 804 $\boxed{\div}$ 324 $\boxed{=}$ gives 21.

1. Calculate each of the following using your calculator.
   How can you be sure your answer is correct?
   Describe and use different methods to check your calculator answer.
   (a) 6 804 ÷ 324 (b) 6 318 ÷ 234
   (c) 32 136 ÷ 78 (d) 5 408 ÷ 13
   (e) 7 353 ÷ 9 (f) 9 963 ÷ 123

Vusi and Busi check their written and calculator answers by using **estimation** before or after calculation.

Their method is to round off the numbers, so that they can easily calculate the estimate *mentally*. For example, to check 6 804 ÷ 324 they work as follows:

- Vusi rounds 6 804 ÷ 324 to 6 000 ÷ 300 and calculates it *mentally* as 20.
- Busi rounds 6 804 ÷ 324 to 6 600 ÷ 300 and calculates it *mentally* as 22.

Then the calculator answer 6 804 $\boxed{\div}$ 324 $\boxed{=}$ 21 seems about right.

2. Complete this table by first doing some mental calculation before finding the calculator answer:

Calculation	Mental estimation	Calculator answer
6 804 ÷ 324	6 000 ÷ 300 = 20	21
4 248 ÷ 236		
675 ÷ 15		
3 584 ÷ 32		
5 705 ÷ 163		
5 781 ÷ 47		
8 118 ÷ 66		

6.7 Broken keys: estimate and improve

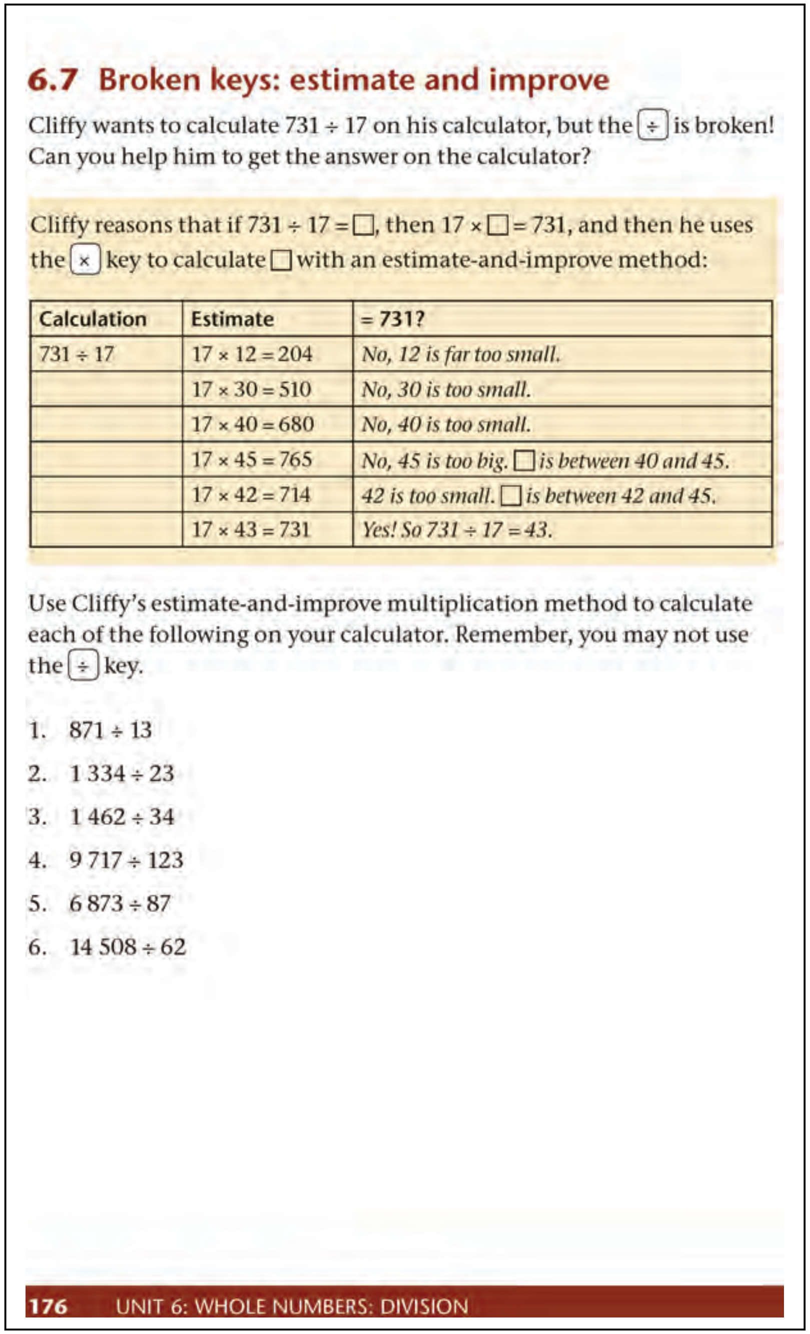
Mathematical notes

The “estimate and improve” method for doing division, as described in the shaded passage, is also used in solving number sentences. See, for example, pages 345 and 346 of the Learner Book.

Answers

Learners’ estimates will differ. The final answers are:

1. 67
2. 58
3. 43
4. 79
5. 79
6. 234



6.7 Broken keys: estimate and improve

Cliffy wants to calculate 731 ÷ 17 on his calculator, but the ÷ is broken! Can you help him to get the answer on the calculator?

Cliffy reasons that if 731 ÷ 17 = □, then 17 × □ = 731, and then he uses the × key to calculate □ with an estimate-and-improve method:

Calculation	Estimate	= 731?
731 ÷ 17	17 × 12 = 204	*No, 12 is far too small.*
	17 × 30 = 510	*No, 30 is too small.*
	17 × 40 = 680	*No, 40 is too small.*
	17 × 45 = 765	*No, 45 is too big. □ is between 40 and 45.*
	17 × 42 = 714	*42 is too small. □ is between 42 and 45.*
	17 × 43 = 731	*Yes! So 731 ÷ 17 = 43.*

Use Cliffy’s estimate-and-improve multiplication method to calculate each of the following on your calculator. Remember, you may not use the ÷ key.

1. 871 ÷ 13
2. 1 334 ÷ 23
3. 1 462 ÷ 34
4. 9 717 ÷ 123
5. 6 873 ÷ 87
6. 14 508 ÷ 62

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Grade 6 Term 2 Unit 7 Decimals

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
7.1 Fifths and tenths and hundredths	Measuring lengths in fractional units	177 to 178
7.2 A different notation for fractions	Being introduced to decimal notation	179 to 181
7.3 Place value parts and number names	Writing decimal numbers in place value parts	181 to 182
7.4 Counting in tenths in both notations	Developing number sense for decimals by counting in tenths using both notations	183 to 185
7.5 Counting in hundredths in both notations	Developing number sense for decimals by counting in hundredths using both notations	185 to 187
7.6 From fractions to decimals to fractions	Converting between notations	187 to188
7.7 Comparing decimals	Understanding the place value of digits in decimals	188 to189
7.8 Reading scales	Reading scales to promote understanding of the place value parts of decimals	190 to 191
7.9 Addition of decimals	Adding decimals by breaking up numbers into place value parts	191 to 193
7.10 Subtraction with decimals	Subtracting decimals by breaking up numbers into place value parts	194 to 195
7.11 Problem solving with decimals	Solving word problems with decimals	196
7.12 Using the calculator to understand decimals	Using calculators to promote understanding of decimal notation	197 to 199

CAPS time allocation	10 hours
CAPS page references	16 to 17 and 252

Mathematical background

Fractions can be represented in different ways: in words, in common fraction notation, in decimal notation, and in percentage notation. For example, "7 twenty-fifths" can be represented as $\frac{7}{25}$ or as 0,28 (which is the decimal notation for $\frac{2}{10} + \frac{8}{100}$) or as 28%, which means $\frac{28}{100}$.

Resources

Cardboard Yellowsticks and Greysticks; calculators

7.1 Fifths and tenths and hundredths

Teaching guidelines

This section takes learners back to the concept of **equivalent fractions**. Learners will focus mainly on tenths, hundredths and fractions that can easily be expressed as tenths and/or hundredths. Without a conceptual understanding of equivalent fractions, learners may struggle to understand decimals and percentages.

Remind learners of the work they did in Term 1 Unit 4, Section 4.2 where they measured lengths accurately with Yellowsticks. Now get learners to work with the longer Greysticks, each of which can be divided into equal parts as small as hundredths.

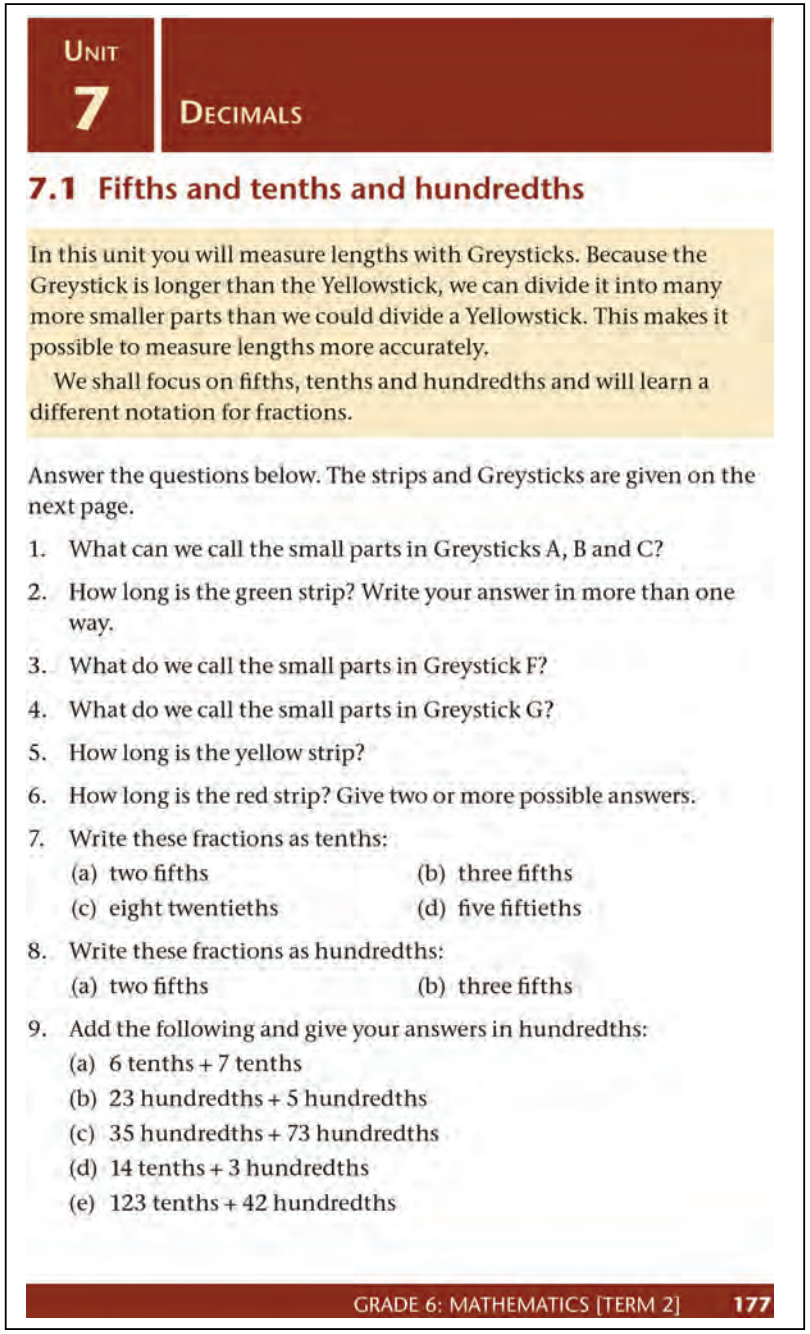
Point out to learners that the questions in Section 7.1 refer to the rulers on the following page in the Learner Book. We suggest that you let learners work in pairs so that they can have one textbook open on page 177 and the other on page 178.

Notes on questions

Question 9 requires learners to add fractions. In order to do this, the fractions must be expressed in the same unit. If they are not expressed in the same unit, learners must convert them to an equivalent form first. It is therefore vital that learners understand the concept of equivalent fractions, as it will assist them in adding and subtracting fractions with different denominators.

Answers

1. Greystick A: tenths; Greystick B: fifths; Greystick C: twentieths
2. 7 tenths; 14 twentieths; 35 fiftieths; 70 hundredths
3. Fiftieths
4. Hundredths
5. 38 fiftieths; 76 hundredths
6. 6 tenths; 60 hundredths; 3 fifths
7. (a) 4 tenths (b) 6 tenths
   (c) 4 tenths (d) 1 tenth
8. (a) 40 hundredths (b) 60 hundredths
9. (a) 130 hundredths (b) 28 hundredths
   (c) 108 hundredths (d) 143 hundredths
   (e) 1 272 hundredths



UNIT 7 DECIMALS

7.1 Fifths and tenths and hundredths

In this unit you will measure lengths with Greysticks. Because the Greystick is longer than the Yellowstick, we can divide it into many more smaller parts than we could divide a Yellowstick. This makes it possible to measure lengths more accurately.

We shall focus on fifths, tenths and hundredths and will learn a different notation for fractions.

Answer the questions below. The strips and Greysticks are given on the next page.

1. What can we call the small parts in Greysticks A, B and C?
2. How long is the green strip? Write your answer in more than one way.
3. What do we call the small parts in Greystick F?
4. What do we call the small parts in Greystick G?
5. How long is the yellow strip?
6. How long is the red strip? Give two or more possible answers.
7. Write these fractions as tenths:
   (a) two fifths (b) three fifths
   (c) eight twentieths (d) five fiftieths
8. Write these fractions as hundredths:
   (a) two fifths (b) three fifths
9. Add the following and give your answers in hundredths:
   (a) 6 tenths + 7 tenths
   (b) 23 hundredths + 5 hundredths
   (c) 35 hundredths + 73 hundredths
   (d) 14 tenths + 3 hundredths
   (e) 123 tenths + 42 hundredths

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Mathematical notes

Note that here learners also work with denominators that are factors of 100. This may develop and further improve learners' understanding when working with percentage notation and decimal notation of fractions.

Teaching guidelines

Finding equivalent fractions often involves subdividing fractional parts into smaller parts and then renaming them.

Greysticks are used to measure different lengths in various units and are thus divided into various fractional parts. As with finding equivalent fractions, these parts must then be named according to the number of subdivisions they have. Ask learners how they would express the length of a Greystick. Also ask them: "How do we name a fraction?" to make the function of the denominator, i.e. the unit of the fraction, clearer.

In this section, learners gain practical experience in expressing the same quantity in different units, i.e. the very essence of equivalence. Learners have the opportunity to learn through experience that 5 tenths, 10 twentieths, 50 hundredths and 1 half all are equivalent. One half ($\frac{1}{2}$) is a particularly useful fraction as it can be used to estimate lengths visually. Any fraction can then be compared to a half to establish its relative size, i.e. is it greater or smaller than $\frac{1}{2}$. Therefore, $\frac{25}{48}$ is more than $\frac{25}{50}$, and $\frac{22}{40}$ is more than $\frac{20}{40}$.

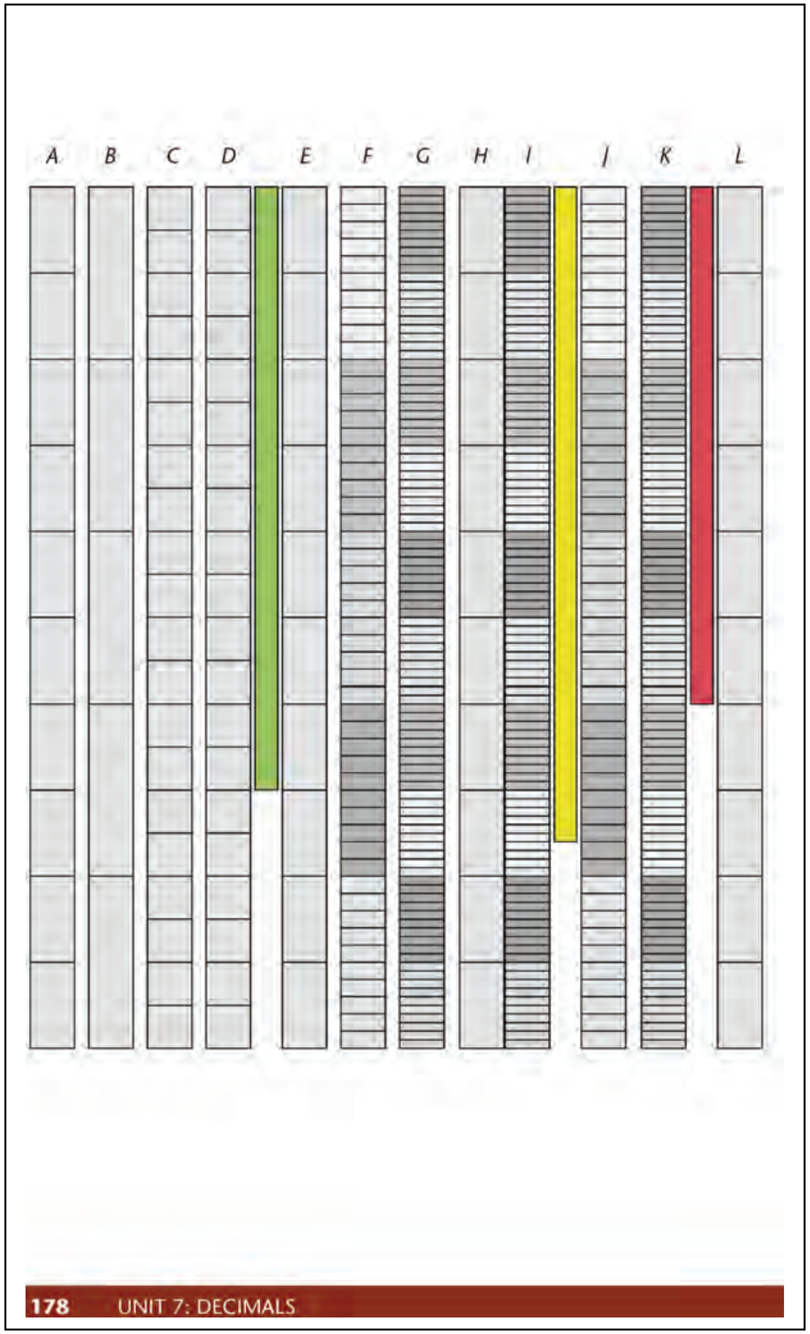
Possible misconceptions

Make sure learners understand the difference between "hundreds", i.e. 200, 300, 400, etc., and "hundredths", i.e. $\frac{2}{100}$, $\frac{3}{100}$ and $\frac{4}{100}$. You can achieve this by stressing the last sounds of each word. Apart from "halves" and "quarters", the names of fractions, i.e. "small parts", are very obvious and meaningful. For example, if there are 16 equal parts, then we are working with "sixteenths".

Notes on questions

If time permits, ask learners to estimate or measure the length and breadth of a page from their textbooks using Yellowsticks and Greysticks. Ask learners to measure the length of a Greystick using Yellowsticks.

At this point, do not encourage learners to use rulers as the aim is not so much about measuring accurately; it is about learners getting used to the meaning of fractions. However, centimetres are hundredths of a metre and will therefore play a more important role at a later stage.



178 UNIT 7: DECIMALS

7.2 A different notation for fractions

Mathematical notes

The decimal notation of a fraction is an extension of the place value notation for whole numbers. For example: 500,46 = 500 + 0,4 + 0,06 (5 hundreds + 4 tenths + 6 hundredths). It is important (and useful) to realise that 46 hundredths is equal to 4 tenths plus 6 hundredths.

Before fractions can be written as decimals, they need to be converted to tenths, hundredths, thousandths, etc.

Teaching guidelines

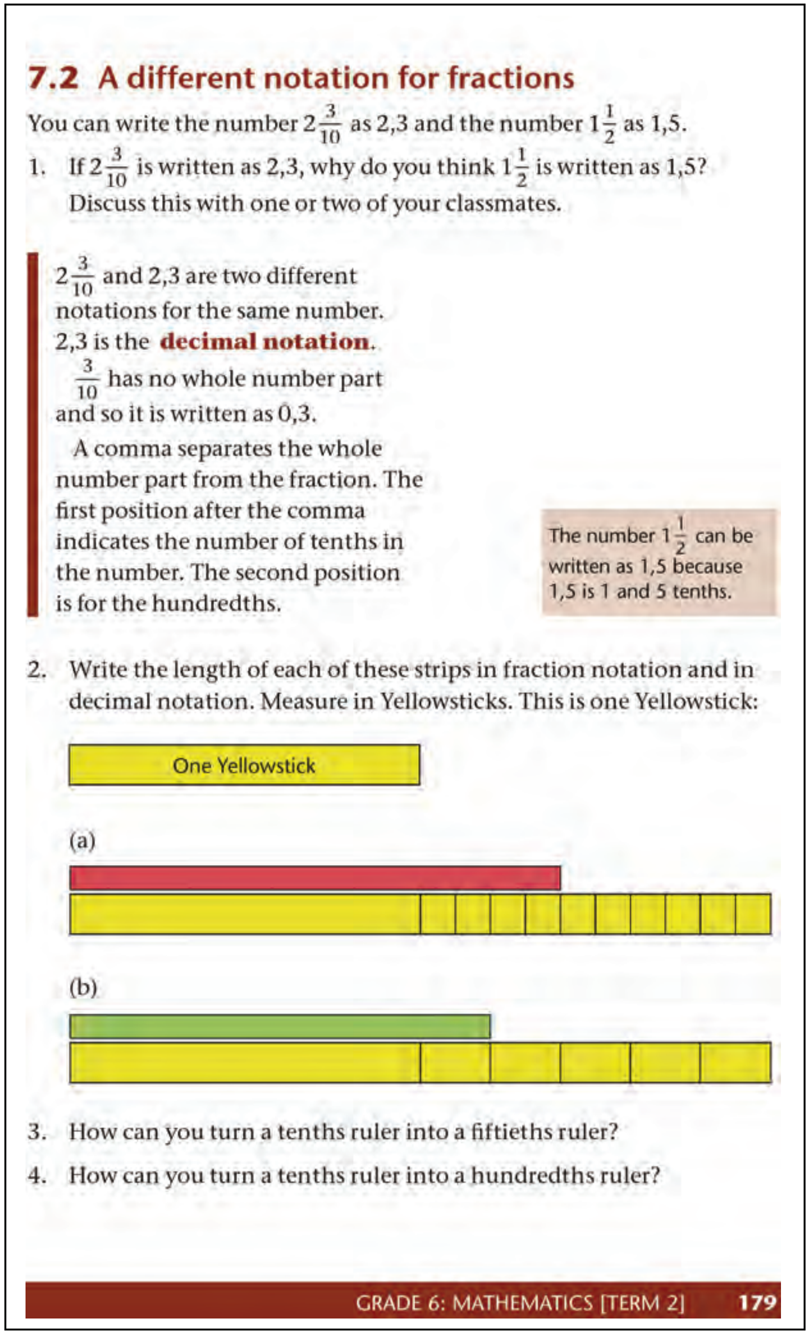
Now that learners are more comfortable working with tenths and hundredths, they can learn how to write these fractions in a different notation. For example, learners are told that $2\frac{3}{10}$ can be written as 2,3, and $1\frac{1}{2}$ can be written as 1,5. Encourage learners to figure out how this new notation works, i.e. one tenth is 0,1 and two tenths is 0,2. Ask them to discuss in pairs why $1\frac{1}{2}$ can be written as 1 and 5 tenths. Get feedback from several pairs of learners before you look at the text alongside the summary bar (i.e. the vertical brown line) on page 179.

Questions 3 and 4 show that the marks $\frac{3}{10}$ and $\frac{6}{10}$ can also indicate $\frac{15}{50}$ and $\frac{30}{50}$ when every unit of measurement has been subdivided.

Answers

1. The 3 in 2,3 denotes tenths. Before fractions can be written as decimals, they need to be converted to tenths, hundredths, thousandths, etc. Thus $1\frac{1}{2} = 1\frac{5}{10}$; therefore it is written as 1,5 and not as 1,1.
2. (a) Red strip: $1\frac{4}{10}$ or 1,4 of a Yellowstick

   (b) Green strip: $1\frac{1}{5}$ or 1,2 of a Yellowstick
3. Divide each of the ten equal parts into five equal parts.
4. Divide each of the ten equal parts into ten equal parts.



7.2 A different notation for fractions

You can write the number $2\frac{3}{10}$ as 2,3 and the number $1\frac{1}{2}$ as 1,5.

1. If $2\frac{3}{10}$ is written as 2,3, why do you think $1\frac{1}{2}$ is written as 1,5? Discuss this with one or two of your classmates.

$2\frac{3}{10}$ and 2,3 are two different notations for the same number. 2,3 is the **decimal notation**.

$\frac{3}{10}$ has no whole number part and so it is written as 0,3.

A comma separates the whole number part from the fraction. The first position after the comma indicates the number of tenths in the number. The second position is for the hundredths.

The number $1\frac{1}{2}$ can be written as 1,5 because 1,5 is 1 and 5 tenths.

2. Write the length of each of these strips in fraction notation and in decimal notation. Measure in Yellowsticks. This is one Yellowstick:

One Yellowstick

(a)

(b)

3. How can you turn a tenths ruler into a fiftieths ruler?
4. How can you turn a tenths ruler into a hundredths ruler?

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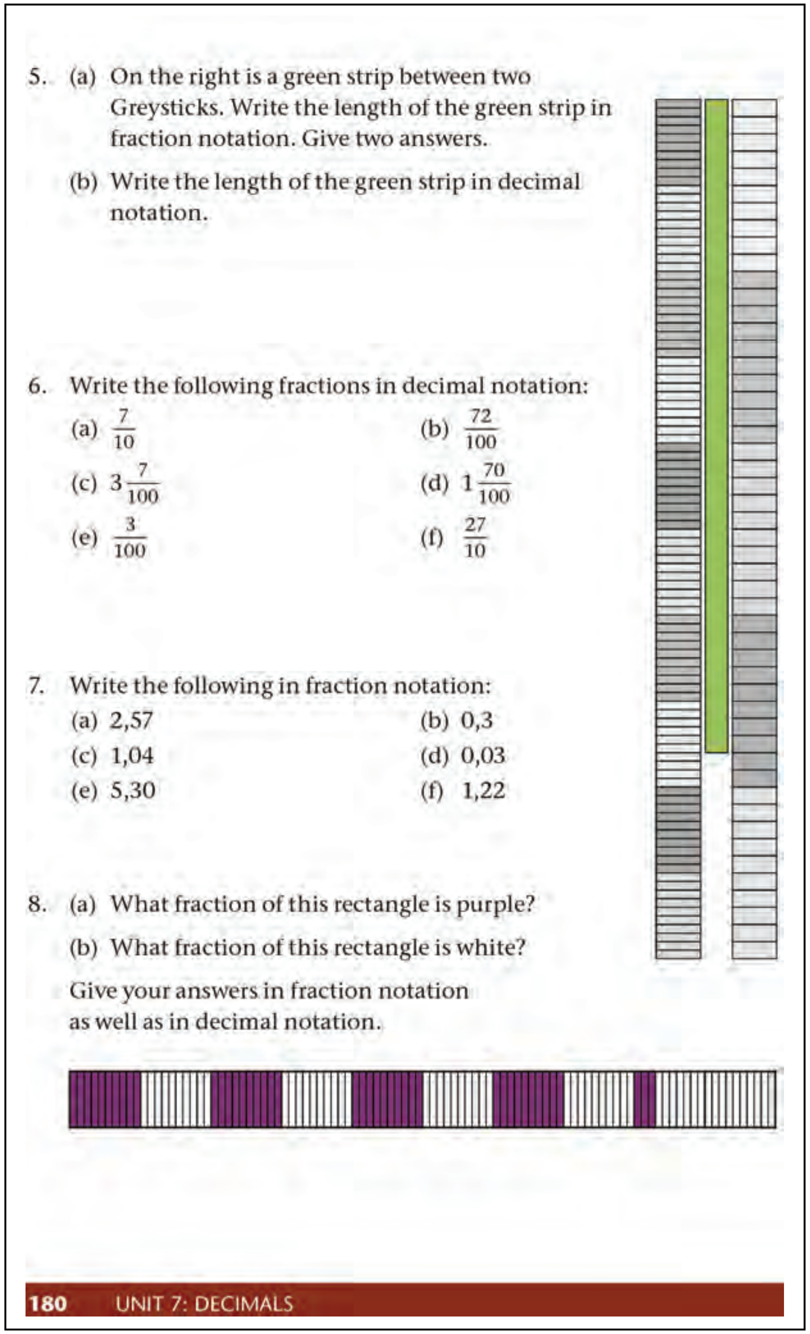
Notes on questions

A zero as last digit does not influence the value of a decimal: for example 1,7 = 1,70

Answers

5. (a) $\frac{76}{100}$; $\frac{38}{50}$
   (b) 0,76
6. (a) 0,7 (b) 0,72
   (c) 3,07 (d) 1,7
   (e) 0,03 (f) 2,7
7. (a) $2\frac{57}{100}$ (b) $\frac{3}{10}$
   (c) $1\frac{4}{100}$ (d) $\frac{3}{100}$
   (e) $5\frac{3}{10}$ (or $5\frac{30}{100}$) (f) $1\frac{22}{100}$
8. (a) $\frac{43}{100}$; 0,43 (b) $\frac{57}{100}$; 0,57



5. (a) On the right is a green strip between two Greysticks. Write the length of the green strip in fraction notation. Give two answers.
   (b) Write the length of the green strip in decimal notation.

6. Write the following fractions in decimal notation:
   (a) $\frac{7}{10}$ (b) $\frac{72}{100}$
   (c) $3\frac{7}{100}$ (d) $1\frac{70}{100}$
   (e) $\frac{3}{100}$ (f) $\frac{27}{10}$

7. Write the following in fraction notation:
   (a) 2,57 (b) 0,3
   (c) 1,04 (d) 0,03
   (e) 5,30 (f) 1,22

8. (a) What fraction of this rectangle is purple?
   (b) What fraction of this rectangle is white?

   Give your answers in fraction notation as well as in decimal notation.

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Answers

9. (a) $\frac{52}{100}$; 0,52 (b) $\frac{23}{100}$; 0,23 (c) $\frac{25}{100}$ ($\frac{1}{4}$); 0,25

10. (a) $\frac{50}{100}$ ($\frac{1}{2}$); 0,5 (b) $\frac{7}{100}$; 0,07 (c) $\frac{43}{100}$; 0,43

11. (a) $\frac{26}{100}$; 0,26 (b) $\frac{29}{100}$; 0,29 (c) $\frac{45}{100}$; 0,45

7.3 Place value parts and number names

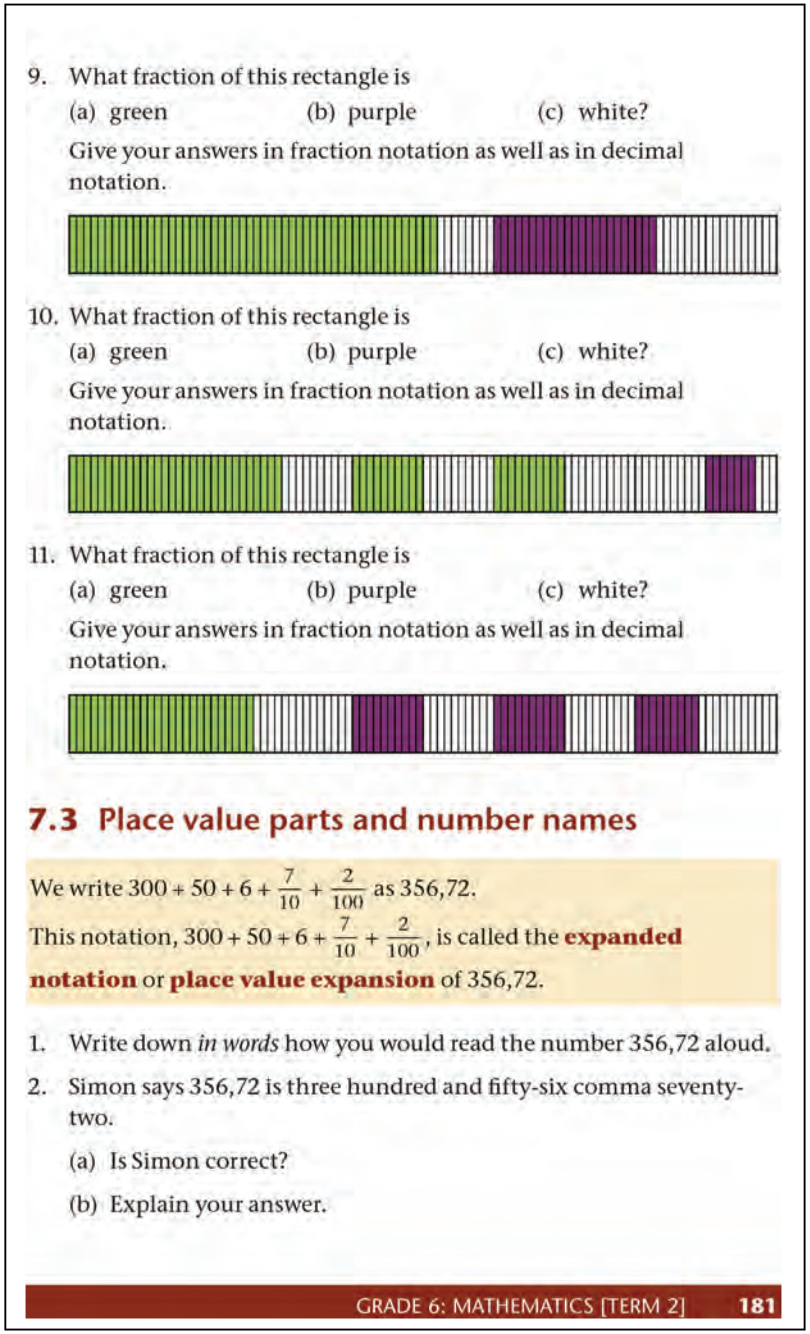
Teaching guidelines

Demonstrate on the board that a number written in decimal notation can also be written in expanded notation, just like whole numbers. You may use the example given in the shaded passage.

In questions 1 and 2 learners need to refrain from reading the decimal part in 356,72 as "seventy-two". It should be read as "seven tenths and two hundredths" or "seventy-two hundredths" or "seven two".

Answers

1. Three hundred and fifty-six comma seven two
2. (a) Simon is not correct, but three hundred and fifty-six and seventy-two hundredths would be correct.
   (b) The digits 7 and 2 after the comma denote tenths and hundredths, not tens and units.



9. What fraction of this rectangle is
   (a) green (b) purple (c) white?
   Give your answers in fraction notation as well as in decimal notation.

10. What fraction of this rectangle is
   (a) green (b) purple (c) white?
   Give your answers in fraction notation as well as in decimal notation.

11. What fraction of this rectangle is
   (a) green (b) purple (c) white?
   Give your answers in fraction notation as well as in decimal notation.

7.3 Place value parts and number names

We write $300 + 50 + 6 + \frac{7}{10} + \frac{2}{100}$ as 356,72.
This notation, $300 + 50 + 6 + \frac{7}{10} + \frac{2}{100}$, is called the **expanded notation** or **place value expansion** of 356,72.

1. Write down *in words* how you would read the number 356,72 aloud.
2. Simon says 356,72 is three hundred and fifty-six comma seventy-two.
   (a) Is Simon correct?
   (b) Explain your answer.

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Teaching guidelines

Demonstrate to learners that when a number is expressed in the decimal notation, the place value parts include tenths and hundredths along with units, tens, hundreds, thousands, etc. You may use the example in the shaded passage and/or other examples.

Answers

3. (a) Three hundred and sixty-two comma seven four

   $300 + 60 + 2 + \frac{7}{10} + \frac{4}{100}$

   (b) One thousand two hundred and eight comma five

   $1\,000 + 200 + 8 + \frac{5}{10}$ ($+ \frac{0}{100}$ in case the 0 is a significant digit)

   (c) Seventy comma three six

   $70 + \frac{3}{10} + \frac{6}{100}$

   (d) One hundred and fifty-four comma one two

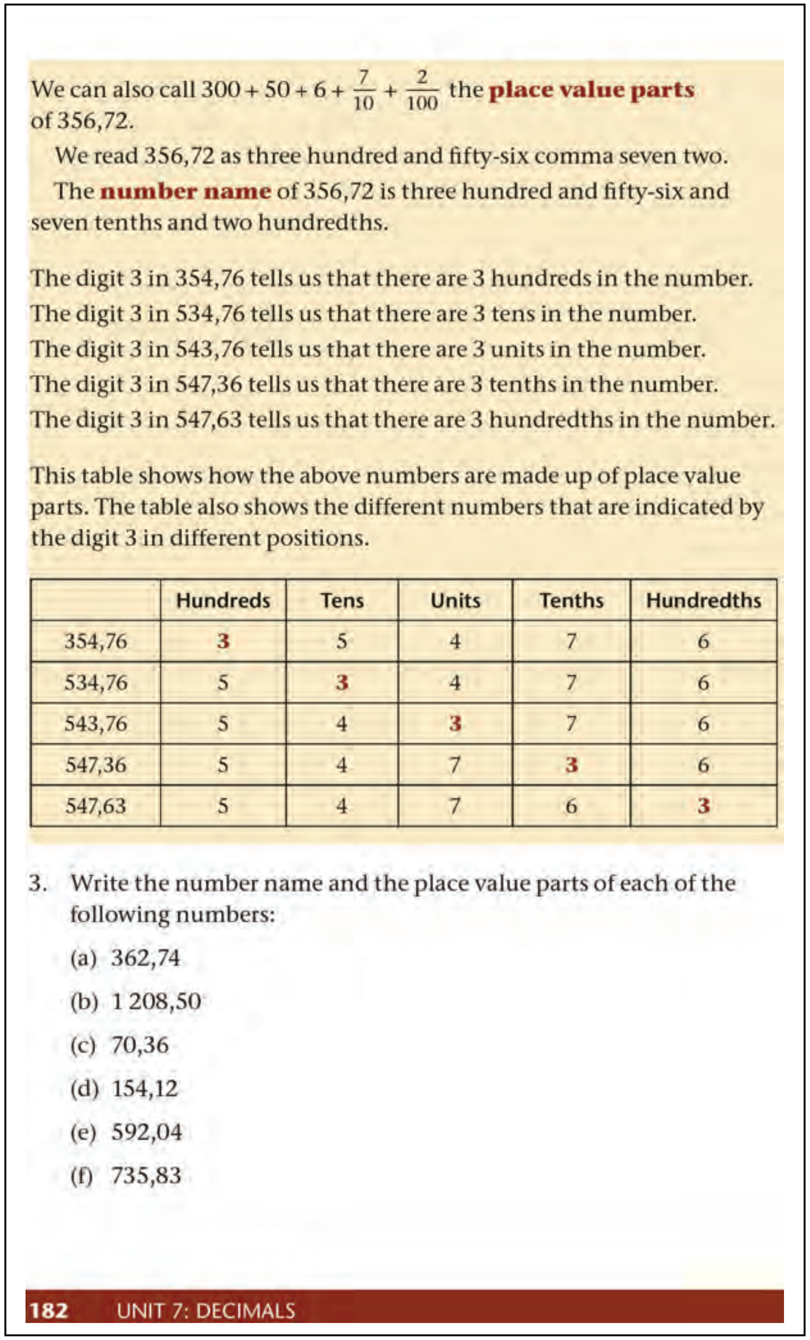
   $100 + 50 + 4 + \frac{1}{10} + \frac{2}{100}$

   (e) Five hundred and ninety-two comma zero four

   $500 + 90 + 2 + \frac{4}{100}$

   (f) Seven hundred and thirty-five comma eight three

   $700 + 30 + 5 + \frac{8}{10} + \frac{3}{100}$



We can also call $300 + 50 + 6 + \frac{7}{10} + \frac{2}{100}$ the **place value parts** of 356,72.

We read 356,72 as three hundred and fifty-six comma seven two.

The **number name** of 356,72 is three hundred and fifty-six and seven tenths and two hundredths.

The digit 3 in 354,76 tells us that there are 3 hundreds in the number.
The digit 3 in 534,76 tells us that there are 3 tens in the number.
The digit 3 in 543,76 tells us that there are 3 units in the number.
The digit 3 in 547,36 tells us that there are 3 tenths in the number.
The digit 3 in 547,63 tells us that there are 3 hundredths in the number.

This table shows how the above numbers are made up of place value parts. The table also shows the different numbers that are indicated by the digit 3 in different positions.

	Hundreds	Tens	Units	Tenths	Hundredths
354,76	**3**	5	4	7	6
534,76	5	**3**	4	7	6
543,76	5	4	**3**	7	6
547,36	5	4	7	**3**	6
547,63	5	4	7	6	**3**

3. Write the number name and the place value parts of each of the following numbers:
   (a) 362,74
   (b) 1 208,50
   (c) 70,36
   (d) 154,12
   (e) 592,04
   (f) 735,83

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7.4 Counting in tenths in both notations

Notes on questions

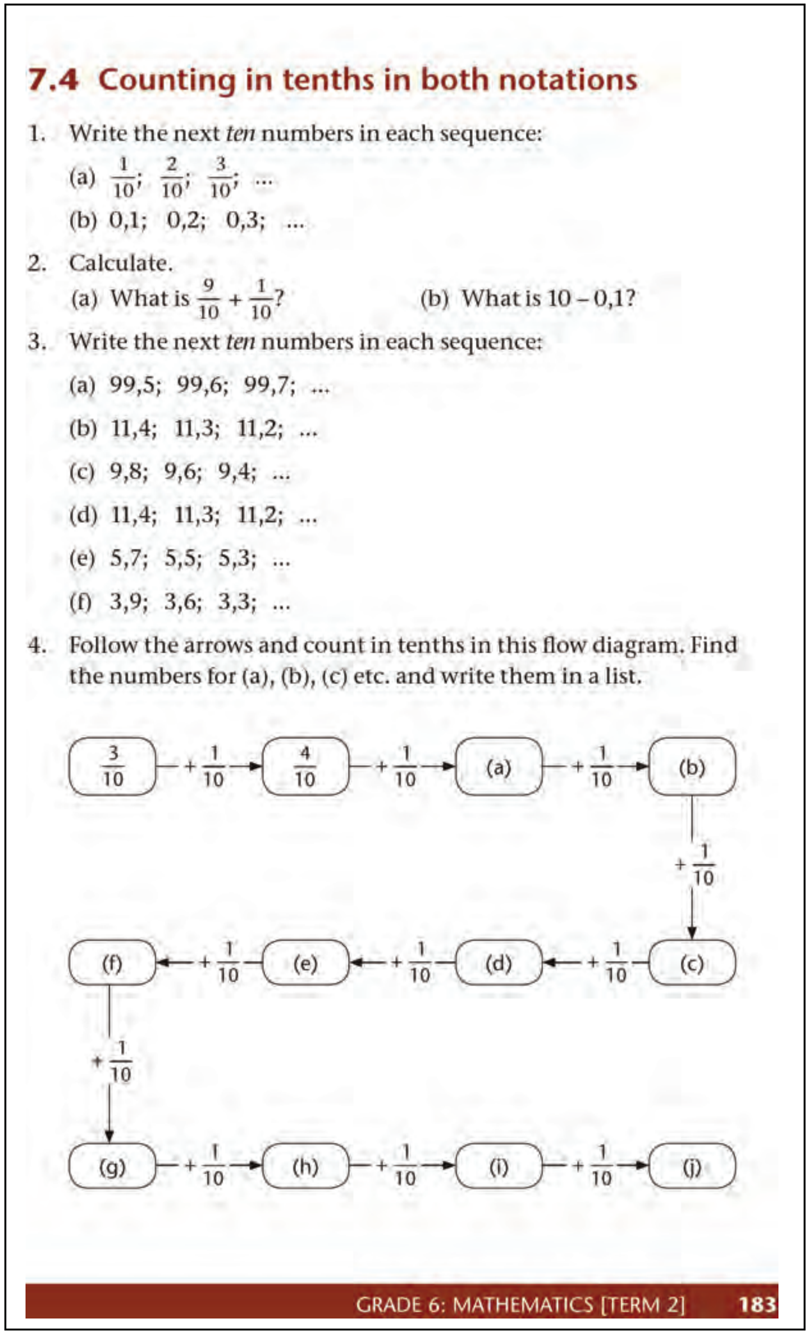
It is important for learners to realise that 10 tenths is the same as 1 (one whole). Question 2(a) illustrates this in a different way. If all the parts are present, we have one whole: 20 twentieths is one unit; 40 twentieths is two units; 83 twentieths would be four units plus another 3 twentieths.

In question 2(b) learners are required to break up a number: $10 - 0{,}1$ is $(9 + \frac{10}{10}) - \frac{1}{10}$.

In question 4 learners work with mixed numbers and improper fractions.

Answers

1. (a) $\frac{4}{10}$; $\frac{5}{10}$; $\frac{6}{10}$; $\frac{7}{10}$; $\frac{8}{10}$; $\frac{9}{10}$; 1; $1\frac{1}{10}$; $1\frac{2}{10}$; $1\frac{3}{10}$
   (b) 0,4; 0,5; 0,6; 0,7; 0,8; 0,9; 1; 1,1; 1,2; 1,3
2. (a) 1 (b) 9,9
3. (a) 99,8; 99,9; 100; 100,1; 100,2; 100,3; 100,4; 100,5; 100,6; 100,7
   (b) 11,1; 11; 10,9; 10,8; 10,7; 10,6; 10,5; 10,4; 10,3; 10,2
   (c) 9,2; 9,0; 8,8; 8,6; 8,4; 8,2; 8,0; 7,8; 7,6; 7,4
   (d) 11,1; 11,0; 10,9; 10,8; 10,7; 10,6; 10,5; 10,4; 10,3; 10,2
   (e) 5,1; 4,9; 4,7; 4,5; 4,3; 4,1; 3,9; 3,7; 3,5; 3,3
   (f) 3,0; 2,7; 2,4; 2,1; 1,8; 1,5; 1,2; 0,9; 0,6; 0,3
4. (a) $\frac{5}{10}$ (b) $\frac{6}{10}$ (c) $\frac{7}{10}$ (d) $\frac{8}{10}$ (e) $\frac{9}{10}$
   (f) $\frac{10}{10}$ or 1 (g) $1\frac{1}{10}$ or $\frac{11}{10}$ (h) $1\frac{2}{10}$ or $\frac{12}{10}$ (i) $1\frac{3}{10}$ or $\frac{13}{10}$ (j) $1\frac{4}{10}$ or $\frac{14}{10}$



7.4 Counting in tenths in both notations

1. Write the next *ten* numbers in each sequence:
   (a) $\frac{1}{10}$; $\frac{2}{10}$; $\frac{3}{10}$; ...
   (b) 0,1; 0,2; 0,3; ...
2. Calculate.
   (a) What is $\frac{9}{10} + \frac{1}{10}$? (b) What is $10 - 0{,}1$?
3. Write the next *ten* numbers in each sequence:
   (a) 99,5; 99,6; 99,7; ...
   (b) 11,4; 11,3; 11,2; ...
   (c) 9,8; 9,6; 9,4; ...
   (d) 11,4; 11,3; 11,2; ...
   (e) 5,7; 5,5; 5,3; ...
   (f) 3,9; 3,6; 3,3; ...
4. Follow the arrows and count in tenths in this flow diagram. Find the numbers for (a), (b), (c) etc. and write them in a list.

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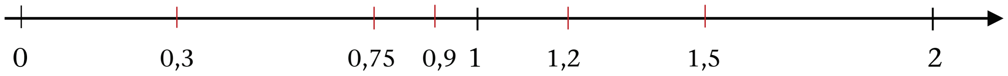
Mathematical notes

Apart from having to place the given numbers correctly on the number line, question 6 may provide learners with the sense that there may be many numbers between any two whole numbers. This awareness will become very important when they engage with mathematics at higher levels in higher grades.

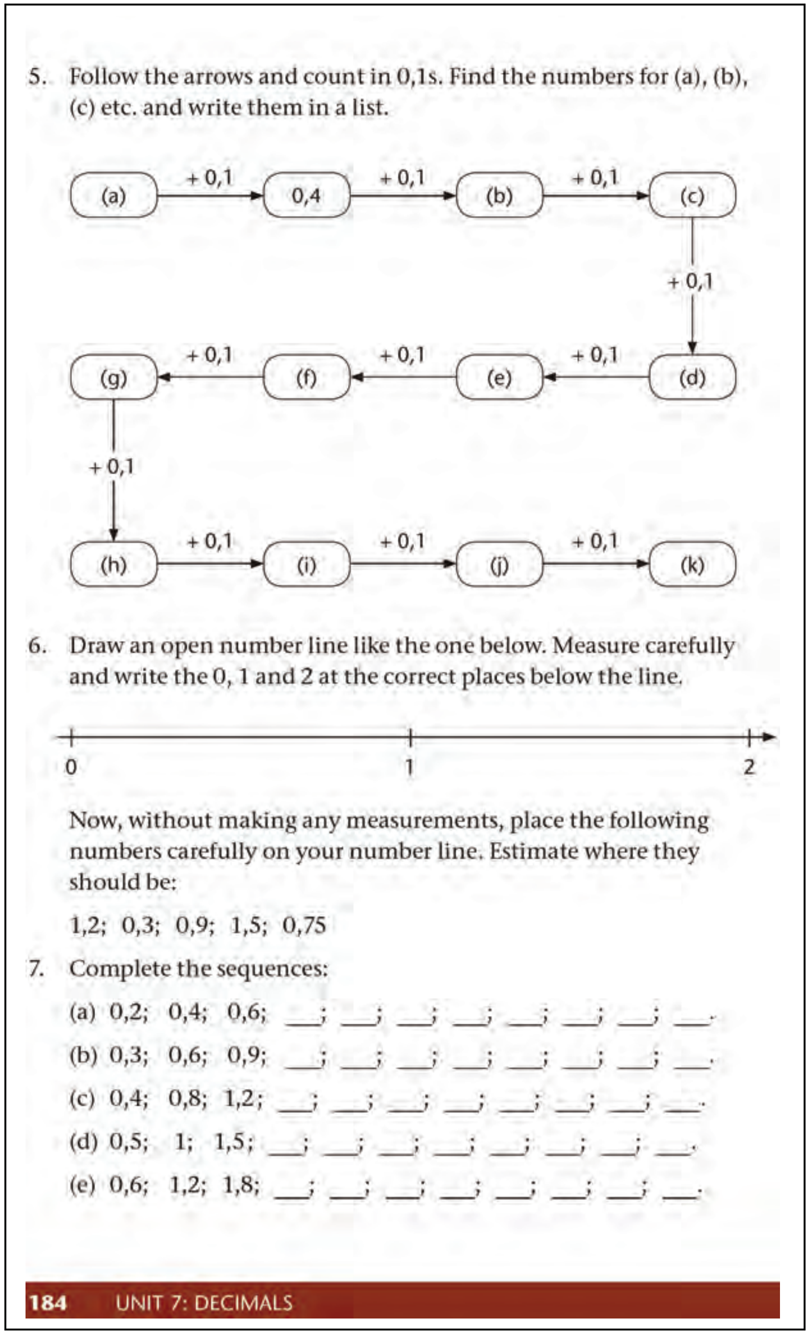
Answers

5. (a) 0,3 (b) 0,5 (c) 0,6 (d) 0,7
   (e) 0,8 (f) 0,9 (g) 1,0 (h) 1,1
   (i) 1,2 (j) 1,3 (k) 1,4

6.



7. (a) 0,8; 1,0; 1,2; 1,4; 1,6; 1,8; 2,0; 2,2
   (b) 1,2; 1,5; 1,8; 2,1; 2,4; 2,7; 3,0; 3,3
   (c) 1,6; 2,0; 2,4; 2,8; 3,2; 3,6; 4,0; 4,4
   (d) 2,0; 2,5; 3,0; 3,5; 4,0; 4,5; 5,0; 5,5
   (e) 2,4; 3,0; 3,6; 4,2; 4,8; 5,4; 6,0; 6,6



5. Follow the arrows and count in 0,1s. Find the numbers for (a), (b), (c) etc. and write them in a list.

6. Draw an open number line like the one below. Measure carefully and write the 0, 1 and 2 at the correct places below the line.

   Now, without making any measurements, place the following numbers carefully on your number line. Estimate where they should be:

   1,2; 0,3; 0,9; 1,5; 0,75

7. Complete the sequences:

   (a) 0,2; 0,4; 0,6; __; __; __; __; __; __; __; __.
   (b) 0,3; 0,6; 0,9; __; __; __; __; __; __; __; __.
   (c) 0,4; 0,8; 1,2; __; __; __; __; __; __; __; __.
   (d) 0,5; 1; 1,5; __; __; __; __; __; __; __; __.
   (e) 0,6; 1,2; 1,8; __; __; __; __; __; __; __; __.

Notes on questions

Question 8(b) gives learners the opportunity to reason as they would when performing grouping as a form of division. For example: "How many . . . in . . . ?"

Answers

8. (a) 0,8; 1,0; 1,2; 1,4; 1,6; 1,8; 2,0; 2,2
   (b) 5
   (c) 1,2; 1,5; 1,8; 2,1; 2,4; 2,7; 3,0
   (d) 1,2; 1,6; 2,0; 2,4; 2,8; 3,2; 3,6; 4,0; 4,4
   (e) 5
   (f) 1; 1,5; 2,0; 2,5; 3,0; 3,5; 4,0; 4,5; 5,0; 5,5
   (g) 1,2; 1,8; 2,4; 3,0; 3,6; 4,2; 4,8; 5,4; 6,0; 6,6

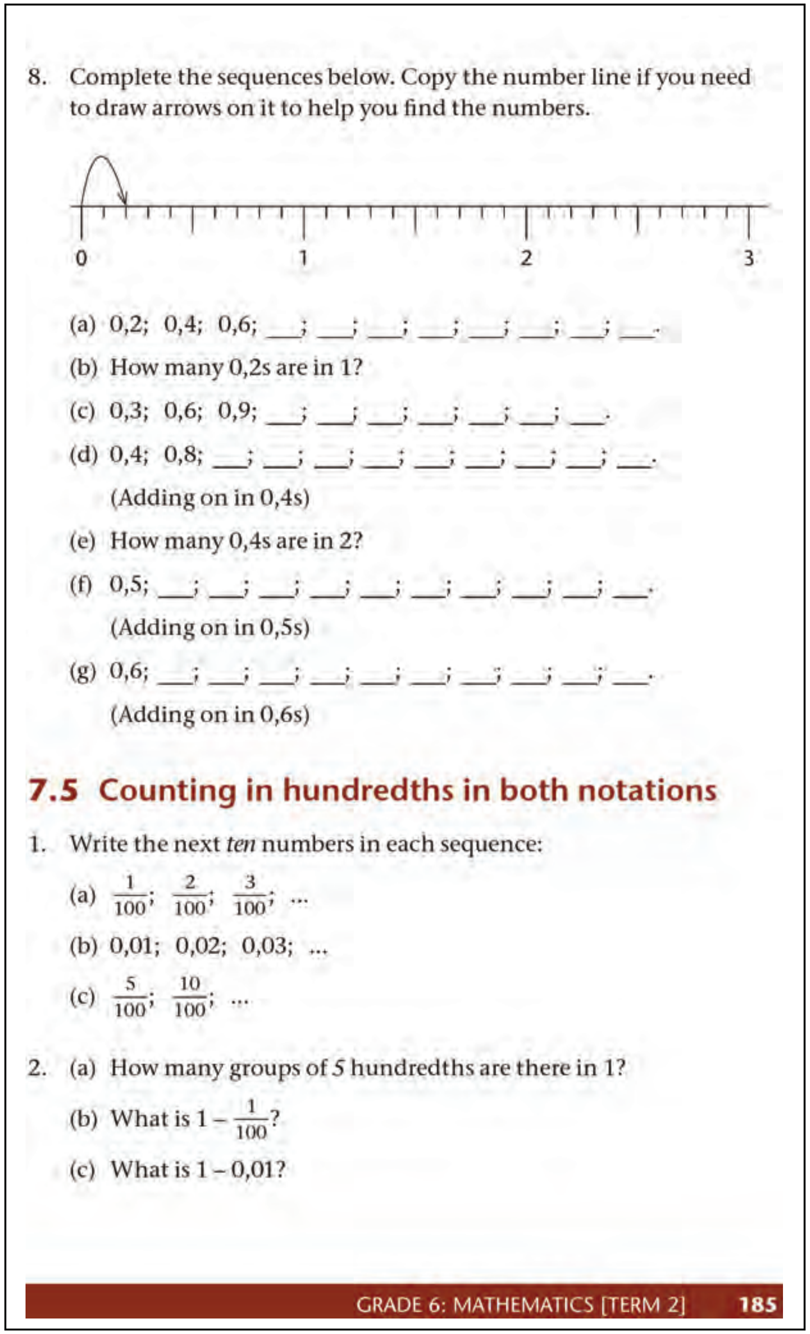
7.5 Counting in hundredths in both notations

Notes on questions

Questions 2(b) and (c) are exactly the same, but the answers must be expressed in the same format as the questions. These different formats stimulate learners' different thinking processes.

Answers

1. (a) $\frac{4}{100}$; $\frac{5}{100}$; $\frac{6}{100}$; $\frac{7}{100}$; $\frac{8}{100}$; $\frac{9}{100}$; $\frac{10}{100}$; $\frac{11}{100}$; $\frac{12}{100}$; $\frac{13}{100}$
   (b) 0,04; 0,05; 0,06; 0,07; 0,08; 0,09; 0,1; 0,11; 0,12; 0,13
   (c) $\frac{15}{100}$; $\frac{20}{100}$; $\frac{25}{100}$; $\frac{30}{100}$; $\frac{35}{100}$; $\frac{40}{100}$; $\frac{45}{100}$; $\frac{50}{100}$; $\frac{55}{100}$; $\frac{60}{100}$
2. (a) 20
   (b) $\frac{99}{100}$
   (c) 0,99



8. Complete the sequences below. Copy the number line if you need to draw arrows on it to help you find the numbers.

(a) 0,2; 0,4; 0,6; ___; ___; ___; ___; ___; ___; ___; ___.
(b) How many 0,2s are in 1?
(c) 0,3; 0,6; 0,9; ___; ___; ___; ___; ___; ___; ___.
(d) 0,4; 0,8; ___; ___; ___; ___; ___; ___; ___; ___; ___.
(Adding on in 0,4s)
(e) How many 0,4s are in 2?
(f) 0,5; ___; ___; ___; ___; ___; ___; ___; ___; ___; ___.
(Adding on in 0,5s)
(g) 0,6; ___; ___; ___; ___; ___; ___; ___; ___; ___; ___.
(Adding on in 0,6s)

7.5 Counting in hundredths in both notations

1. Write the next *ten* numbers in each sequence:
   (a) $\frac{1}{100}$; $\frac{2}{100}$; $\frac{3}{100}$; ...
   (b) 0,01; 0,02; 0,03; ...
   (c) $\frac{5}{100}$; $\frac{10}{100}$; ...
2. (a) How many groups of 5 hundredths are there in 1?
   (b) What is $1 - \frac{1}{100}$?
   (c) What is 1 – 0,01?

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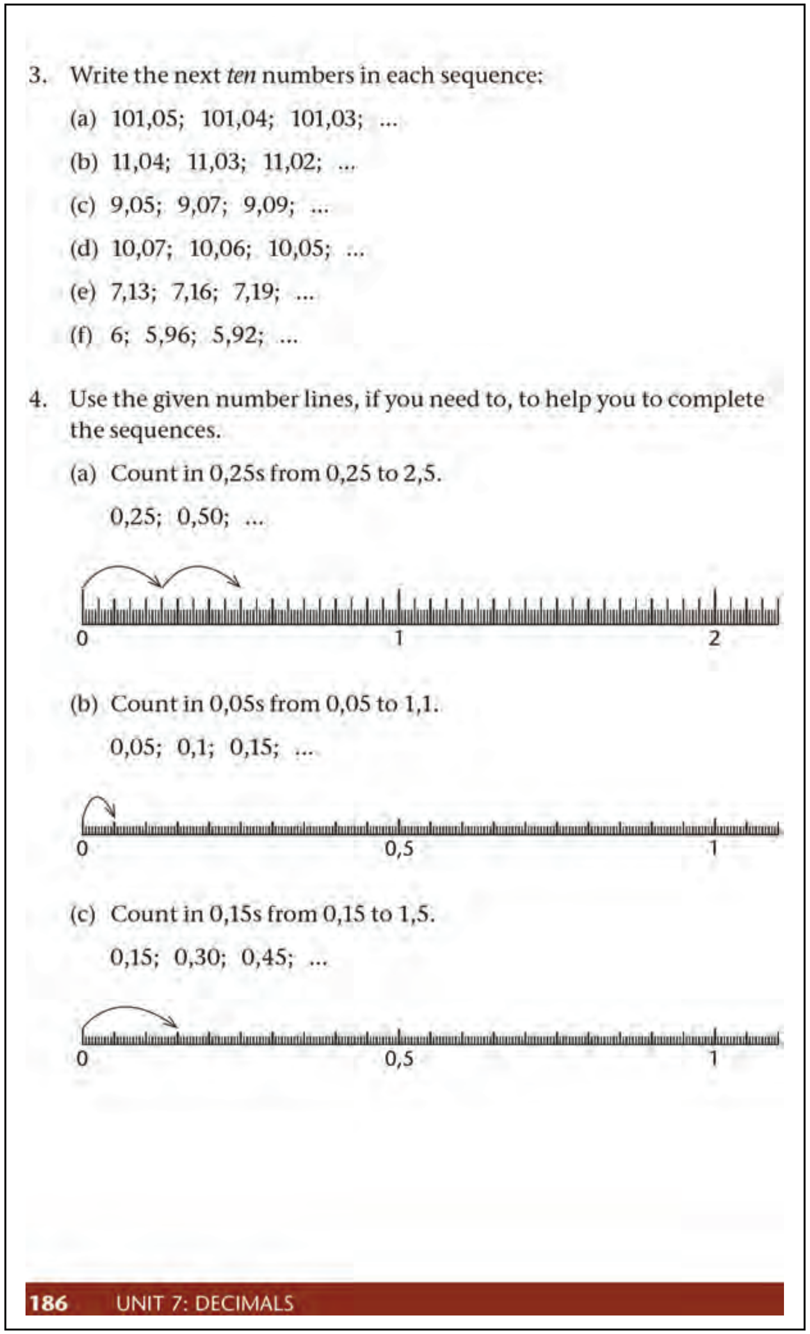
Possible misconceptions

Although the questions require learners to write the next ten numbers, it is important that they also count beyond the next ten; or only from 1 to 2, to 3 (if counting in 0,1s). We do not want to confuse them into thinking that the tenth point is the "end point".

Answers

3. (a) 101,02; 101,01; 101,0; 100,09; 100,08; 100,07; 100,06; 100,05; 100,04; 100,03
   (b) 11,01; 11,0; 10,09; 10,08; 10,07; 10,06; 10,05; 10,04; 10,03; 10,02
   (c) 9,11; 9,13; 9,15; 9,17; 9,19; 9,21; 9,23; 9,25; 9,27; 9,29
   (d) 10,04; 10,03; 10,02; 10,01; 10,0; 9,09; 9,08; 9,07; 9,06; 9,05
   (e) 7,22; 7,25; 7,28; 7,31; 7,34; 7,37; 7,4; 7,43; 7,46; 7,49
   (f) 5,88; 5,84; 5,8; 5,76; 5,72; 5,68; 5,64; 5,6; 5,56; 5,52
4. (a) 0,75; 1; 1,25; 1,50; 1,75; 2; 2,25; 2,5
   (b) 0,2; 0,25; 0,3; 0,35; 0,4; 0,45; 0,5; 0,55; 0,6; 0,65; 0,7; 0,75; 0,8; 0,85; 0,9; 0,95; 1; 1,05; 1,1
   (c) 0,6; 0,75; 0,9; 1,05; 1,2; 1,35; 1,5



3. Write the next *ten* numbers in each sequence:
   (a) 101,05; 101,04; 101,03; ...
   (b) 11,04; 11,03; 11,02; ...
   (c) 9,05; 9,07; 9,09; ...
   (d) 10,07; 10,06; 10,05; ...
   (e) 7,13; 7,16; 7,19; ...
   (f) 6; 5,96; 5,92; ...

4. Use the given number lines, if you need to, to help you to complete the sequences.
   (a) Count in 0,25s from 0,25 to 2,5.
   0,25; 0,50; ...

   (b) Count in 0,05s from 0,05 to 1,1.
   0,05; 0,1; 0,15; ...

   (c) Count in 0,15s from 0,15 to 1,5.
   0,15; 0,30; 0,45; ...

Answers

5. (a) $\frac{95}{100}$ (b) $\frac{96}{100}$ (c) $\frac{97}{100}$ (d) $\frac{98}{100}$

(e) $\frac{99}{100}$ (f) 1 (g) $1\frac{1}{100}$ (or $\frac{101}{100}$) (h) $1\frac{2}{100}$ (or $\frac{102}{100}$)

(i) $1\frac{3}{100}$ (j) $1\frac{4}{100}$ (k) $1\frac{5}{100}$

Refer to the comment made on mixed numbers and improper fractions in Section 7.4 when learners completed a fraction diagram and counted in tenths.

7.6 From fractions to decimals to fractions

Mathematical notes

Show learners how to write 100 as a product of its factors in different ways. Factors of 100 are 1, 2, 4, 5, 10, 20, 25, 50 and 100.

For example:

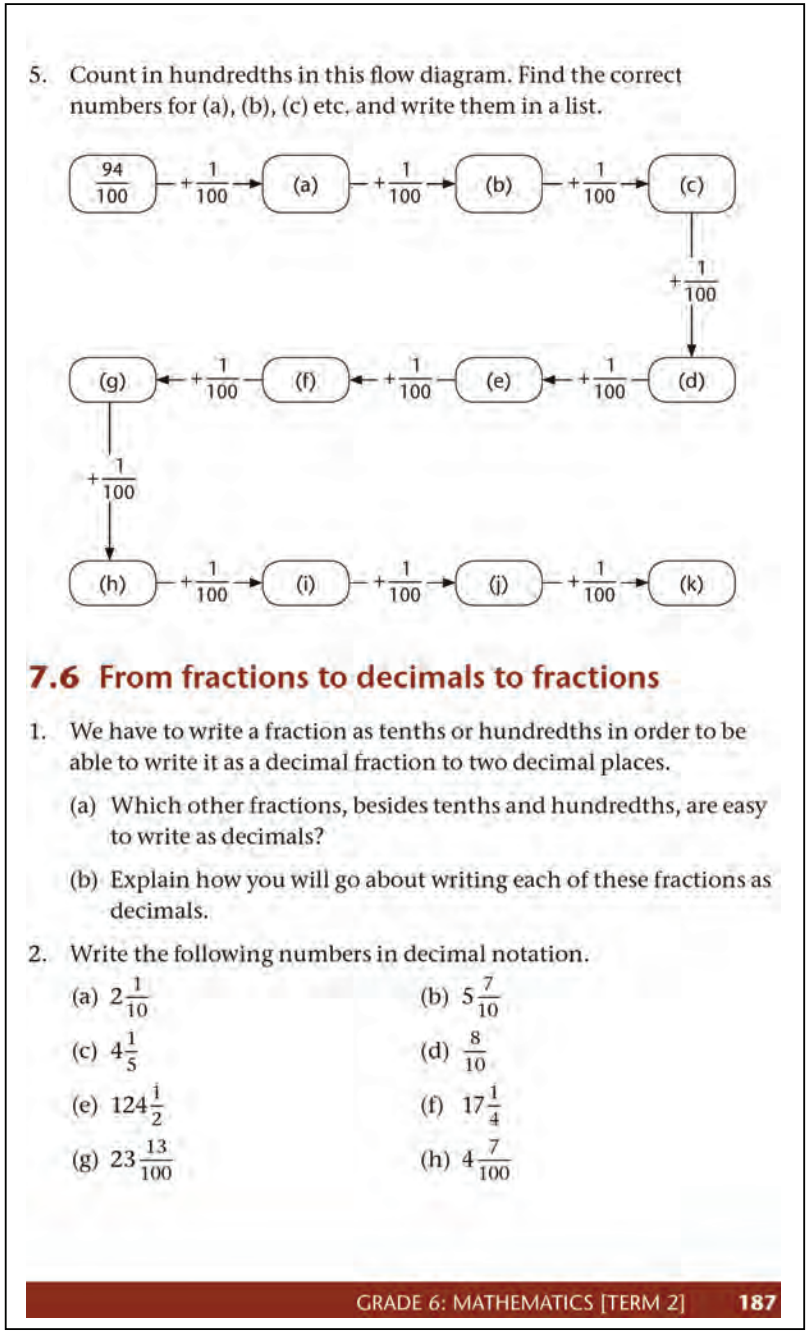
- 1 × 100, 2 × 50, 4 × 25, 5 × 20, 10 × 10
- 4 × 25 can also be shown as 2 × 2 × 5 × 5 or 2 × 2 × 25

Common misconceptions

If there are fewer than 10 hundredths, learners sometimes make the mistake of omitting the "0" that is necessary to write the number of hundredths in the correct column. For example, question 2(h) is sometimes answered as 4,7 instead of 4,07. This is a good opportunity to compare it with 2(b). Every 10 hundredths is one tenth. Therefore, 78 hundredths must be regarded as 7 tenths plus 8 hundredths; 7 hundredths would be 0 tenths and 7 hundredths.

Answers

1. (a) Halves, fifths, twentieths – i.e. fractions with denominators of 2s or 5s as factors.
   (b) Write as an equivalent fraction with ten or hundred as the denominator.
2. (a) 2,1 (b) 5,7
   (c) 4,2 (d) 0,8
   (e) 124,5 (f) 17,25
   (g) 23,13 (h) 4,07



5. Count in hundredths in this flow diagram. Find the correct numbers for (a), (b), (c) etc. and write them in a list.

7.6 From fractions to decimals to fractions

1. We have to write a fraction as tenths or hundredths in order to be able to write it as a decimal fraction to two decimal places.
   (a) Which other fractions, besides tenths and hundredths, are easy to write as decimals?
   (b) Explain how you will go about writing each of these fractions as decimals.
2. Write the following numbers in decimal notation.
   (a) $2\frac{1}{10}$ (b) $5\frac{7}{10}$
   (c) $4\frac{1}{5}$ (d) $\frac{8}{10}$
   (e) $124\frac{1}{2}$ (f) $17\frac{1}{4}$
   (g) $23\frac{13}{100}$ (h) $4\frac{7}{100}$

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Answers

3. (a) $3 + \frac{2}{10}$ (b) $4 + \frac{2}{10} + \frac{7}{100}$

   (c) $7 + \frac{5}{10} + \frac{3}{100}$ (d) $10 + 2 + \frac{3}{100}$

   (e) $50 + \frac{3}{10}$ (f) $3 + \frac{2}{10} + \frac{5}{100}$

   (g) $50 + 6 + \frac{2}{10}$ (h) $20 + \frac{5}{10}$

   (i) $10 + 1 + \frac{7}{10} + \frac{5}{100}$ (j) $\frac{8}{10}$

4. (a) 9,4; 9,2; 9; 8,8; 8,6; 8,4; 8,2

   $9\frac{4}{10}$; $9\frac{2}{10}$; 9; $8\frac{8}{10}$; $8\frac{6}{10}$; $8\frac{4}{10}$; $8\frac{2}{10}$

   (b) 0,6; 0,75; 0,9; 1,05; 1,2; 1,35 (counting in 0,15s)

   $\frac{6}{10}$; $\frac{75}{100}$; $\frac{9}{10}$; $1\frac{5}{100}$; $1\frac{2}{10}$; $1\frac{35}{100}$

   (Learners may also give the fractions as hundredths instead of tenths, for example: $\frac{90}{100}$ instead of $\frac{9}{10}$.)

7.7 Comparing decimals

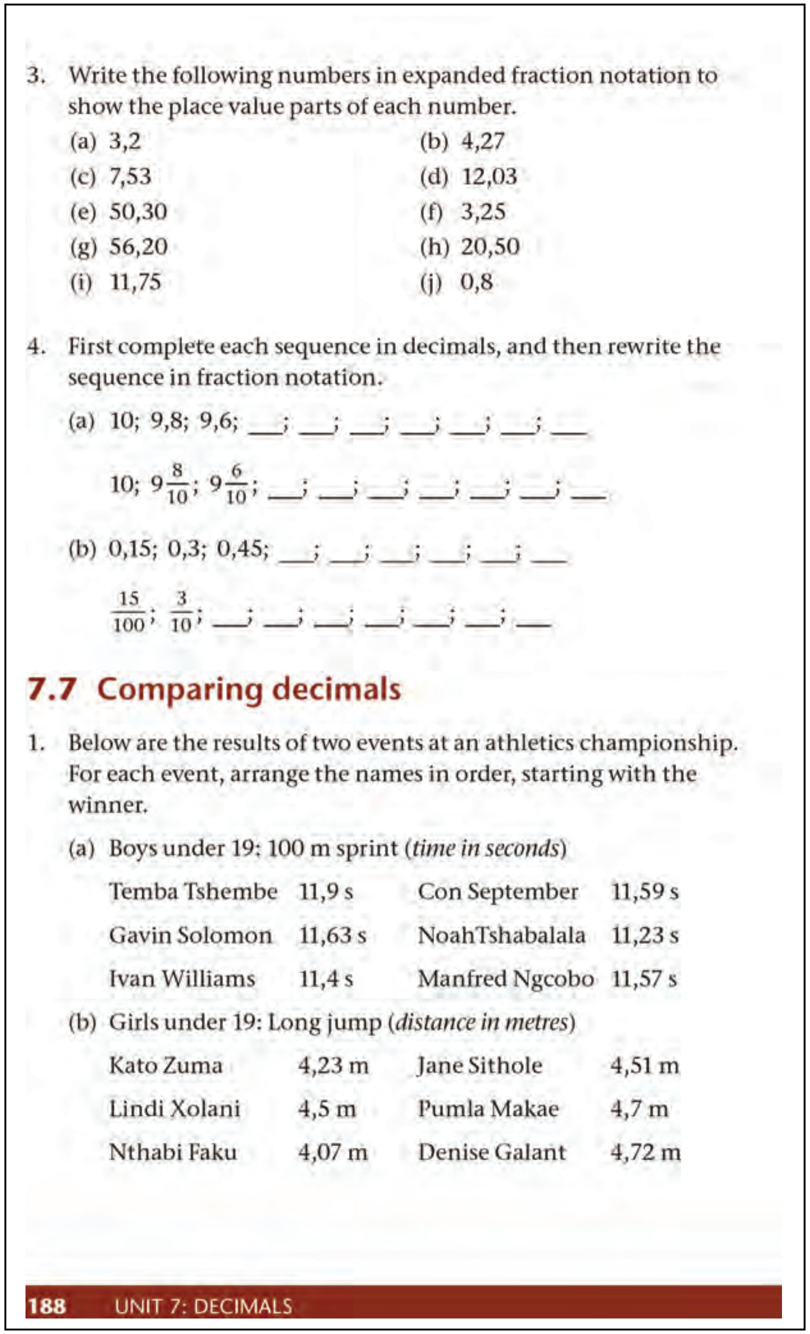
Answers

1. (a)

1.	Noah Tshabalala	11,23 s
2.	Ivan Williams	11,4 s
3.	Manfred Ngcobo	11,57 s
4.	Con September	11,59 s
5.	Gavin Solomon	11,63 s
6.	Temba Tshembe	11,9 s

(b)

1.	Denise Galant	4,72 m
2.	Pumla Makae	4,7 m
3.	Jane Sithole	4,51 m
4.	Lindi Xolani	4,5 m
5.	Kato Zuma	4,23 m
6.	Nthabi Faku	4,07 m



3. Write the following numbers in expanded fraction notation to show the place value parts of each number.
   (a) 3,2 (b) 4,27
   (c) 7,53 (d) 12,03
   (e) 50,30 (f) 3,25
   (g) 56,20 (h) 20,50
   (i) 11,75 (j) 0,8

4. First complete each sequence in decimals, and then rewrite the sequence in fraction notation.
   (a) 10; 9,8; 9,6; ___; ___; ___; ___; ___; ___; ___

   10; $9\frac{8}{10}$; $9\frac{6}{10}$; ___; ___; ___; ___; ___; ___; ___

   (b) 0,15; 0,3; 0,45; ___; ___; ___; ___; ___; ___

   $\frac{15}{100}$; $\frac{3}{10}$; ___; ___; ___; ___; ___; ___; ___

7.7 Comparing decimals

1. Below are the results of two events at an athletics championship. For each event, arrange the names in order, starting with the winner.

   (a) Boys under 19: 100 m sprint (*time in seconds*)

Temba Tshembe	11,9 s	Con September	11,59 s
Gavin Solomon	11,63 s	NoahTshabalala	11,23 s
Ivan Williams	11,4 s	Manfred Ngcobo	11,57 s

   (b) Girls under 19: Long jump (*distance in metres*)

Kato Zuma	4,23 m	Jane Sithole	4,51 m
Lindi Xolani	4,5 m	Pumla Makae	4,7 m
Nthabi Faku	4,07 m	Denise Galant	4,72 m

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Notes on questions

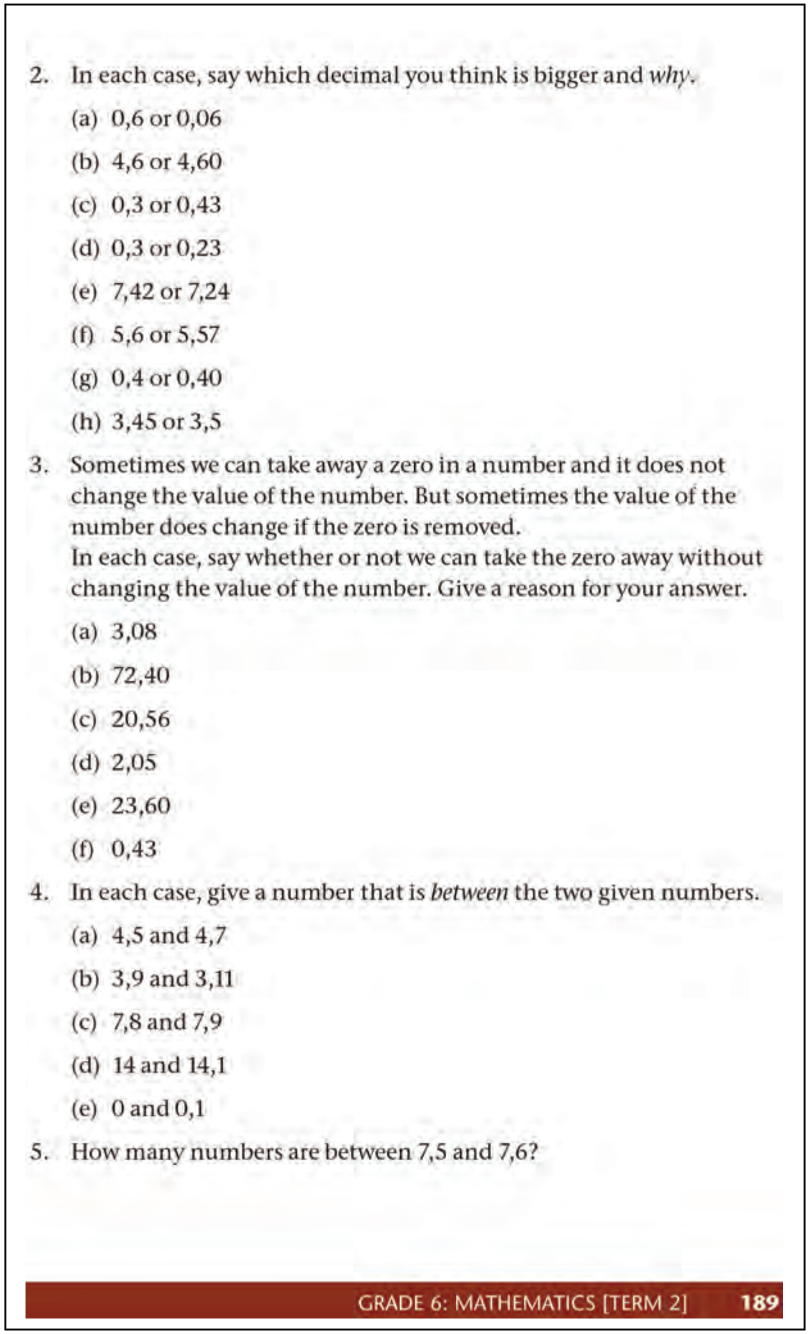
Question 2 shows that a fraction that is expressed with a longer string of meaningful, i.e. non-zero, digits after the comma (in this case two digits), is not necessarily the biggest. For example, 4,23 is smaller than 4,7. If all the digits to the left of the comma are equal, the tenths are the most significant, followed by the hundredths (compared only if the tenths are equal). In long jump or high jump, for example, a higher number indicates a longer distance or higher height. This therefore means a good performance. On the other hand, in a 100 m sprint, a smaller number indicates a faster time. This therefore means a better performance.

In question 2(f), for example, possible justifications can include: "5,6 has 6 tenths, which is more than the 5 tenths of 5,57. The fact that 5,57 has another 7 hundredths as well, is not important, because less than 10 hundredths is less than one tenth." You can expect shorter, less detailed answers from learners, but use this opportunity to hold a class discussion around this.

Question 3 also gives learners the opportunity to think about what zero means in different positions. There are many possible answers to question 4. While there is merit in being able to work out the number exactly in the middle of two numbers, it is not the point of this exercise.

Answers

2. (a) 0,6 = 6 tenths and 0,06 = 6 hundredths
   (b) 4,6 = 4,60 (6 tenths is the same size as 60 hundredths)
   (c) 0,43 is 4 tenths + 3 hundredths, and 0,3 is 3 tenths
   (d) 0,3 is 3 tenths, and 0,23 is 2 tenths + 3 hundredths
   (e) 7,42 is 7 and 4 tenths + 2 hundredths, and 7,24 is 7 and 2 tenths + 4 hundredths
   (f) 5,6 is 5 and 6 tenths, and 5,57 is 5 and 5 tenths + 7 hundredths
   (g) 0,4 = 0,40 – i.e. they are the same, because 4 tenths is the same as 40 hundredths
   (h) 3,5 is 3 and 5 tenths, and 3,45 is 3 and 4 tenths + 5 hundredths
3. (a) 3,08 No, the 0 is a placeholder for tenths.
   (b) 72,40 Yes, it is not a placeholder.
   (c) 20,56 No, the 0 is a placeholder for units.
   (d) 2,05 No, the 0 is a placeholder for tenths.
   (e) 23,60 Yes, it is not a placeholder.
   (f) 0,43 Yes, it is not a placeholder.
4. (a) 4,6 (b) 3,95 (c) 7,85 (d) 14,05 (e) 0,05
5. An infinite number (take time to discuss this using a number line).



2. In each case, say which decimal you think is bigger and *why*.
   (a) 0,6 or 0,06
   (b) 4,6 or 4,60
   (c) 0,3 or 0,43
   (d) 0,3 or 0,23
   (e) 7,42 or 7,24
   (f) 5,6 or 5,57
   (g) 0,4 or 0,40
   (h) 3,45 or 3,5
3. Sometimes we can take away a zero in a number and it does not change the value of the number. But sometimes the value of the number does change if the zero is removed.
   In each case, say whether or not we can take the zero away without changing the value of the number. Give a reason for your answer.
   (a) 3,08
   (b) 72,40
   (c) 20,56
   (d) 2,05
   (e) 23,60
   (f) 0,43
4. In each case, give a number that is *between* the two given numbers.
   (a) 4,5 and 4,7
   (b) 3,9 and 3,11
   (c) 7,8 and 7,9
   (d) 14 and 14,1
   (e) 0 and 0,1
5. How many numbers are between 7,5 and 7,6?

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7.8 Reading scales

Mathematical notes

If learners have a good understanding of place value in decimals, they will be able to read scales with relative ease.

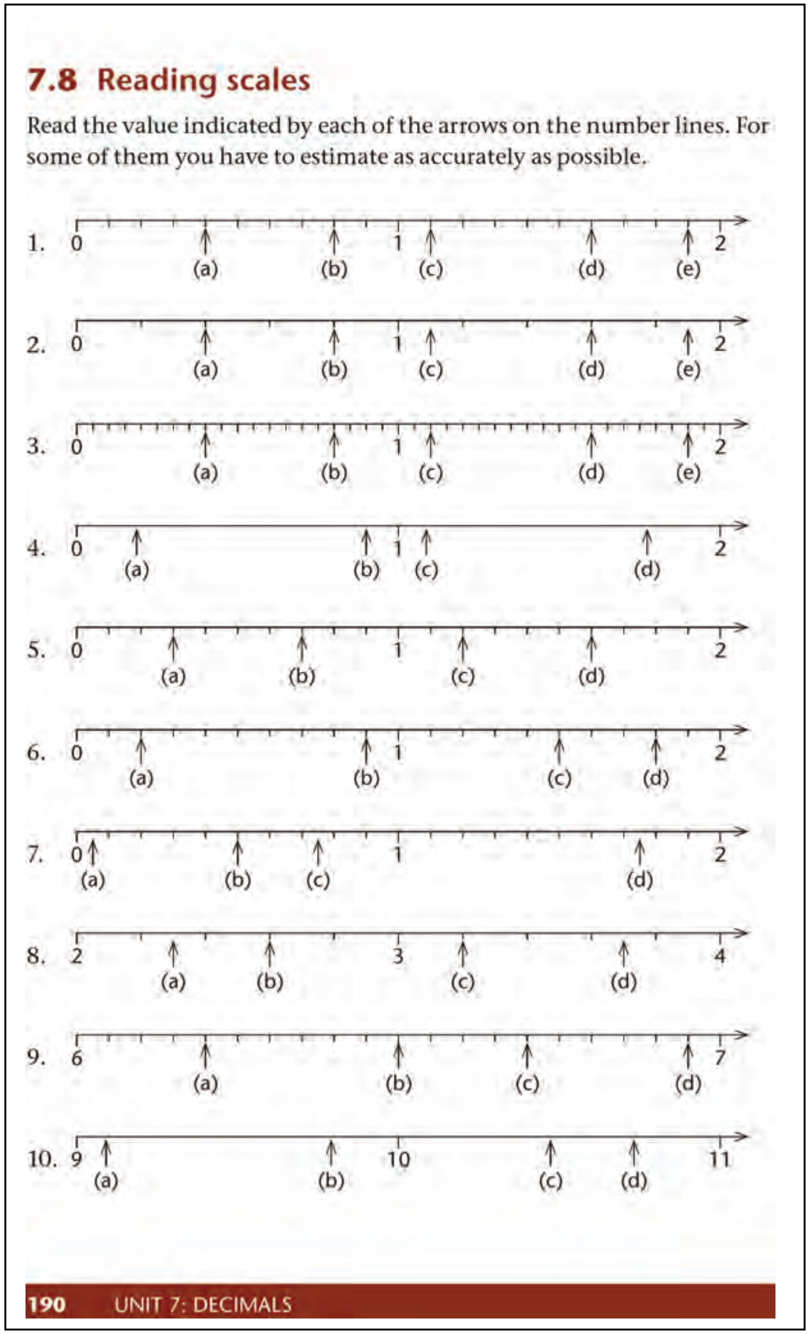
In most of the questions, learners simply have to consider the number of divisions in a unit to determine the value indicated by the arrow. In other questions, such as 2(c) and (e), the arrow is halfway between two divisions. In this instance, learners have to decide what the value is at that point.

Notes on questions

If we look at question 2(a), for example, learners must count into how many parts a unit has been divided (5), and then count how many of those parts are included in the length (2). The fraction (2 fifths) is then converted to tenths or hundredths before learners write it in decimal notation.

Answers

1.	(a) 0,4	(b) 0,8	(c) 1,1	(d) 1,6	(e) 1,9
2.	(a) 0,4	(b) 0,8	(c) 1,1	(d) 1,6	(e) 1,9
3.	(a) 0,4	(b) 0,8	(c) 1,1	(d) 1,6	(e) 1,9
4.	(a) 0,2	(b) 0,9	(c) 1,1	(d) 1,8	
5.	(a) 0,3	(b) 0,7	(c) 1,2	(d) 1,6	
6.	(a) 0,2	(b) 0,9	(c) 1,5	(d) 1,8	
7.	(a) 0,05	(b) 0,5	(c) 0,75	(d) 1,75	
8.	(a) 2,5	(b) 2,6	(c) 3,2	(d) 3,7	
9.	(a) 6,2	(b) 6,5	(c) 6,7	(d) 6,95	
10.	(a) 9,1	(b) 9,8	(c) 10,5	(d) 10,75	



7.8 Reading scales

Read the value indicated by each of the arrows on the number lines. For some of them you have to estimate as accurately as possible.

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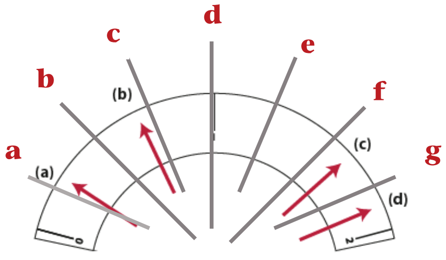
Notes on questions

Question 11 (Scale D) challenges learners as it uses a measuring instrument that has not been finely marked. Learners can mentally mark it into halves, then those halves into quarters until a mark close enough to the value to be read is found. This is a practical way for learners to find a number between two other numbers.

It is also a little like drawing freehand fraction strips. Here, line **d** (red letter) halves the scale and has a value of 1; letter **b** is therefore 0,5 and letter **f** is 1,5; letter **a** would then be 0,25; letter **c** is 0,75; letter **e** is 1,25 and letter **g** is 1,75.



Scale D

Answers

11.	Scale A	(a)	0,2	(b)	0,55	(c)	1,05	(d)	1,75	(e)	1,95
	Scale B	(a)	0,41	(b)	1,5	(c)	2,6	(d)	3,3	(e)	4,9
	Scale C	(a)	0,04	(b)	0,09	(c)	0,18	(d)	0,25	(e)	0,42
	Scale D	(a)	0,25	(b)	0,7	(c)	1,7	(d)	1,9		

7.9 Addition of decimals

Notes on questions

Question 1(a) is an example of how fractions with the same denominator can be added. Question 1(b) is an example illustrating the positional notation system.

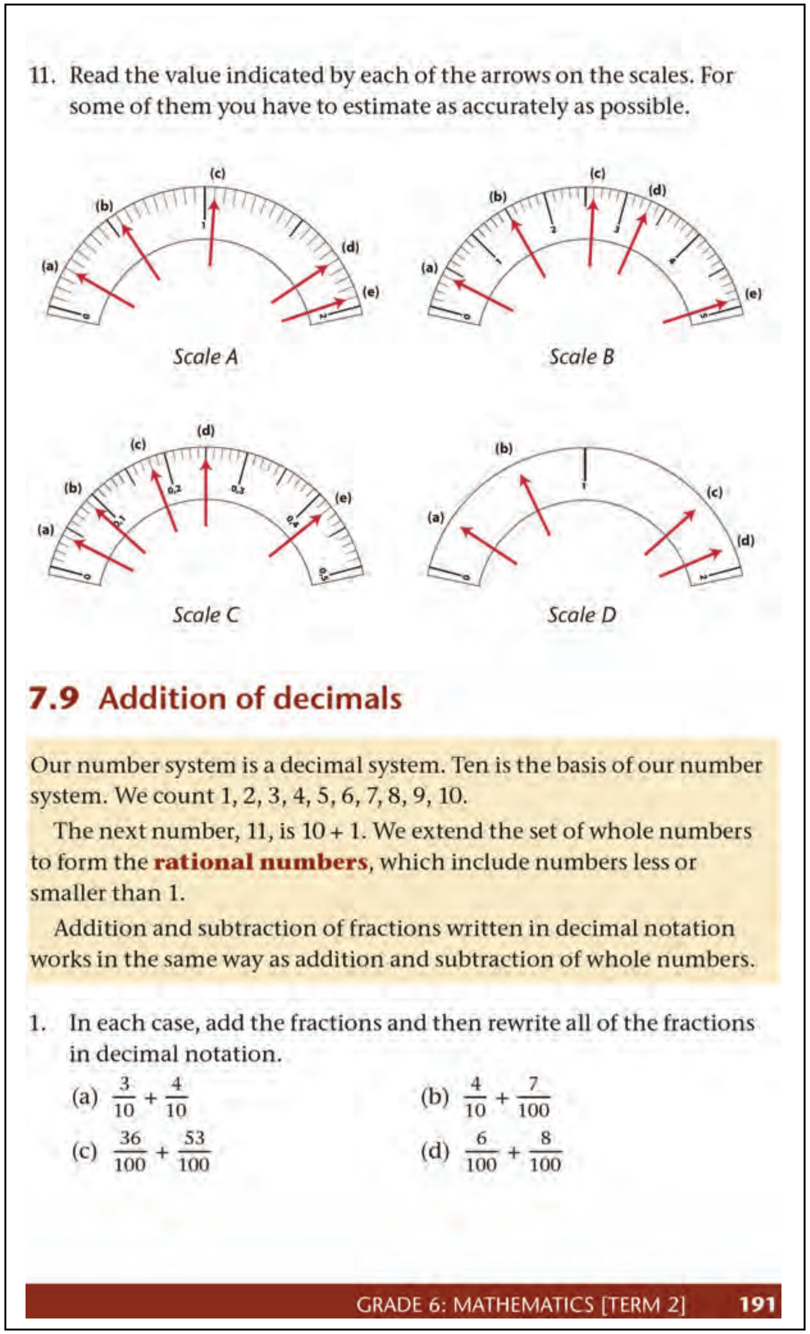
Answers

1. (a) $\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$
   $0,3 + 0,4 = 0,7$

   (b) $\frac{4}{10} + \frac{7}{100} = \frac{47}{100}$
   $0,4 + 0,07 = 0,47$

   (c) $\frac{36}{100} + \frac{53}{100} = \frac{89}{100}$
   $0,36 + 0,53 = 0,89$

   (d) $\frac{6}{100} + \frac{8}{100} = \frac{14}{100}$
   $0,06 + 0,08 = 0,14$



11. Read the value indicated by each of the arrows on the scales. For some of them you have to estimate as accurately as possible.

7.9 Addition of decimals

Our number system is a decimal system. Ten is the basis of our number system. We count 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

The next number, 11, is 10 + 1. We extend the set of whole numbers to form the **rational numbers**, which include numbers less or smaller than 1.

Addition and subtraction of fractions written in decimal notation works in the same way as addition and subtraction of whole numbers.

1. In each case, add the fractions and then rewrite all of the fractions in decimal notation.

   (a) $\frac{3}{10} + \frac{4}{10}$

   (b) $\frac{4}{10} + \frac{7}{100}$

   (c) $\frac{36}{100} + \frac{53}{100}$

   (d) $\frac{6}{100} + \frac{8}{100}$

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Notes on questions

Question 2 is not about finding the right answers as quickly as possible; it focuses on the **process** of finding the answer. Learners have the opportunity to move from the powerful, but more abstract decimal notation to the more concrete expanded notation.

In question 3 learners can count on to the following term in the chain, or subtract each number in the chain from the next one.

Although they should not be used as a replacement for mental arithmetic, calculators should be used here to check the answers. Learners should reflect on any errors and try to correct them.

Answers

2. (a) $10 + 4 + \frac{3}{10} + \frac{5}{100} + 20 + 3 + \frac{4}{10} + \frac{1}{100} = 30 + 7 + \frac{7}{10} + \frac{6}{100} = 37,76$

   (b) $10 + 2 + \frac{1}{10} + \frac{4}{100} + 300 + 20 + 4 + \frac{7}{10} = 300 + 30 + 6 + \frac{8}{10} + \frac{4}{100} = 336, 84$

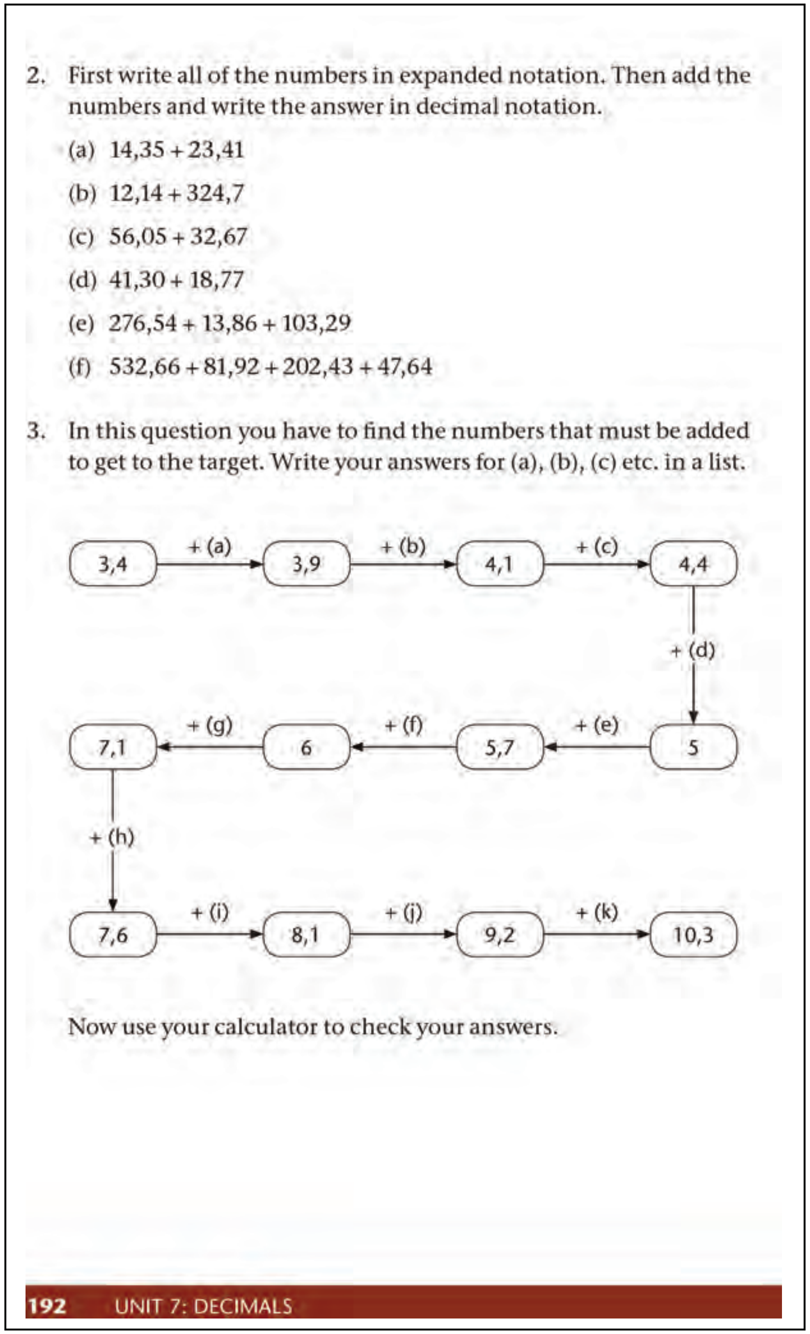
   (c) $50 + 6 + \frac{5}{100} + 30 + 2 + \frac{6}{10} + \frac{7}{100} = 80 + 8 + \frac{7}{10} + \frac{2}{100} = 88,72$

   (d) $40 + 1 + \frac{3}{10} + 10 + 8 + \frac{7}{10} + \frac{7}{100} = 60 + \frac{7}{100} = 60,07$

   (e) $200 + 70 + 6 + \frac{5}{10} + \frac{4}{100} + 10 + 3 + \frac{8}{10} + \frac{6}{100} + 100 + 3 + \frac{2}{10} + \frac{9}{100} =$
   $300 + 90 + 3 + \frac{6}{10} + \frac{9}{100} = 393,69$

   (f) $500 + 30 + 2 + \frac{6}{10} + \frac{6}{100} + 80 + 1 + \frac{9}{10} + \frac{2}{100} + 200 + 2 + \frac{4}{10} + \frac{3}{100} + 40 + 7 + \frac{6}{10} + \frac{4}{100} =$
   $800 + 60 + 4 + \frac{6}{10} + \frac{6}{100} = 864,66$

3. (a) 0,5 (b) 0,2 (c) 0,3 (d) 0,6 (e) 0,7 (f) 0,3
   (g) 1,1 (h) 0,5 (i) 0,5 (j) 1,1 (k) 1,1



2. First write all of the numbers in expanded notation. Then add the numbers and write the answer in decimal notation.
   (a) 14,35 + 23,41
   (b) 12,14 + 324,7
   (c) 56,05 + 32,67
   (d) 41,30 + 18,77
   (e) 276,54 + 13,86 + 103,29
   (f) 532,66 + 81,92 + 202,43 + 47,64

3. In this question you have to find the numbers that must be added to get to the target. Write your answers for (a), (b), (c) etc. in a list.

Now use your calculator to check your answers.

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Notes on questions

Work through the shaded passage in the Learner Book with the class. Using writing to keep track, as explained in the Learner Book, is a useful method of calculation.

Estimation is also a very useful skill as it is good for learners to have an expectation of what answer the calculator should give. For example, they should notice if the answer is ten times bigger or smaller than their estimate.

One method of estimating addition and subtraction is for learners to look at the largest number that will be added or subtracted – i.e. 532,66 in this case. Learners must then look at what its largest significant place value is (hundreds) and round all numbers to that place value before adding or subtracting – i.e. 500 + 100 + 200 + 0 = 800.

If this method is used, the estimate in question 4(a) would be: 0 + 200 + 100 + 200 = 500.

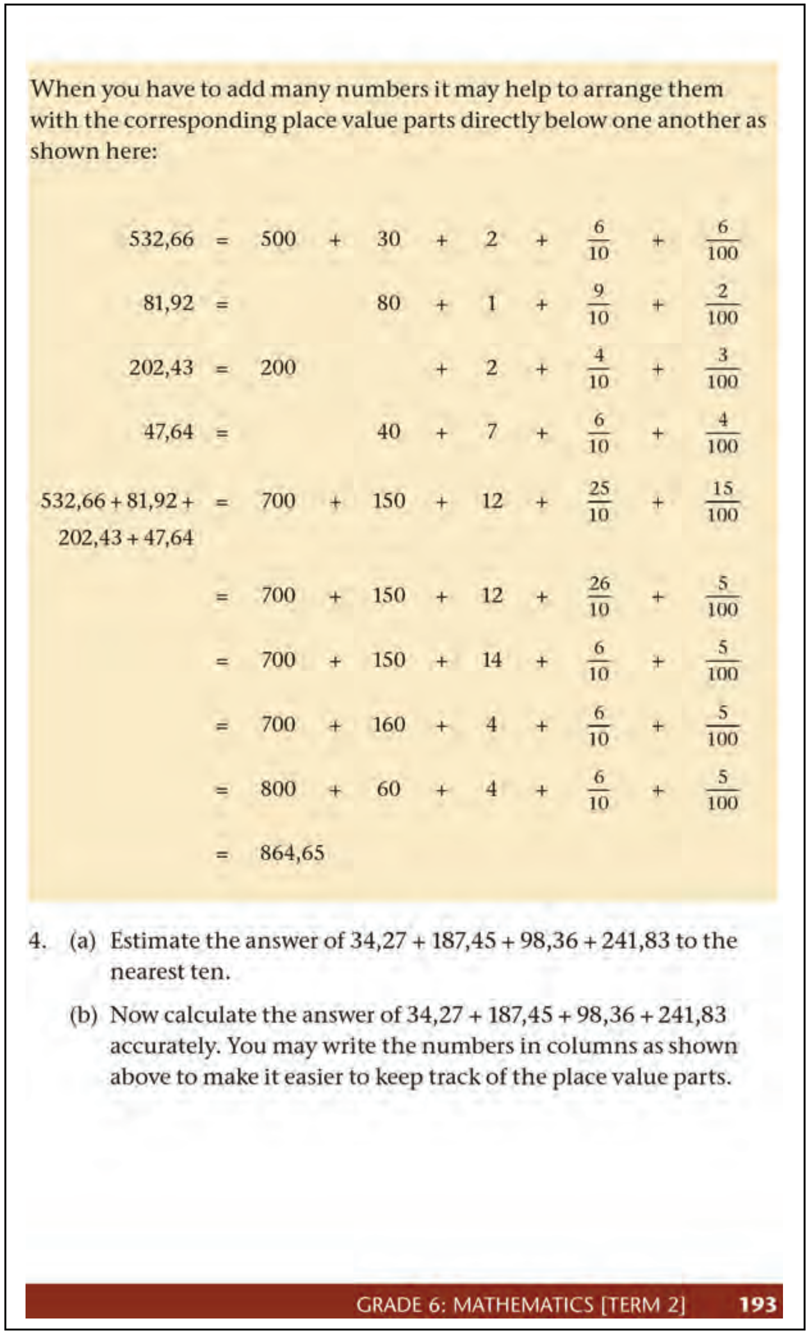
As a further refinement, learners should try to be aware of how many times they have “given” to the total and how many times they have “taken” from the total.

Another way to estimate question 4(a) would be to say: 30 + 190 + 100 + 240 = 560. The numbers used now are already more difficult to add mentally than when all numbers are rounded off to hundreds.

The concept of the “most important digit” is an important one; it refers to the leftmost non-zero digit. The further to the right a number is placed from it (and the comma), the more insignificant it is. Rounding up 47,64 to 50, for example, means that the 7, the 0,6 and then the 0,04 are less and less important.

Answers

4. (a) 34 + 190 + 100 + 240 = 560
   (b) 561,91



When you have to add many numbers it may help to arrange them with the corresponding place value parts directly below one another as shown here:

$$\begin{array}{rcccccccccc}
532{,}66 & = & 500 & + & 30 & + & 2 & + & \frac{6}{10} & + & \frac{6}{100}\\
81{,}92 & = & & & 80 & + & 1 & + & \frac{9}{10} & + & \frac{2}{100}\\
202{,}43 & = & 200 & & & + & 2 & + & \frac{4}{10} & + & \frac{3}{100}\\
47{,}64 & = & & & 40 & + & 7 & + & \frac{6}{10} & + & \frac{4}{100}\\
532{,}66+81{,}92+ & = & 700 & + & 150 & + & 12 & + & \frac{25}{10} & + & \frac{15}{100}\\
202{,}43+47{,}64 & & & & & & & & & & \\
 & = & 700 & + & 150 & + & 12 & + & \frac{26}{10} & + & \frac{5}{100}\\
 & = & 700 & + & 150 & + & 14 & + & \frac{6}{10} & + & \frac{5}{100}\\
 & = & 700 & + & 160 & + & 4 & + & \frac{6}{10} & + & \frac{5}{100}\\
 & = & 800 & + & 60 & + & 4 & + & \frac{6}{10} & + & \frac{5}{100}\\
 & = & 864{,}65 & & & & & & & &
\end{array}$$

4. (a) Estimate the answer of 34,27 + 187,45 + 98,36 + 241,83 to the nearest ten.
   (b) Now calculate the answer of 34,27 + 187,45 + 98,36 + 241,83 accurately. You may write the numbers in columns as shown above to make it easier to keep track of the place value parts.

7.10 Subtraction with decimals

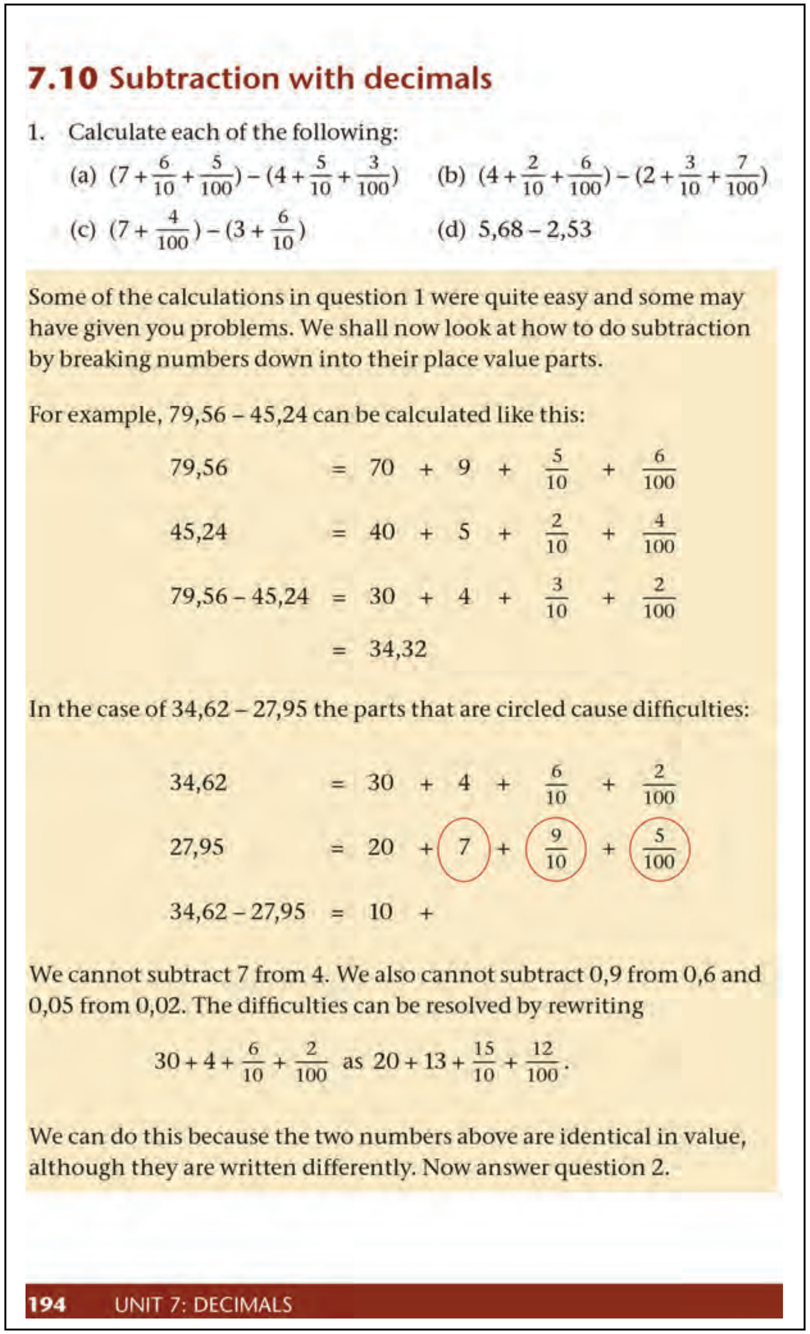
Mathematical notes

The method used and explained in this section is an extension of what was covered in Term 1 Unit 3, Section 3.7, i.e. subtraction of whole numbers.

It is extremely important that all learners understand the explanation of this approach. In this example we start working with the digits with the lowest place value. This allows learners to rewrite the place value parts where difficulties arise. If we look at the second example where $\frac{5}{100}$ must be subtracted from $\frac{2}{100}$, the $\frac{2}{100}$ is rewritten as $\frac{12}{100}$. The extra $\frac{10}{100}$ $(\frac{1}{10})$ is taken from the tenths. The $\frac{6}{10}$ is then also rewritten as $\frac{5}{10}$, which becomes $\frac{15}{10}$, with the extra $\frac{10}{10}$ taken from the units. This is because $\frac{9}{10}$ had to be subtracted from $\frac{6}{10}$, etc.

Answers

1. (a) 3,12 (b) 1,89 (c) 3,44 (d) 3,15



7.10 Subtraction with decimals

1. Calculate each of the following:

(a) $(7 + \frac{6}{10} + \frac{5}{100}) - (4 + \frac{5}{10} + \frac{3}{100})$ (b) $(4 + \frac{2}{10} + \frac{6}{100}) - (2 + \frac{3}{10} + \frac{7}{100})$

(c) $(7 + \frac{4}{100}) - (3 + \frac{6}{10})$ (d) 5,68 – 2,53

Some of the calculations in question 1 were quite easy and some may have given you problems. We shall now look at how to do subtraction by breaking numbers down into their place value parts.

For example, 79,56 – 45,24 can be calculated like this:

$$\begin{array}{lcccccccc} 79{,}56 & = & 70 & + & 9 & + & \frac{5}{10} & + & \frac{6}{100} \\ 45{,}24 & = & 40 & + & 5 & + & \frac{2}{10} & + & \frac{4}{100} \\ 79{,}56 - 45{,}24 & = & 30 & + & 4 & + & \frac{3}{10} & + & \frac{2}{100} \\ & = & 34{,}32 & & & & & & \end{array}$$

In the case of 34,62 – 27,95 the parts that are circled cause difficulties:

$$\begin{array}{lcccccccc} 34{,}62 & = & 30 & + & 4 & + & \frac{6}{10} & + & \frac{2}{100} \\ 27{,}95 & = & 20 & + & \boxed{7} & + & \boxed{\frac{9}{10}} & + & \boxed{\frac{5}{100}} \\ 34{,}62 - 27{,}95 & = & 10 & + & & & & & \end{array}$$

We cannot subtract 7 from 4. We also cannot subtract 0,9 from 0,6 and 0,05 from 0,02. The difficulties can be resolved by rewriting

$$30 + 4 + \frac{6}{10} + \frac{2}{100} \text{ as } 20 + 13 + \frac{15}{10} + \frac{12}{100}.$$

We can do this because the two numbers above are identical in value, although they are written differently. Now answer question 2.

Mathematical notes

Consider the challenging example given in question 2: 34,62 – 27,95. Working from the right, each digit in 34,62 will have to be rewritten. In other words, it should be expressed as: 2 tens + 13 units + $\frac{15}{10}$ + $\frac{12}{100}$. In this way, all the digits are equal to or bigger than the ones that are to be subtracted from them.

Working backwards to confirm our method, we find 12 hundredths. This can be seen as 2 hundredths + 1 tenth. If we add the tenth to the 15 tenths, we have 16 tenths. That can be seen as 6 tenths + 1 unit, giving us a total of 14 units. That can be seen as 4 units and 1 ten, leading us to a total of 3 tens again.

Notes on questions

The purpose of question 3 is not to do the actual subtraction; the focus is on rewriting the number so that subtraction can be done conveniently.

Answers

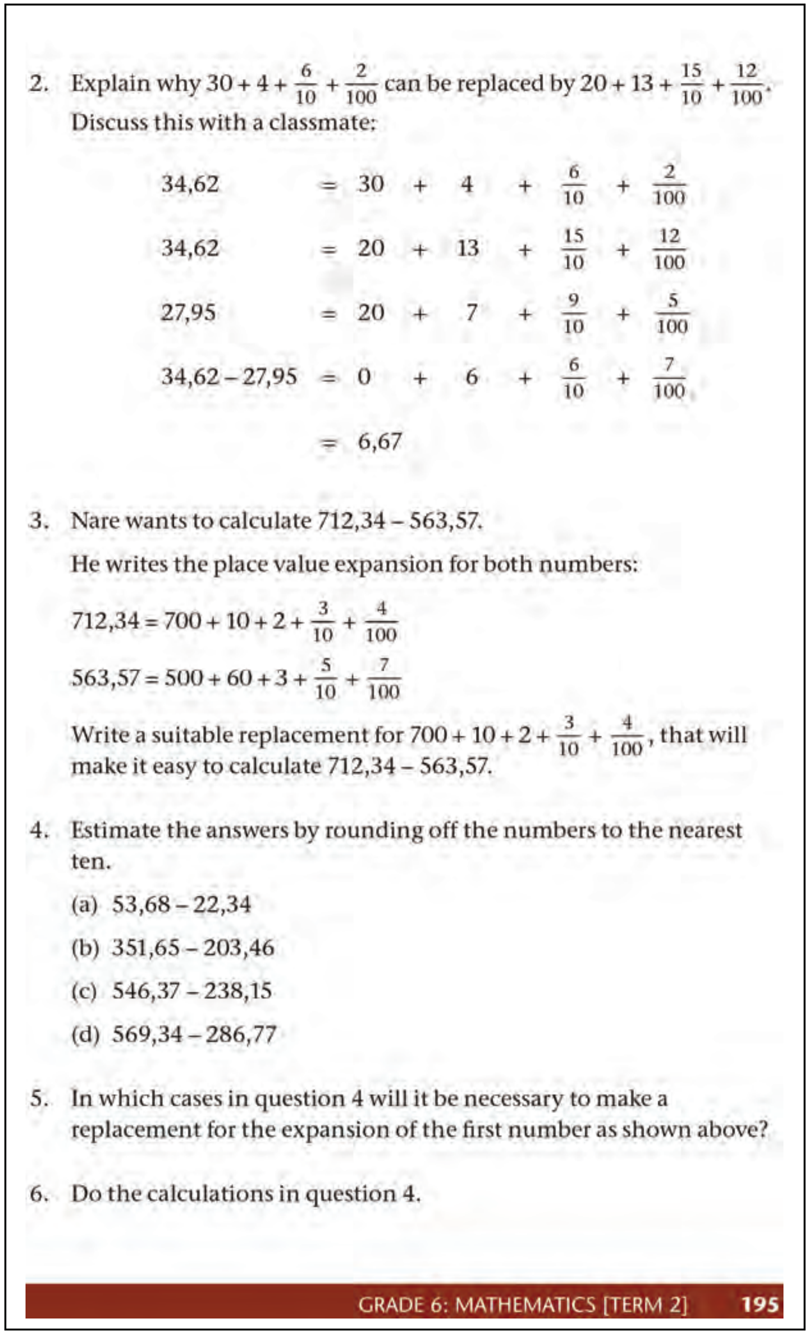
2. $20 + 13 + \frac{15}{10} + \frac{12}{100}$

$$(20 + 10) + 3 + \left(\frac{10}{10} + \frac{5}{10} + \frac{10}{100}\right) + \frac{2}{100}$$

$$= 30 + 4 + \frac{6}{10} + \frac{2}{100}$$

3. $600 + 100 + 11 + \frac{12}{10} + \frac{14}{100}$

4. (a) 50 – 20 = 30 (b) 350 – 200 = 150
   (c) 550 – 240 = 310 (d) 570 – 290 = 280
5. Question 4(d)
6. (a) 31,34 (b) 148,1
   (c) 308,22 (d) 282,57



2. Explain why $30 + 4 + \frac{6}{10} + \frac{2}{100}$ can be replaced by $20 + 13 + \frac{15}{10} + \frac{12}{100}$. Discuss this with a classmate:

$$\begin{aligned}
34{,}62 &= 30 + 4 + \frac{6}{10} + \frac{2}{100}\\
34{,}62 &= 20 + 13 + \frac{15}{10} + \frac{12}{100}\\
27{,}95 &= 20 + 7 + \frac{9}{10} + \frac{5}{100}\\
34{,}62 - 27{,}95 &= 0 + 6 + \frac{6}{10} + \frac{7}{100}\\
&= 6{,}67
\end{aligned}$$

3. Nare wants to calculate 712,34 – 563,57.

   He writes the place value expansion for both numbers:

   $712{,}34 = 700 + 10 + 2 + \frac{3}{10} + \frac{4}{100}$

   $563{,}57 = 500 + 60 + 3 + \frac{5}{10} + \frac{7}{100}$

   Write a suitable replacement for $700 + 10 + 2 + \frac{3}{10} + \frac{4}{100}$, that will make it easy to calculate 712,34 – 563,57.

4. Estimate the answers by rounding off the numbers to the nearest ten.
   (a) 53,68 – 22,34
   (b) 351,65 – 203,46
   (c) 546,37 – 238,15
   (d) 569,34 – 286,77

5. In which cases in question 4 will it be necessary to make a replacement for the expansion of the first number as shown above?

6. Do the calculations in question 4.

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7.11 Problem solving with decimals

Mathematical notes

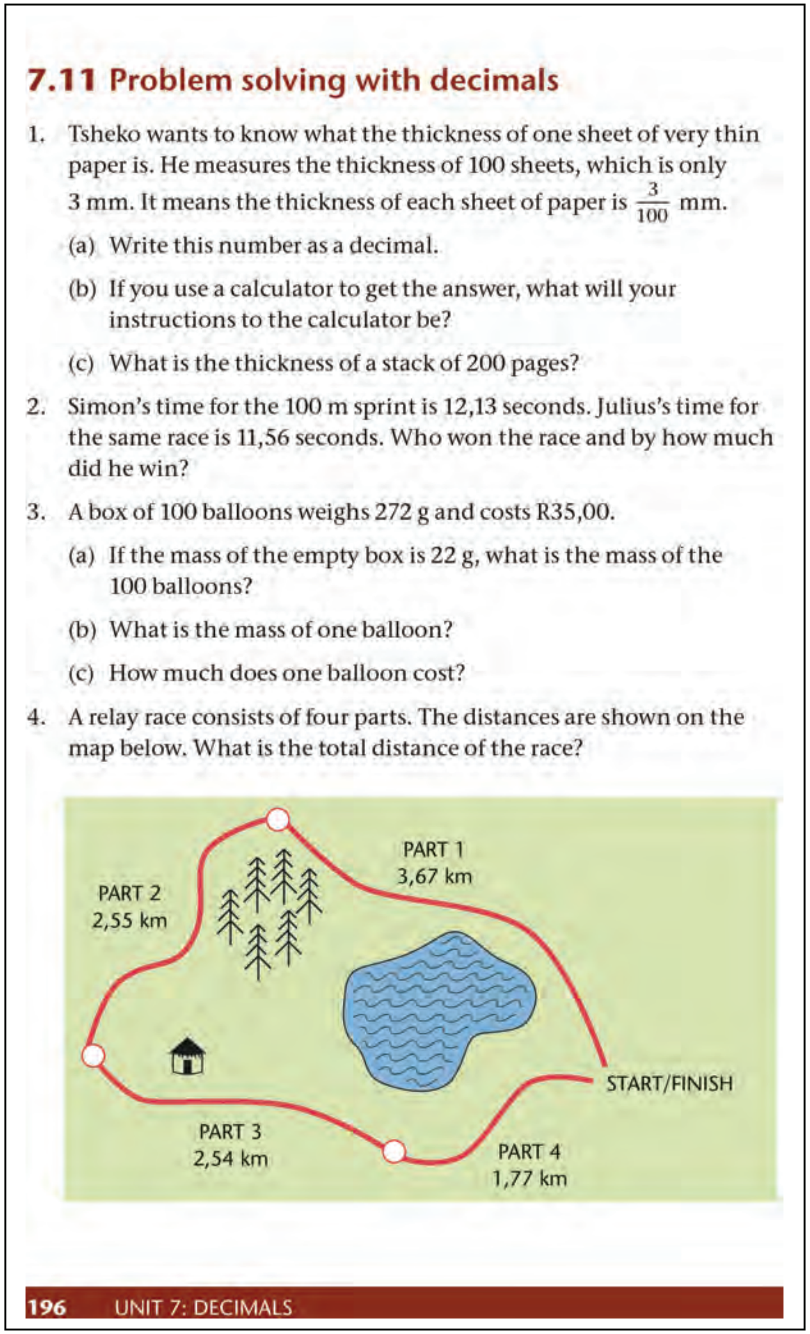
By now, learners should be used to fractions as units of measurement. In this section we will illustrate how fractions and metric units are linked, i.e. 1 mm is $\frac{1}{1\,000}$ m and 1 cm is $\frac{1}{100}$ m. It therefore also follows that 1 mm is $\frac{1}{10}$ of a centimetre. 1 decimetre is $\frac{1}{10}$ of a metre (decimetres are not commonly used). So, for example: 28,73 cm = 287,3 mm = 0,2873 m. Also, 1 m is $\frac{1}{1\,000}$ km. Therefore, 28,73 cm will be 0,0002873 km.

Notes on questions

Question 1(c) shows learners the basics of ratio. Question 2 again demonstrates that the smaller number is sometimes better, and question 3 teaches learners how to make sense of raw data.

Answers

1. (a) 0,03
   (b) The instructions would be: 3 ÷ 100 =
   (c) 6 mm
2. Julius won by 0,57 s.
3. (a) 250 g (b) 2,5 g (c) R0,35 or 35 c
4. 10,53 km



7.11 Problem solving with decimals

1. Tsheko wants to know what the thickness of one sheet of very thin paper is. He measures the thickness of 100 sheets, which is only 3 mm. It means the thickness of each sheet of paper is $\frac{3}{100}$ mm.
   (a) Write this number as a decimal.
   (b) If you use a calculator to get the answer, what will your instructions to the calculator be?
   (c) What is the thickness of a stack of 200 pages?
2. Simon's time for the 100 m sprint is 12,13 seconds. Julius's time for the same race is 11,56 seconds. Who won the race and by how much did he win?
3. A box of 100 balloons weighs 272 g and costs R35,00.
   (a) If the mass of the empty box is 22 g, what is the mass of the 100 balloons?
   (b) What is the mass of one balloon?
   (c) How much does one balloon cost?
4. A relay race consists of four parts. The distances are shown on the map below. What is the total distance of the race?

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7.12 Using the calculator to understand decimals

Teaching guidelines

On most basic calculators it will be sufficient to press the “+” key once before starting to repeatedly press the “=” key. Generally speaking, pressing the “=” key on a basic calculator means that the last instruction must be carried out again.

Instead of pressing the “=” key again, learners can type in another number and then “=”. The operation will then be performed on the number that was just entered.

Different calculators perform in different ways. If learners type “50 + 1 =” on some calculators, then “=” would cause “+ 1” to be repeated. If learners had to type “2 × 50 =” then “=” would cause “2 ×” to be repeated.

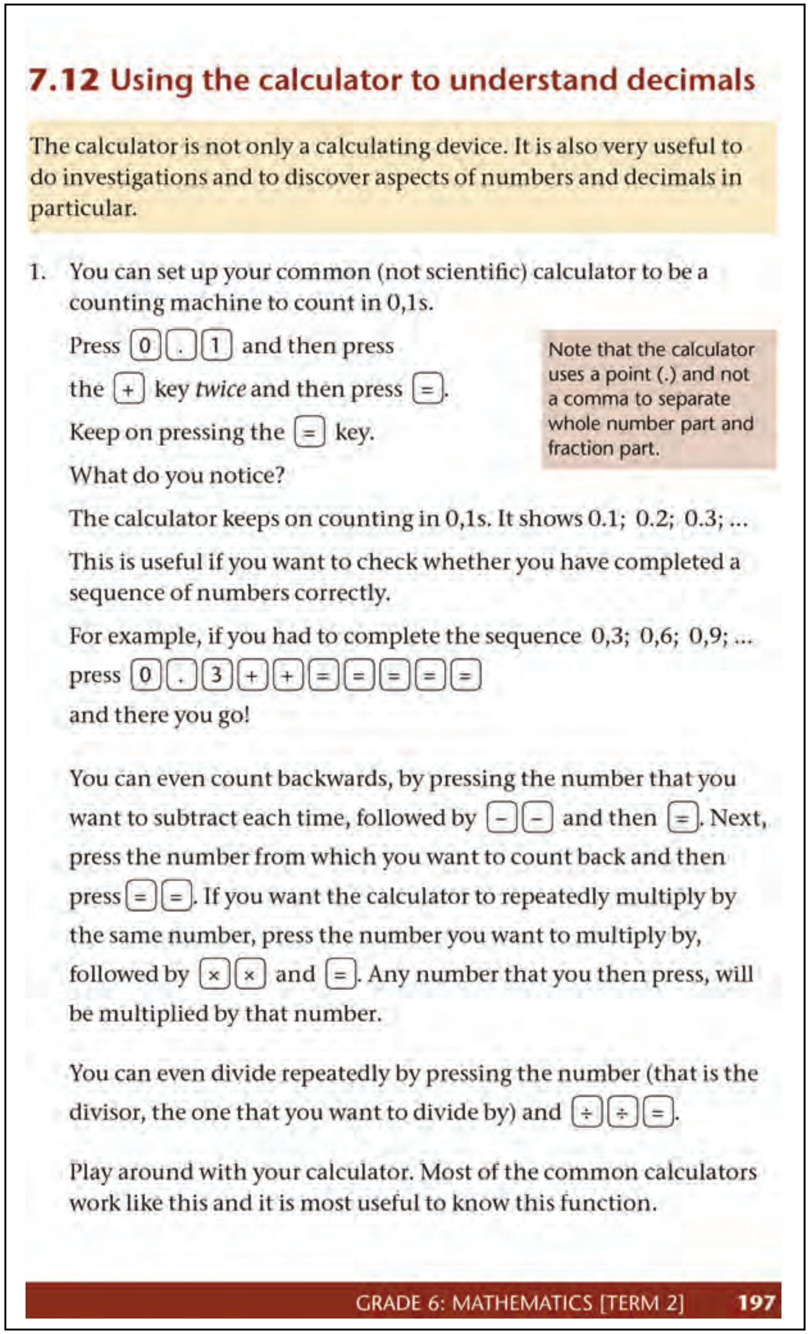
Common calculators are designed to do common calculations as conveniently and quickly as possible, and possibly to repeat them.

Scientific calculators, however, allow for longer calculations. If learners press the “=” sign after the answer has been displayed, the display stays the same.

Another important difference between common and scientific calculators is that when typing in “212 + 10%”, for example, the common calculator assumes that it needs to add 10% of 212. The more “literally-minded” scientific calculator would interpret those keys as “212 + 0,1”.

Answers

1. The calculator counts in 0,1s: 0,1; 0,2; 0,3; ...



7.12 Using the calculator to understand decimals

The calculator is not only a calculating device. It is also very useful to do investigations and to discover aspects of numbers and decimals in particular.

1. You can set up your common (not scientific) calculator to be a counting machine to count in 0,1s.

   Press $\boxed{0}\boxed{.}\boxed{1}$ and then press the $\boxed{+}$ key *twice* and then press $\boxed{=}$.

   Keep on pressing the $\boxed{=}$ key.

   What do you notice?

   Note that the calculator uses a point (.) and not a comma to separate whole number part and fraction part.

   The calculator keeps on counting in 0,1s. It shows 0.1; 0.2; 0.3; ...

   This is useful if you want to check whether you have completed a sequence of numbers correctly.

   For example, if you had to complete the sequence 0,3; 0,6; 0,9; ...

   press $\boxed{0}\boxed{.}\boxed{3}\boxed{+}\boxed{+}\boxed{=}\boxed{=}\boxed{=}\boxed{=}\boxed{=}$

   and there you go!

   You can even count backwards, by pressing the number that you want to subtract each time, followed by $\boxed{-}\boxed{-}$ and then $\boxed{=}$. Next, press the number from which you want to count back and then press $\boxed{=}\boxed{=}$. If you want the calculator to repeatedly multiply by the same number, press the number you want to multiply by, followed by $\boxed{\times}\boxed{\times}$ and $\boxed{=}$. Any number that you then press, will be multiplied by that number.

   You can even divide repeatedly by pressing the number (that is the divisor, the one that you want to divide by) and $\boxed{\div}\boxed{\div}\boxed{=}$.

   Play around with your calculator. Most of the common calculators work like this and it is most useful to know this function.

Notes on questions

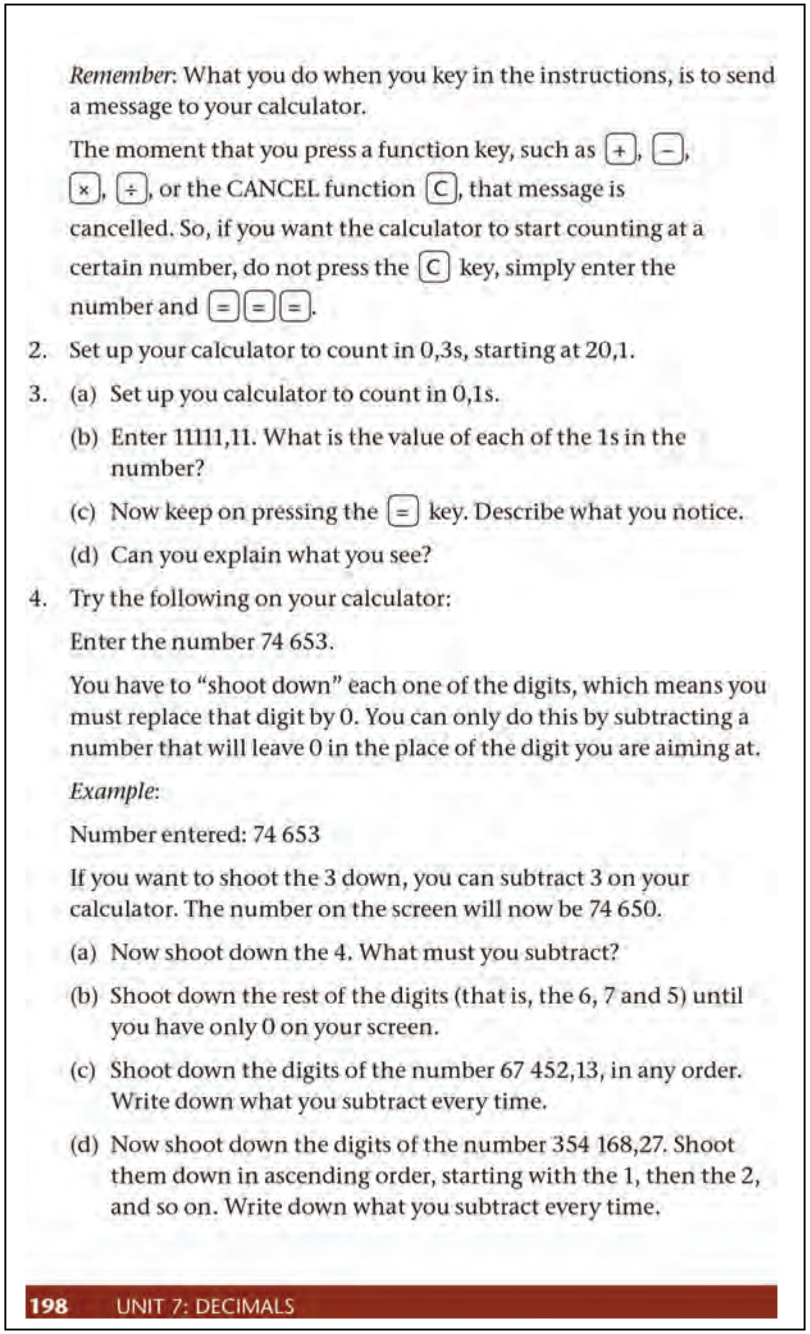
Question 2 can be found by pressing “20,1 + 0,3 =”, followed by “=” again. Question 3(b) assumes that (a) has already been completed satisfactorily. The current sequence is then interrupted to enter the new number, on which the same operation is to be repeated.

Although 3(d) is more focused on getting learners to think and not simply to arrive at a specific answer, it shows them that an operation that takes ten repeats to change the units will have to be repeated many, many, many times to change the digit in the hundred thousands place value position (hundred thousand × more than ten times, to be exact).

Question 4 affords learners the opportunity to recognise their own reasoning – learners who are successful in “shooting down” the digits generally have a good understanding of place value. However, learners who are less successful need to be given more time to work through the questions.

Answers

2. 20,4; 20,7; 21; 21,3; ...
3. (b) The digit after the comma is the one that changes, then it is the unit that changes, and then it immediately goes back to the tenth again: 11111,21; 11111,31; 11111,41; 11111,51; 11111,61; 11111,71; 11111,81; 11111,**9**1; 1111**2**,01; 1111**2**,**1**1; 1111**2**,**2**1; ...
4. (a) 74 653 – 4 000 = 70 653; subtract 4 000 to “shoot down” 4.
   (b) Subtract 600; then 70 000; then 50; then 3.
   (c) For 6, subtract 60 000; for 7, subtract 7 000; for 4, subtract 400; for 5, subtract 50; for 2, subtract 2; for 1, subtract 0,1; for 3, subtract 0,03 (any order is acceptable).
   (d) This number contains all the numbers between 1 and 8 in a variety of orders. Start by “shooting down” 1, then 2, and so on. Subtract 100; then 0,2; then 300 000; then 4 000; then 50 000; then 60; then 0,07; then lastly, 8.



Remember: What you do when you key in the instructions, is to send a message to your calculator.

The moment that you press a function key, such as $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, $\boxed{\div}$, or the CANCEL function $\boxed{C}$, that message is cancelled. So, if you want the calculator to start counting at a certain number, do not press the $\boxed{C}$ key, simply enter the number and $\boxed{=}\boxed{=}\boxed{=}$.

2. Set up your calculator to count in 0,3s, starting at 20,1.
3. (a) Set up you calculator to count in 0,1s.
   (b) Enter 11111,11. What is the value of each of the 1s in the number?
   (c) Now keep on pressing the $\boxed{=}$ key. Describe what you notice.
   (d) Can you explain what you see?
4. Try the following on your calculator:

Enter the number 74 653.

You have to “shoot down” each one of the digits, which means you must replace that digit by 0. You can only do this by subtracting a number that will leave 0 in the place of the digit you are aiming at.

Example:

Number entered: 74 653

If you want to shoot the 3 down, you can subtract 3 on your calculator. The number on the screen will now be 74 650.

(a) Now shoot down the 4. What must you subtract?
(b) Shoot down the rest of the digits (that is, the 6, 7 and 5) until you have only 0 on your screen.
(c) Shoot down the digits of the number 67 452,13, in any order. Write down what you subtract every time.
(d) Now shoot down the digits of the number 354 168,27. Shoot them down in ascending order, starting with the 1, then the 2, and so on. Write down what you subtract every time.

Mathematical notes

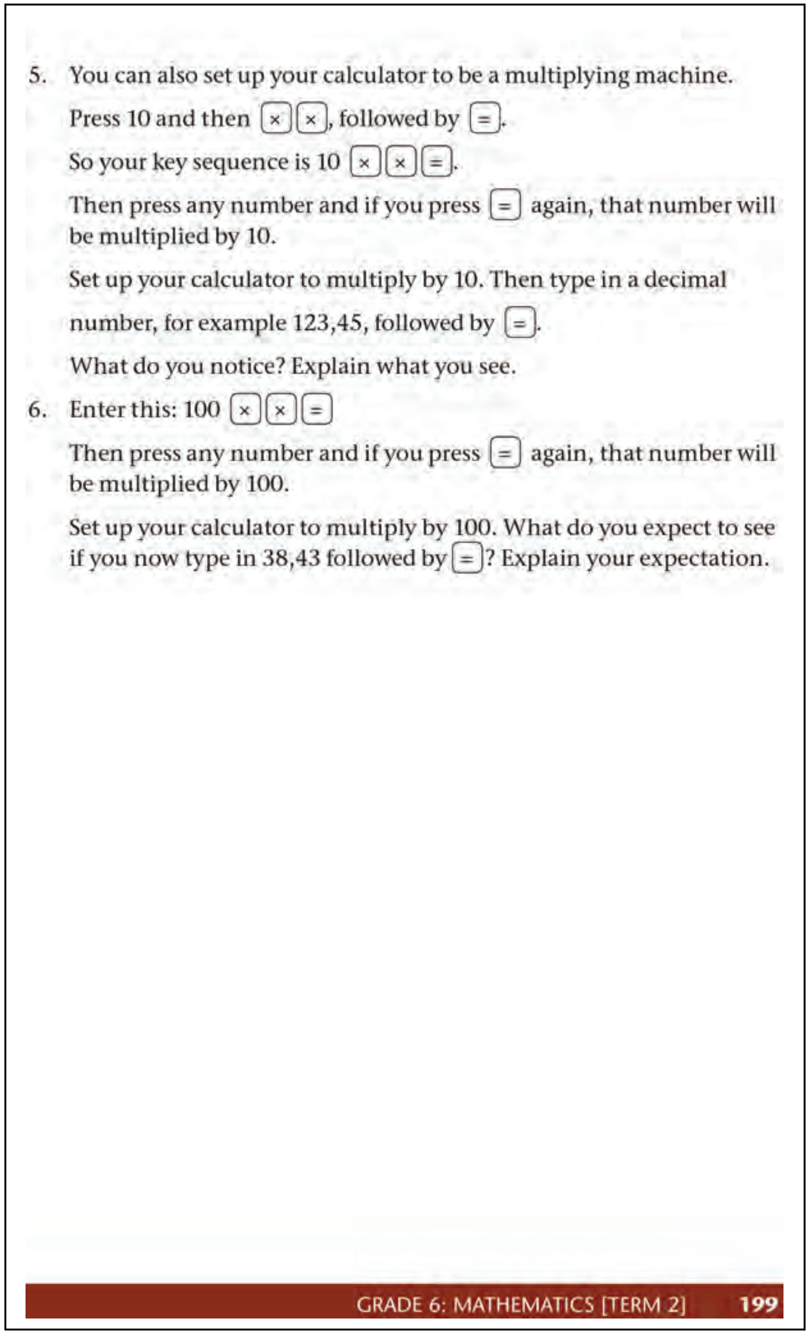
This activity (question 5) illustrates the power of the place value system and can help learners by getting them used to the proportions involved. It also helps them to improve their estimation skills.

Multiplying and dividing by powers of ten are some of the easiest mental arithmetic operations in base 10. In fact, such calculations have become so entrenched in us (also through metric conversions) that we sometimes forget that the similarity in appearance between 255 and 2,55 is merely because of the convention we follow. The reason why we are multiplying or dividing by ten is simply because we work with a base 10 system. It may also be useful to expand a number to illustrate what happens. Have a look at this multiplication in expanded notation:

$123{,}45 \times 10$
$= (1 \times 100 + 2 \times 10 + 3 \times 1 + 4 \times 0{,}1 + 5 \times 0{,}01) \times 10$
(now each part gets multiplied by 10)
$= 1 \times 1\,000 + 2 \times 100 + 3 \times 10 + 4 \times 1 + 5 \times 0{,}1$
$= 1234{,}5$

Answers

5. It seems as if the position of the decimal comma (or decimal point) moves to the right. **Do not teach this as a method**. Each time you press "=", the number becomes ten times bigger.
6. 3843.



5. You can also set up your calculator to be a multiplying machine.

   Press 10 and then $\boxed{\times}\boxed{\times}$, followed by $\boxed{=}$.

   So your key sequence is 10 $\boxed{\times}\boxed{\times}\boxed{=}$.

   Then press any number and if you press $\boxed{=}$ again, that number will be multiplied by 10.

   Set up your calculator to multiply by 10. Then type in a decimal number, for example 123,45, followed by $\boxed{=}$.

   What do you notice? Explain what you see.

6. Enter this: 100 $\boxed{\times}\boxed{\times}\boxed{=}$

   Then press any number and if you press $\boxed{=}$ again, that number will be multiplied by 100.

   Set up your calculator to multiply by 100. What do you expect to see if you now type in 38,43 followed by $\boxed{=}$? Explain your expectation.

Grade 6 Term 2 Unit 8 Capacity and volume

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
8.1 The difference between capacity and volume	Volume measures the amount of space the material of an object or liquid fills up; capacity is the space that is available inside a container	200 to 202
8.2 Containers and measurements	Measuring volume with containers with different shapes	203 to 207
8.3 Work with different units of measurement	Working with and converting between millilitres (ml), litres (ℓ) and kilolitres (kl)	208 to 211

CAPS time allocation	5 hours
CAPS page references	26 and 253 to 256

Mathematical background

This unit will enable learners to:

- look at a quantity of a substance and give a reasonably good estimate of its volume
- estimate using the standard units for measuring volume (of which millilitres and litres are the most common).

Learners will also get to explore how differently shaped or sized containers may or may not have different capacities, or may seem to contain different volumes of liquid. This unit, most importantly, assists learners in attaching **meaning** and **context** to situations involving volumes and volume scales.

Resources

Measuring jug or measuring cylinder; some coarse sand; gravel; rice grains or dried beans; volume scales; various kinds of measuring containers, for example syringes, measuring jugs, etc.; measuring spoons and measuring cups; unusually shaped, clear plastic bottles; marking pen

8.1 The difference between capacity and volume

Mathematical notes

Capacity is the maximum volume that a container can hold. Air automatically fills a container to its full capacity. The volume of air is the full capacity of the container.

When we deal with water (and other liquids) we can fill a container half-full, three-quarters full or right up to its capacity with liquid. In each case, the liquid has a certain volume. If you work with powders such as flour or salt, capacity and volume work as they do for liquids.

What about the volume of any solid? In Grade 5 learners worked with centimetre cubes and they found the volume of blocks with straight sides. However, the potato in the picture does not have straight sides. You could cut the potato into centimetre cubes and count the cubes to get an estimate of the volume. The picture shows a much easier way of finding the volume. The potato pushes aside its own volume of water and we can measure that volume on the jug's scale.

Teaching guidelines

The scale on the jug goes up to 500 ml. Remind learners that one millilitre is the same volume as one cubic centimetre.

Be attuned to how the same volume of water can look different (e.g. have different depths) in different containers. Different amounts of water may also have the same depth in different containers.

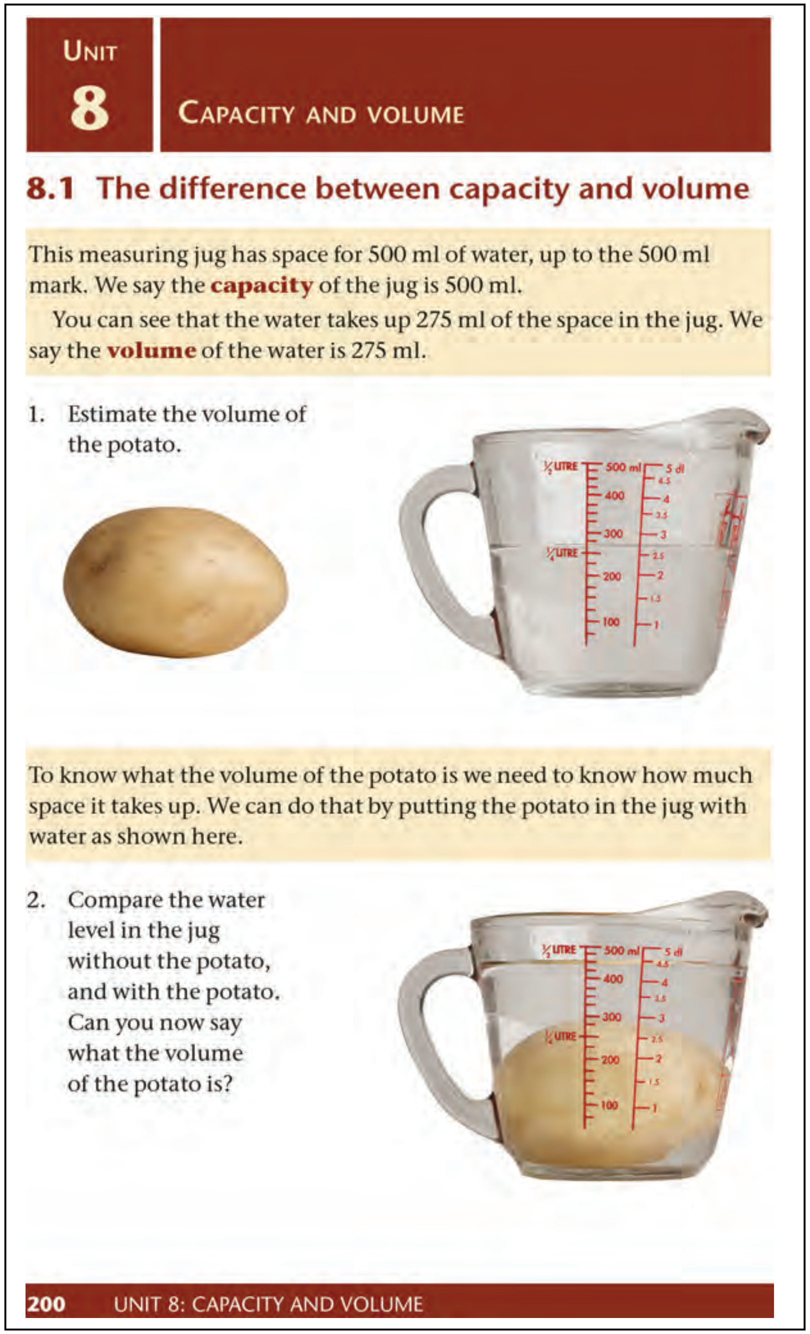
Notes on questions

Question 1 offers a good way to gauge how learners think. The correct answer is not the goal here. Potatoes have different volumes, so ask learners to think about some objects that are about the same volume, for example a small fruit juice box (250 ml) or a cooldrink can (340 ml), or a litre carton of milk (1 000 ml).

Ask learners to look closely at the left side scale on the jug to answer question 2. The added millilitres of the potato have pushed the water up to 450 ml. So, if we work out how many extra millilitres have been pushed up, we say: 450 ml – 275 ml = 175 ml. Learners may ask: *"Why are there two scales on the jug?"* The scale on the right is in "imperial units", the units used in the United States of America.

Answers

1. Learners make an estimate.
2. 175 ml



UNIT 8 CAPACITY AND VOLUME

8.1 The difference between capacity and volume

This measuring jug has space for 500 ml of water, up to the 500 ml mark. We say the **capacity** of the jug is 500 ml.

You can see that the water takes up 275 ml of the space in the jug. We say the **volume** of the water is 275 ml.

1. Estimate the volume of the potato.

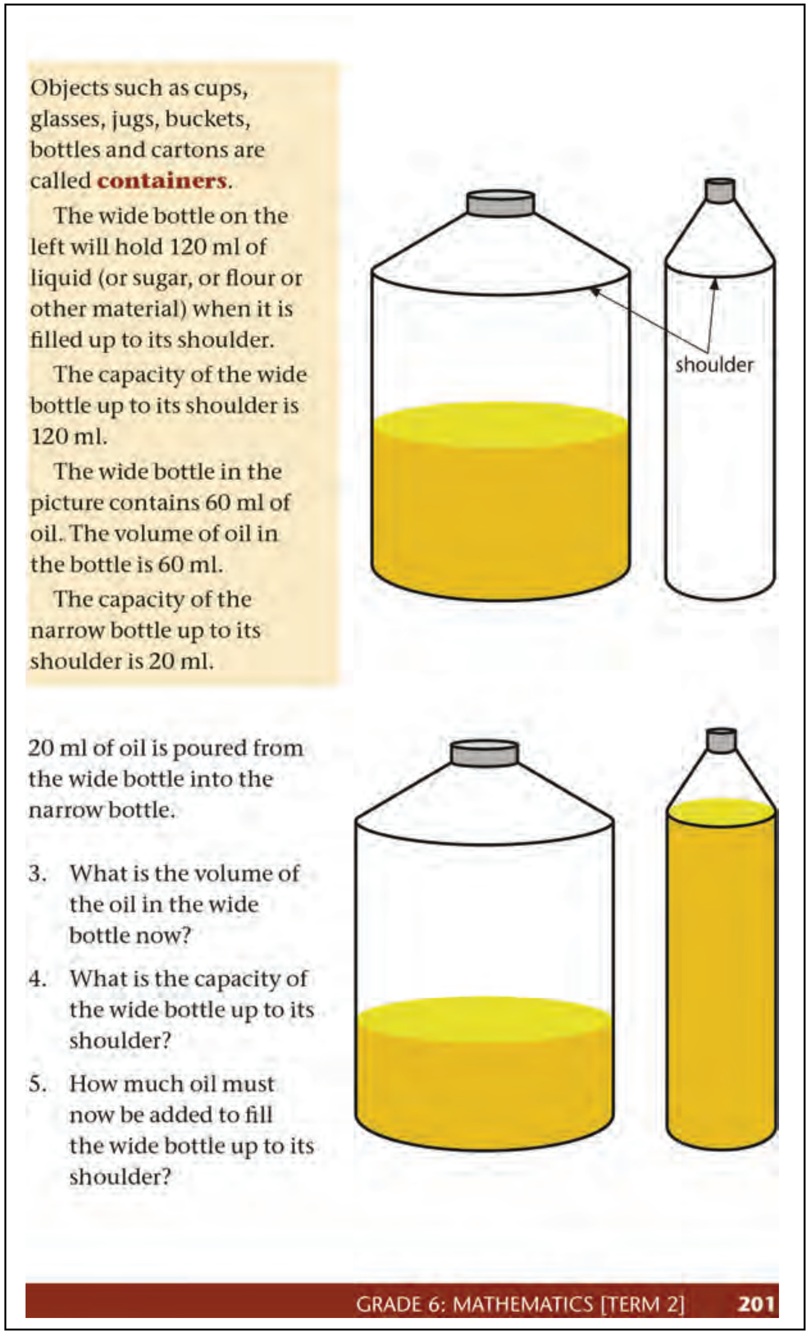
To know what the volume of the potato is we need to know how much space it takes up. We can do that by putting the potato in the jug with water as shown here.

2. Compare the water level in the jug without the potato, and with the potato. Can you now say what the volume of the potato is?

200 UNIT 8: CAPACITY AND VOLUME

Answers

3. The wide bottle had 60 ml of oil and has lost 20 ml that went into the narrow bottle. The wide bottle, therefore, has 40 ml of oil.
4. The capacity is 120 ml, as is stated in the shaded passage.
5. You need another 80 ml to fill it up to 120 ml.



Objects such as cups, glasses, jugs, buckets, bottles and cartons are called **containers**.

The wide bottle on the left will hold 120 ml of liquid (or sugar, or flour or other material) when it is filled up to its shoulder.

The capacity of the wide bottle up to its shoulder is 120 ml.

The wide bottle in the picture contains 60 ml of oil. The volume of oil in the bottle is 60 ml.

The capacity of the narrow bottle up to its shoulder is 20 ml.

20 ml of oil is poured from the wide bottle into the narrow bottle.

3. What is the volume of the oil in the wide bottle now?
4. What is the capacity of the wide bottle up to its shoulder?
5. How much oil must now be added to fill the wide bottle up to its shoulder?

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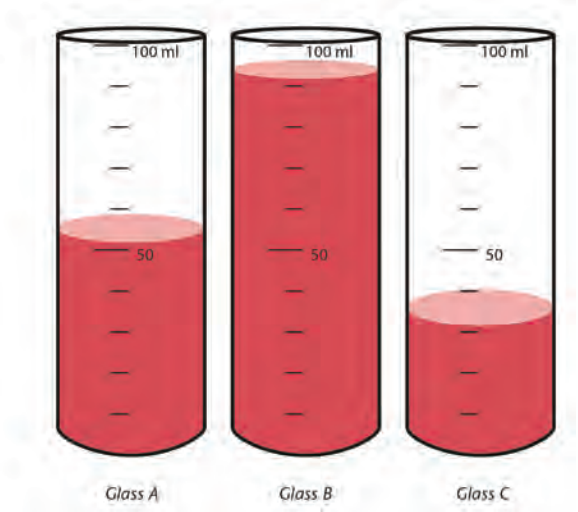
Teaching guidelines

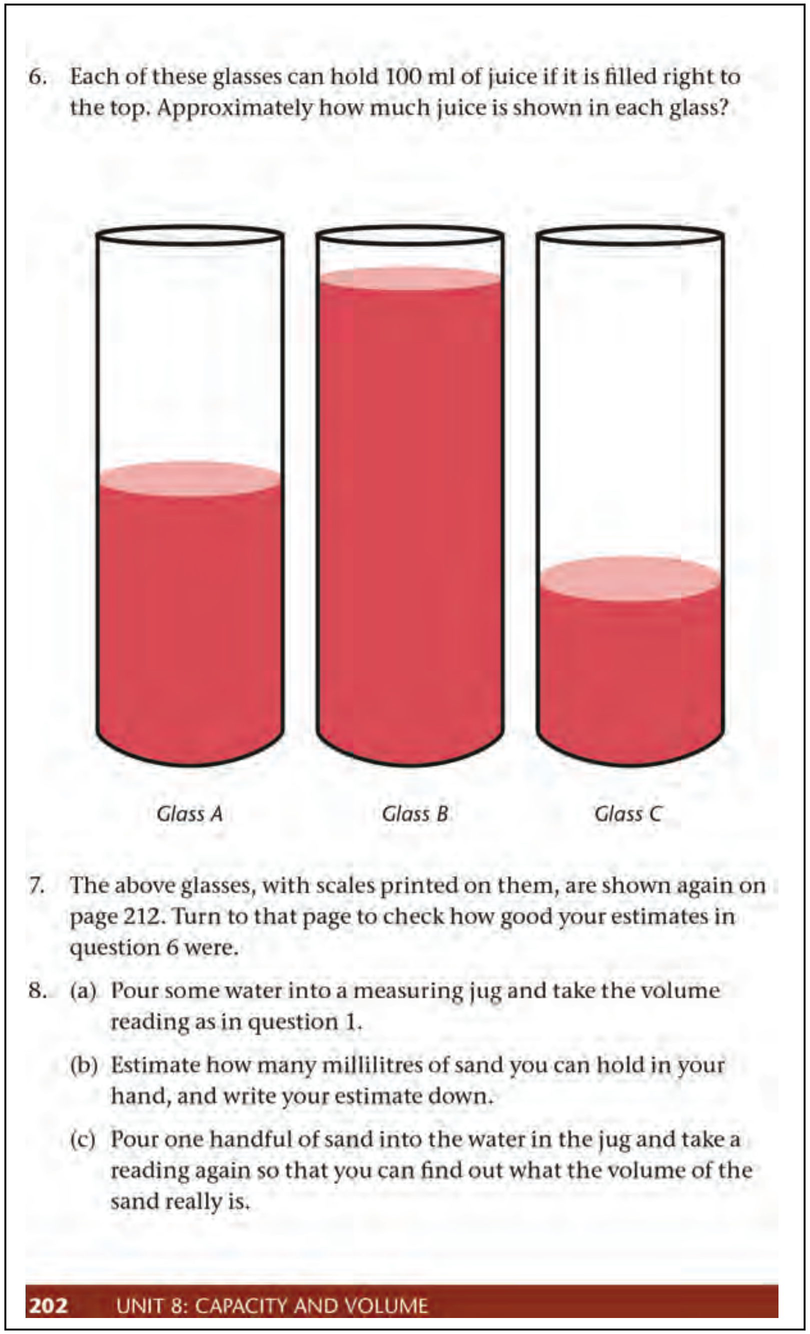
Question 8 is a practical activity. You will need a measuring jug or measuring cylinder. You can find large measuring cylinders in the science kits that are supplied to schools. You will also need some coarse sand, gravel, rice grains or dried beans. (See "Answers" below for further guidelines.)

Answers

6. Consider learners' answers as they will vary: Glass A is about half-full, so the answer is 50 ml; Glass B contains about 90 ml juice; Glass C contains about 30 ml juice.
7. The scales on the glasses on page 212 of the learner book, repeated below, show volumes of A: 51 ml; B: 91 ml and C: 31 ml.



8. (a) Pour water into the measuring jug or cylinder so that it is about $\frac{2}{3}$ full. Ask a learner to read the water level on the scale, and then write the reading on the board.
   (b) Although the Learner Book suggests using sand, you can also use alternative materials such as gravel, dried rice or beans – whatever is more accessible to you. Pour the gravel, dried rice or beans into a learner's hand and ask the learner to estimate the volume. Then record the estimated volume.
   (c) The learner puts all the gravel, beans or rice into the water and reads the new water level on the scale. Subtract the old reading from the new reading: the difference is the volume of the gravel, beans or rice. Ask the class if this volume is close to the estimate.



6. Each of these glasses can hold 100 ml of juice if it is filled right to the top. Approximately how much juice is shown in each glass?

7. The above glasses, with scales printed on them, are shown again on page 212. Turn to that page to check how good your estimates in question 6 were.

8. (a) Pour some water into a measuring jug and take the volume reading as in question 1.
   (b) Estimate how many millilitres of sand you can hold in your hand, and write your estimate down.
   (c) Pour one handful of sand into the water in the jug and take a reading again so that you can find out what the volume of the sand really is.

202 UNIT 8: CAPACITY AND VOLUME

8.2 Containers and measurements

Mathematical notes

It is important that by now learners should have an understanding of the three common units for measuring volume, i.e. millilitres (ml), litres (ℓ) and kilolitres (kl).

In this section specifically, learners get to practise reading volume scales on various kinds of measuring containers, for example syringes, measuring jugs, etc. They will also work with measuring spoons.

Teaching guidelines

Make sure that in your teaching you focus on:

- how big millilitres, litres and kilolitres are
- how to use the scale factor of 1 000 – i.e. 1 000 ml = 1 ℓ; 1 000 ℓ = 1 kl
- the fact that the volume scale, unlike the length scale, on each container is different for each shape of container.

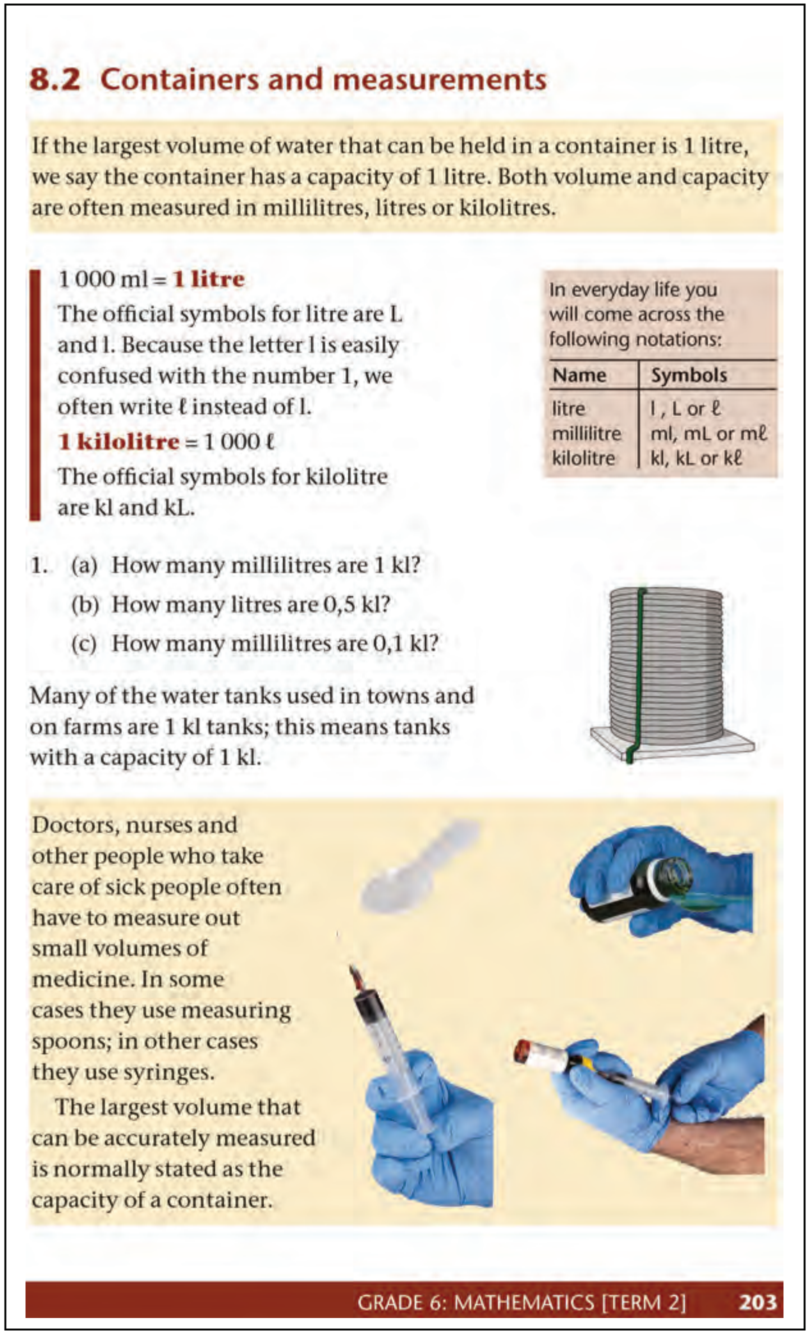
Explain to learners that on a ruler, the gaps (intervals) between centimetre marks are always the same on all rulers. A short, wider container, however, will have millilitre marks close together and a tall, narrow container will have its millilitre marks widely spaced.

Possible misconceptions

Questions such as: *"What is the capacity of a bath in kilolitres, litres or millilitres?"* are useful, and answers such as "100 kilolitres", "1 000 litres", etc. can be assessed by referring to suitable reference units. However, questions such as *"1 000 litres – how many bucketsful is that?"* or *"Can a kilolitre fit in a bath?"* would help to develop learners' sense of scale of the three units. Developing learners' sense of different volume amounts (How much is 1 000 litres?) is important.

Answers

1. (a) There are 1 000 millilitres in a litre. A kilolitre has 1 000 litres, thus a kilolitre has 1 000 × 1 000 millilitres, or 1 000 000 (one million) millilitres.
   (b) 0,5 kilolitre is 0,5 × 1 000 litres, which is 500 litres.
   (c) 0,1 kilolitres is 0,1 × 1 000 litres. Every litre is 1 000 millilitres, so 0,1 kilolitres is 0,1 × 1 000 × 1 000 millilitres, which is 0,1 × 1 000 000 millilitres, which is 100 000 millilitres.



8.2 Containers and measurements

If the largest volume of water that can be held in a container is 1 litre, we say the container has a capacity of 1 litre. Both volume and capacity are often measured in millilitres, litres or kilolitres.

1 000 ml = **1 litre**

The official symbols for litre are L and l. Because the letter l is easily confused with the number 1, we often write ℓ instead of l.

1 kilolitre = 1 000 ℓ

The official symbols for kilolitre are kl and kL.

In everyday life you will come across the following notations:

Name	Symbols
litre	l , L or ℓ
millilitre	ml, mL or mℓ
kilolitre	kl, kL or kℓ

1. (a) How many millilitres are 1 kl?
   (b) How many litres are 0,5 kl?
   (c) How many millilitres are 0,1 kl?

Many of the water tanks used in towns and on farms are 1 kl tanks; this means tanks with a capacity of 1 kl.

Doctors, nurses and other people who take care of sick people often have to measure out small volumes of medicine. In some cases they use measuring spoons; in other cases they use syringes.

The largest volume that can be accurately measured is normally stated as the capacity of a container.

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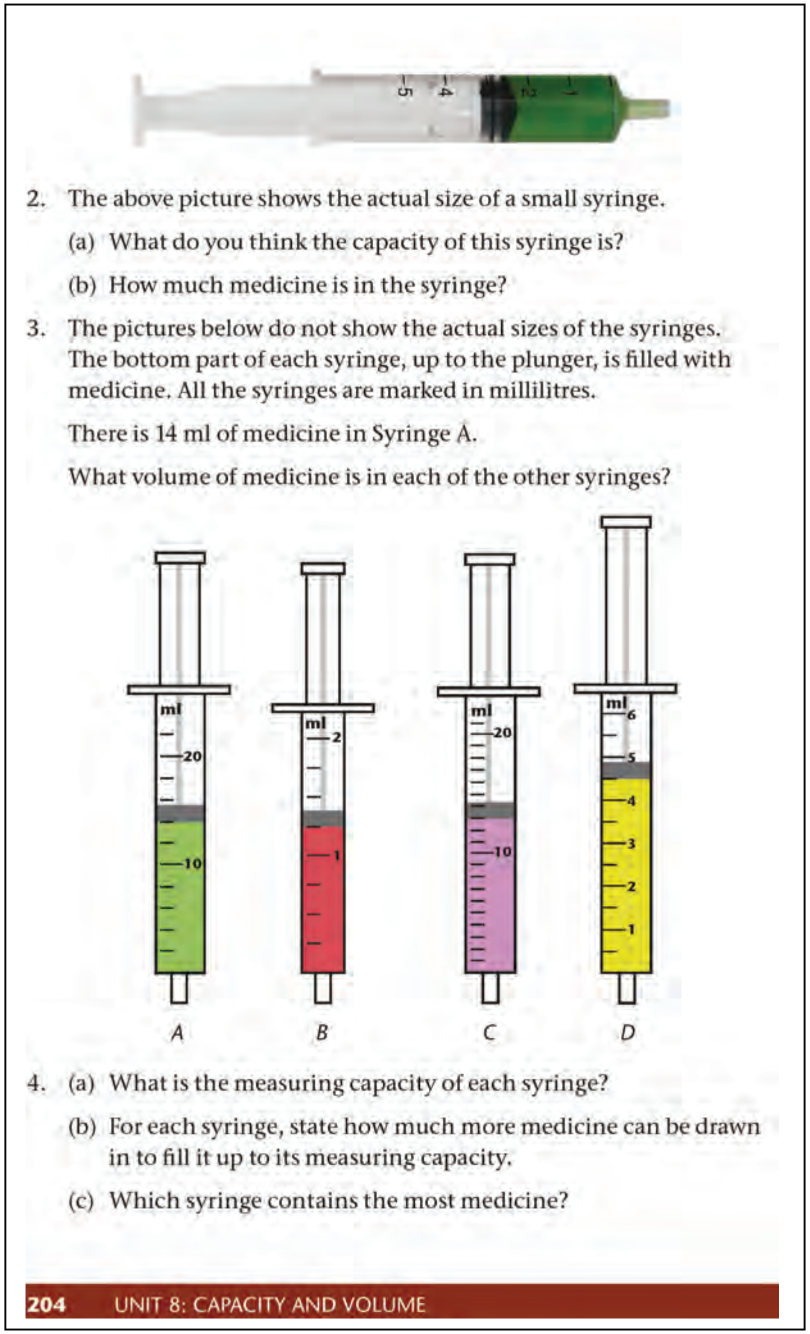
Notes on questions

It is very important to remind learners that the pictures in question 3 do not show the actual sizes of the syringes. For example, syringe B is actually much smaller than syringe C.

Answers

2. (a) The capacity is 5 ml, as shown on the scale, but it might be able to hold more than 5 ml of liquid if you pull the plunger far enough back.
   (b) It seems that there is 2,5 ml to 3 ml of medicine in the syringe.
3. Syringe A: 14 ml (The gaps or intervals on the scale are each 2 ml.)
   Syringe B: $1\frac{1}{4}$ ml (The gaps or intervals on the scale are each $\frac{1}{4}$ ml.)
   Syringe C: 13 ml (The gaps or intervals on the scale are each 1 ml.)
   Syringe D: $4\frac{1}{2}$ ml (The gaps or intervals on the scale are each $\frac{1}{2}$ ml.)
4. (a) A: 20 and $\frac{1}{5}$ ml; B: 2 ml; C: 20 and $\frac{1}{10}$ ml; D: 6 ml
   (b) Syringe A: 20 (and $\frac{1}{5}$) –14 ml of medicine already in the syringe = 6 (and $\frac{1}{5}$) ml
   Syringe B: 2 ml – $1\frac{1}{4}$ ml = $\frac{3}{4}$ ml
   Syringe C: 21 ml – 13 ml = 8 ml
   Syringe D: 6 ml – $4\frac{1}{2}$ ml = $1\frac{1}{2}$ ml
   (c) Syringe A, because it contains 14 ml.



2. The above picture shows the actual size of a small syringe.
   (a) What do you think the capacity of this syringe is?
   (b) How much medicine is in the syringe?
3. The pictures below do not show the actual sizes of the syringes. The bottom part of each syringe, up to the plunger, is filled with medicine. All the syringes are marked in millilitres.
   There is 14 ml of medicine in Syringe A.
   What volume of medicine is in each of the other syringes?

4. (a) What is the measuring capacity of each syringe?
   (b) For each syringe, state how much more medicine can be drawn in to fill it up to its measuring capacity.
   (c) Which syringe contains the most medicine?

204 UNIT 8: CAPACITY AND VOLUME

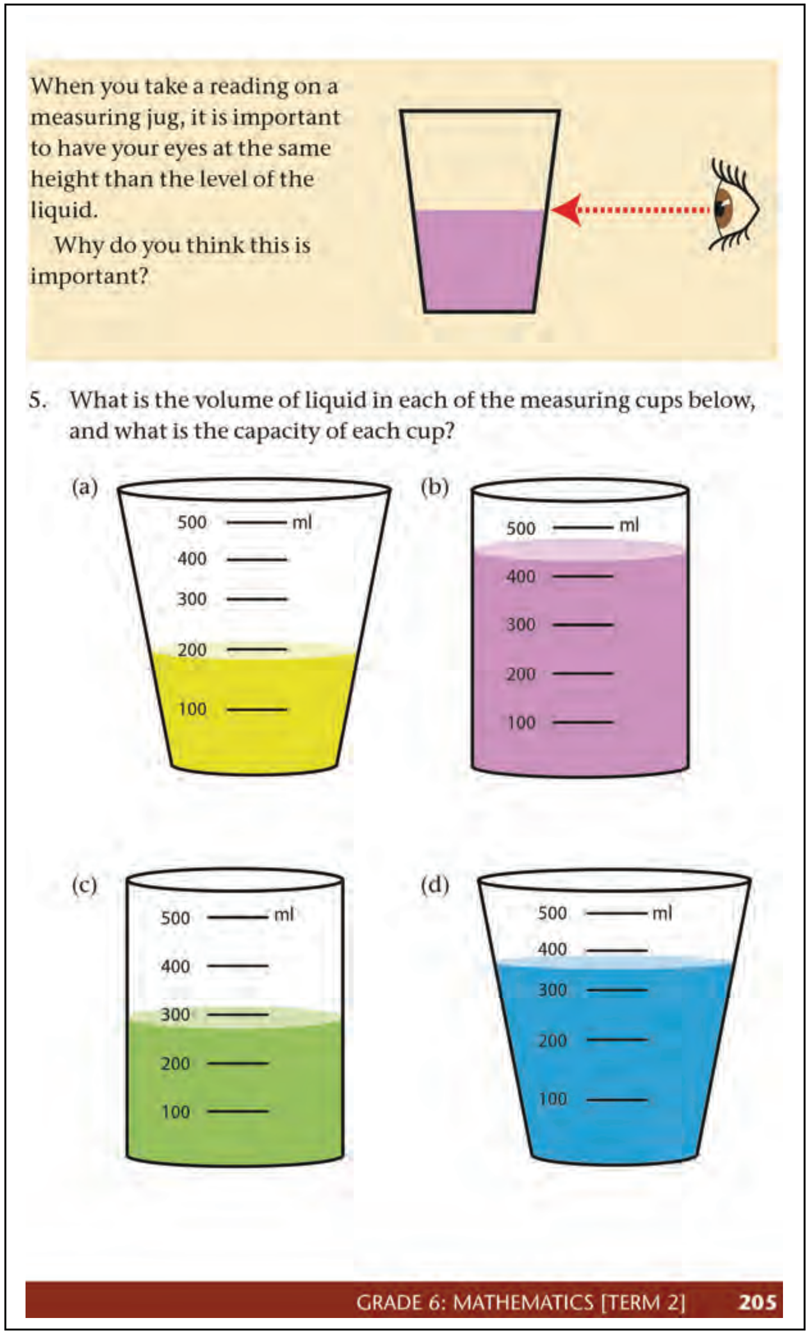
Notes on questions

An excellent enrichment activity would be to source a variety of unusually shaped, clear plastic bottles and ask learners to draw how they think the volume scales for each one may look. For example, a bottle that is ball-shaped (spherical) will have scale lines far apart near the bottom and neck or mouth, while the lines will be closer to each other near the "waist".

Ask learners to look carefully at the cups; they all have the same capacity of slightly more than 500 ml. However, they are not all the same shape. Now ask learners to describe the differences between the markings on the scales. (Answer: The gaps between the markings are the same for the cups that have vertical sides, but the gaps for the cone-shaped cups vary – the gaps are bigger near the bottom and narrower near the top.)

Answers

5. (a) Volume of liquid is about 190 ml, while the cup's capacity is slightly more than 500 ml.
   (b) Volume of liquid is about 420 ml, while the cup's capacity is slightly more than 500 ml.
   (c) Volume of liquid is about 280 ml, while the cup's capacity is slightly more than 500 ml.
   (d) Volume of liquid is about 350 ml, while the cup's capacity is slightly more than 500 ml.



When you take a reading on a measuring jug, it is important to have your eyes at the same height than the level of the liquid.

Why do you think this is important?

5. What is the volume of liquid in each of the measuring cups below, and what is the capacity of each cup?

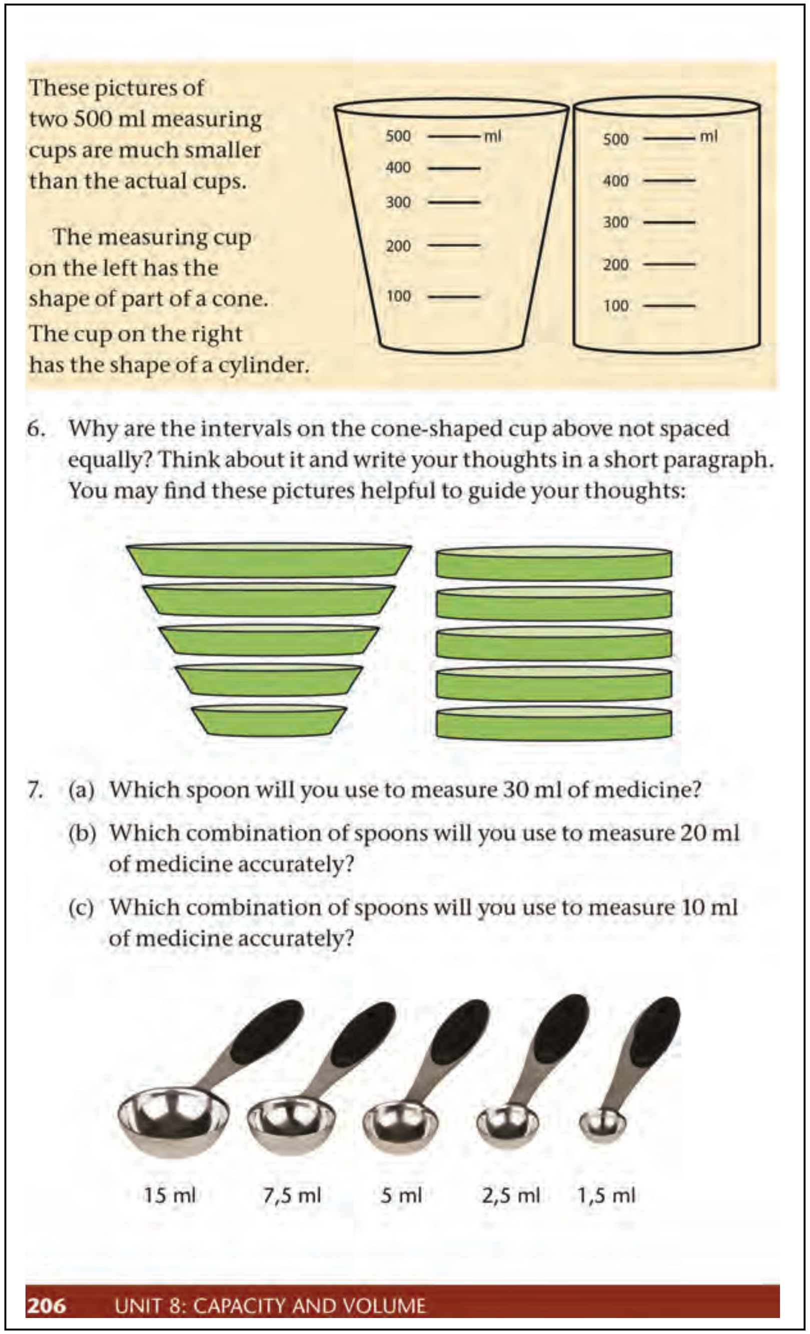
Possible misconceptions

Some learners will struggle to predict how volume scales will look in differently shaped bottles. This is not surprising as learners have to think of two measurements at the same time. One measurement is the area of the base of the container; the other is the height of the liquid. Wide containers have a large base area and narrow containers have a small base area. If the base is large, a volume of 100 ml of liquid, for example, will not go very high. However, if the base area is small, 100 ml of liquid will have to rise higher.

Get learners to add equal amounts of water (say 20 ml each time) to a wide container and a narrow container. Then let them mark each successive water level on the side of the container with a marking pen.

Answers

6. If the intervals or gaps between marks on the cone-shaped cup were equally spaced, you could not measure accurately with such a scale. The green cone slices in the picture show what would happen: the slices are equal in thickness but not equal in volume. The slices near the top have more volume than the slices at the bottom. So the marks must be at greater spacing (i.e. wider intervals) near the bottom, to ensure that the bottom slices have the same volume as the top slices. Ask learners to imagine slicing the 500 ml cup they see on this page. Each slice *must* have a volume of 100 ml. The bottom slice *must* be thicker than the top slice.
7. (a) Learners can suggest the following: Use the 15 ml spoon twice; use the 7,5 ml spoon four times; use the 5 ml spoon six times; use the 2,5 ml spoon 12 times; use the 1,5 ml spoon 20 times. (However, seeing that this is medicine and one would want to measure the prescribed dosis as accurately as possible, it would be best to use the 15 ml spoon twice. It would also be the quickest way.)
   (b) Learners can suggest the following: Use the 15 ml and the 5 ml spoon; use the 7,5 ml spoon twice, and then use the 5 ml spoon.
   (c) Use the 7,5 ml and the 2,5 ml spoons.



These pictures of two 500 ml measuring cups are much smaller than the actual cups.

The measuring cup on the left has the shape of part of a cone. The cup on the right has the shape of a cylinder.

6. Why are the intervals on the cone-shaped cup above not spaced equally? Think about it and write your thoughts in a short paragraph. You may find these pictures helpful to guide your thoughts:

7. (a) Which spoon will you use to measure 30 ml of medicine?
   (b) Which combination of spoons will you use to measure 20 ml of medicine accurately?
   (c) Which combination of spoons will you use to measure 10 ml of medicine accurately?

Answers

8. Ten tablespoons will be 150 ml of water and 20 tablespoons will be 300 ml of water, so the answer must lie between 10 and 20 tablespoons. Let's try 15 tablespoons: 15 ml × 15 = 225 ml. We are now getting closer to 250 ml! Let's add one more tablespoon: 225 ml + 15 ml = 240 ml. Now we need only another 10 ml and that is about $\frac{2}{3}$ of a tablespoon. The answer, therefore, is 16 and $\frac{2}{3}$ tablespoons.

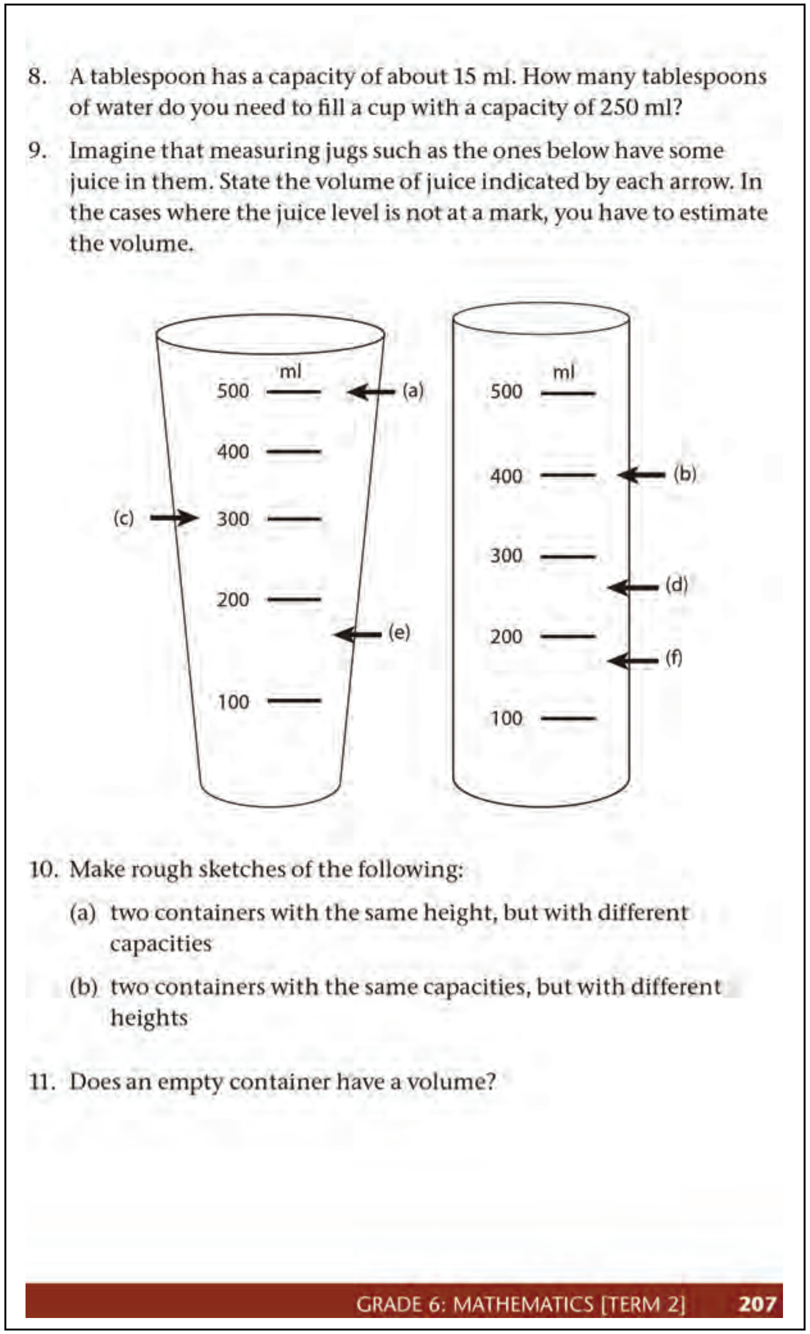
   An approximated answer would be 17 tablespoons.

9. (a) 500 ml (b) About 410 ml (c) 300 ml

   (d) About 270 ml (e) About 170 ml (f) About 170 ml

10. Learners must draw the following containers:
    (a) Two containers with the same height, but one container will be wider than the other. This means, therefore, that it will have a bigger capacity.
    (b) One container must be taller and narrower than the other. Though it is taller it is also narrower, and the narrowness compensates for the greater height.

11. Yes, an empty container is an object with its own volume, like a potato. Ask learners to imagine a clay cup with very thick walls and a bottom. All the clay that was used to make the cup has a volume. You can measure the volume of the cup in a larger container, using the same method as with the potato at the beginning of this unit.



8. A tablespoon has a capacity of about 15 ml. How many tablespoons of water do you need to fill a cup with a capacity of 250 ml?

9. Imagine that measuring jugs such as the ones below have some juice in them. State the volume of juice indicated by each arrow. In the cases where the juice level is not at a mark, you have to estimate the volume.

10. Make rough sketches of the following:
    (a) two containers with the same height, but with different capacities
    (b) two containers with the same capacities, but with different heights

11. Does an empty container have a volume?

GRADE 6: MATHEMATICS [TERM 2] 207

8.3 Work with different units of measurement

Mathematical notes

This section consolidates learners' sense of how big the different units of volume are. Learners need to have a feel for the quantities we measure in millilitres, litres and kilolitres.

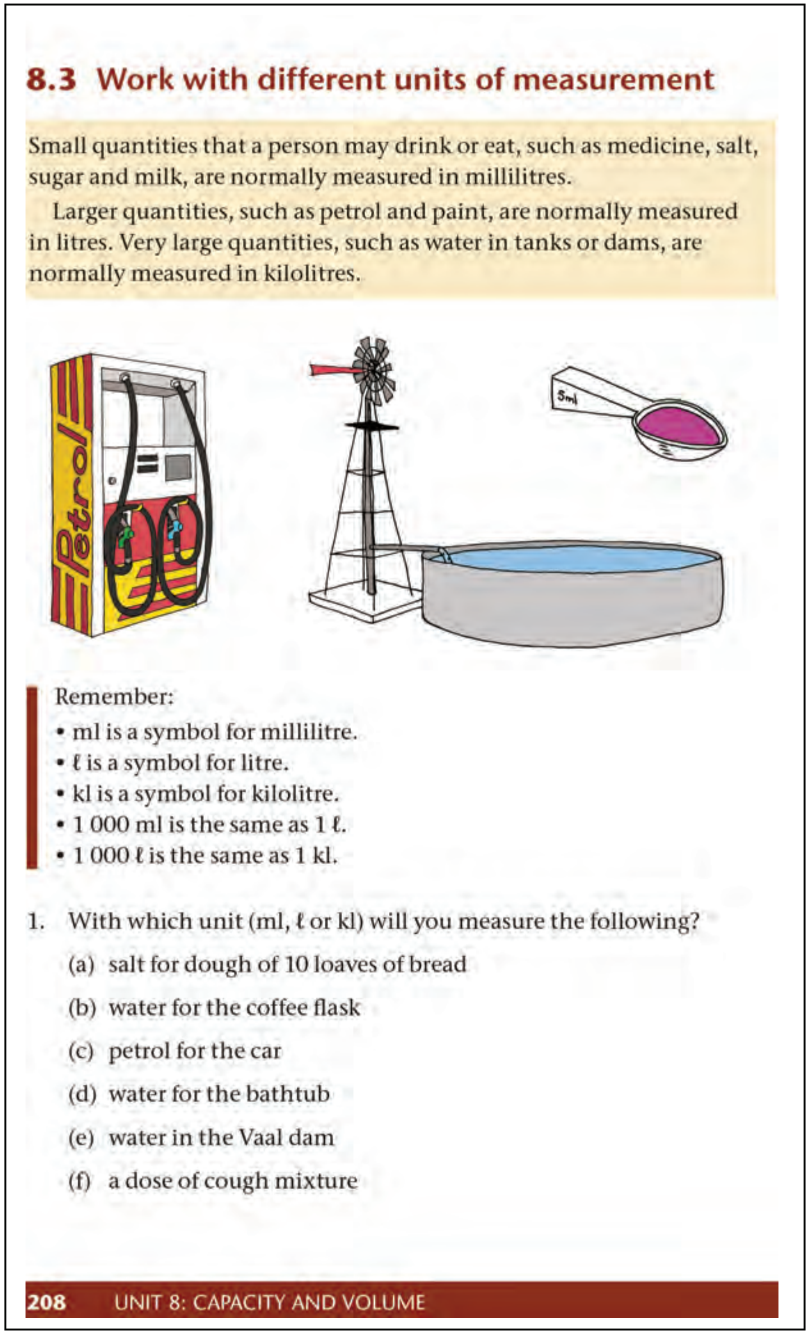
Learners will be given plenty of exercise in converting measurements between these three units.

Teaching guidelines

We know that learners are beginning to grasp the concept of volume when they can estimate measurements of volume. To help develop their understanding further, keep using real-life examples of volume measurements.

Answers

1. (a) millilitres – i.e. ml
   (b) millilitres – i.e. ml
   (c) litres – i.e. ℓ
   (d) litres – i.e. ℓ
   (e) kilolitres – i.e. kl
   (f) millilitres – i.e. ml



8.3 Work with different units of measurement

Small quantities that a person may drink or eat, such as medicine, salt, sugar and milk, are normally measured in millilitres.

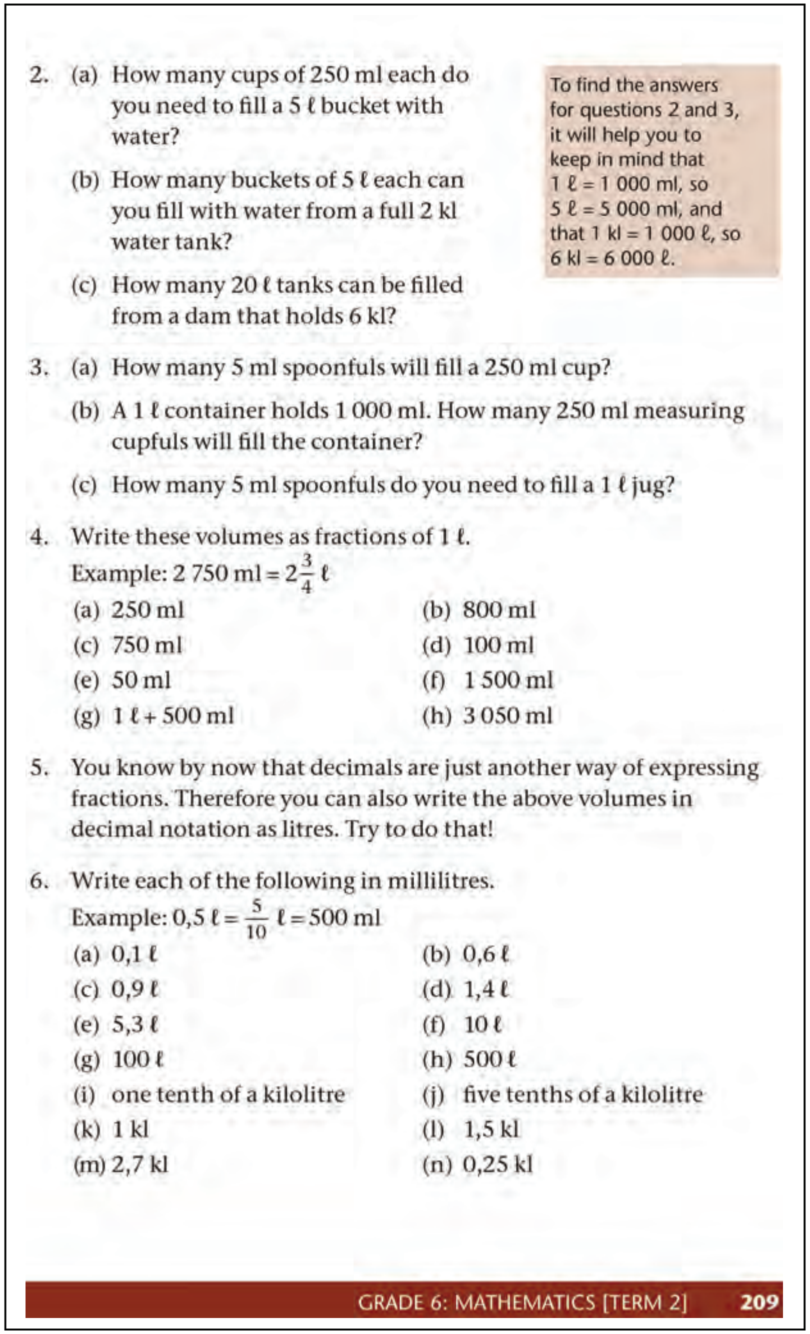
Larger quantities, such as petrol and paint, are normally measured in litres. Very large quantities, such as water in tanks or dams, are normally measured in kilolitres.

Remember:
- ml is a symbol for millilitre.
- ℓ is a symbol for litre.
- kl is a symbol for kilolitre.
- 1 000 ml is the same as 1 ℓ.
- 1 000 ℓ is the same as 1 kl.

1. With which unit (ml, ℓ or kl) will you measure the following?
   (a) salt for dough of 10 loaves of bread
   (b) water for the coffee flask
   (c) petrol for the car
   (d) water for the bathtub
   (e) water in the Vaal dam
   (f) a dose of cough mixture

Answers

2. (a) Four cups of 250 ml will give me 1 ℓ, so for 5 ℓ I need five times that amount, which is 20 cups.
   (b) 2 kl is 2 000 ℓ. How many amounts of 5 ℓ can I get from 2 000 ℓ? The answer is 400 buckets.
   (c) 6 kl is 6 000 ℓ. How many amounts of 20 ℓ can I get from 6 000 ℓ? The answer is 6 000 ℓ ÷ 20 = 300 tanks.
3. (a) 250 ml ÷ 5 = 50 spoonfuls
   (b) It will take 4 cupfuls to fill the container.
   (c) 1 ℓ is 1 000 ml: 1 000 ml ÷ 5 ml = 200 spoonfuls
4. (a) $\frac{1}{4}\ell$ (b) $\frac{8}{10}$ or $\frac{4}{5}\ell$ (c) $\frac{3}{4}\ell$ (d) $\frac{1}{10}\ell$
   (e) $\frac{5}{100}$ or $\frac{1}{20}\ell$ (f) $1\frac{1}{2}\ell$ (g) $1\frac{1}{2}\ell$ (h) $3\frac{50}{1\,000}$ or $3\frac{1}{20}\ell$
5. (a) 0,25 ℓ (b) 0,8 ℓ (c) 0,75 ℓ (d) 0,1 ℓ
   (e) 0,05 ℓ (f) 1,5 ℓ (g) 1,5 ℓ (h) 3,05 ℓ
6. (a) $0{,}1\,\ell = \frac{1}{10}\ell = 100$ ml (b) $0{,}6\,\ell = \frac{6}{10}\ell = 600$ ml
   (c) $0{,}9\,\ell = \frac{9}{10}\ell = 900$ ml (d) $1{,}4\,\ell = 1\frac{4}{10}\ell = 1\,400$ ml
   (e) $5{,}3\,\ell = 5\frac{3}{10}\ell = 5\,300$ ml (f) 10 ℓ = 10 000 ml
   (g) 100 ℓ = 100 000 ml (h) 500 ℓ = 500 000 ml
   (i) $\frac{1}{10}$ kl = 100 ℓ = 100 000 ml (j) $\frac{5}{10}$ kl = 500 ℓ = 500 000 ml
   (k) 1 kl = 1 000 ℓ = 1 000 000 ml (l) 1,5 kl = 1 500 ℓ = 1 500 000 ml
   (m) 2,7 kl = $2\frac{7}{10}$ kl = 2 700 000 ml (n) 0,25 kl = $\frac{1}{4}$ kl = 250 000 ml



2. (a) How many cups of 250 ml each do you need to fill a 5 ℓ bucket with water?
   (b) How many buckets of 5 ℓ each can you fill with water from a full 2 kl water tank?
   (c) How many 20 ℓ tanks can be filled from a dam that holds 6 kl?

To find the answers for questions 2 and 3, it will help you to keep in mind that 1 ℓ = 1 000 ml, so 5 ℓ = 5 000 ml, and that 1 kl = 1 000 ℓ, so 6 kl = 6 000 ℓ.

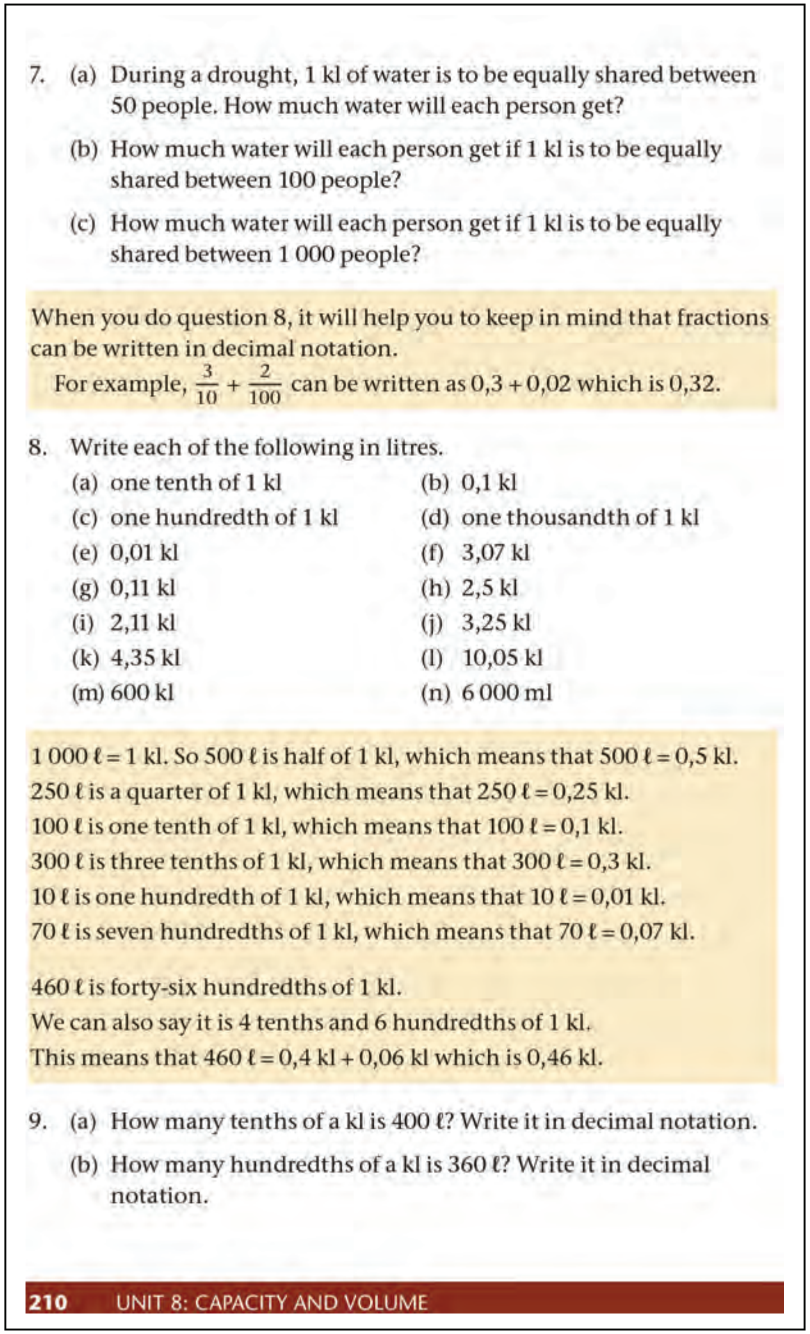
3. (a) How many 5 ml spoonfuls will fill a 250 ml cup?
   (b) A 1 ℓ container holds 1 000 ml. How many 250 ml measuring cupfuls will fill the container?
   (c) How many 5 ml spoonfuls do you need to fill a 1 ℓ jug?
4. Write these volumes as fractions of 1 ℓ.
   Example: 2 750 ml = $2\frac{3}{4}\ell$
   (a) 250 ml (b) 800 ml
   (c) 750 ml (d) 100 ml
   (e) 50 ml (f) 1 500 ml
   (g) 1 ℓ + 500 ml (h) 3 050 ml
5. You know by now that decimals are just another way of expressing fractions. Therefore you can also write the above volumes in decimal notation as litres. Try to do that!
6. Write each of the following in millilitres.
   Example: $0{,}5\,\ell = \frac{5}{10}\,\ell = 500$ ml
   (a) 0,1 ℓ (b) 0,6 ℓ
   (c) 0,9 ℓ (d) 1,4 ℓ
   (e) 5,3 ℓ (f) 10 ℓ
   (g) 100 ℓ (h) 500 ℓ
   (i) one tenth of a kilolitre (j) five tenths of a kilolitre
   (k) 1 kl (l) 1,5 kl
   (m) 2,7 kl (n) 0,25 kl

Answers

7. (a) 1 kl of water is 1 000 ℓ, so divided between 50 people, each person wil get 20 ℓ. (Ask learners how many buckets of water this is.)
   (b) The answer will be half of the answer in (a), i.e. 10 ℓ.
   (c) Each person will get just 1 ℓ of water.

8. (a) 100 ℓ (b) 100 ℓ
   (c) 10 ℓ (d) 1 ℓ
   (e) 10 ℓ (f) 3 070 ℓ
   (g) 110 ℓ (h) 2 500 ℓ
   (i) 2 110 ℓ (j) 3 250 ℓ
   (k) 4 350 ℓ (l) 10 050 ℓ
   (m) 600 000 ℓ (n) 6 ℓ

9. (a) 0,4 kl (b) 0,36 kl



7. (a) During a drought, 1 kl of water is to be equally shared between 50 people. How much water will each person get?
   (b) How much water will each person get if 1 kl is to be equally shared between 100 people?
   (c) How much water will each person get if 1 kl is to be equally shared between 1 000 people?

When you do question 8, it will help you to keep in mind that fractions can be written in decimal notation.
For example, $\frac{3}{10} + \frac{2}{100}$ can be written as 0,3 + 0,02 which is 0,32.

8. Write each of the following in litres.
   (a) one tenth of 1 kl (b) 0,1 kl
   (c) one hundredth of 1 kl (d) one thousandth of 1 kl
   (e) 0,01 kl (f) 3,07 kl
   (g) 0,11 kl (h) 2,5 kl
   (i) 2,11 kl (j) 3,25 kl
   (k) 4,35 kl (l) 10,05 kl
   (m) 600 kl (n) 6 000 ml

1 000 ℓ = 1 kl. So 500 ℓ is half of 1 kl, which means that 500 ℓ = 0,5 kl.
250 ℓ is a quarter of 1 kl, which means that 250 ℓ = 0,25 kl.
100 ℓ is one tenth of 1 kl, which means that 100 ℓ = 0,1 kl.
300 ℓ is three tenths of 1 kl, which means that 300 ℓ = 0,3 kl.
10 ℓ is one hundredth of 1 kl, which means that 10 ℓ = 0,01 kl.
70 ℓ is seven hundredths of 1 kl, which means that 70 ℓ = 0,07 kl.

460 ℓ is forty-six hundredths of 1 kl.
We can also say it is 4 tenths and 6 hundredths of 1 kl.
This means that 460 ℓ = 0,4 kl + 0,06 kl which is 0,46 kl.

9. (a) How many tenths of a kl is 400 ℓ? Write it in decimal notation.
   (b) How many hundredths of a kl is 360 ℓ? Write it in decimal notation.

210 UNIT 8: CAPACITY AND VOLUME

Answers

10\. (a) $\frac{1}{4}$ kl = 0,25 kl (b) $1\frac{1}{4}$ kl = 1,25 kl (c) $2\frac{3}{4}$ kl = 2,75 kl

(d) $\frac{65}{100}$ kl = 0,65 kl (e) $\frac{15}{100}$ kl = 0,15 kl (f) $12\frac{1}{2}$ kl = 12,5 kl

(g) $\frac{37}{100}$ kl = 0,37 kl (h) $6\frac{83}{100}$ kl = 6,83 kl (i) $\frac{8}{100}$ kl = 0,08 kl

(j) $\frac{6}{10}$ kl = 0,6 kl

11\. (a) 7,33 ℓ; 45 100 ml; 639 ℓ; 2,54 kl; 8 kl

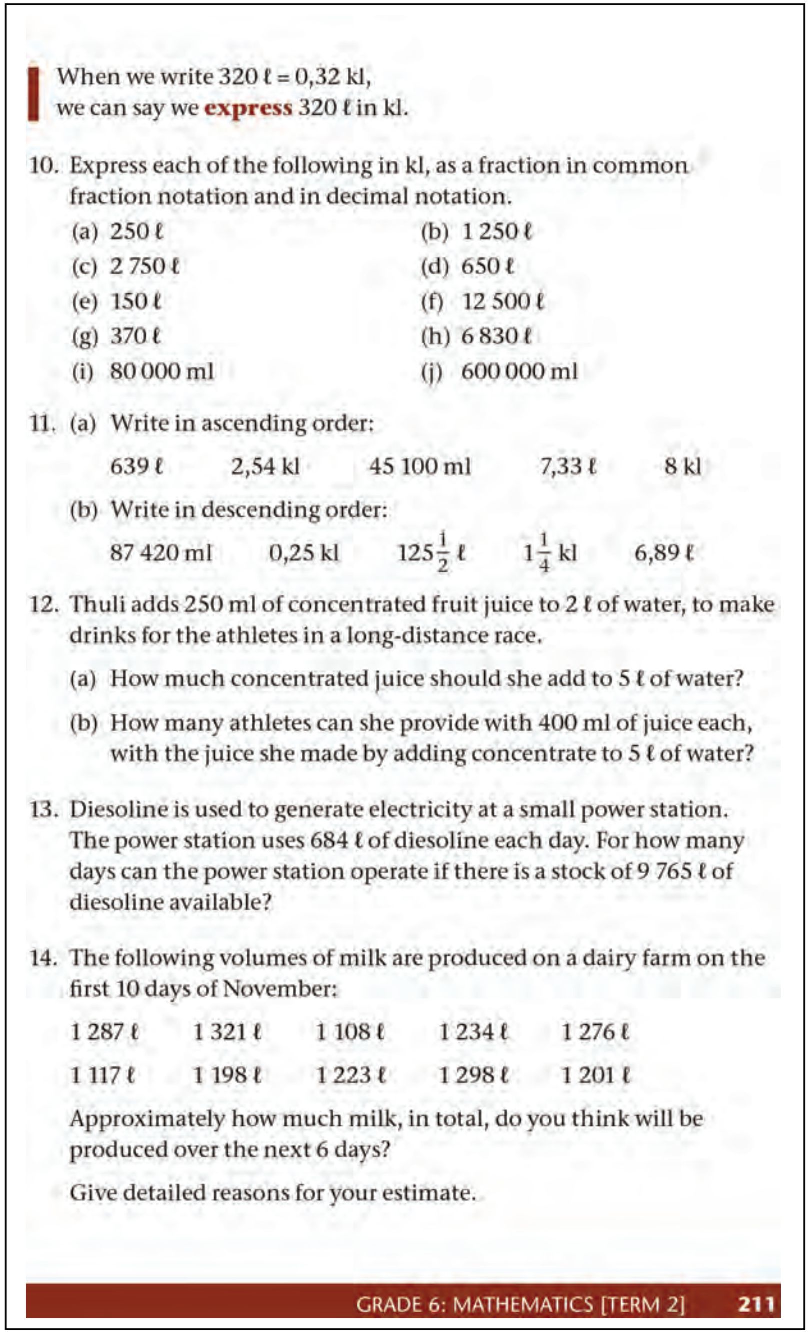
(b) $1\frac{1}{4}$ kl; 0,25 kl; $125\frac{1}{2}$ ℓ; 87 420 ml; 6,89 ℓ

12\. (a) 625 ml: If she must add 250 ml of concentrated juice to 2 ℓ of water, then she must add 125 ml of concentrated juice to 1 ℓ of water. So for 5 ℓ of water she adds 5 times 125 ml of concentrated juice, which is 625 ml.

(b) 14 athletes, because 5 000 ml water + 625 ml concentrate gives 5 625 ml. At 400 ml per athlete, that will be enough for 14 servings.

13\. 14 days

14\. Learners' answers will vary. For example, the total for 10 days is 12 263 ℓ. On average it will be about 1 226 ℓ per day, i.e. approximately 7 357 ℓ in total over the next 6 days.



When we write 320 ℓ = 0,32 kl,
we can say we **express** 320 ℓ in kl.

10\. Express each of the following in kl, as a fraction in common fraction notation and in decimal notation.

(a) 250 ℓ (b) 1 250 ℓ
(c) 2 750 ℓ (d) 650 ℓ
(e) 150 ℓ (f) 12 500 ℓ
(g) 370 ℓ (h) 6 830 ℓ
(i) 80 000 ml (j) 600 000 ml

11\. (a) Write in ascending order:

639 ℓ 2,54 kl 45 100 ml 7,33 ℓ 8 kl

(b) Write in descending order:

87 420 ml 0,25 kl $125\frac{1}{2}$ ℓ $1\frac{1}{4}$ kl 6,89 ℓ

12\. Thuli adds 250 ml of concentrated fruit juice to 2 ℓ of water, to make drinks for the athletes in a long-distance race.

(a) How much concentrated juice should she add to 5 ℓ of water?

(b) How many athletes can she provide with 400 ml of juice each, with the juice she made by adding concentrate to 5 ℓ of water?

13\. Diesoline is used to generate electricity at a small power station. The power station uses 684 ℓ of diesoline each day. For how many days can the power station operate if there is a stock of 9 765 ℓ of diesoline available?

14\. The following volumes of milk are produced on a dairy farm on the first 10 days of November:

1 287 ℓ 1 321 ℓ 1 108 ℓ 1 234 ℓ 1 276 ℓ
1 117 ℓ 1 198 ℓ 1 223 ℓ 1 298 ℓ 1 201 ℓ

Approximately how much milk, in total, do you think will be produced over the next 6 days?

Give detailed reasons for your estimate.

GRADE 6: MATHEMATICS [TERM 2] 211

Term 3

Grade 6 Term 3 Unit 1 Mass

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
1.1 Quiz	Revising the meaning of mass; choosing appropriate instruments and units	215
1.2 Comparing mass measurements	Ordering and comparing the mass of objects, using grams and kilograms	216 to 218
1.3 Reading mass in grams and kilograms	Reading mass on different scales	218 to 219
1.4 Solving problems about mass and quantity	Solving problems in the context of mass	219 to 220

CAPS time allocation	5 hours
CAPS page references	26 and 259 to 261

Mathematical background

Length, **capacity**, **volume**, **area** and **mass** are all different *properties of objects*. **Length**, **capacity**, **volume** and **area** are called *spatial measures*. We can often see how much space something takes up, how much area it covers, or how long it is.

Mass is *not a spatial measure*; it is a *physical measure*. The mass of an object is the property that we feel in our hands. For example, we say an object feels heavy or light. From experience we can remember how heavy a bucket of water is, but we cannot always guess how heavy an object is simply by looking at its size. Young learners often assume that the bigger something is, the heavier it must be. A small piece of iron may, however, be much heavier than a large piece of plastic foam. From this example we can tell that the density of iron is greater than the density of plastic foam.

The heaviness of an object is really the force of gravity that the object and the earth exert on each other. We can use various instruments, such as a bathroom scale, to measure the heaviness of an object. In this instance we can tell the mass of an object because the scale has been marked in grams or kilograms. If a person stands on a bathroom scale, his or her mass may, for example, show 60 kg. Or we may find that a brick has a mass of 1 kg. This mass is useful when somebody needs to calculate how many bricks he can safely load onto his bakkie.

Learners go through four stages when learning to measure; namely:

- identifying and understanding the property they are measuring
- comparing and ordering examples of a particular measure
- using informal or non-standard units to measure
- using formal or standard units to measure.

Formal, standard units allow people all over the world to measure, record, quantify and compare objects using the same units. The focus of measuring mass in Grade 6 is therefore on learning to use standard units of mass. A difficulty that people face with formal measurement, however, is that instruments are often difficult to read.

Resources

For example: 1 kg packet of flour, 1 kg packet of sugar, 1 kg packet of salt, 400 g box of cereal (if possible), empty grocery containers, a kitchen scale, a bathroom scale, cups, rice, tea, sand, stones

1.1 Quiz

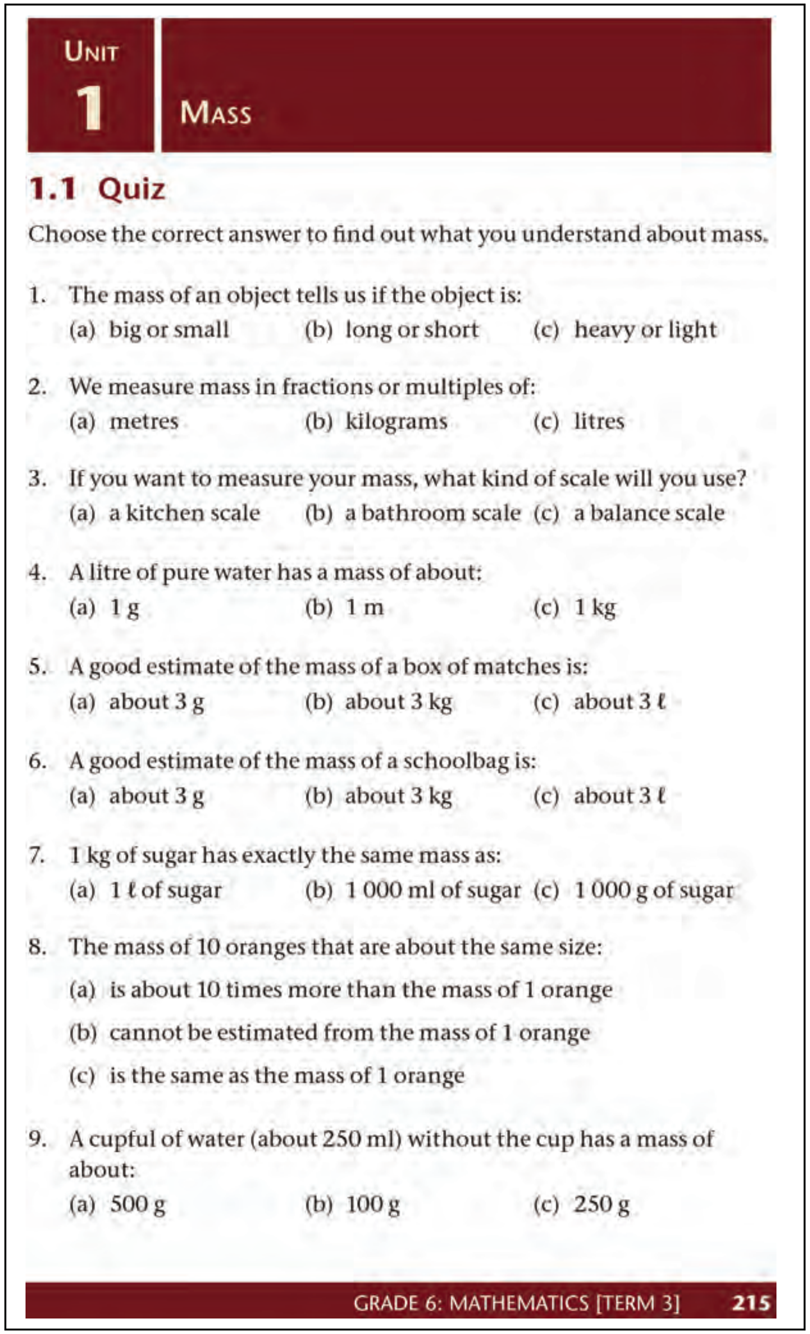
Teaching guidelines

The work that learners have covered in previous grades should enable them to answer all the questions in this first section. By Grade 6, learners should have a sense of how much one gram (1 g) and one kilogram (1 kg) is. This will help them to make sensible estimates of the mass of objects before measuring them.

Use this quiz as a way to consolidate learners' prior knowledge or simply use it as a baseline assessment.

Answers

1. (c) heavy or light
2. (b) kilograms
3. (b) a bathroom scale
4. (c) 1 kg
5. (a) about 3 g
6. (b) about 3 kg
7. (c) 1 000 g of sugar
8. (a) is about 10 times more than the mass of 1 orange
9. (c) 250 g



UNIT
1
MASS

1.1 Quiz

Choose the correct answer to find out what you understand about mass.

1. The mass of an object tells us if the object is:
   (a) big or small (b) long or short (c) heavy or light
2. We measure mass in fractions or multiples of:
   (a) metres (b) kilograms (c) litres
3. If you want to measure your mass, what kind of scale will you use?
   (a) a kitchen scale (b) a bathroom scale (c) a balance scale
4. A litre of pure water has a mass of about:
   (a) 1 g (b) 1 m (c) 1 kg
5. A good estimate of the mass of a box of matches is:
   (a) about 3 g (b) about 3 kg (c) about 3 ℓ
6. A good estimate of the mass of a schoolbag is:
   (a) about 3 g (b) about 3 kg (c) about 3 ℓ
7. 1 kg of sugar has exactly the same mass as:
   (a) 1 ℓ of sugar (b) 1 000 ml of sugar (c) 1 000 g of sugar
8. The mass of 10 oranges that are about the same size:
   (a) is about 10 times more than the mass of 1 orange
   (b) cannot be estimated from the mass of 1 orange
   (c) is the same as the mass of 1 orange
9. A cupful of water (about 250 ml) without the cup has a mass of about:
   (a) 500 g (b) 100 g (c) 250 g

1.2 Comparing mass measurements

Mathematical notes

This section focuses on two issues. Firstly, volume and mass are only proportional if comparing the same substances (see question 2). Sometimes objects with the same mass can have different volumes (see question 1). Sometimes objects with larger volumes have a smaller mass than objects with smaller volumes (see question 3). As mentioned earlier, objects have different densities. Grade 6 learners are not expected to have a deep understanding of density at this point, but they should know that certain substances are heavier than others when comparing equal volumes of each substance. Secondly, learners revise the relationship between grams and kilograms.

Teaching guidelines

For question 1, try to bring a 1 kg packet of flour, a 1 kg packet of sugar, a 1 kg packet of salt and a 400 g box of cereal. This will help you to visually demonstrate to learners that larger objects do not always have a greater mass.

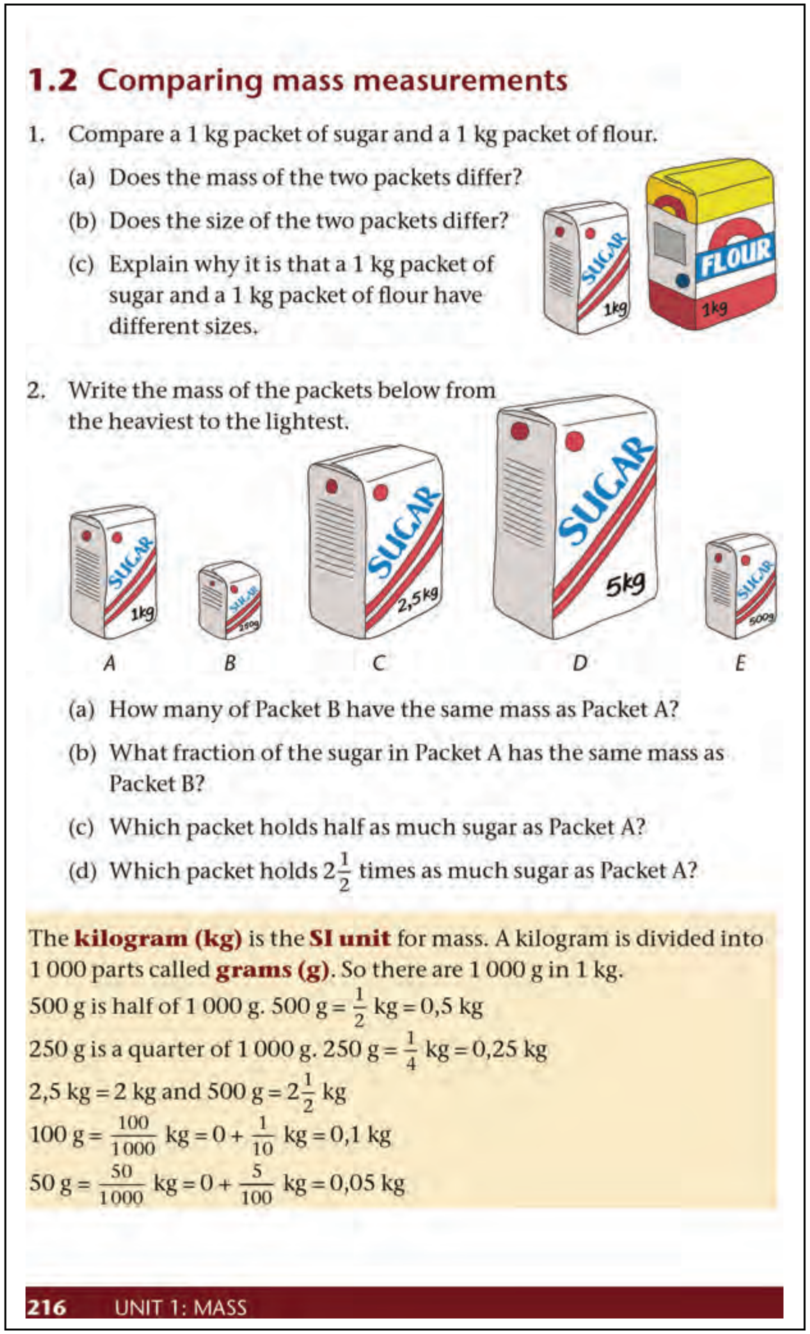
Notes on questions

In question 1 we see that substances can have the same mass but different volumes. Learners see further examples of this in question 3. In question 2, the bigger packets are heavier, but this is only because all the packets contain sugar.

Answers

1. (a) No
   (b) Yes, the 1 kg packet of flour is bigger than the 1 kg packet of sugar.
   (c) Consider learners' explanations. A reasonable answer would be, for example: Flour is a different substance than sugar; you need more flour to make up a kilogram mass compared to the amount of sugar needed to make up a kilogram mass.

   Learners might simply say that "sugar is heavier than flour". If they say this, ask them if the small packet of sugar in question 2 is heavier than the big packet of flour in question 1. (The answer to this is no, because the packet of flour is heavier than the small packet of sugar.) So, it is incorrect to say "sugar is heavier than flour". We always have to compare the heaviness of equal volumes of a substance.
2. D: 5 kg; C: 2,5 kg; A: 1 kg; E: 500 g; B: 250 g
   (a) 4 (b) Quarter (c) Packet E (d) Packet C



1.2 Comparing mass measurements

1. Compare a 1 kg packet of sugar and a 1 kg packet of flour.
   (a) Does the mass of the two packets differ?
   (b) Does the size of the two packets differ?
   (c) Explain why it is that a 1 kg packet of sugar and a 1 kg packet of flour have different sizes.
2. Write the mass of the packets below from the heaviest to the lightest.
   (a) How many of Packet B have the same mass as Packet A?
   (b) What fraction of the sugar in Packet A has the same mass as Packet B?
   (c) Which packet holds half as much sugar as Packet A?
   (d) Which packet holds $2\frac{1}{2}$ times as much sugar as Packet A?

The **kilogram (kg)** is the **SI unit** for mass. A kilogram is divided into 1 000 parts called **grams (g)**. So there are 1 000 g in 1 kg.

500 g is half of 1 000 g. 500 g = $\frac{1}{2}$ kg = 0,5 kg

250 g is a quarter of 1 000 g. 250 g = $\frac{1}{4}$ kg = 0,25 kg

2,5 kg = 2 kg and 500 g = $2\frac{1}{2}$ kg

100 g = $\frac{100}{1000}$ kg = 0 + $\frac{1}{10}$ kg = 0,1 kg

50 g = $\frac{50}{1000}$ kg = 0 + $\frac{5}{100}$ kg = 0,05 kg

216 UNIT 1: MASS

Teaching guidelines

You can use the shaded passage on page 216 to remind learners about the relationship between grams and kilograms, as well as how this relationship can be expressed in common fractions and decimal fractions.

Remind learners that in order to get an accurate reading, they need to stand directly in front of the scale (analogue scale) when reading the mass. If they stand too far to the right or the left of the dial, they will get an inaccurate reading.

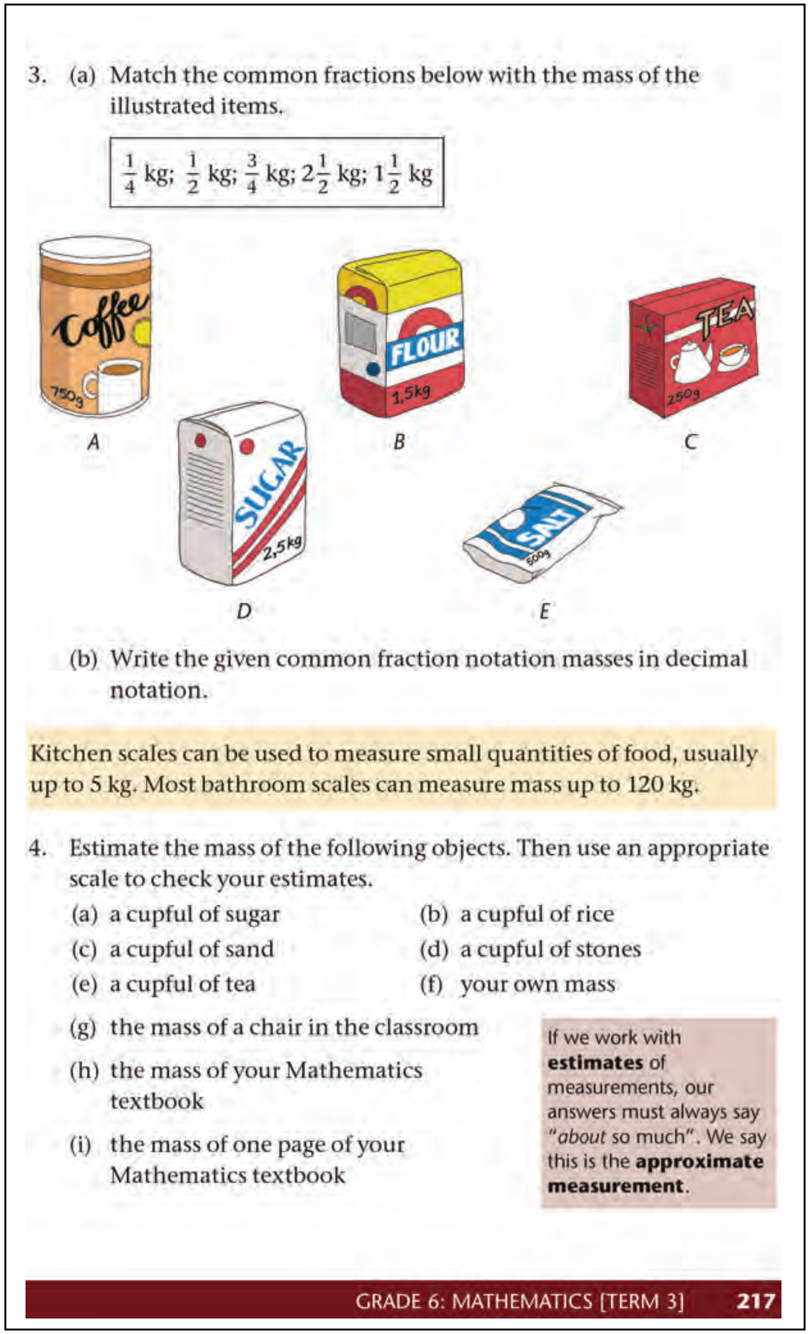
Notes on questions

If possible, try to bring **empty grocery containers** like those shown in question 3 to class. For question 4, bring a **kitchen** and **bathroom scale** to class if you can. Learners will use a kitchen scale to measure the quantities in questions (a)–(e) and (h). They will use a bathroom scale to measure the quantities in questions (f) and (g). Also see if you can source cups, sugar, rice, tea, sand and stones, and bring these to class.

Learners will need to think about how to find the mass of a chair. One option is for them to hold the chair above their heads while they stand on the scale. Then they can subtract their own mass from the combined mass of the chair and themselves.

Answers

3. (a) A: $\frac{3}{4}$ kg B: $1\frac{1}{2}$ kg C: $\frac{1}{4}$ kg D: $2\frac{1}{2}$ kg E: $\frac{1}{2}$ kg
   (b) A: 0,75 kg B: 1,5 kg C: 0,25 kg D: 2,5 kg E: 0,5 kg
4. Learners' answers will vary as different kinds of sugar, rice, sand, stones, tea, etc. have different masses. The masses given below are only approximations based on a 250 ml cup.
   (a) About 200 g
   (b) About 200 g
   (c) About 375 g
   (d) It depends on the kind and size of the stones, but probably lighter than sand – i.e. less than 375 g
   (e) About 250 g
   (f) Learners' masses will differ
   (g) The mass of a chair will differ from classroom to classroom
   (h) About 600 g
   (i) 630 g ÷ 190 ≈ 3 g (this is based on dividing the total number of pages plus the cover, i.e. 369 numbered + 7 unnumbered pages by 2 because by page we mean front and back = 376 ÷ 2 = 188; add to this 2 pages for the cover: 188 + 2 = 190.)



3. (a) Match the common fractions below with the mass of the illustrated items.

$\frac{1}{4}$ kg; $\frac{1}{2}$ kg; $\frac{3}{4}$ kg; $2\frac{1}{2}$ kg; $1\frac{1}{2}$ kg

(b) Write the given common fraction notation masses in decimal notation.

Kitchen scales can be used to measure small quantities of food, usually up to 5 kg. Most bathroom scales can measure mass up to 120 kg.

4. Estimate the mass of the following objects. Then use an appropriate scale to check your estimates.
   (a) a cupful of sugar
   (b) a cupful of rice
   (c) a cupful of sand
   (d) a cupful of stones
   (e) a cupful of tea
   (f) your own mass
   (g) the mass of a chair in the classroom
   (h) the mass of your Mathematics textbook
   (i) the mass of one page of your Mathematics textbook

If we work with **estimates** of measurements, our answers must always say "*about* so much". We say this is the **approximate measurement**.

GRADE 6: MATHEMATICS [TERM 3] 217

Answers

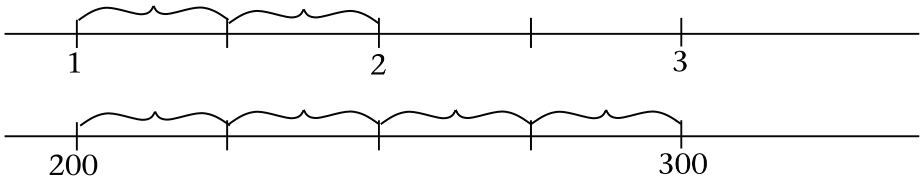
5. (a) 2 800 g (b) 500 g (c) 1 500 g
(d) 20 000 g (e) 60 000 g

6. (a) 2 kg (b) $\frac{1}{4}$ kg or 0,25 kg (c) $\frac{1}{10}$ kg or 0,1 kg
(d) $\frac{3}{4}$ kg or 0,75 kg (e) $5\frac{1}{2}$ kg or 5,5 kg (f) $3\frac{1}{4}$ kg or 3,25 kg

1.3 Reading mass in grams and kilograms

Mathematical notes

Draw, for example, the following two number lines on the board to demonstrate the understanding of intervals to learners.

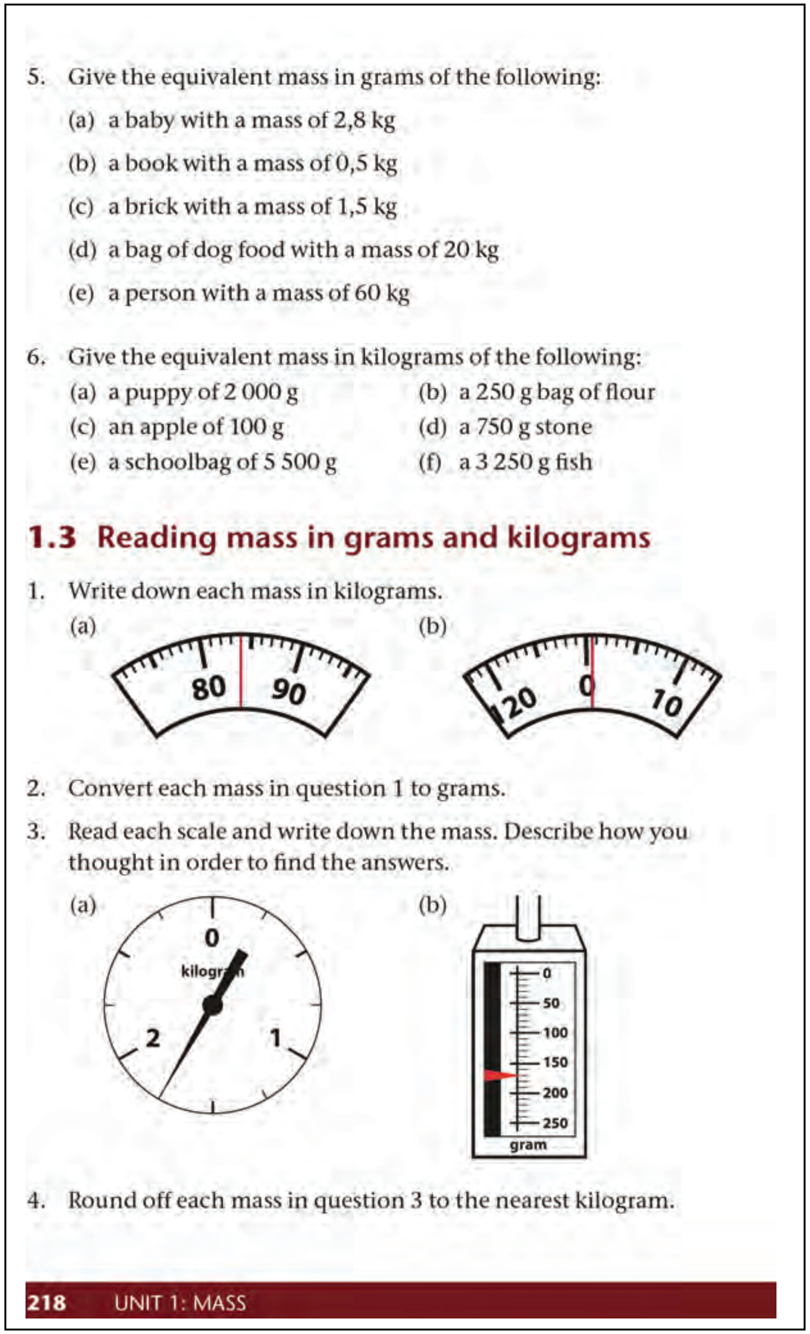


The first number line shows two numbered intervals with two unnumbered intervals/units in between. One can deduce that the value of each unit of measurement between the numbered intervals is equal to half or 0,5.

The second number line shows one numbered interval with four unnumbered intervals/units in between. One can deduce that the value of each unit of measurement between the numbered intervals is equal to 50.

Answers

1. (a) 84 kg (b) $\frac{1}{2}$ kg or 0,5 kg
2. (a) 84 000 g (b) 500 g
3. (a) 1,75 kg or $1\frac{3}{4}$ kg: There are four intervals of measurement between each numbered mark. One numbered interval equals 1 kg. Therefore each interval represents a quarter of a kg.
(b) 170 g: Each unit of measurement equals 10 g.
4. (a) 1,75 kg ≈ 2 kg (b) 170 g ≈ 0 kg



5. Give the equivalent mass in grams of the following:
(a) a baby with a mass of 2,8 kg
(b) a book with a mass of 0,5 kg
(c) a brick with a mass of 1,5 kg
(d) a bag of dog food with a mass of 20 kg
(e) a person with a mass of 60 kg

6. Give the equivalent mass in kilograms of the following:
(a) a puppy of 2 000 g (b) a 250 g bag of flour
(c) an apple of 100 g (d) a 750 g stone
(e) a schoolbag of 5 500 g (f) a 3 250 g fish

1.3 Reading mass in grams and kilograms

1. Write down each mass in kilograms.
(a) (b)

2. Convert each mass in question 1 to grams.
3. Read each scale and write down the mass. Describe how you thought in order to find the answers.
(a) (b)

4. Round off each mass in question 3 to the nearest kilogram.

218 UNIT 1: MASS

Mathematical notes

Everything has mass. Sometimes an instrument and the unit in which it is calibrated is not sensitive enough to show the mass. For example, if a paper clip has a mass of 2 g and you place a single paper clip on a bathroom scale, it will show 0 g. This does not mean that the paper clip has no mass. If you place the same paper clip on a digital kitchen scale it will show 2 g. If you place 500 paper clips on the bathroom scale, it will show 1 kg.

When we round off the mass of something that weighs between 0 kg and 1 kg, it is possible that the mass is nearer to zero than to one. In the case of the paper clip, the rounded mass is 0 kg.

Notes on questions

As a challenge you can ask learners to round off the masses in questions 5(b) and (c).

Answers

5. (a) $80\frac{1}{10}$ kg (b) $\frac{25}{100}$ kg (c) $\frac{5}{100}$ kg (d) $34\frac{87}{100}$ kg

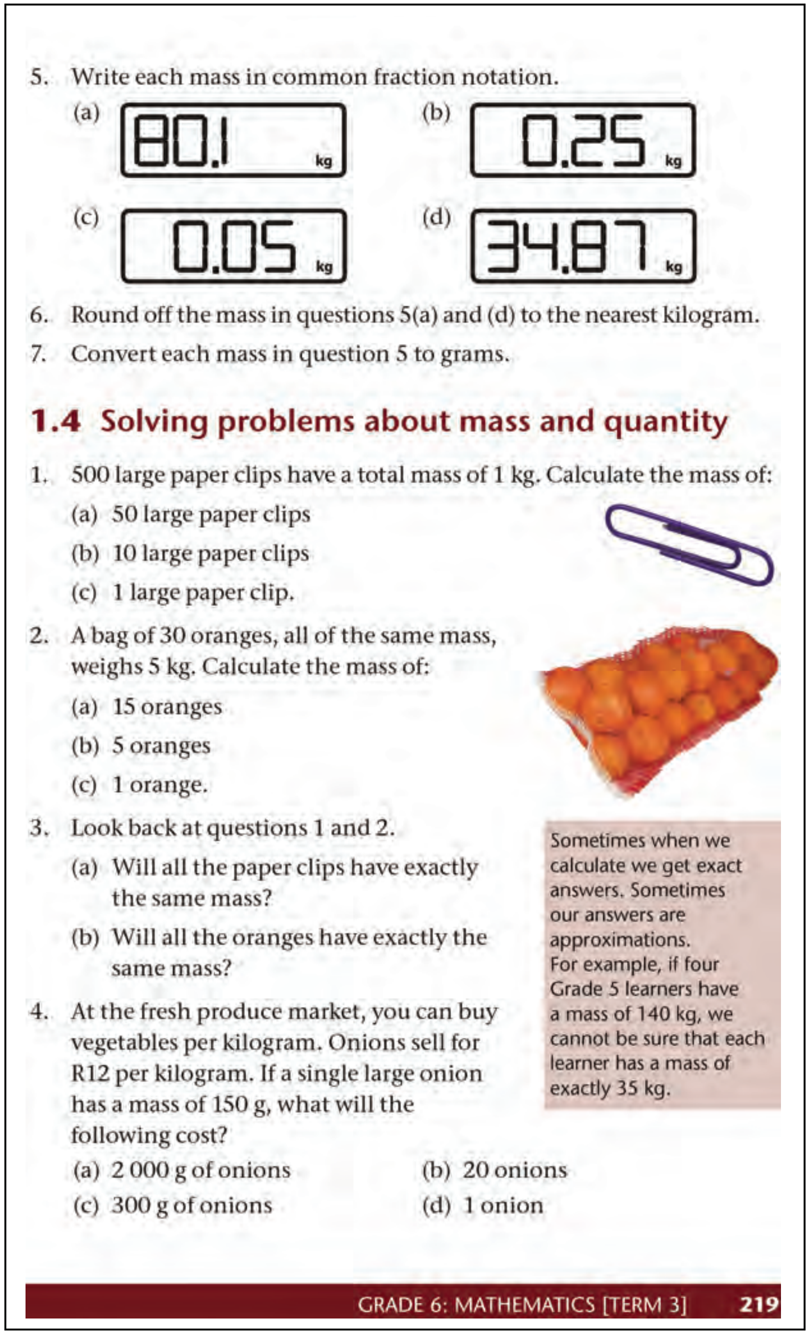
6. (a) 80 kg (d) 35 kg

7. (a) 80 100 g (b) 250 g (c) 50 g (d) 34 870 g

1.4 Solving problems about mass and quantity

Answers

1. (a) 500 paper clips have a mass of 1 000 g, so 50 paper clips have a mass of $\frac{1\,000}{10}$ g = 100 g
   (b) Mass of 10 paper clips = 20 g
   (c) Mass of one paper clip = 2 g
2. (a) 15 oranges have a mass of 5 000 g ÷ 2 = 2 500 g or $2\frac{1}{2}$ kg
   (b) 5 oranges have a mass of 5 000 g ÷ 6 = 833 rem 2 g, or about 833 g
   (c) 1 orange has a mass of 5 000 g ÷ 30 = 166 rem 20 g, or about 166 g
3. (a) Yes
   (b) No, the masses of individual oranges will vary.
4. (a) 2 000 g = 2 kg, so 2 kg × R12 = R24
   (b) 20 × 150 g = 3 000 g = 3 kg, so 3 kg × R12 = R36
   (c) 300 g = 0,3 kg, so 0,3 kg × R12 = R3,60
   (d) 150 g = 0,15 kg, so 0,15 kg × R12 = R1,80



5. Write each mass in common fraction notation.
   (a) 80.1 kg (b) 0.25 kg
   (c) 0.05 kg (d) 34.87 kg
6. Round off the mass in questions 5(a) and (d) to the nearest kilogram.
7. Convert each mass in question 5 to grams.

1.4 Solving problems about mass and quantity

1. 500 large paper clips have a total mass of 1 kg. Calculate the mass of:
   (a) 50 large paper clips
   (b) 10 large paper clips
   (c) 1 large paper clip.
2. A bag of 30 oranges, all of the same mass, weighs 5 kg. Calculate the mass of:
   (a) 15 oranges
   (b) 5 oranges
   (c) 1 orange.
3. Look back at questions 1 and 2.
   (a) Will all the paper clips have exactly the same mass?
   (b) Will all the oranges have exactly the same mass?
4. At the fresh produce market, you can buy vegetables per kilogram. Onions sell for R12 per kilogram. If a single large onion has a mass of 150 g, what will the following cost?
   (a) 2 000 g of onions (b) 20 onions
   (c) 300 g of onions (d) 1 onion

Sometimes when we calculate we get exact answers. Sometimes our answers are approximations. For example, if four Grade 5 learners have a mass of 140 kg, we cannot be sure that each learner has a mass of exactly 35 kg.

GRADE 6: MATHEMATICS [TERM 3] 219

Notes on questions

Question 6 requires that learners do a lot of very similar calculations. To ensure that learners complete this question, you can divide the class into four groups. Group 1 calculates the grams required at two months. Group 2 calculates the grams required at three months. Group 3 calculates the grams required at four months. Group 4 calculates the grams required at five and six months (it is the same calculation). Each group can write their answers on the board. Let one group check another group's calculations. All learners can then individually calculate the answers to questions 6(b) and (c).

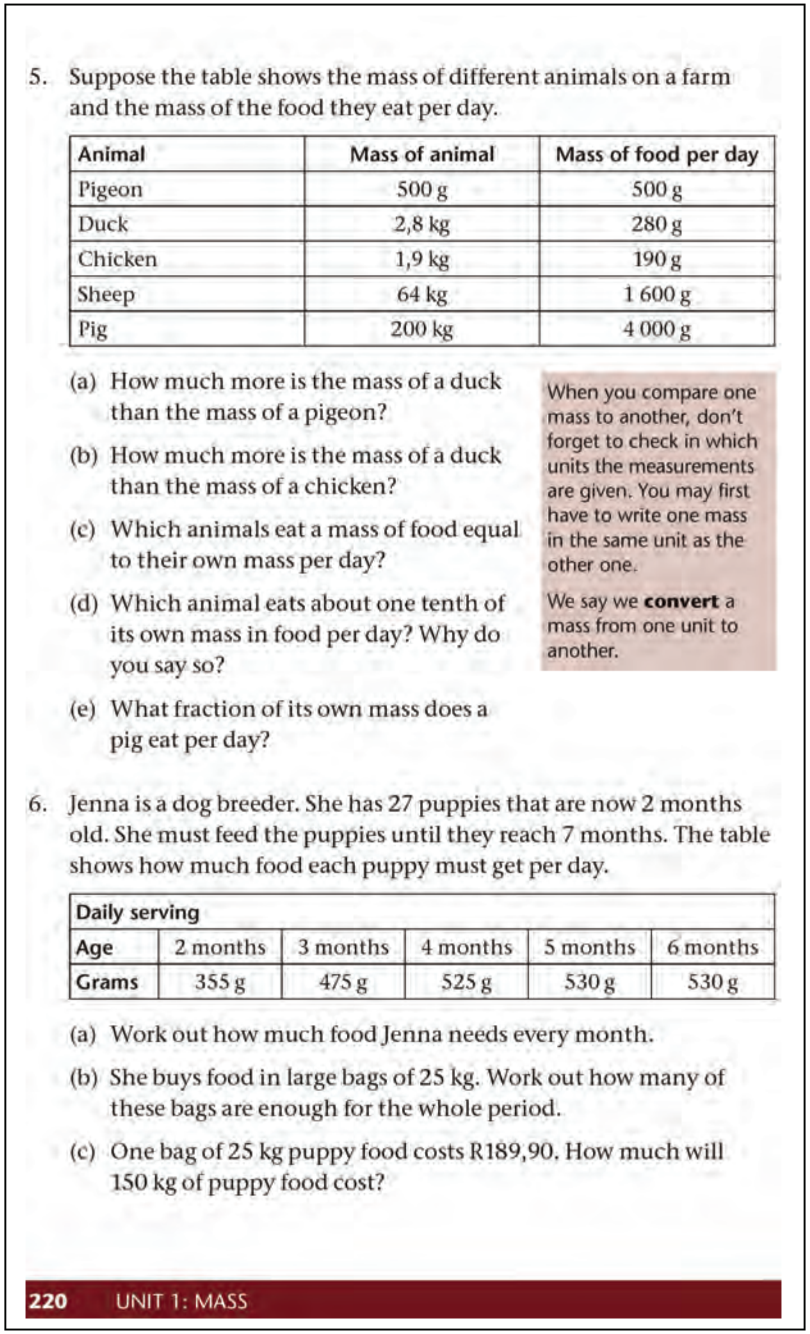
Answers

5. (a) 2,8 kg = 2 800 g, so 2 800 g – 500 g = 2 300 g = 2,3 kg
   (b) 2,8 kg – 1,9 kg = 2 800 g – 1 900 g = 900 g
   (c) A pigeon
   (d) A duck: 280 g × 10 = 2 800 g = 2,8 kg
   A chicken: 190 g × 10 = 1 900 g = 1,9 kg
   (e) $\frac{4}{200} = \frac{2}{100} = \frac{1}{50}$

6.

Daily serving					
Age	2 months	3 months	4 months	5 months	6 months
Grams	355 g	475 g	525 g	530 g	530 g
Grams for all 27 puppies per day	355 g × 27 = 9 585 g	475 g × 27 = 12 825 g	525 g × 27 = 14 175 g	530 g × 27 = 14 310 g	530 g × 27 = 14 310 g
Grams for all 27 puppies per month (assume 30 days)	9 585 g × 30 = 287 550 g	12 825 g × 30 = 384 750 g	14 175 g × 30 = 425 250 g	14 310 g × 30 = 429 300 g	14 310 g × 30 = 429 300 g

(a) Total amount of puppy food needed: 1 956 150 g or 1 956,15 kg
(b) 1 956 150 g ÷ 25 000 g = 78 rem 6 150 g. Jenna needs to buy 79 bags.
(c) 150 kg ÷ 25 kg × R189,90 = R1 139,40



5. Suppose the table shows the mass of different animals on a farm and the mass of the food they eat per day.

Animal	Mass of animal	Mass of food per day
Pigeon	500 g	500 g
Duck	2,8 kg	280 g
Chicken	1,9 kg	190 g
Sheep	64 kg	1 600 g
Pig	200 kg	4 000 g

(a) How much more is the mass of a duck than the mass of a pigeon?
(b) How much more is the mass of a duck than the mass of a chicken?
(c) Which animals eat a mass of food equal to their own mass per day?
(d) Which animal eats about one tenth of its own mass in food per day? Why do you say so?
(e) What fraction of its own mass does a pig eat per day?

When you compare one mass to another, don't forget to check in which units the measurements are given. You may first have to write one mass in the same unit as the other one.

We say we **convert** a mass from one unit to another.

6. Jenna is a dog breeder. She has 27 puppies that are now 2 months old. She must feed the puppies until they reach 7 months. The table shows how much food each puppy must get per day.

Daily serving					
Age	2 months	3 months	4 months	5 months	6 months
Grams	355 g	475 g	525 g	530 g	530 g

(a) Work out how much food Jenna needs every month.
(b) She buys food in large bags of 25 kg. Work out how many of these bags are enough for the whole period.
(c) One bag of 25 kg puppy food costs R189,90. How much will 150 kg of puppy food cost?

220 UNIT 1: MASS

Grade 6 Term 3 Unit 2 Whole numbers

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
2.1 Order, compare and describe big numbers	The magnitude of numbers	221
2.2 Represent 6-digit to 9-digit numbers	Representing numbers with symbols and words	222
2.3 Multiples and factors	Breaking numbers down into factors and building numbers up by multiplying	223

CAPS time allocation	1 hour
CAPS page references	13 to 15 and 262

The questions in Section 2.3 all provide practice in **Mental Mathematics**.

Mathematical background

The spaces that are sometimes used in writing the symbols for large numbers, for example 356 723 852 instead of 356723852, are helpful to make sense of and compare large numbers.

356 723 852

356 millions 723 thousands 852 units

Thinking of large numbers in terms of a number of millions, a number of thousands and a number of units is also reflected in the number names:
Three hundred and fifty-six **million** seven hundred and twenty-three **thousand** and eight hundred and fifty-two.
The millions-thousands-units structure of number names is also reflected in the third row of the table below.

The powers of 10 provide the scaffold within which we conceive of and represent large numbers. This structure can be represented as follows:

1 000 000 000	100 000 000	10 000 000	1 000 000	100 000	10 000	1 000	100	10	1
10 hundred millions	10 ten millions	10 millions	10 hundred thousands	10 ten thousands	10 thousands	10 hundreds	10 tens	10 units	
1 000 millions	100 millions	10 millions	1 000 thousands	100 thousands	10 thousands	1 000 units	100 units	10 units	

2.1 Order, compare and describe big numbers

Teaching guidelines

Once learners have completed question 1, you may put alternative representations like the following on the board. This promotes understanding of the "power of 10" structure that we use to name and represent numbers.

$$\underbrace{10\quad 10\quad 10\quad 10\quad 10\quad 10\quad 10\quad 10\quad 10\quad 10}_{100}$$

$$\underbrace{100\quad 100\quad 100\quad 100\quad 100\quad 100\quad 100\quad 100\quad 100\quad 100}_{1\ 000}$$

Note that questions 2 and 3 involve only some of a wide variety of this kind of questions that can be asked as a mental mathematics activity in class.

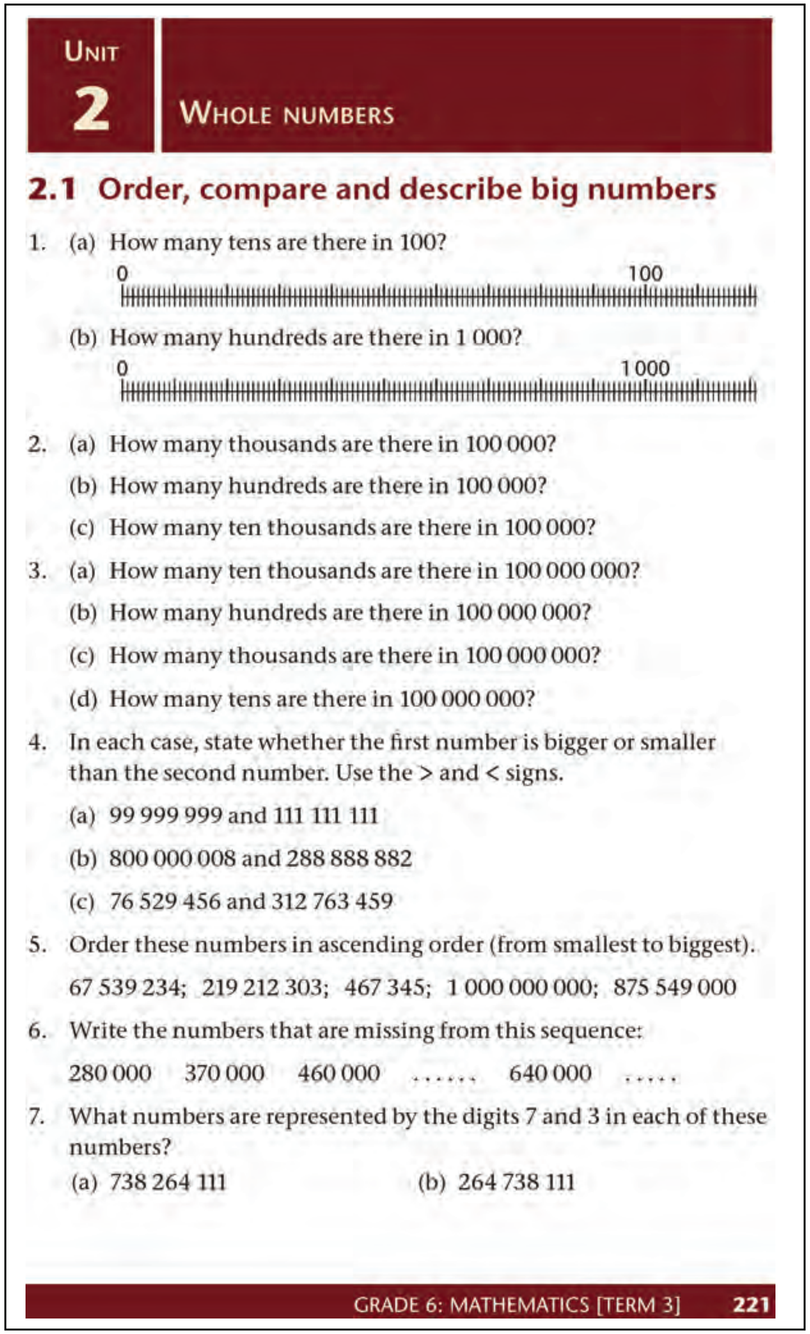
Learners who are challenged by question 4 may be supported by suggesting that they read the numbers aloud, saying the number names. Having to do this will force them to interpret the number symbols in terms of a number of millions, a number of thousands and a number of units. It will also help to have a representation like the following on the board so that that learners can refer to it while doing questions 4 and 5.

356 ¦ 723 ¦ 852

356 millions 723 thousands 852 units

Answers

1. (a) 10 (b) 10
2. (a) 100 (b) 1 000 (c) 10
3. (a) 10 000 (b) 1 000 000 (c) 100 000 (d) 10 000 000
4. (a) 99 999 999 < 111 111 111
   (b) 800 000 008 > 288 888 882
   (c) 76 529 456 < 312 763 459
5. 467 345; 67 539 234; 219 212 303; 875 549 000; 1 000 000 000
6. 280 000 370 000 460 000 550 000 640 000 730 000
7. (a) 738 264 111: 700 000 000 and 30 000 000
   (b) 264 738 111: 700 000 and 30 000



UNIT 2 WHOLE NUMBERS

2.1 Order, compare and describe big numbers

1. (a) How many tens are there in 100?

   (b) How many hundreds are there in 1 000?

2. (a) How many thousands are there in 100 000?
   (b) How many hundreds are there in 100 000?
   (c) How many ten thousands are there in 100 000?
3. (a) How many ten thousands are there in 100 000 000?
   (b) How many hundreds are there in 100 000 000?
   (c) How many thousands are there in 100 000 000?
   (d) How many tens are there in 100 000 000?
4. In each case, state whether the first number is bigger or smaller than the second number. Use the > and < signs.
   (a) 99 999 999 and 111 111 111
   (b) 800 000 008 and 288 888 882
   (c) 76 529 456 and 312 763 459
5. Order these numbers in ascending order (from smallest to biggest).
   67 539 234; 219 212 303; 467 345; 1 000 000 000; 875 549 000
6. Write the numbers that are missing from this sequence:
   280 000 370 000 460 000 ...... 640 000 .....
7. What numbers are represented by the digits 7 and 3 in each of these numbers?
   (a) 738 264 111 (b) 264 738 111

GRADE 6: MATHEMATICS [TERM 3] 221

2.2 Represent 6-digit to 9-digit numbers

Teaching guidelines

Questions 1 to 4 can be used for diagnostic assessment purposes. Monitor learners and engage with learners who need support.

Answers

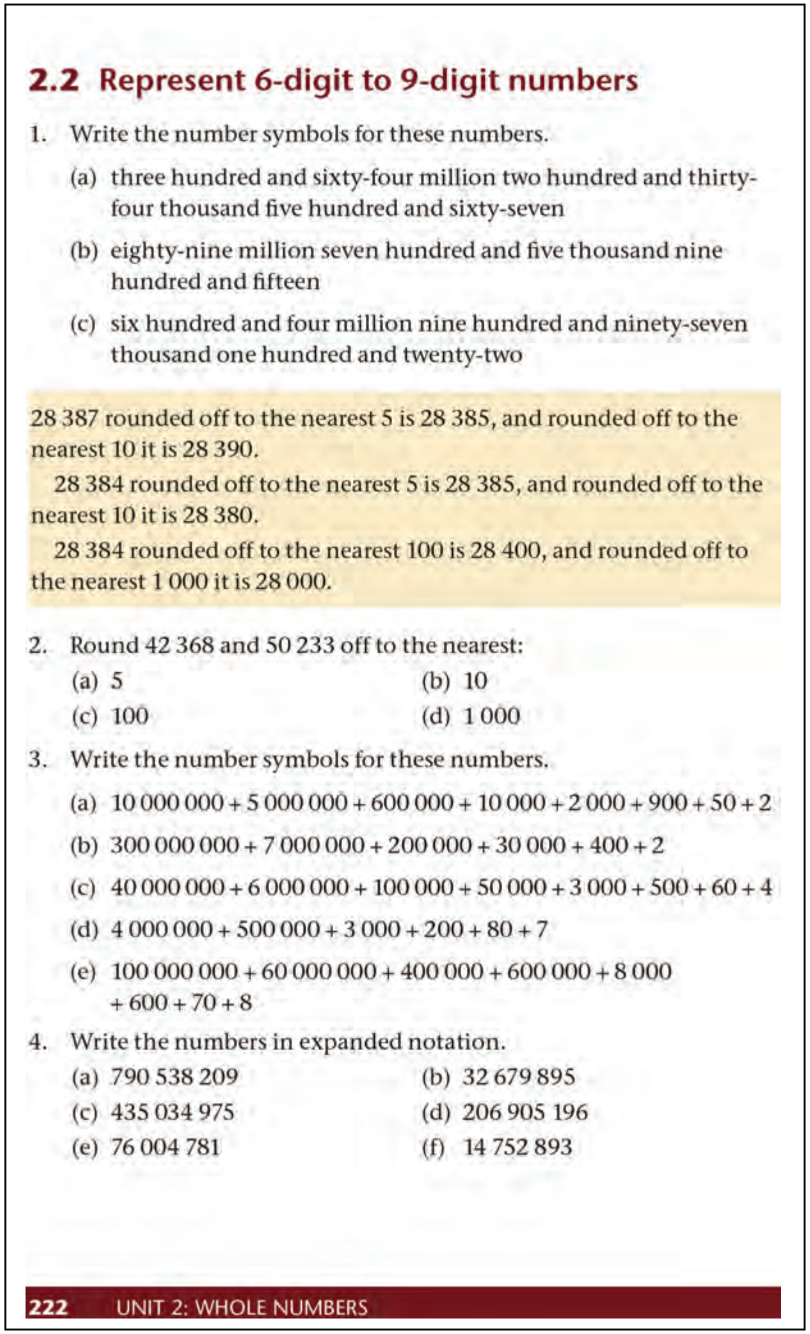
1. (a) 364 234 567 (b) 89 705 915 (c) 604 997 122

2.

Rounded off to the nearest …	(a) 5	(b) 10	(c) 100	(d) 1 000
42 368	42 370	42 370	42 400	42 000
50 233	50 235	50 230	50 200	50 000

3. (a) 15 612 952 (b) 307 230 402 (c) 46 153 564
   (d) 4 503 287 (e) 161 008 678

4. (a) 700 000 000 + 90 000 000 + 500 000 + 30 000 + 8 000 + 200 + 9
   (b) 30 000 000 + 2 000 000 + 600 000 + 70 000 + 9 000 + 800 + 90 + 5
   (c) 400 000 000 + 30 000 000 + 5 000 000 + 30 000 + 4 000 + 900 + 70 + 5
   (d) 200 000 000 + 6 000 000 + 900 000 + 5 000 + 100 + 90 + 6
   (e) 70 000 000 + 6 000 000 + 4 000 + 700 + 80 + 1
   (f) 10 000 000 + 4 000 000 + 700 000 + 50 000 + 2 000 + 800 + 90 + 3



2.2 Represent 6-digit to 9-digit numbers

1. Write the number symbols for these numbers.
   (a) three hundred and sixty-four million two hundred and thirty-four thousand five hundred and sixty-seven
   (b) eighty-nine million seven hundred and five thousand nine hundred and fifteen
   (c) six hundred and four million nine hundred and ninety-seven thousand one hundred and twenty-two

28 387 rounded off to the nearest 5 is 28 385, and rounded off to the nearest 10 it is 28 390.

28 384 rounded off to the nearest 5 is 28 385, and rounded off to the nearest 10 it is 28 380.

28 384 rounded off to the nearest 100 is 28 400, and rounded off to the nearest 1 000 it is 28 000.

2. Round 42 368 and 50 233 off to the nearest:
   (a) 5 (b) 10
   (c) 100 (d) 1 000
3. Write the number symbols for these numbers.
   (a) 10 000 000 + 5 000 000 + 600 000 + 10 000 + 2 000 + 900 + 50 + 2
   (b) 300 000 000 + 7 000 000 + 200 000 + 30 000 + 400 + 2
   (c) 40 000 000 + 6 000 000 + 100 000 + 50 000 + 3 000 + 500 + 60 + 4
   (d) 4 000 000 + 500 000 + 3 000 + 200 + 80 + 7
   (e) 100 000 000 + 60 000 000 + 400 000 + 600 000 + 8 000 + 600 + 70 + 8
4. Write the numbers in expanded notation.
   (a) 790 538 209 (b) 32 679 895
   (c) 435 034 975 (d) 206 905 196
   (e) 76 004 781 (f) 14 752 893

222 UNIT 2: WHOLE NUMBERS

2.3 Multiples and factors

Teaching guidelines

“Factor”, “multiple” and “product” are technical words that do not form part of everyday language, hence learners are very dependent on instruction to learn the meaning of these words. In addition to the information provided in the first shaded passage, you may inform learners that a product is a multiple of each of its factors:

15 is a multiple of 5, 15 is also a multiple of 3

It may also be useful to point out to learners that we can think of a number, for example 72, in two ways:

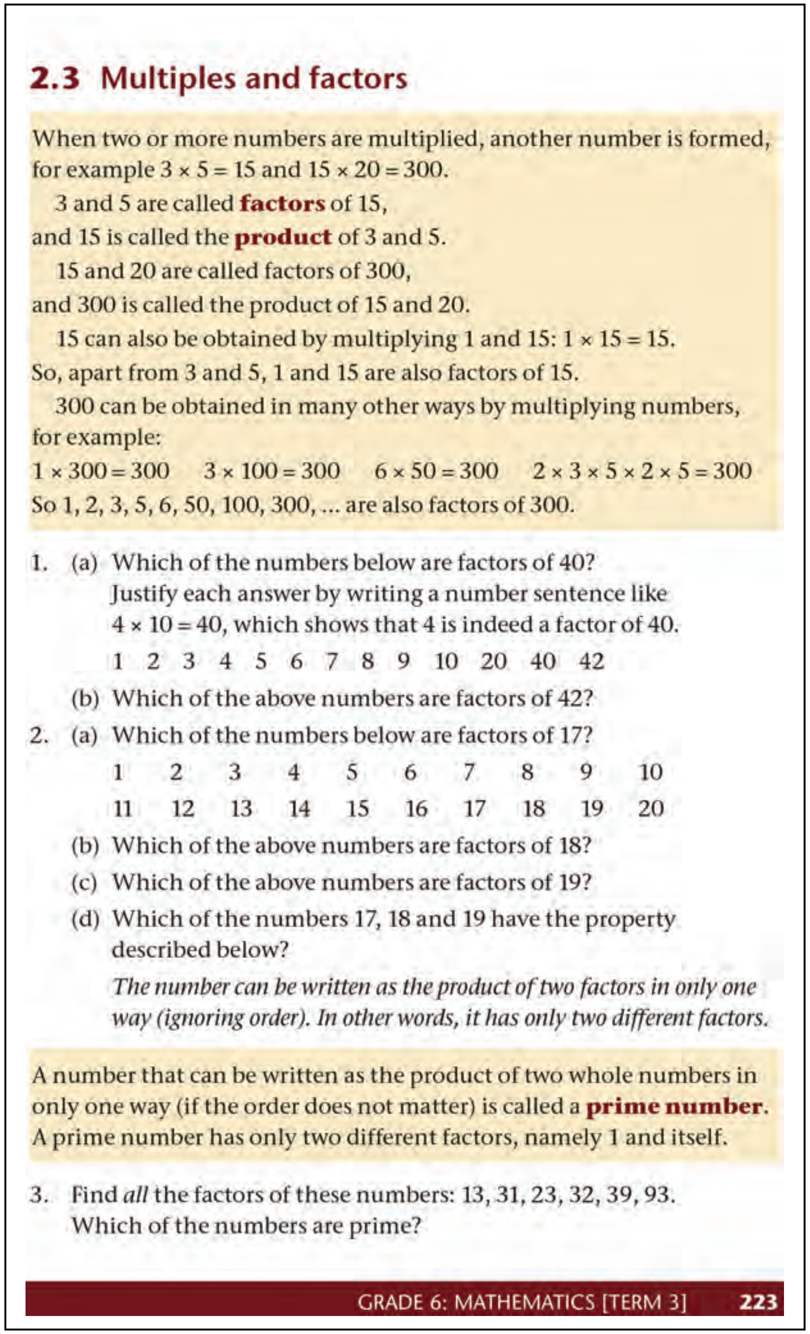
- We can ask how the number can be **built up by adding** other numbers together: 72 = 70 + 2
- We can ask how the number can be **built up by multiplying** other numbers: 72 = 2 × 2 × 2 × 3 × 3

Answers

1. (a) 1 × 40 = 40   2 × 20 = 40   4 × 10 = 40   5 × 8 = 40
8 × 5 = 40   10 × 4 = 40   20 × 2 = 40   40 × 1 = 40
(b) 1; 2; 3; 6; 7; 42

2. (a) 1 and 17
(b) 1; 2; 3; 6; 9; 18
(c) 1 and 19
(d) 17 and 19

3. 13: 1 and 13   31: 1 and 31
23: 1 and 23   32: 1; 2; 4; 8; 16; 32
39: 1; 3; 13; 39   93: 1; 3; 31; 93
The prime numbers are 13; 23; 31.



2.3 Multiples and factors

When two or more numbers are multiplied, another number is formed, for example 3 × 5 = 15 and 15 × 20 = 300.

3 and 5 are called **factors** of 15,
and 15 is called the **product** of 3 and 5.

15 and 20 are called factors of 300,
and 300 is called the product of 15 and 20.

15 can also be obtained by multiplying 1 and 15: 1 × 15 = 15.
So, apart from 3 and 5, 1 and 15 are also factors of 15.

300 can be obtained in many other ways by multiplying numbers, for example:
1 × 300 = 300   3 × 100 = 300   6 × 50 = 300   2 × 3 × 5 × 2 × 5 = 300
So 1, 2, 3, 5, 6, 50, 100, 300, ... are also factors of 300.

1. (a) Which of the numbers below are factors of 40?
Justify each answer by writing a number sentence like 4 × 10 = 40, which shows that 4 is indeed a factor of 40.
1 2 3 4 5 6 7 8 9 10 20 40 42
(b) Which of the above numbers are factors of 42?

2. (a) Which of the numbers below are factors of 17?
1 2 3 4 5 6 7 8 9 10
11 12 13 14 15 16 17 18 19 20
(b) Which of the above numbers are factors of 18?
(c) Which of the above numbers are factors of 19?
(d) Which of the numbers 17, 18 and 19 have the property described below?
The number can be written as the product of two factors in only one way (ignoring order). In other words, it has only two different factors.

A number that can be written as the product of two whole numbers in only one way (if the order does not matter) is called a **prime number**. A prime number has only two different factors, namely 1 and itself.

3. Find *all* the factors of these numbers: 13, 31, 23, 32, 39, 93.
Which of the numbers are prime?

GRADE 6: MATHEMATICS [TERM 3] 223

Grade 6 Term 3 Unit 3 Whole numbers: Addition and subtraction

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
3.1 Revision	Estimating sums and differences, and checking with a calculator	224 to 225
3.2 Addition and subtraction in financial contexts	Doing extensive calculations relating to two realistic situations	226 to 227
3.3 Add and subtract measurements	Doing a variety of calculations with measurements of different physical qualities	228 to 229
3.4 Calculations using a calculator	Solving word problems in a variety of contexts	230

CAPS time allocation	8 hours
CAPS page references	13 to 15 and 262 to 263

Mathematical background

Although this unit does not contain any new mathematical content, the focus is on using addition and subtraction in a variety of practical contexts, including finance and measurement. The word problems in this unit reflect the following meanings of addition and subtraction:

- Adding up different components of a quantity to determine the total, for example: *"John spent R534 on food, R892 on accommodation and R254 on travel. How much did he spend in total?"*
- Finding the missing component in a combination of quantities, for example: *"Mary bought juice, vegetables and bread for R286. She spent R89 on juice and R132 on vegetables. How much did she spend on bread?"*
- Increasing a quantity, for example: *"In January 2016, there were 10 438 taxpayers registered in a municipality. During 2016, 8 786 new taxpayers registered. How many taxpayers were registered by the end of 2016?"*
- Decreasing a quantity, for example: *"At 08:00 one morning there was 34 879 ℓ of water in a reservoir. During the day, 12 341 ℓ flowed out of the reservoir. How much water was left in the reservoir?"*
- Finding the difference between two quantities, for example: *"There are 3,46 million voters in province A and 8,26 million voters in province B. How many more voters are there in province B than in province A?"*
- Establishing a shortfall, for example: *"A municipality has R9,82 million available for building a new library and the estimated cost is R16,23 million. How much money must still be found to pay for the library?"*

Resources

Calculators

3.1 Revision

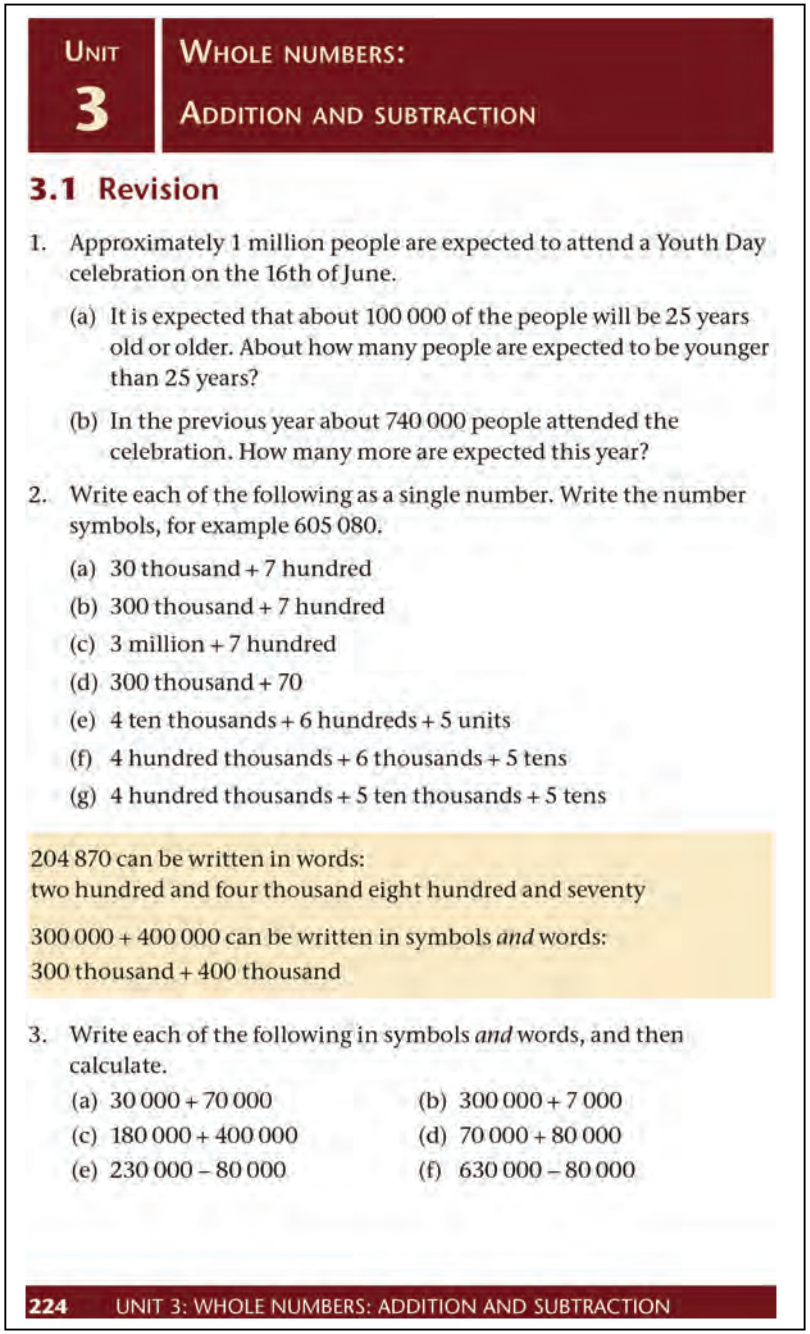
Teaching guidelines

Questions 1(a) and (b) demonstrate two different meanings of subtraction. The first meaning is finding the missing part of a total and the second is establishing by how much a quantity has increased. The purpose of the question is not to assess learners' reading ability but to provide opportunities for learners to identify that subtraction is appropriate in such situations. Therefore, the best way to approach this is to explain the situation to learners by "telling the story", writing the numbers on the board and then asking learners to calculate and call out the answers.

Questions 2 and 3 consist of mental mathematics where learners do simple addition with large numbers.

Answers

1. (a) 900 000 (b) 260 000
2. (a) 30 700 (b) 300 700 (c) 3 000 700 (d) 300 070
   (e) 40 605 (f) 406 050 (g) 450 050
3. (a) 30 thousand + 70 thousand = 100 thousand
   (b) 300 thousand + 7 thousand = 307 thousand
   (c) 180 thousand + 400 thousand = 580 thousand
   (d) 70 thousand + 80 thousand = 150 thousand
   (e) 230 thousand – 80 thousand = 150 thousand
   (f) 630 thousand – 80 thousand = 550 thousand



UNIT 3

WHOLE NUMBERS: ADDITION AND SUBTRACTION

3.1 Revision

1. Approximately 1 million people are expected to attend a Youth Day celebration on the 16th of June.
   (a) It is expected that about 100 000 of the people will be 25 years old or older. About how many people are expected to be younger than 25 years?
   (b) In the previous year about 740 000 people attended the celebration. How many more are expected this year?
2. Write each of the following as a single number. Write the number symbols, for example 605 080.
   (a) 30 thousand + 7 hundred
   (b) 300 thousand + 7 hundred
   (c) 3 million + 7 hundred
   (d) 300 thousand + 70
   (e) 4 ten thousands + 6 hundreds + 5 units
   (f) 4 hundred thousands + 6 thousands + 5 tens
   (g) 4 hundred thousands + 5 ten thousands + 5 tens

204 870 can be written in words:
two hundred and four thousand eight hundred and seventy

300 000 + 400 000 can be written in symbols *and* words:
300 thousand + 400 thousand

3. Write each of the following in symbols *and* words, and then calculate.
   (a) 30 000 + 70 000 (b) 300 000 + 7 000
   (c) 180 000 + 400 000 (d) 70 000 + 80 000
   (e) 230 000 – 80 000 (f) 630 000 – 80 000

224 UNIT 3: WHOLE NUMBERS: ADDITION AND SUBTRACTION

Teaching guidelines

Remind learners that in question 4 they are required to first estimate their answers to the nearest ten thousand. Let them make their estimates and write them down. Then get them to do the actual calculations with a calculator, and write the answers next to the estimates. Finally, let them calculate the differences between their estimates and the actual answers without using a calculator.

Notes on questions

In question 5 the focus is on the accuracy of learners' calculations without them using a calculator. In cases where learners' answers with or without the calculator differ, they should redo the calculations without the calculator until they get it right.

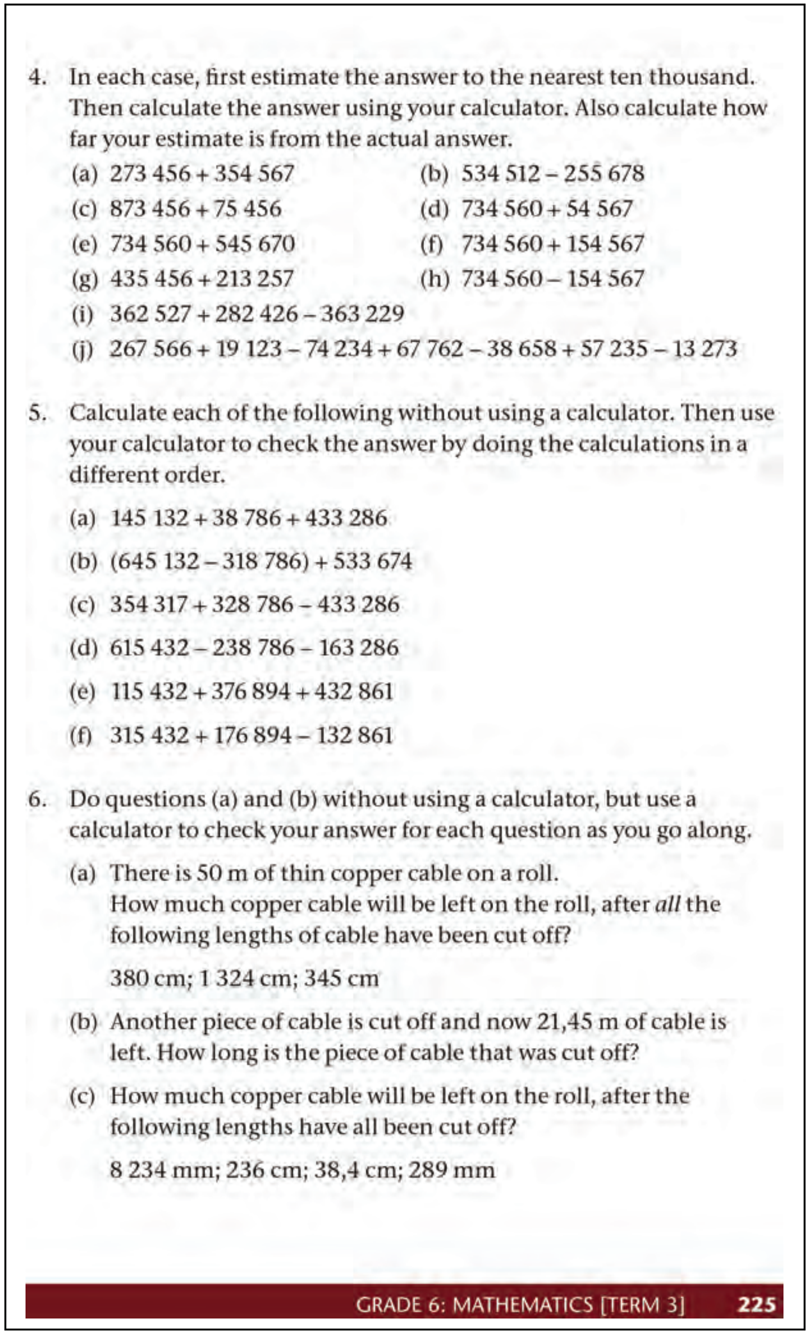
Question 6 can be given to learners as a homework project.

Answers

4.	Estimated:	Calculated:	Difference:
(a)	620 000	628 023	8 023
(b)	270 000	278 834	8 834
(c)	950 000	948 912	1 088
(d)	780 000	789 127	9 127
(e)	1 280 000	1 280 230	230
(f)	880 000	889 127	9 127
(g)	650 000	648 713	1 287
(h)	580 000	579 993	7
(i)	280 000	281 724	1 724
(j)	300 000	285 521	14 479

5. (a) 617 204 (b) 860 020 (c) 249 817
(d) 213 360 (e) 925 187 (f) 359 465

6. (a) 2 951 cm or 29,51 m
(b) 806 cm or 8,06 m
(c) 1 018,3 cm or 10,183 m or 10 183 mm



4. In each case, first estimate the answer to the nearest ten thousand. Then calculate the answer using your calculator. Also calculate how far your estimate is from the actual answer.
   - (a) 273 456 + 354 567
   - (b) 534 512 – 255 678
   - (c) 873 456 + 75 456
   - (d) 734 560 + 54 567
   - (e) 734 560 + 545 670
   - (f) 734 560 + 154 567
   - (g) 435 456 + 213 257
   - (h) 734 560 – 154 567
   - (i) 362 527 + 282 426 – 363 229
   - (j) 267 566 + 19 123 – 74 234 + 67 762 – 38 658 + 57 235 – 13 273

5. Calculate each of the following without using a calculator. Then use your calculator to check the answer by doing the calculations in a different order.
   - (a) 145 132 + 38 786 + 433 286
   - (b) (645 132 – 318 786) + 533 674
   - (c) 354 317 + 328 786 – 433 286
   - (d) 615 432 – 238 786 – 163 286
   - (e) 115 432 + 376 894 + 432 861
   - (f) 315 432 + 176 894 – 132 861

6. Do questions (a) and (b) without using a calculator, but use a calculator to check your answer for each question as you go along.
   - (a) There is 50 m of thin copper cable on a roll. How much copper cable will be left on the roll, after *all* the following lengths of cable have been cut off?
     380 cm; 1 324 cm; 345 cm
   - (b) Another piece of cable is cut off and now 21,45 m of cable is left. How long is the piece of cable that was cut off?
   - (c) How much copper cable will be left on the roll, after the following lengths have all been cut off?
     8 234 mm; 236 cm; 38,4 cm; 289 mm

GRADE 6: MATHEMATICS [TERM 3] 225

3.2 Addition and subtraction in financial contexts

Resources

Calculators

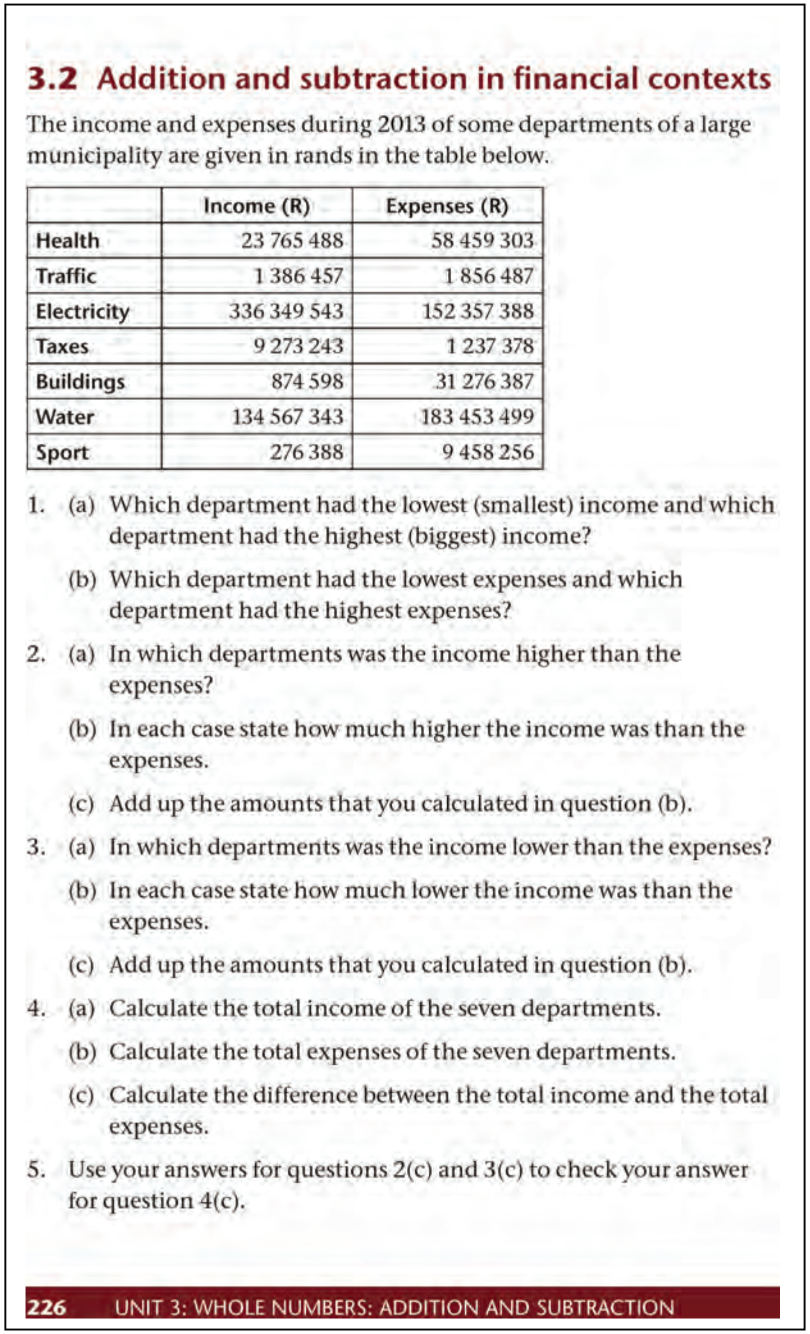
Teaching guidelines

Explain to learners that all the questions in this section relate to the same context. Let them read the page first and then take some time to conduct a class discussion around the structure of this activity as a whole. Let learners rewrite the table in their exercise books and add two columns in which they can write the answers for questions 2(b) and 3(b). Ask learners to think about how they will do question 5. Only then let them start to work out and record their answers.

Learners may use calculators for all the work.

Answers

1. (a) Lowest income: Sport Highest income: Electricity
   (b) Lowest expenses: Taxes Highest expenses: Water
2. (a) Electricity and Taxes
   (b) Electricity: R183 992 155
   Taxes: R8 035 865
   (c) R192 028 020
3. (a) Health; Traffic; Buildings; Water and Sport
   (b) Health: R34 693 815
   Traffic: R470 030
   Buildings: R30 401 789
   Water: R48 886 156
   Sport: R9 181 868
   (c) R123 633 658
4. (a) R506 493 060
   (b) R438 098 698
   (c) R68 394 362
5. R192 028 020 – R123 633 658 = R68 394 362



3.2 Addition and subtraction in financial contexts

The income and expenses during 2013 of some departments of a large municipality are given in rands in the table below.

	Income (R)	Expenses (R)
Health	23 765 488	58 459 303
Traffic	1 386 457	1 856 487
Electricity	336 349 543	152 357 388
Taxes	9 273 243	1 237 378
Buildings	874 598	31 276 387
Water	134 567 343	183 453 499
Sport	276 388	9 458 256

1. (a) Which department had the lowest (smallest) income and which department had the highest (biggest) income?
   (b) Which department had the lowest expenses and which department had the highest expenses?
2. (a) In which departments was the income higher than the expenses?
   (b) In each case state how much higher the income was than the expenses.
   (c) Add up the amounts that you calculated in question (b).
3. (a) In which departments was the income lower than the expenses?
   (b) In each case state how much lower the income was than the expenses.
   (c) Add up the amounts that you calculated in question (b).
4. (a) Calculate the total income of the seven departments.
   (b) Calculate the total expenses of the seven departments.
   (c) Calculate the difference between the total income and the total expenses.
5. Use your answers for questions 2(c) and 3(c) to check your answer for question 4(c).

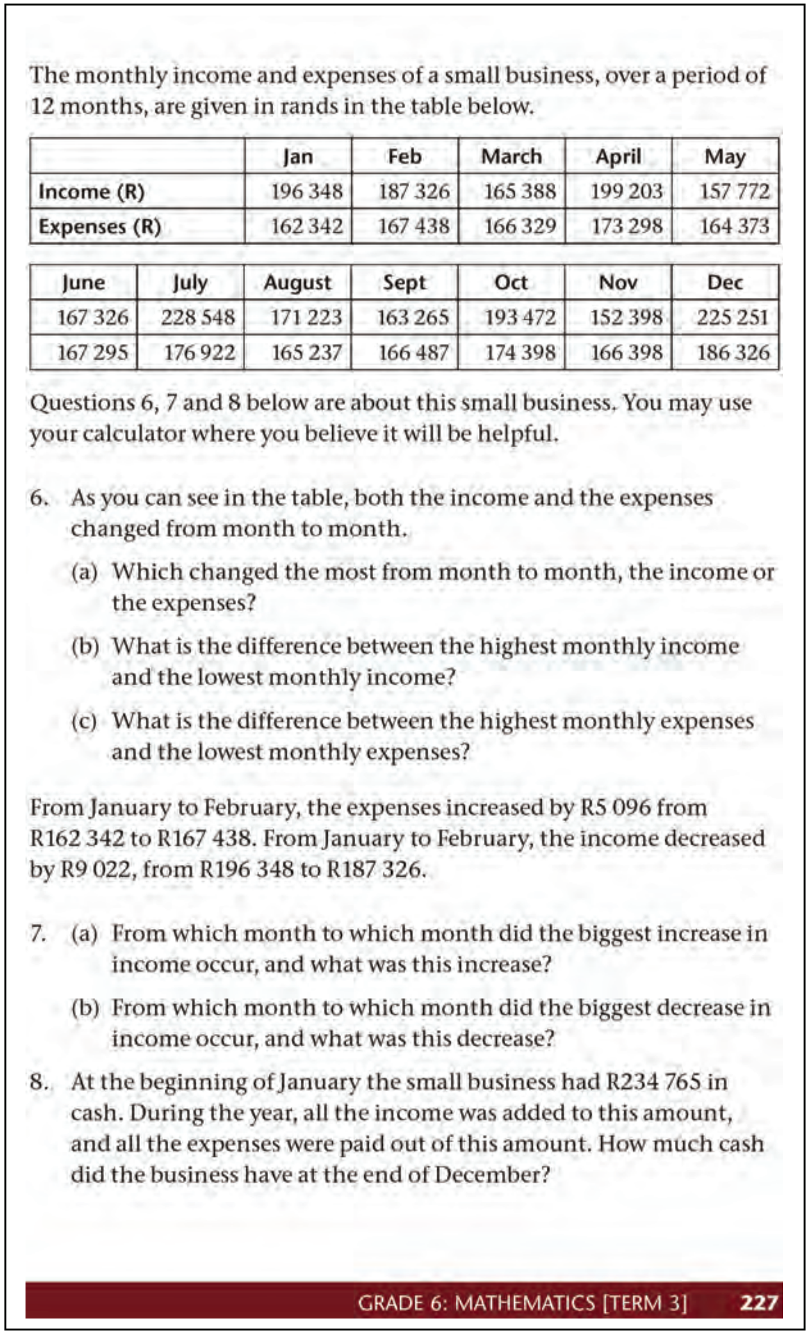
226 UNIT 3: WHOLE NUMBERS: ADDITION AND SUBTRACTION

Teaching guidelines

The questions on this page are similar to the questions on page 226 in that they all relate to the same context. You may decide to give these questions as a homework project, possibly stretching over a few days. Learners can hand the "project" in on loose sheets of paper for assessment purposes.

Answers

6. (a) The income changed the most.
   (b) R228 548 – R152 398 = R76 150
   (c) R186 326 – R162 342 = R23 984
7. (a) From November to December; an increase of R72 853
   (b) From July to August; a decrease of R57 325
8. R234 765 + R2 207 520 – R2 036 843 = R405 442



The monthly income and expenses of a small business, over a period of 12 months, are given in rands in the table below.

	Jan	Feb	March	April	May	June	July	August	Sept	Oct	Nov	Dec
Income (R)	196 348	187 326	165 388	199 203	157 772	167 326	228 548	171 223	163 265	193 472	152 398	225 251
Expenses (R)	162 342	167 438	166 329	173 298	164 373	167 295	176 922	165 237	166 487	174 398	166 398	186 326

Questions 6, 7 and 8 below are about this small business. You may use your calculator where you believe it will be helpful.

6. As you can see in the table, both the income and the expenses changed from month to month.
   (a) Which changed the most from month to month, the income or the expenses?
   (b) What is the difference between the highest monthly income and the lowest monthly income?
   (c) What is the difference between the highest monthly expenses and the lowest monthly expenses?

From January to February, the expenses increased by R5 096 from R162 342 to R167 438. From January to February, the income decreased by R9 022, from R196 348 to R187 326.

7. (a) From which month to which month did the biggest increase in income occur, and what was this increase?
   (b) From which month to which month did the biggest decrease in income occur, and what was this decrease?
8. At the beginning of January the small business had R234 765 in cash. During the year, all the income was added to this amount, and all the expenses were paid out of this amount. How much cash did the business have at the end of December?

3.3 Add and subtract measurements

Resources

Calculators

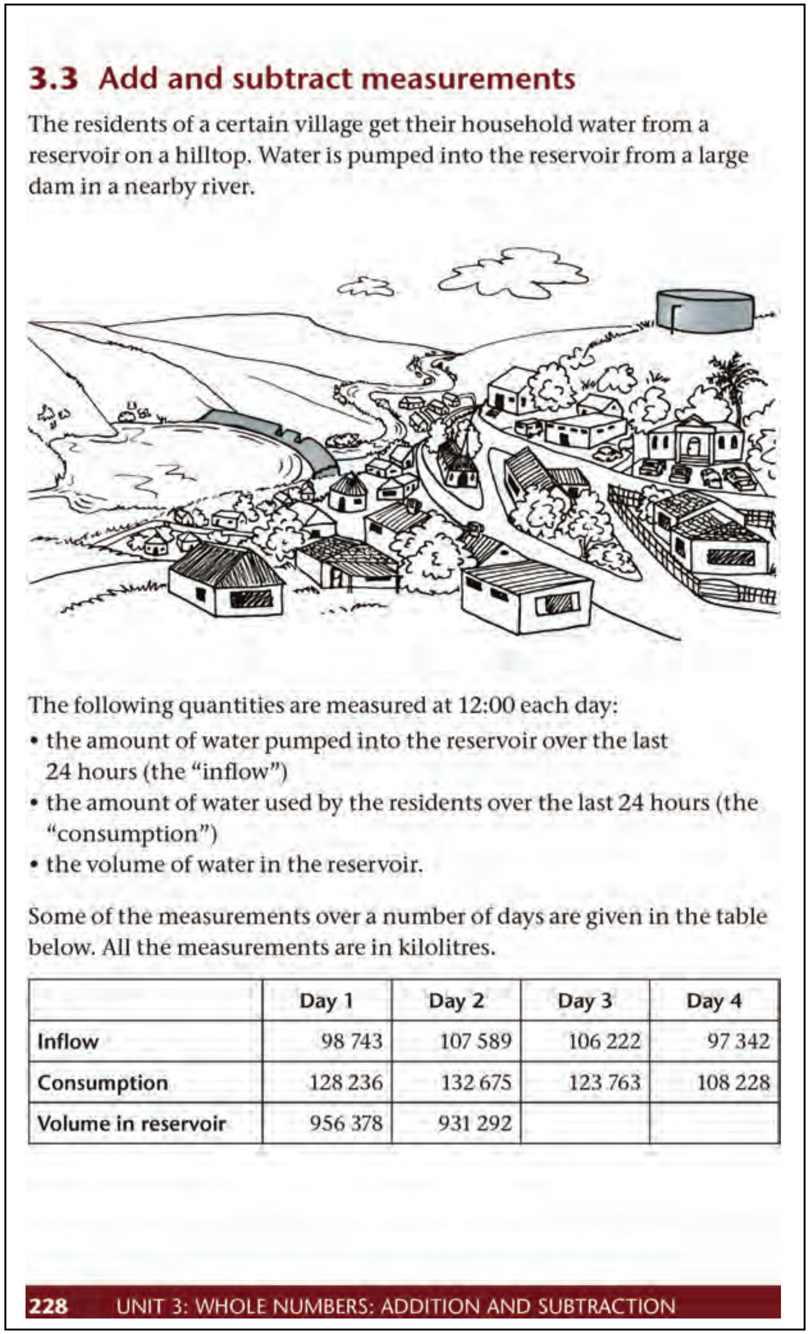
Mathematical background

In this extended activity relating to water supply and usage in a village, learners engage with three related variable quantities, i.e. the inflow of water into a reservoir, the outflow, and the water level in the reservoir.

Teaching guidelines

Begin this section by holding a class discussion on the context of the situation discussed in the Learner Book. Then provide learners with an overview of what they will have to do in the different questions. Note that question 5 is of a different nature than the preceding questions, i.e. it is about the issue of whether or not water may be leaking from the system. Water is a very scarce resource in South Africa and any loss, especially due to leakage, is a major problem that impacts service delivery in many municipal areas.

Learners may use calculators for all the work.



3.3 Add and subtract measurements

The residents of a certain village get their household water from a reservoir on a hilltop. Water is pumped into the reservoir from a large dam in a nearby river.

The following quantities are measured at 12:00 each day:

- the amount of water pumped into the reservoir over the last 24 hours (the "inflow")
- the amount of water used by the residents over the last 24 hours (the "consumption")
- the volume of water in the reservoir.

Some of the measurements over a number of days are given in the table below. All the measurements are in kilolitres.

	Day 1	Day 2	Day 3	Day 4
Inflow	98 743	107 589	106 222	97 342
Consumption	128 236	132 675	123 763	108 228
Volume in reservoir	956 378	931 292		

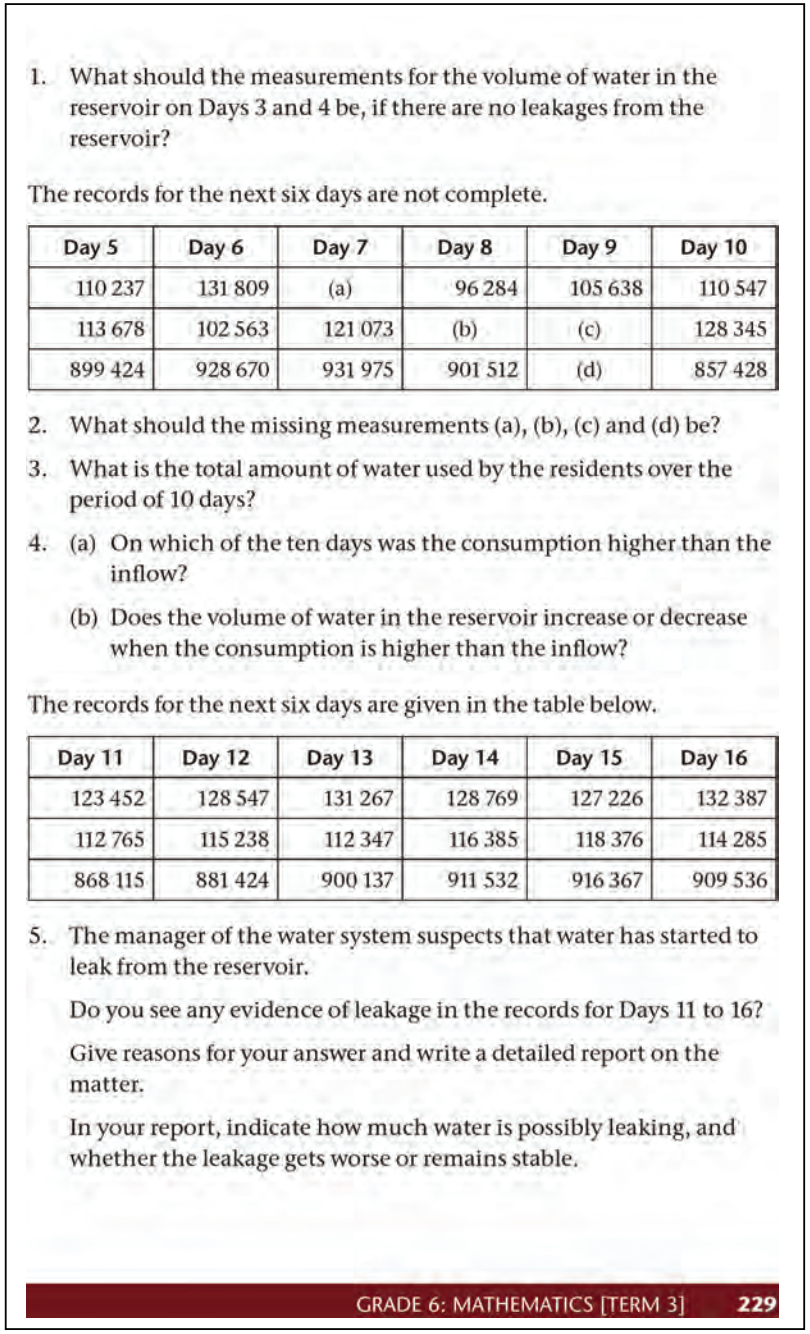
Teaching guidelines

Ensure that learners understand that the tables are continuations of the table on the previous page. The first row represents the inflow of water, the second row represents the outflow and the third row represents the volume in the reservoir.

Answers

1. Day 3: 913 751 kl; Day 4: 902 865 kl
2. (a) 124 378 kl
   (b) 126 747 kl
   (c) 131 924 kl
   (d) 875 226 kl
3. 1 217 232 kl
4. (a) Day 1, 2, 3, 4, 5, 8, 9 and 10
   (b) The volume of water decreases.
5. Learners write their own reports on the situation. It is important that they refer to the numbers in the table in their report to justify their reasoning. Making a table like the one below will be an excellent response. The leakage starts on Day 13 and clearly gets worse as time progresses.

	Day 10	Day 11	Day 12	Day 13	Day 14	Day 15	Day 16
Inflow		123 452	128 547	131 267	128 769	127 226	132 387
Outflow		112 765	115 238	112 347	116 385	118 376	114 285
Volume	857428	868 115	881 424	900 137	911 532	916 367	909 536
Volume if no leakage		868 115	881 424	900 344	912 521	920 382	934 469
Total leakage		0	0	207	989	4 015	24 933
Leakage on day				207	782	3 026	20 918



1. What should the measurements for the volume of water in the reservoir on Days 3 and 4 be, if there are no leakages from the reservoir?

The records for the next six days are not complete.

Day 5	Day 6	Day 7	Day 8	Day 9	Day 10
110 237	131 809	(a)	96 284	105 638	110 547
113 678	102 563	121 073	(b)	(c)	128 345
899 424	928 670	931 975	901 512	(d)	857 428

2. What should the missing measurements (a), (b), (c) and (d) be?
3. What is the total amount of water used by the residents over the period of 10 days?
4. (a) On which of the ten days was the consumption higher than the inflow?
   (b) Does the volume of water in the reservoir increase or decrease when the consumption is higher than the inflow?

The records for the next six days are given in the table below.

Day 11	Day 12	Day 13	Day 14	Day 15	Day 16
123 452	128 547	131 267	128 769	127 226	132 387
112 765	115 238	112 347	116 385	118 376	114 285
868 115	881 424	900 137	911 532	916 367	909 536

5. The manager of the water system suspects that water has started to leak from the reservoir.

   Do you see any evidence of leakage in the records for Days 11 to 16?

   Give reasons for your answer and write a detailed report on the matter.

   In your report, indicate how much water is possibly leaking, and whether the leakage gets worse or remains stable.

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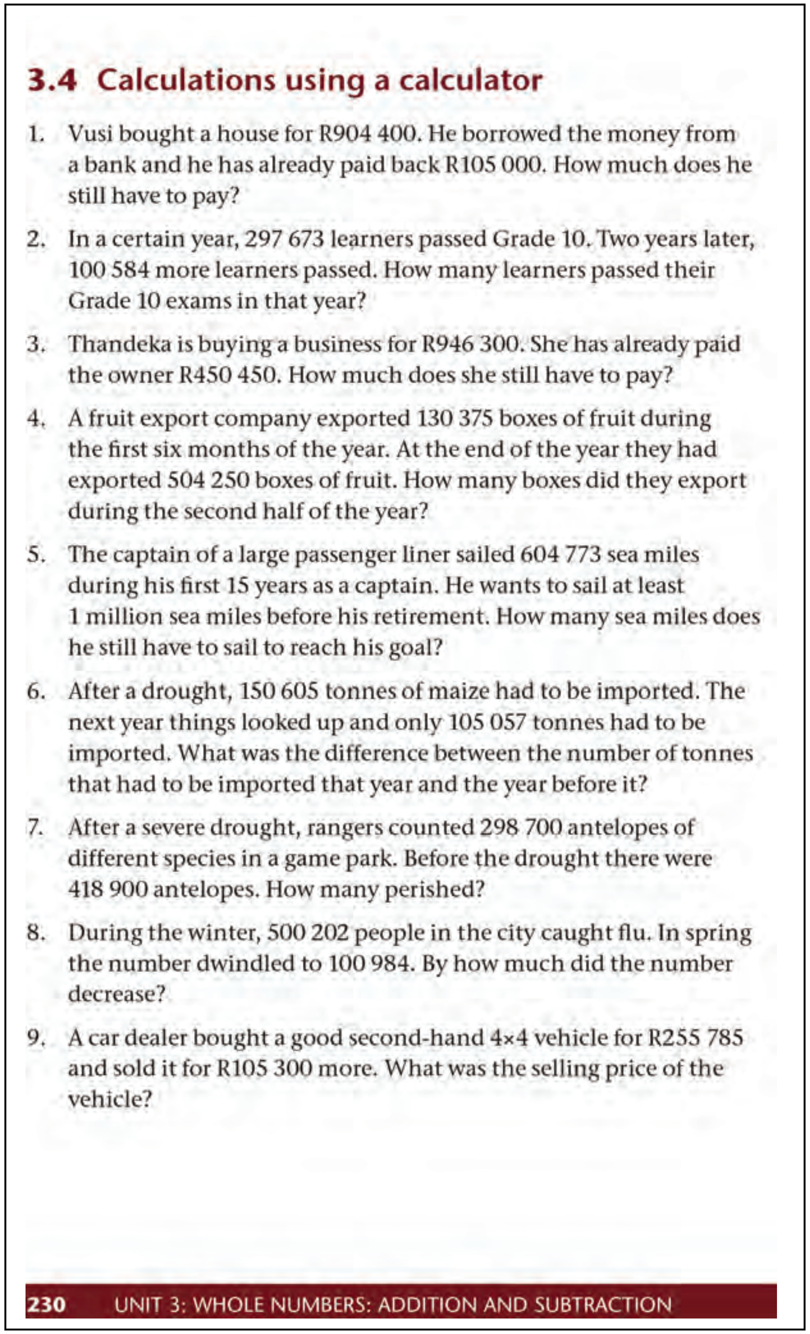
3.4 Calculations using a calculator

Teaching guidelines

Challenge learners to read these questions themselves without your assistance. Suggest that they make estimates before they do any calculations.

Answers

1. R799 400
2. 398 257 learners
3. R495 850
4. 373 875 boxes
5. 395 227 sea miles
6. 45 548 tonnes
7. 120 200 antelopes
8. 399 218 people
9. R361 085



3.4 Calculations using a calculator

1. Vusi bought a house for R904 400. He borrowed the money from a bank and he has already paid back R105 000. How much does he still have to pay?
2. In a certain year, 297 673 learners passed Grade 10. Two years later, 100 584 more learners passed. How many learners passed their Grade 10 exams in that year?
3. Thandeka is buying a business for R946 300. She has already paid the owner R450 450. How much does she still have to pay?
4. A fruit export company exported 130 375 boxes of fruit during the first six months of the year. At the end of the year they had exported 504 250 boxes of fruit. How many boxes did they export during the second half of the year?
5. The captain of a large passenger liner sailed 604 773 sea miles during his first 15 years as a captain. He wants to sail at least 1 million sea miles before his retirement. How many sea miles does he still have to sail to reach his goal?
6. After a drought, 150 605 tonnes of maize had to be imported. The next year things looked up and only 105 057 tonnes had to be imported. What was the difference between the number of tonnes that had to be imported that year and the year before it?
7. After a severe drought, rangers counted 298 700 antelopes of different species in a game park. Before the drought there were 418 900 antelopes. How many perished?
8. During the winter, 500 202 people in the city caught flu. In spring the number dwindled to 100 984. By how much did the number decrease?
9. A car dealer bought a good second-hand 4×4 vehicle for R255 785 and sold it for R105 300 more. What was the selling price of the vehicle?

230 UNIT 3: WHOLE NUMBERS: ADDITION AND SUBTRACTION

Grade 6 Term 3 Unit 4 Viewing objects

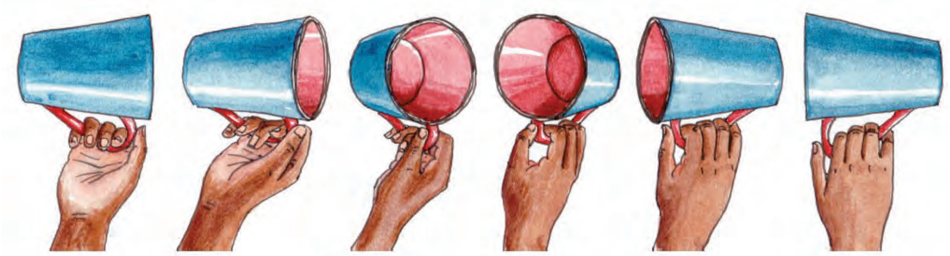
Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
4.1 Different views of the same object	Seeing goal posts from different positions on a soccer/hockey field	231
4.2 Different views of a collection of objects	Seeing a group of four objects on a table top from different positions around the table	232
4.3 Different views of a stack of cubes	Seeing a stack of identical cubes from different positions	233
4.4 Different views of composite objects	Seeing a composite object from different positions	234 to 235
4.5 Different views of more stacks of cubes	Seeing a group of five stacks of cubes on a table top from different positions	236

CAPS time allocation	3 hours
CAPS page references	23 and 263

Mathematical background

This unit is about taking careful notice of how the same object (simple or composite) or collection of objects can look very different when viewed from different positions. This awareness is important to developing spatial sense of three-dimensional objects. It is also important when one has to draw a three-dimensional object, especially if the object is not a simple one. One will then draw the object as seen from a number of different positions. Together the drawings become a useful tool to understand the total spatial form of the object. Such drawings are routinely used in the technical fields (e.g. civil and mechanical engineering) during the design process.

Allow learners to proceed at their own pace through the sections. Some learners may progress quickly and finish all five sections within 3 hours. Other learners may progress quite slowly and only complete the first three sections.



Resources

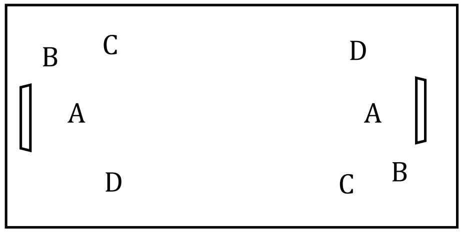
A range of objects such as cups, glasses, plastic bottles, books, small boxes etc. that can be used for practical work as suggested in Section 4.2

4.1 Different views of the same object

Teaching guidelines

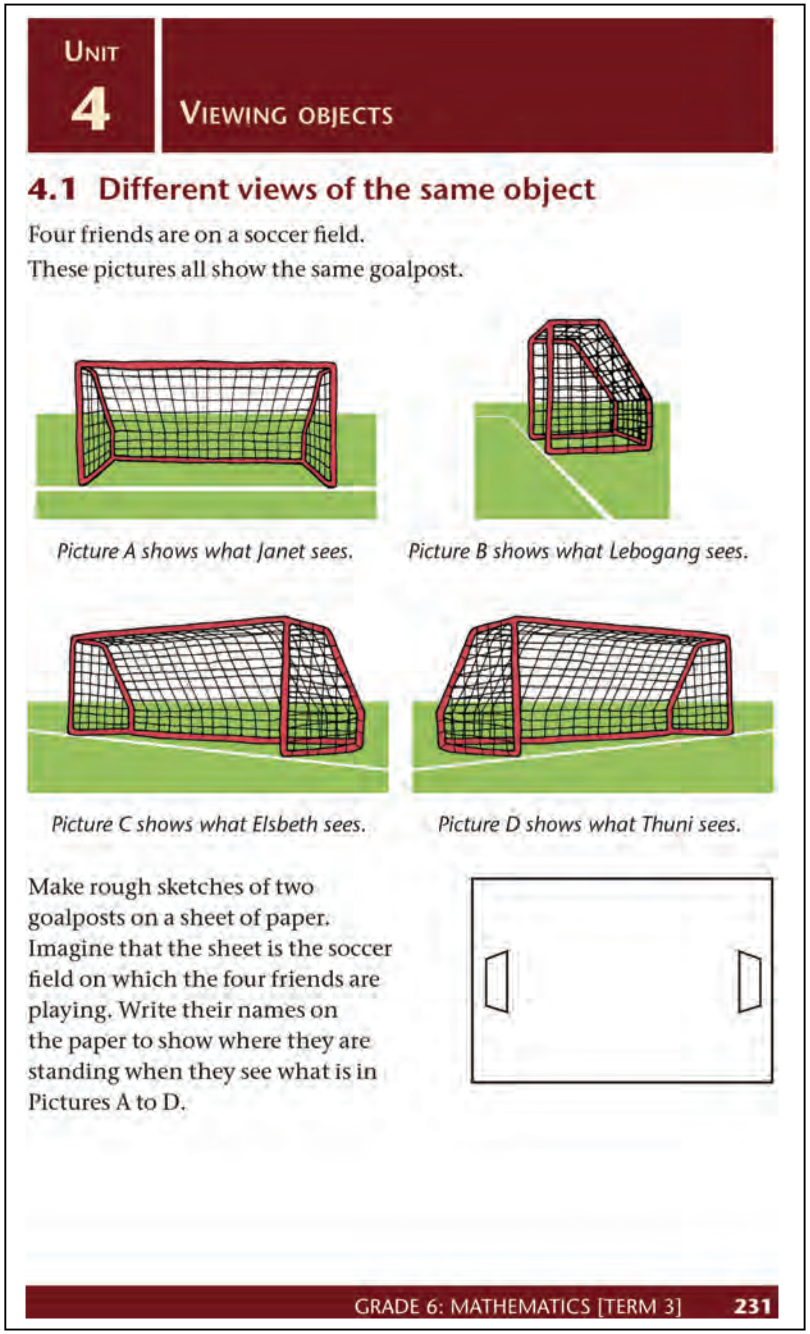
Some learners may find this activity very challenging. You could suggest that they make the drawing, then imagine the sheet of paper to be a soccer or hockey field on which they are playing. They should then ask themselves where they need to be on the field to see the posts as in Picture A. Once they have figured out where on the field they should be to see the posts as in one picture, they will find it easier to figure out the positions for the other pictures.

Answers



Additional questions

Some learners may finish this activity quite quickly while others may take a long time. Learners who complete it quickly may be asked to make neat drawings of the red frame of the posts in the different positions.



UNIT 4

VIEWING OBJECTS

4.1 Different views of the same object

Four friends are on a soccer field.

These pictures all show the same goalpost.

Picture A shows what Janet sees.

Picture B shows what Lebogang sees.

Picture C shows what Elsbeth sees.

Picture D shows what Thuni sees.

Make rough sketches of two goalposts on a sheet of paper. Imagine that the sheet is the soccer field on which the four friends are playing. Write their names on the paper to show where they are standing when they see what is in Pictures A to D.

GRADE 6: MATHEMATICS [TERM 3] 231

4.2 Different views of a collection of objects

Mathematical notes

When groups of objects are viewed, all or parts of some objects will be obscured by the objects closer to the viewer. We only see the parts of the individual objects that lie in front, and only see the front parts of objects that lie along the lines of sight from our eyes at our viewing position.

Teaching guidelines

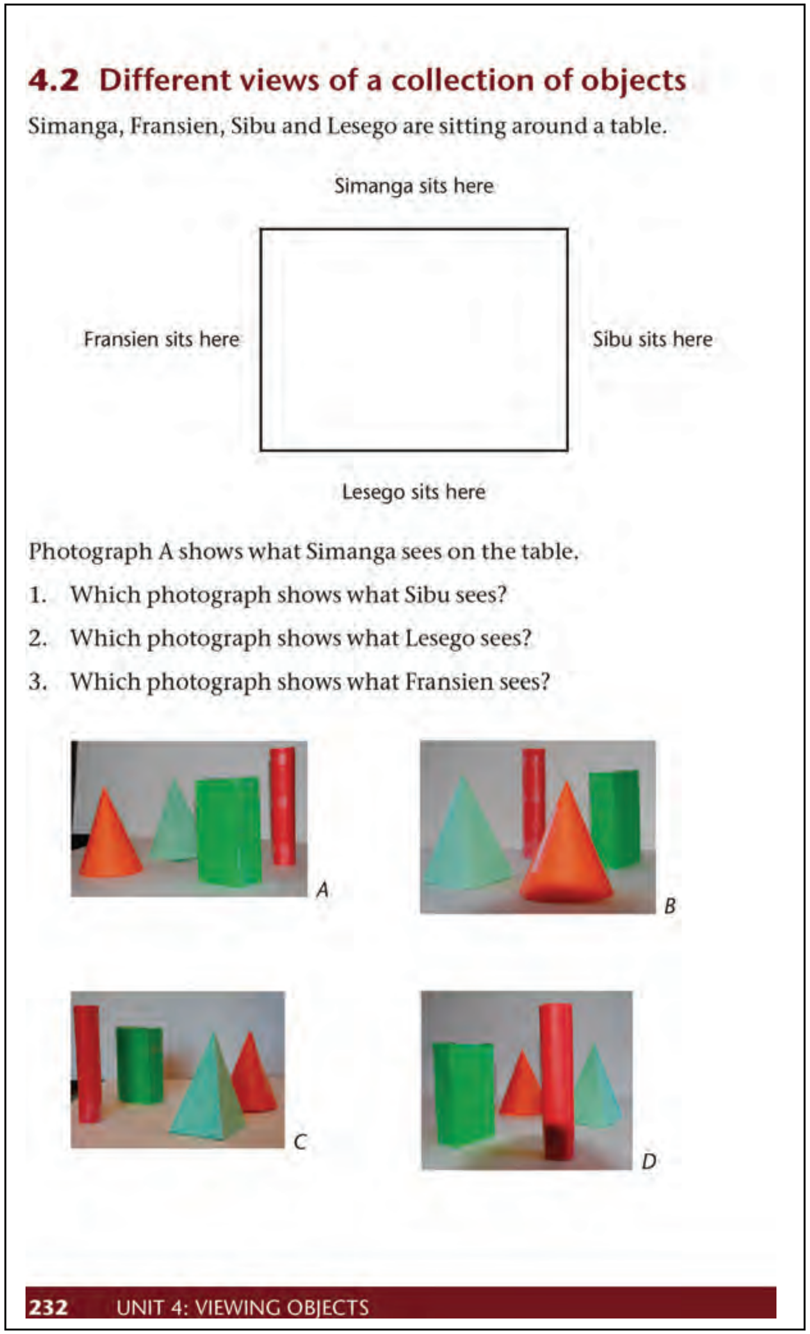
The activity in this section is challenging. It lends itself to a two-step approach. First allow learners to grapple with the challenge on their own. As individual learners find the answers, set them up in small groups with others who have finished and let them discuss how they reasoned out their responses.

Possible misconceptions

Some learners may be seriously challenged by the activity. It may help them to experiment with four actual objects, for example a bottle, a cup, a small box and a book. They should try to arrange the objects in positions similar to those shown in the four photographs.

Answers

1. Photograph B
2. Photograph C
3. Photograph D



4.2 Different views of a collection of objects

Simanga, Fransien, Sibu and Lesego are sitting around a table.

Simanga sits here

Fransien sits here

Sibu sits here

Lesego sits here

Photograph A shows what Simanga sees on the table.

1. Which photograph shows what Sibu sees?
2. Which photograph shows what Lesego sees?
3. Which photograph shows what Fransien sees?

232 UNIT 4: VIEWING OBJECTS

4.3 Different views of a stack of cubes

Mathematical notes

In this case some edges and vertices of some cubes are visible from one position, but obscured from other viewing positions.

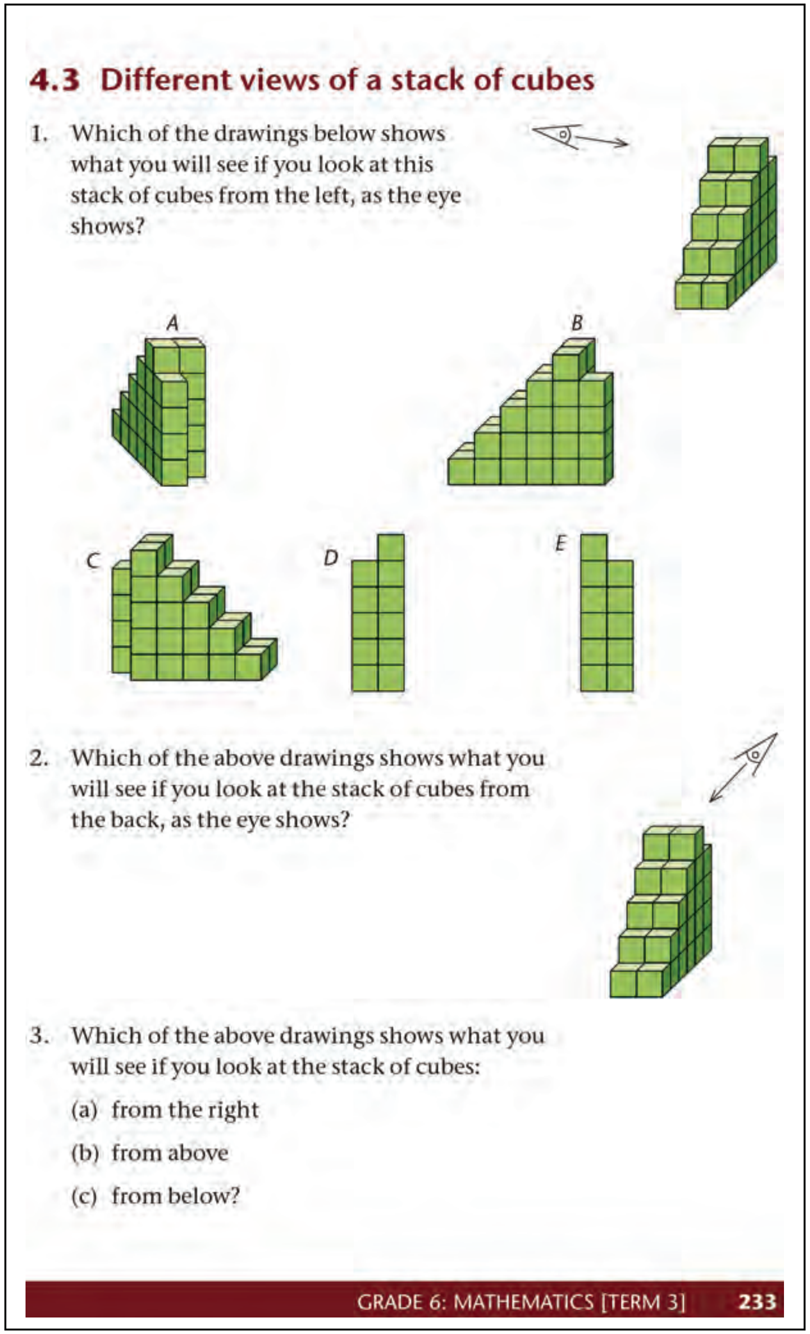
Teaching guidelines

Learners who do not manage this activity may find Sections 4.4 and 4.5 easier. Allow them to do these sections first and then return to Section 4.3.

If learners still do not manage to do Section 4.3, give them cubes so they can build the stack and look at it as indicated by the drawing.

Answers

1. Drawing C
2. Drawing A
3. (a) Drawing B
   (b) Drawing D
   (c) Drawing E



4.3 Different views of a stack of cubes

1. Which of the drawings below shows what you will see if you look at this stack of cubes from the left, as the eye shows?

2. Which of the above drawings shows what you will see if you look at the stack of cubes from the back, as the eye shows?

3. Which of the above drawings shows what you will see if you look at the stack of cubes:
   (a) from the right
   (b) from above
   (c) from below?

GRADE 6: MATHEMATICS [TERM 3] 233

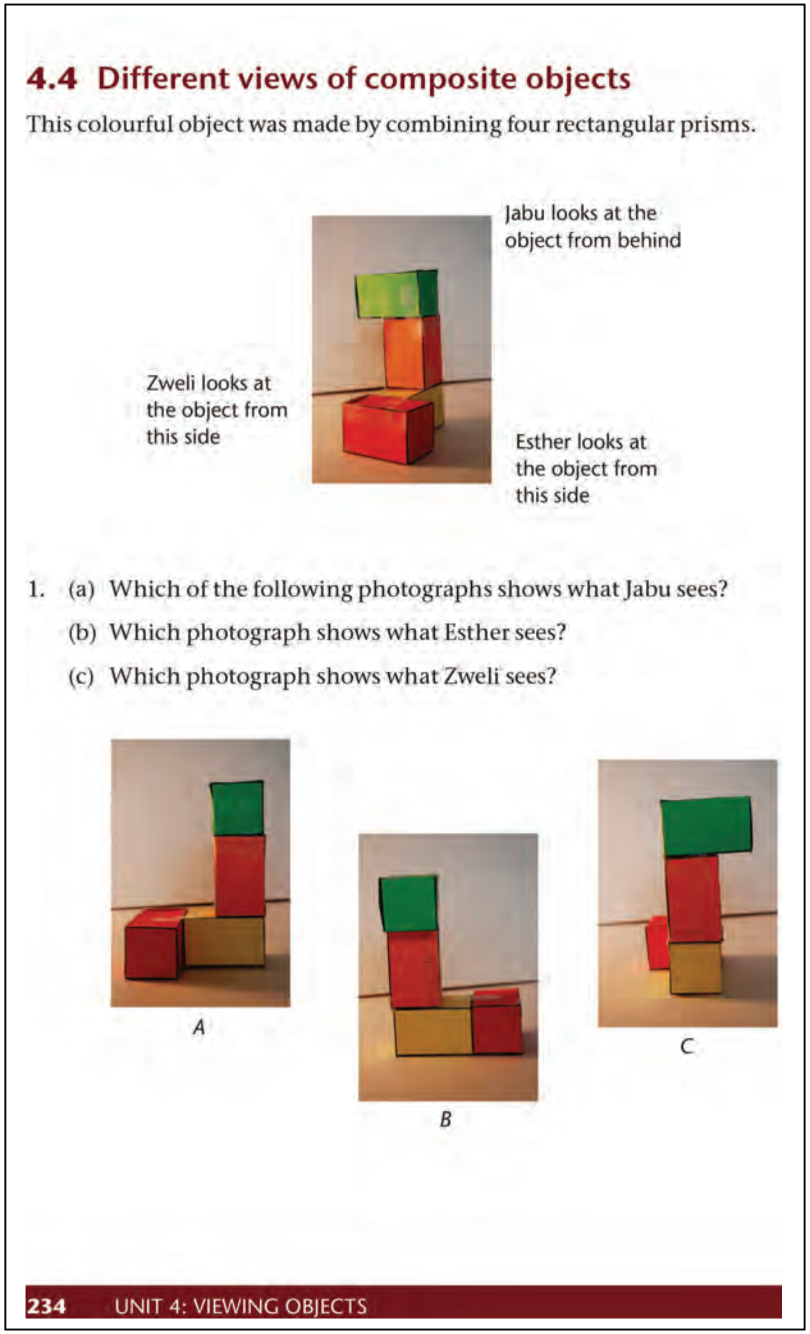
4.4 Different views of composite objects

Mathematical notes

Composite objects are just simple objects put together (stuck or attached) to form bigger objects. We can imagine an irregular 3-D object to be a composite of two or more simple objects (e.g. an L-shaped prism can be imagined to be a composite of two rectangular prisms).

Answers

1. (a) Photograph C
   (b) Photograph A
   (c) Photograph B



4.4 Different views of composite objects

This colourful object was made by combining four rectangular prisms.

Jabu looks at the object from behind

Zweli looks at the object from this side

Esther looks at the object from this side

1. (a) Which of the following photographs shows what Jabu sees?
   (b) Which photograph shows what Esther sees?
   (c) Which photograph shows what Zweli sees?

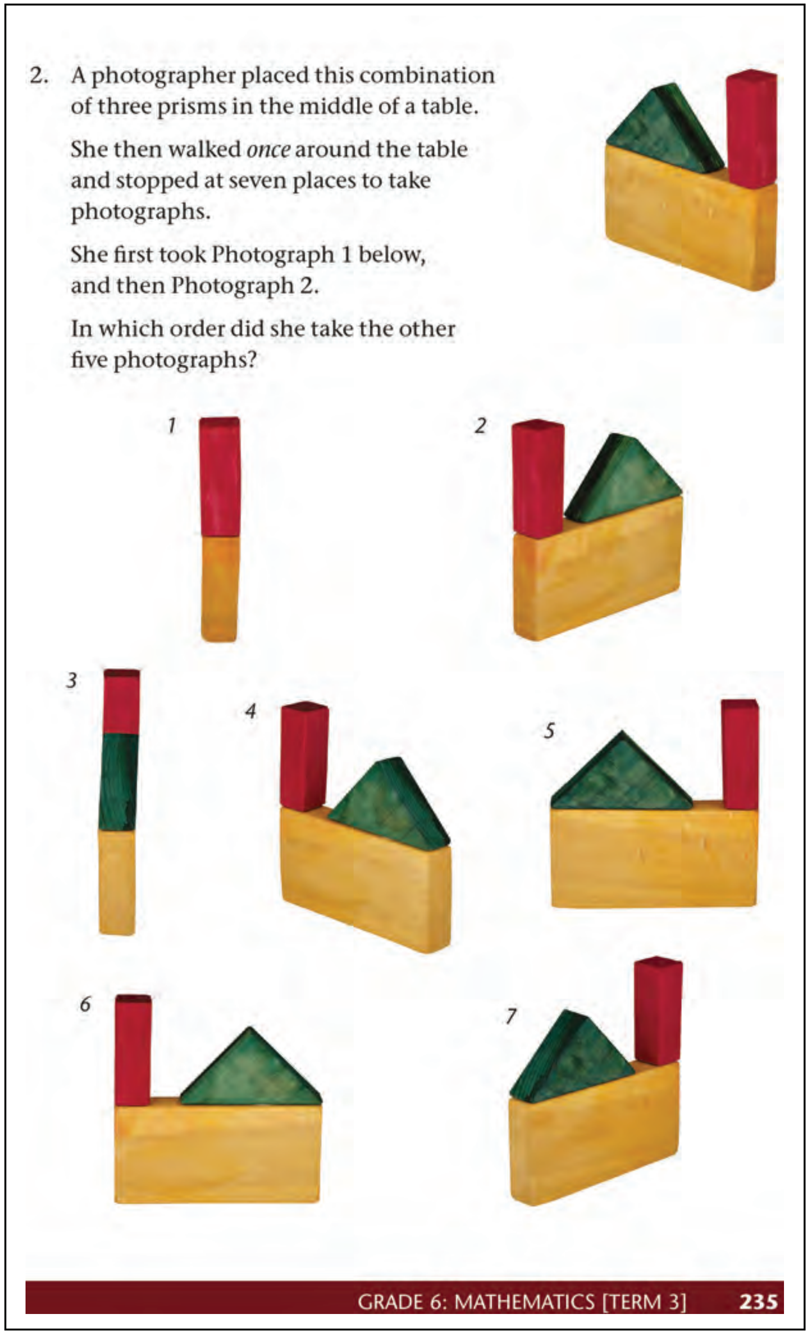
A

B

C

Answers

2. She first took 1, then 2, followed by 6, 4, 3, 7, 5.



2. A photographer placed this combination of three prisms in the middle of a table.

   She then walked *once* around the table and stopped at seven places to take photographs.

   She first took Photograph 1 below, and then Photograph 2.

   In which order did she take the other five photographs?

GRADE 6: MATHEMATICS [TERM 3] 235

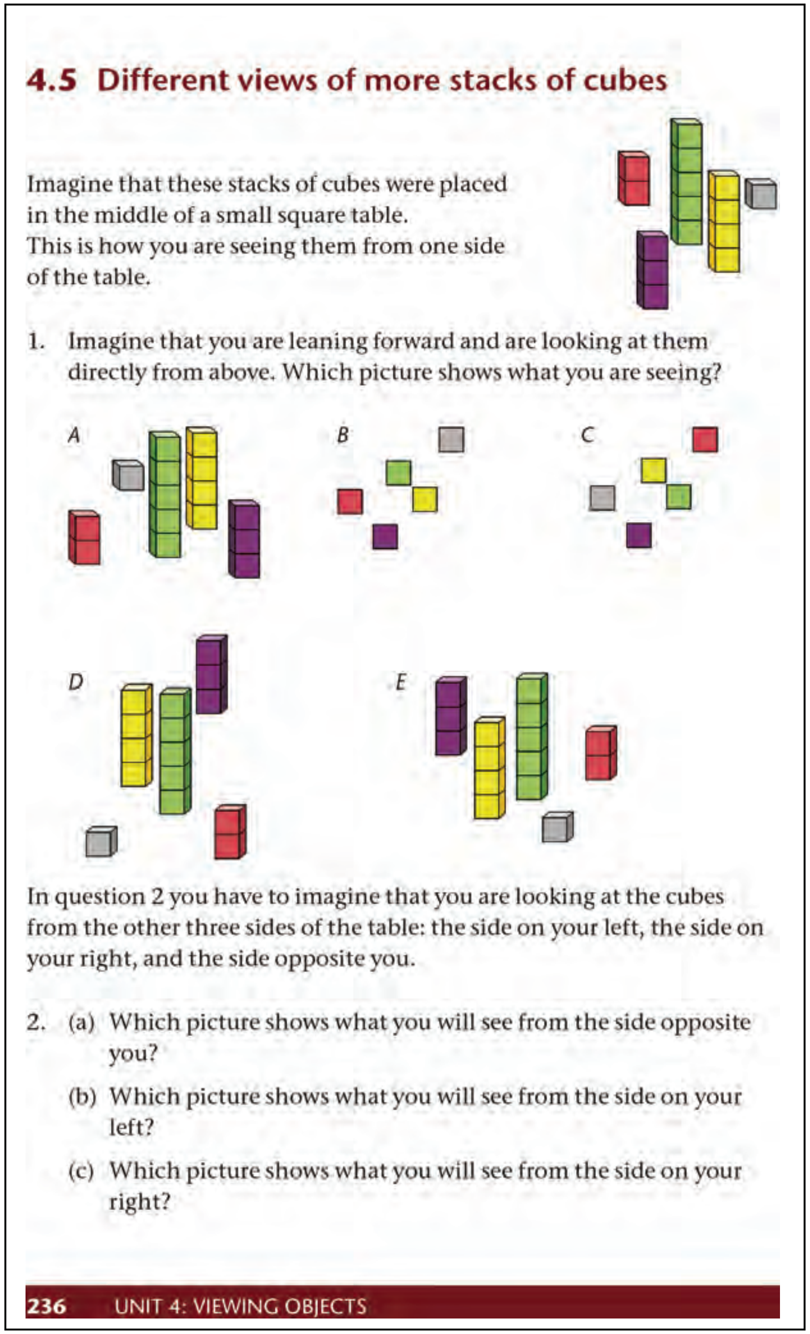
4.5 Different views of more stacks of cubes

Teaching guidelines

This activity is very similar to the activities in Sections 4.3 and 4.4.

Answers

1. Picture B
2. (a) Picture D
   (b) Picture A
   (c) Picture E



4.5 Different views of more stacks of cubes

Imagine that these stacks of cubes were placed in the middle of a small square table. This is how you are seeing them from one side of the table.

1. Imagine that you are leaning forward and are looking at them directly from above. Which picture shows what you are seeing?

In question 2 you have to imagine that you are looking at the cubes from the other three sides of the table: the side on your left, the side on your right, and the side opposite you.

2. (a) Which picture shows what you will see from the side opposite you?
   (b) Which picture shows what you will see from the side on your left?
   (c) Which picture shows what you will see from the side on your right?

236 UNIT 4: VIEWING OBJECTS

Grade 6 Term 3 Unit 5 Properties of two-dimensional shapes

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
5.1 Some revision	Some revision of closed shapes, looking at sides and angles	237
5.2 Polygons	Comparing different polygons and then focusing on hexagons	238 to 240
5.3 Drawing circles and patterns in circles	Introducing a very important tool in geometry: the pair of compasses	241 to 243
5.4 Patterns with circles	Exploring circles drawn in a regular grid	244 to 246

CAPS time allocation	4 hours
CAPS page references	21 to 22 and 264

Mathematical background

This unit begins with revision that involves identifying and naming polygons and angles. It then focuses on regular hexagons and other important polygons. Learners dissect regular hexagons in various ways and uncover “hidden” polygons in the hexagons. Finally learners work with pairs of compasses: drawing both circles and patterns with intersecting circles.

Resources

Tracing paper

Compasses

5.1 Some revision

Teaching guidelines

This section can be used to assess whether learners are able to identify straight and curved sides in two-dimensional figures and whether they can identify right angles and reflex angles.

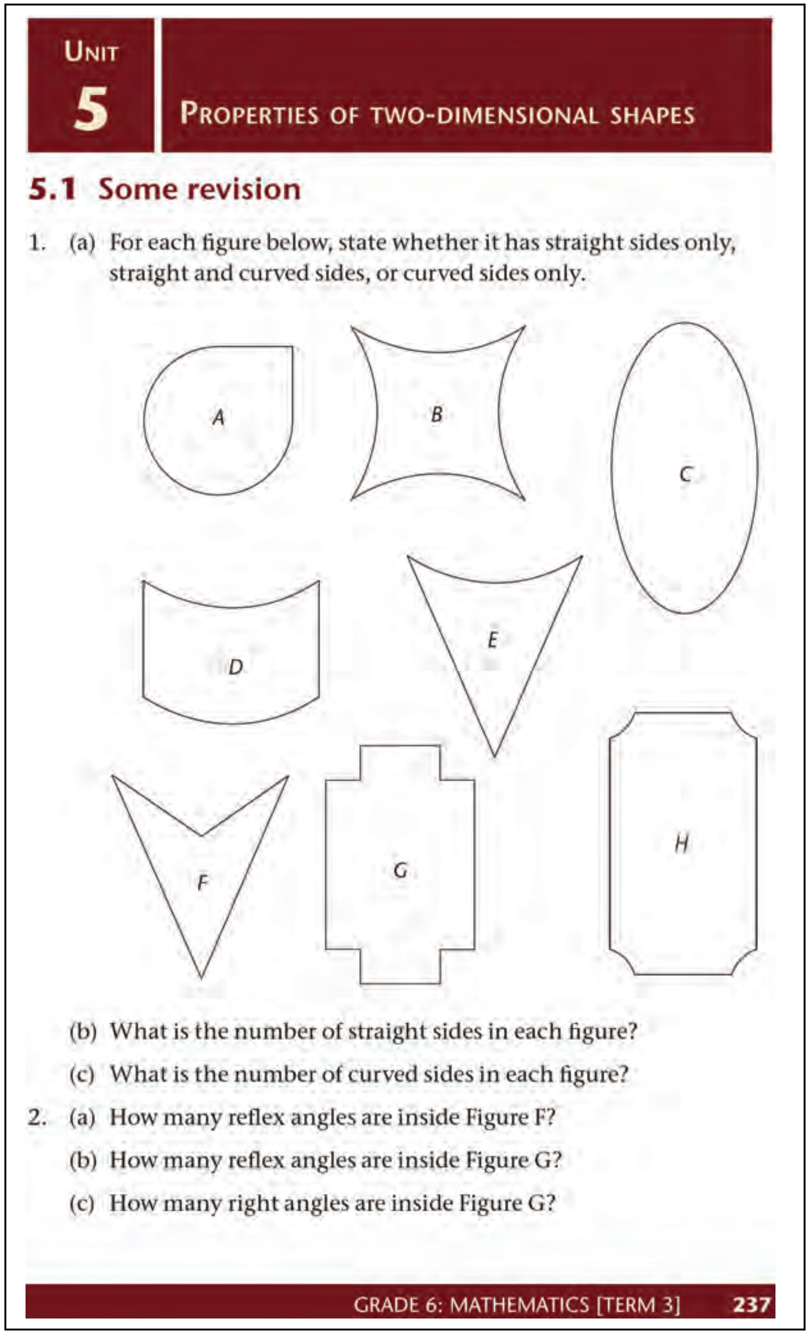
While learners are working you could draw copies of the figures on the board. These will be useful when learners present their answers. In question 2 learners could show which angles are reflex angles in Figures F and G, and which angles are right angles in Figure G.

Answers

1. (a) Figure A – straight and curved
Figure B – curved
Figure C – curved
Figure D – straight and curved
Figure E – straight and curved
Figure F – straight
Figure G – straight
Figure H – straight and curved

(b)	A: 2	B: 0	C: 0	D: 2	E: 2	F: 4	G: 12	H: 4
(c)	A: 1	B: 4	C: 1	D: 2	E: 1	F: 0	G: 0	H: 4

2. (a) 1 (b) 4 (c) 8



UNIT 5

PROPERTIES OF TWO-DIMENSIONAL SHAPES

5.1 Some revision

1. (a) For each figure below, state whether it has straight sides only, straight and curved sides, or curved sides only.

(b) What is the number of straight sides in each figure?

(c) What is the number of curved sides in each figure?

2. (a) How many reflex angles are inside Figure F?

(b) How many reflex angles are inside Figure G?

(c) How many right angles are inside Figure G?

5.2 Polygons

Mathematical notes

After a brief revision of the polygons, the focus quickly shifts to regular hexagons. Connecting selected, or all, the vertices of a hexagon divides it into a number of smaller triangles, quadrilaterals, pentagons, hexagons and more (depending on the choice of lines drawn).

Teaching guidelines

You can use question 1 to assess learners' ability to identify polygons, and question 2 to assess whether learners can identify right angles. In question 2 learners can use a right-angle template to assess which angles are right angles, which are smaller than right angles and which are bigger than right angles. Since the polygons are not regular, the pentagon has two "almost right angles". You could also ask learners to name angles smaller than right angles, and angles bigger than right angles.

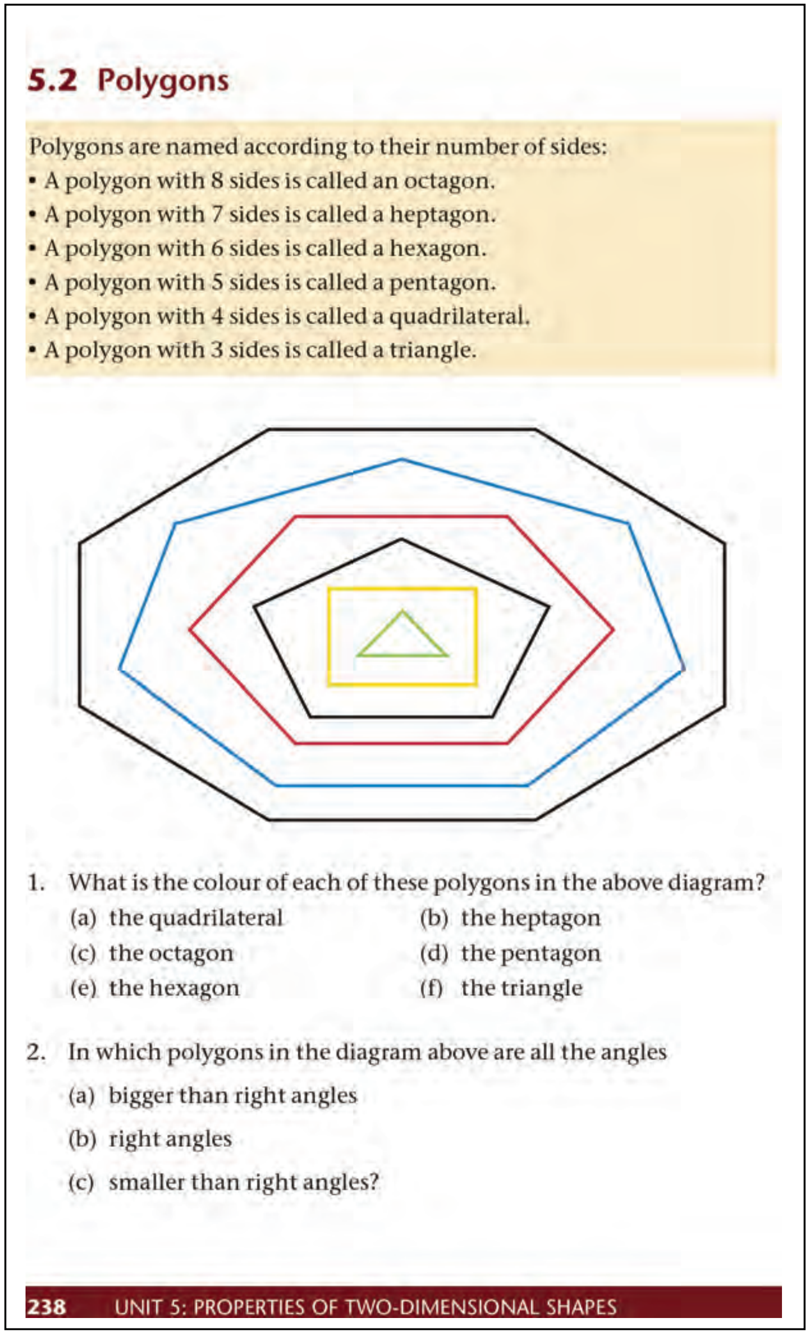
It is important to allow learners the time and give them the necessary support to "see" the many different polygons that form when a few extra lines are drawn to connect some, or all, of the vertices of the hexagons. This skill is about selecting some of the lines in the mind's eye while ignoring others for the moment. This is the purpose behind the invitation to learners in question 3 to shade particular polygons formed by the extra lines.

Notes on questions

If the figures on page 238 are regular polygons, the identical angles at the vertices become progressively bigger from the triangle in the centre to the octagon on the outside.

Answers

1. (a) Yellow (b) Blue (c) Black (outside)
   (d) Black (inside) (e) Red (f) Green
2. (a) Octagon (black), heptagon (blue), hexagon (red), pentagon (black)
   (b) Rectangle (yellow)
   (c) Triangle (green)



5.2 Polygons

Polygons are named according to their number of sides:
- A polygon with 8 sides is called an octagon.
- A polygon with 7 sides is called a heptagon.
- A polygon with 6 sides is called a hexagon.
- A polygon with 5 sides is called a pentagon.
- A polygon with 4 sides is called a quadrilateral.
- A polygon with 3 sides is called a triangle.

1. What is the colour of each of these polygons in the above diagram?
   (a) the quadrilateral (b) the heptagon
   (c) the octagon (d) the pentagon
   (e) the hexagon (f) the triangle
2. In which polygons in the diagram above are all the angles
   (a) bigger than right angles
   (b) right angles
   (c) smaller than right angles?

238 UNIT 5: PROPERTIES OF TWO-DIMENSIONAL SHAPES

Notes on questions: tracing

Tracing paper is the easiest to work with. However, the tracing required in question 3 can be done in several other ways if real tracing paper is not available:

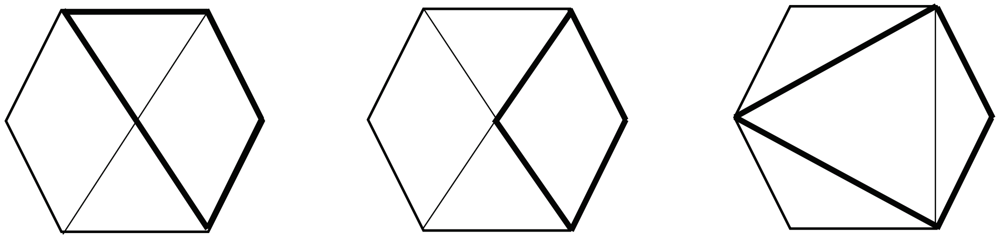
- Photocopies of the six hexagons provided on page 446 in the Addendum can be made, if you have access to a copier.
- Lunch wrap or waxed baking paper can be used as tracing paper.
- 60 g paper is available in most stationary shops and is transparent enough to allow tracing without backlighting.

Do not be tempted to photocopy or let learners trace the small hexagons with internal lines in questions 4 to 10. Learners will find it easier to see the figures within lines if they work with the bigger copies.

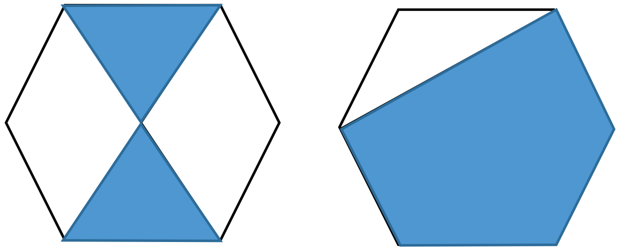
Answers

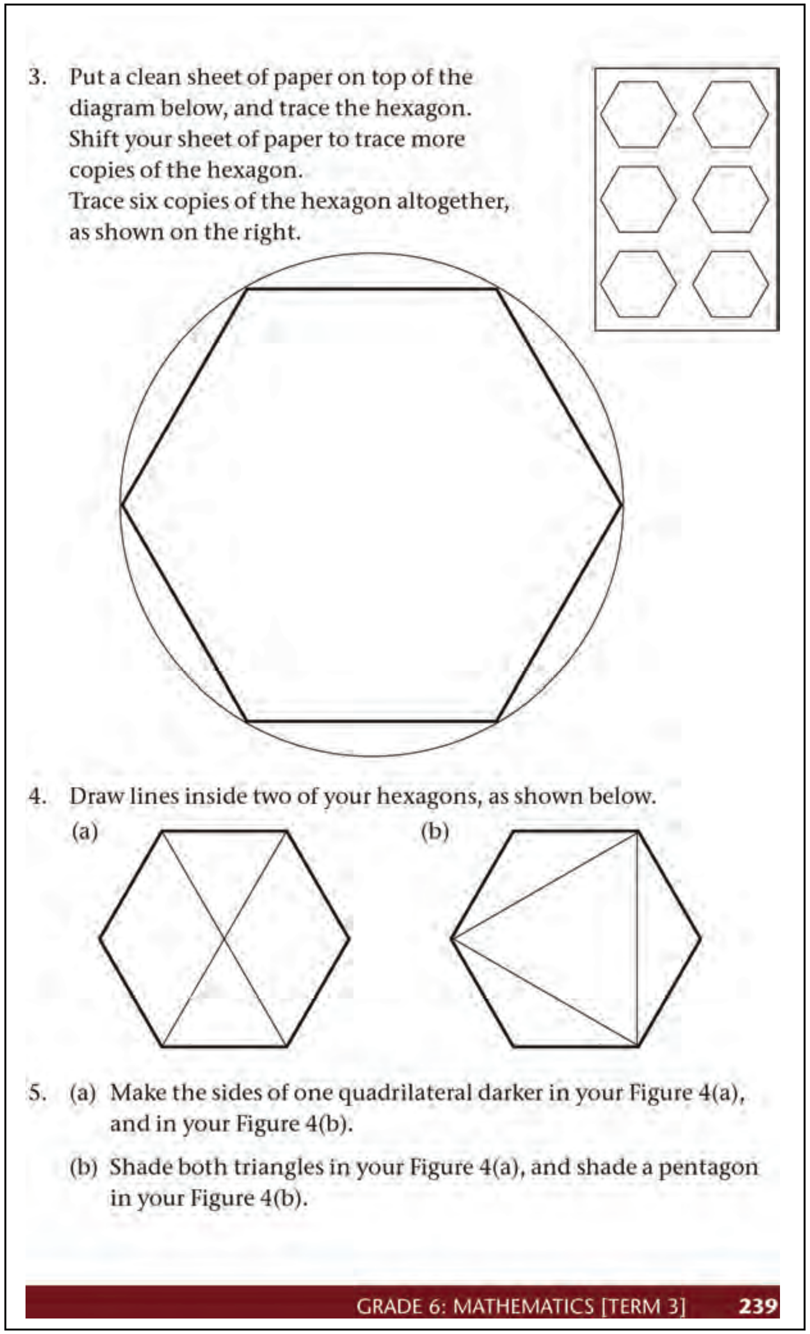
3. Learners' own drawings of hexagons
4. (a)–(b) Learners' own drawings
5. Learner's own drawings. Some possibilities are:

(a)



(b)





3. Put a clean sheet of paper on top of the diagram below, and trace the hexagon. Shift your sheet of paper to trace more copies of the hexagon. Trace six copies of the hexagon altogether, as shown on the right.

4. Draw lines inside two of your hexagons, as shown below.

(a) (b)

5. (a) Make the sides of one quadrilateral darker in your Figure 4(a), and in your Figure 4(b).

(b) Shade both triangles in your Figure 4(a), and shade a pentagon in your Figure 4(b).

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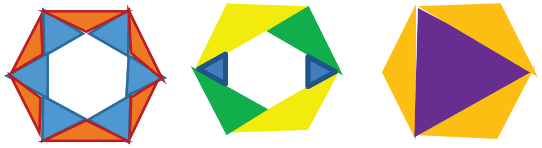
Possible misconceptions

Seeing figures within figures is an important skill in geometry. Learners will use this a lot when solving problems in the Senior and FET Phases. Learning to focus on specific lines or shapes is an important skill to learn.

Learners may be confused by the "jumble" of lines and may not be able to focus sufficiently to "see" the different polygons "hiding" in the figure. It is very important to establish which learners face this challenge and to support them in untangling the "jumble" and seeing structure. To help them, it may be good to give them extra printouts of the figures and ask them to highlight selected lines that show up particular polygons against the background of crisscrossing lines.

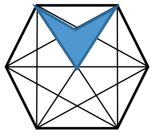
Answers

6. Learners' own drawings of the figures given in the Learner Book
7. (a) Some of the triangles that learners might shade:

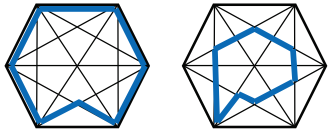


   (b) If only the small triangles are shaded, then a hexagon will remain unshaded in the centre. However, if all triangles are shaded, no polygon remains unshaded.

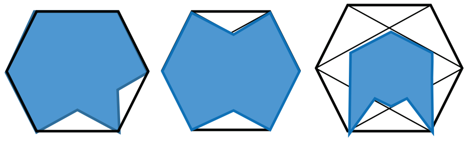
   (c) Learners might shade any combination of three triangles.
8. Learners' own drawings of the figures given in the Learner Book
9. (a) One possibility is:



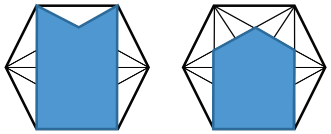
(b) Two possibilities are:



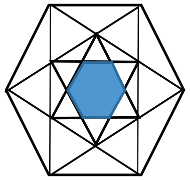
(c) Three possibilities are:

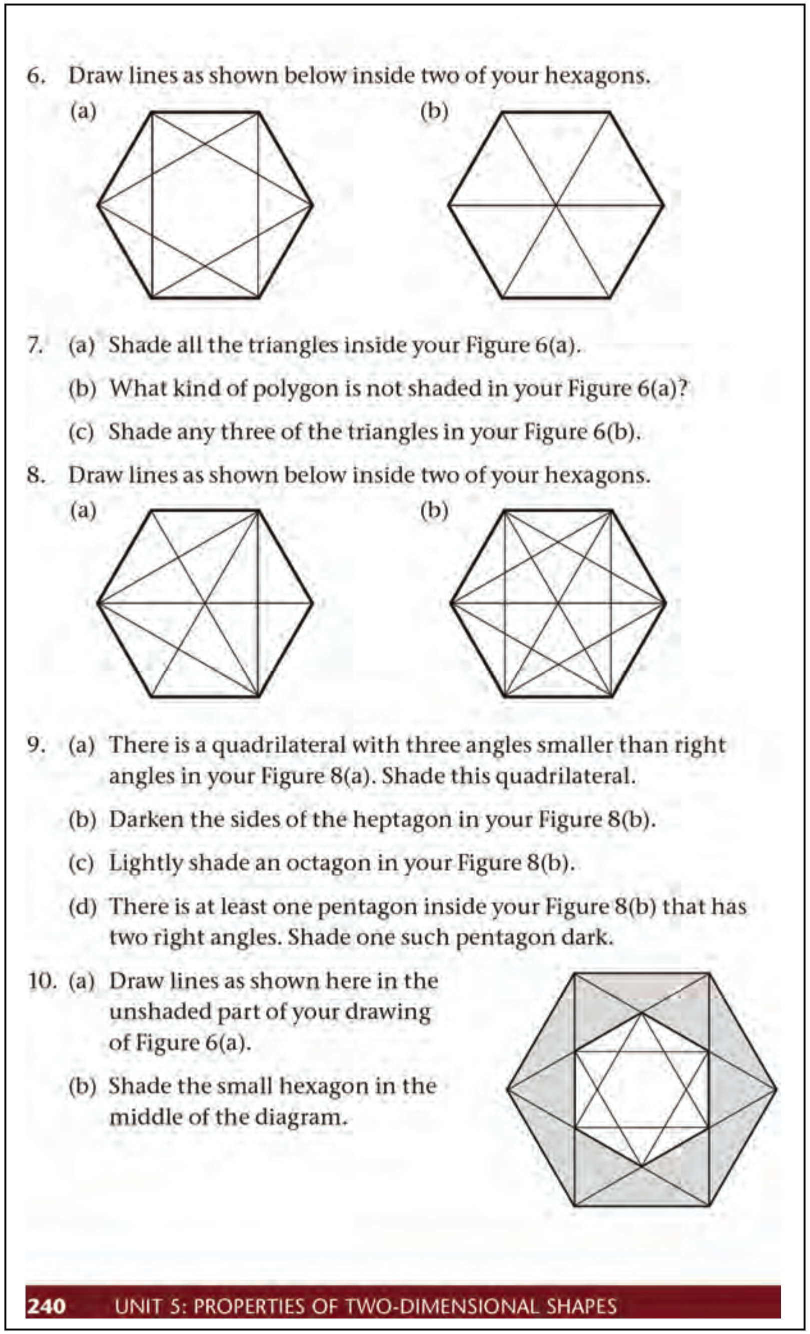


(d) Two possibilities are:



10. (a)–(b)





6. Draw lines as shown below inside two of your hexagons.

(a) (b)

7. (a) Shade all the triangles inside your Figure 6(a).
   (b) What kind of polygon is not shaded in your Figure 6(a)?
   (c) Shade any three of the triangles in your Figure 6(b).
8. Draw lines as shown below inside two of your hexagons.

(a) (b)

9. (a) There is a quadrilateral with three angles smaller than right angles in your Figure 8(a). Shade this quadrilateral.
   (b) Darken the sides of the heptagon in your Figure 8(b).
   (c) Lightly shade an octagon in your Figure 8(b).
   (d) There is at least one pentagon inside your Figure 8(b) that has two right angles. Shade one such pentagon dark.
10. (a) Draw lines as shown here in the unshaded part of your drawing of Figure 6(a).
    (b) Shade the small hexagon in the middle of the diagram.

5.3 Drawing circles and patterns in circles

Mathematical notes

Compasses are introduced in this section. Compasses have the obvious purpose of drawing circles or parts of circles (called arcs).

A circle is made up of a set of points that are all the same distance from the centre of the circle. Compasses allow you to set the distance between the centre (where the tip of the compasses is placed) and the circle. This allows you to draw bigger or smaller circles.

Teaching guidelines

This section has two focuses. The first focus is to allow learners to develop some skill in using compasses. This is not an easy skill for most learners to master. Encourage them to use *one hand only* when using their compasses. They should exert a small amount of pressure on the sharp tip so it does not slip. While they do this, they should use their thumb and index finger to grip the handle of the compasses and roll it, causing the pencil tip to gently turn around the sharp tip. Let them do many trial runs before asking them to do the exercises and activities in this section.

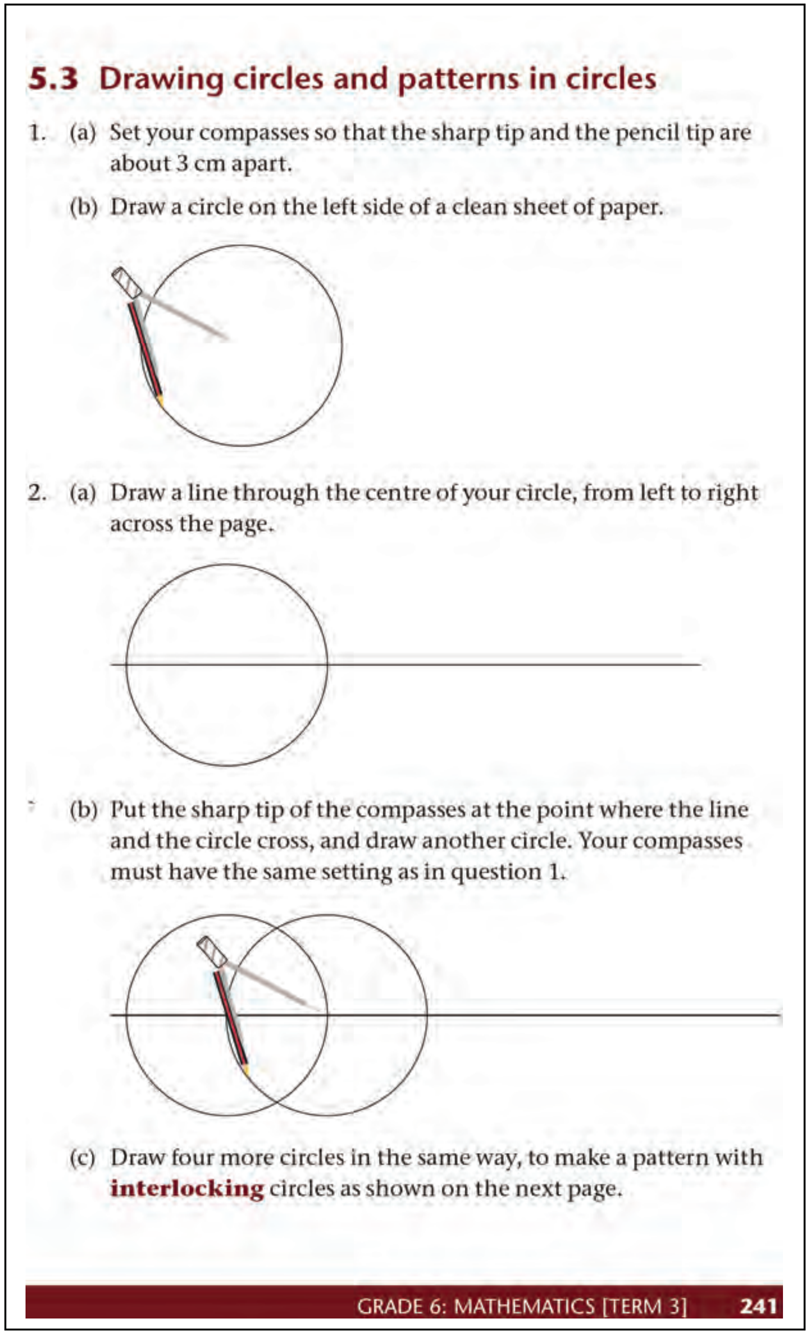
The second focus is drawing patterns with circles. In this section and the next patterns will be made by drawing overlapping circles of the same size.

The exercises and activities in this section are exploratory. Nonetheless, stop your learners from time to time and ask them to think about what they are doing. The objective is not only to draw many circles, but also to see that particular patterns and shapes arise as they do so. You can ask learners to imagine straight lines connecting points where the circles touch or intersect (cross each other). If they do this they may be able to see/imagine different kinds of polygons "hidden" in the patterns.

It is also possible to make patterns with circles of different sizes in which circles are placed inside each other. If learners have time, they can explore what happens if they keep the centres of circles in the same place, but increase the size of the circles. They could also draw a set of circles of different sizes that all touch at one point on the edge of the circles.

Answers

1. (a)–(b) Learners' own drawings
2. (a)–(c) Learners' own drawings



5.3 Drawing circles and patterns in circles

1. (a) Set your compasses so that the sharp tip and the pencil tip are about 3 cm apart.
   (b) Draw a circle on the left side of a clean sheet of paper.

2. (a) Draw a line through the centre of your circle, from left to right across the page.
   (b) Put the sharp tip of the compasses at the point where the line and the circle cross, and draw another circle. Your compasses must have the same setting as in question 1.
   (c) Draw four more circles in the same way, to make a pattern with **interlocking** circles as shown on the next page.

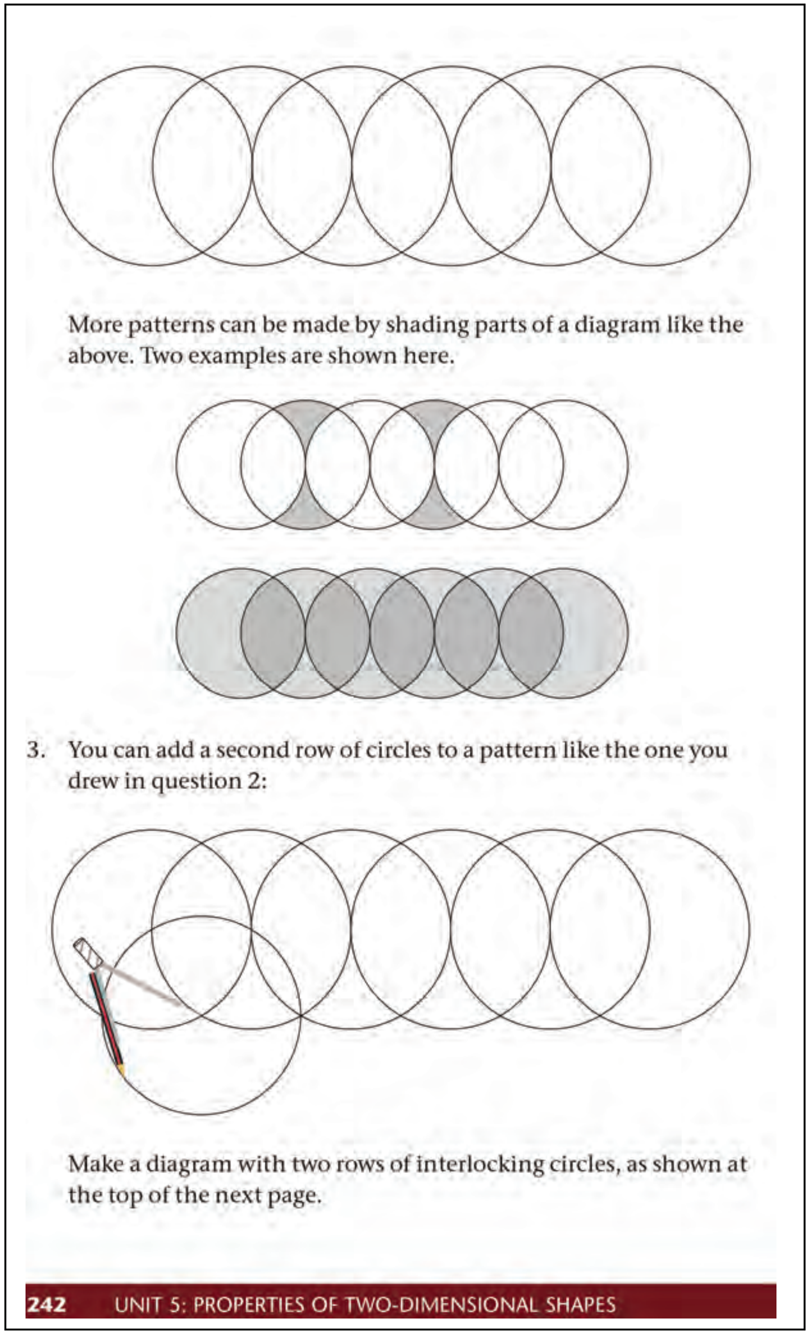
GRADE 6: MATHEMATICS [TERM 3] **241**

Notes on questions

In question 3 the tip of the compasses should be placed where two circles in the top row intersect.

Answers

3. Learners' own drawings



More patterns can be made by shading parts of a diagram like the above. Two examples are shown here.

3. You can add a second row of circles to a pattern like the one you drew in question 2:

Make a diagram with two rows of interlocking circles, as shown at the top of the next page.

242 UNIT 5: PROPERTIES OF TWO-DIMENSIONAL SHAPES

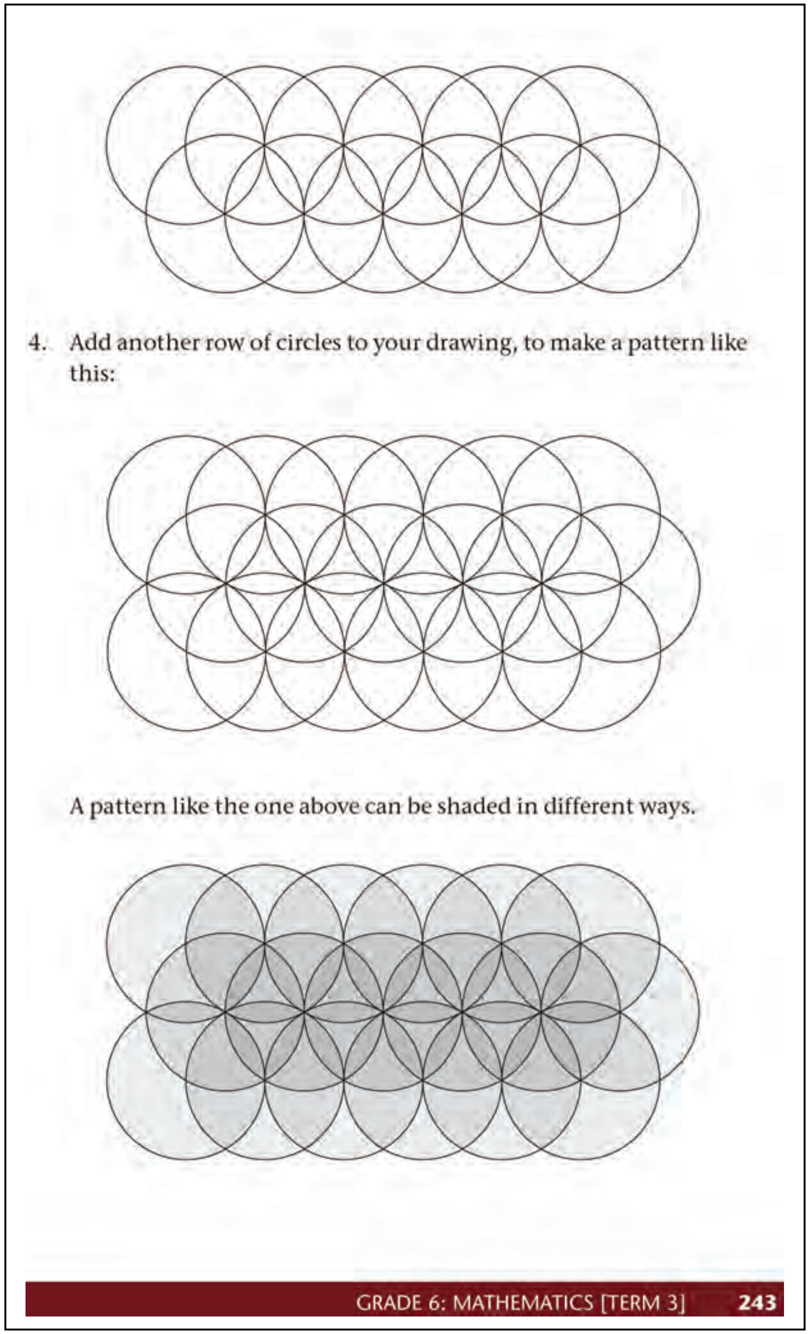
Notes on questions

In question 4 the tip of the compasses should be placed where two circles in the row above intersect, as was done in question 3.

Ask learners to shade their circles to form their own patterns.

Answers

4. Learners' own drawings



4. Add another row of circles to your drawing, to make a pattern like this:

A pattern like the one above can be shaded in different ways.

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5.4 Patterns with circles

Teaching guidelines

In this section learners continue to practise drawing circles with compasses. They also follow instructions to make different patterns. Learners usually enjoy making the patterns.

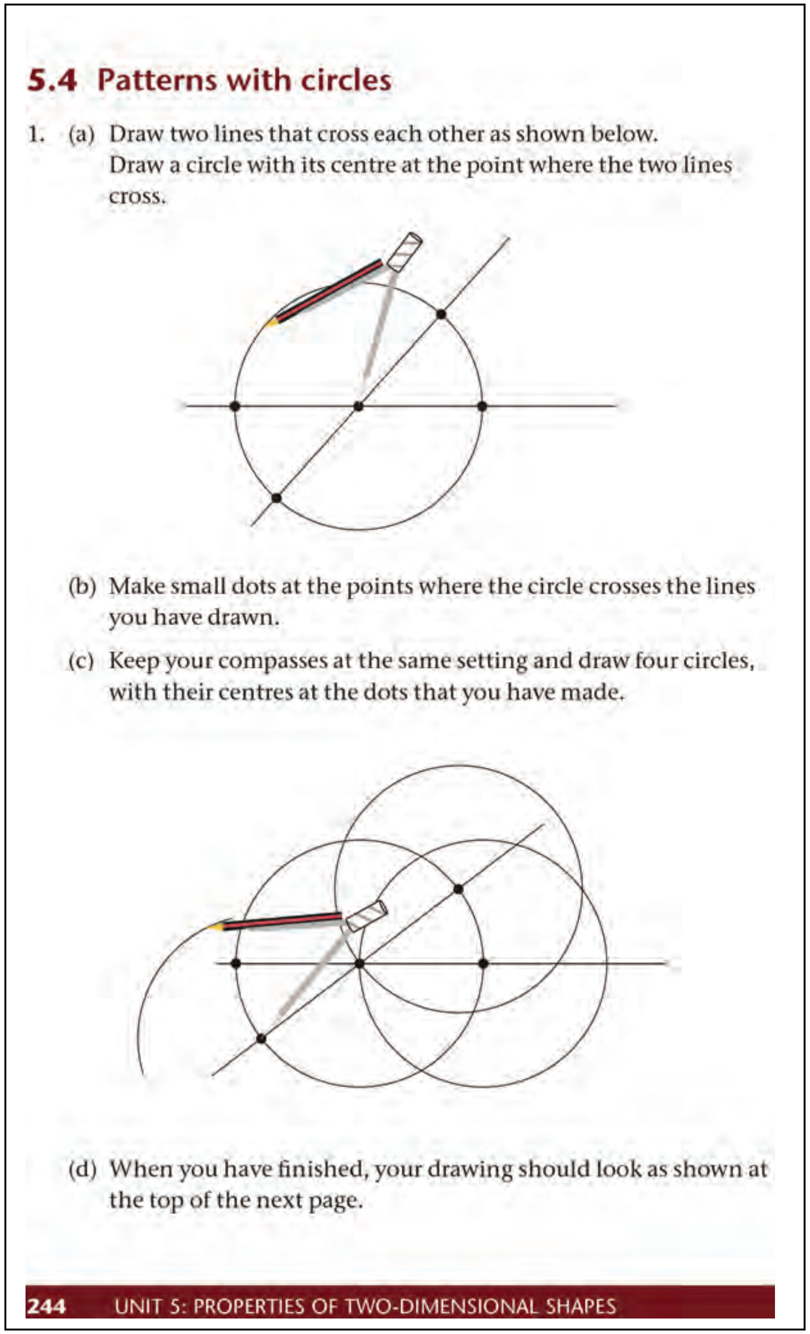
In both questions 1 and 2 learners should shade their final drawing to create their own patterns.

As you did in Section 5.3, you can stop learners and ask them to take notice of any patterns or relationships they see between the parts of their drawings. In particular, ask them whether they see any symmetries emerging.

You can also ask them to imagine lines connecting the points where circles intersect, and then whether this helps them to "see" hidden polygons. They can name these polygons.

Answers

1. (a)–(d) Learners' own drawings



5.4 Patterns with circles

1. (a) Draw two lines that cross each other as shown below. Draw a circle with its centre at the point where the two lines cross.

(b) Make small dots at the points where the circle crosses the lines you have drawn.

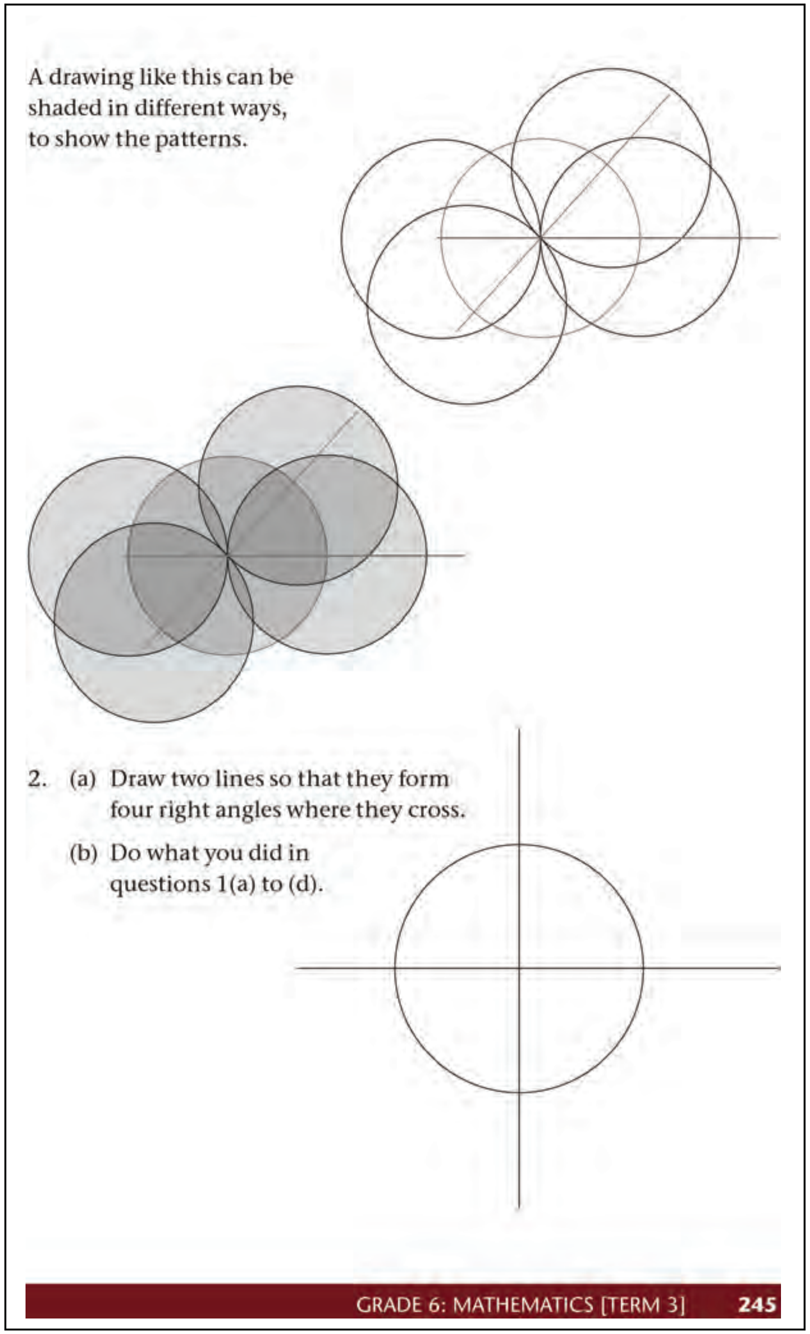
(c) Keep your compasses at the same setting and draw four circles, with their centres at the dots that you have made.

(d) When you have finished, your drawing should look as shown at the top of the next page.

244 UNIT 5: PROPERTIES OF TWO-DIMENSIONAL SHAPES

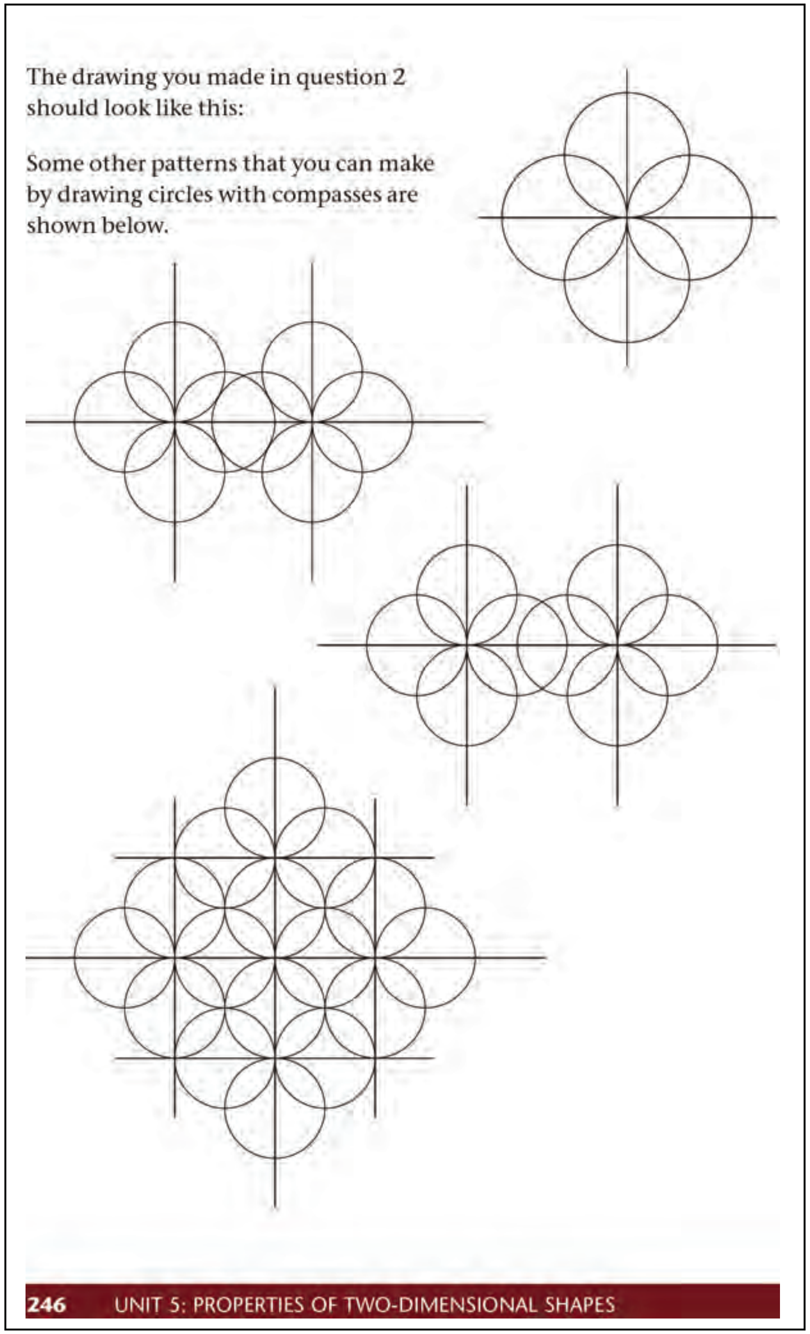
Answers

2. (a)–(b) Learners' own drawings



A drawing like this can be shaded in different ways, to show the patterns.

2. (a) Draw two lines so that they form four right angles where they cross.
   (b) Do what you did in questions 1(a) to (d).



The drawing you made in question 2 should look like this:

Some other patterns that you can make by drawing circles with compasses are shown below.

Grade 6 Term 3 Unit 6 Transformations

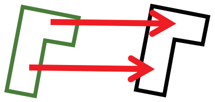
Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
6.1 Rotations, reflections and translations	Introducing the three basic transformations	247 to 250
6.2 Describing patterns	Looking at repeating patterns and seeing the basic transformations in them	251 to 253
6.3 Symmetry in patterns	Identifying symmetries in patterns	254 to 257

CAPS time allocation	3 hours
CAPS page references	23 and 265

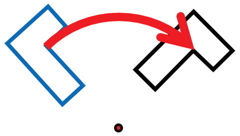
Mathematical background

Any relocation of a figure can be achieved by a combination of three types of movement, called "transformations":

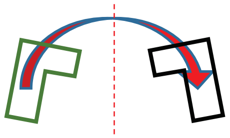
- Translation: slide in a particular direction, through a particular distance, without rotating.



- Rotation: rotating around a particular point outside or on the figure, through a particular angle.



- Reflection: flipping it over (reflecting it), i.e. picking it up, turning it over and putting it down again.

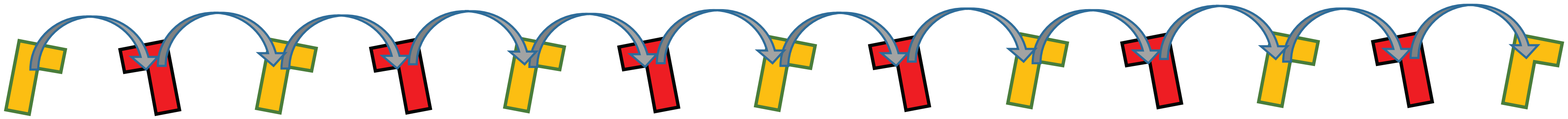


Reflection of a figure always produces symmetry. The axis of reflection (*the broken line in the above figure*) is the line of symmetry.

If two identical figures lie on the same flat surface it is always possible to get one of the two figures to fit exactly on top of the other figure by performing a translation, rotation or reflection, or a translation and a reflection (a so-called "glide-reflection").

Translations, rotations and reflections do not change the shape or size of a figure. Other kinds of transformations, for example enlargements, change the size. There are also transformations, for example stretching in one direction, that change the figure.

Patterns are formed when the same transformation or set of transformations is repeatedly applied to the same figure, for example:



6.1 Rotations, reflections and translations

Mathematical notes

Translations happen when a figure is moved (or can be imagined to be moved) from one place to another without turning it or flipping it over. Translations happen along a straight line and over a certain distance.

Rotations happen when a figure is turned around a fixed point (or imagined to be turned about a fixed point) called the centre of rotation. Each part of the figure is assumed to go through the same turn. A rotation can be through any angle (bigger angles mean a bigger rotation). Experience with rotations can contribute to the development of a sense of angle size in learners' minds.

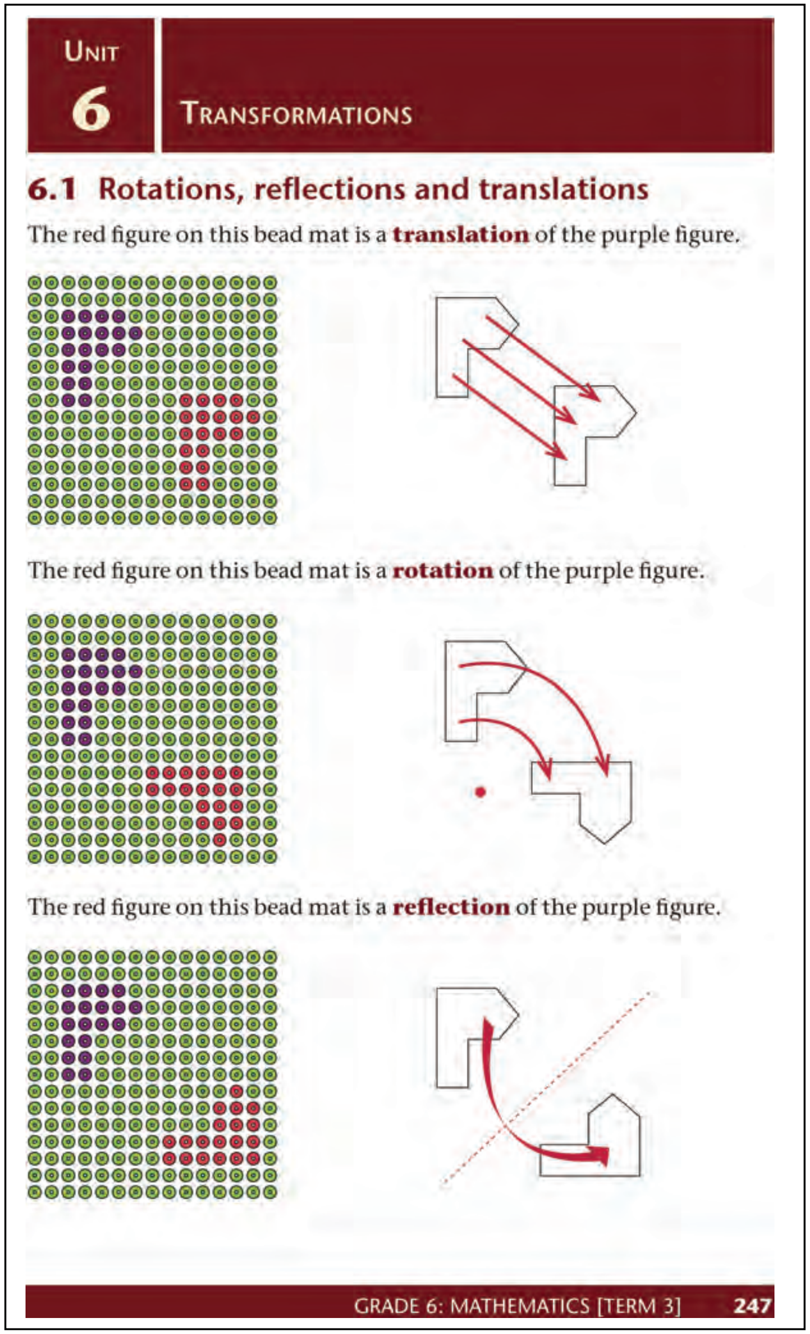
Reflections happen when a figure is flipped over (or is imagined to be flipped over) a fixed line, the line of symmetry. Each part of the figure is assumed to go through the same flip.

Teaching guidelines

It may be useful to talk about "different ways in which an object can be moved", and to request learners to move one of their hands in each of the three ways described on page 247. It is important that they slide or rotate their hands on the surface of a desk when they perform translations and rotations. By walking round the class and observing learners performing a translation, rotation or reflection is a quick way to assess whether they correctly distinguish between the three kinds of movements.

Possible misconceptions

Some learners may confuse rotations with reflections (or confuse all three). This is probably inexperience with "imagining" or "seeing" the transformations. Such learners will need more opportunities to rotate and flip over a figure to compare the results. Allow them to use whatever they need to begin to distinguish between the three types of transformation (e.g. moving their hands between two figures to "act out" the transformation). These are necessary first steps to being able to think through the transformations without any tools or gestures. The following two sections will reinforce this.



UNIT
6

TRANSFORMATIONS

6.1 Rotations, reflections and translations

The red figure on this bead mat is a **translation** of the purple figure.

The red figure on this bead mat is a **rotation** of the purple figure.

The red figure on this bead mat is a **reflection** of the purple figure.

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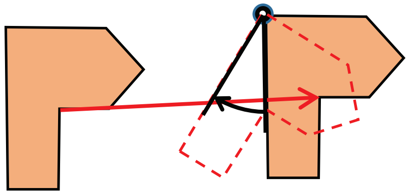
Notes on questions

Allow learners to become aware that there are often different transformation sets that will result in the same end transformation of a figure. They could compare their descriptions in question 1(b) to become aware of the different transformation sets possible for the transformation of the same figure.

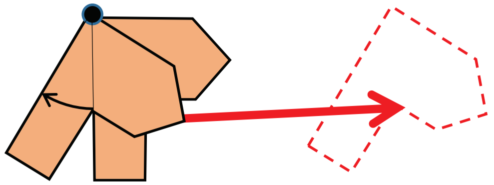
Answers

1. (a) No

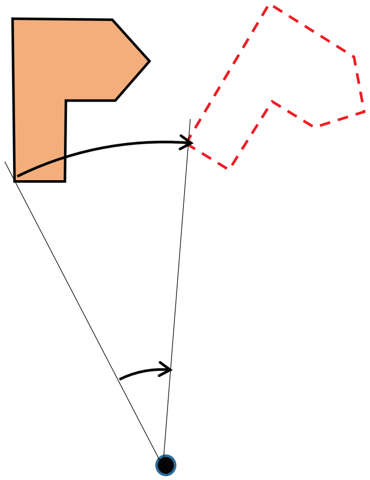
   (b) The template can be translated slightly upwards and to the right as indicated by the red arrow, then rotated to the left around the black point (there are many other possibilities).

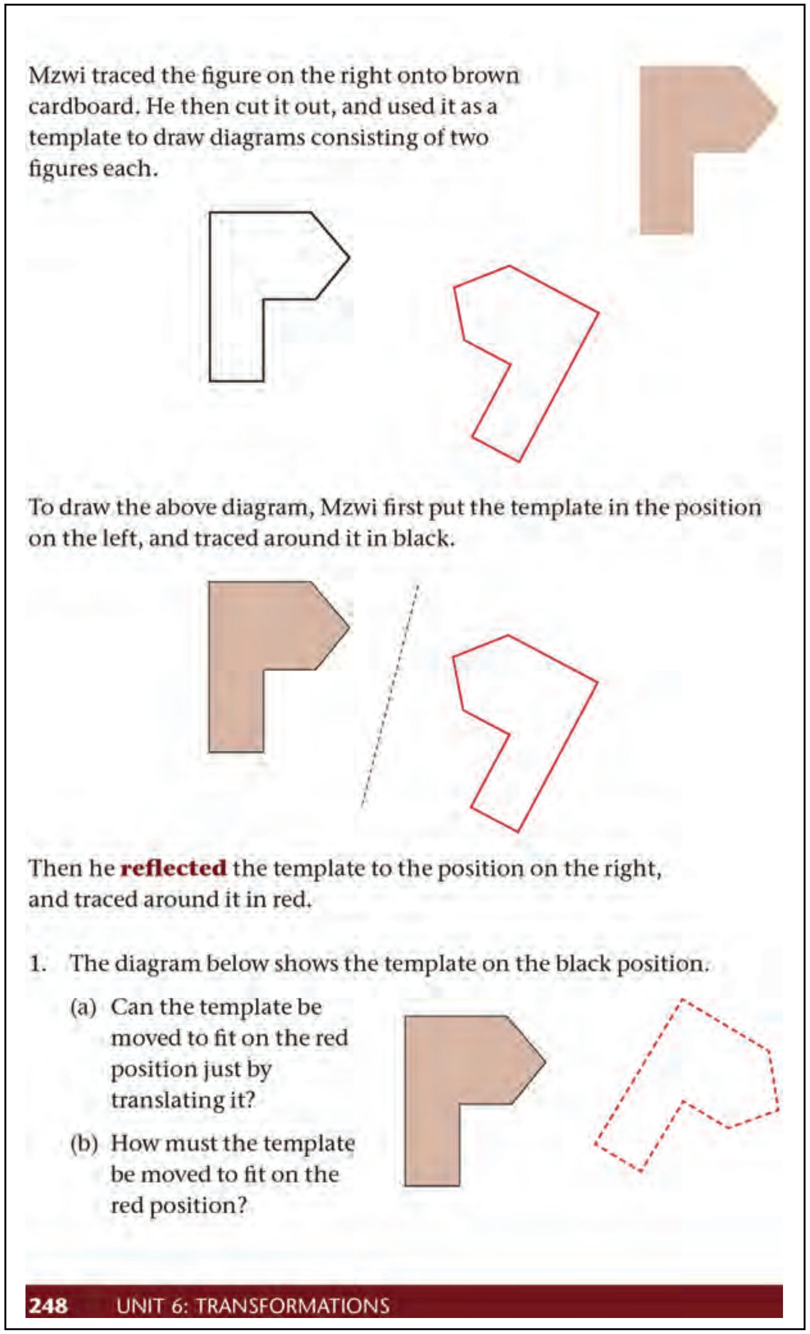


It can also be rotated first, then translated.



Alternatively, the template can simply be rotated around the point indicated on the right below.





Mzwi traced the figure on the right onto brown cardboard. He then cut it out, and used it as a template to draw diagrams consisting of two figures each.

To draw the above diagram, Mzwi first put the template in the position on the left, and traced around it in black.

Then he **reflected** the template to the position on the right, and traced around it in red.

1. The diagram below shows the template on the black position.

   (a) Can the template be moved to fit on the red position just by translating it?

   (b) How must the template be moved to fit on the red position?

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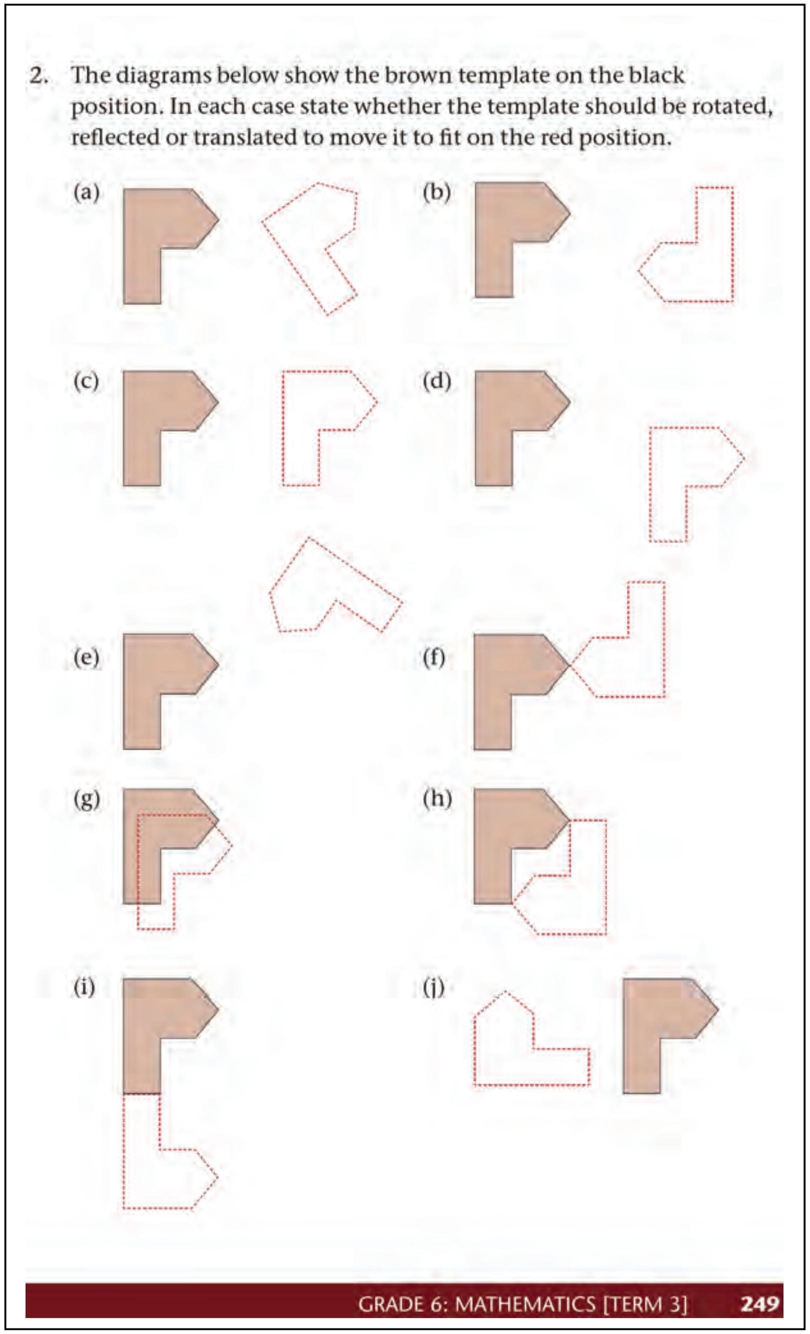
Teaching guidelines

You may suggest to learners that they imagine moving the brown template to the indicated position, and should try to think what kind of movement they have to make with the hand holding the template.

Answers

2. (a) Translated to the right, then rotated
   (b) Translated to the right, then rotated
   (c) Translated to the right
   (d) Translated downwards to the right
   (e) Reflected
   (f) Rotated
   (g) Translated downwards to the right
   (h) Rotated, then translated
   (i) Reflected
   (j) Rotated, then translated



2. The diagrams below show the brown template on the black position. In each case state whether the template should be rotated, reflected or translated to move it to fit on the red position.

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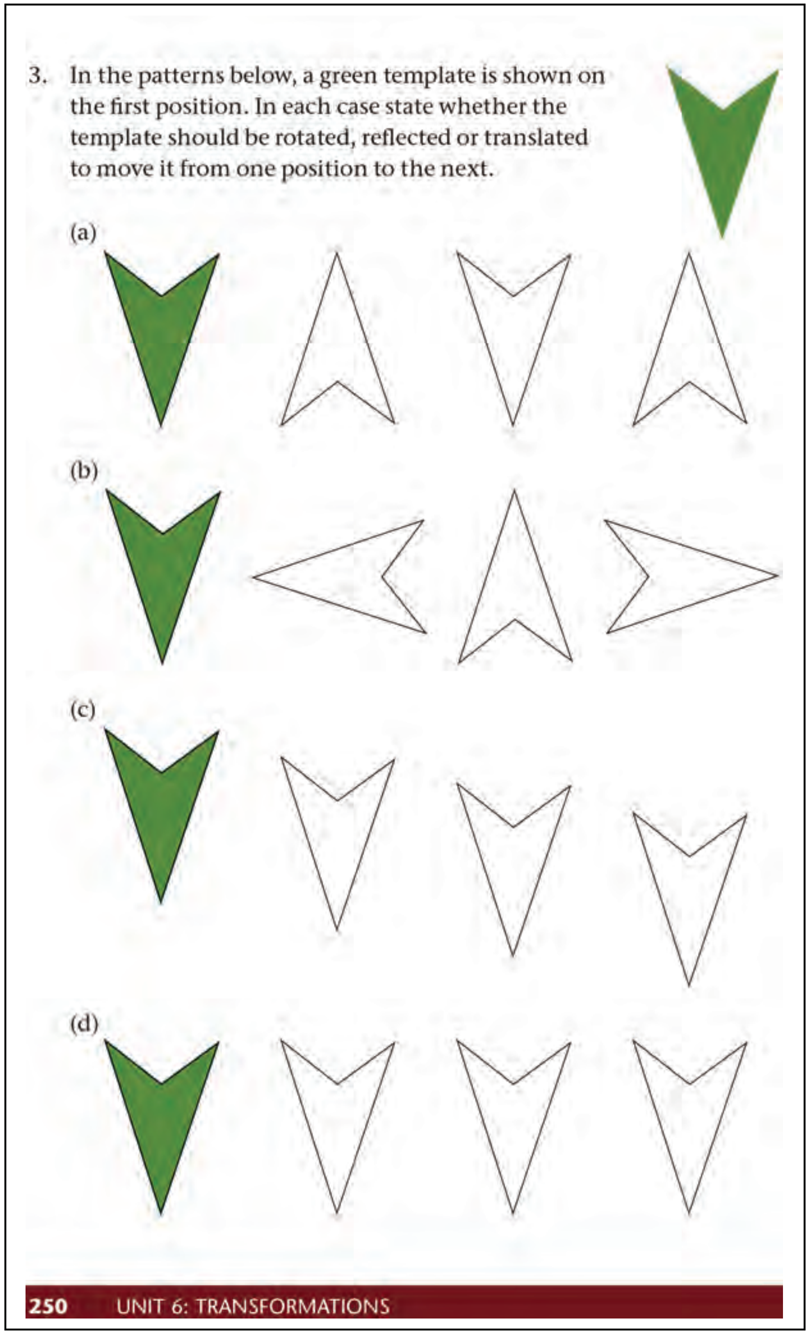
Teaching guidelines

Point out to learners that they should identify **one** kind of movement that is repeated for each row of kites. Learners should really try to figure this out by using their imagination. They may cut out a rough template and move it to copy the patterns.

Answers

3. (a) Rotated (halfway) around a point between the first two figures, again, and again.
   (b) Rotated (quarter of a revolution, through a right angle) around a point between the first two figures, again, and again.
   (c) Translated (slightly downwards) to the right, again, and again, along a straight line.
   (d) Translated to the right, again, and again, along a straight line.



3. In the patterns below, a green template is shown on the first position. In each case state whether the template should be rotated, reflected or translated to move it from one position to the next.

(a)

(b)

(c)

(d)

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6.2 Describing patterns

Mathematical notes

Patterns are formed when figures are arranged in a regular, repetitive way. Patterns are the natural result of repeated applications of the three basic transformations on one or more figures.

Teaching guidelines

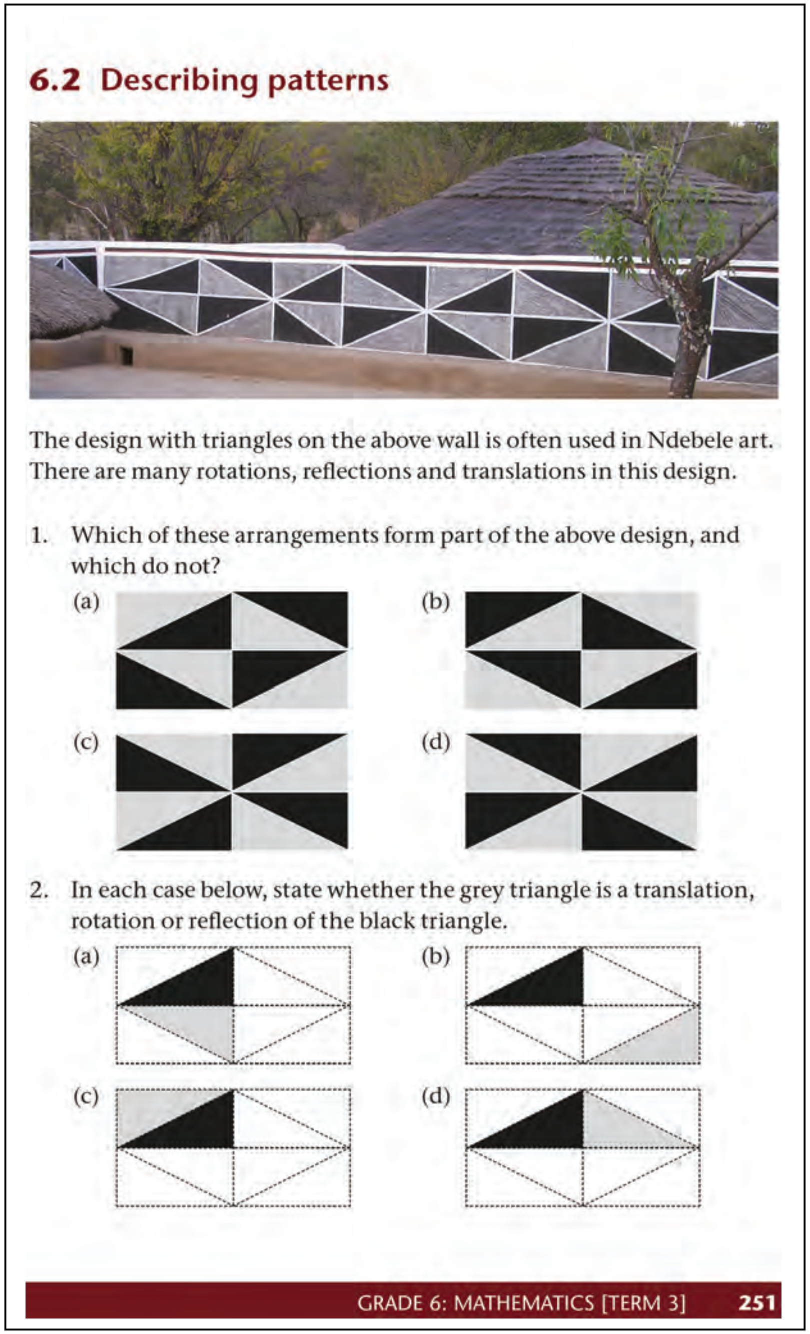
This section allows learners to consolidate their understanding of the three basic transformations. Encourage them to engage with the meaning of what they are doing, and not just to complete the exercises by following the instructions.

Possible misconceptions

As before, learners who confuse the transformations will require focused support. Give such learners a figure and ask them to describe to a fellow learner a set of transformations that they are performing on the figure. A description in their own words where the terms translation, rotation and reflection are not used/used incorrectly must be attended to.

Answers

1. Arrangements (a) and (d) form part of the design, arrangements (b) and (d) do not.
2. (a) Reflection (b) Translation
   (c) Rotation (c) Reflection



6.2 Describing patterns

The design with triangles on the above wall is often used in Ndebele art. There are many rotations, reflections and translations in this design.

1. Which of these arrangements form part of the above design, and which do not?

2. In each case below, state whether the grey triangle is a translation, rotation or reflection of the black triangle.

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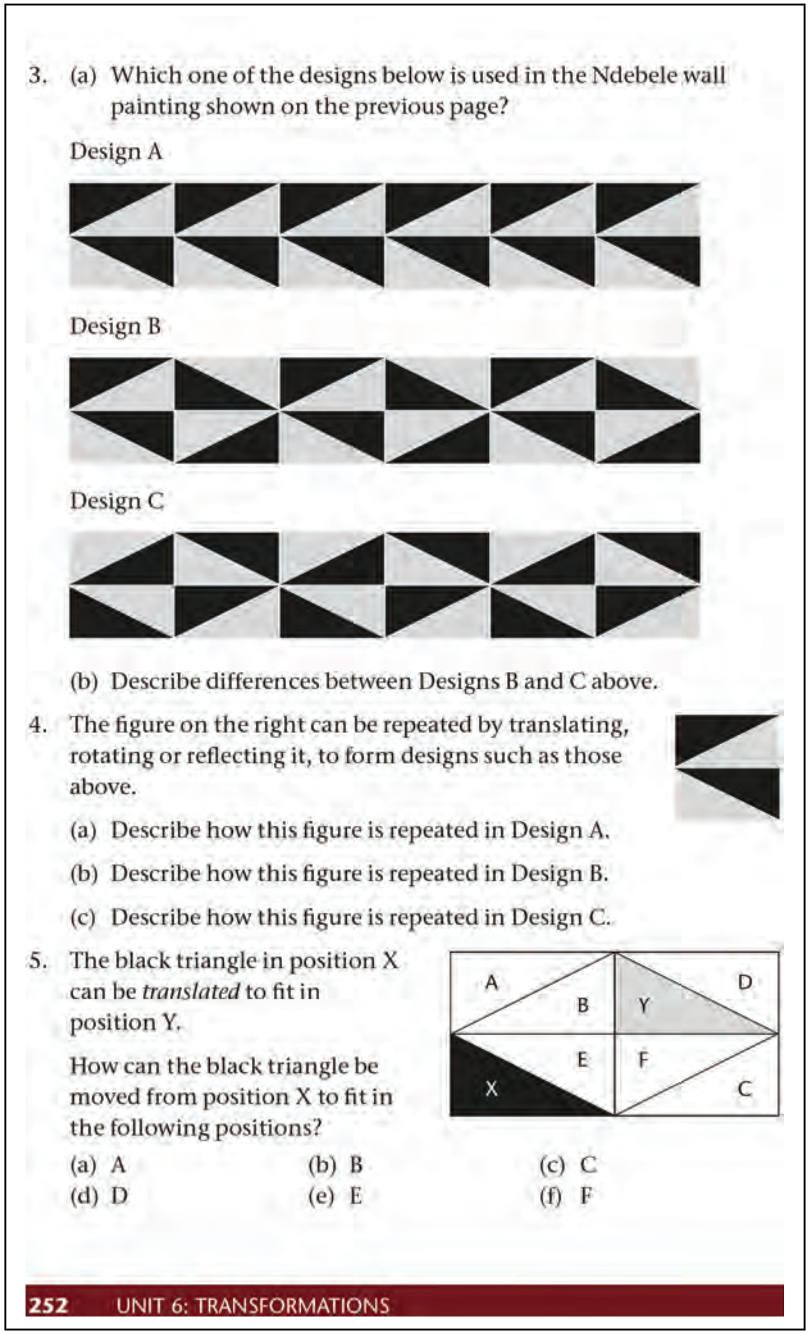
Answers

3. (a) Design C
   (b) They are the same except the one is upside down. They are reflections of each other. Design B is a reflection of Design C.
4. (a) The figure is repeated through a translation to the right.
   (b) The figure is repeated through a rotation.
   (c) The reflection (along a vertical line through its middle) of the figure is repeatedly rotated.
5. (a) Reflected
   (b) Reflected then translated
   (c) Reflected
   (d) Rotated
   (e) Rotated
   (f) Translated then reflected



3. (a) Which one of the designs below is used in the Ndebele wall painting shown on the previous page?

Design A

Design B

Design C

(b) Describe differences between Designs B and C above.

4. The figure on the right can be repeated by translating, rotating or reflecting it, to form designs such as those above.

(a) Describe how this figure is repeated in Design A.

(b) Describe how this figure is repeated in Design B.

(c) Describe how this figure is repeated in Design C.

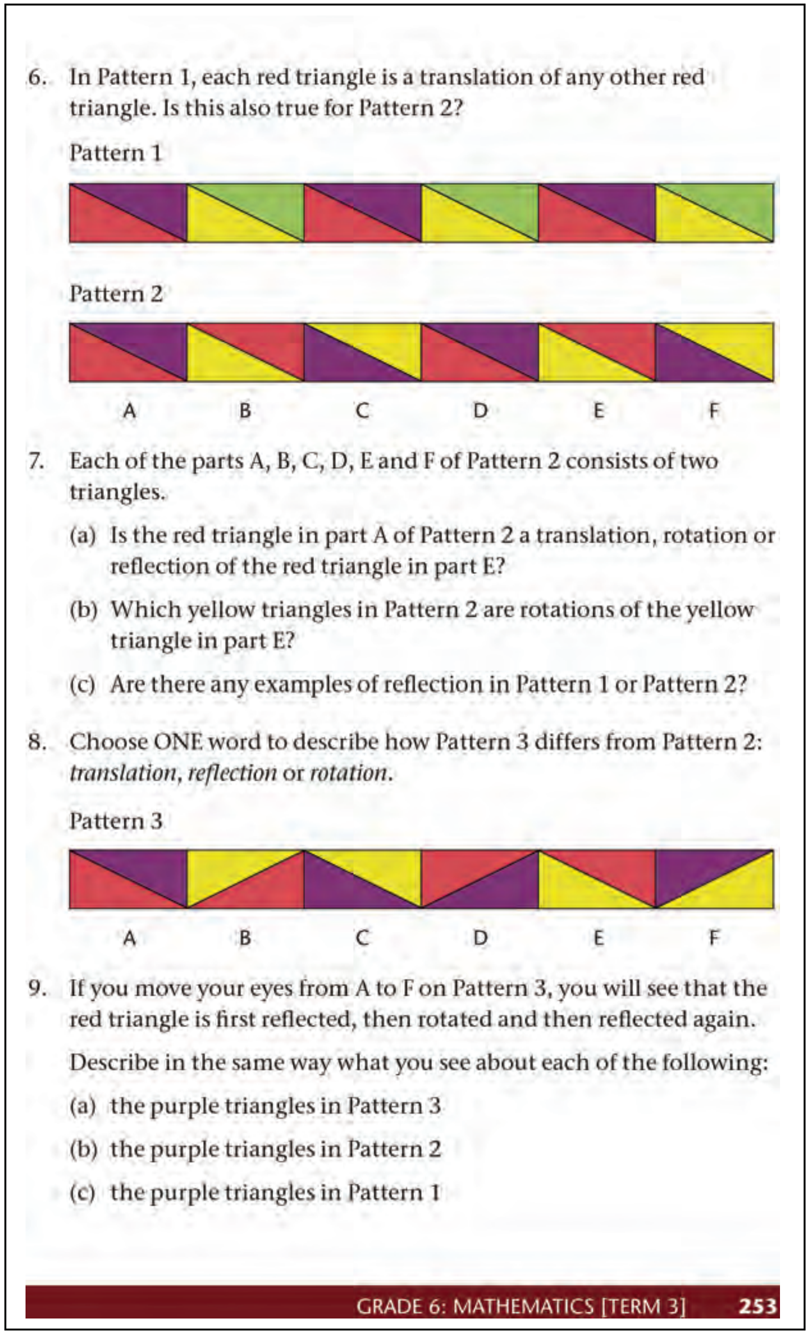
5. The black triangle in position X can be *translated* to fit in position Y.

How can the black triangle be moved from position X to fit in the following positions?

(a) A (b) B (c) C
(d) D (e) E (f) F

Answers

6. No
7. (a) Rotation
   (b) Parts C and F
   (c) No
8. Reflection (of B, D and F)
9. (a) Rotated and translated, then reflected, then translated and rotated.
   (b) Rotated and translated, then rotated, then rotated and translated.
   (c) Only translated.



6. In Pattern 1, each red triangle is a translation of any other red triangle. Is this also true for Pattern 2?

Pattern 1

Pattern 2

A B C D E F

7. Each of the parts A, B, C, D, E and F of Pattern 2 consists of two triangles.
   (a) Is the red triangle in part A of Pattern 2 a translation, rotation or reflection of the red triangle in part E?
   (b) Which yellow triangles in Pattern 2 are rotations of the yellow triangle in part E?
   (c) Are there any examples of reflection in Pattern 1 or Pattern 2?

8. Choose ONE word to describe how Pattern 3 differs from Pattern 2: *translation*, *reflection* or *rotation*.

Pattern 3

A B C D E F

9. If you move your eyes from A to F on Pattern 3, you will see that the red triangle is first reflected, then rotated and then reflected again.

   Describe in the same way what you see about each of the following:
   (a) the purple triangles in Pattern 3
   (b) the purple triangles in Pattern 2
   (c) the purple triangles in Pattern 1

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6.3 Symmetry in patterns

Mathematical notes

The nouns **rotation**, **translation** and **reflection** and their verb counterparts **rotate**, **translate** and **reflect** are used as follows:

- The verbs describe how objects can be moved, for example: “A template placed on the green hexagon can be reflected to fit on the red hexagon.”
- The nouns indicate the result of a movement, for example: “The red hexagon is a reflection of the green hexagon.”

Teaching guidelines

If learners do not experience problems with the way in which the words rotation, translation and reflection are used in question 1, there is no need to conduct a classroom discussion about the two ways in which these words can be used (see above).

Some learners may struggle to “see” symmetry, or the absence of symmetry. Allow them to “act out” the folding of one side of the figure onto the other using their hands. Remind them that the line of symmetry is like a fold line. Also, it would be very helpful to provide cut-outs of figures and ask learners to investigate whether the figures can be folded symmetrically (which will identify the lines of symmetry that will lie along the fold lines).

Notes on questions

Questions 1 and 2 are important in helping to establish whether a learner has a clear understanding of the different transformations. Again, let learners describe the transformations of the figures by mimicking the transformations with their hands.

Answers

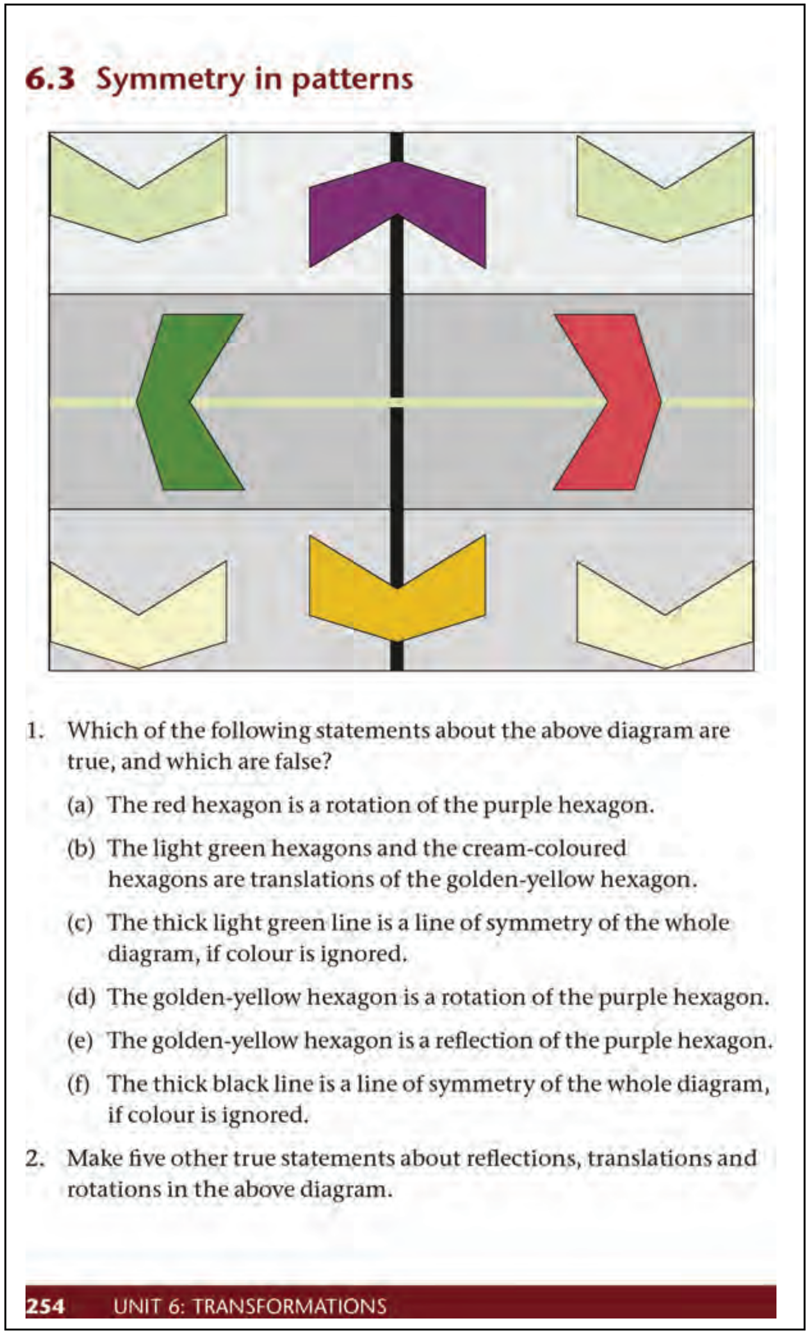
1. (a) True (b) True (c) False
   (d) True (e) True (f) True
2. Any of the eight hexagons can be obtained through a rotation of any other hexagon in the design, in some cases quarter-revolutions, in other cases half-revolutions.

   Any of the light green, light yellow and golden yellow hexagons can be obtained through a translation of any of the others.

   The purple hexagon can be obtained through a reflection of the golden-yellow hexagon.

   The dark green hexagon can be obtained through a reflection of the red hexagon.

   The purple hexagon is a rotation of the red hexagon.

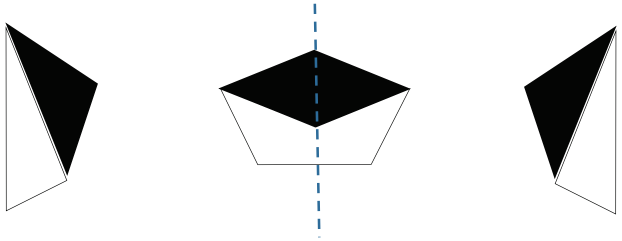


6.3 Symmetry in patterns

1. Which of the following statements about the above diagram are true, and which are false?
   (a) The red hexagon is a rotation of the purple hexagon.
   (b) The light green hexagons and the cream-coloured hexagons are translations of the golden-yellow hexagon.
   (c) The thick light green line is a line of symmetry of the whole diagram, if colour is ignored.
   (d) The golden-yellow hexagon is a rotation of the purple hexagon.
   (e) The golden-yellow hexagon is a reflection of the purple hexagon.
   (f) The thick black line is a line of symmetry of the whole diagram, if colour is ignored.
2. Make five other true statements about reflections, translations and rotations in the above diagram.

Answers

3. A rough drawing of these figures:



4. (a) Examples:

   The white cross can be translated to move it from a red square to a blue square.

   The black-and-white pentagon can be translated to move it from the blue square to a yellow square.

   The black-and-white quadrilateral can be reflected in a diagonal of the left red square in the third row to move it from the yellow square in the middle left to the blue square second from left in the third row.

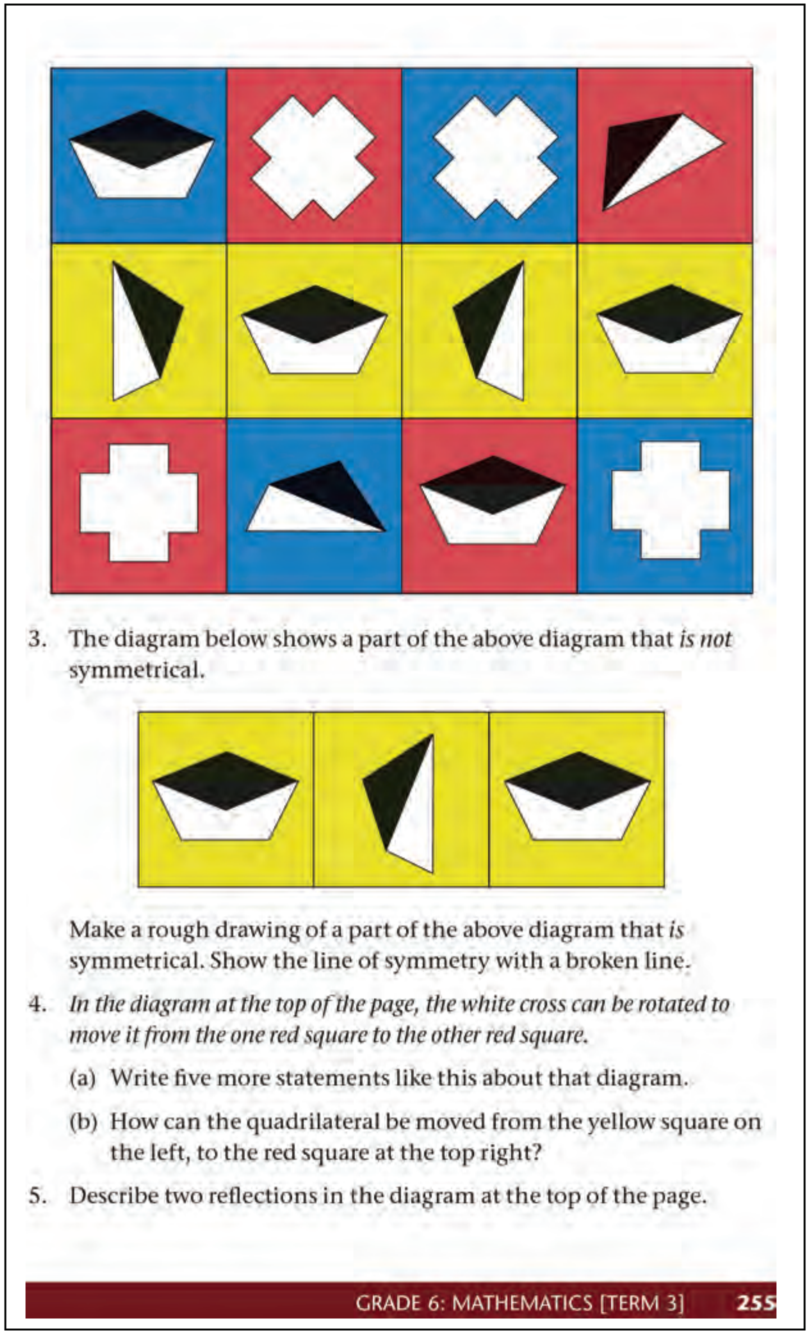
   The black-and-white quadrilateral can be reflected in a vertical line to move it from the one yellow square to the other yellow square.

   The black-and-white pentagon can be translated to move it from one yellow square to another yellow square.

   (b) It can be rotated anti-clockwise through an obtuse angle.

5. The non-symmetrical black-and-white quadrilateral can be reflected to move it from the one yellow square to the other yellow square.

   The white cross can be reflected to move it from the red square to the blue square, in the top row and also in the bottom row.



3. The diagram below shows a part of the above diagram that *is not* symmetrical.

   Make a rough drawing of a part of the above diagram that *is* symmetrical. Show the line of symmetry with a broken line.

4. *In the diagram at the top of the page, the white cross can be rotated to move it from the one red square to the other red square.*

   (a) Write five more statements like this about that diagram.

   (b) How can the quadrilateral be moved from the yellow square on the left, to the red square at the top right?

5. Describe two reflections in the diagram at the top of the page.

Answers

6. (a) Examples:

If the colours are ignored, the blue dotted line is a line of symmetry, and so is the red dotted line.

The red arrow is a reflection of the yellow arrow, around the blue dotted line.

The red arrow is also an anti-clockwise rotation of the yellow arrow, around the point where the two dotted lines intersect.

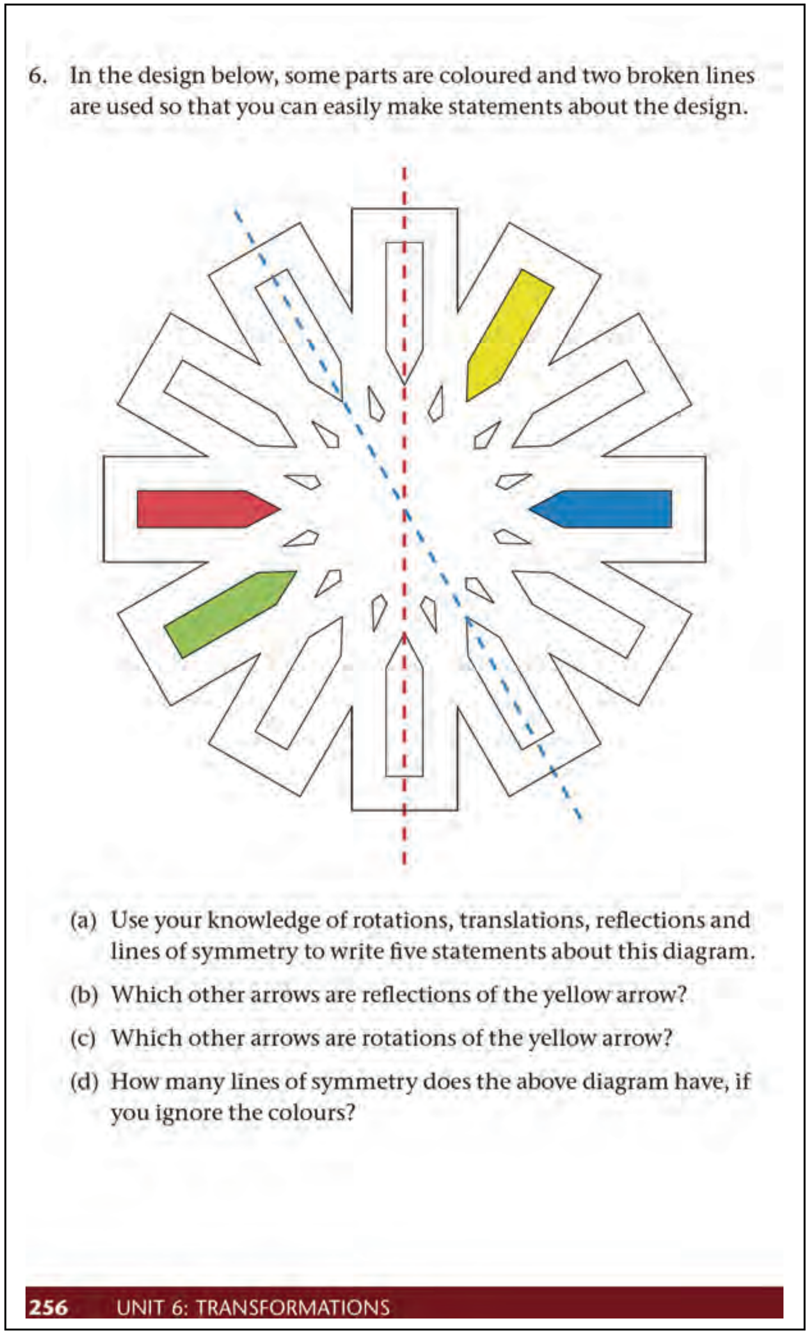
Each big arrow is a rotation of any other big arrow.

Each small arrow is a rotation of any other small arrow.

(b) All the other arrows

(c) All the other arrows

(d) 12 lines of symmetry



6. In the design below, some parts are coloured and two broken lines are used so that you can easily make statements about the design.

(a) Use your knowledge of rotations, translations, reflections and lines of symmetry to write five statements about this diagram.

(b) Which other arrows are reflections of the yellow arrow?

(c) Which other arrows are rotations of the yellow arrow?

(d) How many lines of symmetry does the above diagram have, if you ignore the colours?

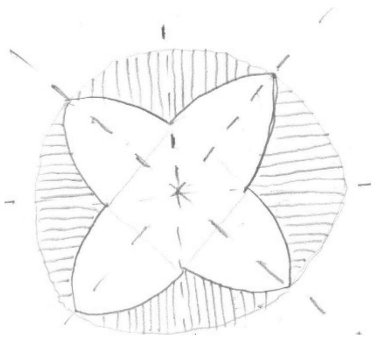
Notes on questions

Note that the placemat in question 7 is not symmetrical as a whole. It is, however, symmetrical in certain details/parts (but even these symmetries are real-life "almost symmetries"). Allow learners to identify and discuss the different transformations that are visible in the design of the placemat. Note that the placemat weaver might not have thought of these transformations when he/she made the design. However, knowledge of the transformations can help the viewer to appreciate the artwork.

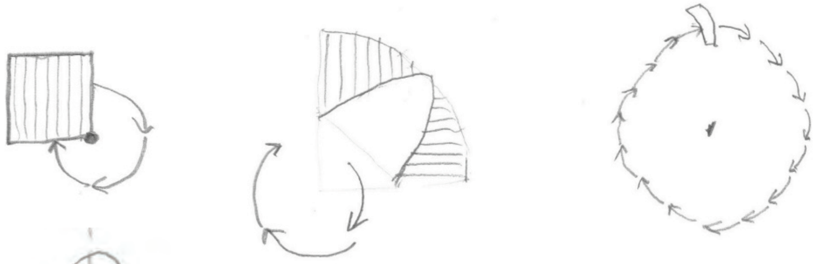
Answers

Note that the sketches below are just examples of many different ways in which the transformations in the design may be highlighted. Learners' sketches may look quite different.

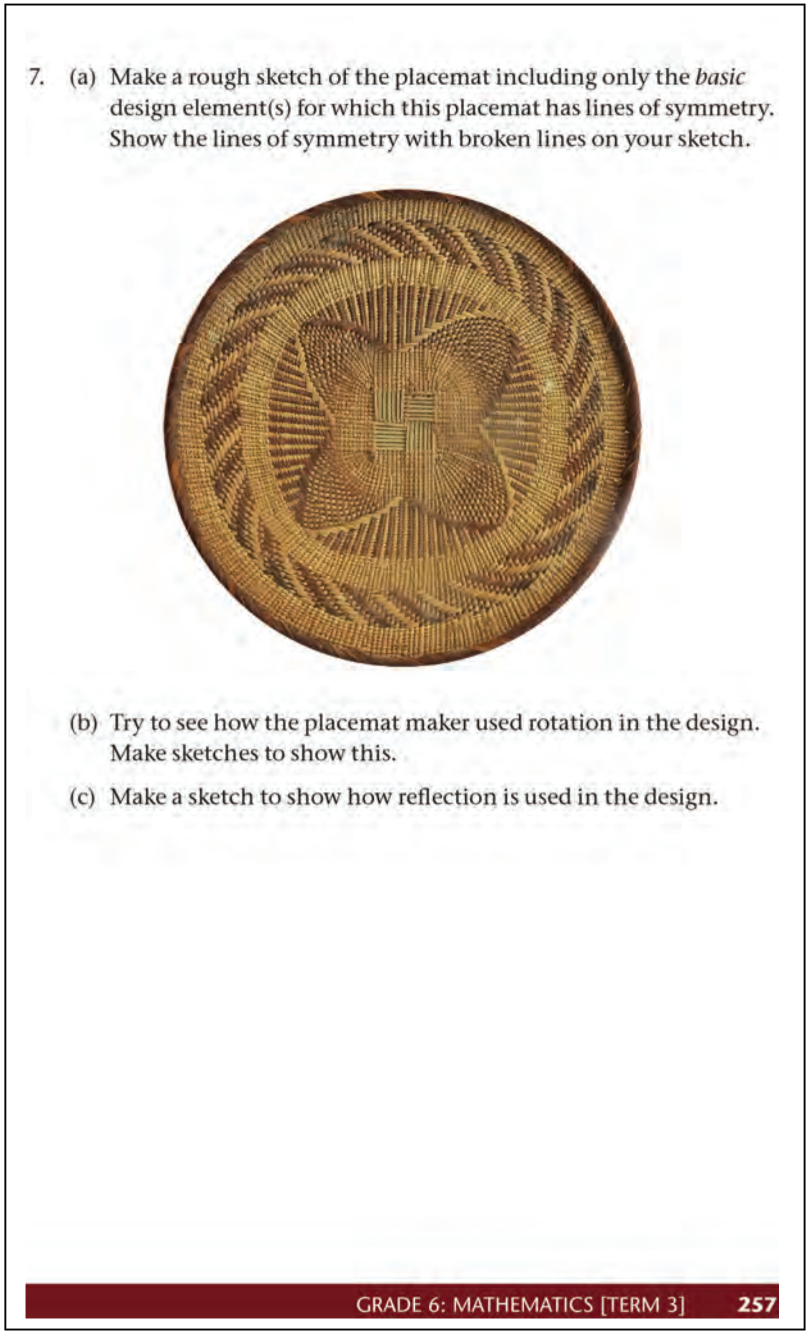
7. (a)



(b)



(c)



7. (a) Make a rough sketch of the placemat including only the *basic* design element(s) for which this placemat has lines of symmetry. Show the lines of symmetry with broken lines on your sketch.

(b) Try to see how the placemat maker used rotation in the design. Make sketches to show this.

(c) Make a sketch to show how reflection is used in the design.

Grade 6 Term 3 Unit 7 Temperature

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
7.1 The Celsius scale and medical thermometers	Understanding what a healthy person's temperature is; understanding the scales on a thermometer	258 to 259
7.2 Daily temperature	Using the unit of degree Celsius (°C) to describe or imagine temperatures in places where people live	260 to 261

CAPS time allocation	1 hour
CAPS page references	28 and 266

Mathematical background

In Term 3 Unit 1 we saw that a standard unit of mass is very useful, for example when we need to tell someone how much sugar to buy. Standard measuring units allow for the correct message to be communicated between people. For example, if John's mother asks for 250 g of butter, John knows that he needs to buy 250 g and not 500 g of butter.

With hot and cold things – as in the temperature of something – it is difficult to report how hot or cold they are because people feel temperature differently. However, we can communicate if everyone agrees on a unit of temperature, for example the degree Celsius (°C). This unit of measurement makes it easy for people to read recipes, especially when instructed to "heat the oven to 140 °C". Temperature as a topic in Mathematics involves:

- reading scales on thermometers, marked in degrees
- understanding fractions of a degree
- showing temperature changes on a graph
- understanding that each type of thermometer has a temperature range (most thermometers cannot measure very high or very low temperatures).

We can subtract temperatures from each other to find differences, and to measure how much a temperature has increased or decreased. We cannot, however, add temperatures of substances to find their combined temperature. If we have a cup of hot water at 60 °C and another at 40 °C, and we pour them into a jug, we do not get water at 100 °C. This may be a nice experiment for learners to do and work out for themselves.

We also cannot multiply a temperature by a temperature, nor divide a temperature by a temperature.

Resources

Thermometer

7.1 The Celsius scale and medical thermometers

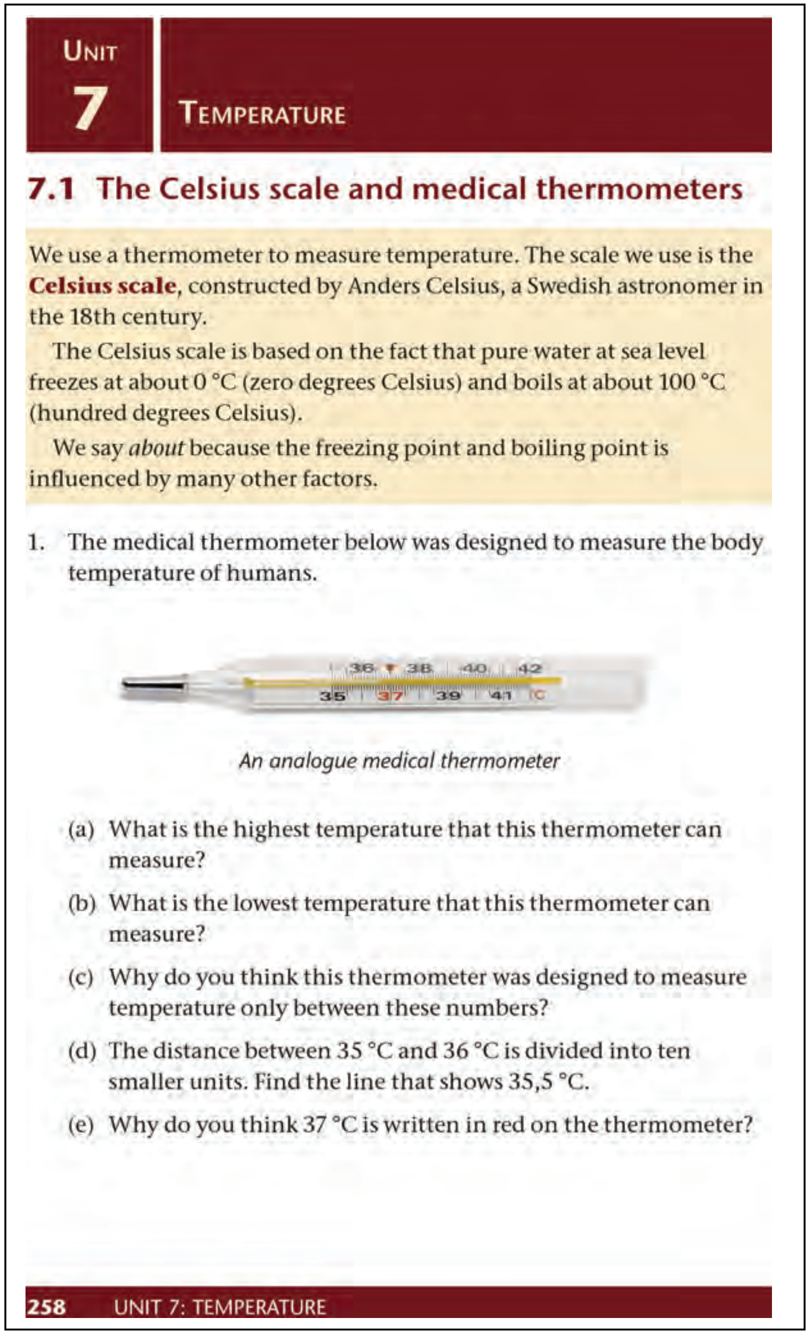
Teaching guidelines

Bring a thermometer to class if at all possible. If you cannot, draw one on the board.

Explain to learners that in order to measure temperature, we measure the length of a column of liquid. You can demonstrate this by taking a reading without touching the bulb of the thermometer. Then rub the thumb of one hand against the other fingers of your hand to warm your fingers up. Hold the thermometer in that hand for a few minutes.

Answers

1. (a) 42 °C
   (b) 35 °C
   (c) This thermometer is used to measure body temperature only.
   (d) Learners find the line that shows 35,5 °C.
   (e) 37 °C is generally the normal body temperature.



UNIT 7 TEMPERATURE

7.1 The Celsius scale and medical thermometers

We use a thermometer to measure temperature. The scale we use is the **Celsius scale**, constructed by Anders Celsius, a Swedish astronomer in the 18th century.

The Celsius scale is based on the fact that pure water at sea level freezes at about 0 °C (zero degrees Celsius) and boils at about 100 °C (hundred degrees Celsius).

We say *about* because the freezing point and boiling point is influenced by many other factors.

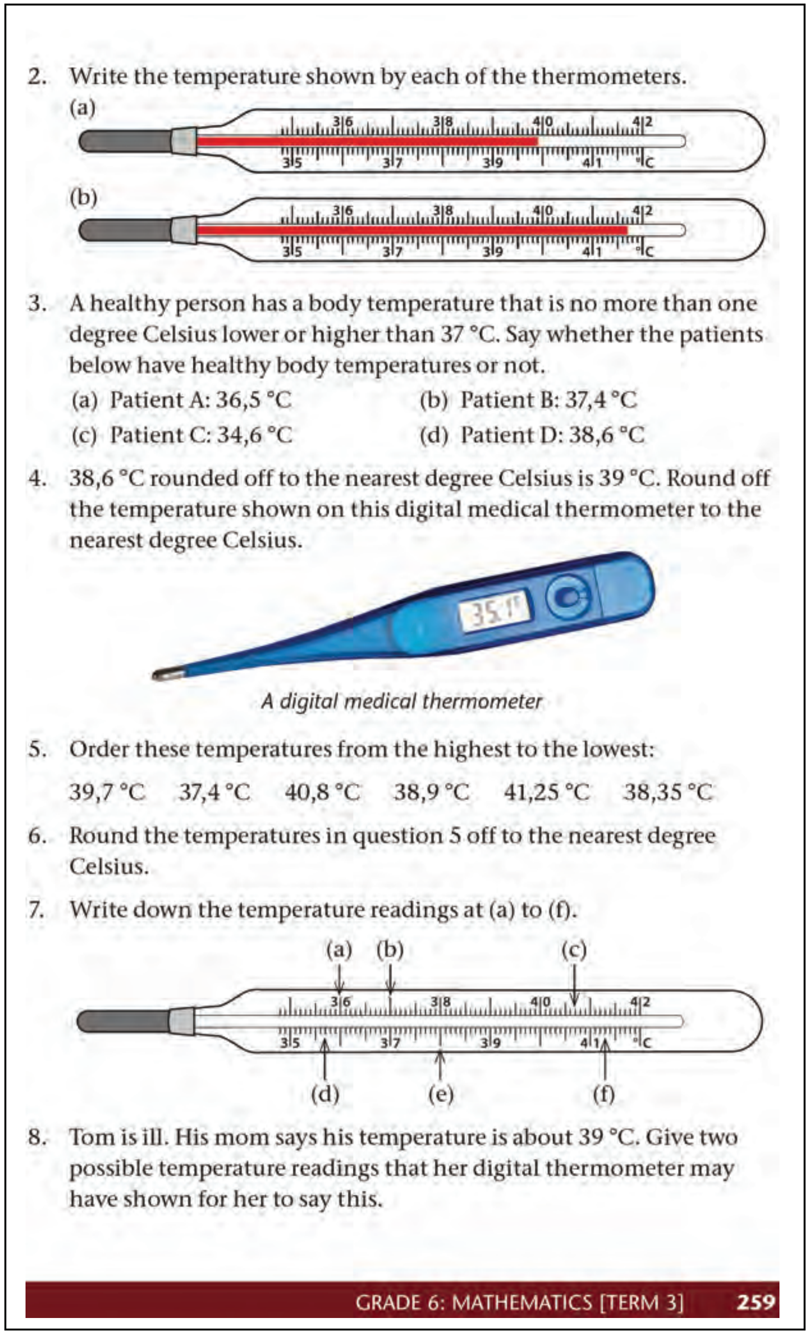
1. The medical thermometer below was designed to measure the body temperature of humans.

An analogue medical thermometer

   (a) What is the highest temperature that this thermometer can measure?
   (b) What is the lowest temperature that this thermometer can measure?
   (c) Why do you think this thermometer was designed to measure temperature only between these numbers?
   (d) The distance between 35 °C and 36 °C is divided into ten smaller units. Find the line that shows 35,5 °C.
   (e) Why do you think 37 °C is written in red on the thermometer?

Answers

2. (a) 39,9 °C
   (b) 41,7 °C
3. (a) Healthy
   (b) Healthy
   (c) Not well
   (d) Feverish
4. 35 °C
5. 41,25 °C; 40,8 °C; 39,7 °C; 38,9 °C; 38,35 °C; 37,4 °C
6. 39,7 °C ≈ 40 °C 37,4 °C ≈ 37 °C 40,8 °C ≈ 41 °C
   38,9 °C ≈ 39 °C 41,25 °C ≈ 41 °C 38,35 °C ≈ 38 °C
7. (a) 36 °C (b) 37 °C (c) 40,7 °C
   (d) 35,7 °C (e) 38 °C (f) 41,3 °C
8. Any reading from 38,5 °C to 39,4 °C. The reason is that his temperature is *about* 39 °C.
   38,5 °C could be rounded *up* to 39 °C; so could 38,6 °C; 38,7 °C; 38,8 °C and 38,9 °C.
   39,4 °C could be rounded *down* to 39 °C; so could 39,3 °C; 39,2 °C and 39,1 °C.



2. Write the temperature shown by each of the thermometers.

3. A healthy person has a body temperature that is no more than one degree Celsius lower or higher than 37 °C. Say whether the patients below have healthy body temperatures or not.
   (a) Patient A: 36,5 °C (b) Patient B: 37,4 °C
   (c) Patient C: 34,6 °C (d) Patient D: 38,6 °C
4. 38,6 °C rounded off to the nearest degree Celsius is 39 °C. Round off the temperature shown on this digital medical thermometer to the nearest degree Celsius.

A digital medical thermometer

5. Order these temperatures from the highest to the lowest:
   39,7 °C 37,4 °C 40,8 °C 38,9 °C 41,25 °C 38,35 °C
6. Round the temperatures in question 5 off to the nearest degree Celsius.
7. Write down the temperature readings at (a) to (f).

8. Tom is ill. His mom says his temperature is about 39 °C. Give two possible temperature readings that her digital thermometer may have shown for her to say this.

7.2 Daily temperature

Teaching guidelines

It would be ideal to have a thermometer in class and use it to do some experiments. Let learners take readings each morning and at midday for a week. Learners can make a chart to record their readings. Stick the chart on the wall. Apart from learning about temperature, this is an excellent example of a variable quantity. The experience will promote learners' intuitive awareness of variable quantities, which is a critical prerequisite for making sense of algebra from Grade 7 onwards.

You have to inform learners that temperatures sometimes become colder than 0 °C (freezing point) and are notated as a negative number (showing a "–" sign), for example –18,6 °C. Thus we say these temperatures are below freezing point, but we will only focus on temperatures at and above freezing point.

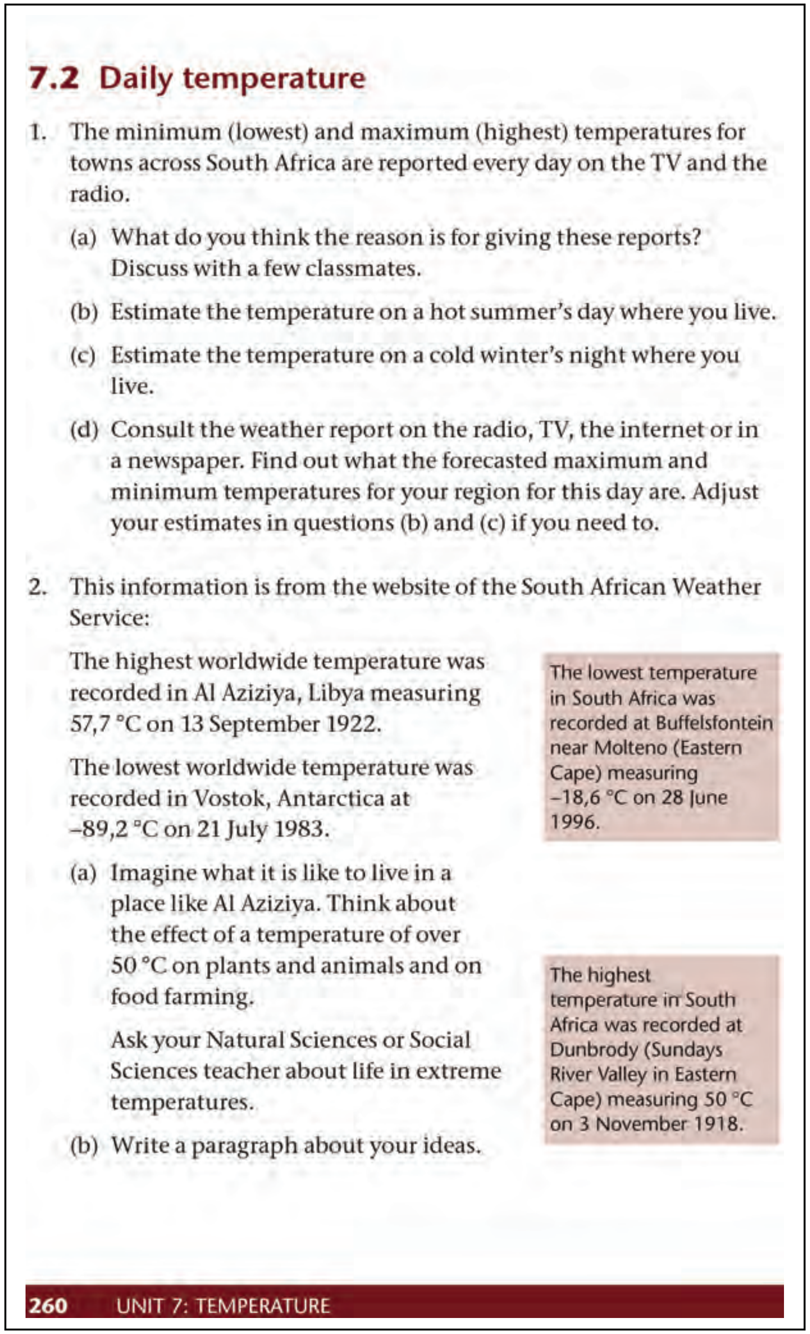
Notes on questions

For question 1(d) learners have to consult a weather report on the radio, TV, the internet or in a newspaper. They need to find out what the forecasted maximum and minimum temperatures for their region, for the day you teach this lesson, are.

In question 2, learners investigate and write a paragraph about what it is like to live in a very hot place like Al Aziziya. Make sure the learners know that the temperature of 57,7 °C is the temperature *inside* a house or classroom, not out in the sunshine. You can prompt learners' writing with questions such as: *"What clothes would you wear?", "How much would you need to drink?", "If you left a bottle of water on a table, what would the temperature of the water be?", "What could you do to cool off?"*

Answers

1. (a) Learners discuss reasons for national, daily temperature reports with a few classmates. Reasons could be, for example, that farmers might want to bring sheep into a sheltered place if snow is coming, or they might worry about frost at night that will spoil the fruit crop, or people might want to know whether they should wear warm or cool clothes to work.
   (b) Estimates will differ and depend on the region where learners live. Typical good estimates could be 30 °C to 40 °C.
   (c) Estimates will differ and depend on the region where learners live. Typical good estimates will be –5 °C to 5 °C.
   (d) Learners must adjust their estimates in questions (b) and (c) if necessary.
2. (a)–(b) Learners investigate and write a paragraph about what it is like to live in a very hot place like Al Aziziya.



7.2 Daily temperature

1. The minimum (lowest) and maximum (highest) temperatures for towns across South Africa are reported every day on the TV and the radio.
   (a) What do you think the reason is for giving these reports? Discuss with a few classmates.
   (b) Estimate the temperature on a hot summer's day where you live.
   (c) Estimate the temperature on a cold winter's night where you live.
   (d) Consult the weather report on the radio, TV, the internet or in a newspaper. Find out what the forecasted maximum and minimum temperatures for your region for this day are. Adjust your estimates in questions (b) and (c) if you need to.

2. This information is from the website of the South African Weather Service:

   The highest worldwide temperature was recorded in Al Aziziya, Libya measuring 57,7 °C on 13 September 1922.

   The lowest worldwide temperature was recorded in Vostok, Antarctica at –89,2 °C on 21 July 1983.

   (a) Imagine what it is like to live in a place like Al Aziziya. Think about the effect of a temperature of over 50 °C on plants and animals and on food farming.

   Ask your Natural Sciences or Social Sciences teacher about life in extreme temperatures.

   (b) Write a paragraph about your ideas.

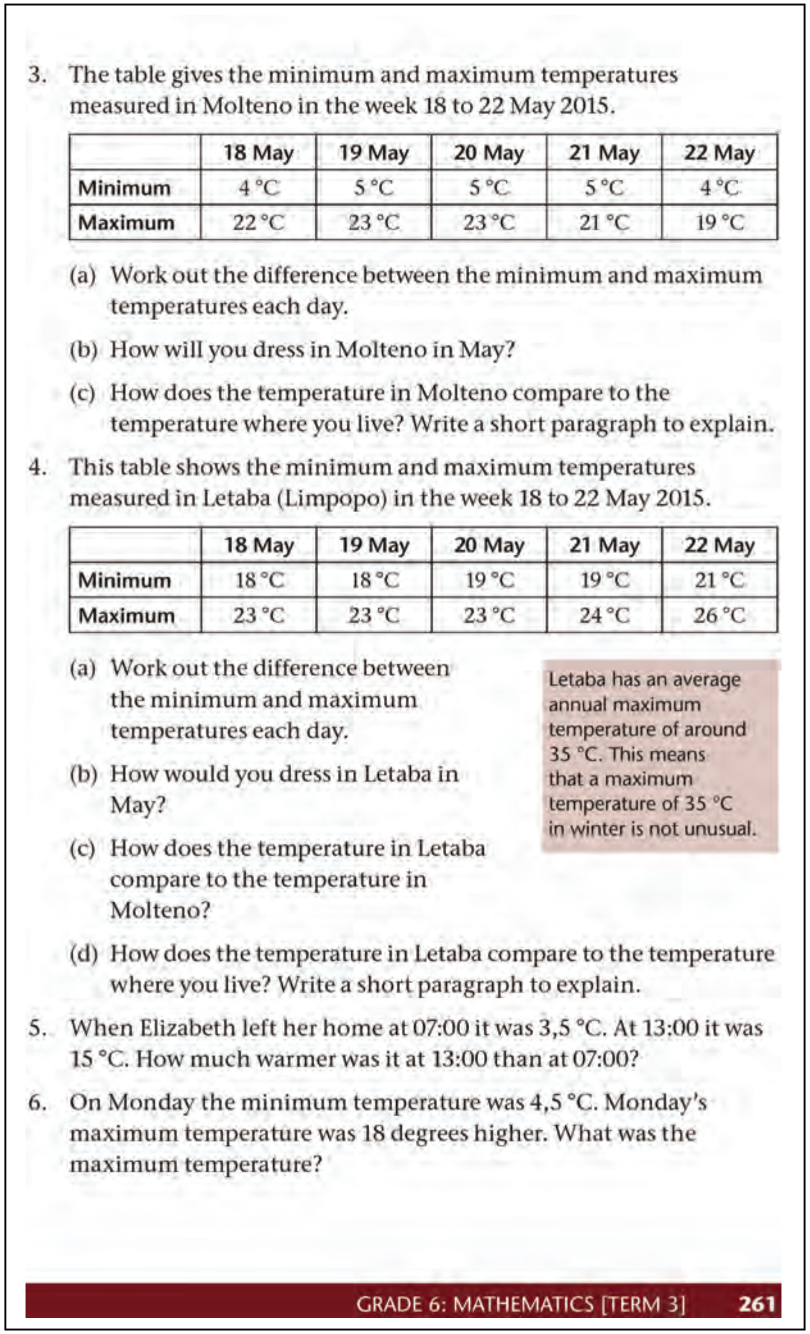
The lowest temperature in South Africa was recorded at Buffelsfontein near Molteno (Eastern Cape) measuring –18,6 °C on 28 June 1996.

The highest temperature in South Africa was recorded at Dunbrody (Sundays River Valley in Eastern Cape) measuring 50 °C on 3 November 1918.

260 UNIT 7: TEMPERATURE

Answers

3. (a) 18 May: 18 °C; 19 May: 18 °C; 20 May: 18 °C; 21 May: 16 °C; 22 May: 15 °C
   (b) Dress up warmly for the mornings and evenings, and probably take off the warm sweater or jersey in the middle of the day.
   (c) Learners write a short paragraph to compare the temperature of where they live to that of Molteno.
4. (a) 18 May: 5 °C; 19 May: 5 °C; 20 May: 4 °C; 21 May: 5 °C; 22 May: 5 °C
   (b) Learners should suggest wearing cool clothes; more summer-oriented clothing.
   (c) The temperature difference in Letaba is much smaller, and the evenings and early mornings are not as cold as in Molteno.
   (d) Learners write a short paragraph to compare the temperature of where they live to that of Letaba.
5. 11,5 °C
6. 22,5 °C



3. The table gives the minimum and maximum temperatures measured in Molteno in the week 18 to 22 May 2015.

	18 May	19 May	20 May	21 May	22 May
Minimum	4 °C	5 °C	5 °C	5 °C	4 °C
Maximum	22 °C	23 °C	23 °C	21 °C	19 °C

   (a) Work out the difference between the minimum and maximum temperatures each day.
   (b) How will you dress in Molteno in May?
   (c) How does the temperature in Molteno compare to the temperature where you live? Write a short paragraph to explain.

4. This table shows the minimum and maximum temperatures measured in Letaba (Limpopo) in the week 18 to 22 May 2015.

	18 May	19 May	20 May	21 May	22 May
Minimum	18 °C	18 °C	19 °C	19 °C	21 °C
Maximum	23 °C	23 °C	23 °C	24 °C	26 °C

   (a) Work out the difference between the minimum and maximum temperatures each day.
   (b) How would you dress in Letaba in May?
   (c) How does the temperature in Letaba compare to the temperature in Molteno?
   (d) How does the temperature in Letaba compare to the temperature where you live? Write a short paragraph to explain.

Letaba has an average annual maximum temperature of around 35 °C. This means that a maximum temperature of 35 °C in winter is not unusual.

5. When Elizabeth left her home at 07:00 it was 3,5 °C. At 13:00 it was 15 °C. How much warmer was it at 13:00 than at 07:00?

6. On Monday the minimum temperature was 4,5 °C. Monday's maximum temperature was 18 degrees higher. What was the maximum temperature?

GRADE 6: MATHEMATICS [TERM 3] 261

Grade 6 Term 3 Unit 8 Percentages

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
8.1 Working with hundredths	Understanding the meaning of percentage	262 to 263
8.2 Finding percentages of whole numbers	Finding the percentage of a whole number	263 to 265
8.3 Apply your knowledge	Word problems involving percentage	265 to 267

CAPS time allocation	5 hours
CAPS page references	16 to 17 and 267

Mathematical background

Percentage is yet another way to represent fractions.

- In the common fraction notation, any denominator can be used, for example $\frac{2}{5}$, $\frac{4}{25}$ or $\frac{5}{8}$.
- In the percentage notation, only 100 is used as a denominator, hence $\frac{2}{5}$, $\frac{4}{25}$ and $\frac{5}{8}$ are represented as $\frac{40}{100}$, $\frac{16}{100}$ and $\frac{62,5}{100}$. Written in the percentage notation that is 40%, 16% and 62,5%.

The percentage symbol (%) means hundredths.

Percentage is especially useful when fraction parts of different quantities have to be compared. For example, you may want to compare the following test scores.

A learner scored 43 out of 50 marks in Test 1, and the same learner scored 23 out of 25 marks in Test 2. In which test did the learner perform better?

Test 1: $\frac{43}{50} = \frac{86}{100} = 86\%$

Test 2: $\frac{23}{25} = \frac{92}{100} = 92\%$

Therefore the learner performed better in Test 2.

8.1 Working with hundredths

Critical knowledge

The key to understanding percentage is to know that “percent” and “hundredths” are synonyms. To say “*I attained 60% for a test*” means exactly the same as “*I attained 60 hundredths of the marks*”.

However, percentage is not only used in situations where a whole quantity is divided into 100 equal parts (hundredths). If a loaf of bread is divided into 5 equal pieces each piece is one fifth of the loaf, which can also be described as 20 hundredths or 20% of the loaf.

Possible misconceptions

Because percentages are given as whole numbers, learners may forget that they refer to parts of a whole. The diagrams in question 2 are intended to provide a visual image of the relative size of a range of percentages, and to emphasise that percentages are fractions.

Teaching guidelines

Question 1 is intended to activate knowledge that learners may already have about percentage, and should be followed by some class discussion. You may ask further questions like: “*If you were given 12 out of 20 for a test, what percentage is that?*”

Notes on questions

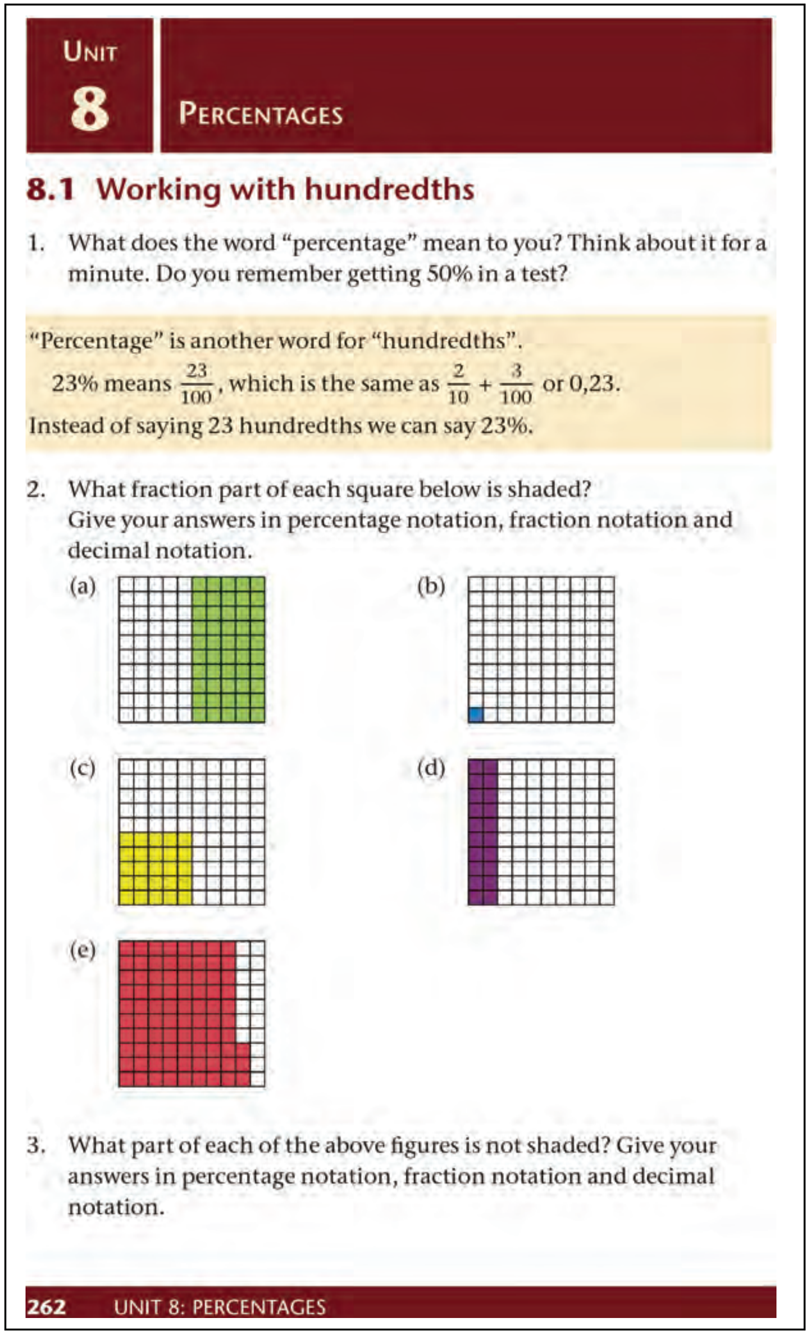
For answers 2(a)–(e) learners can count the small squares, but they can also work it out by multiplying the number of columns by the number of rows. For example, in question 2(c) there are 5 × 5 = 25 small squares.

Answers

1. Learners give their own definitions of percentage.

2. (a) 50%; $\frac{50}{100}$; 0,50 (b) 1% ; $\frac{1}{100}$; 0,01 (c) 25%; $\frac{25}{100}$; 0,25
(d) 20%; $\frac{20}{100}$; 0,20 (e) 83%; $\frac{83}{100}$; 0,83

3. (a) 50%; $\frac{50}{100}$; 0,50 (b) 99%; $\frac{99}{100}$; 0,99 (c) 75%; $\frac{75}{100}$; 0,75
(d) 80%, $\frac{80}{100}$; 0,8 (e) 17%; $\frac{17}{100}$; 0,17



UNIT 8 PERCENTAGES

8.1 Working with hundredths

1. What does the word “percentage” mean to you? Think about it for a minute. Do you remember getting 50% in a test?

“Percentage” is another word for “hundredths”.
23% means $\frac{23}{100}$, which is the same as $\frac{2}{10} + \frac{3}{100}$ or 0,23.
Instead of saying 23 hundredths we can say 23%.

2. What fraction part of each square below is shaded?
Give your answers in percentage notation, fraction notation and decimal notation.

3. What part of each of the above figures is not shaded? Give your answers in percentage notation, fraction notation and decimal notation.

262 UNIT 8: PERCENTAGES

Mathematical notes

A decimal with two digits after the decimal comma, for example 0,62, can be interpreted in two ways: as tenths and hundredths ($\frac{6}{10} + \frac{2}{100}$) or as hundredths only ($\frac{62}{100}$).

To convert a decimal to the percentage notation, it should be interpreted as a number of hundredths. For example 0,62 is 62 hundredths and that is 62%.

To convert a common fraction to the percentage notation, the fraction must be expressed in terms of hundredths. This means that an equivalent fraction with denominator 100 must be formed. For example, to express $\frac{9}{25}$ as a percentage, you need to realise that it is equivalent to 36 hundredths, which can be expressed as 36%.

Teaching guidelines

Demonstrate the conversion of decimals and common fractions to percentage notation with a few examples before the learners engage with questions 4 and 5.

Answers

4. (a) 45% (b) 70% (c) 3% (d) 95% (e) 20% (f) 250%
5. (a) 40% (b) 70% (c) 75% (d) 250% (e) 65% (f) 122%
(g) 56% (h) 120%

8.2 Finding percentages of whole numbers

Teaching guidelines

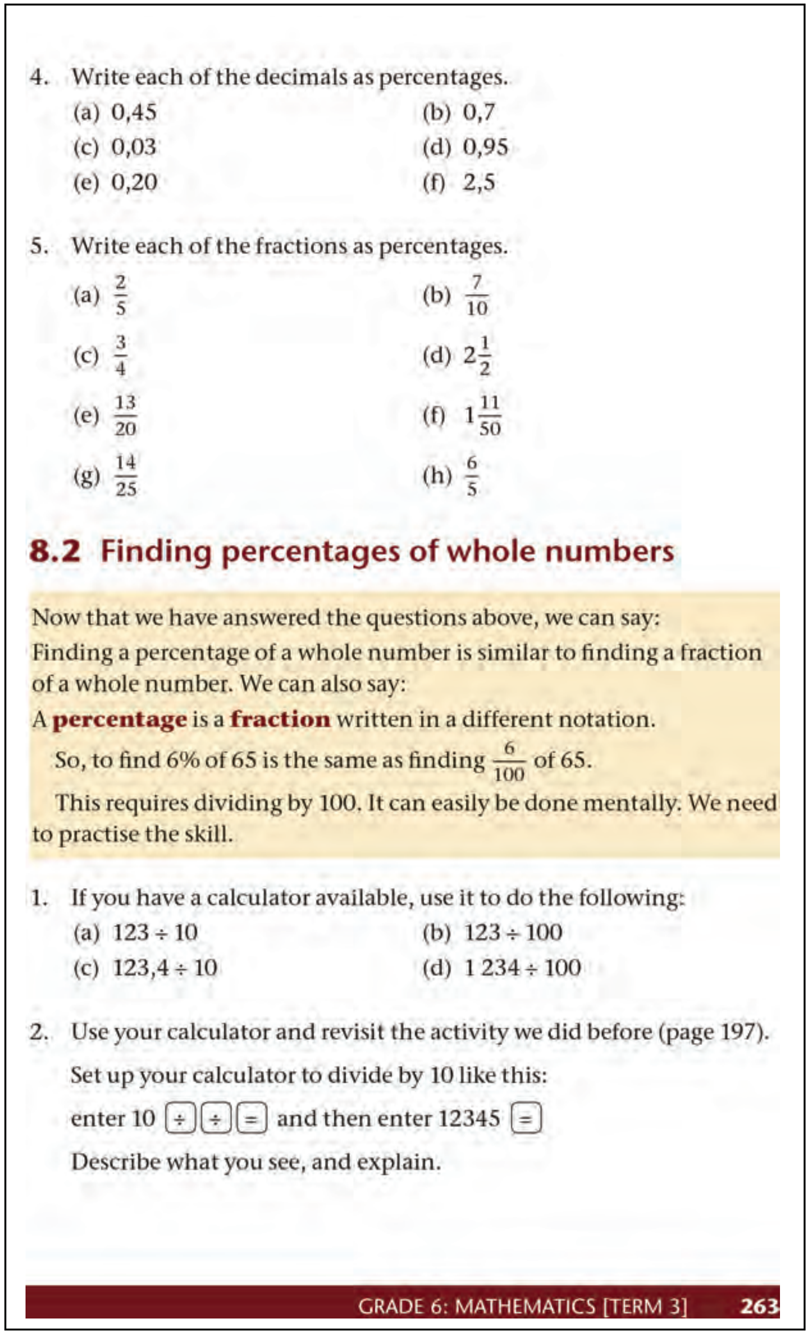
It is critical that learners come to understand a **percentage** of a whole number as a **fraction** of the whole number. To promote this, let learners calculate some simple fraction parts of whole numbers and only then introduce percentages of whole numbers. For example, let learners calculate the following:

$\frac{3}{4}$ of R24 $\frac{2}{5}$ of R40 $\frac{4}{10}$ of R40 $\frac{3}{10}$ of R700 $\frac{7}{100}$ of R600

Demonstrate some of these calculations on the board and emphasise that the number is divided by the denominator of the fraction to find out how big each of the fraction parts is, then multiplied by the numerator to establish how much the given fraction of the number is. For example, to calculate $\frac{3}{4}$ of R24 we first calculate 24 ÷ 4 to establish how much one quarter of 24 is, then multiply by 3 to establish how much 3 quarters of 24 is.

Answers

1. (a) 12,3 (b) 1,23 (c) 12,34 (d) 12,34



4. Write each of the decimals as percentages.
   (a) 0,45 (b) 0,7
   (c) 0,03 (d) 0,95
   (e) 0,20 (f) 2,5

5. Write each of the fractions as percentages.
   (a) $\frac{2}{5}$ (b) $\frac{7}{10}$
   (c) $\frac{3}{4}$ (d) $2\frac{1}{2}$
   (e) $\frac{13}{20}$ (f) $1\frac{11}{50}$
   (g) $\frac{14}{25}$ (h) $\frac{6}{5}$

8.2 Finding percentages of whole numbers

Now that we have answered the questions above, we can say:
Finding a percentage of a whole number is similar to finding a fraction of a whole number. We can also say:
A **percentage** is a **fraction** written in a different notation.

So, to find 6% of 65 is the same as finding $\frac{6}{100}$ of 65.

This requires dividing by 100. It can easily be done mentally. We need to practise the skill.

1. If you have a calculator available, use it to do the following:
   (a) 123 ÷ 10 (b) 123 ÷ 100
   (c) 123,4 ÷ 10 (d) 1 234 ÷ 100

2. Use your calculator and revisit the activity we did before (page 197).
   Set up your calculator to divide by 10 like this:
   enter 10 $\boxed{\div}\boxed{\div}\boxed{=}$ and then enter 12345 $\boxed{=}$
   Describe what you see, and explain.

Mental mathematics activity

The following mental mathematics activity may be useful at the beginning of the section. It may help learners to think about ideas on how to multiply and divide by powers of 10. You may write these questions on the board.

How much is:

8 × 10	80 ÷ 10	80 × 10	800 ÷ 10	60 × 10	60 ÷ 10
7 × 100	70 × 10	7 × 1 000	700 ÷ 10	7 000 ÷ 10	70 ÷ 10
34 × 10	340 × 10	34 × 100	34 × 1 000	340 × 100	340 × 1 000
340 ÷ 10	3 400 ÷ 100	34 000 ÷ 100	3 400 ÷ 10	34 000 ÷ 100	

Mathematical notes

Teachers sometimes inappropriately refer to the "shifting of the comma" when dividing by 10, 100 and higher powers of 10. In reality it is the place value of the digits that decrease when divided. It is useful to think of dividing each place value part of the number separately by 10, 100, etc. For example:

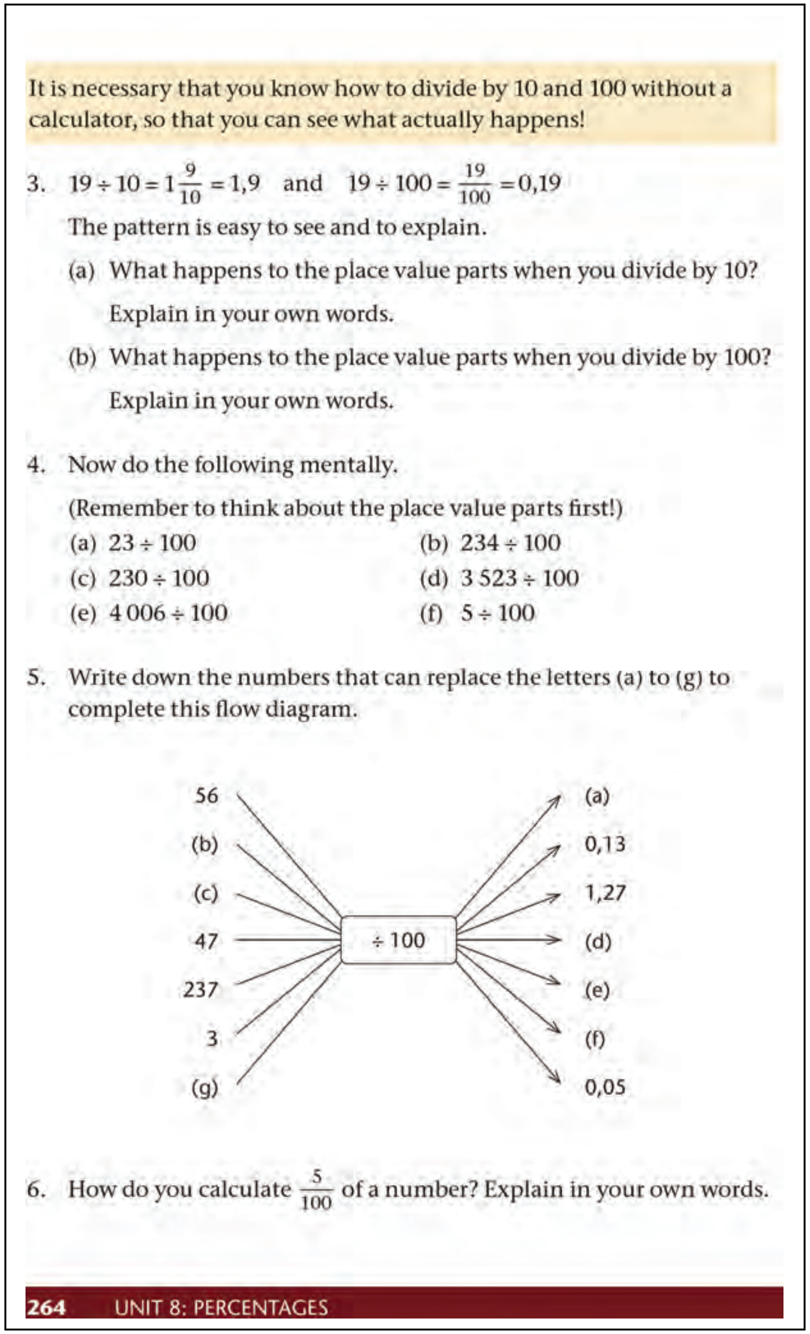
$$\begin{aligned} 563 \div 10 &= (500 + 60 + 3) \div 10 \\ &= 500 \div 10 + 60 \div 10 + 3 \div 10 \\ &= 50 + 6 + \tfrac{3}{10} \text{ or } 0{,}3 = 56{,}3 \end{aligned}$$

Teaching guidelines

Demonstrate the calculation of 563 ÷ 10 by breaking down and building up as shown above on the board after learners have completed question 3. Also demonstrate the calculation of 563 ÷ 100 in the same way.

Answers

3. (a)–(b) Learners may describe the apparent shifting of the digits to other place value positions, or explain that the number indicated by each digit changes because it is reduced 10 times or 100 times.
4. (a) 0,23 (b) 2,34 (c) 2,3 (d) 35,23 (e) 40,06 (f) 0,05
5. (a) 0,56 (b) 13 (c) 127 (d) 0,47 (e) 2,37 (f) 0,03
   (g) 5
6. Divide the number by 100 (to establish how much one hundredth of the number is), then multiply by 5. Alternatively, multiply the number by 5 then divide by 100.



It is necessary that you know how to divide by 10 and 100 without a calculator, so that you can see what actually happens!

3. $19 \div 10 = 1\frac{9}{10} = 1{,}9$ and $19 \div 100 = \frac{19}{100} = 0{,}19$

   The pattern is easy to see and to explain.

   (a) What happens to the place value parts when you divide by 10? Explain in your own words.

   (b) What happens to the place value parts when you divide by 100? Explain in your own words.

4. Now do the following mentally.

   (Remember to think about the place value parts first!)

   (a) 23 ÷ 100 (b) 234 ÷ 100
   (c) 230 ÷ 100 (d) 3 523 ÷ 100
   (e) 4 006 ÷ 100 (f) 5 ÷ 100

5. Write down the numbers that can replace the letters (a) to (g) to complete this flow diagram.

6. How do you calculate $\frac{5}{100}$ of a number? Explain in your own words.

Answers

7. (a) 10 (b) 150 (c) 3,8
(d) 1,6 (e) 10

8. (a) 3,9 (b) 60 (c) 54
(d) 61 (e) 1,8 (f) R21

8.3 Apply your knowledge

Notes on questions

In question 1 learners can first calculate 20% of the original amount and then subtract it from of the original amount (which is 100%). Alternatively, 20% off means 80% is left in all the cases in question 1, so time can be saved by simply calculating the remaining 80%.

100% of R400 – 20% of R400	or you can calculate 80% of R400
= R400 – R80	80% of R400
= R320	= R320

Possible misconceptions

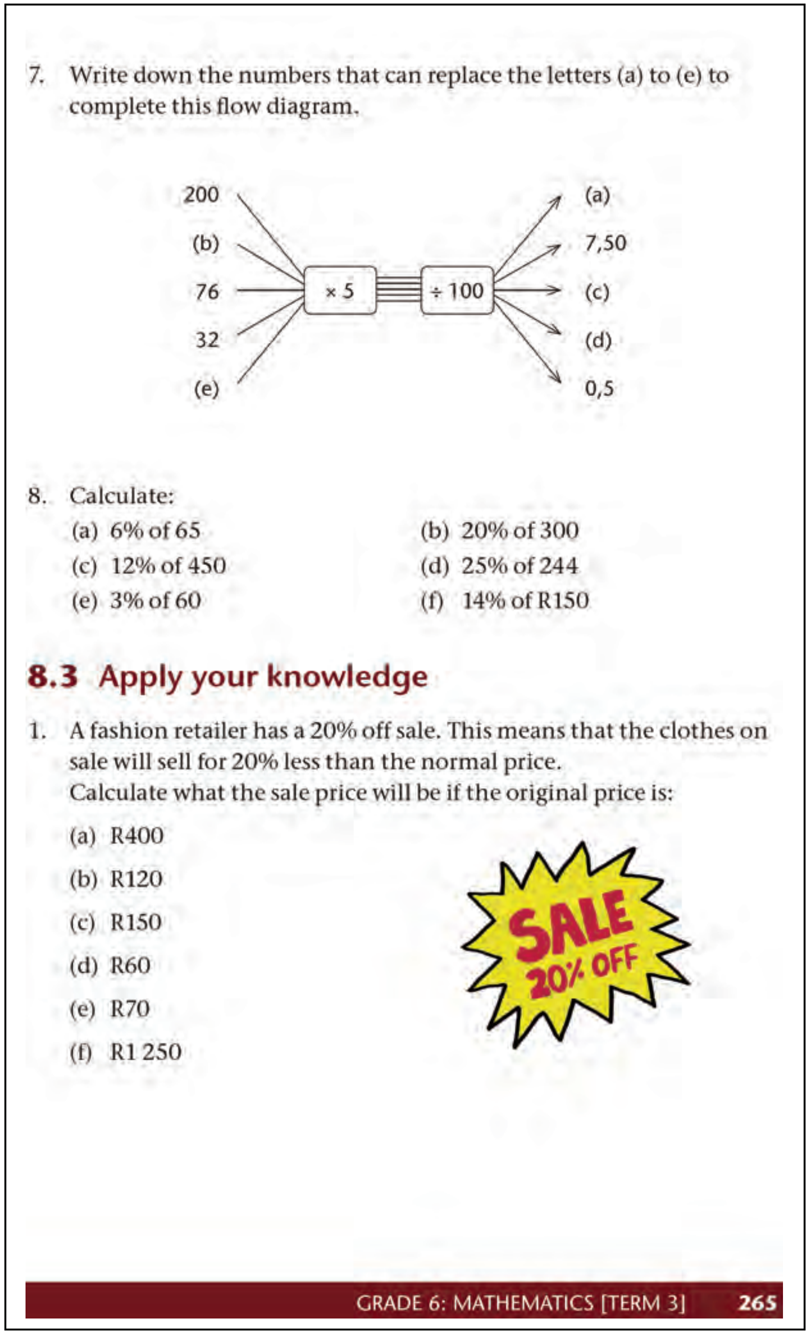
When people say “the price has gone up by 25%”, they mean that 25% of the original price has been added. The price is now 125% of what it was.

For example, if a price of R4 000 increases by 25%, it will increase with R1 000 and become R5 000.

Note that if the price was to go down to R4 000 again, it would only be a 20% decrease, as R1 000 is 20% of R5 000.

Answers

1. (a) R320 (b) R96 (c) R120 (d) R48 (e) R56 (f) R1 000



7. Write down the numbers that can replace the letters (a) to (e) to complete this flow diagram.

8. Calculate:
(a) 6% of 65 (b) 20% of 300
(c) 12% of 450 (d) 25% of 244
(e) 3% of 60 (f) 14% of R150

8.3 Apply your knowledge

1. A fashion retailer has a 20% off sale. This means that the clothes on sale will sell for 20% less than the normal price.
Calculate what the sale price will be if the original price is:
(a) R400
(b) R120
(c) R150
(d) R60
(e) R70
(f) R1 250

Possible misconceptions

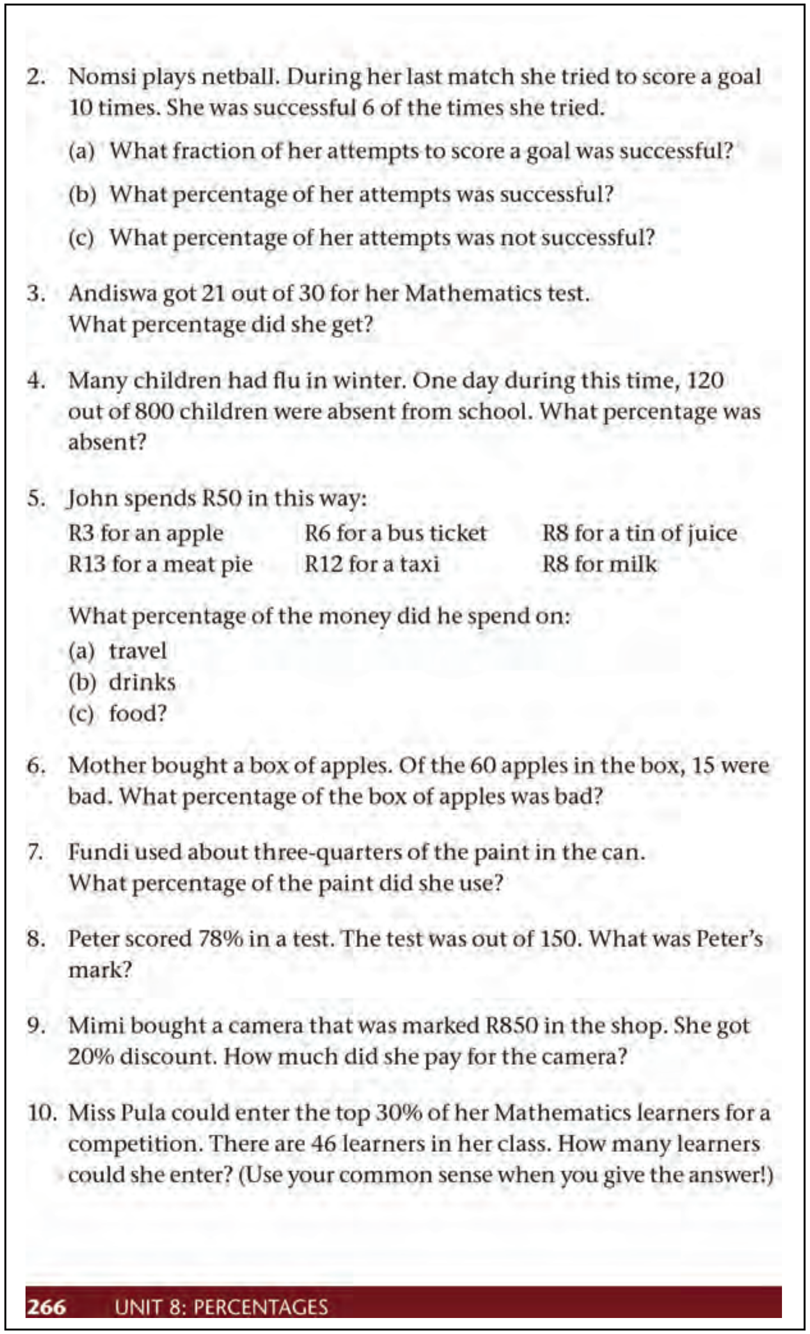
Some learners may find question 7 confusing because the size of the can is not indicated. One does not need to know how big the can is to know that $\frac{3}{4}$ of the can will be left after $\frac{1}{4}$ of the can has been subtracted. In this question the can is a unit of measurement, and $\frac{3}{4}$ and $\frac{75}{100}$ represent the same amount of the can.

Notes on questions

Question 10 shows that Miss Pula could enter 13,8 learners for a competition. In some cases it is clear-cut whether such an answer should be rounded up or down, as in: "*How many workers can be afforded at full pay*?" In question 10 it could be argued that the competition probably rounded up the learners to the nearest one, i.e. 14 learners. On the other hand, the competition might have limited space, so only 13 learners can go.

Answers

2. (a) $\frac{6}{10}$ (b) 60% (c) 40%
3. 70%
4. 15%
5. (a) 36% (b) 32% (c) 32%
6. 25%
7. 75%
8. 117 (out of 150)
9. R680
10. 13 or 14 learners, depending on Miss Pula's justification; it cannot be "13,8 learners".



2. Nomsi plays netball. During her last match she tried to score a goal 10 times. She was successful 6 of the times she tried.
   (a) What fraction of her attempts to score a goal was successful?
   (b) What percentage of her attempts was successful?
   (c) What percentage of her attempts was not successful?
3. Andiswa got 21 out of 30 for her Mathematics test. What percentage did she get?
4. Many children had flu in winter. One day during this time, 120 out of 800 children were absent from school. What percentage was absent?
5. John spends R50 in this way:
   R3 for an apple R6 for a bus ticket R8 for a tin of juice
   R13 for a meat pie R12 for a taxi R8 for milk

   What percentage of the money did he spend on:
   (a) travel
   (b) drinks
   (c) food?
6. Mother bought a box of apples. Of the 60 apples in the box, 15 were bad. What percentage of the box of apples was bad?
7. Fundi used about three-quarters of the paint in the can. What percentage of the paint did she use?
8. Peter scored 78% in a test. The test was out of 150. What was Peter's mark?
9. Mimi bought a camera that was marked R850 in the shop. She got 20% discount. How much did she pay for the camera?
10. Miss Pula could enter the top 30% of her Mathematics learners for a competition. There are 46 learners in her class. How many learners could she enter? (Use your common sense when you give the answer!)

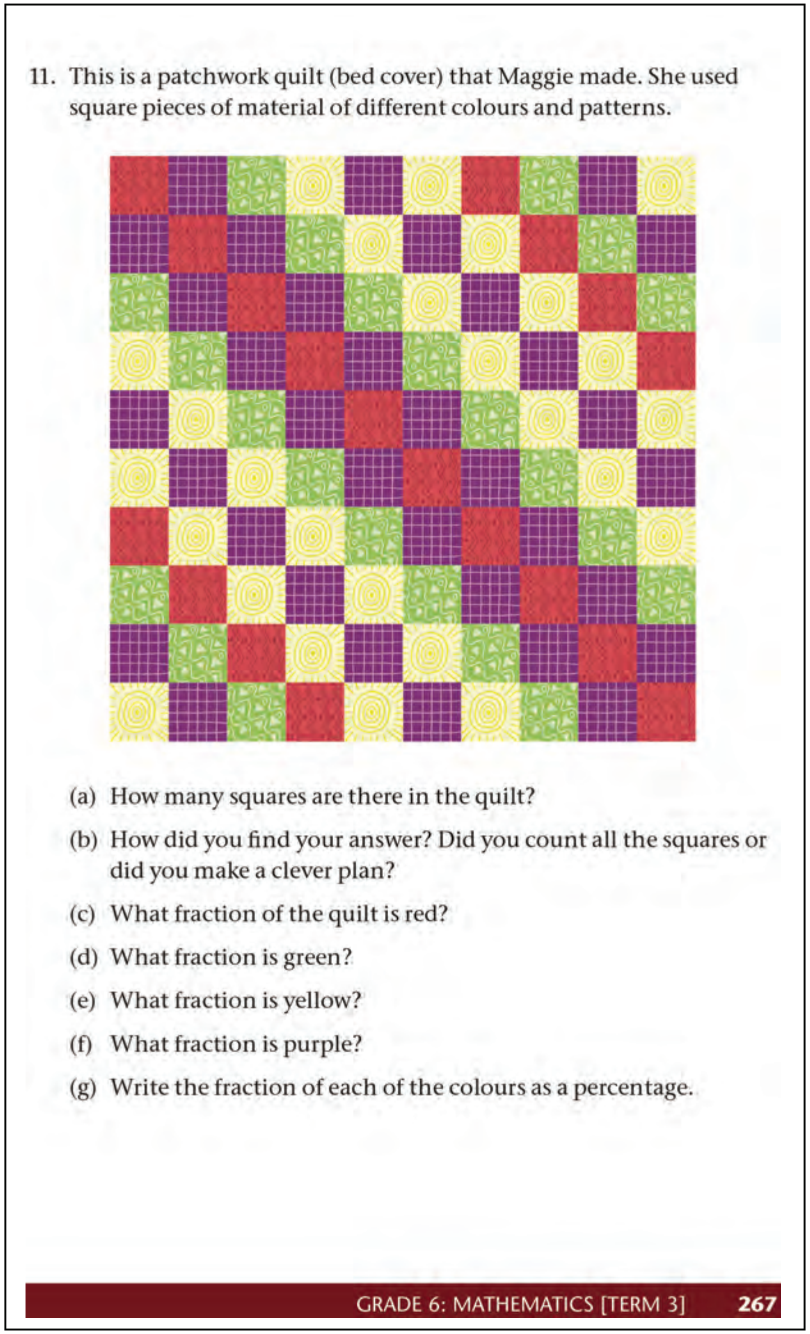
266 UNIT 8: PERCENTAGES

Teaching guidelines

The figure is symmetrical about the red diagonal as well as the other (less conspicuous) diagonal from bottom left to top right. The sets of various colours, other than red, can be counted on one side of the red diagonal and multiplied by 2.

Answers

11. (a) 100
    (b) One clever plan is to count the rows and the columns and then multiply them.
    (c) $\frac{9}{50}$
    (d) $\frac{11}{50}$
    (e) $\frac{13}{50}$
    (f) $\frac{17}{50}$
    (g) Red: 18%
    Green: 22%
    Yellow: 26%
    Purple: 34%



11. This is a patchwork quilt (bed cover) that Maggie made. She used square pieces of material of different colours and patterns.

(a) How many squares are there in the quilt?

(b) How did you find your answer? Did you count all the squares or did you make a clever plan?

(c) What fraction of the quilt is red?

(d) What fraction is green?

(e) What fraction is yellow?

(f) What fraction is purple?

(g) Write the fraction of each of the colours as a percentage.

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Grade 6 Term 3 Unit 9 Data handling

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
9.1 Representing data	Drawing a double bar graph; finding and interpreting the median value	268 to 270
9.2 Analysing and interpreting data in a pictograph	Understanding one-to-many pictographs; interpreting trends across different categories	270 to 271
9.3 Interpreting and reporting data	Comparing pie charts; estimating percentages; judging the correctness of statements; writing short data reports	272 to 276
9.4 Project	Designing a questionnaire, gathering, representing, analysing and interpreting data	277

CAPS time allocation	9 hours
CAPS page references	30 to 31 and 268 to 269

Mathematical background

Data are bits of information about a particular context. We ask questions about a situation or context, which lead to the collection of information. The way in which the data are organised and represented (and the further questions that we ask) allow us to see trends in the data.

In data handling we work with large amounts of information related to particular contexts. Instead of focusing on each bit of information separately, the way we organise, represent and analyse the data provide us with ways to talk about it in general. We look at the data in a global way and identify trends or characteristics that describe it.

Some ideas that differentiate data handling from other topics in Mathematics:

- The answer to data questions is in the information from lots of data gathered.

  Data handling is necessary where measurements and frequencies vary. One measurement cannot provide accurate information about a situation. Lots of different data can be confusing, so we organise the data that we collect in different ways. Different representations make different trends more visible.
- The numbers we use in data handling always have some unit of measurement, or some description of the category to which they belong.

  In Mathematics, learners work mostly with abstract numbers. In data handling the numbers must be interpreted in a context. The number 13 can be 13 learners or 13 cm or 13 goals.
- Data questions are always answered with a story about the context.

  Data handling starts when we need to answer a question about a situation where the property we look at varies. The numerical answers we get through data handling must be interpreted to answer the question about the situation.

Resources

Graph or grid paper (see Addendum); a soccer or netball ball and measuring tape (optional)

9.1 Representing data

Mathematical notes

The way in which data are organised and represented, impacts on the visibility of the trends in the data. It is easier to see that the ball tends to bounce higher at higher temperatures in the bar graph on page 269 than in the table on page 268 of the Learner Book. Graphs provide a picture of the data. This picture facilitates analysis.

In this data set the bounces are the categories. The heights are the numerical data.

We can also analyse data by looking for typical values that represent the data and by examining how clustered or spread the data is. The median and the mode are both summary values that can be used to represent a data set. It is important to note that the characteristics of the data set will determine whether and how you should use the mode, mean or median to identify central tendencies.

Teaching guidelines

Prepare the table and graphs on the board or on posters to use for class discussions. As learners work through the data remind them that they are trying to find both the general trend of the data and typical characteristics of the data.

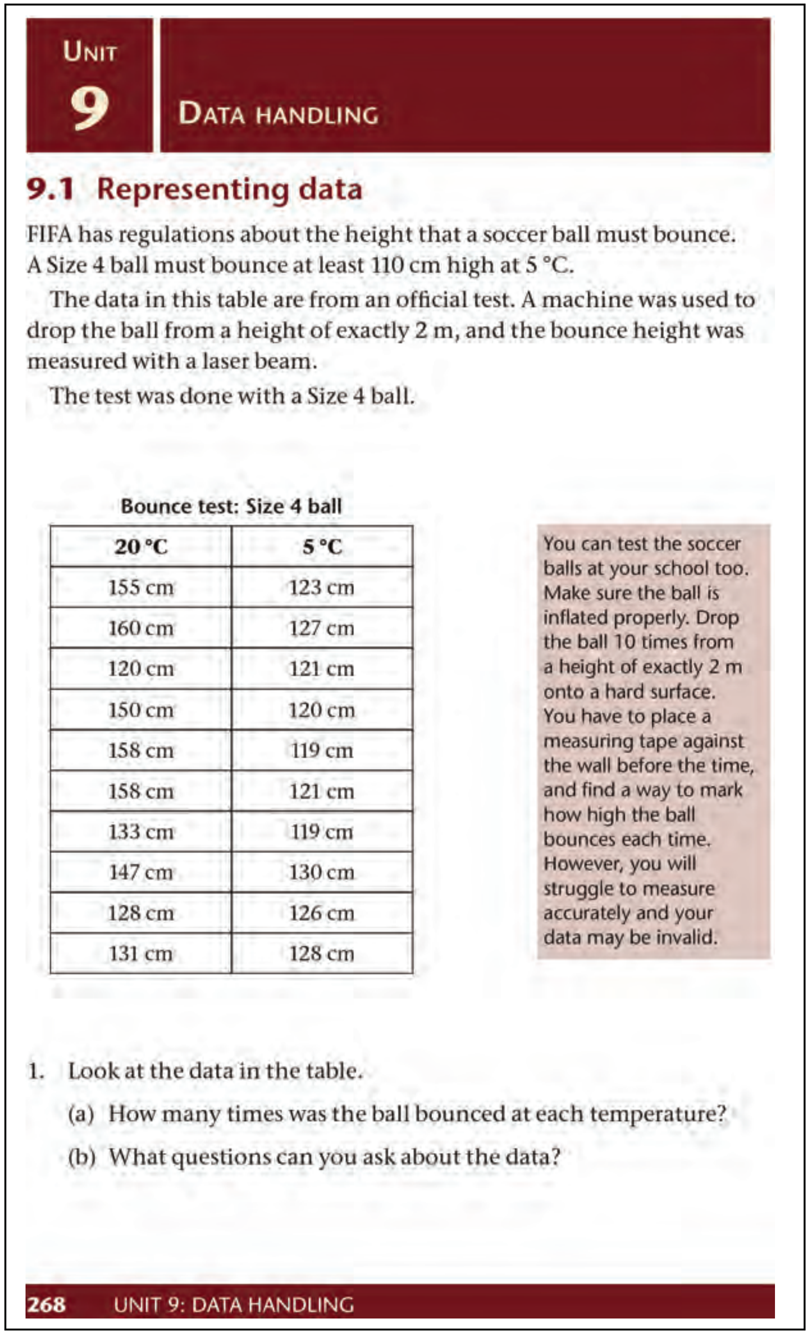
Possible misconceptions

Learners may only pay attention to the very long bars on the graph and make hasty conclusions. Learners may also use the highest bounce at 20 °C, or the lowest bounce at 5 °C, to summarise the bounce heights, rather than a representative height.

In data handling we look for trends in the data. Some learners may be concerned that the ball bounces higher at the lower temperature on bounce number 3. Explain that this one deviation does not distract from the general trend shown by the other nine bounces.

Answers

1. (a) 10 times
   (b) Answers will differ. The aim is to let learners discuss what information they could get from the data. Some examples of statistical questions are:
   How high does a ball typically bounce at 5 °C? And at 20 °C? (*Here we guide learners to look at central tendencies that best describe the height at 5 °C.*)
   How much higher does a ball bounce at 20 °C than at 5 °C?
   What is the range of heights the ball bounces at 5 °C?
   What is the range of heights the ball bounces at 20 °C?
   What is the most common height the ball bounces at 5 °C? And at 20 °C?



UNIT 9

DATA HANDLING

9.1 Representing data

FIFA has regulations about the height that a soccer ball must bounce. A Size 4 ball must bounce at least 110 cm high at 5 °C.

The data in this table are from an official test. A machine was used to drop the ball from a height of exactly 2 m, and the bounce height was measured with a laser beam.

The test was done with a Size 4 ball.

Bounce test: Size 4 ball

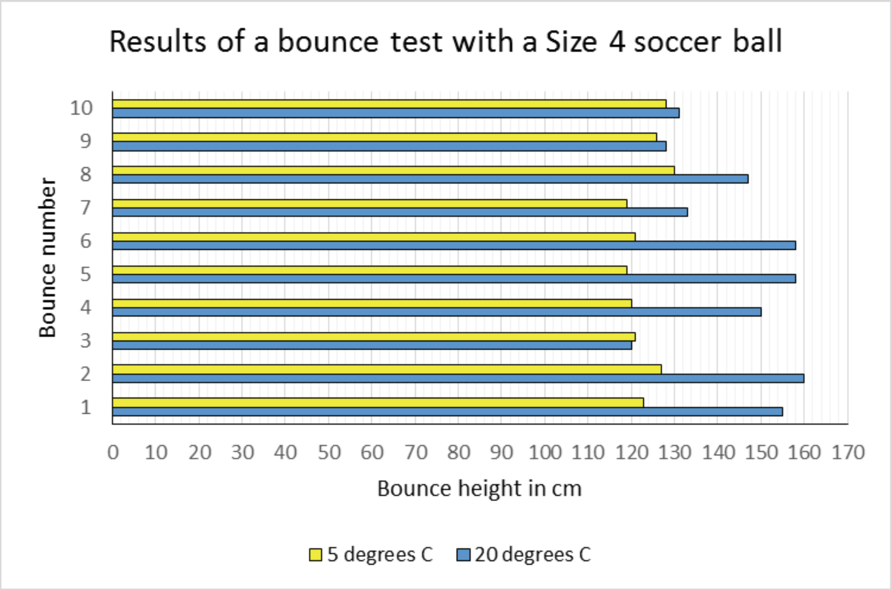
20 °C	5 °C
155 cm	123 cm
160 cm	127 cm
120 cm	121 cm
150 cm	120 cm
158 cm	119 cm
158 cm	121 cm
133 cm	119 cm
147 cm	130 cm
128 cm	126 cm
131 cm	128 cm

You can test the soccer balls at your school too. Make sure the ball is inflated properly. Drop the ball 10 times from a height of exactly 2 m onto a hard surface. You have to place a measuring tape against the wall before the time, and find a way to mark how high the ball bounces each time. However, you will struggle to measure accurately and your data may be invalid.

1. Look at the data in the table.
   (a) How many times was the ball bounced at each temperature?
   (b) What questions can you ask about the data?

Answers

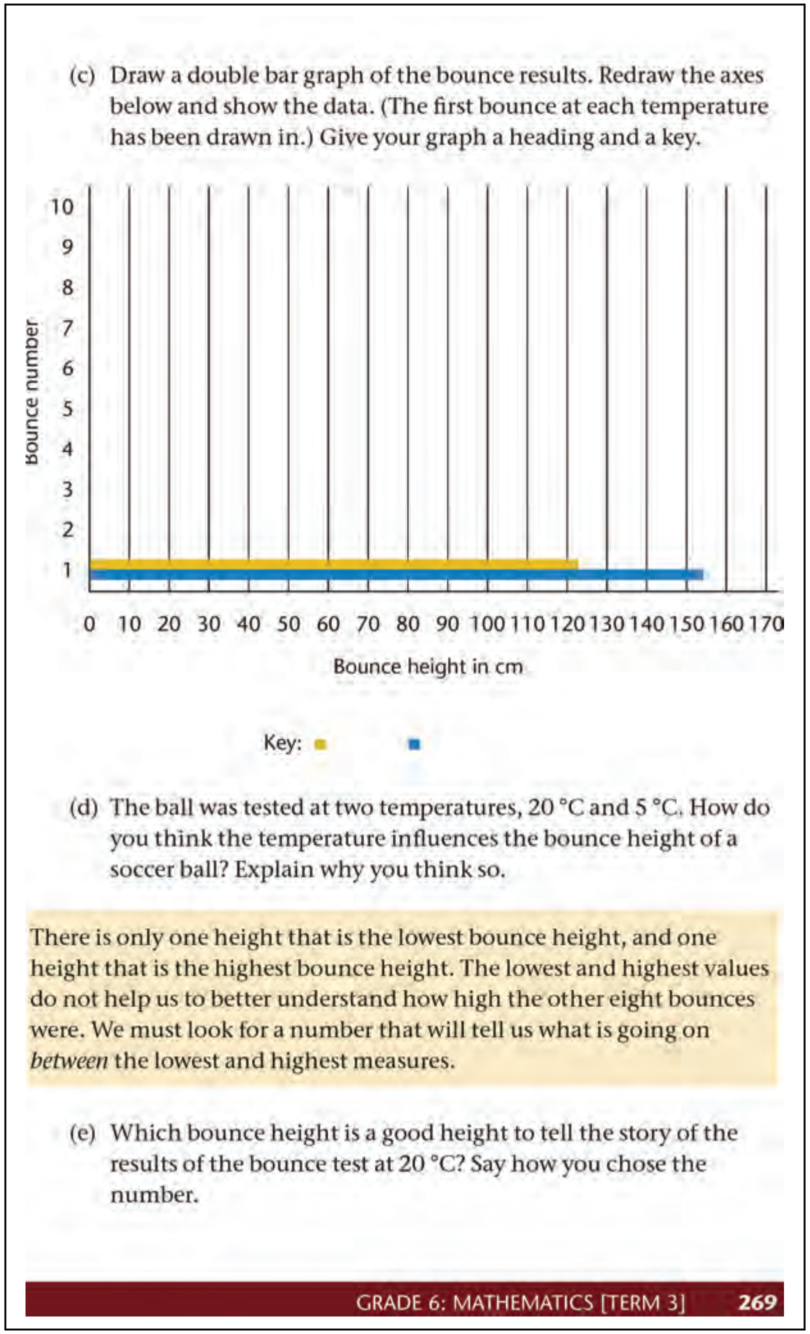
1. (c) Provide learners with suitable grid paper to draw the graph.



(d) Answers should interpret the graph and the raw data. For example:

In *five cases* the bounce height at 20 °C was *much higher* than at 5 °C – bounces number 1, 2, 4, 5 and 6 were all *more than 30 cm higher* at the high temperature than at the low temperature. For *another two* bounces (bounces number 7 and 8) the bounce heights were *about 15 cm higher* at the high temperature than at the low temperature. But in *two cases* (bounces number 9 and 10) the bounces at 20 °C were *only* a *few centimetres higher* than the bounces at 5 °C.
So, in *nine out of ten bounces* the bounce height at 20 °C was *higher* than at 5 °C, and in *one case out of ten* the bounce height at 20 °C was slightly lower than at the lower temperature. The test convinces us that the temperature makes a difference to bounce heights, and that in general balls bounce higher at higher temperatures.

(e) Answers may differ. About 125 cm is a good summary estimate for bounce heights at 5 °C. A value halfway between 120 cm and 160 cm (i.e. about 140 cm) is a good estimate of a representative bounce height at 20 °C. Some learners may use the highest or lowest bounce heights, but they are not representative. Other learners may choose the mode (158 cm is the mode for bounce heights at 20 °C). The data for 5 °C have two modes (119 cm and 121 cm occur twice). These modes are not representative either.



(c) Draw a double bar graph of the bounce results. Redraw the axes below and show the data. (The first bounce at each temperature has been drawn in.) Give your graph a heading and a key.

Key: 🟨 🟦

(d) The ball was tested at two temperatures, 20 °C and 5 °C. How do you think the temperature influences the bounce height of a soccer ball? Explain why you think so.

There is only one height that is the lowest bounce height, and one height that is the highest bounce height. The lowest and highest values do not help us to better understand how high the other eight bounces were. We must look for a number that will tell us what is going on *between* the lowest and highest measures.

(e) Which bounce height is a good height to tell the story of the results of the bounce test at 20 °C? Say how you chose the number.

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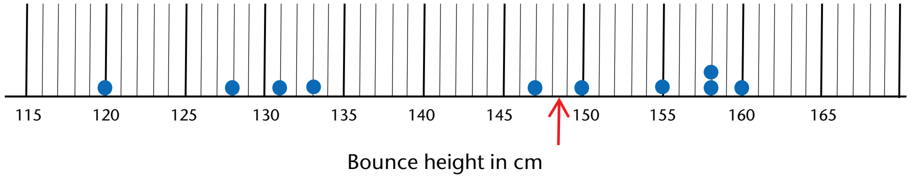
Answers

(f) The median bounce height at 20 °C is halfway between 147 cm and 150 cm. The median is 148,5 cm.



(g) At 20 ˚C the bounce heights ranged from 120 cm to 160 cm. Half of the bounce heights were higher than 148,5 cm, and half were lower than 148,5 cm.

2. Yes. The minimum and maximum bounce heights at 5 °C are taken correctly off the table. The median is correct – it is the height halfway between 121 cm and 123 cm.
3. The ball may bounce up to about 40 cm lower on cold mornings than on warm afternoons. We can see this by comparing the lengths that the blue bars extend beyond the yellow bars on the graph in 1(c).

9.2 Analysing and interpreting data in a pictograph

Mathematical notes

Pictographs are a visual representation of data using icons, pictures, symbols, etc. They are also referred to as pictograms, pictorial charts, pictorial graphs or picture graphs. The key that is used in pictographs may include a many to one representation (e.g. 🍎 = 20 apples; ◗ = 10 apples) or a one to one representation (e.g. 🍎 = 1 apple). The pictograph includes a title and labels for the axis. The columns or rows in pictographs reflect icons or pictures instead of bars.

Teaching guidelines

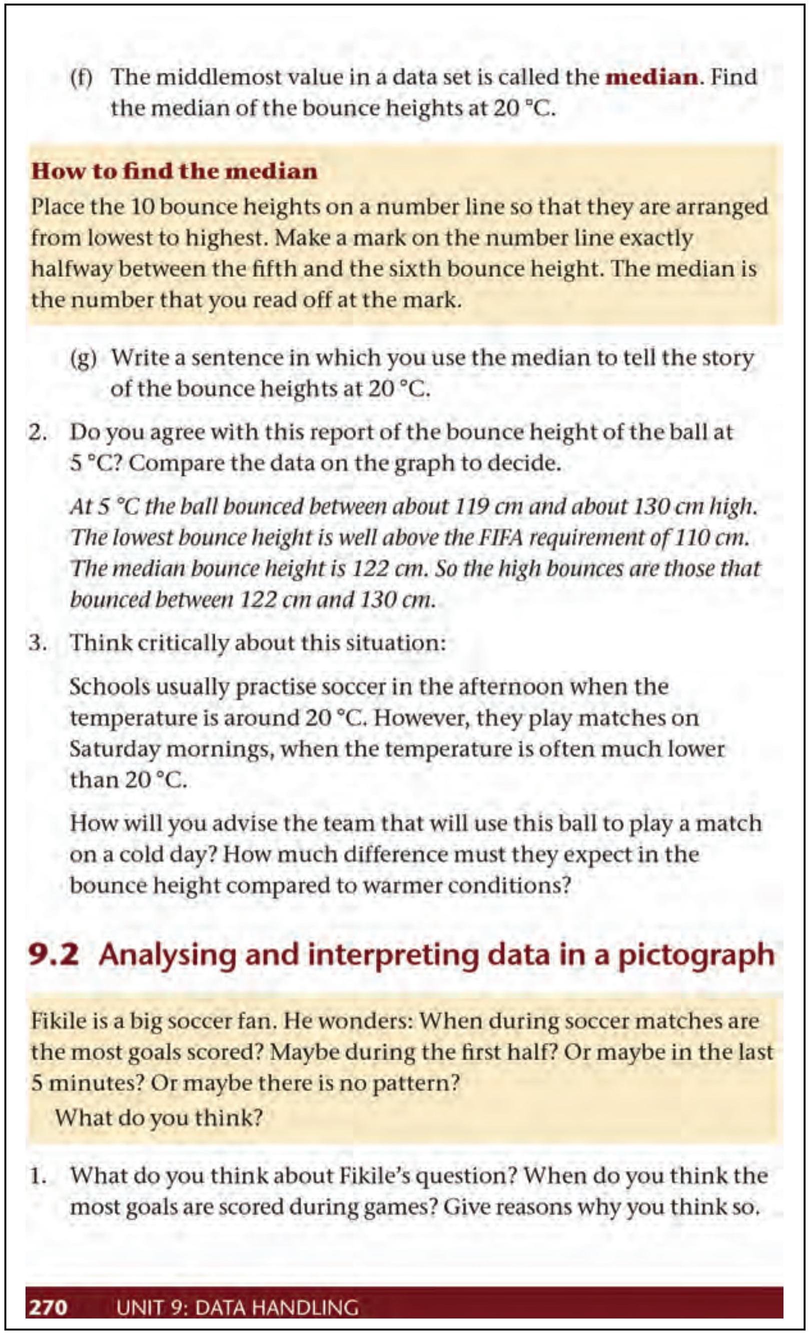
Prepare the graph on the board or on a poster for use during class discussion.

Notes on questions

In question 1 learners will have to give their opinions; they do not have enough evidence to reach a common conclusion. Further evidence is provided on page 271 of the Learner Book.

Answers

1. Opinions will differ. Learners may think that more goals are scored in the beginning of the first half when the players are fresh; or at the end of the first half because they want to end the first half in the lead; or at the end of the second half because they give everything to win. They need data to come to an agreement.



(f) The middlemost value in a data set is called the **median**. Find the median of the bounce heights at 20 °C.

How to find the median

Place the 10 bounce heights on a number line so that they are arranged from lowest to highest. Make a mark on the number line exactly halfway between the fifth and the sixth bounce height. The median is the number that you read off at the mark.

(g) Write a sentence in which you use the median to tell the story of the bounce heights at 20 °C.

2. Do you agree with this report of the bounce height of the ball at 5 °C? Compare the data on the graph to decide.

   At 5 °C the ball bounced between about 119 cm and about 130 cm high. The lowest bounce height is well above the FIFA requirement of 110 cm. The median bounce height is 122 cm. So the high bounces are those that bounced between 122 cm and 130 cm.

3. Think critically about this situation:

   Schools usually practise soccer in the afternoon when the temperature is around 20 °C. However, they play matches on Saturday mornings, when the temperature is often much lower than 20 °C.

   How will you advise the team that will use this ball to play a match on a cold day? How much difference must they expect in the bounce height compared to warmer conditions?

9.2 Analysing and interpreting data in a pictograph

Fikile is a big soccer fan. He wonders: When during soccer matches are the most goals scored? Maybe during the first half? Or maybe in the last 5 minutes? Or maybe there is no pattern?

What do you think?

1. What do you think about Fikile's question? When do you think the most goals are scored during games? Give reasons why you think so.

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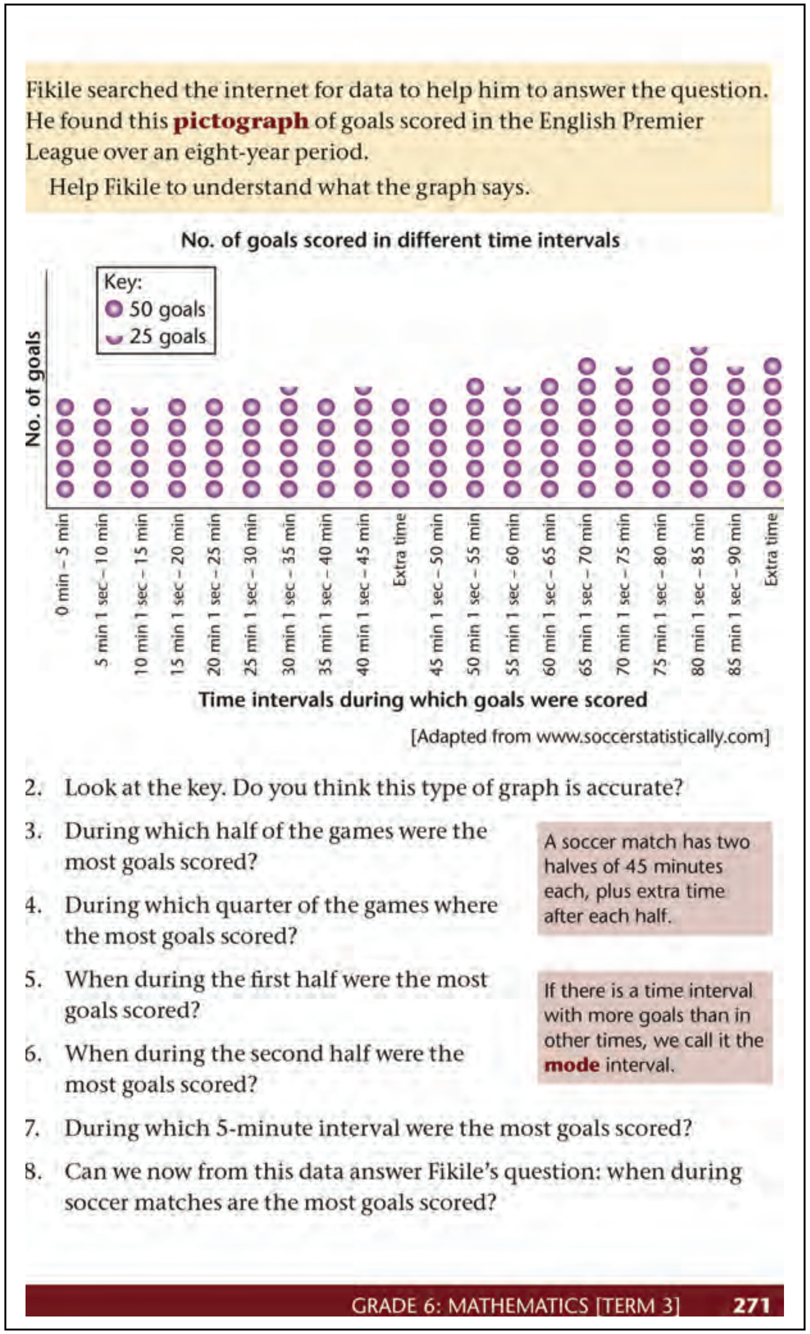
Mathematical notes

In pictographs the data needs to be rounded off. This means that all values are approximate values rather than accurate values. The approximated values do, however, make it easier for us to see the overall trends.

Answers

2. No. The data of many games were rounded to fit the key of multiples of 50. It is unlikely that the goals scored in 8 000 games were scored in multiples of 50.
3. Second half. Without counting we can see that almost all the stacks of balls in the second half are higher than those in the first half.
4. During the fourth quarter. That is between about 65 minutes and 90 minutes, excluding extra time.
5. Between about 30 minutes and 45 minutes. That is roughly in the second quarter of the game.
6. Between about 65 minutes and 85 minutes
7. 80 minutes to 85 minutes
8. Yes, most goals are scored towards the end of games, in the last quarter, and usually in the last 10 to 15 minutes of normal play.



Fikile searched the internet for data to help him to answer the question. He found this **pictograph** of goals scored in the English Premier League over an eight-year period.

Help Fikile to understand what the graph says.

No. of goals scored in different time intervals

[Adapted from www.soccerstatistically.com]

2. Look at the key. Do you think this type of graph is accurate?
3. During which half of the games were the most goals scored?
4. During which quarter of the games where the most goals scored?
5. When during the first half were the most goals scored?
6. When during the second half were the most goals scored?
7. During which 5-minute interval were the most goals scored?
8. Can we now from this data answer Fikile's question: when during soccer matches are the most goals scored?

A soccer match has two halves of 45 minutes each, plus extra time after each half.

If there is a time interval with more goals than in other times, we call it the **mode** interval.

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9.3 Interpreting and reporting data

Mathematical notes

Previously learners have used fractions to quantify the relative proportions of the “slices” (sectors) of pie charts. Since learners have now done work on percentages, they can use percentages to state the relative proportions of the sectors of pie charts.

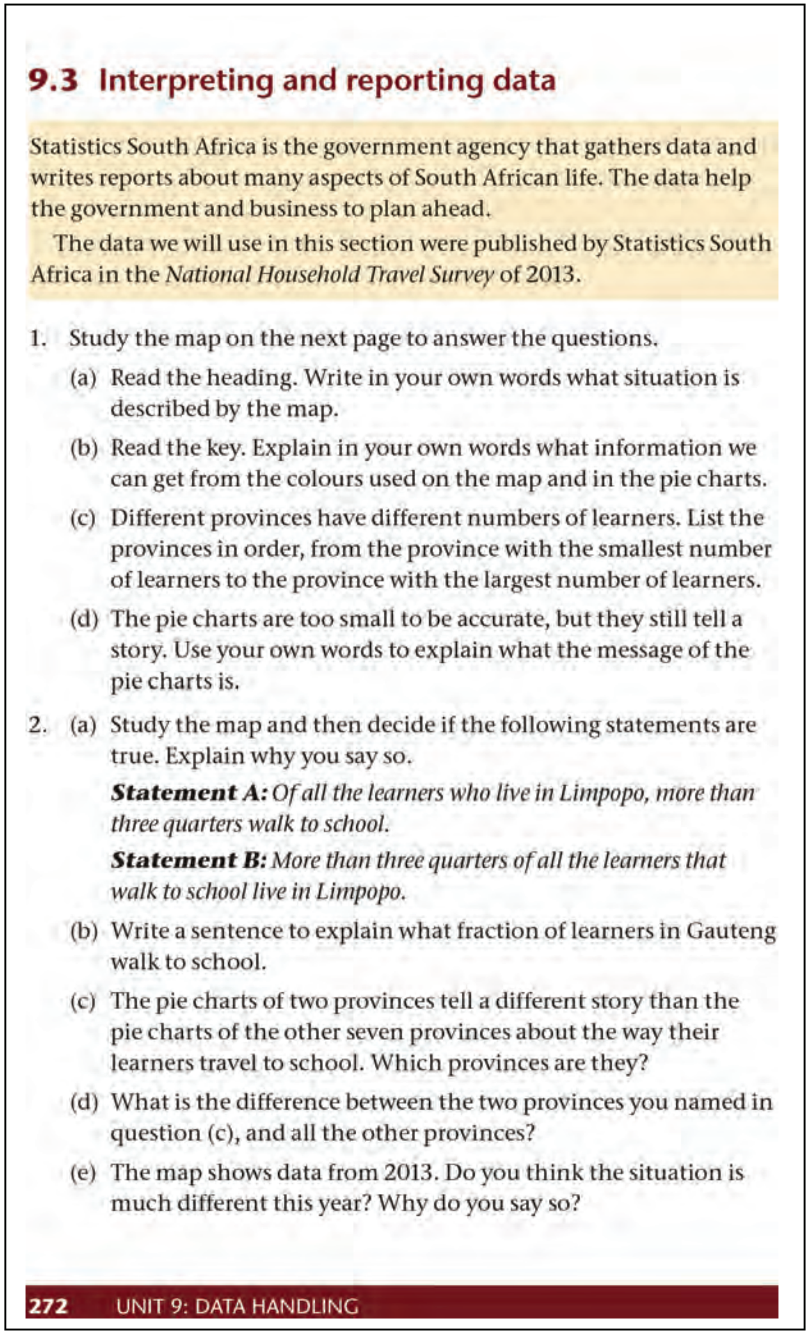
Questions on fractions and percentages often compare fractions and percentages of the same whole. Sometimes you need to compare fractions or percentages of different wholes: for example $\frac{1}{2}$ of R10 is less than $\frac{1}{4}$ of R100. The pie charts on the maps on page 273 of the Learner Book and those on page 276 show proportions of learners who use different kinds of transport in the different provinces. However, the actual number of children in each province differs. It is possible that a smaller percentage of a larger number of children can work out to be more than a bigger percentage of a smaller number of children.

Teaching guidelines

Discuss the relationship between the percentages of different quantities when you discuss the map with the class. Use the opportunity to strengthen knowledge of percentages.

Answers

1. (a) The map shows how many learners attended school in each province in 2013, and the main mode of travel that they used.
   (b) The colour of each province shows how many learners attended school in that province in 2013.
   The colours on the pie charts show the main form of transport that learners used.
   (c) Northern Cape
   Free State and North West
   Western Cape and Mpumalanga
   Limpopo and Gauteng
   Eastern Cape
   KwaZulu-Natal
   (d) In 2013, in all provinces the most common way in which learners travelled was by walking. In most provinces between about two thirds and about three quarters of learners walked to school. In the Western Cape and Gauteng a smaller proportion of learners walked to school, but it was still the most common mode of transport in these provinces.
2. See next page.



9.3 Interpreting and reporting data

Statistics South Africa is the government agency that gathers data and writes reports about many aspects of South African life. The data help the government and business to plan ahead.

The data we will use in this section were published by Statistics South Africa in the *National Household Travel Survey* of 2013.

1. Study the map on the next page to answer the questions.
   (a) Read the heading. Write in your own words what situation is described by the map.
   (b) Read the key. Explain in your own words what information we can get from the colours used on the map and in the pie charts.
   (c) Different provinces have different numbers of learners. List the provinces in order, from the province with the smallest number of learners to the province with the largest number of learners.
   (d) The pie charts are too small to be accurate, but they still tell a story. Use your own words to explain what the message of the pie charts is.
2. (a) Study the map and then decide if the following statements are true. Explain why you say so.
   Statement A: *Of all the learners who live in Limpopo, more than three quarters walk to school.*
   Statement B: *More than three quarters of all the learners that walk to school live in Limpopo.*
   (b) Write a sentence to explain what fraction of learners in Gauteng walk to school.
   (c) The pie charts of two provinces tell a different story than the pie charts of the other seven provinces about the way their learners travel to school. Which provinces are they?
   (d) What is the difference between the two provinces you named in question (c), and all the other provinces?
   (e) The map shows data from 2013. Do you think the situation is much different this year? Why do you say so?

Mathematical notes

It is important that learners distinguish between base categories when they compare percentages. For example, they must notice the difference between the following questions:

- *"What percentage of children in KZN walk to school?"* The base category (population) is all the school-going children in KZN.
- *"What percentage of children who walk to school live in KZN?"* The base category (population) is all the children in SA who walk to school.

In much of data handling there is more uncertainty than there is in other areas of mathematics. This can make learners nervous. Learners need to use the evidence provided in their analysis to back up their arguments.

Teaching guidelines

Spend time during class discussions to ask more questions that refer to different base categories.

Notes on questions

Question 2(e) asks learners to think beyond the data provided and to consider the current situation. Here learners cannot merely draw on their own opinions; they will need to consider whether they know of anything that could have changed the situation. You could also look up current data from STATSSA to see whether the situation has changed much. However, this should not distract from learners first giving their opinions.

Answers

2. (a) ***Statement A:*** True; for Limpopo the light blue sector is bigger than three quarters of the circle.

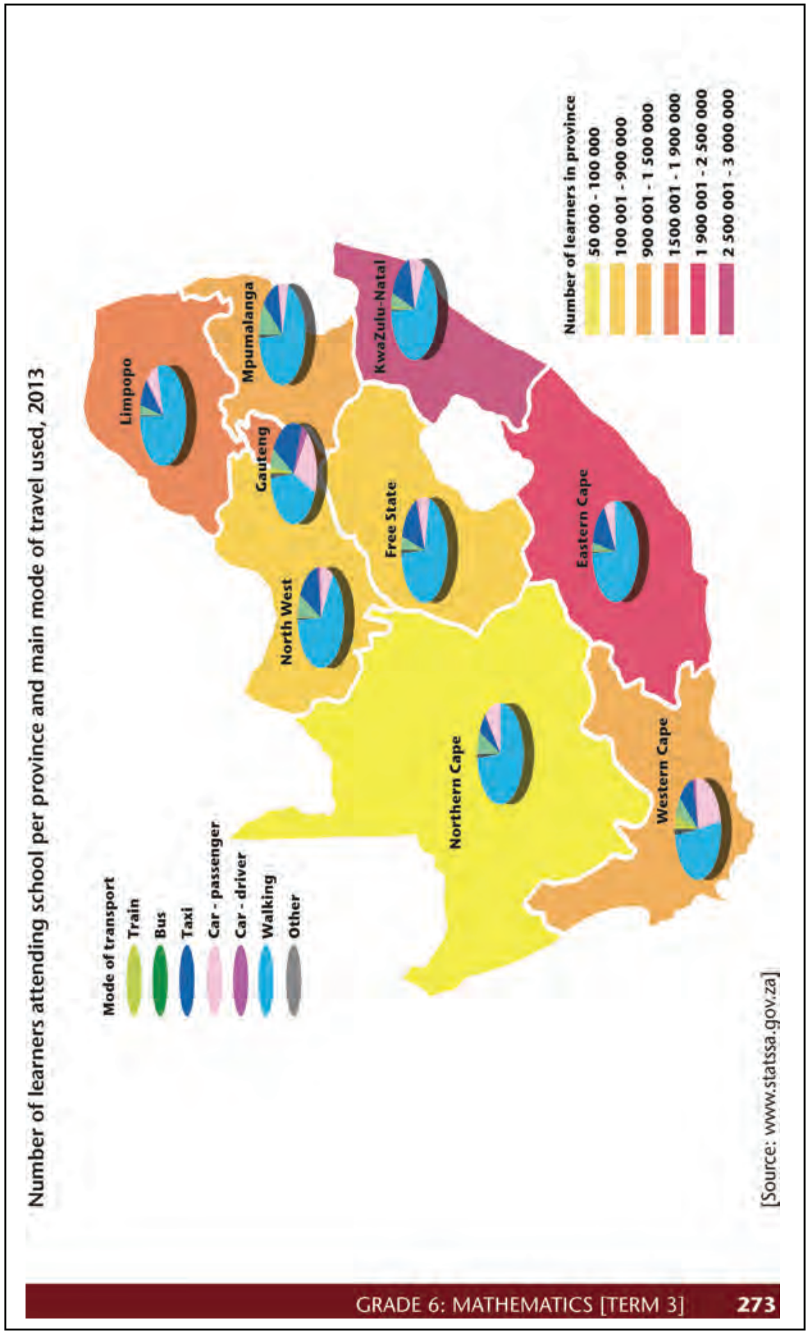
   Statement B: False; there is no information given on this map that counts all children who walk to school and then says in which provinces these children live.

   (b) More than one third but less than half of learners in Gauteng walk to school (light blue sector).

   (c) Gauteng and the Western Cape

   (d) Gauteng and the Western Cape are the only provinces where half, or less than half of the children walk to school.

   (e) Opinions may differ. Reasons must support the opinion. For example, have they heard in the news of large scale bus services or other possible changes in transport that have been implemented since 2013?



Number of learners attending school per province and main mode of travel used, 2013

[Source: www.statssa.gov.za]

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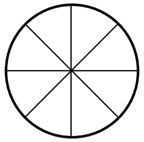
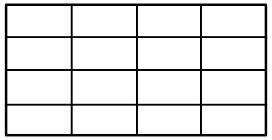
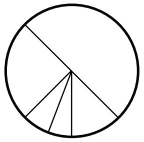
Mathematical notes

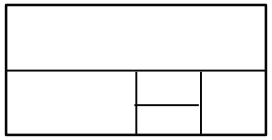
When learners are shown diagrams divided into fraction parts, often all the parts show the same fractions, as in the two examples alongside. Learners can then simply count the parts and use this to name the fractions. This may lead learners to ignore the sizes of the parts. They may not check whether all the pieces are the same size, i.e. whether all the pieces represent the same fraction.





If learners are given diagrams that are divided into different fraction parts (see the examples alongside), they need to use proportional thinking to work out the size of one fraction part in relation to other fraction parts. When learners need to interpret the size of the sectors in the pie chart on page 274 they need to use this skill.

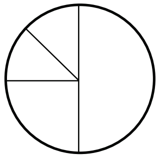




Teaching guidelines

Prepare the pie chart on the board with cut-outs of the different sectors to demonstrate how to estimate the fraction sizes of the sectors.

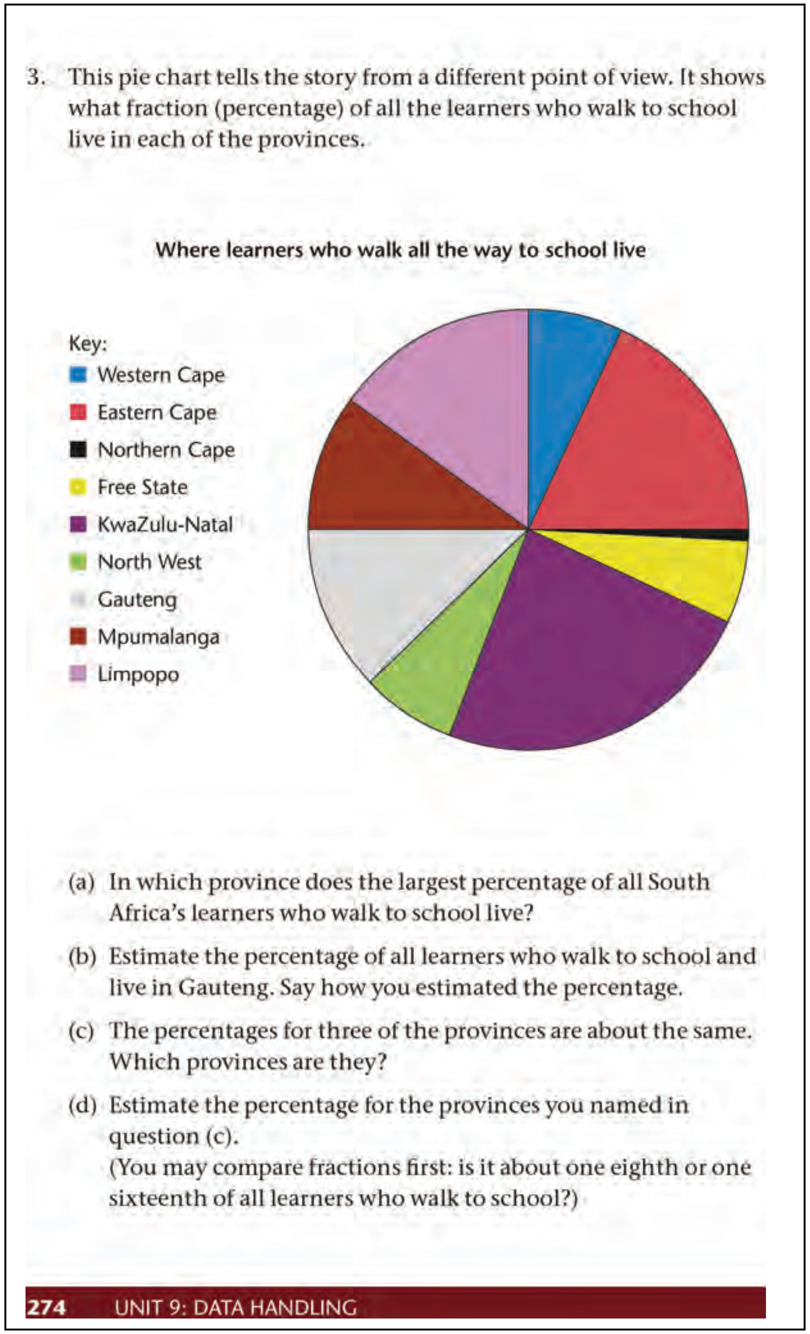
If learners struggle to identify the percentages in the answers, ask them to draw circles and divide them into halves, and to divide the one half into quarters and then one of the quarters into eighths. You can then ask them to write the size of each sector as a percentage.



Answers

3. (a) KwaZulu-Natal (the purple sector)
   (b) Gauteng (grey sector) is about half of a quarter. That is half of 25% of all learners who walk to school, i.e. 12,5%.
   (c) The sectors for Gauteng and Limpopo are about the same size; the sector for Mpumalanga is a little smaller, but not much.
   (d) Draw lines or trace and cut out the sectors to see how many times they fit in the circle to estimate the fraction.
   Gauteng 12,5% (one eighth), Limpopo: 12,5% (one eighth) and Mpumalanga about 10% (one tenth)

   In total about 35% of children who walk to school live in Gauteng, KwaZulu-Natal and Mpumalanga.



3. This pie chart tells the story from a different point of view. It shows what fraction (percentage) of all the learners who walk to school live in each of the provinces.

(a) In which province does the largest percentage of all South Africa's learners who walk to school live?

(b) Estimate the percentage of all learners who walk to school and live in Gauteng. Say how you estimated the percentage.

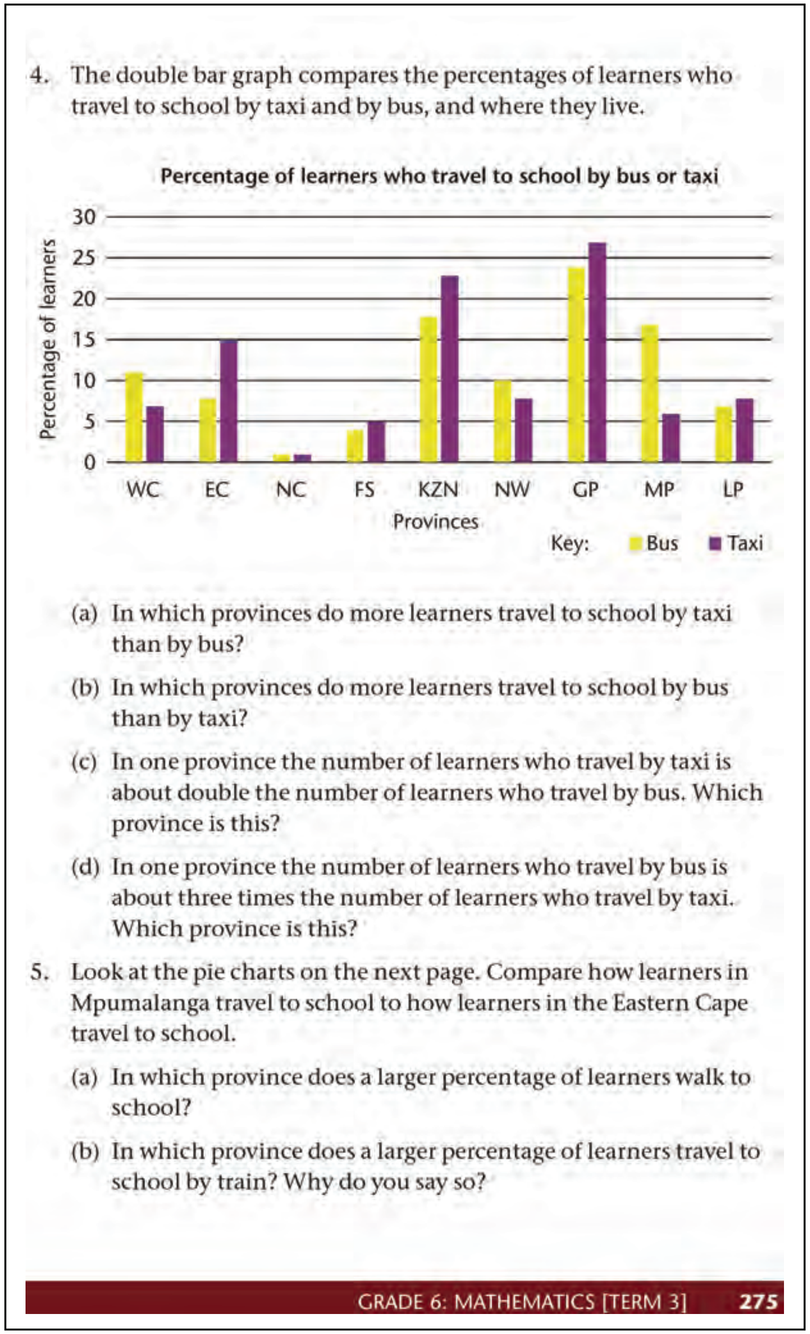
(c) The percentages for three of the provinces are about the same. Which provinces are they?

(d) Estimate the percentage for the provinces you named in question (c).
(You may compare fractions first: is it about one eighth or one sixteenth of all learners who walk to school?)

Answers

4. (a) Provinces where the purple bar is higher: Eastern Cape, Free State, KwaZulu-Natal, Gauteng and Limpopo.
   (b) Provinces where the yellow bar is higher: Western Cape, North West and Mpumalanga.
   (c) Eastern Cape
   (d) Mpumalanga

5. (a) In the Eastern Cape a larger percentage of learners (about 75%) walk to school than in Mpumalanga (about 70%).
   (b) In the Eastern Cape a larger percentage of learners (1%) take the train to school than in Mpumalanga. The number of learners who take the train to school in Mpumalanga is not reflected, which means it must be less than 1%.



4. The double bar graph compares the percentages of learners who travel to school by taxi and by bus, and where they live.

Percentage of learners who travel to school by bus or taxi

   (a) In which provinces do more learners travel to school by taxi than by bus?
   (b) In which provinces do more learners travel to school by bus than by taxi?
   (c) In one province the number of learners who travel by taxi is about double the number of learners who travel by bus. Which province is this?
   (d) In one province the number of learners who travel by bus is about three times the number of learners who travel by taxi. Which province is this?

5. Look at the pie charts on the next page. Compare how learners in Mpumalanga travel to school to how learners in the Eastern Cape travel to school.
   (a) In which province does a larger percentage of learners walk to school?
   (b) In which province does a larger percentage of learners travel to school by train? Why do you say so?

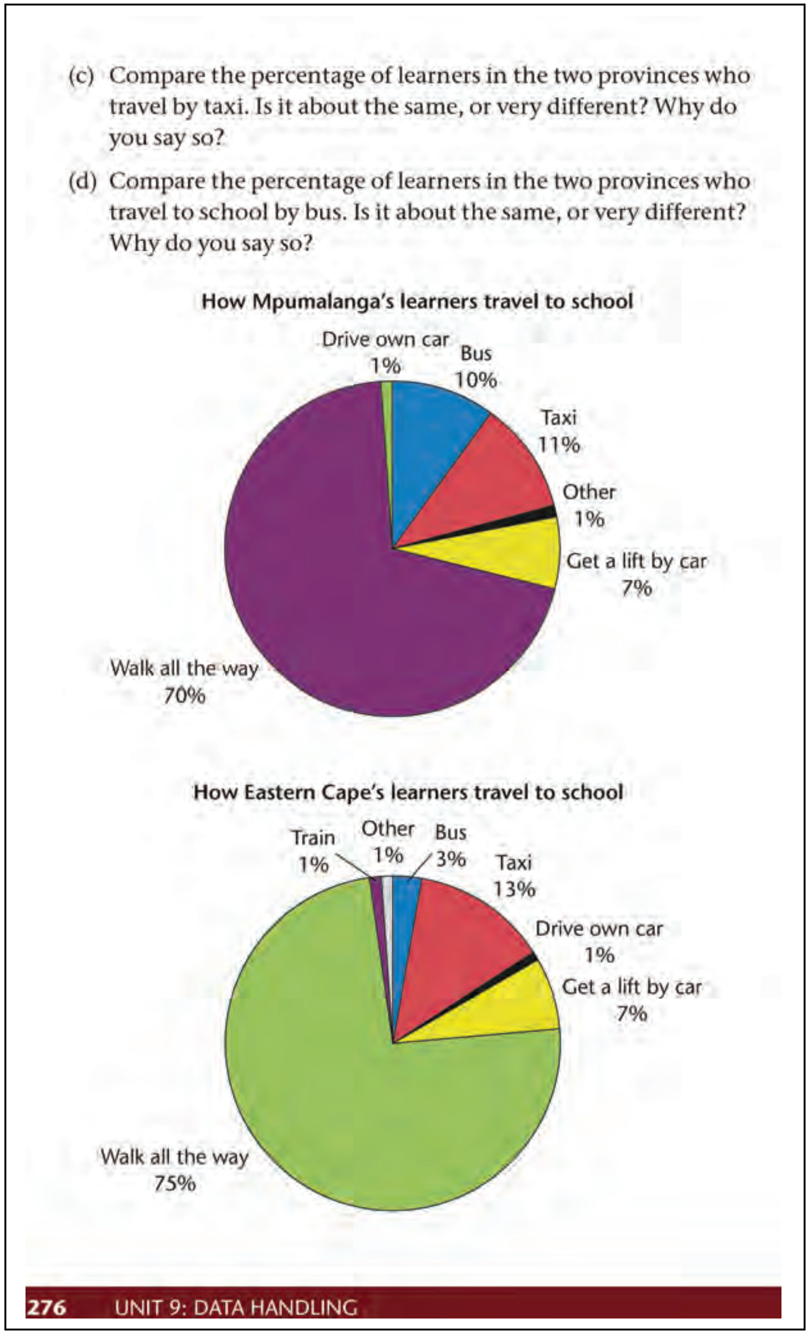
Possible misconceptions

Learners may confuse percentages with actual numbers. Learners may, for example, automatically assume that 10% of learners in Mpumalanga are more than 3% of learners in the Eastern Cape because the number 10 is bigger than the number 3. However, learners should remember that there are more learners in the Eastern Cape than there are in Mpumalanga (see the map on page 273). There are probably more learners in Mpumalanga who travel by bus and taxi than in the Eastern Cape. However, listen carefully to the language that learners use, and check that they are comparing percentages and not total numbers.

Answers

5. (c) In the Eastern Cape 13% of learners travel to school by taxi, and in Mpumalanga it is 11%. That is about the same percentage.

   (d) In Mpumalanga about three times more learners travel to school by bus (10%) than in the Eastern Cape (3%).



(c) Compare the percentage of learners in the two provinces who travel by taxi. Is it about the same, or very different? Why do you say so?

(d) Compare the percentage of learners in the two provinces who travel to school by bus. Is it about the same, or very different? Why do you say so?

276 UNIT 9: DATA HANDLING

9.4 Project

Teaching guidelines

This project will take about three weeks to complete. Learners must plan the project in the first week. During the second week they must gather and record the data. During the third week they must represent, analyse and interpret the data.

Week 1

- Share your assessment criteria with learners. An example is provided on the next page.
- Help learners with the following preparations:
  - Plan and design a questionnaire.

    The questionnaire must ask for biographical data like age, grade and sex.

    For question 1, the questionnaire must indicate different kinds of transport that each participant must tick. Only one option per participant may be ticked. The question must be clear that it is the transport that is *used most*.
  - Try out the questionnaire and adapt it if necessary.
  - Form groups and decide how they will share the work to gather relevant data to answer all the questions, including their own question.

    Think ahead to the organising of the data and number the questionnaires, and let learners write their names on the questionnaires they administrate.
  - Plan a time to gather the data. For example, arrange with the relevant teacher to visit their classes on a specific day.

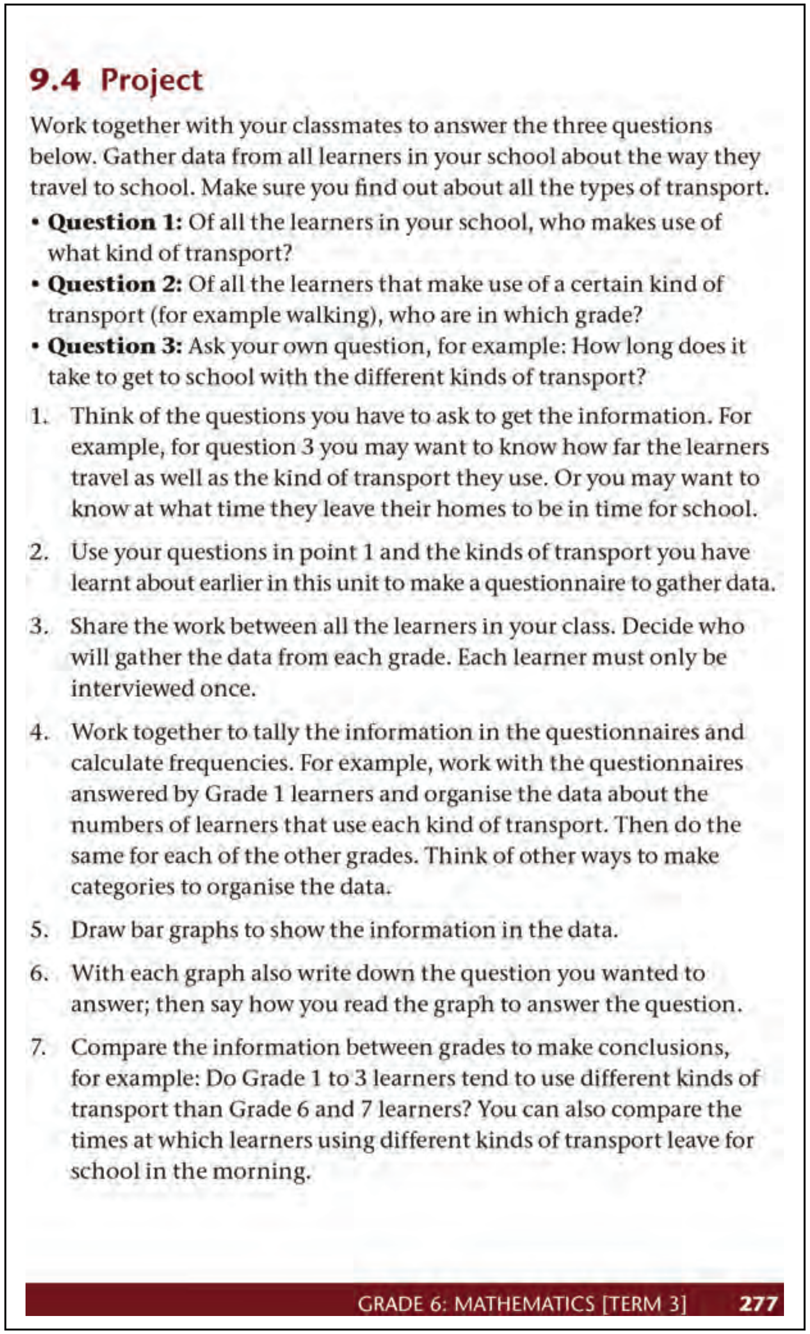
Week 2

- Learners gather data as planned.

Week 3

- Help learners to decide on headings for the different parts of their reports. They may use headings like:
  1. **The question we want to answer**
  2. **Data gathering**
     (Here they tell how and where they gathered data.)
  3. **Representations of the data**
     (Here they provide their tables and graphs.)

(Continued on the next page)



9.4 Project

Work together with your classmates to answer the three questions below. Gather data from all learners in your school about the way they travel to school. Make sure you find out about all the types of transport.

- **Question 1:** Of all the learners in your school, who makes use of what kind of transport?
- **Question 2:** Of all the learners that make use of a certain kind of transport (for example walking), who are in which grade?
- **Question 3:** Ask your own question, for example: How long does it take to get to school with the different kinds of transport?

1. Think of the questions you have to ask to get the information. For example, for question 3 you may want to know how far the learners travel as well as the kind of transport they use. Or you may want to know at what time they leave their homes to be in time for school.
2. Use your questions in point 1 and the kinds of transport you have learnt about earlier in this unit to make a questionnaire to gather data.
3. Share the work between all the learners in your class. Decide who will gather the data from each grade. Each learner must only be interviewed once.
4. Work together to tally the information in the questionnaires and calculate frequencies. For example, work with the questionnaires answered by Grade 1 learners and organise the data about the numbers of learners that use each kind of transport. Then do the same for each of the other grades. Think of other ways to make categories to organise the data.
5. Draw bar graphs to show the information in the data.
6. With each graph also write down the question you wanted to answer; then say how you read the graph to answer the question.
7. Compare the information between grades to make conclusions, for example: Do Grade 1 to 3 learners tend to use different kinds of transport than Grade 6 and 7 learners? You can also compare the times at which learners using different kinds of transport leave for school in the morning.

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4. **Analysis and summary of the data**
   (Here they provide calculations and summary values that are relevant to the question.)
5. **Interpretation of the data**
   (Here they write a paragraph to interpret the data and summaries.)

- Provide learners with paper to prepare their graphs.
- Arrange for learners to present their findings to each other, or to an audience of school mates and teachers.

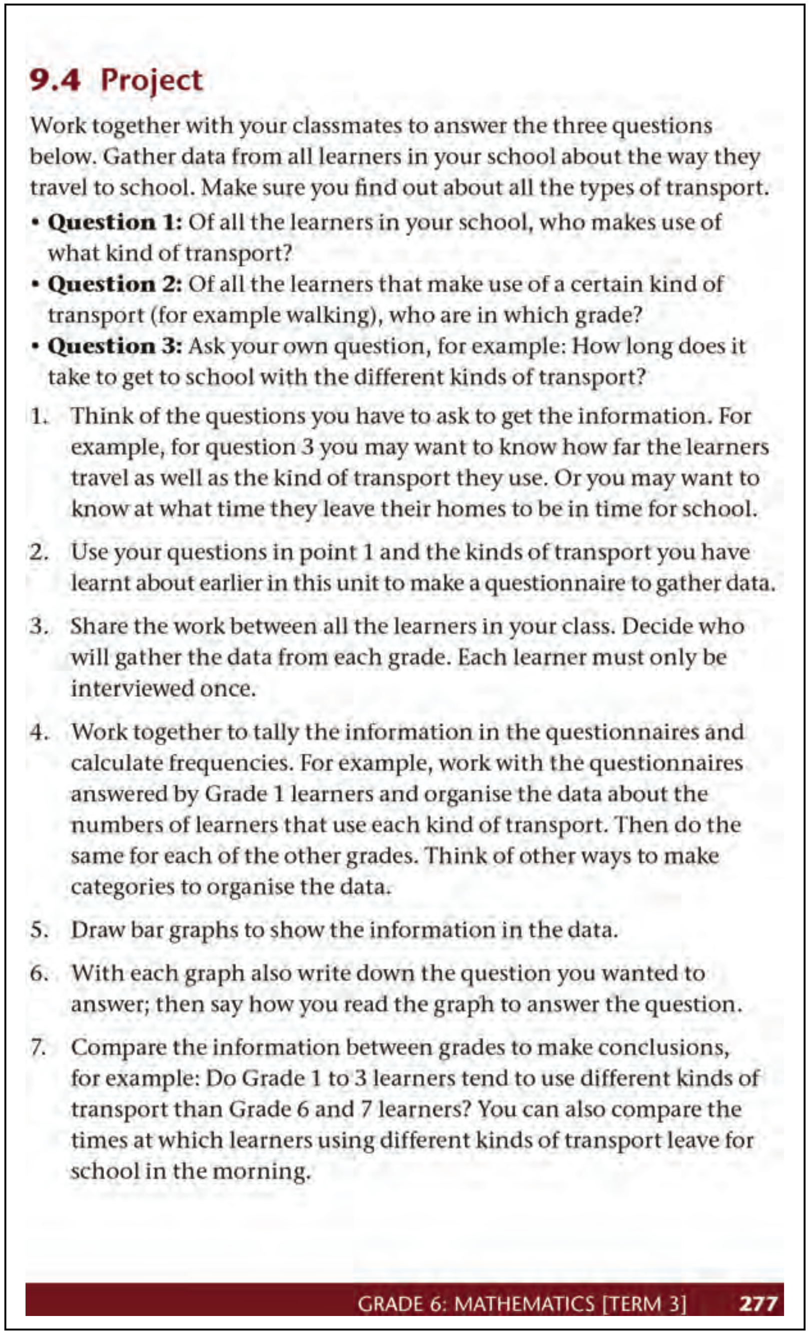
Suggested assessment criteria

Data gathering: The questionnaire must be clear and unambiguous. The data gathering plan must ensure that all participants are reached, and no one is asked more than once. (10 marks)

Data organisation: The data from the questionnaires must be organised in tables with clear headings. (5 marks)

Data representation and analysis: The data must be represented in bar graphs, pictographs or double bar graphs. The graphs must have a heading and the axes must be labelled. The scale must be correct. The bars must be drawn accurately. The graphs must be neat and easy to read. (10 marks)

Data interpretation and reporting: Each group must write a report to answer the questions. The report must say how and where they gathered the data, and what they found. The graphs and summaries must be used as evidence of findings. (10 marks)



9.4 Project

Work together with your classmates to answer the three questions below. Gather data from all learners in your school about the way they travel to school. Make sure you find out about all the types of transport.

- **Question 1:** Of all the learners in your school, who makes use of what kind of transport?
- **Question 2:** Of all the learners that make use of a certain kind of transport (for example walking), who are in which grade?
- **Question 3:** Ask your own question, for example: How long does it take to get to school with the different kinds of transport?

1. Think of the questions you have to ask to get the information. For example, for question 3 you may want to know how far the learners travel as well as the kind of transport they use. Or you may want to know at what time they leave their homes to be in time for school.
2. Use your questions in point 1 and the kinds of transport you have learnt about earlier in this unit to make a questionnaire to gather data.
3. Share the work between all the learners in your class. Decide who will gather the data from each grade. Each learner must only be interviewed once.
4. Work together to tally the information in the questionnaires and calculate frequencies. For example, work with the questionnaires answered by Grade 1 learners and organise the data about the numbers of learners that use each kind of transport. Then do the same for each of the other grades. Think of other ways to make categories to organise the data.
5. Draw bar graphs to show the information in the data.
6. With each graph also write down the question you wanted to answer; then say how you read the graph to answer the question.
7. Compare the information between grades to make conclusions, for example: Do Grade 1 to 3 learners tend to use different kinds of transport than Grade 6 and 7 learners? You can also compare the times at which learners using different kinds of transport leave for school in the morning.

Grade 6 Term 3 Unit 10 Numeric patterns

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
10.1 Finding input and output numbers	Decreasing sequences with a constant difference (“subtraction”)	278 to 279
10.2 Using patterns to solve problems	Applied sequences in everyday situations	280
10.3 From tables to rules	Families of sequences with a constant difference in table form	281 to 282
10.4 Adding sequences	Mathematical problem solving	283
10.5 Multiplying sequences	Mathematical problem solving	284

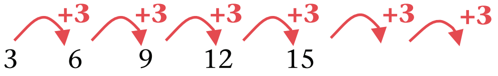
CAPS time allocation	5 hours
CAPS page references	18 to 19 and 270 to 272

Mathematical background

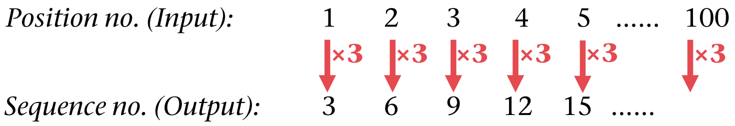
Numeric patterns, as part of the Content Area “Patterns, Functions and Algebra”, should serve as building blocks to develop the basic concepts of algebra in the Senior and FET phases. The study of numeric patterns should develop the concepts of **variable**, **relationships** and **functions**. The function concept is captured in the notion of the triad **input numbers** → **rule** → **output numbers**. (See page 116 of this guide.)

Much of our pattern work focuses on methods to find the **calculation plan (rule)**, because it is useful to find **output numbers** and **input numbers**. The following two approaches to pattern work should be emphasised throughout:

- **Recursive (“horizontal”) patterns** in sequences describing the relationship between any two consecutive numbers in a sequence, and then continuing the sequence for a few next numbers. For example:



- **Functional (“vertical”) patterns** describing the constant relationship between two sets (the two **variables**, i.e. the input and output variables), and then applying this pattern to calculate further-lying values (e.g. the 100th number). For example:



These two ideas (**recursive** and **functional relationships**) are important horizon knowledge, i.e. important for future mathematical concepts. Recursion leads to the important mathematical concepts of the gradient of a straight line and the derivative of a function. The function concept underlies all of high school algebra and the Grade 12 work on rate of change (calculus).

Resources

Calculators

10.1 Finding input and output numbers

Mathematical notes

The underlying mathematical concept in this section, which consists of one activity, is that of a decreasing function (the candles become shorter). We are handling it as a numeric sequence – the candles burn at 2 cm/h or 1 cm/h.

Teaching guidelines

Learners use their knowledge of sequences (e.g. the rules describing the functions) to solve contextual problems, such as how long a candle will burn and which candle will last the longest. To solve these problems learners need to do things such as finding input and output values, finding rules, solving equations, etc.

This is also a critical activity for learning all the underlying mathematical concepts and procedures. Make sure that all learners attempt all the questions.

Learners should at this stage very explicitly have the following meta-knowledge (*knowing how to use knowledge gained*):

- That they can easily continue a sequence using **recursion** (horizontal patterns).
- That for larger values it will be much easier to use the functional relationship (vertical rule) or to develop and use more sophisticated recursive strategies, for example:

Candle A

		+1	+1	+1	+1	+1	+5
Time (hours)	0	1	2	3	4	5	10
Length (cm)	36	34	32	30	28	26	16
		−2	−2	−2	−2	−2	5×−2

Learners should at this stage also be aware of the mistake to continue the horizontal pattern over missing values (the break in the sequence of input numbers) in a table. For example:

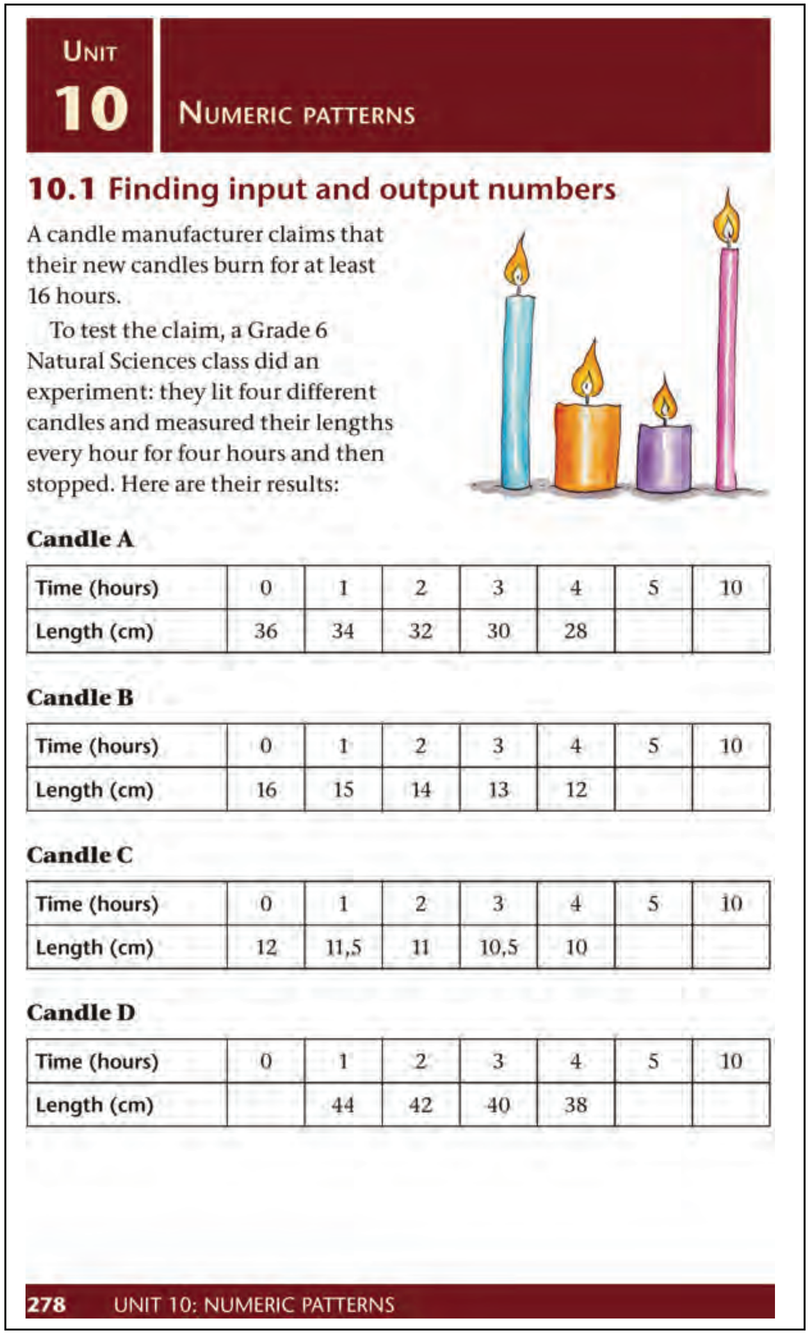
Candle A

Time (hours)	0	1	2	3	4	5	10
Length (cm)	36	34	32	30	28	26	24 ✗
		−2	−2	−2	−2	−2	−2

They should also realise that they cannot introduce properties that are not valid for all values. For example:

Candle A

							×2
Time (hours)	0	1	2	3	4	5	10
Length (cm)	36	34	32	30	28	26	52 ✗
							×2



UNIT 10 NUMERIC PATTERNS

10.1 Finding input and output numbers

A candle manufacturer claims that their new candles burn for at least 16 hours.

To test the claim, a Grade 6 Natural Sciences class did an experiment: they lit four different candles and measured their lengths every hour for four hours and then stopped. Here are their results:

Candle A

Time (hours)	0	1	2	3	4	5	10
Length (cm)	36	34	32	30	28		

Candle B

Time (hours)	0	1	2	3	4	5	10
Length (cm)	16	15	14	13	12		

Candle C

Time (hours)	0	1	2	3	4	5	10
Length (cm)	12	11,5	11	10,5	10		

Candle D

Time (hours)	0	1	2	3	4	5	10
Length (cm)		44	42	40	38		

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Notes on questions

If learners struggle with question 3, refer to page 115 of the Learner Book, where they did similar work.

One way to tackle question 5(a) is to substitute the known *Output number* 10 into the rule for Candle A to obtain the **equation** 10 = 36 – (2 × *Time*) and then to find out what *Time* will make the equation true, either through reasoning or trial and improvement.

Answers

1. Consider learners' explanations.

Candle A

Time (hours)	0	1	2	3	4	5	10
Length (cm)	36	34	32	30	28	26	16

Candle B

Time (hours)	0	1	2	3	4	5	10
Length (cm)	16	15	14	13	12	11	6

Candle C

Time (hours)	0	1	2	3	4	5	10
Length (cm)	12	11,5	11	10,5	10	9,5	7

Candle D

Time (hours)	0	1	2	3	4	5	10
Length (cm)	46	44	42	40	38	36	26

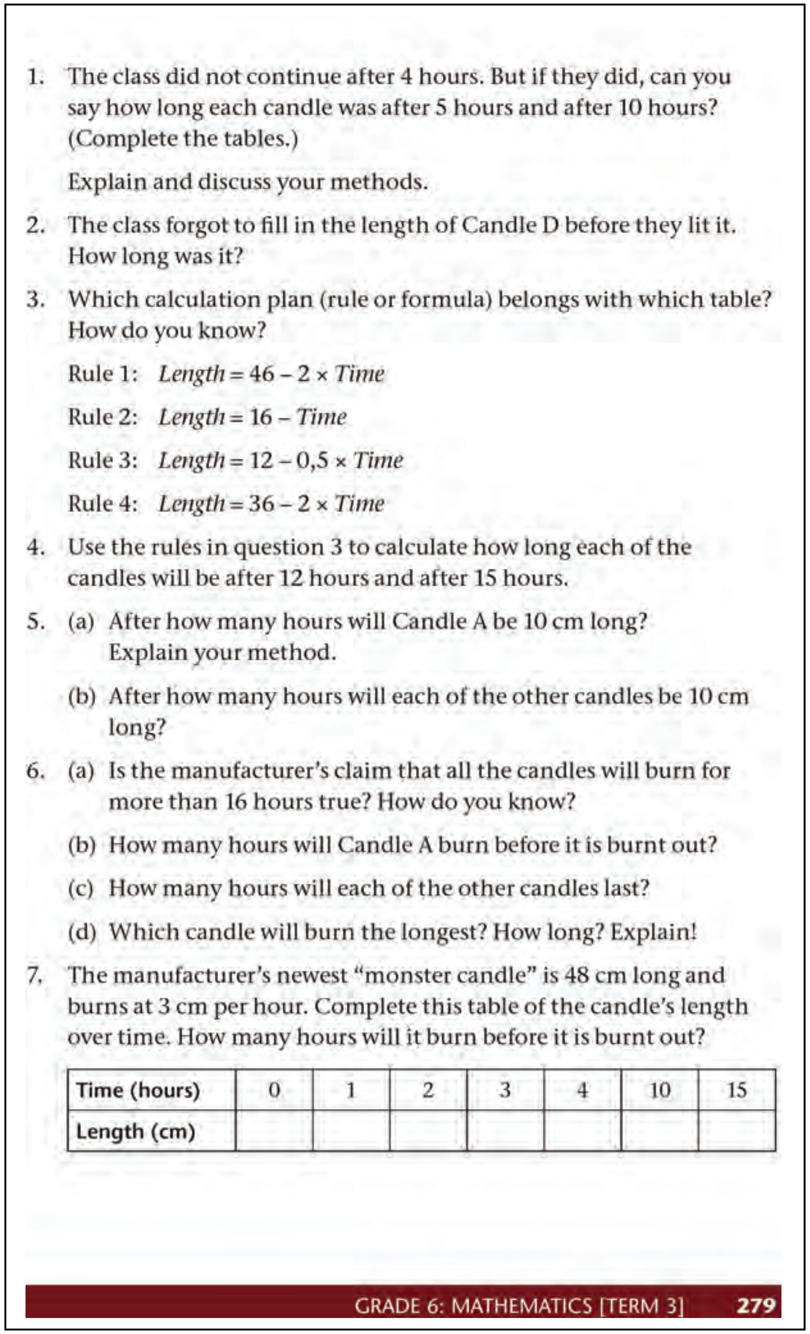
2. 46 cm
3. Rule 1: Candle D Rule 2: Candle B Rule 3: Candle C Rule 4: Candle A
4. After 12 hours: After 15 hours:

Candle A: Length = 36 – 2×12 = 12 cm Length = 36 – 2×15 = 6 cm
Candle B: Length = 16 – 12 = 4 cm Length = 16 – 15 = 1 cm
Candle C: Length = 12 – 0,5×12 = 6 cm Length = 12 – 0,5×15 = 4,5 cm
Candle D: Length = 46 – 2×12 = 22 cm Length = 46 – 2×15 = 16 cm

5. (a) 10 = 36 – 2×*Time*, so 2×*Time* = 26, so *Time* = 13 hours
   (b) Candle B: 6 hours Candle C: 4 hours Candle D: 18 hours
6. (a) True; Candle B will burn for 16 hours and all the others for more than 16 hours.
   (b) 0 = 36 – 2×*Time*, so 2×*Time* = 36, so *Time* = 18 hours
   (c) Candle B: 16 hours Candle C: 24 hours Candle D: 23 hours
   (d) Candle C: 24 hours. Consider learners' explanations.
7.

Time (hours)	0	1	2	3	4	10	15
Length (cm)	48	45	42	39	36	18	3

16 hours



1. The class did not continue after 4 hours. But if they did, can you say how long each candle was after 5 hours and after 10 hours? (Complete the tables.)

   Explain and discuss your methods.
2. The class forgot to fill in the length of Candle D before they lit it. How long was it?
3. Which calculation plan (rule or formula) belongs with which table? How do you know?

   Rule 1: *Length* = 46 – 2 × *Time*

   Rule 2: *Length* = 16 – *Time*

   Rule 3: *Length* = 12 – 0,5 × *Time*

   Rule 4: *Length* = 36 – 2 × *Time*
4. Use the rules in question 3 to calculate how long each of the candles will be after 12 hours and after 15 hours.
5. (a) After how many hours will Candle A be 10 cm long? Explain your method.
   (b) After how many hours will each of the other candles be 10 cm long?
6. (a) Is the manufacturer's claim that all the candles will burn for more than 16 hours true? How do you know?
   (b) How many hours will Candle A burn before it is burnt out?
   (c) How many hours will each of the other candles last?
   (d) Which candle will burn the longest? How long? Explain!
7. The manufacturer's newest "monster candle" is 48 cm long and burns at 3 cm per hour. Complete this table of the candle's length over time. How many hours will it burn before it is burnt out?

Time (hours)	0	1	2	3	4	10	15
Length (cm)							

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10.2 Using patterns to solve problems

Teaching guidelines

You should make sure that learners understand the contexts and the problems, for example that they understand that in question 2 the height of the seedling on Day 0 is 0 mm, and after 1 day it will be 1,5 mm high. (It is the same concept as the candles in the previous section which also started at 0 hours.)

Answers

1. (a)

Number of doughnuts	1	2	3	4	5	6	7	8	9	10	100
Total cost (in cents)	25	50	75	100	125	150	175	200	225	250	2 500

(b) 25 × 25c = 625c or R6,25

(c) *Total cost* (in cents) = *Number of doughnuts* × 25

(d) Check with given numbers in table.

(e) *Number of doughnuts* = *Total cost* ÷ 25c per doughnut

So 550c ÷ 25c/doughnut = 22 doughnuts

2. (a) 3 mm over two days, so daily growth = 1,5 mm

(b) 10,5 mm is exactly between 9 mm and 12 mm, so exactly between Day 6 and Day 8 is Day 7.

Or, Height = 1,5 mm × *Day number.*

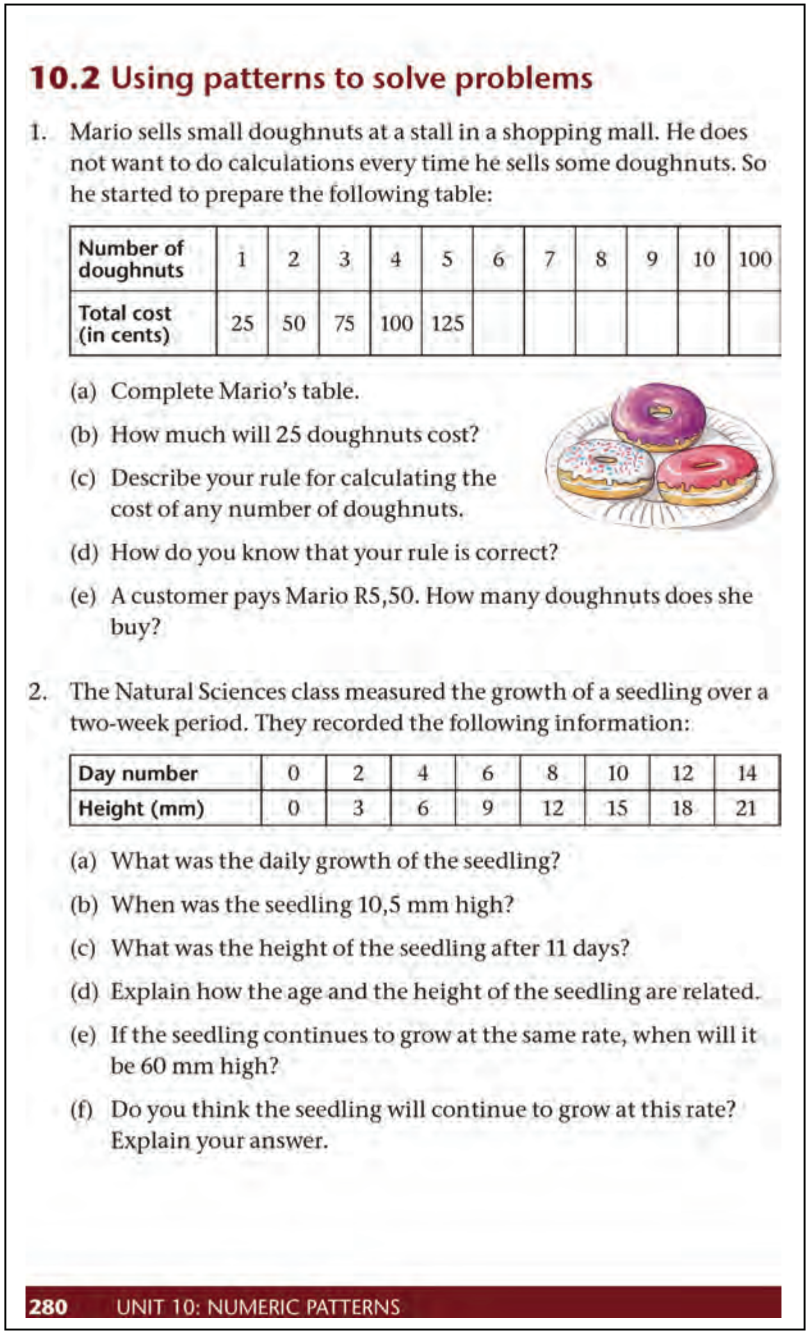
So 10,5 = 1,5 × *Day number*, so Day number = 10,5 ÷ 1,5 = 7.

(c) 16,5 mm

(d) The seedling grows at a rate of 1,5 mm per day. The seedling grows higher as it grows older.

(e) The seedling grows 3 mm in 2 days, so 30 mm in 20 days and 60 mm in 40 days.

(f) In theory yes, but not in reality. The growth rate of plants slows down as time passes, and eventually the plant will stop growing.



10.2 Using patterns to solve problems

1. Mario sells small doughnuts at a stall in a shopping mall. He does not want to do calculations every time he sells some doughnuts. So he started to prepare the following table:

Number of doughnuts	1	2	3	4	5	6	7	8	9	10	100
Total cost (in cents)	25	50	75	100	125						

(a) Complete Mario's table.

(b) How much will 25 doughnuts cost?

(c) Describe your rule for calculating the cost of any number of doughnuts.

(d) How do you know that your rule is correct?

(e) A customer pays Mario R5,50. How many doughnuts does she buy?

2. The Natural Sciences class measured the growth of a seedling over a two-week period. They recorded the following information:

Day number	0	2	4	6	8	10	12	14
Height (mm)	0	3	6	9	12	15	18	21

(a) What was the daily growth of the seedling?

(b) When was the seedling 10,5 mm high?

(c) What was the height of the seedling after 11 days?

(d) Explain how the age and the height of the seedling are related.

(e) If the seedling continues to grow at the same rate, when will it be 60 mm high?

(f) Do you think the seedling will continue to grow at this rate? Explain your answer.

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10.3 From tables to rules

This activity returns to the mathematical notion of a "family of functions", i.e. families of sequences with the same constant difference. It is presented here in the context of a number table (grid). In order to predict numbers further down in the table, learners need to construct rules for the sequences in each column and find input and output numbers.

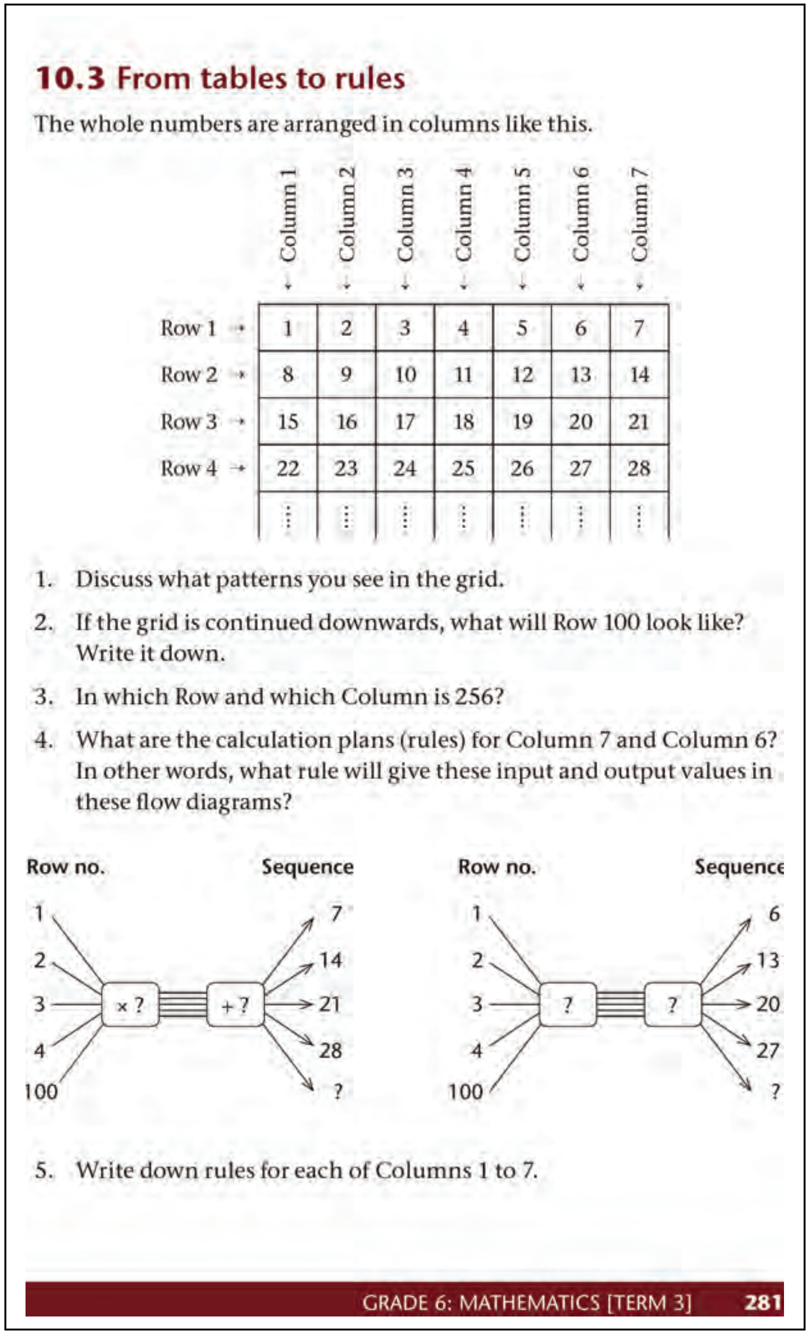
This is exactly the work on families of functions we did in the Term 1. In Term 1 the sequences were in horizontal format, but now they are given vertically.

	Sequence	**Description in words**	**Flow diagram/Rule**	
−1	3, 7, 11, 15, 19, 23, 27, ...	One less than multiples of 4	$-\boxed{\times 4}-\boxed{-1}\rightarrow$	−1
+2	**4, 8, 12, 16, 20, 24, 28, ...**	**Multiples of 4**	$-\boxed{\times 4}\rightarrow$	+2
	6, 10, 14, 18, 22, 26, 30, ...	Two more than multiples of 4	$-\boxed{\times 4}-\boxed{+2}\rightarrow$	

As in Term 1, learners will find it useful to identify the column in which the numbers are multiples, and then deduce the rule for each of the other columns through their relationship with the multiples column. We use tables and multiples of 7 (question 1) and multiples of 6 (question 6), but you can design similar activities for other multiples as needed.

Answers

1. The difference between the rows is 7, in all columns.
2. The last number will be 100×7 = 700, then: 694, 695, 696, 697, 698, 699, 700
3. 252 ÷ 7 = 36, so 252 is a multiple of 7 and in Column 7, Row 36.
   So 256 will be in Row 37, Column 4: 253, 254, 255, 256, 257, 258, 259
4. Column 7: Row number × 7 1, 2, 3, 4, 100 $-\boxed{\times 7}-\boxed{+0}\rightarrow$ 7, 14, 21, 28, 700
   Column 6: Row number × 7 – 1 1, 2, 3, 4, 100 $-\boxed{\times 7}-\boxed{-1}\rightarrow$ 6, 13, 20, 27, 699
5. Start with Column 7, the multiples of 7!
   Column 1: Row number × 7 – 6
   Column 2: Row number × 7 – 5
   Column 3: Row number × 7 – 4
   Column 4: Row number × 7 – 3
   Column 5: Row number × 7 – 2
   Column 6: Row number × 7 – 1
   Column 7: Row number × 7 – 0



10.3 From tables to rules

The whole numbers are arranged in columns like this.

	Column 1	Column 2	Column 3	Column 4	Column 5	Column 6	Column 7
Row 1 →	1	2	3	4	5	6	7
Row 2 →	8	9	10	11	12	13	14
Row 3 →	15	16	17	18	19	20	21
Row 4 →	22	23	24	25	26	27	28
	⋮	⋮	⋮	⋮	⋮	⋮	⋮

1. Discuss what patterns you see in the grid.
2. If the grid is continued downwards, what will Row 100 look like? Write it down.
3. In which Row and which Column is 256?
4. What are the calculation plans (rules) for Column 7 and Column 6? In other words, what rule will give these input and output values in these flow diagrams?

5. Write down rules for each of Columns 1 to 7.

Notes on questions

The vertical sequences all have a constant difference of 6. Once we identify Column 6 as the multiples of 6, we know its rule is *Row number* × 6.

Column 5 is then 1 less than a multple of 6, so its rule is *Row number* × 6 – 1.

	5 less than multiples of 6	4 less than multiples of 6	3 less than multiples of 6	2 less than multiples of 6	1 less than multiples of 6	Multiples of 6: Rule is ×6
Row 1	1	2	3	4	5	6
Row 2	7	8	9	10	11	12
Row 3	13	14	15	16	17	18
Row 4	19	20	21	22	23	24

–1 –1 –1 –1 –1

Answers

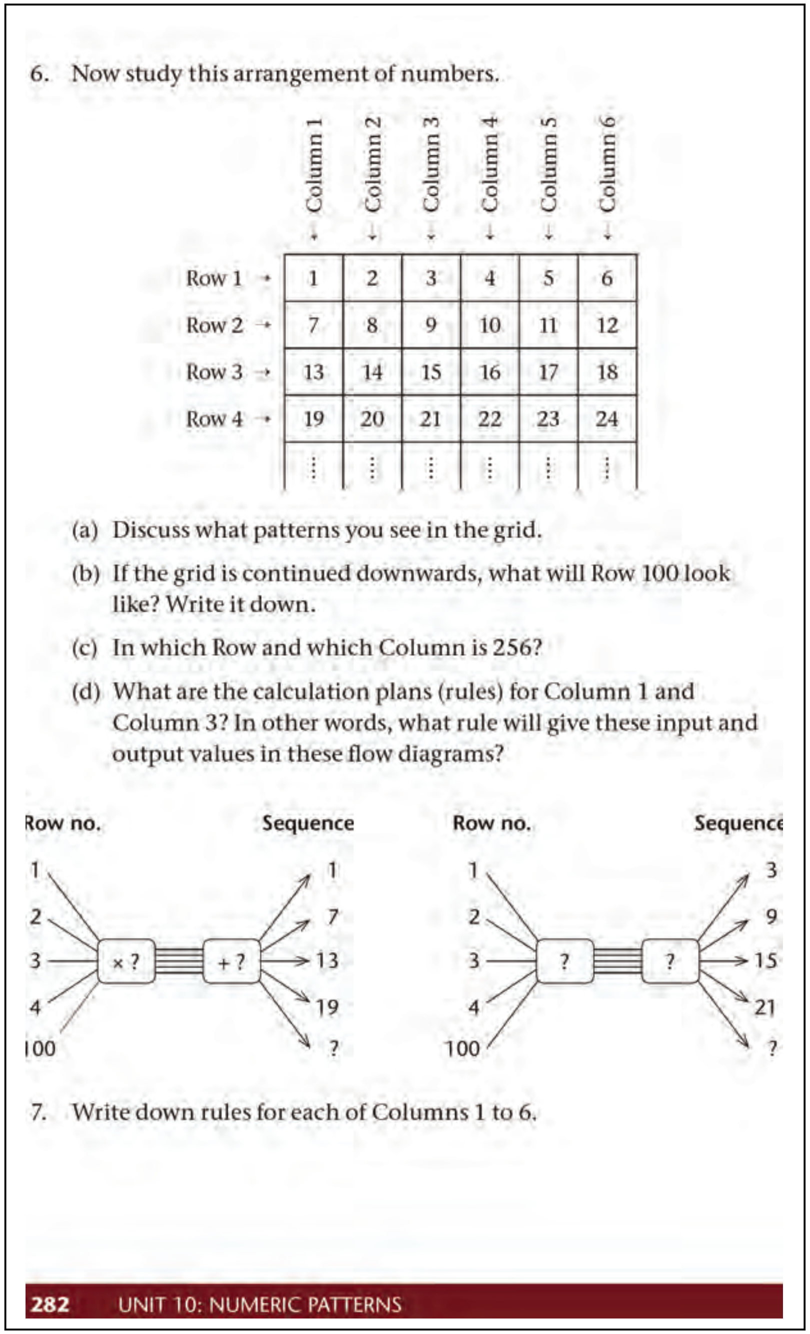
6. (a) The vertical difference in all columns is 6.

   (b) The last number in Row 100 is 100 × 6 = 600. So 595, 596, 597, 598, 599, 600

   (c) 252 ÷ 6 = 42, so 252 is a multiple of 6 and in Row 42, Column 6.
   So 256 will be in Row 43, Column 4: 253, 254, 255, 256, 257, 258

   (d) Column 1: 1, 2, 3, 4, 100 $\rightarrow \boxed{\times 6} \rightarrow \boxed{-5} \rightarrow$ 1, 7, 13, 19, 595
   Column 3: 1, 2, 3, 4, 100 $\rightarrow \boxed{\times 6} \rightarrow \boxed{-3} \rightarrow$ 3, 9, 15, 21, 597

7. Start with Column 6, the multiples of 6!
   Column 1 = Row number × 6 – 5
   Column 2 = Row number × 6 – 4
   Column 3 = Row number × 6 – 3
   Column 4 = Row number × 6 – 2
   Column 5 = Row number × 6 – 1
   Column 6 = Row number × 6 – 0



6. Now study this arrangement of numbers.

	Column 1 ↓	Column 2 ↓	Column 3 ↓	Column 4 ↓	Column 5 ↓	Column 6 ↓
Row 1 →	1	2	3	4	5	6
Row 2 →	7	8	9	10	11	12
Row 3 →	13	14	15	16	17	18
Row 4 →	19	20	21	22	23	24
	⋮	⋮	⋮	⋮	⋮	⋮

(a) Discuss what patterns you see in the grid.

(b) If the grid is continued downwards, what will Row 100 look like? Write it down.

(c) In which Row and which Column is 256?

(d) What are the calculation plans (rules) for Column 1 and Column 3? In other words, what rule will give these input and output values in these flow diagrams?

7. Write down rules for each of Columns 1 to 6.

282 UNIT 10: NUMERIC PATTERNS

10.4 Adding sequences

Teaching guidelines

This activity is an investigation where learners, through engagement, will generalise an underlying theorem to predict the nature of the resulting sequence, without having to construct the addends.

To make sure that learners understand the context and know what they must do (how to add the sequences, etc.) it will be good if you present the shaded passage and establish through question-and-answer if learners do understand before they engage with the task.

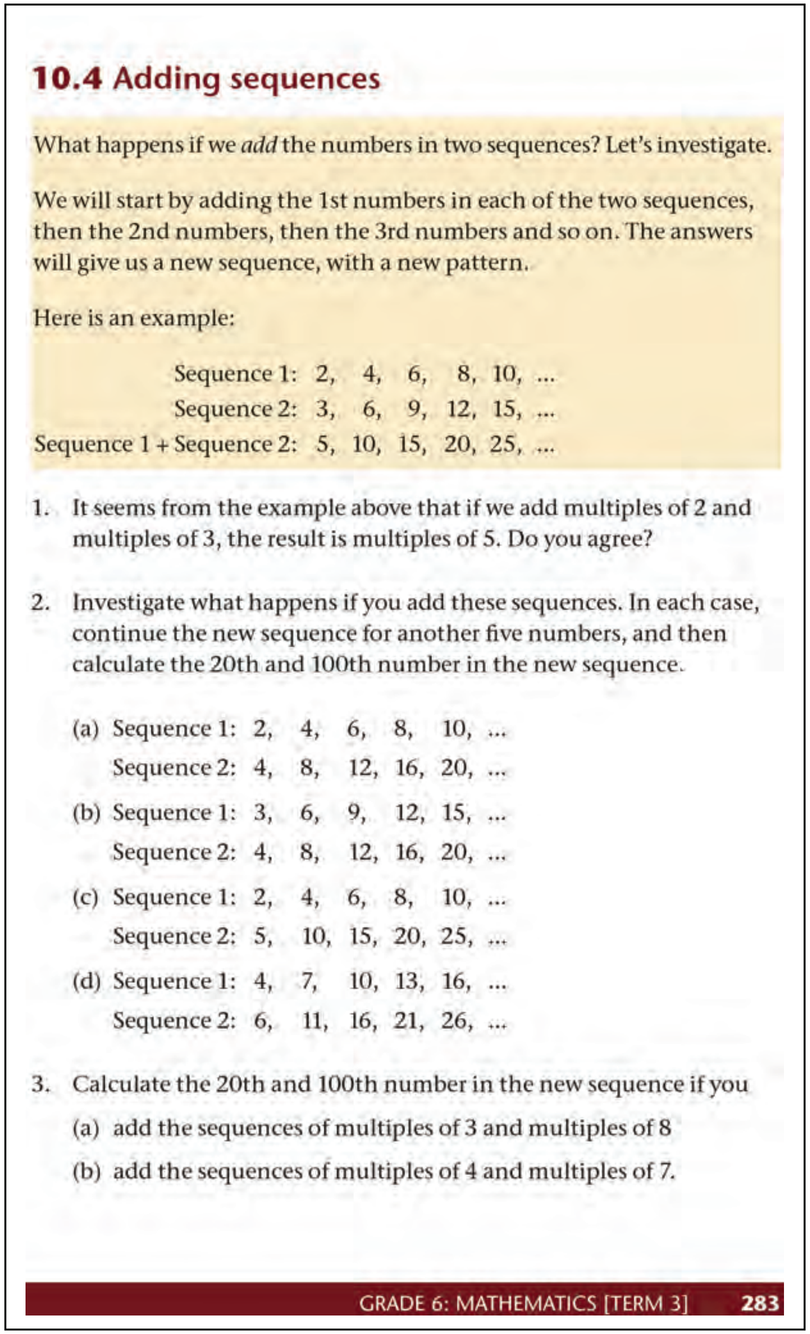
Mathematical notes

From the examples it seems that if we add two sequences with a constant difference, the new sequence again has a constant difference. We can say (mentally) exactly what the new sequence is without actually having or adding the two sequences. If we add multiples of 3 and multiples of 5, the result is multiples of 8. You don't need the sequences of multiples of 3 and multiples of 5. In terms of high school mathematics we can say that if we add two arithmetic sequences, the result is again an arithmetic sequence.

For teachers only: If the two arithmetic sequences are represented as $an + b$ and $pn + q$, then the sum is $(a + p)n + (b + q)$, which again is an arithmetic sequence.

Answers

1. Yes
2. (a) 6, 12, 18, 24, 30, 36, 42, 48, 54, 60
   Multiples of 6, so 20th number = 20×6 = 120; 100th number = 100×6 = 600
   (b) 7, 14, 21, 28, 35, 42, 49, 56, 63, 70
   Multiples of 7, so 20th number = 20×7 = 140; 100th number = 100×7 = 700
   (c) 7, 14, 21, 28, 35, 42, 49, 56, 63, 70
   Multiples of 7, so 20th number = 20×7 = 140; 100th number = 100×7 = 700
   (d) 10, 18, 26, 34, 42, 50, 58, 66, 74, 82
   Rule: *Position number* × 8 + 2. So 20th number = 20×8 + 2 = 162;
   100th number = 100×8 + 2 = 802
3. (a) It will be multiples of 3+8 = 11, with rule *Position* number×11.
   20th number = 20×11 = 220; 100th number = 100×11 = 1 100
   (b) It will be multiples of 4+7 = 11, with rule *Position* number×11.
   20th number = 20×11 = 220; 100th number = 100×11 = 1 100



10.4 Adding sequences

What happens if we *add* the numbers in two sequences? Let's investigate.

We will start by adding the 1st numbers in each of the two sequences, then the 2nd numbers, then the 3rd numbers and so on. The answers will give us a new sequence, with a new pattern.

Here is an example:

Sequence 1: 2, 4, 6, 8, 10, ...
Sequence 2: 3, 6, 9, 12, 15, ...
Sequence 1 + Sequence 2: 5, 10, 15, 20, 25, ...

1. It seems from the example above that if we add multiples of 2 and multiples of 3, the result is multiples of 5. Do you agree?
2. Investigate what happens if you add these sequences. In each case, continue the new sequence for another five numbers, and then calculate the 20th and 100th number in the new sequence.
   (a) Sequence 1: 2, 4, 6, 8, 10, ...
   Sequence 2: 4, 8, 12, 16, 20, ...
   (b) Sequence 1: 3, 6, 9, 12, 15, ...
   Sequence 2: 4, 8, 12, 16, 20, ...
   (c) Sequence 1: 2, 4, 6, 8, 10, ...
   Sequence 2: 5, 10, 15, 20, 25, ...
   (d) Sequence 1: 4, 7, 10, 13, 16, ...
   Sequence 2: 6, 11, 16, 21, 26, ...
3. Calculate the 20th and 100th number in the new sequence if you
   (a) add the sequences of multiples of 3 and multiples of 8
   (b) add the sequences of multiples of 4 and multiples of 7.

10.5 Multiplying sequences

Teaching guidelines

This section is an extension of the previous section, with exactly the same mindset, but now using multiplication instead of addition. **This section is intended for enrichment only, and only to be attempted by the fastest learners.** Learners will have to be very inventive in establishing a vertical relationship that can be used to find the 20th and 100th numbers. We give some examples below.

For teachers only: If we multiply two arithmetic sequences, the result is a quadratic sequence, i.e. a sequence that does not have a constant difference, but an increasing difference such as 2, 4, 6, 8, 10 ...

Answers

1. (a) New sequence: 4, 16, 36, 64, 100, 144, 196, 256, 324, 400

This can be re-written as:

Position number	1	2	3	4	5	6	7
Output number	2^2	4^2	6^2	8^2	10^2	12^2	14^2

$20^{th} = (2\times20)^2 =$ 1 600 100th: $(2\times100)^2 =$ 40 000

(b) New sequence: 2, 8, 18, 32, 50, 72, 98, 128, 162, 200

Position no.	**1**	**2**	**3**	**4**	**5**	. . .	**20**	. . .	**100**
	↓×2	↓×4	↓×6	↓×8	↓×10		↓×40		↓×200
Sequence	2	8	18	32	50		800		20 000

(c) New sequence: 6, 24, 54, 96, 150, 216, 294, 384, 486, 600

Position no.	**1**	**2**	**3**	**4**	**5**	. . .	**20**	. . .	**100**
	↓×6	↓×12	↓×18	↓×24	↓×30		↓×120		↓×600
Sequence	6	24	54	96	150		2 400		60 000

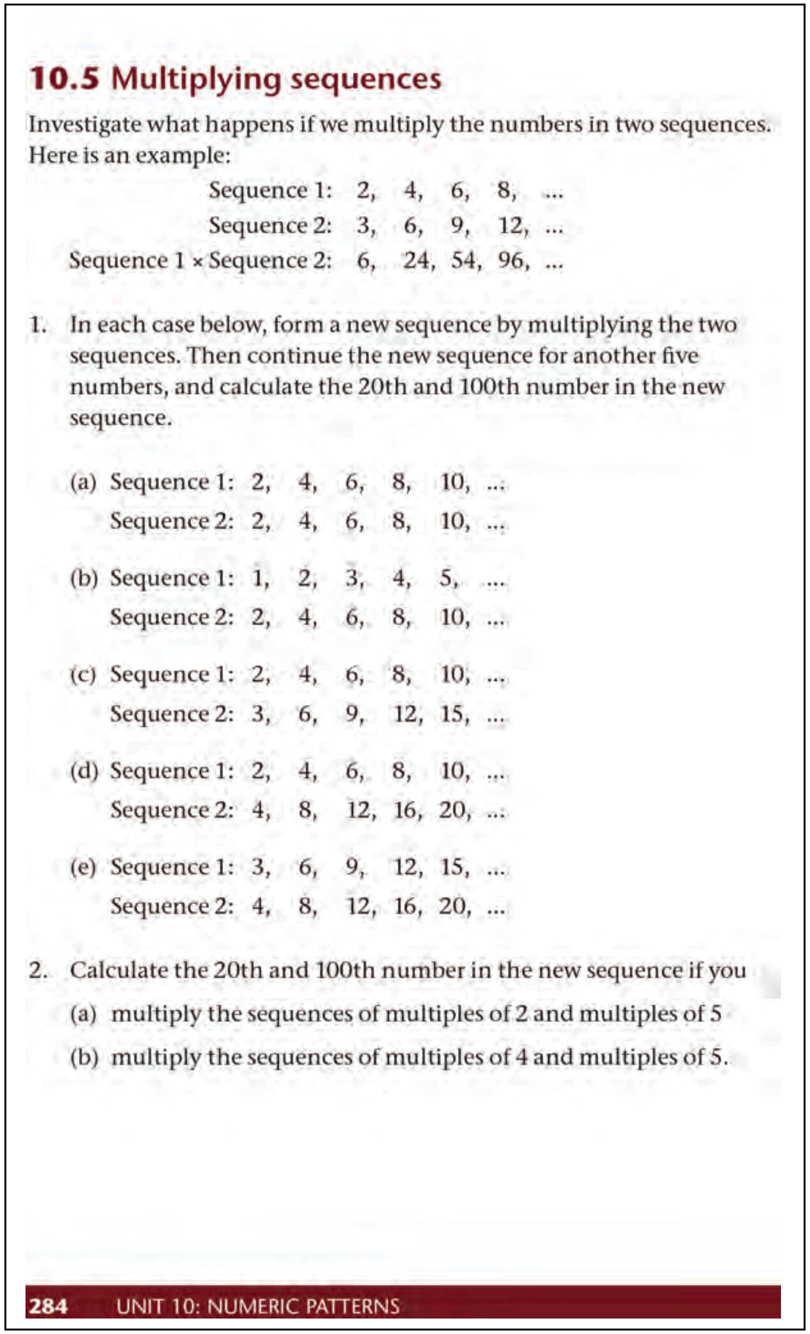
(d) New sequence: 8, 32, 72, 128, 200, 288, 392, 512, 648, 800

Position no.	**1**	**2**	**3**	**4**	**5**	. . .	**20**	. . .	**100**
	↓×8	↓×16	↓×24	↓×32	↓×40		↓×160		↓×800
Sequence	8	32	72	128	200		3 200		80 000

(e) New sequence: 12, 48, 108, 192, 300, 432, 588, 768, 972, 1 200

Position no.	**1**	**2**	**3**	**4**	**5**	. . .	**20**	. . .	**100**
	↓×12	↓×24	↓×36	↓×48	↓×60		↓×160		↓×1 200
Sequence	8	32	72	128	2 000		3 200		120 000

2. (a) 20th: 4 000; 100th: 100 000 (b) 20th: 8 000; 100th: 200 000



10.5 Multiplying sequences

Investigate what happens if we multiply the numbers in two sequences. Here is an example:

Sequence 1: 2, 4, 6, 8, ...
Sequence 2: 3, 6, 9, 12, ...
Sequence 1 × Sequence 2: 6, 24, 54, 96, ...

1. In each case below, form a new sequence by multiplying the two sequences. Then continue the new sequence for another five numbers, and calculate the 20th and 100th number in the new sequence.

(a) Sequence 1: 2, 4, 6, 8, 10, ...
Sequence 2: 2, 4, 6, 8, 10, ...

(b) Sequence 1: 1, 2, 3, 4, 5, ...
Sequence 2: 2, 4, 6, 8, 10, ...

(c) Sequence 1: 2, 4, 6, 8, 10, ...
Sequence 2: 3, 6, 9, 12, 15, ...

(d) Sequence 1: 2, 4, 6, 8, 10, ...
Sequence 2: 4, 8, 12, 16, 20, ...

(e) Sequence 1: 3, 6, 9, 12, 15, ...
Sequence 2: 4, 8, 12, 16, 20, ...

2. Calculate the 20th and 100th number in the new sequence if you
(a) multiply the sequences of multiples of 2 and multiples of 5
(b) multiply the sequences of multiples of 4 and multiples of 5.

Grade 6 Term 3 Unit 11 Length

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
11.1 Estimate, measure, compare and order	Estimating and measuring length, choose appropriate units and instruments	285 to 287
11.2 Write in different units	Converting between metric units of length	288 to 289
11.3 Calculations	Doing calculations in the context of length	290
11.4 Rounding off	Rounding off to 5, 10, 100 and 1 000, as well as particular units of length	291 to 293
11.5 Problem solving	Solving problems within the context of length	293 to 294

CAPS time allocation	5 hours
CAPS page references	25 and 272 to 274

Mathematical background

In this unit learners will explore the lengths of objects. We can use lengths to compare and order objects. For example, we can use our sense of length to say that the teacher's desk is wider than the classroom door. Knowledge of length allows us to solve more complex problems and calculations. For example, if a roll of string is 500 m long, will there be enough string to give each Grade 6 learner 2 m of string if there are 4 classes of 40 learners each?

Learners go through four stages when learning to measure. These four stages consist of:

1. identifying and understanding the property they are measuring
2. comparing and ordering examples of a particular measure
3. using informal or non-standard units to measure
4. using formal or standard units to measure.

Using standard units ensures that people everywhere in the world can measure, quantify and compare objects using the same measure. The focus for Grade 6 learners is on using standard units of length. By Grade 6, most learners can comfortably use a ruler to measure in centimetres and millimetres. Some are also comfortable with using a metre stick. However, many may still find it difficult to use builders' tape measures, and many have little experience of using a trundle wheel. The more opportunities learners have to estimate distances, measure them, and then compare the difference between their estimates and measurements, the better they will become at both estimating and measuring. It is important that learners estimate before measuring, rather than simply rounding off their measurements.

Resources

Long jump record sheet; different measuring instruments for practice, i.e. metre sticks, trundle wheel, builders' tape measure, measuring tapes

Each learner must have a ruler.

11.1 Estimate, measure, compare and order

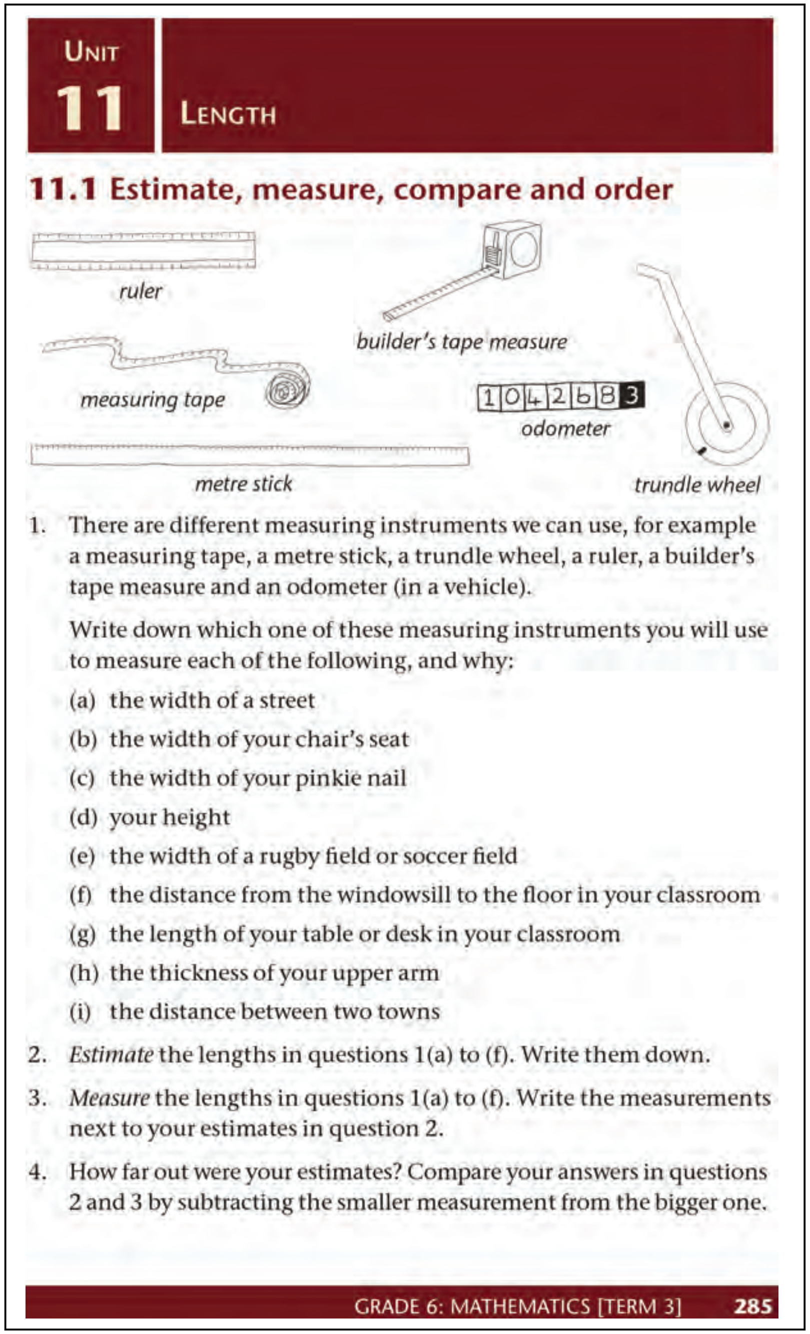
Teaching guidelines

It is essential to demonstrate the use of a metre stick.

Learners might not recognise the odometer in the picture, so you'll have to show a variety of real-life examples to learners.

Answers

1. (a) A builders' tape measure or trundle wheel; most streets are more than 2 m, but less than 10 m wide.
   (b) A ruler, metre stick or measuring tape; most school chairs are narrow enough to measure with a ruler.
   (c) A ruler; the width of a pinkie fingernail is a couple of millimetres.
   (d) A measuring tape; most Grade 6 learners are between 1 m and 2 m tall.
   (e) A trundle wheel; the width of a field is normally between 50 m and 75 m.
   (f) A metre stick or measuring tape; this distance is normally more than a ruler length but less than a metre.
   (g) A metre stick or a measuring tape; this distance is normally more than a ruler length but less than a metre.
   (h) A measuring tape, because it can bend around a person's arm.
   (i) An odometer; the distance between towns is usually many kilometres.
2. (a) Between 8 m and 10 m.
   (b) Probably varies between 30 cm and 50 cm.
   (c) Between 5 mm and 10 mm.
   (d) Average for boys in Grade 6 is 147 cm to 157 cm. Average for girls in Grade 6 is 152 cm to 160 cm. (The boys catch up at about age 15 and become taller on average than the girls.)
   (e) Standard fields: Soccer 64 m to 73 m and Rugby 68 m to 70 m.
   (f) This will vary from classroom to classroom.
   (g) This will vary from classroom to classroom.
   (h) Varies from learner to learner, possibly between 15 cm and 35 cm.
   (i) Most learners are not able to measure this during a normal school day. Varies from town to town.
3. Practical activity. Learners' answers will differ.
4. Learners' answers will differ.



UNIT 11 LENGTH

11.1 Estimate, measure, compare and order

1. There are different measuring instruments we can use, for example a measuring tape, a metre stick, a trundle wheel, a ruler, a builder's tape measure and an odometer (in a vehicle).

   Write down which one of these measuring instruments you will use to measure each of the following, and why:
   (a) the width of a street
   (b) the width of your chair's seat
   (c) the width of your pinkie nail
   (d) your height
   (e) the width of a rugby field or soccer field
   (f) the distance from the windowsill to the floor in your classroom
   (g) the length of your table or desk in your classroom
   (h) the thickness of your upper arm
   (i) the distance between two towns
2. *Estimate* the lengths in questions 1(a) to (f). Write them down.
3. *Measure* the lengths in questions 1(a) to (f). Write the measurements next to your estimates in question 2.
4. How far out were your estimates? Compare your answers in questions 2 and 3 by subtracting the smaller measurement from the bigger one.

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Mathematical notes

In question 6 learners work with rulers scaled in different units of measurement, for example centimetres, tenths of metres and tenths of kilometres. This question helps us to lay a basis for proportional reasoning and also for understanding the scaling that learners will need to do when they draw graphs in the Senior Phase.

Teaching guidelines

In question 6 learners first need to work out what the longer divisions represent.
They need to:

- subtract the first numbered division from the second numbered division, for example: in the top ruler this is 8 cm – 3 cm = 5 cm;
- count the number of intervals marked by longer lines between the numbered divisions – in the top ruler this is 5;
- divide the measurement obtained in the first step by the number of intervals, for example: 5 cm ÷ 5 divisions = each longer division is 1 cm; and
- count the number of smaller intervals between the larger intervals, which in this case is 10 (each smaller division in the top ruler is 1 mm).

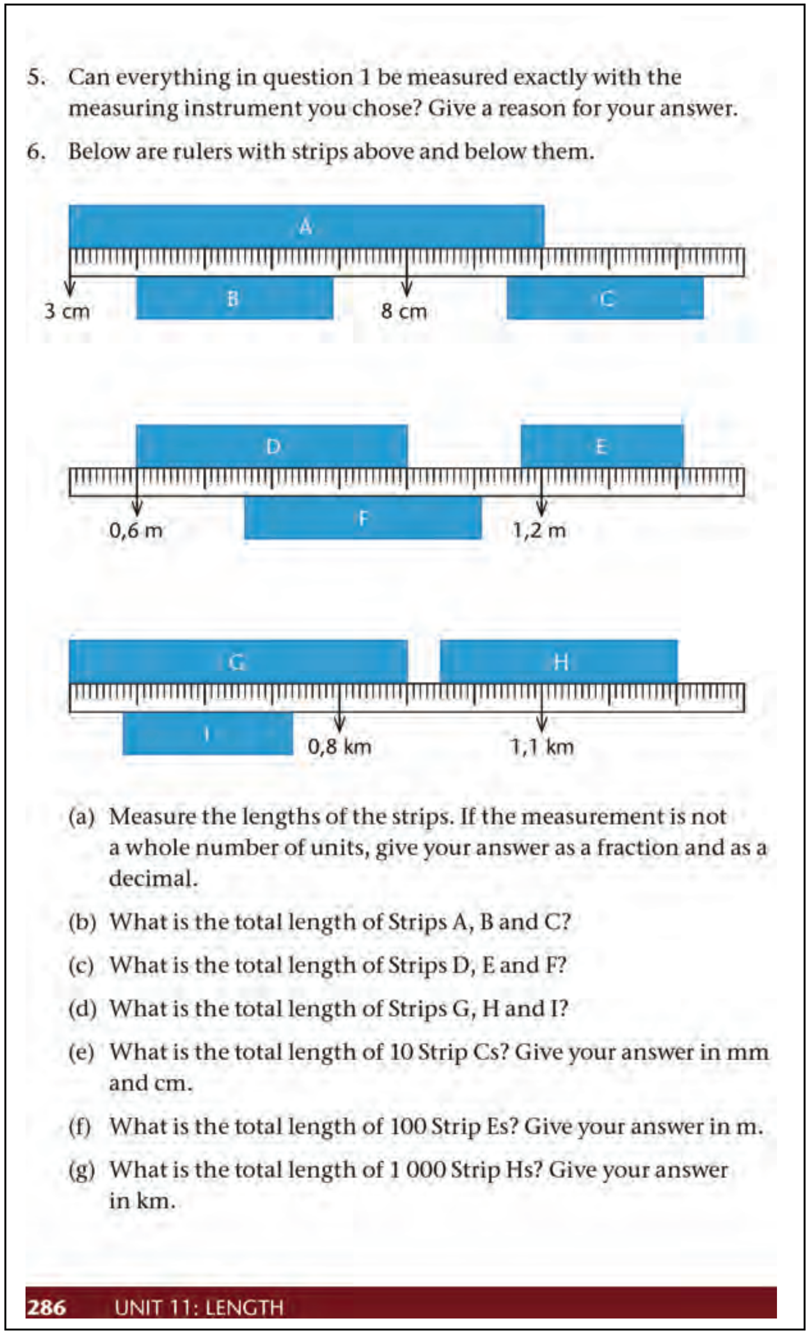
Possible misconceptions

Remind learners that the scaling on each ruler is different.

Answers

5. Learners' answers will differ, but most measurements are not exact. Motor vehicle parts must fit tightly and must therefore be measured very accurately, with an error of no more than 0,01 mm. The white lines on a sports field, however, can have an error of 10 mm in their width and most of the time it will go unnoticed.

6. (a) A: 7 cm B: $2\frac{9}{10}$ cm = 2,9 cm C: $2\frac{9}{10}$ cm = 2,9 cm
D: $\frac{4}{10}$ m = 0,4 m E: $\frac{24}{100}$ m = 0,24 m F: $\frac{35}{100}$ m = 0,35 m
G: 0,5 km H: $\frac{35}{100}$ km = 0,35 km I: $\frac{25}{100}$ km = 0,25 km
(b) 7 cm + 2,9 cm + 2,9 cm = 12,8 cm (c) 0,4 m + 0,24 m + 0,35 m = 0,99 m
(d) 0,5 km + 0,35 km + 0,25 km = 1,1 km (e) 2,9 mm × 10 = 29 mm = 2 cm + 9 mm
(f) 0,24 m × 100 = 24 m (g) 0,35 km × 1 000 = 350 km



5. Can everything in question 1 be measured exactly with the measuring instrument you chose? Give a reason for your answer.
6. Below are rulers with strips above and below them.

(a) Measure the lengths of the strips. If the measurement is not a whole number of units, give your answer as a fraction and as a decimal.
(b) What is the total length of Strips A, B and C?
(c) What is the total length of Strips D, E and F?
(d) What is the total length of Strips G, H and I?
(e) What is the total length of 10 Strip Cs? Give your answer in mm and cm.
(f) What is the total length of 100 Strip Es? Give your answer in m.
(g) What is the total length of 1 000 Strip Hs? Give your answer in km.

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Mathematical notes

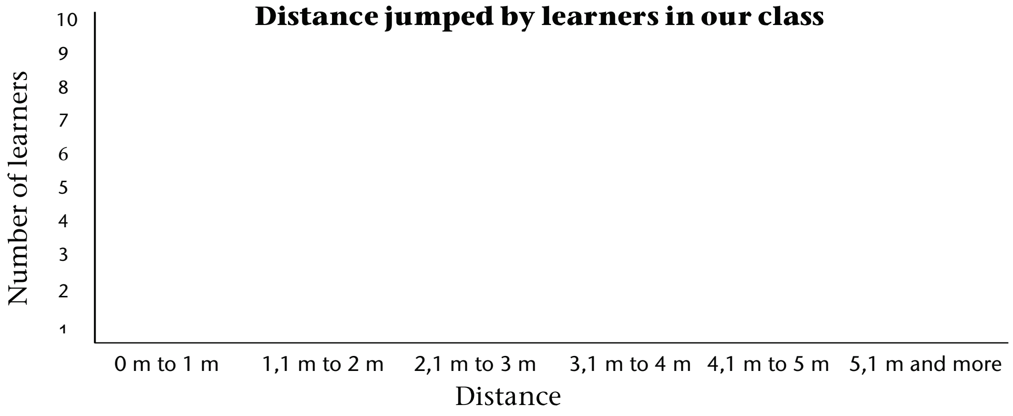
The examples in questions 7 and 8 are designed to help learners visualise and estimate lengths. Whenever learners take measurements that vary, these measurements can be used as data. Keep the data of learners' long jump efforts. Learners can organise, analyse and represent this data. You may need to assist learners in deciding on labels and scaling for the axes.

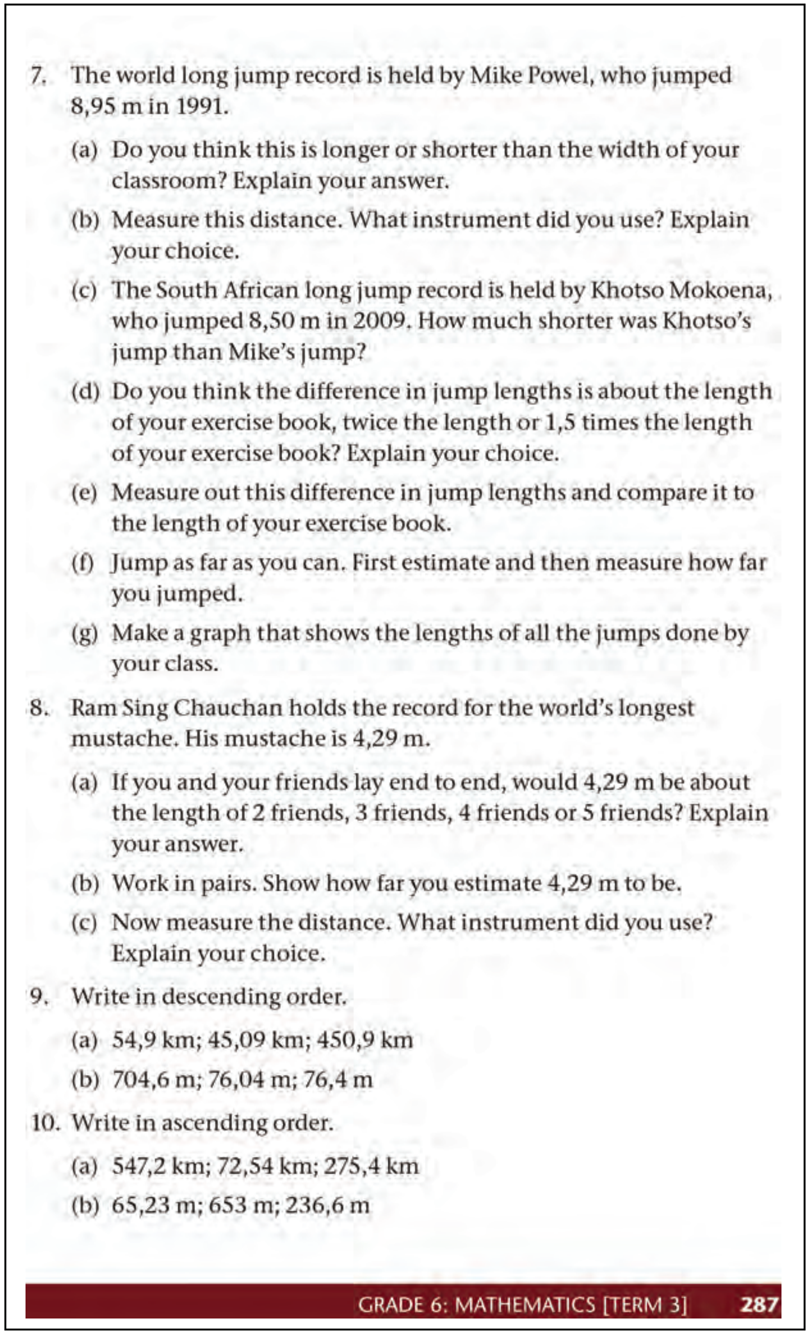
Notes on question 7(f)

Divide learners into groups of ten to do the long jump activity. You could ask a parent or teaching assistant to supervise it. Let learners work in pairs. One learner jumps, the other marks the end points. They then estimate and measure the distance. Then the other learner jumps. Give learners a recording sheet that includes space for the estimated and measured distances.

Answers

7. (a) Learners' answers will differ, partly because the sizes of classrooms differ.
   (b) Practical activity. Measuring tapes or metre sticks are sensible instruments.
   (c) 0,45 m or 45 cm
   (d) Learners' answers will differ. Consider all explanations. Learners could think of the measurement as about half a metre, or 45 cm. The length of an A4 page is about 30 cm. 45 cm is about $1\frac{1}{2}$ times the length of an A4 page.
   (e) Practical activity
   (f) Practical activity
   (g) Graphs will differ from class to class. An example of how to label the axes is provided below.





7. The world long jump record is held by Mike Powel, who jumped 8,95 m in 1991.
   (a) Do you think this is longer or shorter than the width of your classroom? Explain your answer.
   (b) Measure this distance. What instrument did you use? Explain your choice.
   (c) The South African long jump record is held by Khotso Mokoena, who jumped 8,50 m in 2009. How much shorter was Khotso's jump than Mike's jump?
   (d) Do you think the difference in jump lengths is about the length of your exercise book, twice the length or 1,5 times the length of your exercise book? Explain your choice.
   (e) Measure out this difference in jump lengths and compare it to the length of your exercise book.
   (f) Jump as far as you can. First estimate and then measure how far you jumped.
   (g) Make a graph that shows the lengths of all the jumps done by your class.
8. Ram Sing Chauchan holds the record for the world's longest mustache. His mustache is 4,29 m.
   (a) If you and your friends lay end to end, would 4,29 m be about the length of 2 friends, 3 friends, 4 friends or 5 friends? Explain your answer.
   (b) Work in pairs. Show how far you estimate 4,29 m to be.
   (c) Now measure the distance. What instrument did you use? Explain your choice.
9. Write in descending order.
   (a) 54,9 km; 45,09 km; 450,9 km
   (b) 704,6 m; 76,04 m; 76,4 m
10. Write in ascending order.
   (a) 547,2 km; 72,54 km; 275,4 km
   (b) 65,23 m; 653 m; 236,6 m

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Answers*(continue)*

8. (a) Learners' answers will differ. Most Grade 6 learners are about $1\frac{1}{2}$ m tall; 4,29 m is longer than two learners but shorter than three learners.
   (b) Practical activity
   (c) Practical activity. Metre sticks or measuring tapes are suitable instruments.
9. (a) 450,9 km; 54,9 km; 45,09 km (b) 704,6 m; 76,4 m; 76,04 m
10. (a) 72,54 km; 275,4 km; 547,2 km (b) 65,23 m; 236,6 m; 653 m

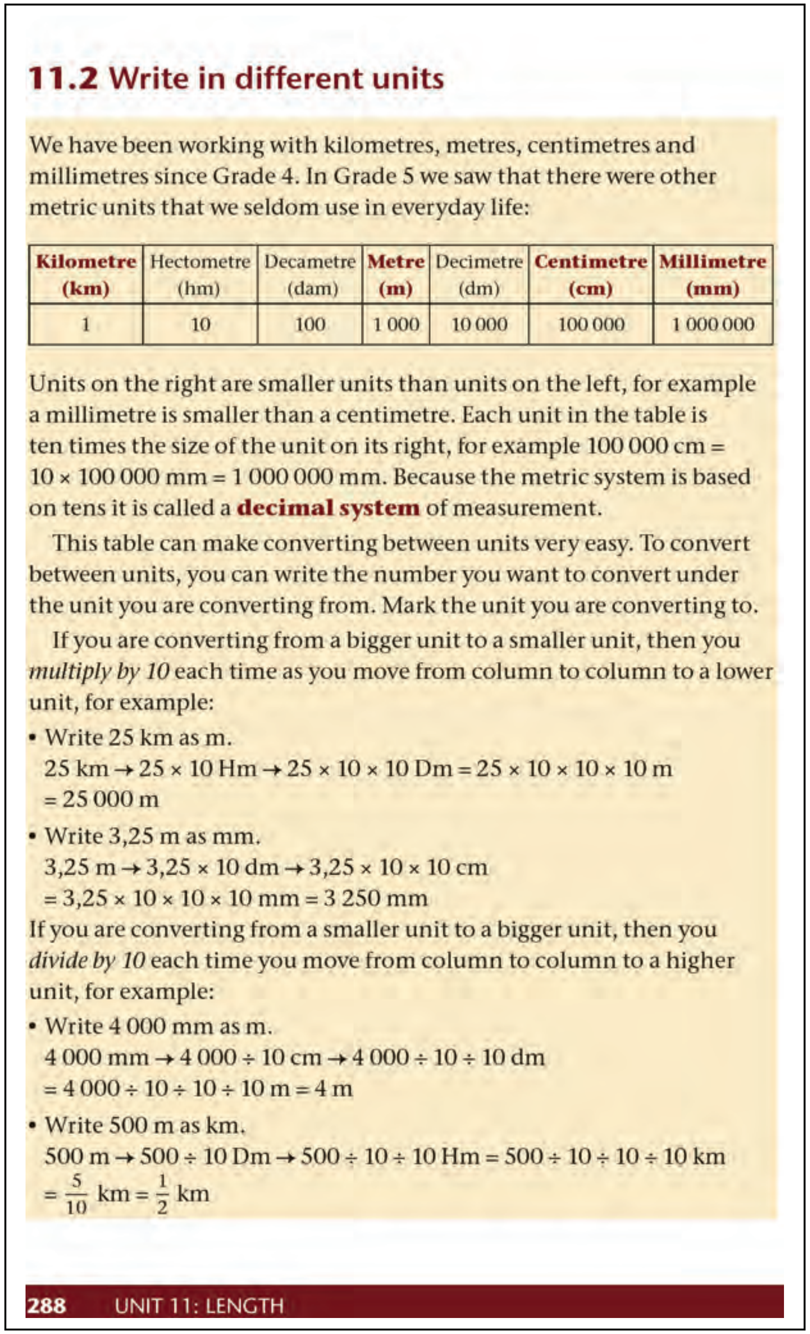
11.2 Write in different units

Mathematical notes

In Grade 6 learners convert from:

- km to m, cm and mm
- m to km, cm and mm
- cm to km, m and mm
- mm to cm, m and km.

Learners can learn the conversion factors off by heart. However, as with everything learnt off by heart, learners will sometimes forget the conversion factors and use an incorrect one. It is better for learners to understand how the relationship between metric units works in general. The table in the shaded passage in the Learner Book helps learners to understand how to convert units.



11.2 Write in different units

We have been working with kilometres, metres, centimetres and millimetres since Grade 4. In Grade 5 we saw that there were other metric units that we seldom use in everyday life:

Kilometre (km)	Hectometre (hm)	Decametre (dam)	**Metre (m)**	Decimetre (dm)	**Centimetre (cm)**	**Millimetre (mm)**
1	10	100	1 000	10 000	100 000	1 000 000

Units on the right are smaller units than units on the left, for example a millimetre is smaller than a centimetre. Each unit in the table is ten times the size of the unit on its right, for example 100 000 cm = 10 × 100 000 mm = 1 000 000 mm. Because the metric system is based on tens it is called a **decimal system** of measurement.

This table can make converting between units very easy. To convert between units, you can write the number you want to convert under the unit you are converting from. Mark the unit you are converting to.

If you are converting from a bigger unit to a smaller unit, then you *multiply by 10* each time as you move from column to column to a lower unit, for example:

- Write 25 km as m.
  25 km → 25 × 10 Hm → 25 × 10 × 10 Dm = 25 × 10 × 10 × 10 m
  = 25 000 m
- Write 3,25 m as mm.
  3,25 m → 3,25 × 10 dm → 3,25 × 10 × 10 cm
  = 3,25 × 10 × 10 × 10 mm = 3 250 mm

If you are converting from a smaller unit to a bigger unit, then you *divide by 10* each time you move from column to column to a higher unit, for example:

- Write 4 000 mm as m.
  4 000 mm → 4 000 ÷ 10 cm → 4 000 ÷ 10 ÷ 10 dm
  = 4 000 ÷ 10 ÷ 10 ÷ 10 m = 4 m
- Write 500 m as km.
  500 m → 500 ÷ 10 Dm → 500 ÷ 10 ÷ 10 Hm = 500 ÷ 10 ÷ 10 ÷ 10 km
  = $\frac{5}{10}$ km = $\frac{1}{2}$ km

Teaching guidelines

You can use the approach in the shaded passage on page 288 to explain to learners how to use the expanded table to convert between units. Learners draw a table like the one below.

km	hm	dam	**m**	dm	**cm**	**mm**

Be mindful of the prescribed units of measurement (printed in bold) and note that additional units are indicated in the table above.

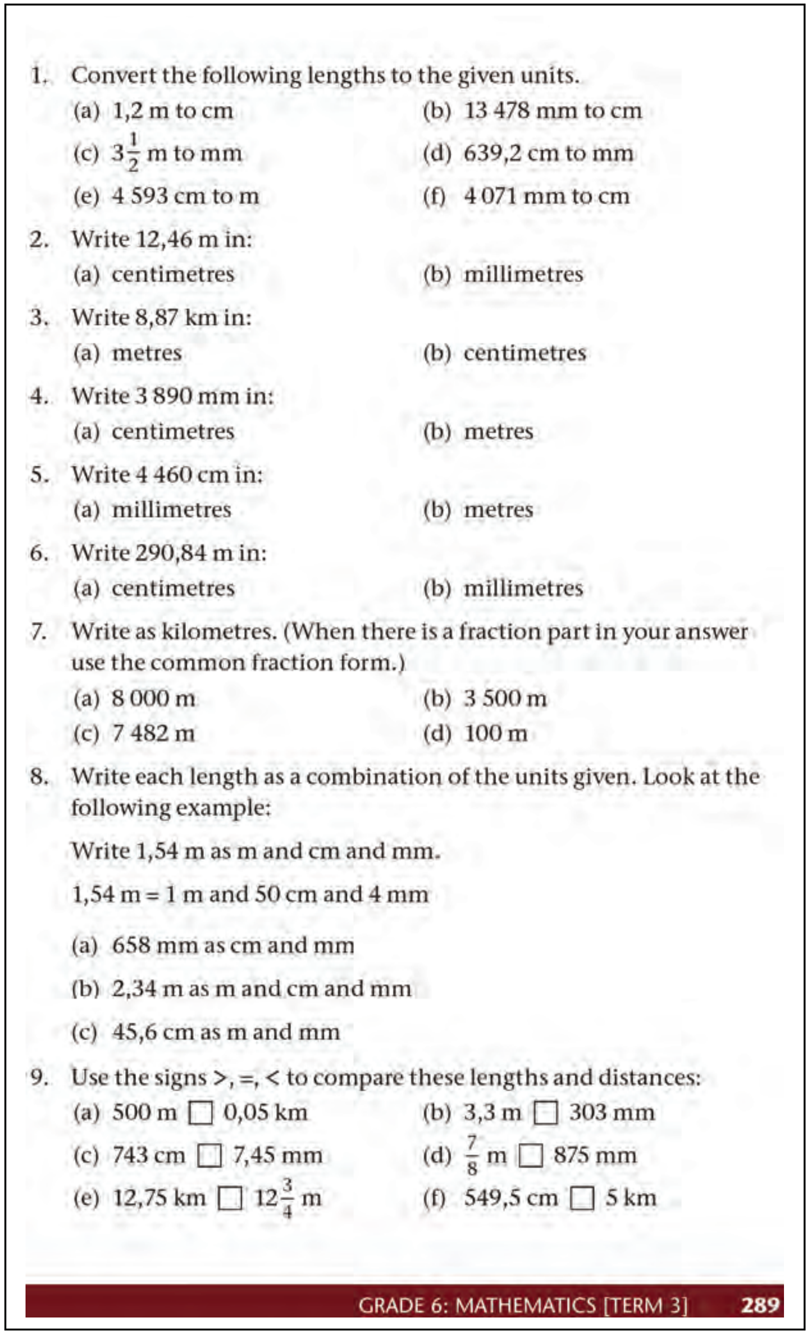
Learners must:

- write the number under the correct unit and then mark to which unit they are converting;
- when converting from a larger unit to a smaller unit, multiply by 10 each time they move to a smaller unit; and
- when converting from a smaller unit to larger unit, divide by 10 each time they move to a larger unit.

Teaching learners a mnemonic will help them to remember the units (their names, their sequence and the numerical relationships between them), even though they are not required to "work" with all seven of them. You could make any sentence you like with words that begin with the letters k, h, d, m, d, c, m. The Department of Basic Education, for example, provides the mnemonic "Kids Have Dreams Making Dad Chocolate Muffins" (DBE (2015). *Annual National Assessment of 2014. Diagnostic report. Intermediate and Senior Phases. Mathematics.* Government Printers. Pretoria, p. 37).

Answers

1. (a) 120 cm (b) 1 347,8 mm (c) 3 500 mm
   (d) 6 392 mm (e) 45,93 m (f) 407,1 cm
2. (a) 1 246 cm (b) 12 460 mm
3. (a) 8 870 m (b) 887 000 cm
4. (a) 389 cm (b) 3,89 m
5. (a) 44 600 mm (b) 44,6 m
6. (a) 29 084 cm (b) 290 840 mm
7. (a) 8 km (b) $3\frac{1}{2}$ km (c) $7\frac{482}{1\,000}$ km (d) $\frac{1}{10}$ km
8. (a) 65 cm and 8 mm (b) 2 m 34 cm and 0 mm (c) 0 m and 456 mm
9. (a) 500 m > 0,05 km (b) 3,3 m > 303 mm (c) 743 cm > 7,45 mm
   (d) $\frac{7}{8}$ m = 875 mm (e) 12,75 km > $12\frac{3}{4}$ m (f) 549,5 cm < 5 km



1. Convert the following lengths to the given units.
   (a) 1,2 m to cm (b) 13 478 mm to cm
   (c) $3\frac{1}{2}$ m to mm (d) 639,2 cm to mm
   (e) 4 593 cm to m (f) 4 071 mm to cm
2. Write 12,46 m in:
   (a) centimetres (b) millimetres
3. Write 8,87 km in:
   (a) metres (b) centimetres
4. Write 3 890 mm in:
   (a) centimetres (b) metres
5. Write 4 460 cm in:
   (a) millimetres (b) metres
6. Write 290,84 m in:
   (a) centimetres (b) millimetres
7. Write as kilometres. (When there is a fraction part in your answer use the common fraction form.)
   (a) 8 000 m (b) 3 500 m
   (c) 7 482 m (d) 100 m
8. Write each length as a combination of the units given. Look at the following example:

   Write 1,54 m as m and cm and mm.

   1,54 m = 1 m and 50 cm and 4 mm

   (a) 658 mm as cm and mm
   (b) 2,34 m as m and cm and mm
   (c) 45,6 cm as m and mm
9. Use the signs >, =, < to compare these lengths and distances:
   (a) 500 m ☐ 0,05 km (b) 3,3 m ☐ 303 mm
   (c) 743 cm ☐ 7,45 mm (d) $\frac{7}{8}$ m ☐ 875 mm
   (e) 12,75 km ☐ $12\frac{3}{4}$ m (f) 549,5 cm ☐ 5 km

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11.3 Calculations

Teaching guidelines

The easiest way for learners to find the answer to question 6, is to count in intervals of $1\frac{1}{2}$, for example, $1\frac{1}{2}$; 3; $4\frac{1}{2}$; 6, etc. until they get to just beyond 20. Learners can record this on a number line.

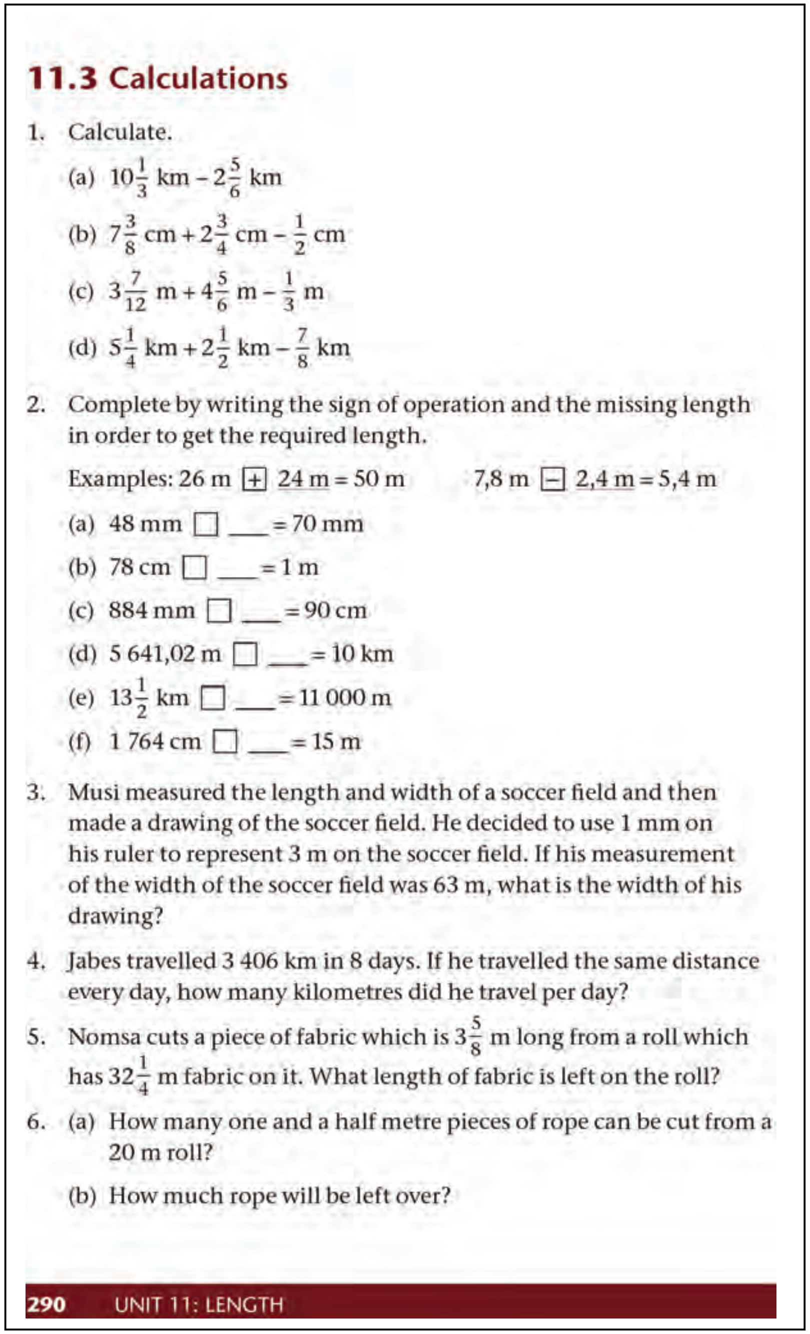
In question 6(a) learners cannot just divide 20 by $1\frac{1}{2}$ and take $13\frac{1}{3}$ length of rope as the answer. Explain to learners that the answer must be a whole number of lengths of rope of $1\frac{1}{2}$ m.

Notes on questions

Question 3 uses ratio; question 4 is an example of rate. In question 3 the real width of the field was 63 m. Musi cannot fit 63 m onto a sheet of paper so he does a scale drawing. 1 mm on the paper will represent 3 m on the ground. How many 3 m lengths are there in 63 m? When learners have answered that question, they need only a small step to the answer of 21 mm.

Answers

1. (a) $7\frac{1}{2}$ km (b) $9\frac{5}{8}$ cm
   (c) $8\frac{1}{12}$ m (d) $6\frac{7}{8}$ km
2. (a) + 22 mm (b) + 22 cm (c) + 16 mm
   (d) + 4 358,98 m (e) – $2\frac{1}{2}$ km (f) – 264 cm
3. 21 mm
4. 425,75 km, because 3 406 km ÷ 8 days = 425,75 km per day (i.e. km in every day)
5. $28\frac{5}{8}$ m, because $32\frac{1}{4}$ m – $3\frac{5}{8}$ is the same as $32\frac{2}{8} - 3\frac{5}{8}$, which is the same as $31 + \frac{10}{8} - 3\frac{5}{8}$, which is the same as $28 + \frac{5}{8}$, which is $28\frac{5}{8}$
6. (a) 13 lengths
   (b) $\frac{1}{2}$ m rope left over



11.3 Calculations

1. Calculate.
   (a) $10\frac{1}{3}$ km – $2\frac{5}{6}$ km
   (b) $7\frac{3}{8}$ cm + $2\frac{3}{4}$ cm – $\frac{1}{2}$ cm
   (c) $3\frac{7}{12}$ m + $4\frac{5}{6}$ m – $\frac{1}{3}$ m
   (d) $5\frac{1}{4}$ km + $2\frac{1}{2}$ km – $\frac{7}{8}$ km
2. Complete by writing the sign of operation and the missing length in order to get the required length.
   Examples: 26 m $\boxed{+}$ <u>24 m</u> = 50 m 7,8 m $\boxed{-}$ <u>2,4 m</u> = 5,4 m
   (a) 48 mm ☐ ___ = 70 mm
   (b) 78 cm ☐ ___ = 1 m
   (c) 884 mm ☐ ___ = 90 cm
   (d) 5 641,02 m ☐ ___ = 10 km
   (e) $13\frac{1}{2}$ km ☐ ___ = 11 000 m
   (f) 1 764 cm ☐ ___ = 15 m
3. Musi measured the length and width of a soccer field and then made a drawing of the soccer field. He decided to use 1 mm on his ruler to represent 3 m on the soccer field. If his measurement of the width of the soccer field was 63 m, what is the width of his drawing?
4. Jabes travelled 3 406 km in 8 days. If he travelled the same distance every day, how many kilometres did he travel per day?
5. Nomsa cuts a piece of fabric which is $3\frac{5}{8}$ m long from a roll which has $32\frac{1}{4}$ m fabric on it. What length of fabric is left on the roll?
6. (a) How many one and a half metre pieces of rope can be cut from a 20 m roll?
   (b) How much rope will be left over?

290 UNIT 11: LENGTH

11.4 Rounding off

Mathematical notes

In this section learners round off to the nearest 5, 10, 100 and 1 000. Learners also round off to specified units of measurement.

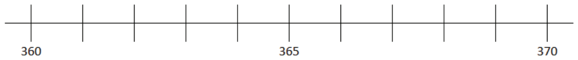
Measurement provides a useful context for learners to understand rounding off. Questions that ask *"is it closer to ... than to ...",* assist learners in further understanding the concept of rounding off.

Teaching guidelines

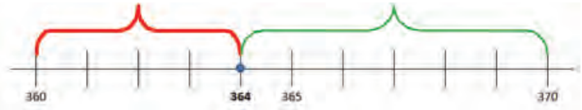
The understanding of the concept of place value will be useful when rounding off is applied. The following strategy may help learners to apply rounding off:

Example: Round 364 off to the nearest 10.

- Draw a number line that includes 10 intervals, starting with 360 and ending with 370.



- Mark 364 on the number line and determine whether it is closer to 360 or closer to 370.



- 364 is closer to 360
- 364 ≈ 360

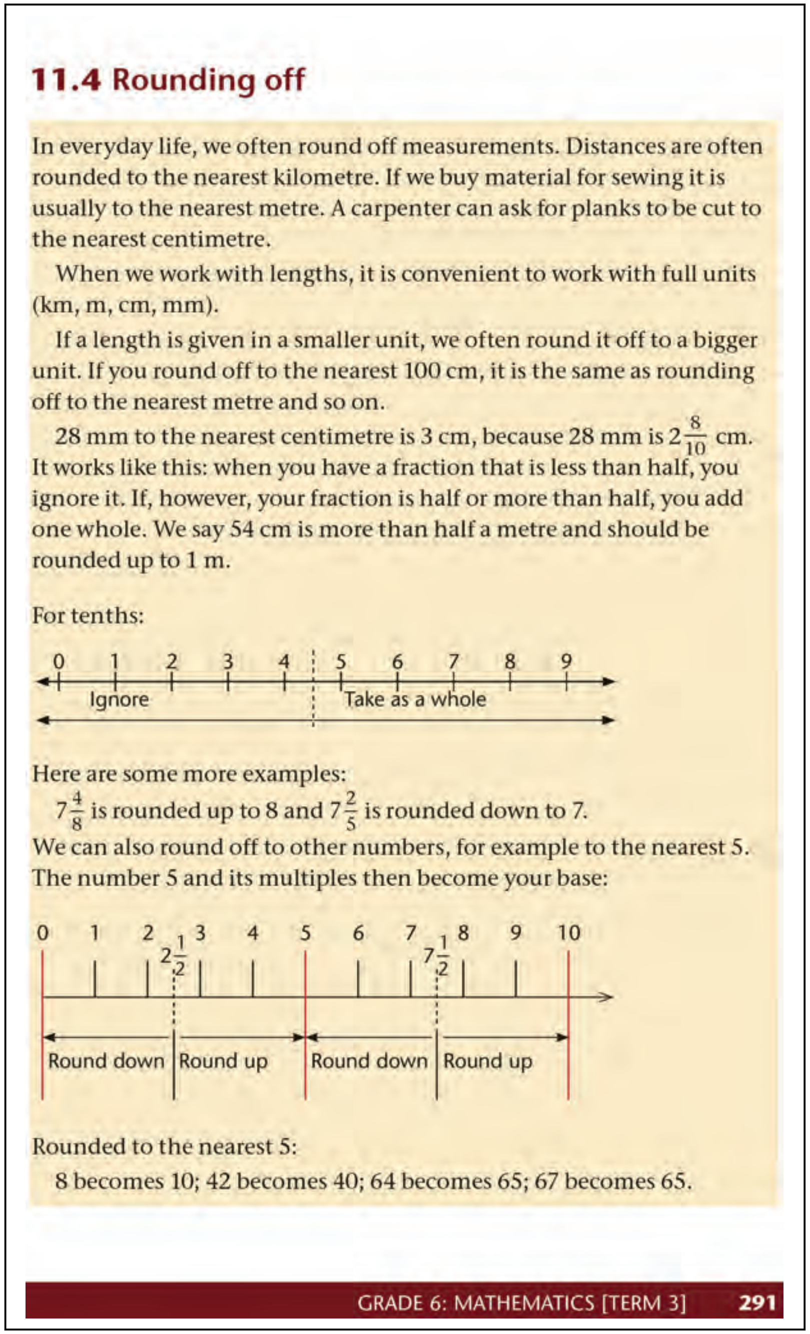
You can also use the approach in the shaded passage to explain rounding off to the nearest unit of measurement. If a length is given in a smaller unit, we often round it off to a bigger unit. If we round off to the nearest 100 cm, it is the same as rounding off to the nearest metre. Rounding off to the nearest 10 mm is the same as rounding off to the nearest centimetre. Rounding off to the nearest 1 000 m is the same as rounding off to the nearest kilometre, and so on.

So, 46 mm rounded off to the nearest centimetre is 5 cm. There are 10 mm in 1 cm and 46 mm is closer to 50 mm than to 40 mm.

Similarly, 2 592 m rounded off to the nearest kilometre is 3 km. There are 1 000 m in 1 km and 2 592 m is closer to 3 000 m than to 2 000 m.

In this section learners get plenty of practice in converting units.

This section is fairly long. You could consider splitting up questions for classwork and homework. Questions 5 and 6 could be given as homework.



11.4 Rounding off

In everyday life, we often round off measurements. Distances are often rounded to the nearest kilometre. If we buy material for sewing it is usually to the nearest metre. A carpenter can ask for planks to be cut to the nearest centimetre.

When we work with lengths, it is convenient to work with full units (km, m, cm, mm).

If a length is given in a smaller unit, we often round it off to a bigger unit. If you round off to the nearest 100 cm, it is the same as rounding off to the nearest metre and so on.

28 mm to the nearest centimetre is 3 cm, because 28 mm is $2\frac{8}{10}$ cm. It works like this: when you have a fraction that is less than half, you ignore it. If, however, your fraction is half or more than half, you add one whole. We say 54 cm is more than half a metre and should be rounded up to 1 m.

For tenths:

Here are some more examples:

$7\frac{4}{8}$ is rounded up to 8 and $7\frac{2}{5}$ is rounded down to 7.

We can also round off to other numbers, for example to the nearest 5. The number 5 and its multiples then become your base:

Rounded to the nearest 5:

8 becomes 10; 42 becomes 40; 64 becomes 65; 67 becomes 65.

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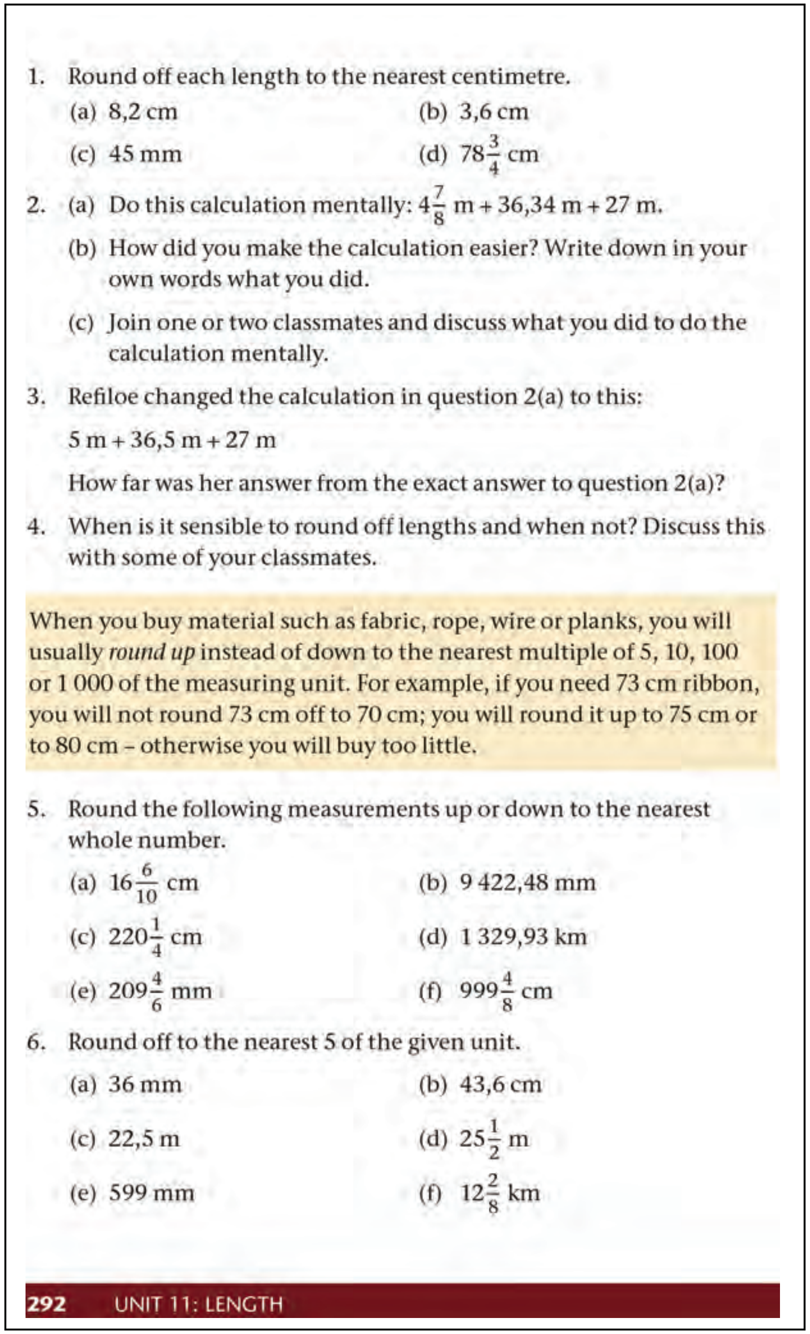
Notes on questions

Questions 1, 3 and 4 help learners to decide when it is appropriate to round down and in which contexts they should ignore the mathematical rule and round up. You might like to read through all the questions and the shaded passage that follows. Explain to learners that when estimating how much money they will need for their shopping, for example, it is advisable to round up the prices of the goods they want to buy. In this way, they won't be short of money; if anything, they will have more money than needed. The same applies to budgets. When the school secretary estimates how much the school needs to buy a photocopier, it is advisable for the secretary to round up, and not down.

Answers

1. (a) 8 cm (b) 4 cm (c) 5 cm (d) 79 cm
2. (a) Mental calculation; answer = 68,215 m
   (b) Learners' answers will differ. Some learners might round off all the numbers to whole numbers. Some learners might round up the numbers to the nearest whole number, then add 0,34 m to the answer, and then subtract $\frac{1}{8}$ from that answer.
   (c) Learners share their answers.
3. 0,285 m or 285 cm more than the calculation
4. Learners' own discussions (refer to the shaded passage on page 292)
5. (a) 17 cm (b) 9 422 mm (c) 220 cm
   (d) 1 330 km (e) 210 mm (f) 1 000 cm
6. (a) 35 mm (b) 45 cm (c) 25 m
   (d) 30 m (e) 600 mm (f) 10 km



1. Round off each length to the nearest centimetre.
   (a) 8,2 cm (b) 3,6 cm
   (c) 45 mm (d) $78\frac{3}{4}$ cm
2. (a) Do this calculation mentally: $4\frac{7}{8}$ m + 36,34 m + 27 m.
   (b) How did you make the calculation easier? Write down in your own words what you did.
   (c) Join one or two classmates and discuss what you did to do the calculation mentally.
3. Refiloe changed the calculation in question 2(a) to this:
   5 m + 36,5 m + 27 m
   How far was her answer from the exact answer to question 2(a)?
4. When is it sensible to round off lengths and when not? Discuss this with some of your classmates.

When you buy material such as fabric, rope, wire or planks, you will usually *round up* instead of down to the nearest multiple of 5, 10, 100 or 1 000 of the measuring unit. For example, if you need 73 cm ribbon, you will not round 73 cm off to 70 cm; you will round it up to 75 cm or to 80 cm – otherwise you will buy too little.

5. Round the following measurements up or down to the nearest whole number.
   (a) $16\frac{6}{10}$ cm (b) 9 422,48 mm
   (c) $220\frac{1}{4}$ cm (d) 1 329,93 km
   (e) $209\frac{4}{6}$ mm (f) $999\frac{4}{8}$ cm
6. Round off to the nearest 5 of the given unit.
   (a) 36 mm (b) 43,6 cm
   (c) 22,5 m (d) $25\frac{1}{2}$ m
   (e) 599 mm (f) $12\frac{2}{8}$ km

Notes on questions

In question 7 learners need to remember the order of operations. Remind learners that they must first work out the brackets, then do any division or multiplication, and finally do any addition and subtraction.

Answers

7. (a) Calculated answer: 642,5 cm; rounded off answer: 6 400 mm
   (b) Calculated answer: 13 800 mm; rounded off answer: 14 000 mm
   (c) Calculated answer: 2 544 m; rounded off answer: 3 km
8. (a) 40 008 km
   (b) At poles: 40 000 km; at the equator: 40 100 km

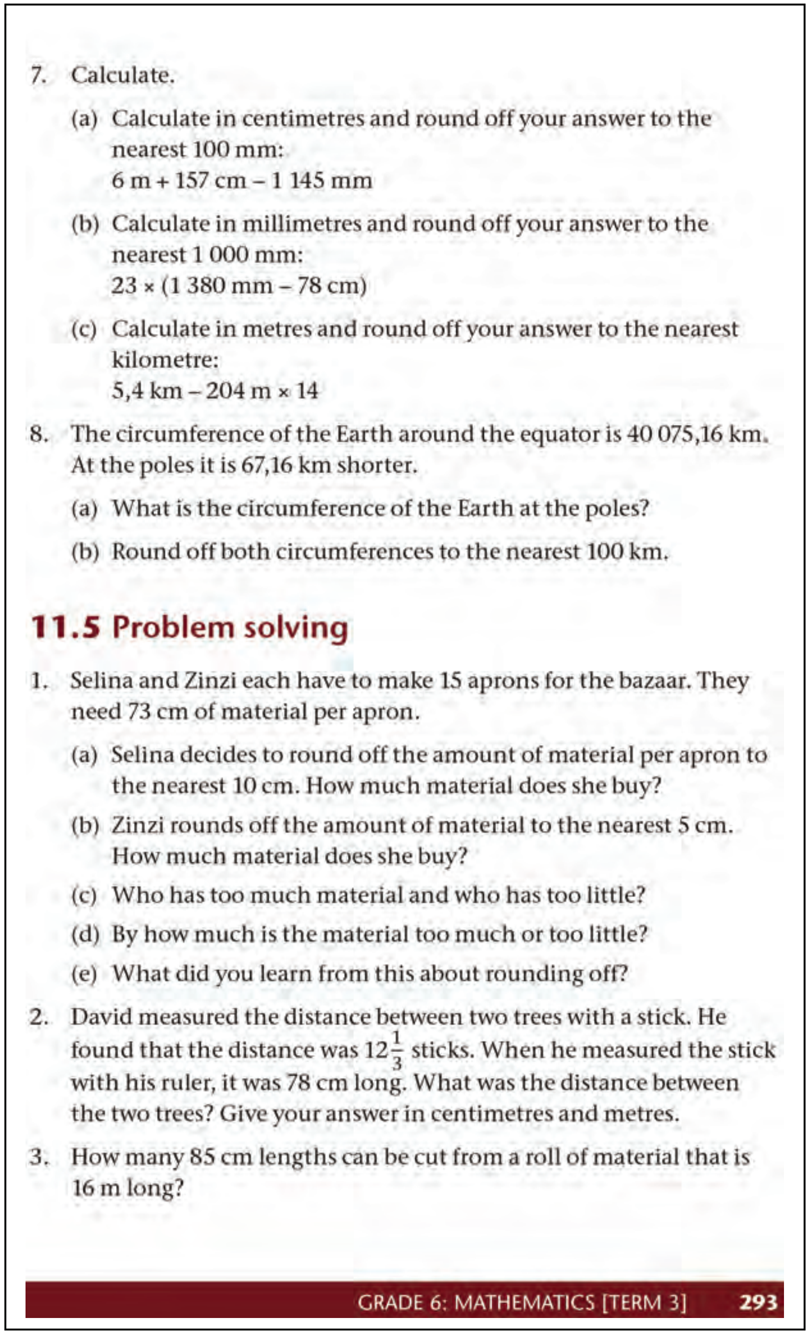
11.5 Problem solving

Notes on questions

Question 1(e) is addressed in the second shaded passage on rounding off on page 292. You may need to suggest that for question 2, learners translate the question into a whole number calculation and a calculation of a fraction of a whole. In other words, learners can multiply the whole number part first (12 × 78), and then take $\frac{1}{3}$ of 78.

Answers

1. (a) 1 050 cm; the amount to the nearest 10 cm per apron = 70 cm.
   So, 70 × 15 = 1 050 cm
   (b) 1 125 cm; the amount to the nearest 5 cm per apron = 75 cm (assuming it is per apron again).
   So, 75 × 15 = 1 125 cm
   (c) Selina has too little; Zinzi has too much. This is true because for 15 aprons they need 73 cm × 15, which is 1 095 cm of material.
   (d) Selina has 45 cm too little; Zinzi has 30 cm too much.
   (e) If rounding off when buying material in lengths, you need to round up. When you round off, the bigger the unit you round off to, the greater the difference between the actual answer and the rounded off answer.
2. $12\frac{1}{3} \times 78$ cm = (12 × 78 cm) + ($\frac{1}{3}$ of 78 cm) = 936 cm + 26 cm = 962 cm
3. 18 lengths (ignore the remainder, i.e. the remnants of the material)



7. Calculate.
   (a) Calculate in centimetres and round off your answer to the nearest 100 mm:
   6 m + 157 cm – 1 145 mm
   (b) Calculate in millimetres and round off your answer to the nearest 1 000 mm:
   23 × (1 380 mm – 78 cm)
   (c) Calculate in metres and round off your answer to the nearest kilometre:
   5,4 km – 204 m × 14
8. The circumference of the Earth around the equator is 40 075,16 km. At the poles it is 67,16 km shorter.
   (a) What is the circumference of the Earth at the poles?
   (b) Round off both circumferences to the nearest 100 km.

11.5 Problem solving

1. Selina and Zinzi each have to make 15 aprons for the bazaar. They need 73 cm of material per apron.
   (a) Selina decides to round off the amount of material per apron to the nearest 10 cm. How much material does she buy?
   (b) Zinzi rounds off the amount of material to the nearest 5 cm. How much material does she buy?
   (c) Who has too much material and who has too little?
   (d) By how much is the material too much or too little?
   (e) What did you learn from this about rounding off?
2. David measured the distance between two trees with a stick. He found that the distance was $12\frac{1}{3}$ sticks. When he measured the stick with his ruler, it was 78 cm long. What was the distance between the two trees? Give your answer in centimetres and metres.
3. How many 85 cm lengths can be cut from a roll of material that is 16 m long?

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Notes on questions

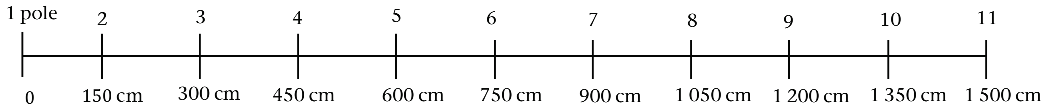
In question 5, make sure learners understand that there are *seven* trips from East London to Cape Town, and back to East London again. The distance given is only for East London to Cape Town. Learners need to double this to get the distance for one trip.

For question 6, it is important that learners draw the fence (see below). If they simply write a number sentence, for example 1 500 cm ÷ 150 cm, they will get the wrong answer. This calculation does not account for the pole needed at the start of the fence. Learners also need to convert 15 m to centimetres.

Question 7 draws on learners' knowledge of perimeter.

Questions 8, 9 and 10 provide practice in rate and ratio.

Answers

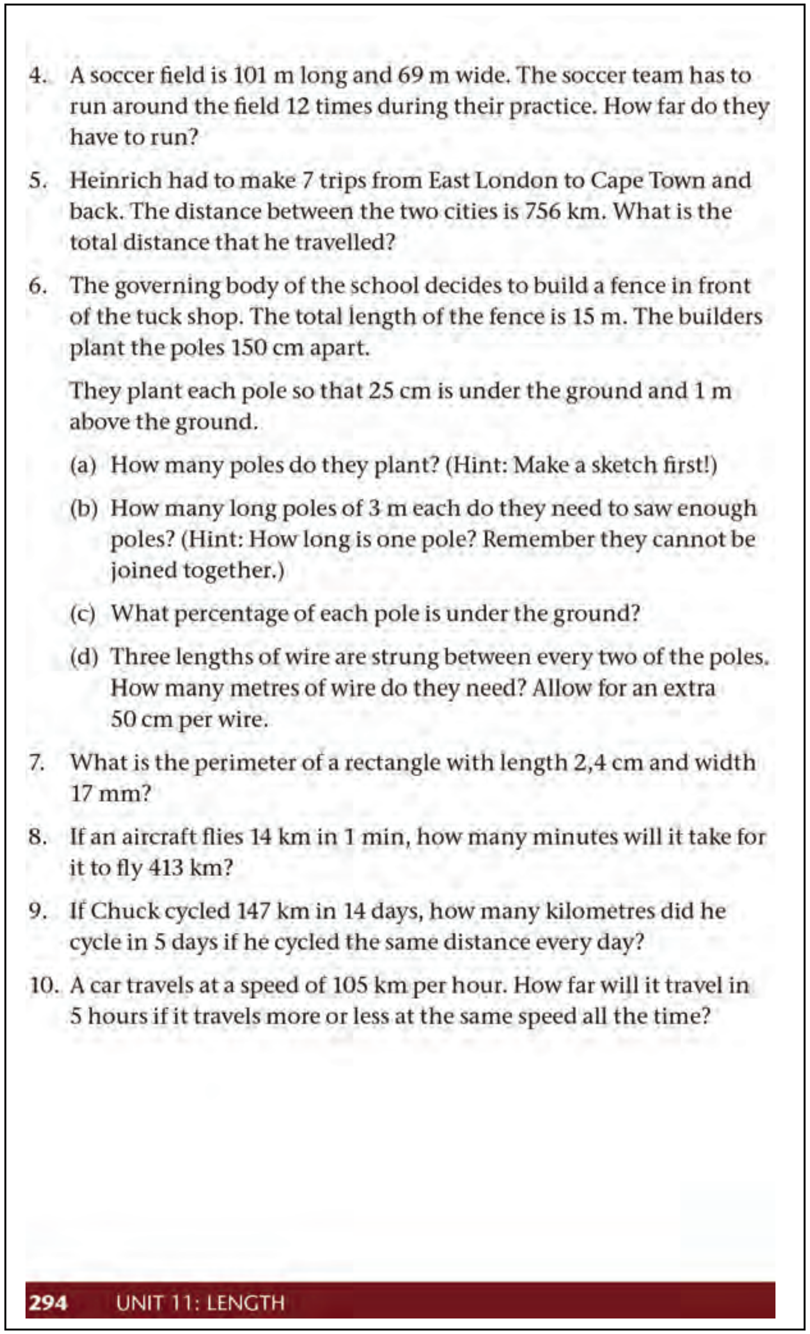
4. Perimeter of field = 340 m; 12 × 340 m = 4 080 m
5. 10 584 km
6. 

   (a) 11 poles

   (b) 6 poles (Each fence pole is 125 cm long, so 6 long poles are needed; only 2 fence poles can be cut from a 3 m pole.)

   (c) $\frac{25}{125} = \frac{5}{25} = \frac{1}{5} = 20\%$

   (d) 10 spaces × 3 wires × 150 cm = 4 500 cm; 4 500 cm + 50 × 3 wires = 4 650 cm
7. 24 mm + 17 mm + 24 mm + 17 mm = 82 mm = 8,2 cm
8. 413 ÷ 14 = 29,5 minutes
9. 147 ÷ 14 = 10,5 km per day; 10,5 × 5 = 52,5 km in 5 days
10. 105 km/h × 5 hours = 525 km



4. A soccer field is 101 m long and 69 m wide. The soccer team has to run around the field 12 times during their practice. How far do they have to run?
5. Heinrich had to make 7 trips from East London to Cape Town and back. The distance between the two cities is 756 km. What is the total distance that he travelled?
6. The governing body of the school decides to build a fence in front of the tuck shop. The total length of the fence is 15 m. The builders plant the poles 150 cm apart.

   They plant each pole so that 25 cm is under the ground and 1 m above the ground.

   (a) How many poles do they plant? (Hint: Make a sketch first!)

   (b) How many long poles of 3 m each do they need to saw enough poles? (Hint: How long is one pole? Remember they cannot be joined together.)

   (c) What percentage of each pole is under the ground?

   (d) Three lengths of wire are strung between every two of the poles. How many metres of wire do they need? Allow for an extra 50 cm per wire.
7. What is the perimeter of a rectangle with length 2,4 cm and width 17 mm?
8. If an aircraft flies 14 km in 1 min, how many minutes will it take for it to fly 413 km?
9. If Chuck cycled 147 km in 14 days, how many kilometres did he cycle in 5 days if he cycled the same distance every day?
10. A car travels at a speed of 105 km per hour. How far will it travel in 5 hours if it travels more or less at the same speed all the time?

294 UNIT 11: LENGTH

Term 4

Grade 6 Term 4 Unit 1 Whole numbers

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
1.1 Represent, order and compare big numbers	Practice of content covered in Terms 1, 2 and 3	297 to 298
1.2 Investigate even, odd and prime numbers	Investigating a variety of given hypotheses	298 to 299

CAPS time allocation	1 hour
CAPS page references	13 to 15 and 278

Mathematical background

In algebraic equations, the multiplication signs are often omitted. *The use of algebraic notation is used to expand teacher knowledge.*

Any even number can be written in the form $2n$, where n is a natural number, and any number that can be written in the form $2n$ is an even number.
Any even number has 2 as a factor.

Any odd number can be written in the form $2n + 1$, where n is a natural number, and any number that can be written in the form $2n + 1$ is an odd number.
If 2 is not a factor of a number, the number is odd.

The sum of any two even numbers, $2m$ and $2n$, is an even number because $2m + 2n = 2(m + n)$. For example: $2 \times 7 + 2 \times 9 = 2 \times (7 + 9)$
The product of any two even numbers is an even number because it has 2 as a factor.

The sum of any two odd numbers, $2m + 1$ and $2n + 1$, is an even number because $2m + 1 + 2n + 1 = 2m + 2n + 2 = 2(m + n + 1)$.
The product of any two odd numbers is an odd number because it does not have 2 as a factor.

The sum of any odd number, $2m + 1$, and any even number, $2n$, is an odd number because $2m + 1 + 2n = 2m + 2n + 1 = 2(m + n) + 1$.
The product of any odd number and any even number is an even number because it has 2 as a factor.

1.1 Represent, order and compare big numbers

Teaching guidelines

Questions 1 to 7 are practice in work done in Terms 1, 2 and 3. This set of questions may be used as an assessment instrument.

Answers

1. (a) 300 005 000 (b) 300 500 000 (c) 300 050 000 (d) 300 000 500
2. (a) seven hundred million four hundred thousand and thirty
   (b) seven hundred million forty thousand three hundred
   (c) seven hundred million four thousand and thirty
   (d) seven hundred million forty-three thousand
   (e) seven hundred and four million and thirty
   (f) seven hundred million four thousand three hundred

3.

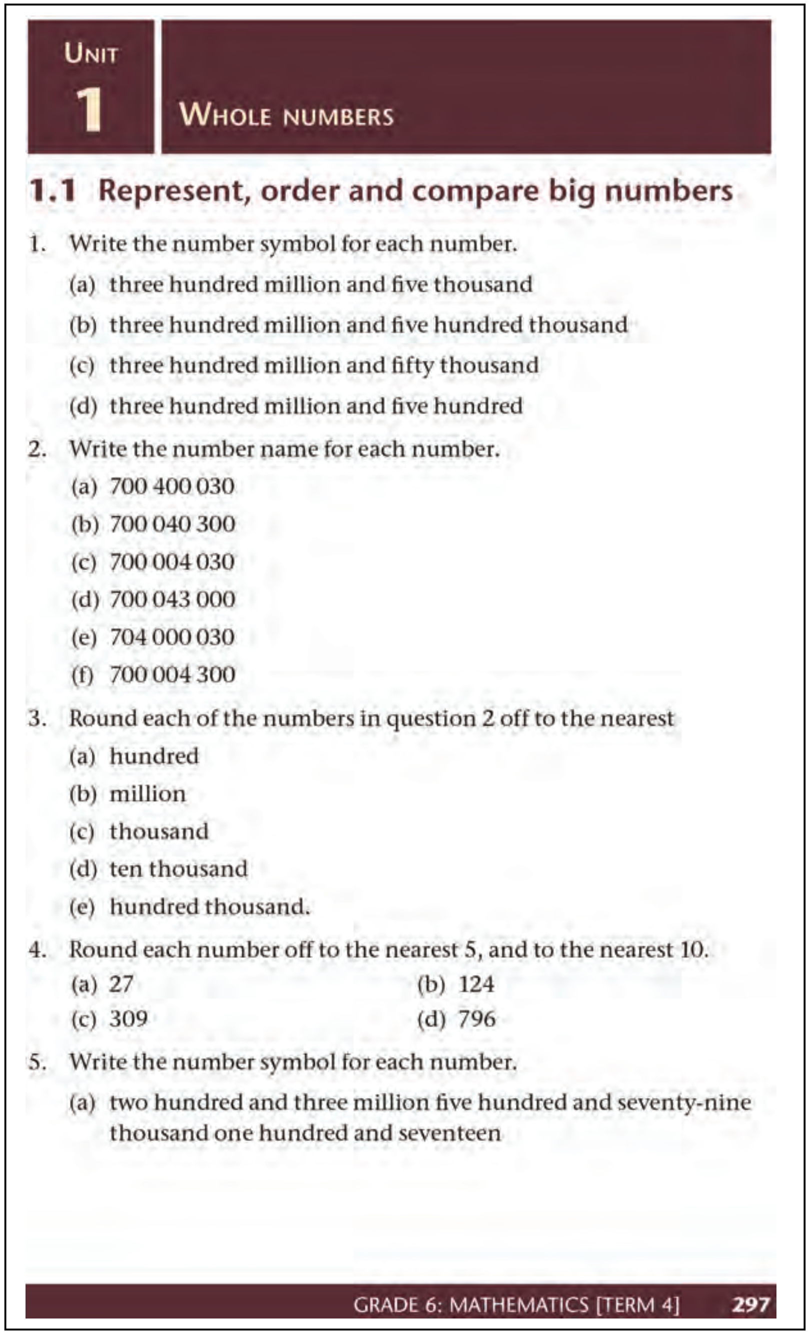
Rounded off to the nearest …	**(a) hundred**	**(b) million**	**(c) thousand**
(a) 700 400 030	700 400 000	700 000 000	700 400 000
(b) 700 040 300	700 040 300	700 000 000	700 040 000
(c) 700 004 030	700 004 000	700 000 000	700 004 000
(d) 700 043 000	700 043 000	700 000 000	700 043 000
(e) 704 000 030	704 000 000	704 000 000	704 000 000
(f) 700 004 300	700 004 300	700 000 000	700 004 000

Rounded off to the nearest …	**(d) ten thousand**	**(e) hundred thousand**
(a) 700 400 030	700 400 000	700 400 000
(b) 700 040 300	700 040 000	700 000 000
(c) 700 004 030	700 000 000	700 000 000
(d) 700 043 000	700 040 000	700 000 000
(e) 704 000 030	704 000 000	700 000 000
(f) 700 004 300	700 000 000	700 000 000

4.

Rounded off to the nearest …	**5**	**10**
(a) 27	25	30
(b) 124	125	120
(c) 309	310	310
(d) 796	795	800

5. (a) 203 579 117



UNIT 1 WHOLE NUMBERS

1.1 Represent, order and compare big numbers

1. Write the number symbol for each number.
   (a) three hundred million and five thousand
   (b) three hundred million and five hundred thousand
   (c) three hundred million and fifty thousand
   (d) three hundred million and five hundred
2. Write the number name for each number.
   (a) 700 400 030
   (b) 700 040 300
   (c) 700 004 030
   (d) 700 043 000
   (e) 704 000 030
   (f) 700 004 300
3. Round each of the numbers in question 2 off to the nearest
   (a) hundred
   (b) million
   (c) thousand
   (d) ten thousand
   (e) hundred thousand.
4. Round each number off to the nearest 5, and to the nearest 10.
   (a) 27 (b) 124
   (c) 309 (d) 796
5. Write the number symbol for each number.
   (a) two hundred and three million five hundred and seventy-nine thousand one hundred and seventeen

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Answers

5. (b) 578 123 467 (c) 98 050 618
(d) 9 876 543 (e) 907 717 014

6. 907 717 014; 578 123 467; 203 579 117; 98 050 618; 9 876 543

7. (a) 3 492 897 < 3 940 289 (b) 6 374 294 = 6 374 294
(c) 102 901 890 < 201 899 013 (d) 1 000 010 = 1 000 010

1.2 Investigate even, odd and prime numbers

Teaching guidelines

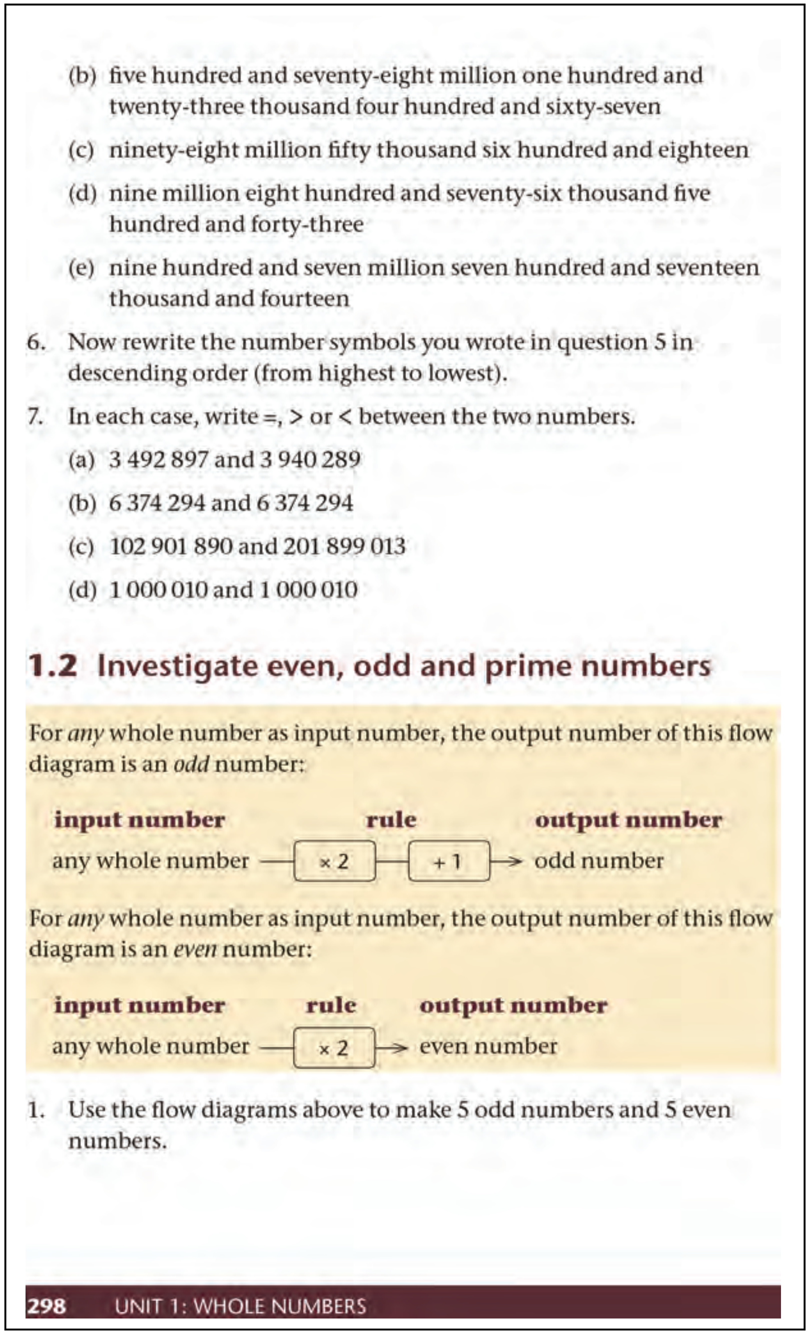
You may start this lesson by asking learners what even numbers are, and what odd numbers are. Many learners may say that even numbers end in 2, 4, 6, 8 or 0, while odd numbers end in 1, 3, 5, 7 or 9. While this is perfectly correct, these definitions are not helpful in forming and testing hypotheses about even and odd numbers, which is the purpose of questions 2, 3 and 7. The definitions given in terms of flow diagrams in the shaded passage are much more useful for this purpose.

Ask learners to try to find whole numbers for which the statements in the shaded passage are not true. Once learners agree with these statements, allow them to engage with the questions.

Answers

1. Learners' numbers will differ, for example:

Chosen input number	× 2 + 1 = odd	× 2 = even
34	69	68
7	15	14
52	105	104
123	247	246
98	197	196



(b) five hundred and seventy-eight million one hundred and twenty-three thousand four hundred and sixty-seven

(c) ninety-eight million fifty thousand six hundred and eighteen

(d) nine million eight hundred and seventy-six thousand five hundred and forty-three

(e) nine hundred and seven million seven hundred and seventeen thousand and fourteen

6. Now rewrite the number symbols you wrote in question 5 in descending order (from highest to lowest).

7. In each case, write =, > or < between the two numbers.
   (a) 3 492 897 and 3 940 289
   (b) 6 374 294 and 6 374 294
   (c) 102 901 890 and 201 899 013
   (d) 1 000 010 and 1 000 010

1.2 Investigate even, odd and prime numbers

For *any* whole number as input number, the output number of this flow diagram is an *odd* number:

input number	**rule**	**output number**
any whole number	→ × 2 → + 1 →	odd number

For *any* whole number as input number, the output number of this flow diagram is an *even* number:

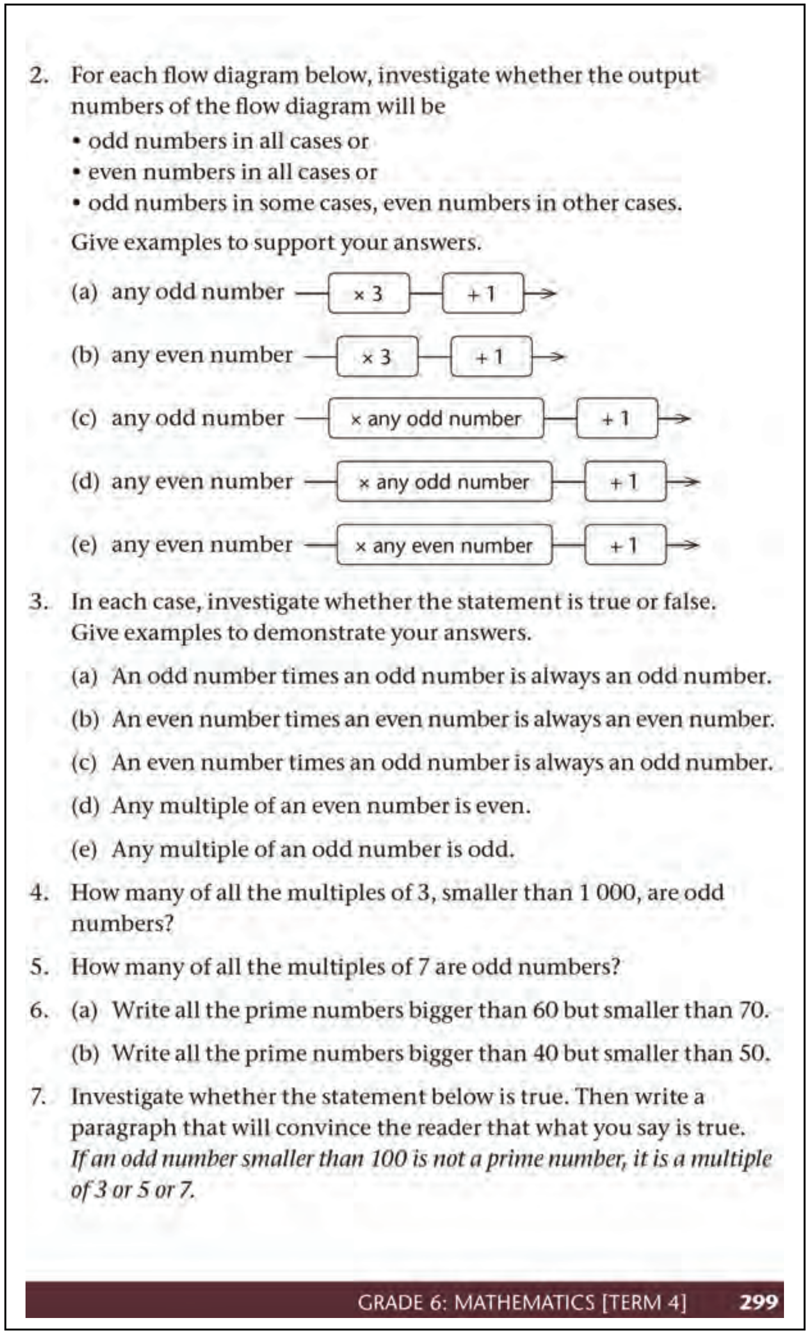
input number	**rule**	**output number**
any whole number	→ × 2 →	even number

1. Use the flow diagrams above to make 5 odd numbers and 5 even numbers.

Answers

2. Learners do the investigations. Numbers will differ but the conclusions will be the same.
   - (a) Will always be even,
     e.g. 3 × 3 + 1 = 10; 41 × 3 + 1 = 124
   - (b) Will always be odd,
     e.g. 12 × 3 + 1 = 37; 2 × 3 + 1 = 7; 82 × 3 + 1 = 247
   - (c) Will always be even,
     e.g. 5 × 7 + 1 = 36; 3 × 3 + 1 = 10; 9 × 5 + 1 = 46
   - (d) Will always be odd,
     e.g. 2 × 5 + 1 = 11; 100 × 3 + 1 = 301; 14 × 41 + 1 = 575
   - (e) Will always be odd,
     e.g. 2 × 4 + 1 = 9; 8 × 8 + 1 = 65; 10 × 2 + 1 = 21
3. Learners investigate and give enough examples to demonstrate their answers.
   - (a) True; e.g. 7 × 3 = 21; 9 × 5 = 45; 13 × 9 = 117
   - (b) True; e.g. 2 × 4 = 8; 4 × 32 = 128; 8 × 10 = 80
   - (c) False; it is always even; e.g. 2 × 3 = 6; 8 × 3 = 24; 4 × 5 = 20
   - (d) True; e.g. multiples of 8: 16; 24; 32; 40; …
     and multiples of 10: 10; 20; 30; 40; …
   - (e) False; only every second multiple is an odd number,
     e.g. multiples of 3: 6; 9; 12; 15; 18; …
     or multiples of 5: 10; 15; 20; 25; 30; …
4. Learner may use the 1 000 chart to solve this question.

   The answer is 167.
5. Half of them
6. (a) 61 and 67
   - (b) 43 and 47
7. The statement is true. A good argument will be that the smallest multiple of 11 that is not a multiple of 7 (77) is 11 × 11 = 121, which is bigger than 100.



2. For each flow diagram below, investigate whether the output numbers of the flow diagram will be
   - odd numbers in all cases or
   - even numbers in all cases or
   - odd numbers in some cases, even numbers in other cases.

   Give examples to support your answers.
   - (a) any odd number → × 3 → + 1 →
   - (b) any even number → × 3 → + 1 →
   - (c) any odd number → × any odd number → + 1 →
   - (d) any even number → × any odd number → + 1 →
   - (e) any even number → × any even number → + 1 →
3. In each case, investigate whether the statement is true or false. Give examples to demonstrate your answers.
   - (a) An odd number times an odd number is always an odd number.
   - (b) An even number times an even number is always an even number.
   - (c) An even number times an odd number is always an odd number.
   - (d) Any multiple of an even number is even.
   - (e) Any multiple of an odd number is odd.
4. How many of all the multiples of 3, smaller than 1 000, are odd numbers?
5. How many of all the multiples of 7 are odd numbers?
6. (a) Write all the prime numbers bigger than 60 but smaller than 70.
   - (b) Write all the prime numbers bigger than 40 but smaller than 50.
7. Investigate whether the statement below is true. Then write a paragraph that will convince the reader that what you say is true.
   If an odd number smaller than 100 is not a prime number, it is a multiple of 3 or 5 or 7.

Grade 6 Term 4 Unit 2 Whole numbers: Multiplication

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
2.1 Revising multiplication	Revising basic multiplication facts and the expanded column notation	300 to 301
2.2 A shorter way of setting out multiplication	Documenting multiplication in one column only	301 to 302
2.3 An even shorter way to set out your work	Omitting some part answers to reduce the length of the column	302 to 303
2.4 Apply and use your knowledge	Word problems	303 to 304
2.5 Use your calculator. But check the answer!	Responsible use of the calculator to multiply	304 to 305

CAPS time allocation	5 hours
CAPS page references	13 to 15 and 278 to 279

Mathematical background

The transition from documenting multiplication with number sentences to documenting it in several columns (the expanded column notation) was addressed in Term 2 Unit 2 (pages 125 to 141 of the Learner Book). The present unit is the transition to the use of only one column, and omitting some of the part answers.

- Documenting multiplication with a succession of **number sentences** that state equivalences between different calculation plans:

Calculation plan A **Calculation plan B**

$$\begin{aligned} 284 \times 378 = (200 + 80 + 4) \times (300 + 70 + 8) &= 200 \times (300 + 70 + 8) + 80 \times (300 + 70 + 8) + 4 \times (300 + 70 + 8) \\ &= 200 \times 300 + 200 \times 70 + 200 \times 8 + 80 \times 300 + 80 \times 70 + 80 \times 8 + 4 \times 300 + 4 \times 70 + 4 \times 8 \ \textbf{(Calculation plan C)} \\ &= 60\,000 + 14\,000 + 1\,600 + 24\,000 + 5\,600 + 640 + 1\,200 + 280 + 32 = 107\,352 \end{aligned}$$

- Documenting multiplication in **columns**:

Several columns

$$\begin{array}{r} 378 \\ \times\ \ \ 4 \\ \hline 32 \\ 280 \\ +\ 1\,200 \\ \hline 1\,512 \end{array} \qquad \begin{array}{r} 378 \\ \times\ \ 80 \\ \hline 640 \\ 5\,600 \\ +\ 24\,000 \\ \hline 30\,240 \end{array} \qquad \begin{array}{r} 378 \\ \times\ 200 \\ \hline 1\,600 \\ 14\,000 \\ +\ 60\,000 \\ \hline 75\,600 \end{array} \qquad \begin{array}{r} 1\,512 \\ 30\,240 \\ +\ 75\,600 \\ \hline 107\,352 \end{array}$$

Single column

$$\begin{array}{r} 378 \\ \times\ 284 \\ \hline 32 \\ 280 \\ 1\,200 \\ 640 \\ 5\,600 \\ 24\,000 \\ 1\,600 \\ 14\,000 \\ +\ 60\,000 \\ \hline 107\,352 \end{array}$$

Reduced single column

$$\begin{array}{r} 378 \\ \times\ 284 \\ \hline 1\,512 \\ 30\,240 \\ +\ 75\,600 \\ \hline 107\,352 \end{array}$$

Documenting in several columns is revised in Section 2.1, and single column notation is introduced and practised in Section 2.2.

The reduced single column notation is introduced in Section 2.3, but it is not compulsory.

All three forms of column exposition can be extended to include the partial products as stated in Calculation plan C (see above), as learners are required to do on page 301.

2.1 Revising multiplication

Teaching guidelines

Question 1 may be utilised as a 30 minute test to assess the level of learners' knowledge and skills with respect to basic multiplication facts.

Answers

1.

×	**30**	**200**	**60**	**80**	**400**	**40**	**600**	**20**
50	1 500	10 000	3 000	4 000	20 000	2 000	30 000	1 000
900	27 000	180 000	54 000	72 000	360 000	36 000	540 000	18 000
700	21 000	140 000	42 000	56 000	280 000	28 000	420 000	14 000
70	2 100	14 000	4 200	5 600	28 000	2 800	42 000	1 400
500	15 000	100 000	30 000	40 000	200 000	20 000	300 000	10 000
90	2 700	18 000	5 400	7 200	36 000	3 600	54 000	1 800
800	24 000	160 000	48 000	64 000	320 000	32 000	480 000	16 000
40	1 200	8 000	2 400	3 200	16 000	1 600	24 000	800
300	9 000	60 000	18 000	24 000	120 000	12 000	180 000	6 000

2.

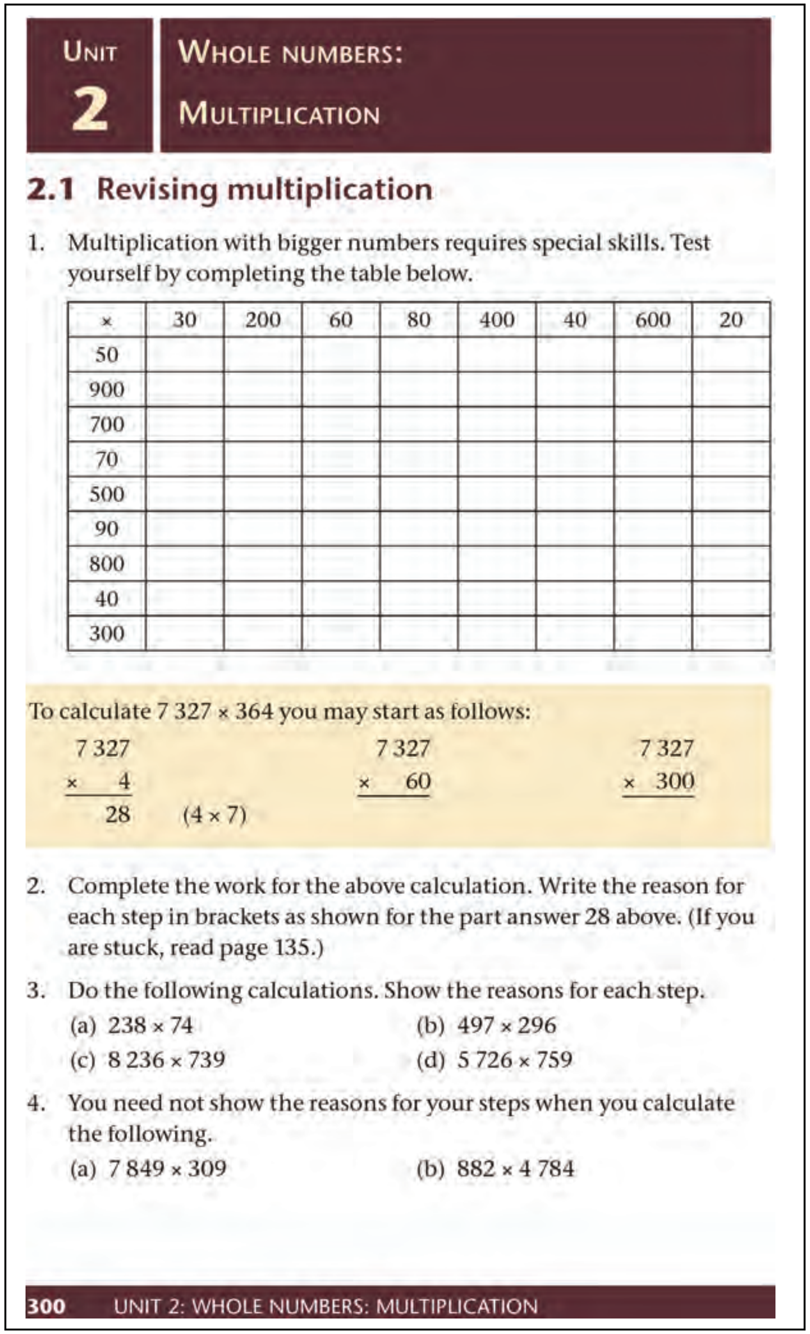
7 327		7 327		7 327	
× 300		× 60		× 4	
2 100	(300 × 7)	420	(60 × 7)	28	(4 × 7)
6 000	(300 × 20)	1 200	(60 × 20)	80	(4 × 20)
90 000	(300 × 300)	18 000	(60 × 300)	1 200	(4 × 300)
2 100 000	(300 × 7 000)	420 000	(60 × 7 000)	28 000	(4 × 7 000)
2 198 100		439 620		29 308	

	2 198 100
	439 620
+	29 308
	2 667 028

3. (a) 17 612 (b) 147 112 (c) 6 086 404 (d) 4 346 034

Check that learners show the reason for each step correctly.

4. (a) 2 425 341 (b) 4 219 488



UNIT 2 WHOLE NUMBERS: MULTIPLICATION

2.1 Revising multiplication

1. Multiplication with bigger numbers requires special skills. Test yourself by completing the table below.

×	30	200	60	80	400	40	600	20
50								
900								
700								
70								
500								
90								
800								
40								
300								

To calculate 7 327 × 364 you may start as follows:

7 327		7 327	7 327
× 4		× 60	× 300
28	(4 × 7)		

2. Complete the work for the above calculation. Write the reason for each step in brackets as shown for the part answer 28 above. (If you are stuck, read page 135.)
3. Do the following calculations. Show the reasons for each step.
   (a) 238 × 74 (b) 497 × 296
   (c) 8 236 × 739 (d) 5 726 × 759
4. You need not show the reasons for your steps when you calculate the following.
   (a) 7 849 × 309 (b) 882 × 4 784

Notes on questions

Question 5 is diagnostic: it tests whether learners are aware of the distributive property and can articulate it in some way. You may ask them to answer the question on loose sheets of paper and to hand it in.

Learners at this level find it very difficult to express properties of operations explicitly. In question 5 it is already quite good if learners can only state that the calculation plans 4 × (2 000 + 300 + 60 + 7) and 4 × 2 000 + 4 × 300 + 4 × 60 + 4 × 7 will have the same answer.

2.2 A shorter way of setting out multiplication

Teaching guidelines

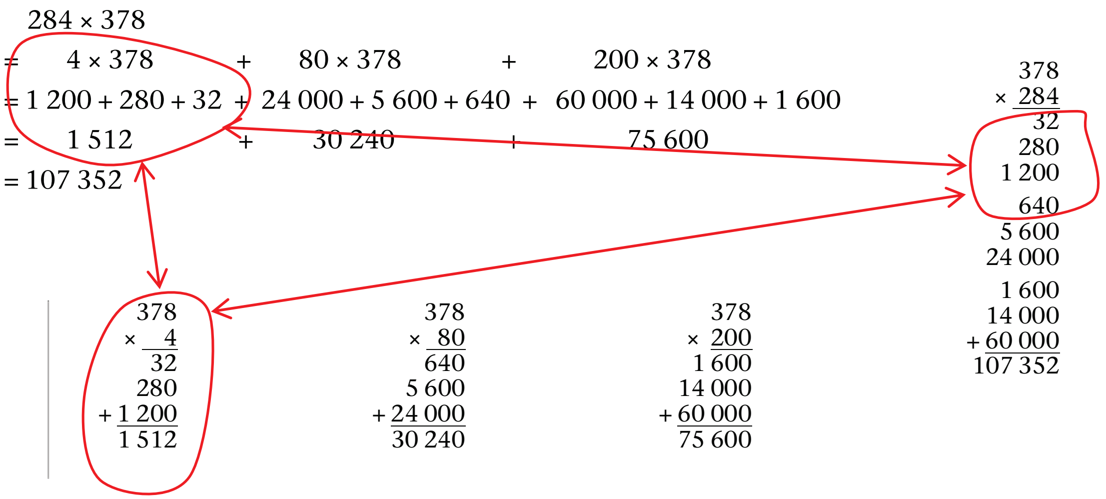
Allow learners freedom in how they document their work for question 1. It will be quite helpful for you to move around while they work in order to get an idea of how learners do multiplication at this stage and how they record their work.

Answers

1. 24 854

Teaching guidelines

Use a lay-out such as the one below or in the shaded passage (use the same or a different example) to introduce the single column format and explain how it relates to setting out multiplication with number sentences and in several columns.



284 × 378
= 4 × 378 + 80 × 378 + 200 × 378
= 1 200 + 280 + 32 + 24 000 + 5 600 + 640 + 60 000 + 14 000 + 1 600
= 1 512 + 30 240 + 75 600
= 107 352

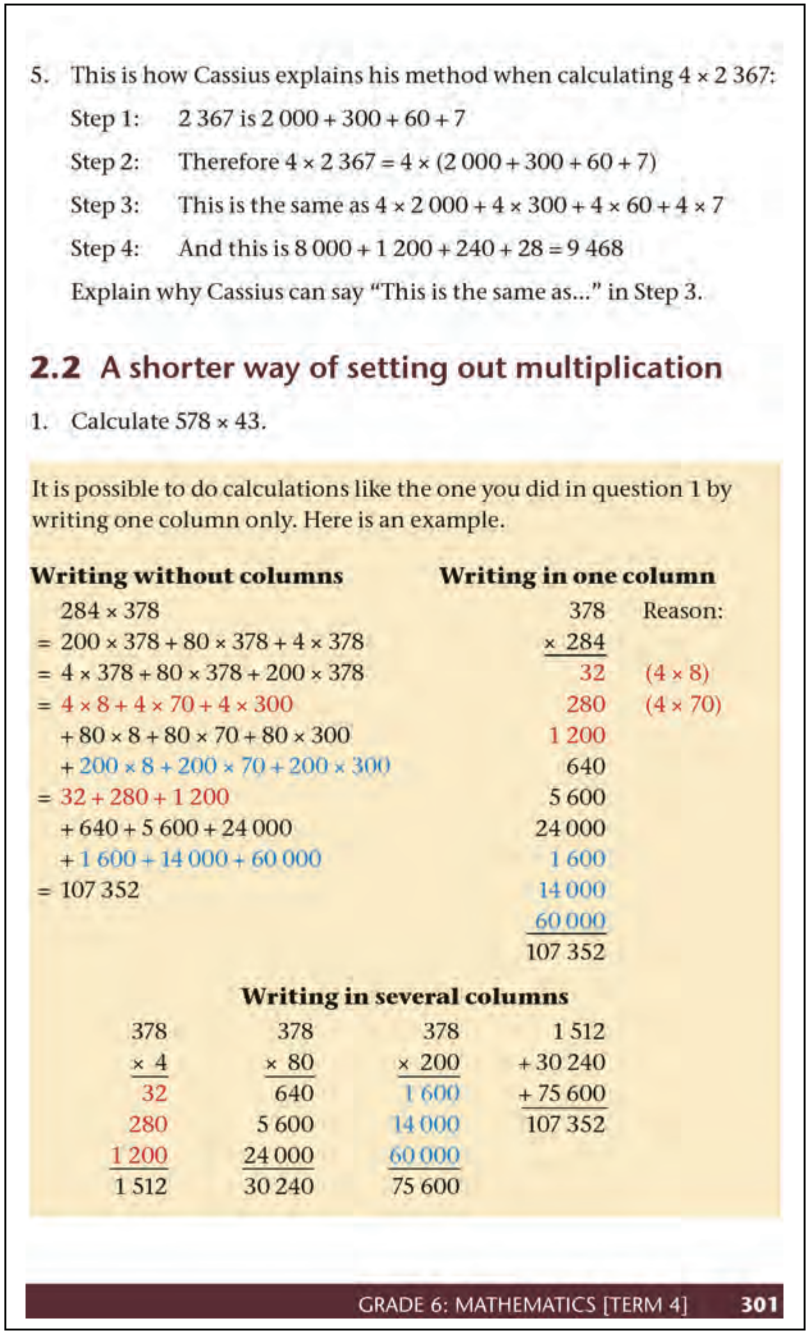
378
× 284
32
280
1 200
640
5 600
24 000
1 600
14 000
+ 60 000
107 352

378
× 4
32
280
+ 1 200
1 512

378
× 80
640
5 600
+ 24 000
30 240

378
× 200
1 600
14 000
+ 60 000
75 600

The three enclosed parts show how one part of the calculation, 4 × 378, is represented in the three different formats. The connections between the representations for 80 × 378 and 200 × 378 can be shown in the same way.



5. This is how Cassius explains his method when calculating 4 × 2 367:

Step 1: 2 367 is 2 000 + 300 + 60 + 7
Step 2: Therefore 4 × 2 367 = 4 × (2 000 + 300 + 60 + 7)
Step 3: This is the same as 4 × 2 000 + 4 × 300 + 4 × 60 + 4 × 7
Step 4: And this is 8 000 + 1 200 + 240 + 28 = 9 468

Explain why Cassius can say "This is the same as..." in Step 3.

2.2 A shorter way of setting out multiplication

1. Calculate 578 × 43.

It is possible to do calculations like the one you did in question 1 by writing one column only. Here is an example.

Writing without columns

284 × 378
= 200 × 378 + 80 × 378 + 4 × 378
= 4 × 378 + 80 × 378 + 200 × 378
= 4 × 8 + 4 × 70 + 4 × 300
+ 80 × 8 + 80 × 70 + 80 × 300
+ 200 × 8 + 200 × 70 + 200 × 300
= 32 + 280 + 1 200
+ 640 + 5 600 + 24 000
+ 1 600 + 14 000 + 60 000
= 107 352

Writing in one column

	Reason:
378	
× 284	
32	(4 × 8)
280	(4 × 70)
1 200	
640	
5 600	
24 000	
1 600	
14 000	
60 000	
107 352	

Writing in several columns

378
× 4
32
280
1 200
1 512

378
× 80
640
5 600
24 000
30 240

378
× 200
1 600
14 000
60 000
75 600

1 512
+ 30 240
+ 75 600
107 352

GRADE 6: MATHEMATICS [TERM 4] 301

Answers

2.

378	
× 284	
32	4 × 8
280	4 × 70
1 200	4 × 300
640	80 × 8
5 600	80 × 70
24 000	80 × 300
1 600	200 × 8
14 000	200 × 70
+ 60 000	200 × 300
107 352	

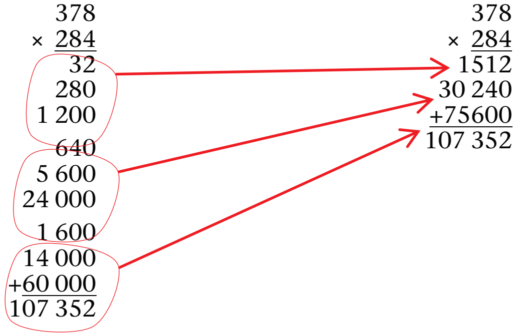
3.

(a)	(b)	(c)	(d)
238	564	5 639	7 694
× 69	× 382	× 94	× 268
72	8	36	32
270	120	120	720
1 800	1 000	2 400	4 800
480	320	20 000	56 000
1 800	4 800	810	240
+12 000	40 000	2 700	5 400
16 422	1 200	54 000	36 000
	18 000	+450 000	420 000
	+150 000	530 066	800
	215 448		18 000
			120 000
			+1 400 000
			2 061 992

2.3 An even shorter way to set out your work

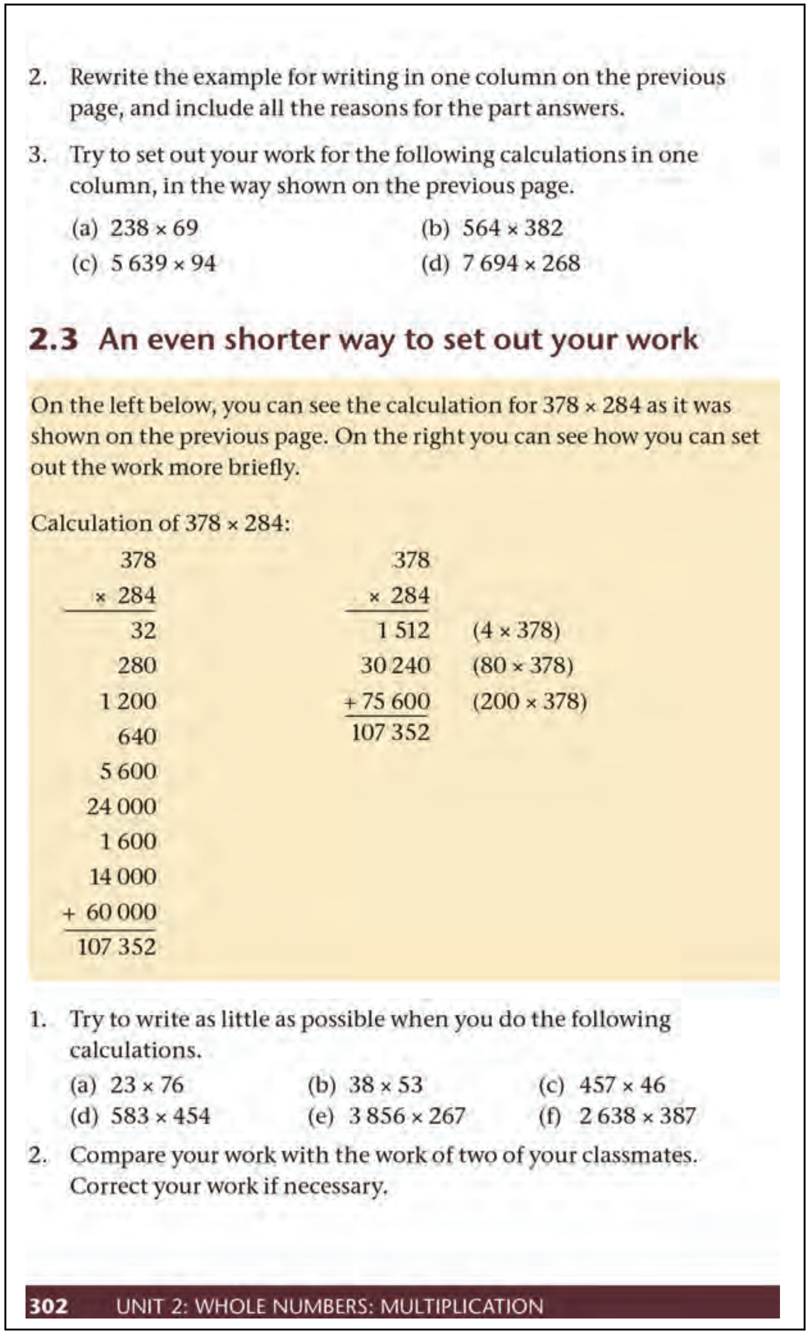
Teaching guidelines

To reduce the number of written part-answers in the column format requires that some steps be done mentally, as indicated below. For example, to produce 1 512 learners will have to write only the 2 of 32 (4 × 8) and remember the 30, then add 30 to the answer 280 of 4 × 70 to get 310, then write the “1” to indicate 10 only and remember the 300, and so on.



Answers

1. (a) 1 748 (b) 2 014 (c) 21 022
   (d) 264 682 (e) 1 029 552 (f) 1 020 906
2. Learners compare their work and make corrections, if necessary.



2. Rewrite the example for writing in one column on the previous page, and include all the reasons for the part answers.
3. Try to set out your work for the following calculations in one column, in the way shown on the previous page.
   (a) 238 × 69 (b) 564 × 382
   (c) 5 639 × 94 (d) 7 694 × 268

2.3 An even shorter way to set out your work

On the left below, you can see the calculation for 378 × 284 as it was shown on the previous page. On the right you can see how you can set out the work more briefly.

Calculation of 378 × 284:

378	378	
× 284	× 284	
32	1 512	(4 × 378)
280	30 240	(80 × 378)
1 200	+ 75 600	(200 × 378)
640	107 352	
5 600		
24 000		
1 600		
14 000		
+ 60 000		
107 352		

1. Try to write as little as possible when you do the following calculations.
   (a) 23 × 76 (b) 38 × 53 (c) 457 × 46
   (d) 583 × 454 (e) 3 856 × 267 (f) 2 638 × 387
2. Compare your work with the work of two of your classmates. Correct your work if necessary.

Answers

3. Ntabi left out these six parts of the nine parts: 300 × 50 = 15 000 300 × 7 = 2 100
40 × 400 = 16 000 40 × 7 = 280 6 × 400 = 2 400 6 × 50 = 300
Corrections:
346 × 457 = 120 000 + 15 000 + 2 100 + 16 000 + 2 000 + 280 + 2 400 + 300 + 42
= 158 122

2.4 Apply and use your knowledge

Notes on questions

Question 2 may appear challenging, as if it requires 36 multi-digit multiplications to be done. In fact, it only requires repeated addition of 274 to complete the first row, and then multiplications by 10 ("adding zeros") to complete the other rows.

Teaching guidelines

Ask learners to start by reading question 1, and to make a plan that will enable them to complete the table quickly and easily.

Inform learners that they can use the results of question 2 to reduce the work required to do question 3.

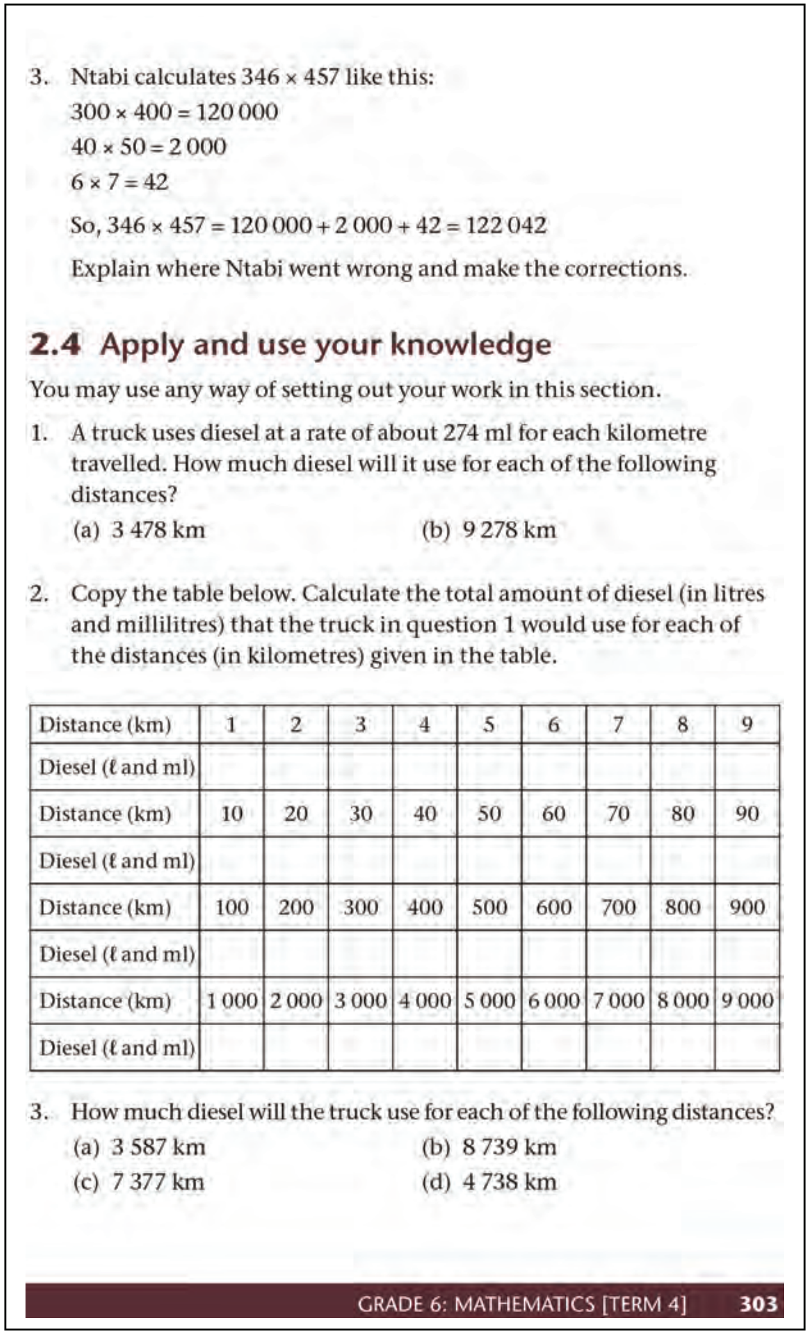
Answers

1. (a) 952 972 ml (b) 2 542 172 ml

2.

Distance (km)	1	2	3	4	5	6	7	8	9
Diesel (ℓ and ml)	274 ml	548 ml	822 ml	1 ℓ 96 ml	1 ℓ 370 ml	1 ℓ 644 ml	1 ℓ 918 ml	2 ℓ 192 ml	2 ℓ 466 ml
Distance (km)	10	20	30	40	50	60	70	80	90
Diesel (ℓ and ml)	2 ℓ 740 ml	5 ℓ 480 ml	8 ℓ 220 ml	10 ℓ 960 ml	13 ℓ 700 ml	16 ℓ 440 ml	19 ℓ 180 ml	21 ℓ 920 ml	24 ℓ 660 ml
Distance (km)	100	200	300	400	500	600	700	800	900
Diesel (ℓ and ml)	27 ℓ 400 ml	54 ℓ 800 ml	82 ℓ 200 ml	109 ℓ 600 ml	137 ℓ	164 ℓ 400 ml	191 ℓ 800 ml	219 ℓ 200 ml	246 ℓ 600 ml
Distance (km)	1 000	2 000	3 000	4 000	5 000	6 000	7 000	8 000	9 000
Diesel (ℓ and ml)	274 ℓ	548 ℓ	822 ℓ	1 096 ℓ	1 370 ℓ	1 644 ℓ	1 918 ℓ	2 192 ℓ	2 466 ℓ

3. (a) 3 587 km = 3 000 km + 500 km + 80 km + 7 km
From table: 822 ℓ + 137 ℓ + 21 ℓ + 920 ml + 1 ℓ + 918 ml = 982 ℓ and 838 ml
(b) 2 394 ℓ and 486 ml (c) 2 021 ℓ and 298 ml (d) 1 298 ℓ and 212 ml



3. Ntabi calculates 346 × 457 like this:
300 × 400 = 120 000
40 × 50 = 2 000
6 × 7 = 42
So, 346 × 457 = 120 000 + 2 000 + 42 = 122 042
Explain where Ntabi went wrong and make the corrections.

2.4 Apply and use your knowledge

You may use any way of setting out your work in this section.

1. A truck uses diesel at a rate of about 274 ml for each kilometre travelled. How much diesel will it use for each of the following distances?
(a) 3 478 km (b) 9 278 km

2. Copy the table below. Calculate the total amount of diesel (in litres and millilitres) that the truck in question 1 would use for each of the distances (in kilometres) given in the table.

Distance (km)	1	2	3	4	5	6	7	8	9
Diesel (ℓ and ml)									
Distance (km)	10	20	30	40	50	60	70	80	90
Diesel (ℓ and ml)									
Distance (km)	100	200	300	400	500	600	700	800	900
Diesel (ℓ and ml)									
Distance (km)	1 000	2 000	3 000	4 000	5 000	6 000	7 000	8 000	9 000
Diesel (ℓ and ml)									

3. How much diesel will the truck use for each of the following distances?
(a) 3 587 km (b) 8 739 km
(c) 7 377 km (d) 4 738 km

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Notes on questions

Learners may make mistakes with question 4 by doing calculations involving the number 125. This number is irrelevant to the question.

In question 6 some learners may fail to notice that they have to multiply by 5.

Question 9 requires more than one calculation and can be done in two different ways:

697 × 2 394 – 697 × 1 090 = 1 668 618 – 759 730 = 908 888 *or*

697 × (2 394 – 1 090) = 697 × 1 304 = 908 888

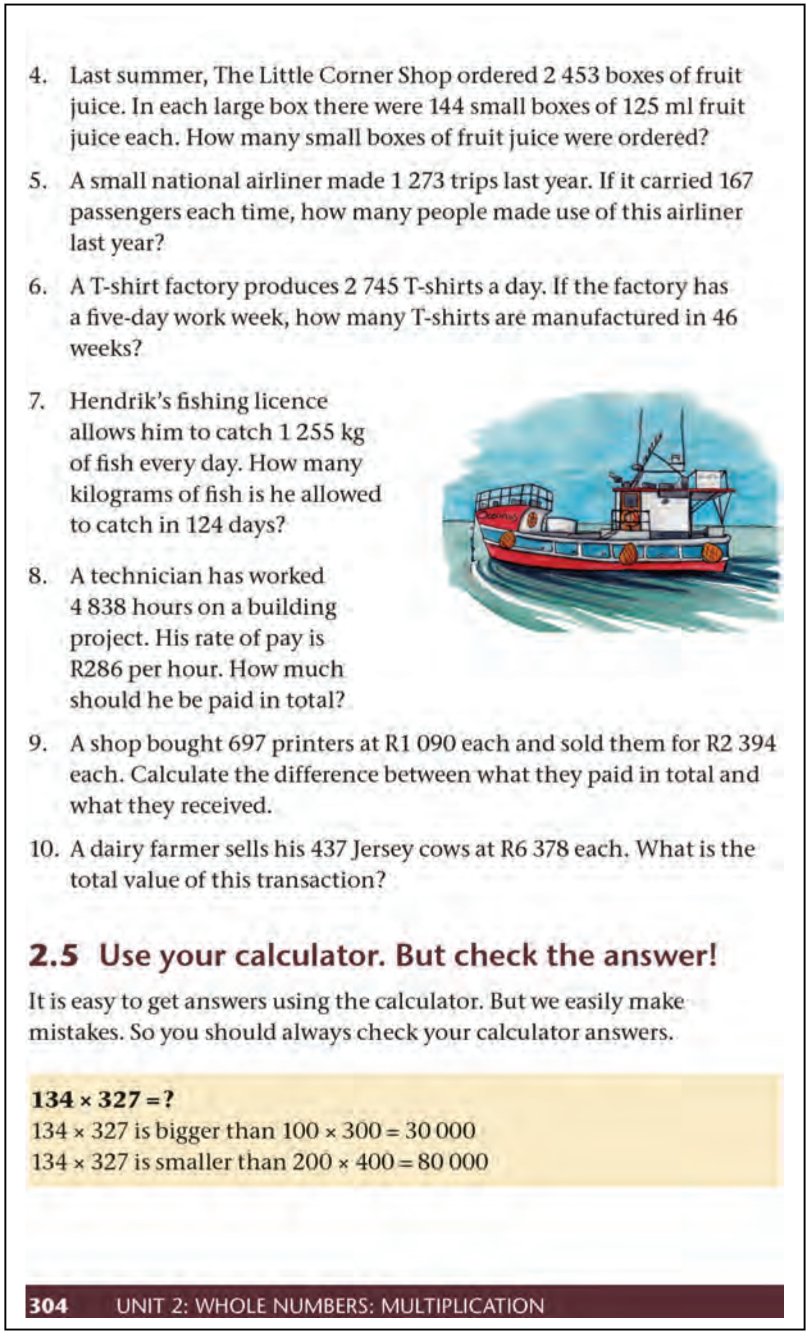
Answers

4. 2 453 × 144 = 353 232 small boxes
5. 1 273 × 167 = 212 591 people
6. 2 745 × 5 × 46 = 631 350 T-shirts
7. 1 255 × 124 = 155 620 kg of fish
8. 4 838 × R286 = R1 383 668
9. R908 888 (see "Notes on questions" above)
10. 437 × R6 378 = R2 787 186

2.5 Use your calculator. But check the answer!

Notes on questions

Probably as a result of their absolute faith in calculators, learners often use them without any thought of whether the answer given by the calculator is correct and/or makes sense. It would be helpful for learners, even before picking up and using the calculator, to develop some sense of the possible answer by making an estimate as shown in the shaded passage.



4. Last summer, The Little Corner Shop ordered 2 453 boxes of fruit juice. In each large box there were 144 small boxes of 125 ml fruit juice each. How many small boxes of fruit juice were ordered?
5. A small national airliner made 1 273 trips last year. If it carried 167 passengers each time, how many people made use of this airliner last year?
6. A T-shirt factory produces 2 745 T-shirts a day. If the factory has a five-day work week, how many T-shirts are manufactured in 46 weeks?
7. Hendrik's fishing licence allows him to catch 1 255 kg of fish every day. How many kilograms of fish is he allowed to catch in 124 days?
8. A technician has worked 4 838 hours on a building project. His rate of pay is R286 per hour. How much should he be paid in total?
9. A shop bought 697 printers at R1 090 each and sold them for R2 394 each. Calculate the difference between what they paid in total and what they received.
10. A dairy farmer sells his 437 Jersey cows at R6 378 each. What is the total value of this transaction?

2.5 Use your calculator. But check the answer!

It is easy to get answers using the calculator. But we easily make mistakes. So you should always check your calculator answers.

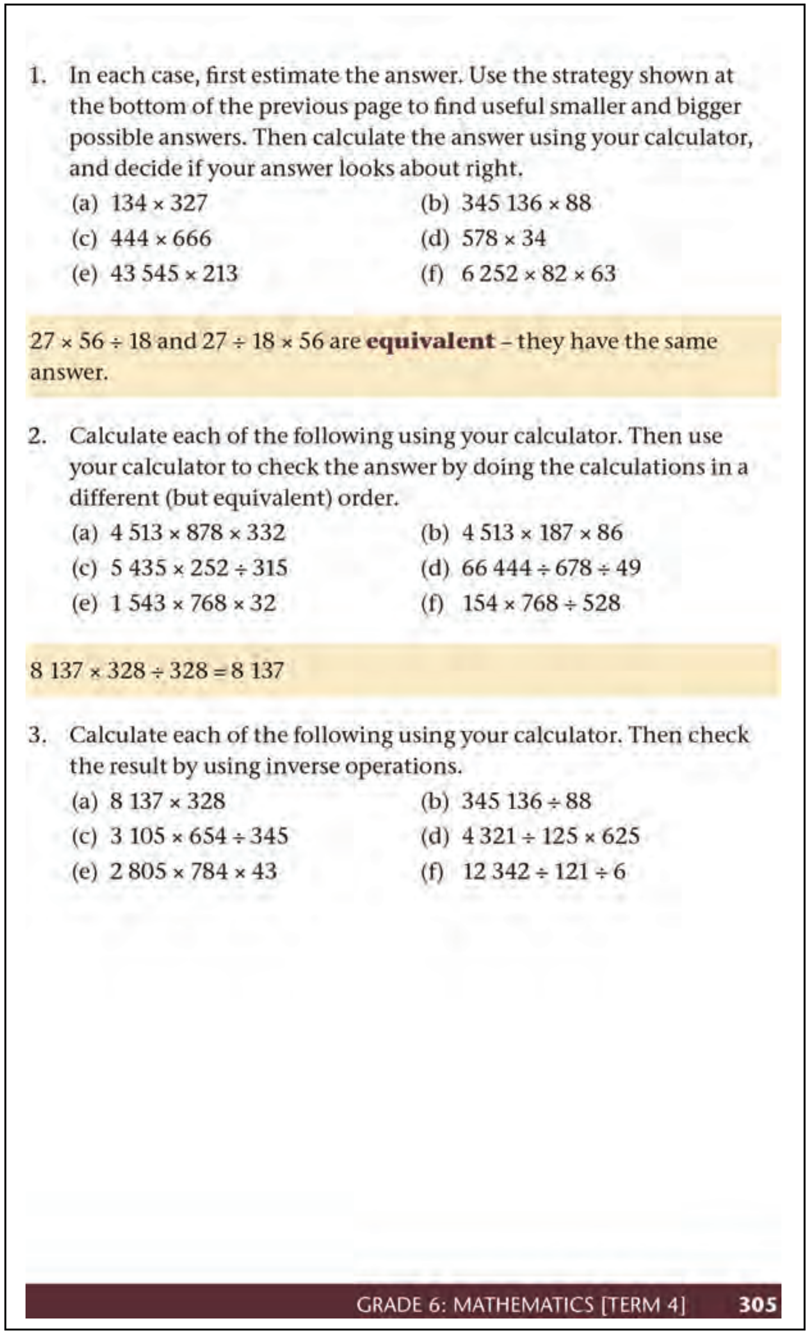
134 × 327 = ?
134 × 327 is bigger than 100 × 300 = 30 000
134 × 327 is smaller than 200 × 400 = 80 000

Answers

1. (a) 43 818 (b) 30 371 968
(c) 295 704 (d) 19 652
(e) 9 275 058 (f) 32 297 832

2. (a) 1 315 521 448 (b) 72 578 066
(c) 4 348 (d) 2
(e) 37 920 768 (f) 224

3. (a) 2 668 936 (b) 3 922
(c) 5 886 (d) 21 605
(e) 94 562 160 (f) 17



1. In each case, first estimate the answer. Use the strategy shown at the bottom of the previous page to find useful smaller and bigger possible answers. Then calculate the answer using your calculator, and decide if your answer looks about right.
(a) 134 × 327 (b) 345 136 × 88
(c) 444 × 666 (d) 578 × 34
(e) 43 545 × 213 (f) 6 252 × 82 × 63

27 × 56 ÷ 18 and 27 ÷ 18 × 56 are **equivalent** – they have the same answer.

2. Calculate each of the following using your calculator. Then use your calculator to check the answer by doing the calculations in a different (but equivalent) order.
(a) 4 513 × 878 × 332 (b) 4 513 × 187 × 86
(c) 5 435 × 252 ÷ 315 (d) 66 444 ÷ 678 ÷ 49
(e) 1 543 × 768 × 32 (f) 154 × 768 ÷ 528

8 137 × 328 ÷ 328 = 8 137

3. Calculate each of the following using your calculator. Then check the result by using inverse operations.
(a) 8 137 × 328 (b) 345 136 ÷ 88
(c) 3 105 × 654 ÷ 345 (d) 4 321 ÷ 125 × 625
(e) 2 805 × 784 × 43 (f) 12 342 ÷ 121 ÷ 6

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Grade 6 Term 4 Unit 3 Common fractions

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
3.1 Fractions of collections	Calculating fractions of whole numbers	306
3.2 Writing the same number in different forms	Working with equivalent fractions and the decimal and percentage notations for fractions	307 to 309
3.3 Equivalent fractions	Forming equivalent fractions and using fraction strips	310 to 311
3.4 Practice	Adding and subtracting fractions	311 to 313
3.5 Using fractions to compare quantities	Working with fractions and ratio	313 to 314

CAPS time allocation	5 hours
CAPS page references	16 to 17 and 280

Mathematical background

Fractions can be represented in three different ways:

- In common fraction notation, where the denominator and numerator can be any whole numbers.
- In decimal notation, which is an extension of the positional notation for whole numbers – the fraction part is expressed as a sum of tenths, hundredths, thousandths, etc. with numerators smaller than ten.
- In percentage notation, where the fraction is expressed as hundredths and the numerator is not limited to whole numbers; it can also be a mixed number expressed as a decimal.

Explain to learners that equivalent fractions provide the basis for adding and subtracting fractions.

3.1 Fractions of collections

Mathematical notes

One of the ways in which we use fractions is to talk about parts of collections. Learners have been using the concept of collections for a number of years already, especially where sharing collections helped to form the basis of their understanding of division. Now there is the further step in that parts of collections are represented by fractions; something that learners partly covered in Term 4 of Grade 5.

Teaching guidelines

The numerical values in Section 3.1 are very simple and learners can do them mentally.

Possible misconceptions

Be mindful of the terms you use when you talk about fractions to learners. Avoid saying things that may confuse learners, for example “one over ten”. Rather say “one tenth”.

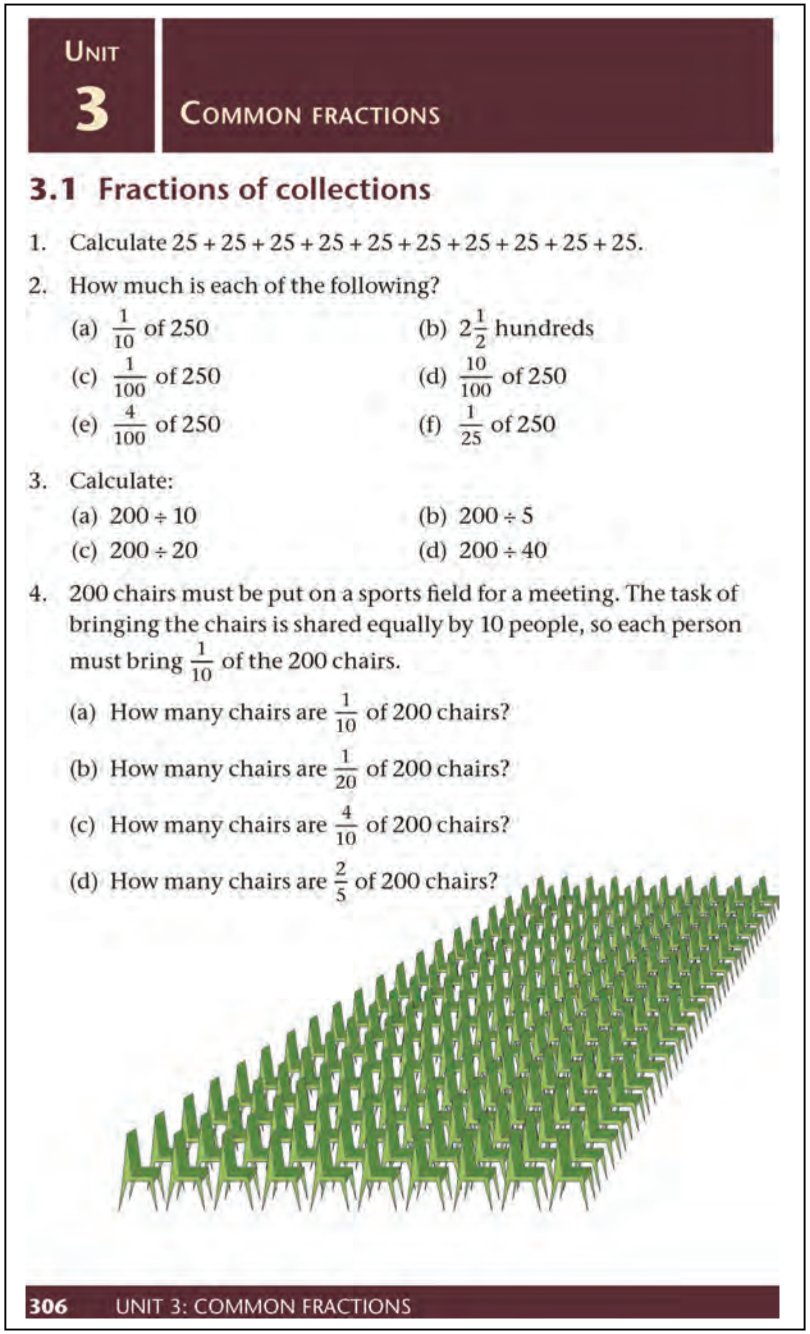
Notes on questions

Questions 1 and 3 involve simple multiplication and division calculations. In question 4 learners are challenged to see the relationship between questions (a) and (b), and that (c) and (d) are equivalent fractions and therefore (d) does not have to be computed. Ask learners if drawing all the chairs helped them to do their calculations.

Point out to learners that in question 2(b) they work with hundreds and not hundred**ths**.

Answers

1. 250
2. (a) 25
   (b) 250 (this is two hundreds and half a hundred)
   (c) 2,5
   (d) 25 (one tenth of 250)
   (e) 10 (one twenty-fifth of 250)
   (f) 10
3. (a) 20 (this is the same as $\frac{1}{10}$ of 200) (b) 40 (this is the same as $\frac{1}{5}$ of 200)
   (c) 10 (d) 5
4. (a) 20 chairs (b) 10 chairs
   (c) 80 chairs (d) 80 chairs



UNIT 3

COMMON FRACTIONS

3.1 Fractions of collections

1. Calculate 25 + 25 + 25 + 25 + 25 + 25 + 25 + 25 + 25 + 25.
2. How much is each of the following?
   (a) $\frac{1}{10}$ of 250 (b) $2\frac{1}{2}$ hundreds
   (c) $\frac{1}{100}$ of 250 (d) $\frac{10}{100}$ of 250
   (e) $\frac{4}{100}$ of 250 (f) $\frac{1}{25}$ of 250
3. Calculate:
   (a) 200 ÷ 10 (b) 200 ÷ 5
   (c) 200 ÷ 20 (d) 200 ÷ 40
4. 200 chairs must be put on a sports field for a meeting. The task of bringing the chairs is shared equally by 10 people, so each person must bring $\frac{1}{10}$ of the 200 chairs.
   (a) How many chairs are $\frac{1}{10}$ of 200 chairs?
   (b) How many chairs are $\frac{1}{20}$ of 200 chairs?
   (c) How many chairs are $\frac{4}{10}$ of 200 chairs?
   (d) How many chairs are $\frac{2}{5}$ of 200 chairs?

306 UNIT 3: COMMON FRACTIONS

3.2 Writing the same number in different forms

Teaching guidelines

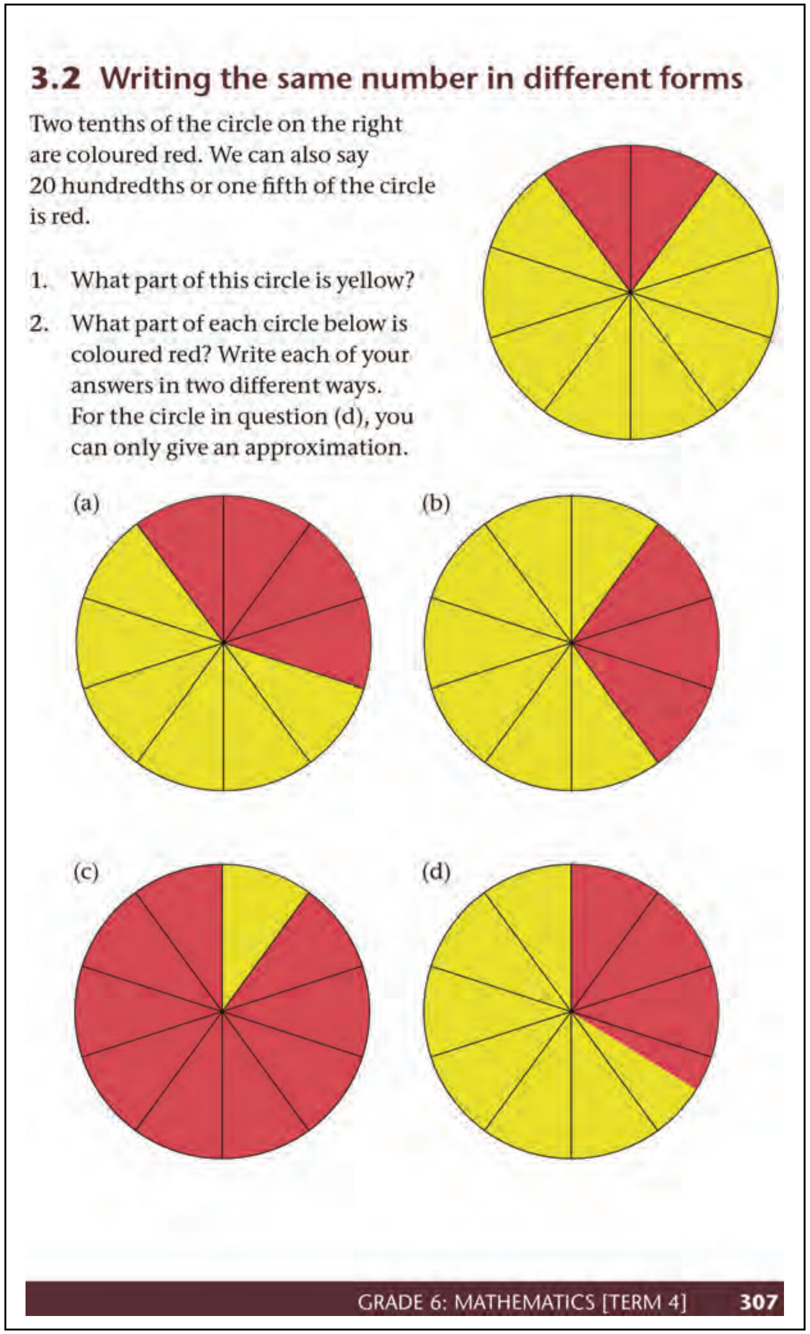
Begin the lesson by discussing the first circle in the Learner Book, i.e. the circle without a number. Ask learners what fraction of the circle is coloured red. Get learners to talk about whether or not it is accurate to say it is 20 hundredths. Encourage learners to think about unusual equivalent fractions here. Explain to learners that because these circles are divided into tenths, they can also be divided into hundredths.

Notes on questions

For question 2(d) learners might think they have to work out a fraction of a fraction, which is one way of doing it. However, a simpler way to do this is to ignore all the lines and to imagine the circle as a whole. You can do this by nearly closing your eyes, and the lines will disappear, but the colours do not.

Answers

1. 8 tenths or 80 hundredths
2. (a) 4 tenths, 2 fifths or 40 hundredths
   (b) 3 tenths or 30 hundredths
   (c) 9 tenths or 90 hundredths
   (d) Approximately 1 third



3.2 Writing the same number in different forms

Two tenths of the circle on the right are coloured red. We can also say 20 hundredths or one fifth of the circle is red.

1. What part of this circle is yellow?
2. What part of each circle below is coloured red? Write each of your answers in two different ways. For the circle in question (d), you can only give an approximation.

GRADE 6: MATHEMATICS [TERM 4] 307

Mathematical notes

Learners spend this part of the section working with fractions, decimals and percentages that represent the same value.

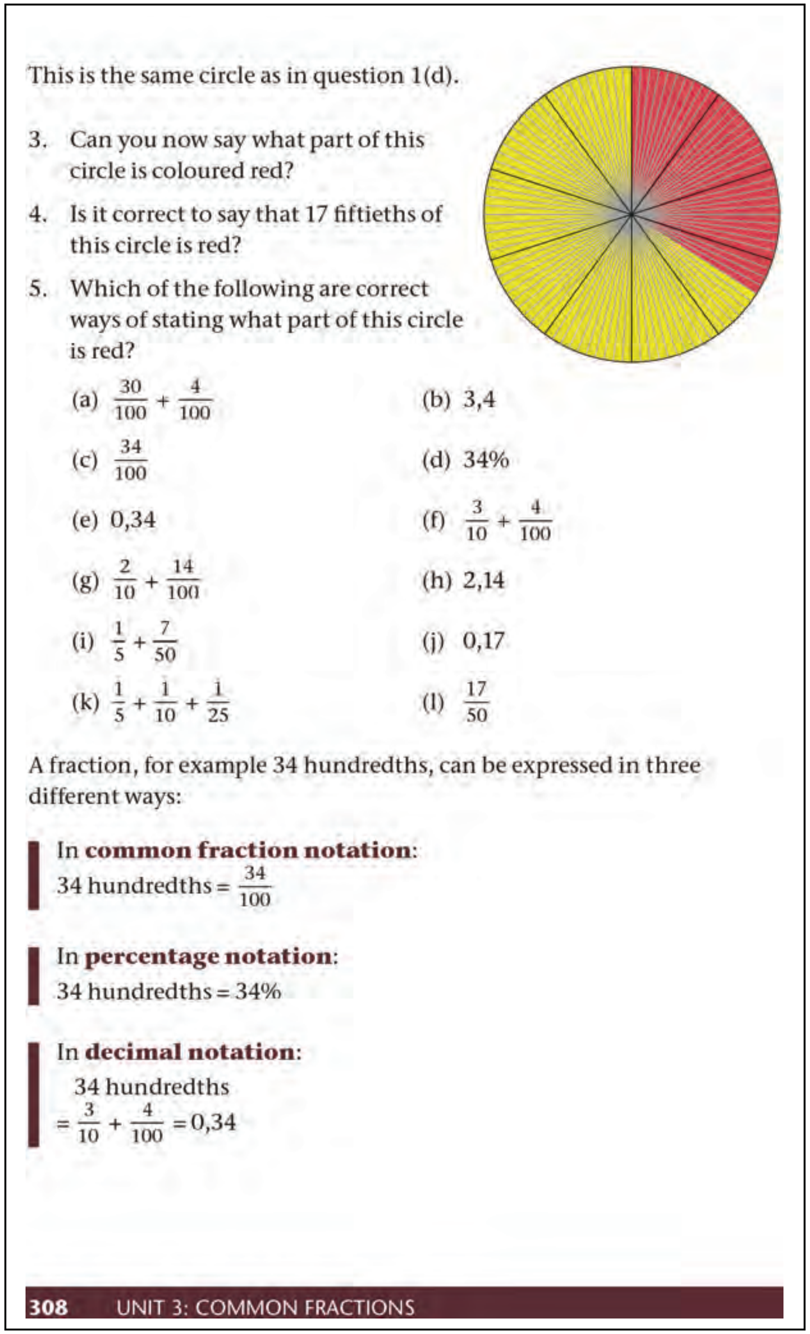
Here learners have the added help of the tenths being divided into tenths themselves. It is therefore possible to work out the exact values of hundredths.

Notes on questions

Questions 3 and 4 follow from question 2(d) on page 307 of the Learner Book. Now that the exact value of the red part can be established, learners can further simplify $\frac{34}{100}$ into fiftieths.

Answers

3. Yes; 34 hundredths
4. Yes; the learners should explain the reason behind their answers.
5. (a) Correct
    (b) Incorrect
    (c) Correct
    (d) Correct
    (e) Correct
    (f) Correct
    (g) Correct
    (h) Incorrect
    (i) Correct
    (j) Incorrect
    (k) Correct
    (l) Correct



This is the same circle as in question 1(d).

3. Can you now say what part of this circle is coloured red?
4. Is it correct to say that 17 fiftieths of this circle is red?
5. Which of the following are correct ways of stating what part of this circle is red?

(a) $\frac{30}{100} + \frac{4}{100}$	(b) 3,4
(c) $\frac{34}{100}$	(d) 34%
(e) 0,34	(f) $\frac{3}{10} + \frac{4}{100}$
(g) $\frac{2}{10} + \frac{14}{100}$	(h) 2,14
(i) $\frac{1}{5} + \frac{7}{50}$	(j) 0,17
(k) $\frac{1}{5} + \frac{1}{10} + \frac{1}{25}$	(l) $\frac{17}{50}$

A fraction, for example 34 hundredths, can be expressed in three different ways:

In **common fraction notation**:
34 hundredths = $\frac{34}{100}$

In **percentage notation**:
34 hundredths = 34%

In **decimal notation**:
34 hundredths
$= \frac{3}{10} + \frac{4}{100} = 0{,}34$

308 UNIT 3: COMMON FRACTIONS

Mathematical notes

Learners will come to understand angle identification better through working on fraction parts of circles.

Notes on questions

In this question the circles can also be divided into tenths and hundredths, allowing learners to work with fractions of tenths and hundredths as decimals and as percentages. You might like to work through question 6(e) with the class as this question requires learners to be more creative, and observe the range of answers.

Answers

6. A: (a) $\frac{4}{10}+\frac{8}{100}$ (b) $\frac{48}{100}$ (c) 0,48 (d) 48% (e) $\frac{2}{5}+\frac{4}{50}$ or $\frac{12}{25}$ or $\frac{2}{5}+\frac{2}{25}$

B: (a) $\frac{2}{10}+\frac{5}{100}$ (b) $\frac{25}{100}$ (c) 0,25 (d) 25% (e) $\frac{1}{5}+\frac{1}{20}$ or $\frac{1}{4}$ or $\frac{1}{5}+\frac{5}{100}$

C: (a) $\frac{3}{10}+\frac{6}{100}$ (b) $\frac{36}{100}$ (c) 0,36 (d) 36% (e) $\frac{3}{10}+\frac{3}{50}$ or $\frac{9}{25}$ or $\frac{18}{50}$

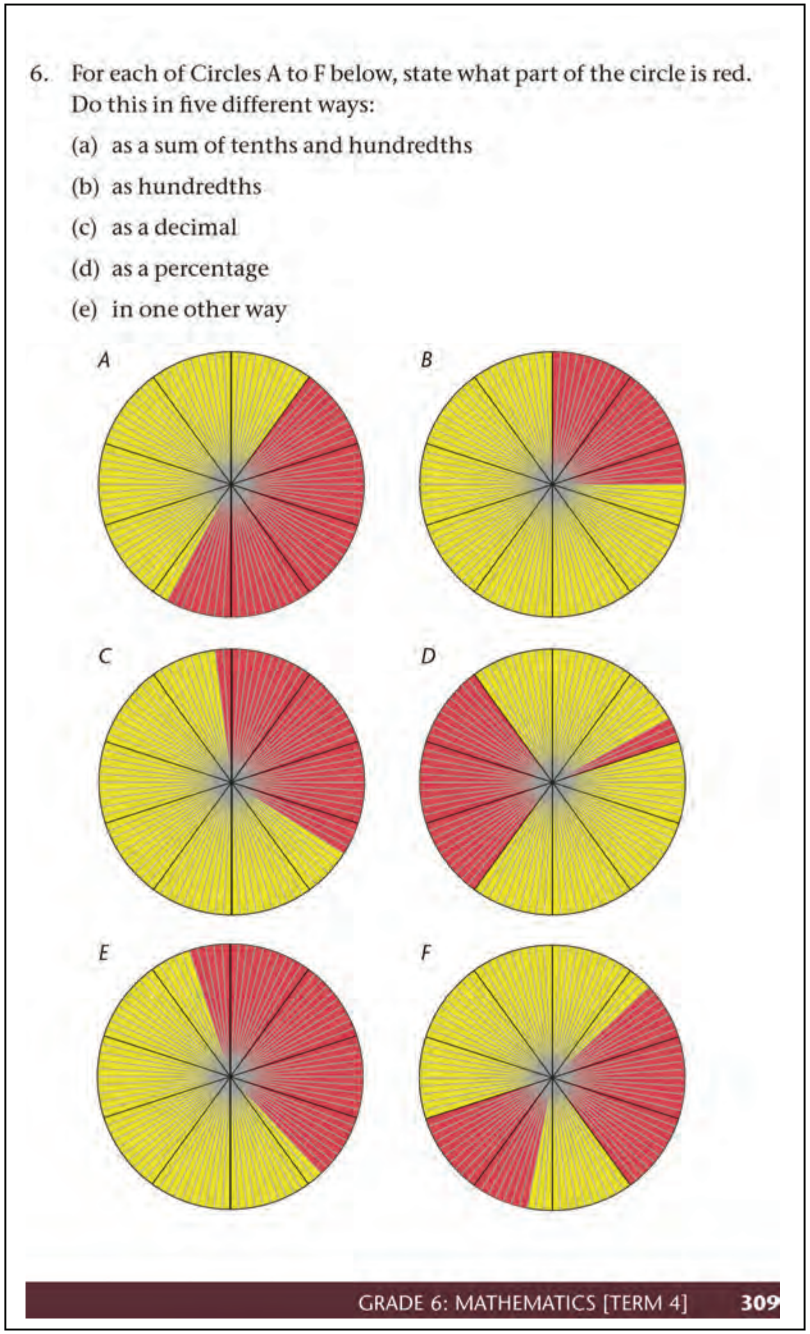
D: (a) $\frac{3}{10}+\frac{3}{100}$ (b) $\frac{33}{100}$ (c) 0,33 (d) 33% (e) $\frac{30}{100}+\frac{3}{100}$

E: (a) $\frac{4}{10}+\frac{3}{100}$ (b) $\frac{43}{100}$ (c) 0,43 (d) 43%

(e) $\frac{2}{5}+\frac{3}{100}$ or $\frac{3}{10}+\frac{8}{100}+\frac{5}{100}$

F: (a) $\frac{4}{10}+\frac{4}{100}$ (b) $\frac{44}{100}$ (c) 0,44 (d) 44%

(e) $\frac{2}{5}+\frac{2}{50}$ or $\frac{3}{10}+\frac{7}{100}+\frac{7}{100}$ or $\frac{11}{25}$



6. For each of Circles A to F below, state what part of the circle is red. Do this in five different ways:
   (a) as a sum of tenths and hundredths
   (b) as hundredths
   (c) as a decimal
   (d) as a percentage
   (e) in one other way

GRADE 6: MATHEMATICS [TERM 4] 309

3.3 Equivalent fractions

Possible misconceptions

A common mistake is for learners to think that they can simply add denominators when adding fractions.

Strongly discourage learners from referring to fractions as “one number over another number”, for example reading $\frac{2}{3}$ as “two over three”. Saying this may contribute to learners thinking that the numerator and denominator are two numbers with similar meanings. So, be mindful when dealing with question 3 and for example in 3(a), talk about 3 eights, 6 sixteenths, 15 fortieths, and 30 eightieths.

Notes on questions

Questions 1 and 2 serve to refresh and consolidate learners’ awareness and understanding of equivalent fractions. They have drawn fraction strips before (in Term 1).

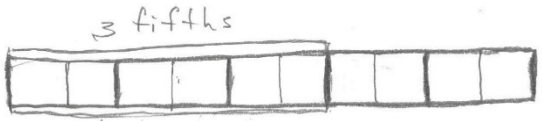
Answers

1. (a)

(b)

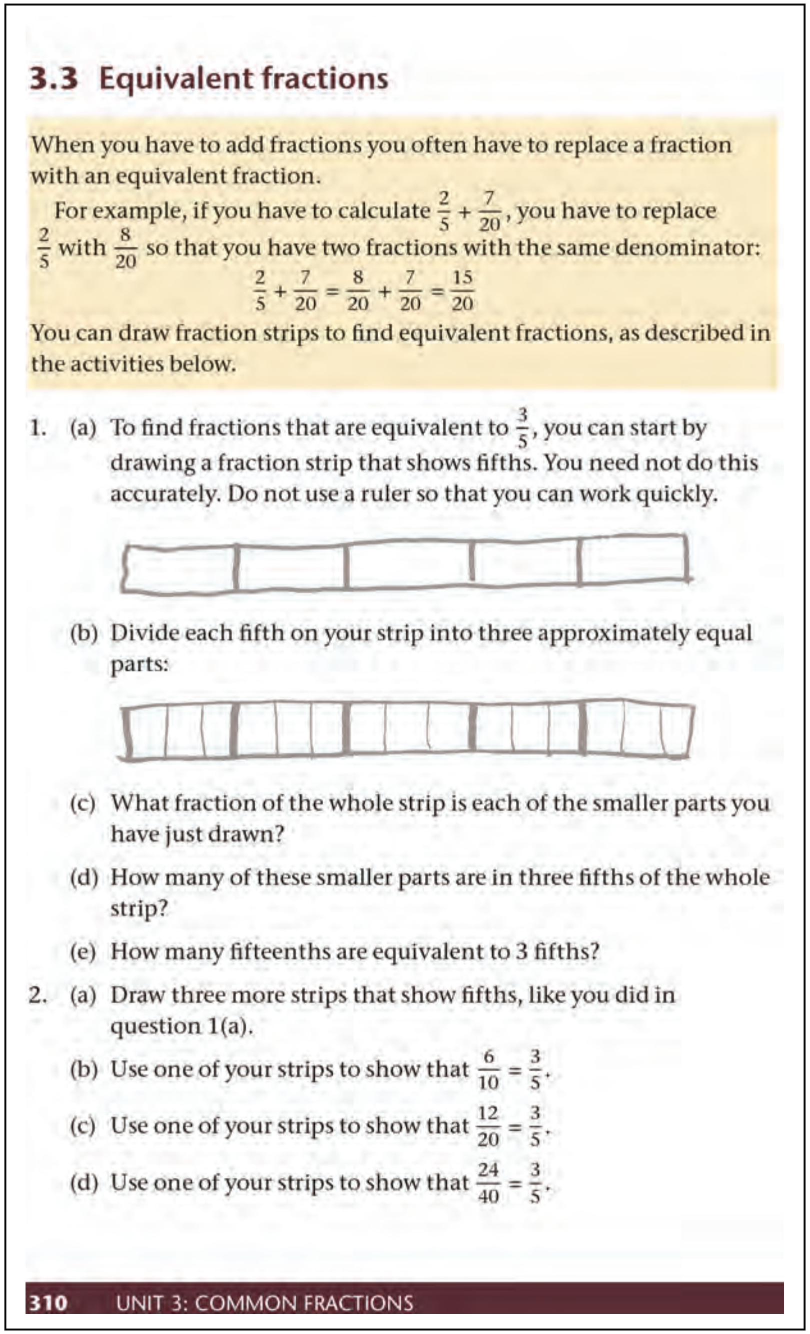
(c) one fifteenth (d) nine (e) 9 fifteenths are equivalent to 3 fifths

2. (b)



(c)

(d)



3.3 Equivalent fractions

When you have to add fractions you often have to replace a fraction with an equivalent fraction.

For example, if you have to calculate $\frac{2}{5} + \frac{7}{20}$, you have to replace $\frac{2}{5}$ with $\frac{8}{20}$ so that you have two fractions with the same denominator:

$$\frac{2}{5} + \frac{7}{20} = \frac{8}{20} + \frac{7}{20} = \frac{15}{20}$$

You can draw fraction strips to find equivalent fractions, as described in the activities below.

1. (a) To find fractions that are equivalent to $\frac{3}{5}$, you can start by drawing a fraction strip that shows fifths. You need not do this accurately. Do not use a ruler so that you can work quickly.

   (b) Divide each fifth on your strip into three approximately equal parts:

   (c) What fraction of the whole strip is each of the smaller parts you have just drawn?

   (d) How many of these smaller parts are in three fifths of the whole strip?

   (e) How many fifteenths are equivalent to 3 fifths?

2. (a) Draw three more strips that show fifths, like you did in question 1(a).

   (b) Use one of your strips to show that $\frac{6}{10} = \frac{3}{5}$.

   (c) Use one of your strips to show that $\frac{12}{20} = \frac{3}{5}$.

   (d) Use one of your strips to show that $\frac{24}{40} = \frac{3}{5}$.

310 UNIT 3: COMMON FRACTIONS

Notes on questions

Question 4 helps to develop learners' intuition of the relationship between equivalent denominators.

Answers

3. A number of examples are given, but consider all learners' answers:
   - (a) $\frac{3}{8} = \frac{6}{16} = \frac{15}{40} = \frac{30}{80}$ (other possible denominators are 32, 48, 56 and 64)
   - (b) $\frac{3}{10} = \frac{6}{20} = \frac{30}{100} = \frac{15}{50}$ (other possible denominators are 30, 40, 60 and 70)
   - (c) $\frac{5}{12} = \frac{10}{24} = \frac{40}{96} = \frac{15}{36}$ (other possible denominators are 48, 60, 72, etc.)
   - (d) $\frac{2}{7} = \frac{6}{21} = \frac{10}{35} = \frac{20}{70}$ (other possible denominators are multiples of 7)
   - (e) $\frac{2}{6} = \frac{1}{3} = \frac{10}{30}$ (other possible denominators are multiples of 6)
   - (f) $\frac{4}{9} = \frac{20}{45} = \frac{40}{90} = \frac{12}{27}$ (also other multiples of 9)
   - (g) $\frac{8}{20} = \frac{4}{10} = \frac{2}{5} = \frac{40}{100}$ (and so on)
   - (h) $\frac{6}{8} = \frac{3}{4} = \frac{75}{100} = \frac{30}{40}$ (and so on)
4. (a) Learners draw a fraction strip that shows eighths.
   - (b) They change the fraction strip in (a) to show fortieths.
   - (c) 5 (d) 3
5. (a) 3 (b) 5

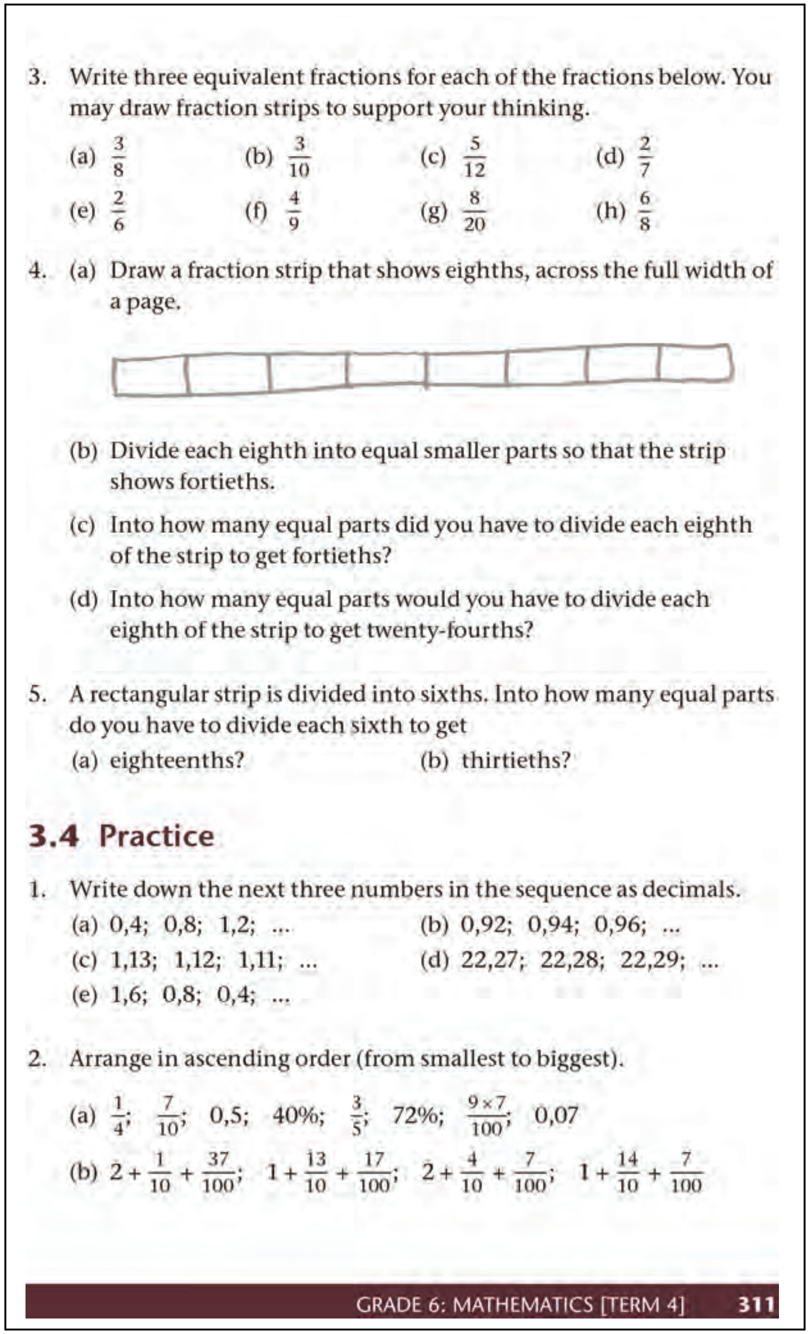
3.4 Practice

Answers

1. (a) 1,6; 2,0; 2,4 (b) 0,98; 1,00; 1,02
   - (c) 1,10; 1,09; 1,08 (d) 22,30; 22,31; 22,32
   - (e) 0,2; 0,1; 0,05
2. (a) The best way to address this question is to convert everything to decimals or fractions with hundredths as denominators.

   0,07; $\frac{1}{4}$; 40%; 0,5; $\frac{3}{5}$; $\frac{9 \times 7}{100}$; $\frac{7}{10}$; 72%

   (b) These are all equivalent: $2 + \frac{1}{10} + \frac{37}{100} = 1 + \frac{13}{10} + \frac{17}{100} = 2 + \frac{4}{10} + \frac{7}{100} = 1 + \frac{14}{10} + \frac{7}{100} = 2,47$



3. Write three equivalent fractions for each of the fractions below. You may draw fraction strips to support your thinking.

   (a) $\frac{3}{8}$ (b) $\frac{3}{10}$ (c) $\frac{5}{12}$ (d) $\frac{2}{7}$

   (e) $\frac{2}{6}$ (f) $\frac{4}{9}$ (g) $\frac{8}{20}$ (h) $\frac{6}{8}$

4. (a) Draw a fraction strip that shows eighths, across the full width of a page.

   (b) Divide each eighth into equal smaller parts so that the strip shows fortieths.

   (c) Into how many equal parts did you have to divide each eighth of the strip to get fortieths?

   (d) Into how many equal parts would you have to divide each eighth of the strip to get twenty-fourths?

5. A rectangular strip is divided into sixths. Into how many equal parts do you have to divide each sixth to get

   (a) eighteenths? (b) thirtieths?

3.4 Practice

1. Write down the next three numbers in the sequence as decimals.

   (a) 0,4; 0,8; 1,2; ... (b) 0,92; 0,94; 0,96; ...

   (c) 1,13; 1,12; 1,11; ... (d) 22,27; 22,28; 22,29; ...

   (e) 1,6; 0,8; 0,4; ...

2. Arrange in ascending order (from smallest to biggest).

   (a) $\frac{1}{4}$; $\frac{7}{10}$; 0,5; 40%; $\frac{3}{5}$; 72%; $\frac{9 \times 7}{100}$; 0,07

   (b) $2 + \frac{1}{10} + \frac{37}{100}$; $1 + \frac{13}{10} + \frac{17}{100}$; $2 + \frac{4}{10} + \frac{7}{100}$; $1 + \frac{14}{10} + \frac{7}{100}$

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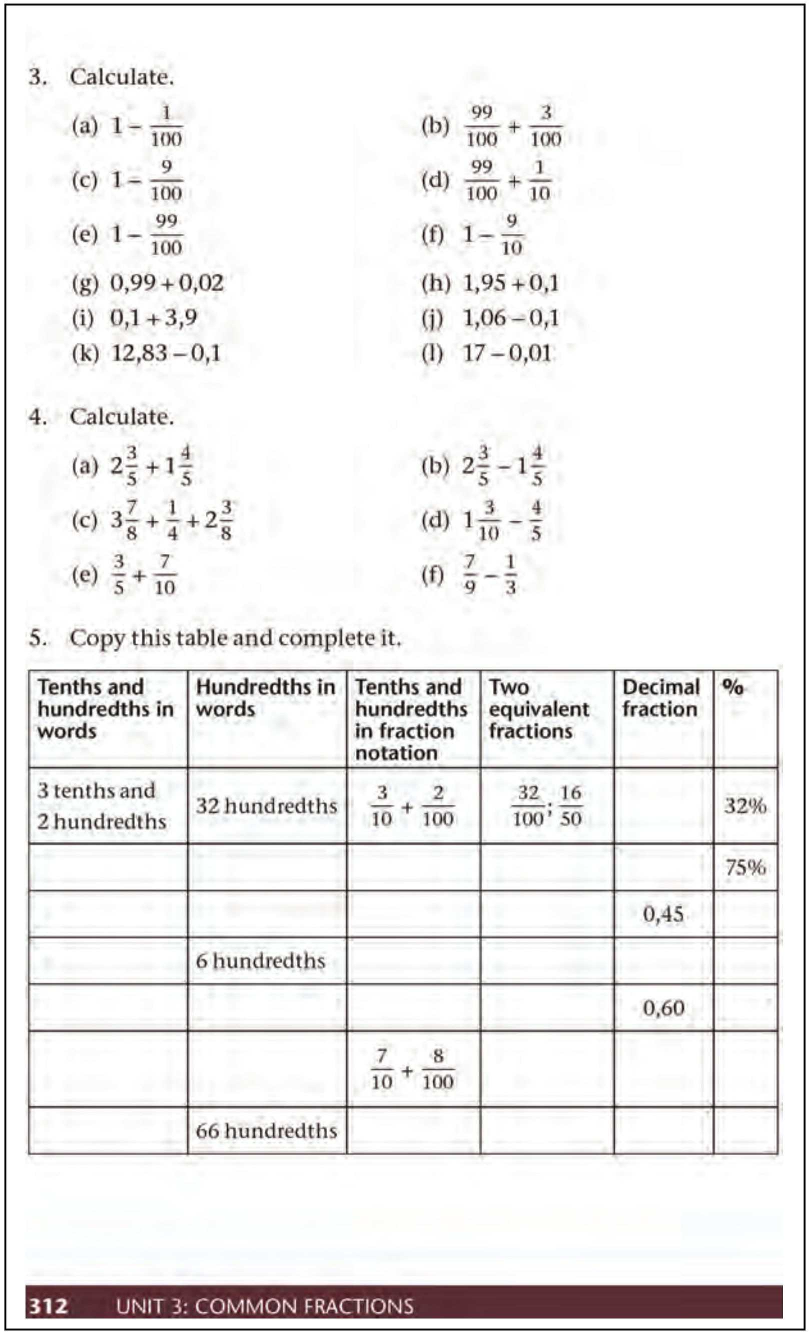
Answers

3. (a) $\frac{99}{100}$ (b) $1\frac{2}{100} = 1\frac{1}{50}$ (c) $\frac{91}{100}$ (d) $1\frac{9}{100}$
(e) $\frac{1}{100}$ (f) $\frac{1}{10}$ (g) 1,01 (h) 2,05
(i) 4 (j) 0,96 (k) 12,73 (l) 16,99

4. (a) $4\frac{2}{5}$ (b) $\frac{4}{5}$ (c) $6\frac{4}{8} = 6\frac{1}{2}$
(d) $\frac{5}{10} = \frac{1}{2}$ (e) $1\frac{3}{10}$ (f) $\frac{4}{9}$

5.

Tenths and hundredths in words	**Hundredths in words**	**Tenths and hundredths in fraction notation**	**Two equivalent fractions**	**Decimal fraction**	**%**
3 tenths and 2 hundredths	32 hundredths	$\frac{3}{10} + \frac{2}{100}$	$\frac{32}{100}$; $\frac{8}{25}$	0,32	32%
7 tenths and 5 hundredths	75 hundredths	$\frac{7}{10} + \frac{5}{100}$	$\frac{75}{100}$; $\frac{3}{4}$	0,75	75%
4 tenths and 5 hundredths	45 hundredths	$\frac{4}{10} + \frac{5}{100}$	$\frac{45}{100}$; $\frac{9}{20}$	0,45	45%
0 tenths and 6 hundredths	6 hundredths	$\frac{0}{10} + \frac{6}{100}$	$\frac{6}{100}$; $\frac{3}{50}$	0,06	6%
6 tenths and 0 hundredths	60 hundredths	$\frac{6}{10} + \frac{0}{100}$	$\frac{60}{100}$; $\frac{3}{5}$	0,60	60%
7 tenths and 8 hundredths	78 hundredths	$\frac{7}{10} + \frac{8}{100}$	$\frac{78}{100}$; $\frac{39}{50}$	0,78	78%
6 tenths and 6 hundredths	66 hundredths	$\frac{6}{10} + \frac{6}{100}$	$\frac{66}{100}$; $\frac{33}{50}$	0,66	66%



3. Calculate.

(a) $1 - \frac{1}{100}$ (b) $\frac{99}{100} + \frac{3}{100}$
(c) $1 - \frac{9}{100}$ (d) $\frac{99}{100} + \frac{1}{10}$
(e) $1 - \frac{99}{100}$ (f) $1 - \frac{9}{10}$
(g) $0,99 + 0,02$ (h) $1,95 + 0,1$
(i) $0,1 + 3,9$ (j) $1,06 - 0,1$
(k) $12,83 - 0,1$ (l) $17 - 0,01$

4. Calculate.

(a) $2\frac{3}{5} + 1\frac{4}{5}$ (b) $2\frac{3}{5} - 1\frac{4}{5}$
(c) $3\frac{7}{8} + \frac{1}{4} + 2\frac{3}{8}$ (d) $1\frac{3}{10} - \frac{4}{5}$
(e) $\frac{3}{5} + \frac{7}{10}$ (f) $\frac{7}{9} - \frac{1}{3}$

5. Copy this table and complete it.

Tenths and hundredths in words	**Hundredths in words**	**Tenths and hundredths in fraction notation**	**Two equivalent fractions**	**Decimal fraction**	**%**
3 tenths and 2 hundredths	32 hundredths	$\frac{3}{10} + \frac{2}{100}$	$\frac{32}{100}$; $\frac{16}{50}$		32%
					75%
				0,45	
	6 hundredths				
				0,60	
		$\frac{7}{10} + \frac{8}{100}$			
	66 hundredths				

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Answers

6. (a) 4,35 (b) 15,73 (c) 0,19
(d) 5 (e) 6,08 (f) 0,94

3.5 Using fractions to compare quantities

Mathematical notes

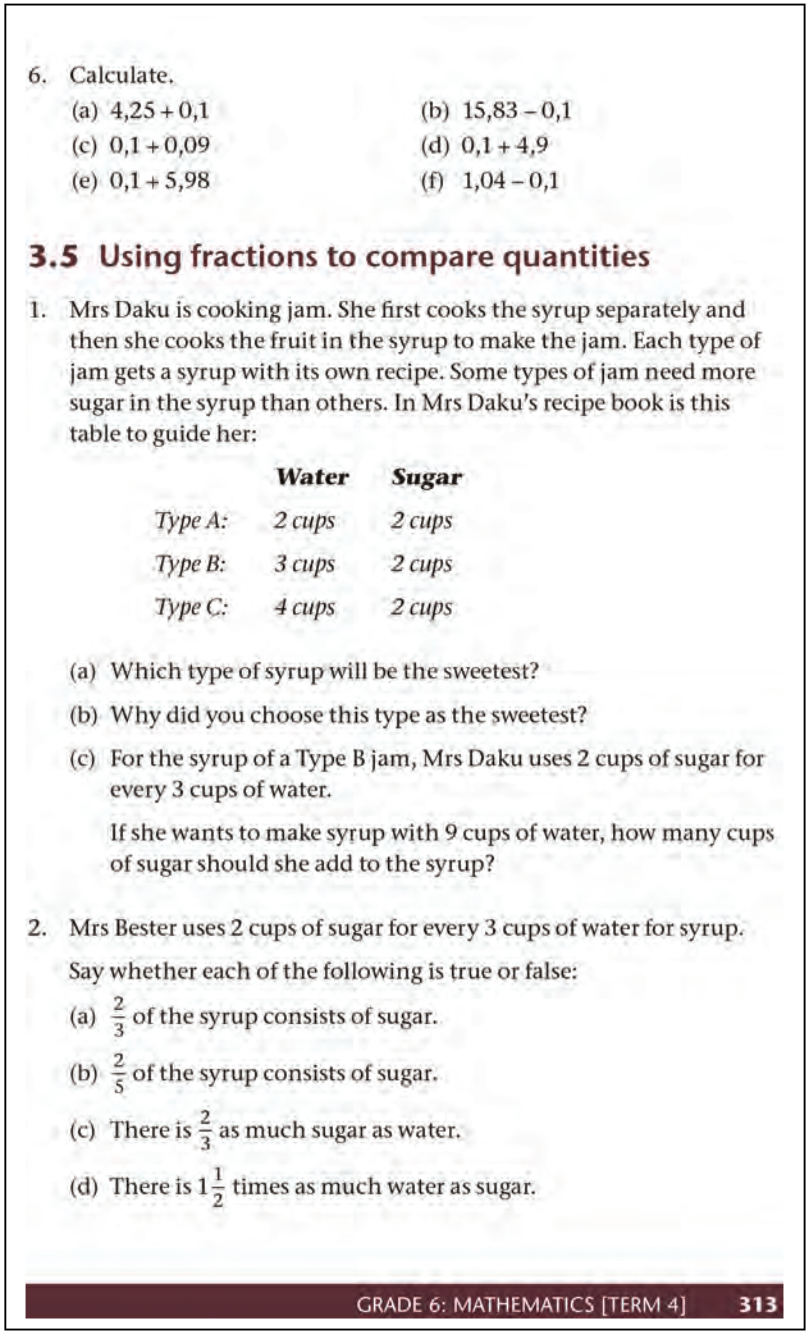
Fractions, among other things, help us to think about proportions. For example, if you make jam that requires two cups of sugar and three cups of water, and you want to triple the recipe, what will the proportion of the sugar be to the water?

Possible misconceptions

When you teach proportions, certain misconceptions may creep in. Look at question 2(a), for example. If there are two cups of sugar to three cups of water, then the total number of cups is five. The proportions would therefore be written as 2:3. This is *not* the same as $\frac{2}{3}$; we are talking about $\frac{2}{5}$ and $\frac{3}{5}$.

Answers

1. (a) Type A, because the proportion of sugar to water is the highest.
   (b) The least amount of water for the same amount of sugar.
   (c) Six cups of sugar (there is three times as much water; therefore there must be three times as much sugar).
2. (a) False (b) True (c) False (d) True



6. Calculate.
   (a) 4,25 + 0,1 (b) 15,83 − 0,1
   (c) 0,1 + 0,09 (d) 0,1 + 4,9
   (e) 0,1 + 5,98 (f) 1,04 − 0,1

3.5 Using fractions to compare quantities

1. Mrs Daku is cooking jam. She first cooks the syrup separately and then she cooks the fruit in the syrup to make the jam. Each type of jam gets a syrup with its own recipe. Some types of jam need more sugar in the syrup than others. In Mrs Daku's recipe book is this table to guide her:

	Water	***Sugar***
Type A:	*2 cups*	*2 cups*
Type B:	*3 cups*	*2 cups*
Type C:	*4 cups*	*2 cups*

   (a) Which type of syrup will be the sweetest?
   (b) Why did you choose this type as the sweetest?
   (c) For the syrup of a Type B jam, Mrs Daku uses 2 cups of sugar for every 3 cups of water.
   If she wants to make syrup with 9 cups of water, how many cups of sugar should she add to the syrup?

2. Mrs Bester uses 2 cups of sugar for every 3 cups of water for syrup.
   Say whether each of the following is true or false:
   (a) $\frac{2}{3}$ of the syrup consists of sugar.
   (b) $\frac{2}{5}$ of the syrup consists of sugar.
   (c) There is $\frac{2}{3}$ as much sugar as water.
   (d) There is $1\frac{1}{2}$ times as much water as sugar.

Mathematical notes

In this section learners will deal with fractions as a ratio in a different context. For example, for every step his father takes, Jody needs to take three steps. The ratio is quite simple, it is 1:3. It can also be described as *three times* and as *one-third*.

Notes on questions

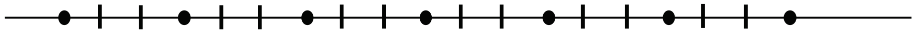
For question 3(b), the copied number line can serve as Jody's father's steps. Then there have to be two marks between each step to show the extra steps Jody takes.

Questions 4(a) and (b) combine addition of fractions with different denominators with the realisation that equivalent fractions mean the same duration of time.

Answers

3. (a) One third; $\frac{1}{3}$

(b)



(c) The father's step is **three** times Jody's step.

(d) Jody's step is **one-third** of his father's step.

4. (a)

Hour	Minutes
$\frac{1}{3}$	20
$\frac{2}{3}$	40
$\frac{1}{5}$	12
$\frac{1}{6}$	10
$\frac{1}{10}$	6

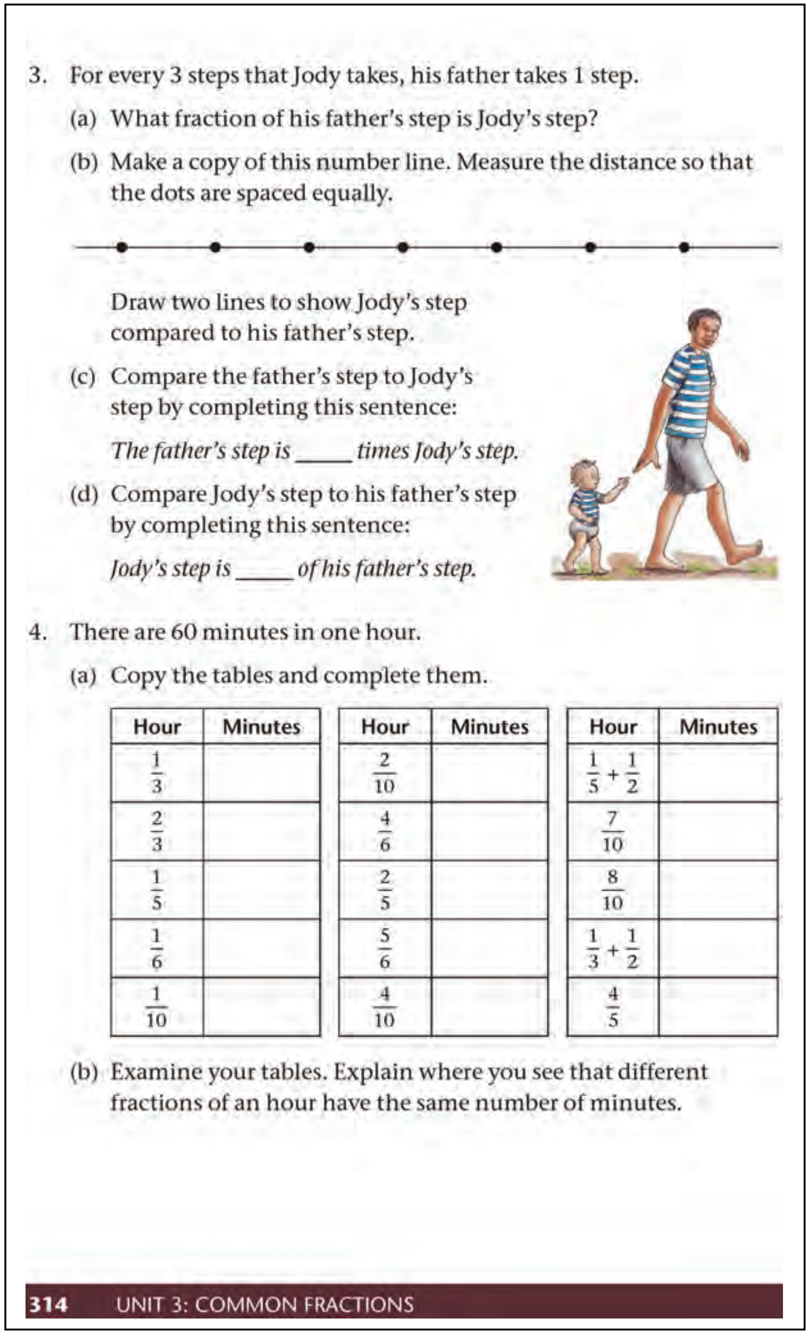
Hour	Minutes
$\frac{2}{10}$	12
$\frac{4}{6}$	40
$\frac{2}{5}$	24
$\frac{5}{6}$	50
$\frac{4}{10}$	24

Hour	Minutes
$\frac{1}{5}+\frac{1}{2}$	42
$\frac{7}{10}$	42
$\frac{8}{10}$	48
$\frac{1}{3}+\frac{1}{2}$	50
$\frac{4}{5}$	48

(b) $\frac{2}{3}=\frac{4}{6}=40$ minutes $\frac{1}{5}=\frac{2}{10}=12$ minutes $\frac{2}{5}=\frac{4}{10}=24$ minutes

$\frac{1}{5}+\frac{1}{2}=\frac{7}{10}=42$ minutes $\frac{8}{10}=\frac{4}{5}=48$ minutes

NB: No calculations are required for question 4(b). Learners only need to make deduction from the table.



3. For every 3 steps that Jody takes, his father takes 1 step.

(a) What fraction of his father's step is Jody's step?

(b) Make a copy of this number line. Measure the distance so that the dots are spaced equally.

Draw two lines to show Jody's step compared to his father's step.

(c) Compare the father's step to Jody's step by completing this sentence:

The father's step is _____ times Jody's step.

(d) Compare Jody's step to his father's step by completing this sentence:

Jody's step is _____ of his father's step.

4. There are 60 minutes in one hour.

(a) Copy the tables and complete them.

Hour	Minutes
$\frac{1}{3}$	
$\frac{2}{3}$	
$\frac{1}{5}$	
$\frac{1}{6}$	
$\frac{1}{10}$	

Hour	Minutes
$\frac{2}{10}$	
$\frac{4}{6}$	
$\frac{2}{5}$	
$\frac{5}{6}$	
$\frac{4}{10}$	

Hour	Minutes
$\frac{1}{5}+\frac{1}{2}$	
$\frac{7}{10}$	
$\frac{8}{10}$	
$\frac{1}{3}+\frac{1}{2}$	
$\frac{4}{5}$	

(b) Examine your tables. Explain where you see that different fractions of an hour have the same number of minutes.

Grade 6 Term 4 Unit 4 Properties of three-dimensional objects

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
4.1 Skeleton models of 3-D objects	Focusing on the edges of 3-D objects	315 to 317
4.2 Drawings and pictures of pyramids	Visible and obscured parts of pyramids as seen from different positions	318 to 320
4.3 Faces, vertices and edges of 3-D objects	Revision of these characteristics of 3-D objects in general	321

CAPS time allocation	5 hours
CAPS page references	22 and 281

Mathematical background

Learners can engage with 3-D objects at three different levels:

- A concrete level; by working with physical objects (models): building or analysing objects.
- A graphic level; by working with 2-D representations of 3-D objects: making or analysing pictures and drawings.
- A language level; by describing 3-D objects or elements of 3-D objects with appropriate language, and by reading text about 3-D objects with comprehension.

Sections 4.1, 4.2 and 4.3 of this unit provide learners with opportunities to work with 3-D objects at the three levels mentioned above respectively.

Resources

Skeletons or other models of various prisms and pyramids

4.1 Skeleton models of 3-D objects

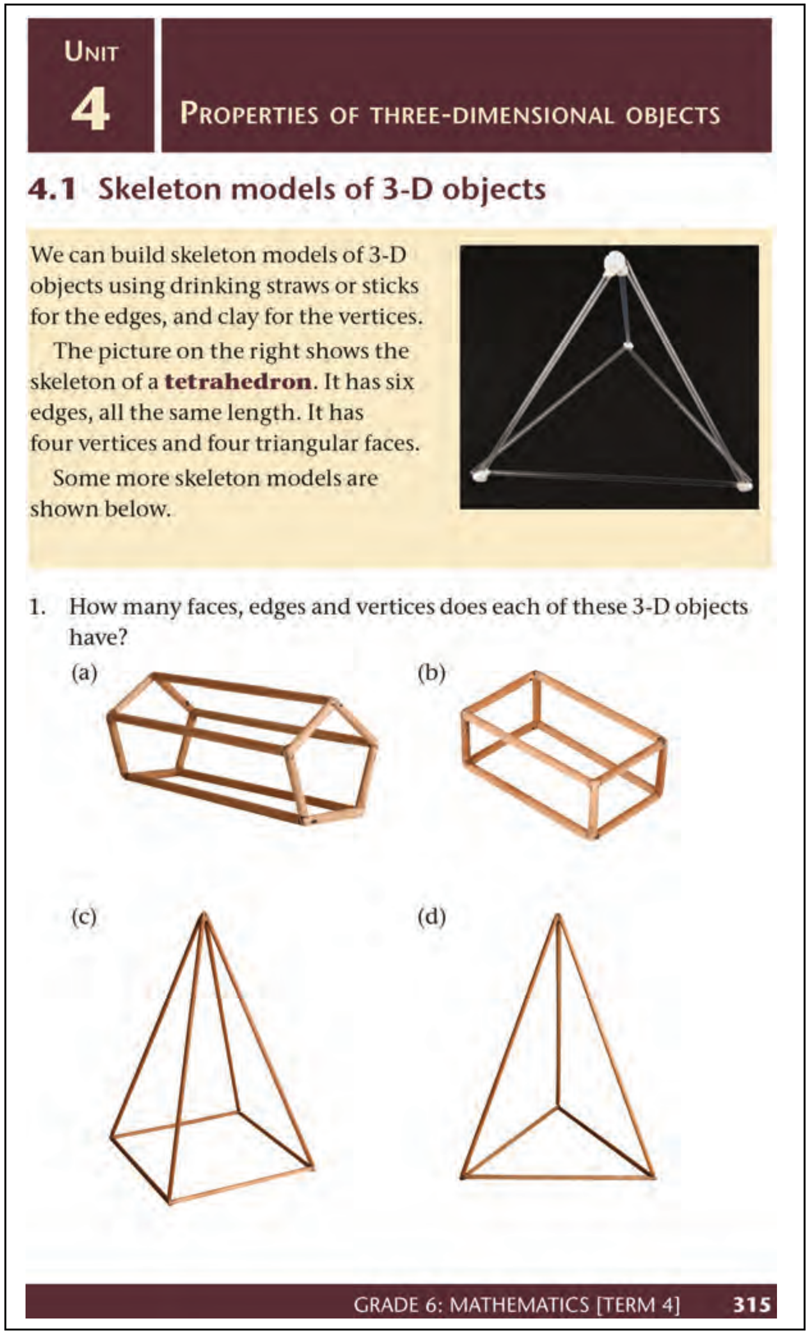
Teaching guidelines

This section builds on the introduction to stick models of 3-D objects with polygonal faces in Term 2 Unit 3 (Section 3.4 on page 148). While it will be useful to display one or two stick/straw models to learners again, it is best to get them to work with question 1 by analysing the given diagrams. It is very important that learners learn to interpret pictures and diagrams of 3-D objects. They will spend substantial time working with actual models when they do questions 2 to 7 on the following pages.

You may have to revise the concepts of face, edge and vertex with respect to 3-D objects. To do this, it is best to have a model available, for example a box.

Answers

1. (a) 7 faces, 15 edges, 10 vertices
   (b) 6 faces, 12 edges, 8 vertices
   (c) 5 faces, 8 edges, 5 vertices
   (d) 4 faces, 6 edges, 4 vertices



UNIT 4

PROPERTIES OF THREE-DIMENSIONAL OBJECTS

4.1 Skeleton models of 3-D objects

We can build skeleton models of 3-D objects using drinking straws or sticks for the edges, and clay for the vertices.

The picture on the right shows the skeleton of a **tetrahedron**. It has six edges, all the same length. It has four vertices and four triangular faces.

Some more skeleton models are shown below.

1. How many faces, edges and vertices does each of these 3-D objects have?

(a) (b)

(c) (d)

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Teaching guidelines

The actual building of the models is not the main purpose of questions 2 to 6, and time constraints will probably not allow the building of all the models in class anyway. However, it is important that learners build the models for at least questions 2 and 3 in class (see the notes on "Possible misconceptions" below).

The main purpose of questions 2 to 7 is to make learners think about the various kinds of 3-D objects and to develop knowledge of the properties of 3-D objects in terms of the numbers of edges, vertices and faces. Section 4.1 provides for the development of concepts and knowledge that will empower learners to work with the higher level tasks in Sections 4.2 and 4.3.

Possible misconceptions

Learners may miss hidden edges and vertices. For example, in question 2(a) learners may mention only 7 vertices (pieces of clay), 3 straws of 10 cm, 2 straws of 8 cm and 2 straws of 5 cm for the yellow prism.

Do not correct learners when they make this mistake. They will realise their error when they start building the model – they will find that they are short of parts. It is much more effective to learn by having to deal with the consequences of their mistakes than when you simply tell them they are wrong.

Answers

2. (a) The yellow prism: 4 straws of 10 cm each, 4 straws of 5 cm each, 4 straws of 8 cm each, 8 pieces of clay

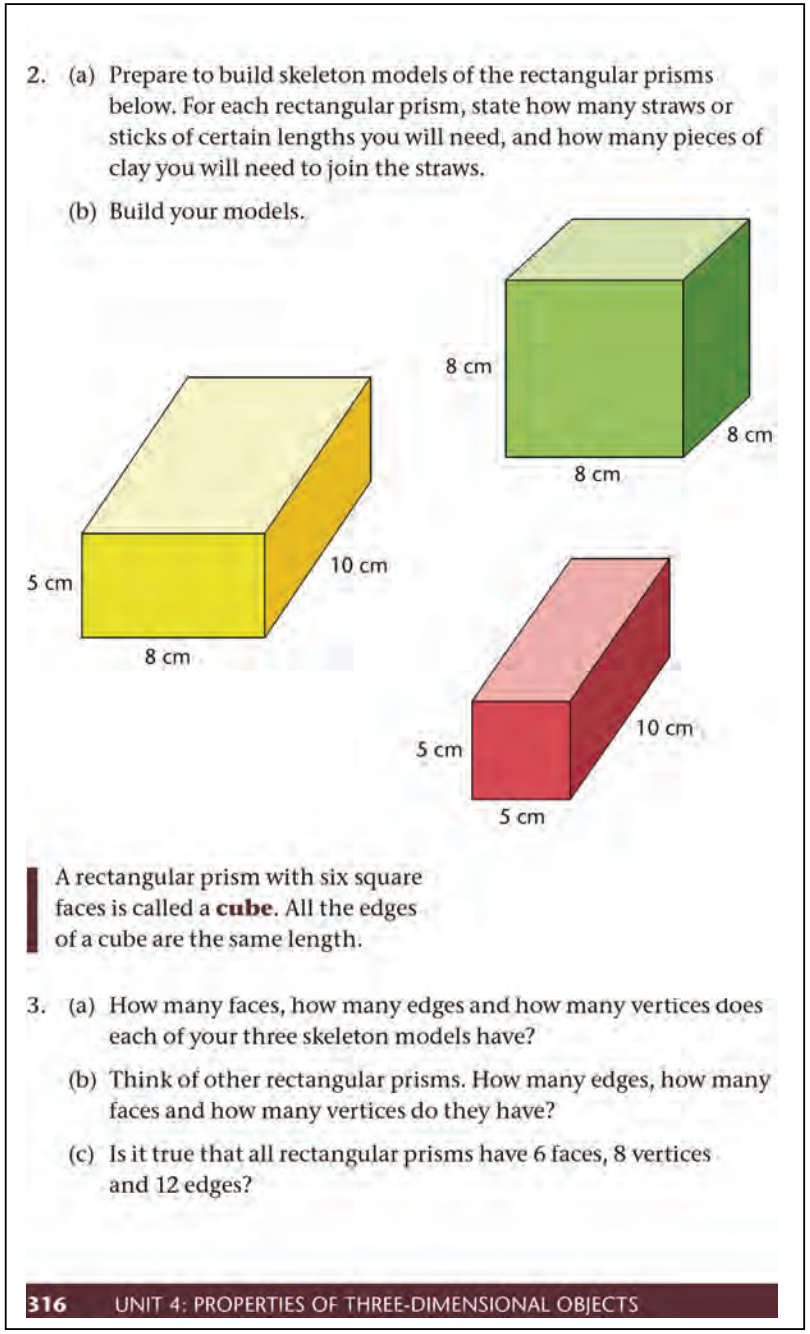
   The green prism: 12 straws of 8 cm each, 8 pieces of clay

   The red prism: 8 straws of 5 cm each, 4 straws of 10 cm each, 8 pieces of clay

3. (a) 6 faces, 12 edges, 8 vertices

   (b) 6 faces, 12 edges, 8 vertices

   (c) Yes



2. (a) Prepare to build skeleton models of the rectangular prisms below. For each rectangular prism, state how many straws or sticks of certain lengths you will need, and how many pieces of clay you will need to join the straws.

   (b) Build your models.

A rectangular prism with six square faces is called a **cube**. All the edges of a cube are the same length.

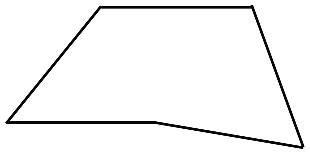
3. (a) How many faces, how many edges and how many vertices does each of your three skeleton models have?

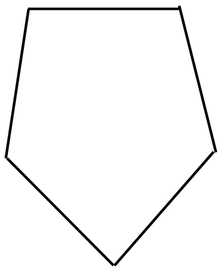
   (b) Think of other rectangular prisms. How many edges, how many faces and how many vertices do they have?

   (c) Is it true that all rectangular prisms have 6 faces, 8 vertices and 12 edges?

Teaching guidelines

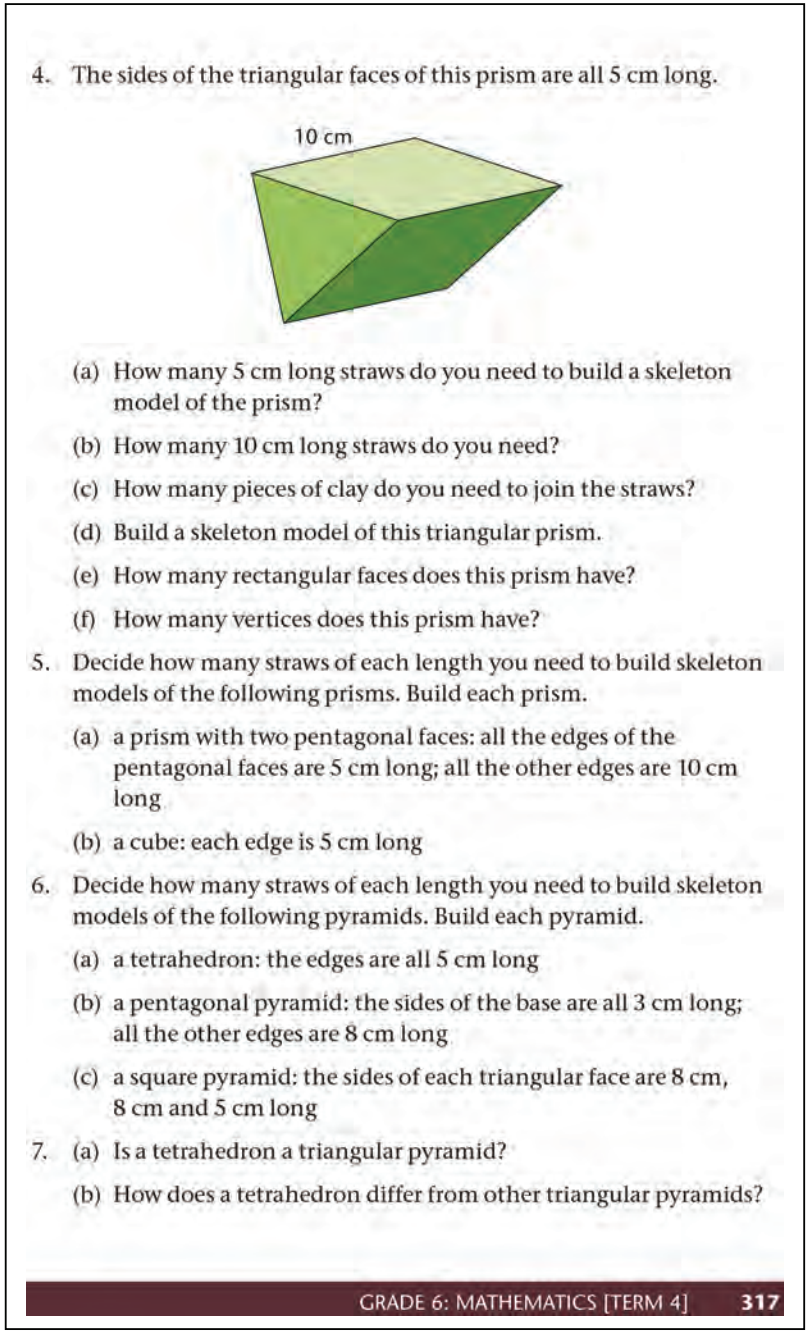
If time permits the building of prisms in questions 4 and 5, learners will have the opportunity to experience an interesting phenomenon. The triangular prism in question 4 is rigid: only one shape is possible. However, the pentagonal prism in question 5 is not rigid: the pentagonal face with sides 5 cm can take many different forms, for example:





Answers

4. (a) 6
   (b) 3
   (c) 6
   (d) Learners' own work
   (e) 3
   (f) 6
5. (a) Ten 5 cm straws and five 10 cm straws
   (b) Twelve 5 cm straws
6. (a) Six 5 cm straws
   (b) Five 3 cm straws and five 8 cm straws
   (c) Four 8 cm straws and four 5 cm straws
7. (a) Yes
   (b) A tetrahedron is a triangular pyramid with all edges the same length.



4. The sides of the triangular faces of this prism are all 5 cm long.

(a) How many 5 cm long straws do you need to build a skeleton model of the prism?
(b) How many 10 cm long straws do you need?
(c) How many pieces of clay do you need to join the straws?
(d) Build a skeleton model of this triangular prism.
(e) How many rectangular faces does this prism have?
(f) How many vertices does this prism have?

5. Decide how many straws of each length you need to build skeleton models of the following prisms. Build each prism.
(a) a prism with two pentagonal faces: all the edges of the pentagonal faces are 5 cm long; all the other edges are 10 cm long
(b) a cube: each edge is 5 cm long

6. Decide how many straws of each length you need to build skeleton models of the following pyramids. Build each pyramid.
(a) a tetrahedron: the edges are all 5 cm long
(b) a pentagonal pyramid: the sides of the base are all 3 cm long; all the other edges are 8 cm long
(c) a square pyramid: the sides of each triangular face are 8 cm, 8 cm and 5 cm long

7. (a) Is a tetrahedron a triangular pyramid?
(b) How does a tetrahedron differ from other triangular pyramids?

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4.2 Drawings and pictures of pyramids

Mathematical notes

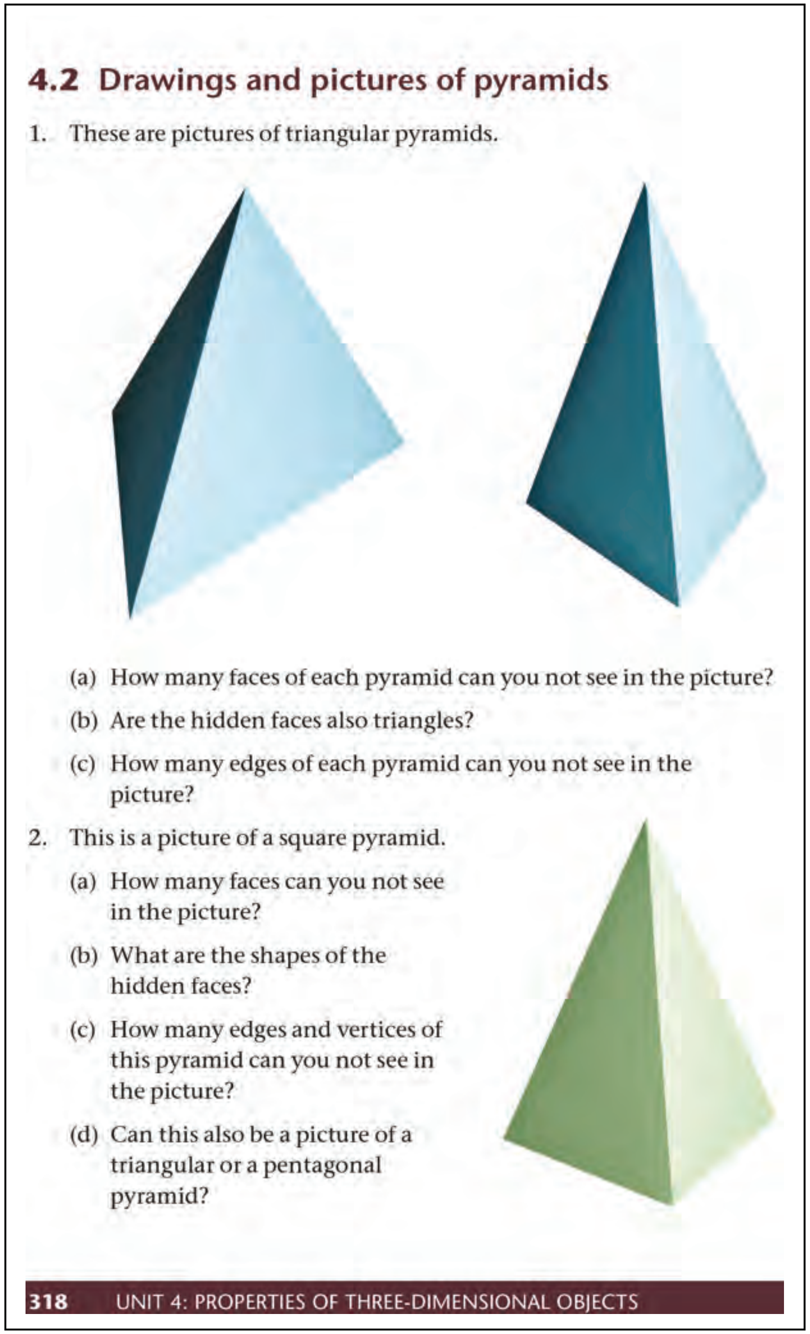
Certain faces, edges and vertices will be hidden behind the parts of the object that lie in the line of sight of the viewer. One way to show this when drawing an object is to show all the visible edges with solid lines and all the hidden edges with broken lines, as shown on page 319.

Teaching guidelines

Encourage learners to answer the questions. However, learners who really battle may need access to physical models in order to strengthen their understanding of the 2-D representations of the 3-D objects.

Answers

1. (a) 2
   (b) Yes
   (c) 1
2. (a) 3
   (b) 2 triangles and 1 square
   (c) 3 edges and 1 vertex
   (d) Yes



4.2 Drawings and pictures of pyramids

1. These are pictures of triangular pyramids.

   (a) How many faces of each pyramid can you not see in the picture?
   (b) Are the hidden faces also triangles?
   (c) How many edges of each pyramid can you not see in the picture?

2. This is a picture of a square pyramid.

   (a) How many faces can you not see in the picture?
   (b) What are the shapes of the hidden faces?
   (c) How many edges and vertices of this pyramid can you not see in the picture?
   (d) Can this also be a picture of a triangular or a pentagonal pyramid?

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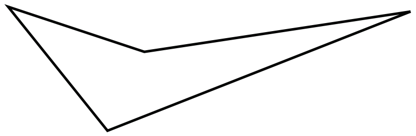
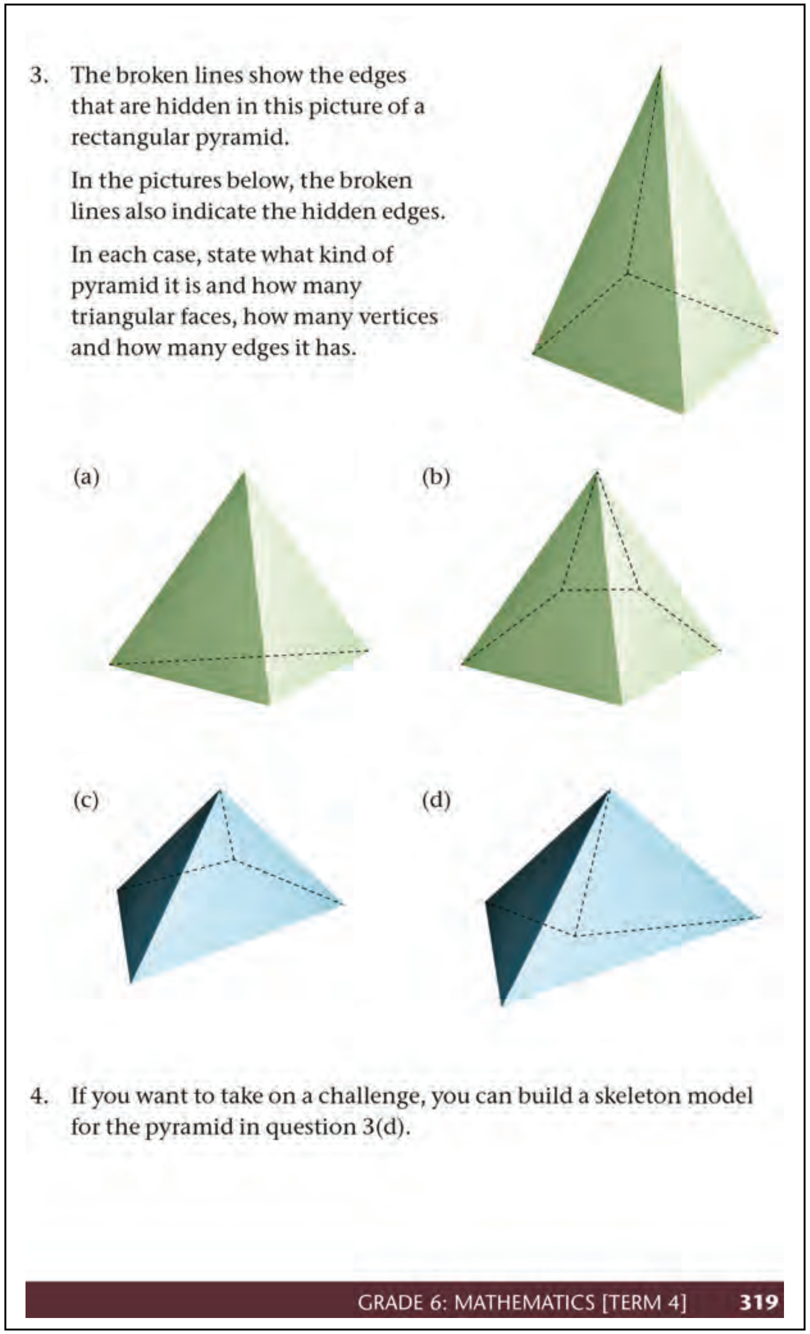
Notes on questions

The base of the pyramid in 3(d) is concave:



Answers

3. (a) Triangular pyramid, 4 triangular faces, 4 vertices, 6 edges
   (b) Pentagonal pyramid, 5 triangular faces, 6 vertices, 10 edges
   (c) Rectangular pyramid, 4 triangular faces, 5 vertices, 8 edges
   (d) Concave pyramid, 4 triangular faces, 5 vertices, 8 edges



3. The broken lines show the edges that are hidden in this picture of a rectangular pyramid.

   In the pictures below, the broken lines also indicate the hidden edges.

   In each case, state what kind of pyramid it is and how many triangular faces, how many vertices and how many edges it has.

(a)

(b)

(c)

(d)

4. If you want to take on a challenge, you can build a skeleton model for the pyramid in question 3(d).

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Teaching guidelines

In questions 6 and 7 it is best if learners draw freehand, i.e. without using a ruler. They need to focus not on accuracy, but on getting the drawing right: showing the hidden edges with dotted or broken lines:

. . . . dotted line

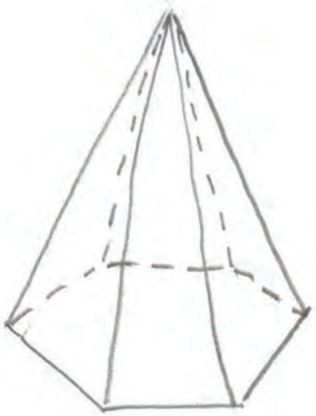
- - - - - - broken line

Answers

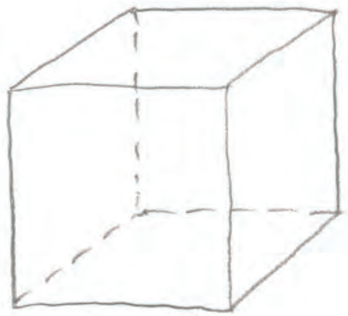
5. Picture A: 4 faces, 3 edges, 0 vertices

   Picture B: 4 faces, 5 edges, 2 vertices

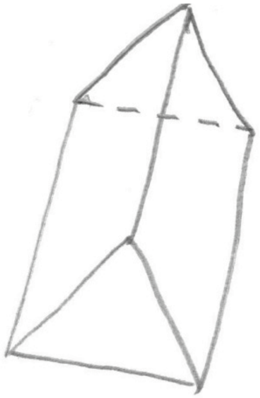
6.

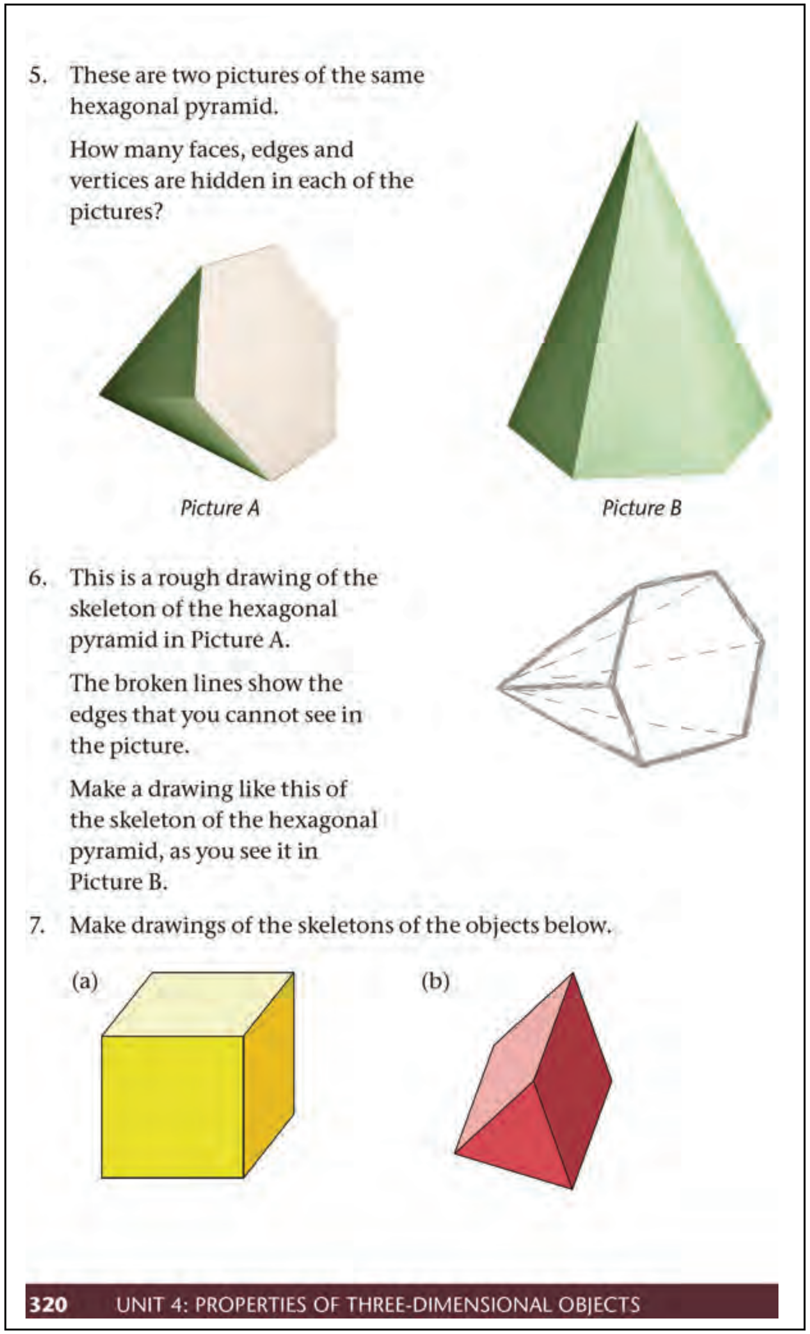


7. (a)



(b)





5. These are two pictures of the same hexagonal pyramid.

   How many faces, edges and vertices are hidden in each of the pictures?

6. This is a rough drawing of the skeleton of the hexagonal pyramid in Picture A.

   The broken lines show the edges that you cannot see in the picture.

   Make a drawing like this of the skeleton of the hexagonal pyramid, as you see it in Picture B.

7. Make drawings of the skeletons of the objects below.

320 UNIT 4: PROPERTIES OF THREE-DIMENSIONAL OBJECTS

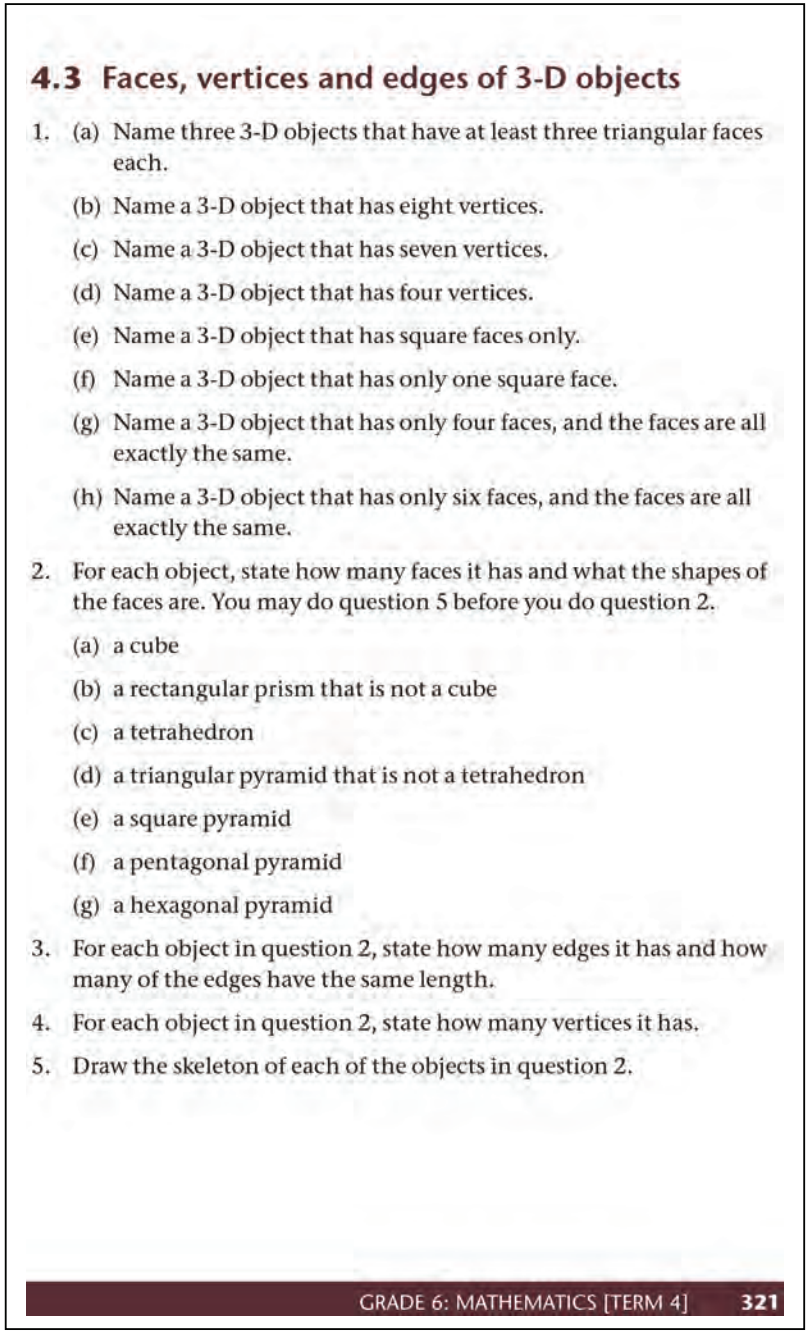
4.3 Faces, vertices and edges of 3-D objects

Teaching guidelines

Learners may find these questions quite demanding; they will need time and perseverance. When they work on question 1, you may suggest that they skip the parts that they find difficult at first, and focus on the easier parts. You may also suggest that they page back through this unit and look for examples of objects with the stated faces.

Answers

1. (a) Tetrahedron, triangular pyramid, square pyramid, any pyramid
   (b) A rectangular prism (c) A hexagonal pyramid
   (d) A tetrahedron, any triangular pyramid
   (e) A cube (f) A square pyramid
   (g) A tetrahedron (h) A cube
2. (a) 6 faces, all squares
   (b) 6 faces, 6 rectangles *or* 6 faces, 2 squares and 4 rectangles
   (c) 4 faces, 4 triangles
   (d) 4 faces, 4 triangles
   (e) 5 faces, 4 triangles and 1 square
   (f) 6 faces, 5 triangles and 1 pentagon
   (g) 7 faces, 6 triangles and 1 hexagon
3. (a) 12 edges, all the same
   (b) 12 edges, 4 the same, another 4 the same, and another 4 the same *or* 12 edges, 8 the same and 4 the same
   (c) 6 edges, all the same
   (d) 6 edges, all may have different lengths
   (e) 8 edges, 4 the same and another 4 the same
   (f) 10 edges, 5 the same and another 5 the same *or* 10 edges, 5 irregular and another 5 the same. (The pentagonal pyramid is not defined as regular and therefore learners may come up with different answers.)
   (g) 12 edges, 6 the same and another 6 the same
4. (a) 8 (b) 8 (c) 4 (d) 4
   (e) 5 (f) 6 (g) 7
5. See next page.

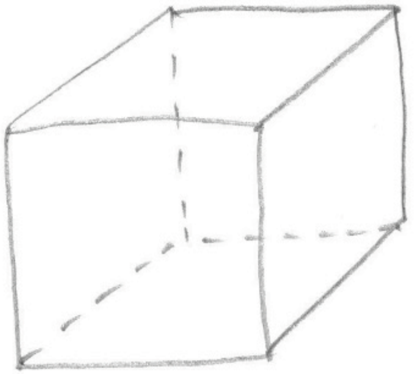


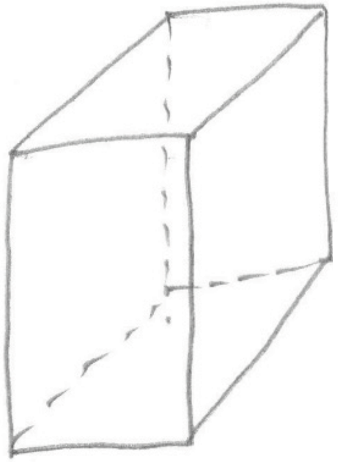
4.3 Faces, vertices and edges of 3-D objects

1. (a) Name three 3-D objects that have at least three triangular faces each.
   (b) Name a 3-D object that has eight vertices.
   (c) Name a 3-D object that has seven vertices.
   (d) Name a 3-D object that has four vertices.
   (e) Name a 3-D object that has square faces only.
   (f) Name a 3-D object that has only one square face.
   (g) Name a 3-D object that has only four faces, and the faces are all exactly the same.
   (h) Name a 3-D object that has only six faces, and the faces are all exactly the same.
2. For each object, state how many faces it has and what the shapes of the faces are. You may do question 5 before you do question 2.
   (a) a cube
   (b) a rectangular prism that is not a cube
   (c) a tetrahedron
   (d) a triangular pyramid that is not a tetrahedron
   (e) a square pyramid
   (f) a pentagonal pyramid
   (g) a hexagonal pyramid
3. For each object in question 2, state how many edges it has and how many of the edges have the same length.
4. For each object in question 2, state how many vertices it has.
5. Draw the skeleton of each of the objects in question 2.

Answers

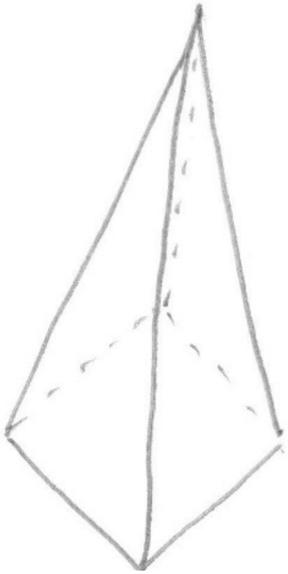
5.

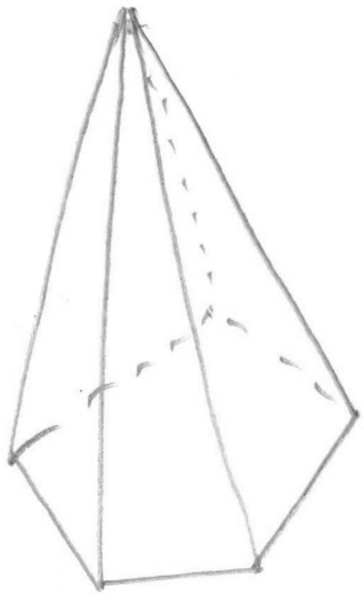

(a)

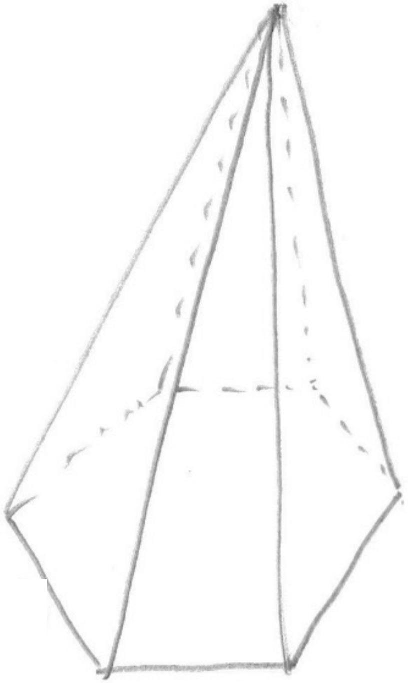

(b)


(c)


(d)


(e)


(f)


(g)

Grade 6 Term 4 Unit 5 Perimeter, area and volume

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
5.1 Perimeter and area	The difference between perimeter and area; measuring perimeter on grids	322 to 326
5.2 Area and perimeter	Measuring area by counting grid squares; perimeter of different shapes with the same area	327 to 329
5.3 Volume and capacity	Measuring volume by counting cubes; the difference between capacity and volume	330 to 331

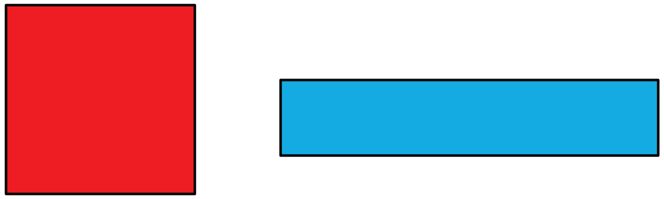
CAPS time allocation	7 hours
CAPS page references	28 and 282 to 283

Mathematical background

The concepts of perimeter, area and volume:

- Perimeter is the distance around an object. For example, the perimeter of a farm could be thought of as the length of fencing needed to fence off the farm completely. Perimeter is measured in the same units as length, for example centimetre and kilometre.
- The concept of area cannot be defined in simple terms. Area is the quantity we can use to describe the size of a surface, for example the surface of a floor. The bigger the area of a floor, the more tiles or paint is needed to cover the floor. It is useful to think of area as the number of small, identical squares laid tightly next to each other without overlapping that is needed to cover the surface of an object.
- The volume of an amount of liquid or a solid object is the amount of space occupied by the liquid or the object. The idea of volume is supported by thinking of an object as being made up of many small, identical cubes stacked tightly together: the volume can be expressed as the number of cubes.

Differentiation between the concepts of perimeter and area presents a conceptual challenge. Perimeter and area are two different indicators of the “size” of a flat object or a face of a 3-D object: the red quadrilateral below is larger than the blue quadrilateral in terms of area, but the blue quadrilateral is larger in terms of perimeter.



Further examples are explored on page 328 of the Learner Book.

Resources

Rulers; measuring tapes (see page 364 of this guide); grid paper

5.1 Perimeter and area

Possible misconceptions

The formulas for the area and perimeter of a rectangle and the volume of a rectangular prism should not be introduced in Grade 6. It may result in learners understanding perimeter only as *2 × (length + breadth)*, area only as *length × breadth* and volume only as *length × breadth × height*, irrespective of the actual shape of the object.

Misconception about measurements on grids

While the grids greatly facilitate effective engagement with perimeter and area, there is one stumbling block: learners may believe that the length of the blue lines can be obtained by counting the number of squares that they pass through. This issue may come up when learners do question 2(d). Instead of using rulers, you may ask them to mark off the length of each blue line on the edge of a sheet of paper and then line the edge up along one of the grid lines to determine its actual length.

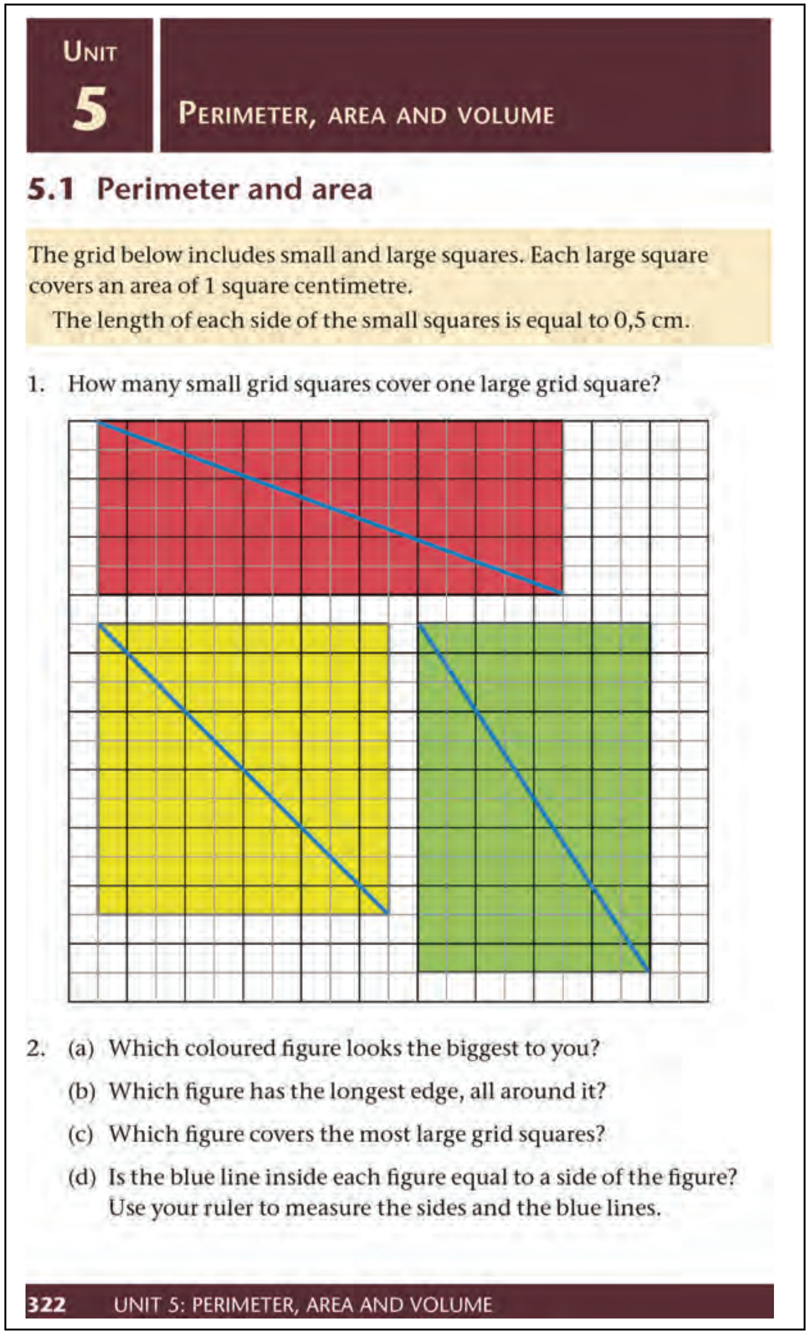
Notes on questions

Questions 2(a), (b) and (c) are designed for learners to see that "larger" can refer to two different things with respect to surfaces: perimeter and area. It is irrelevant what answer learners give to question 2(a), the purpose of the question is to make them think which of the three figures may be the largest, and what "largest" may mean with respect to closed 2-D figures.

Once learners have answered both questions 2(b) and (c), ask them to consider question 2(a) again. Conclude with a classroom discussion in which you clarify that perimeter and area are two different aspects of the size of a surface.

Answers

1. Four small grid squares cover one large grid square.
2. (a) Different learners may give different answers.
   (b) The red rectangle, which has 44 squares around its edge. The yellow and green figures both have 40 squares around the edge.
   (c) The yellow rectangle, which covers 25 large grid squares. The red and green rectangles each cover 24 large grid squares.
   (d) Red figure: two sides 8 cm, two sides 3 cm and blue line approximately 8,5 cm.
   Yellow figure: all four sides 5 cm and blue line approximately 7 cm.
   Green figure: two sides 6 cm, two sides 4 cm and blue line approximately 7,2 cm.



UNIT 5 PERIMETER, AREA AND VOLUME

5.1 Perimeter and area

The grid below includes small and large squares. Each large square covers an area of 1 square centimetre.

The length of each side of the small squares is equal to 0,5 cm.

1. How many small grid squares cover one large grid square?

2. (a) Which coloured figure looks the biggest to you?
   (b) Which figure has the longest edge, all around it?
   (c) Which figure covers the most large grid squares?
   (d) Is the blue line inside each figure equal to a side of the figure? Use your ruler to measure the sides and the blue lines.

322 UNIT 5: PERIMETER, AREA AND VOLUME

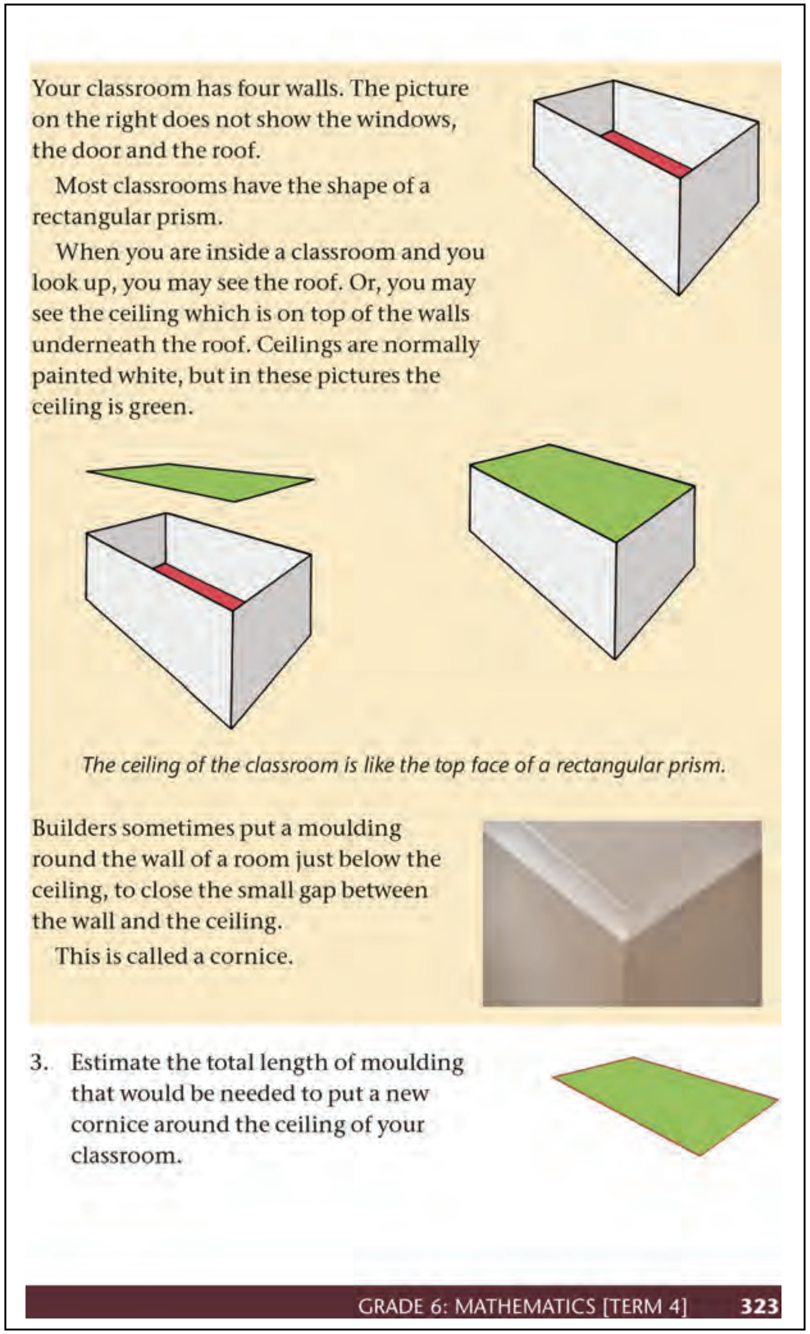
Teaching guidelines

Discuss the classroom as an example of a rectangular prism and use it to explain the concept of perimeter, as suggested in the shaded passage. You can also use it at different times during this unit to explain and revisit the concepts of area, volume and capacity.

The question *"How many identical bricks or rectangular boxes (prisms) of any given size can be packed into the classroom?"* relates to the concept of capacity. The questions *"How many tiles are needed to cover the floor or the ceiling?"* or *"Which wall would require most paint?"* relate to the concept of area.

Answers

3. Learners' answers will differ from classroom to classroom. Help them to reach a consensus.



Your classroom has four walls. The picture on the right does not show the windows, the door and the roof.

Most classrooms have the shape of a rectangular prism.

When you are inside a classroom and you look up, you may see the roof. Or, you may see the ceiling which is on top of the walls underneath the roof. Ceilings are normally painted white, but in these pictures the ceiling is green.

The ceiling of the classroom is like the top face of a rectangular prism.

Builders sometimes put a moulding round the wall of a room just below the ceiling, to close the small gap between the wall and the ceiling.

This is called a cornice.

3. Estimate the total length of moulding that would be needed to put a new cornice around the ceiling of your classroom.

Teaching guidelines

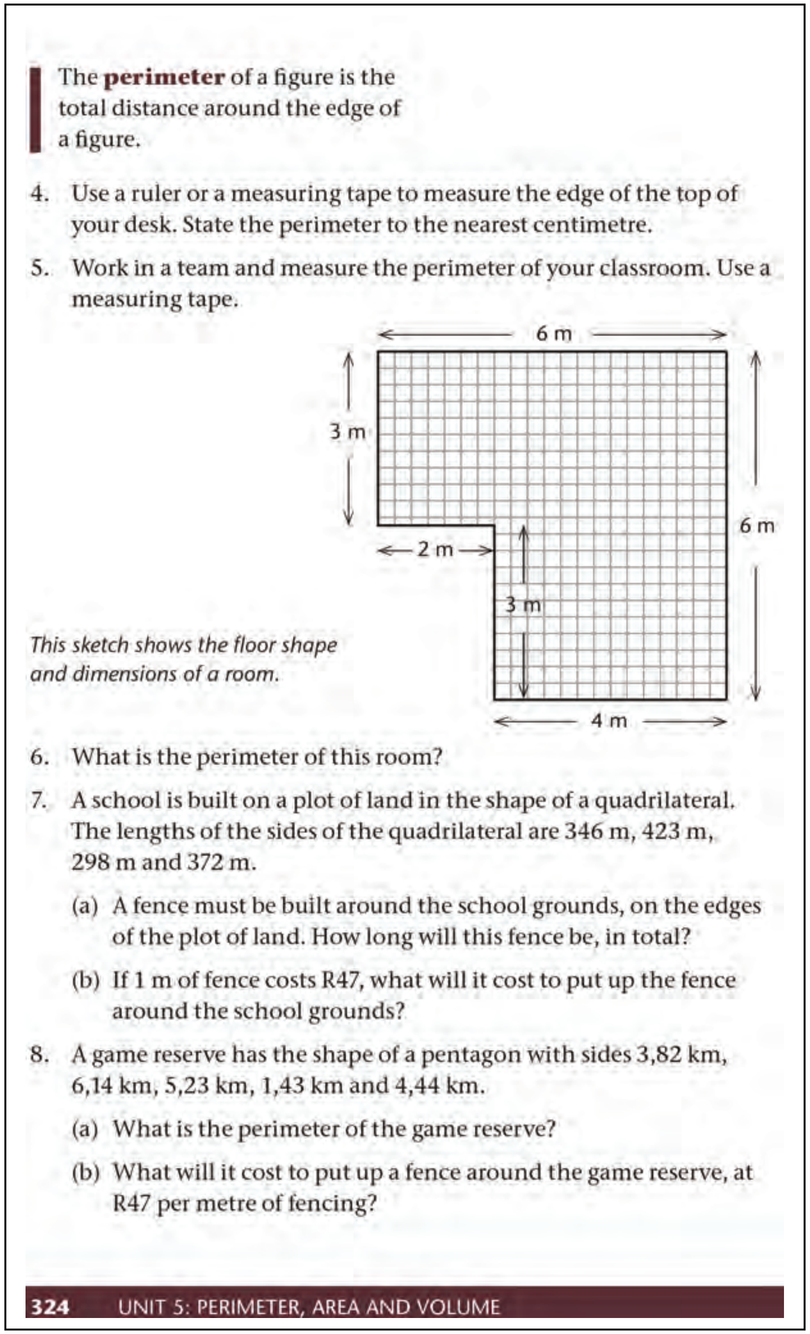
Working with a soft (cloth or plastic) measuring tape provides a powerful experience of perimeter, especially if the tape is longer than the perimeter of the object. In addition to the activities described in the Learner Book, you may make copies of the measuring tapes (rulers) on page 447 in the Addendum and provide each learner with a paper measuring tape. Let them measure the perimeter (circumference) of their wrists and their hands (with or without thumb), and even their various fingers.

To promote differentiation between the concepts of perimeter and area, you may point out that the floor area of the room in the diagram can be measured by counting the number of squares (but do not let learners do this now, it will take up too much time and the squares are too small for accurate counting).

It is critical that learners decide on their own initiative to add the given lengths of the sides in question 7 to calculate the perimeter. If they add the lengths only because someone told them to do so, they may not attend to the idea of perimeter. If learners seem to have difficulty answering question 7(a), suggest that they make a rough sketch of the plot of land – you can even make a sketch on the board.

Answers

4. Practical activity: Learners' answers will differ.
5. Practical activity: Help learners to reach consensus for your classroom.
6. 24 m
7. (a) 1 439 m (b) R67 633
8. (a) 21,06 km = 21 060 m (b) R989 820



The **perimeter** of a figure is the total distance around the edge of a figure.

4. Use a ruler or a measuring tape to measure the edge of the top of your desk. State the perimeter to the nearest centimetre.
5. Work in a team and measure the perimeter of your classroom. Use a measuring tape.

This sketch shows the floor shape and dimensions of a room.

6. What is the perimeter of this room?
7. A school is built on a plot of land in the shape of a quadrilateral. The lengths of the sides of the quadrilateral are 346 m, 423 m, 298 m and 372 m.
   (a) A fence must be built around the school grounds, on the edges of the plot of land. How long will this fence be, in total?
   (b) If 1 m of fence costs R47, what will it cost to put up the fence around the school grounds?
8. A game reserve has the shape of a pentagon with sides 3,82 km, 6,14 km, 5,23 km, 1,43 km and 4,44 km.
   (a) What is the perimeter of the game reserve?
   (b) What will it cost to put up a fence around the game reserve, at R47 per metre of fencing?

324 UNIT 5: PERIMETER, AREA AND VOLUME

Teaching guidelines

The purpose of questions 9, 10 and 11 is to address the possible **misconception** that the length of any line on a grid can be measured by counting the grid squares through which it passes. A learner who holds this misconception will produce 14 cm as the answer for question 9(a).

Questions 9, 10 and 11 may be skipped if this issue was effectively resolved by learners when they did question 2(d) on page 322.

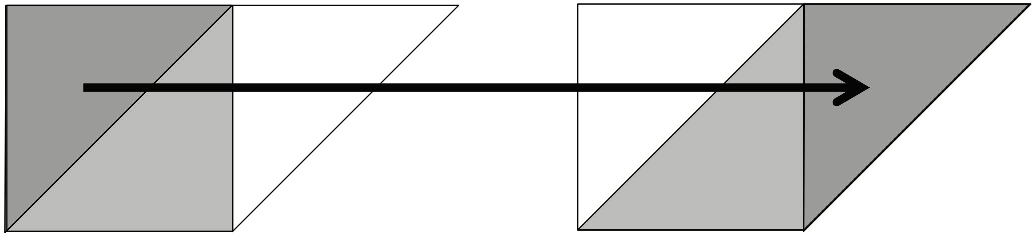
Answers

9. (a) Learners may answer “14”; they should correct themselves when they do (b).
   (b) Red perimeter = 20 cm; blue perimeter = 20 cm
10. (a) Because the diagonal of a grid square is longer than the side of a grid square.
    (b) 20 cm
11. Learners adjust their answer for question 10(b) if necessary.

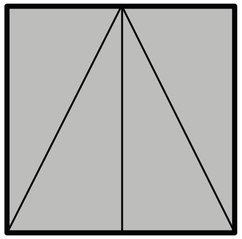
Mathematical notes on questions 12 and 13 on page 326

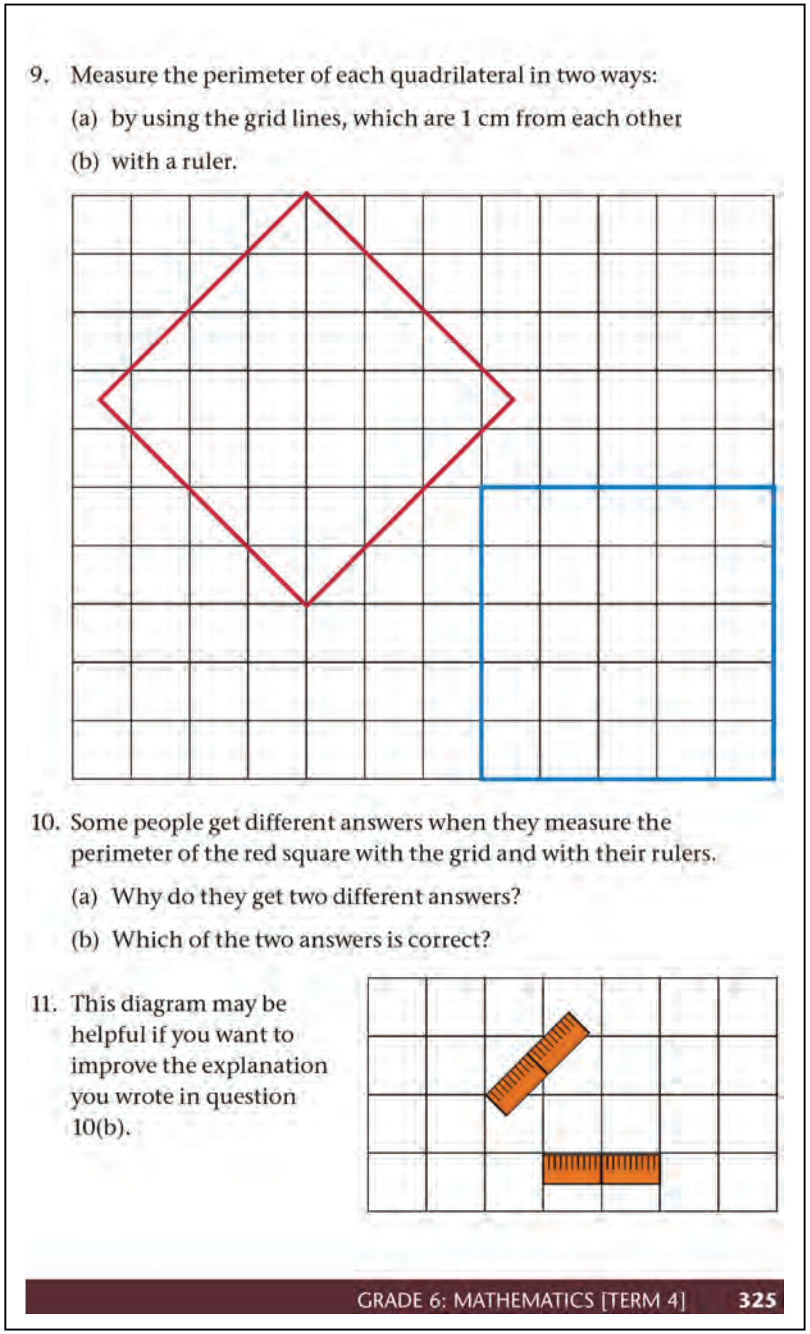
While questions 12 and 13 are primarily intended to provide practice in measuring the perimeter and area of figures, the diagram also expresses some other mathematical ideas.

The square and the parallelogram have equal areas. Below you can see that the dark triangle that forms half of the square can be translated to the right to form half of the parallelogram.



A square can be divided into four identical triangles.





9. Measure the perimeter of each quadrilateral in two ways:
   (a) by using the grid lines, which are 1 cm from each other
   (b) with a ruler.

10. Some people get different answers when they measure the perimeter of the red square with the grid and with their rulers.
    (a) Why do they get two different answers?
    (b) Which of the two answers is correct?

11. This diagram may be helpful if you want to improve the explanation you wrote in question 10(b).

Teaching guidelines

It may be necessary to point out to learners that question 12(a) refers to the square that is part of the coloured diagram on the grid.

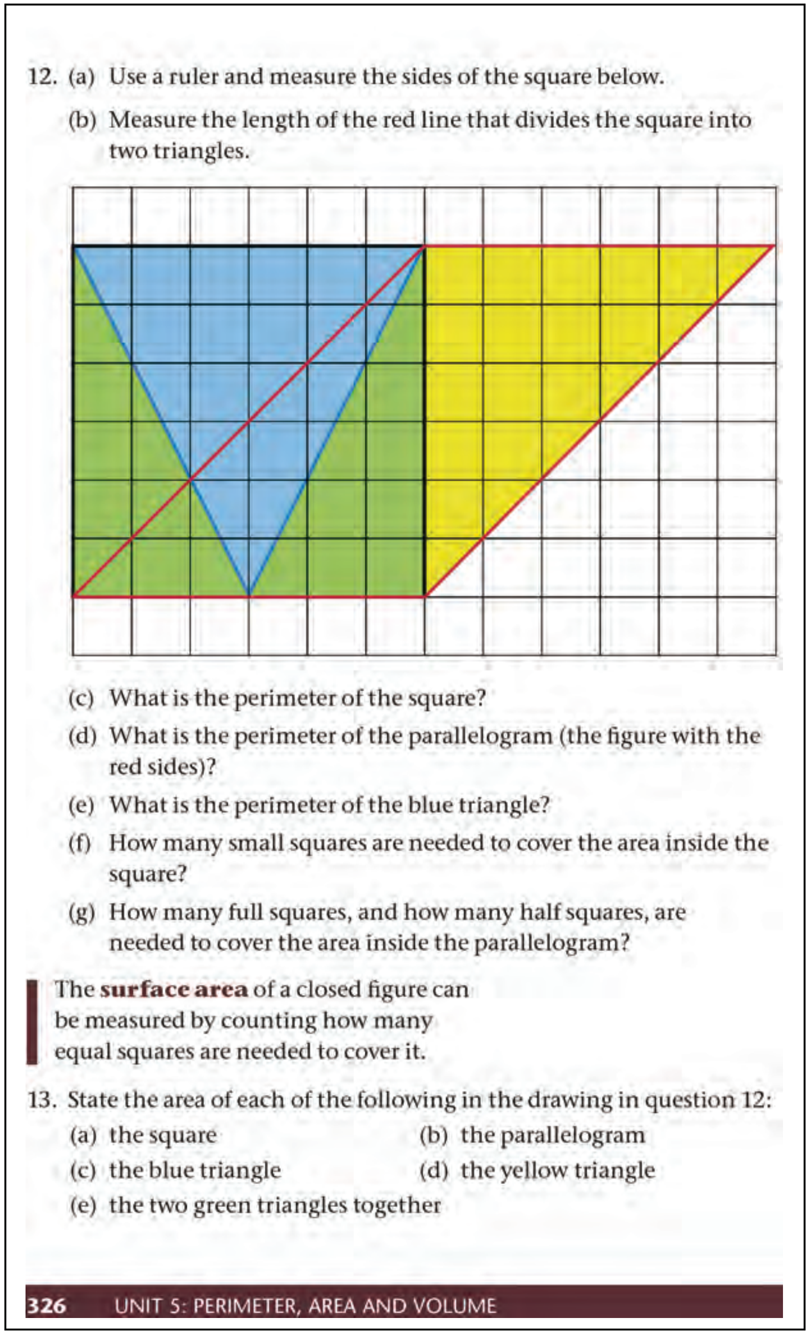
With a view to support learners to analyse the diagram and make sense of the questions, let them make their own copies of the diagram on squared or ruled paper. They need not colour their copies.

It will also be helpful to make a copy of the diagram on the board, with or without the grid, to facilitate effective interactions with learners, and class discussions.

Once learners have completed question 13, you may ask them to reflect on and discuss why the square and the parallelogram have equal areas (see "Mathematical notes" on the previous page).

Answers

12. (a) Each side is 6 cm long.
    - (b) Approximately 8,5 cm
    - (c) 6 cm + 6 cm + 6 cm + 6 cm = 24 cm
    - (d) 8,5 cm + 6 cm + 8,5 cm + 6 cm = 29 cm
    - (e) 6 cm + 6,8 cm + 6,8 cm = 20,4 cm
    - (f) 36 small squares
    - (g) 30 full squares and 12 half squares
13. (a) 36 squares
    - (b) 36 squares
    - (c) 18 squares
    - (d) 18 squares
    - (e) 18 squares



12. (a) Use a ruler and measure the sides of the square below.

(b) Measure the length of the red line that divides the square into two triangles.

(c) What is the perimeter of the square?

(d) What is the perimeter of the parallelogram (the figure with the red sides)?

(e) What is the perimeter of the blue triangle?

(f) How many small squares are needed to cover the area inside the square?

(g) How many full squares, and how many half squares, are needed to cover the area inside the parallelogram?

> The **surface area** of a closed figure can be measured by counting how many equal squares are needed to cover it.

13. State the area of each of the following in the drawing in question 12:

(a) the square

(b) the parallelogram

(c) the blue triangle

(d) the yellow triangle

(e) the two green triangles together

326 UNIT 5: PERIMETER, AREA AND VOLUME

5.2 Area and perimeter

Mathematical notes

While providing practice in measuring the area and perimeter of rectangles, this question also provides learners with an extended experience of how rectangles with the same area can have different perimeters.

Teaching guidelines

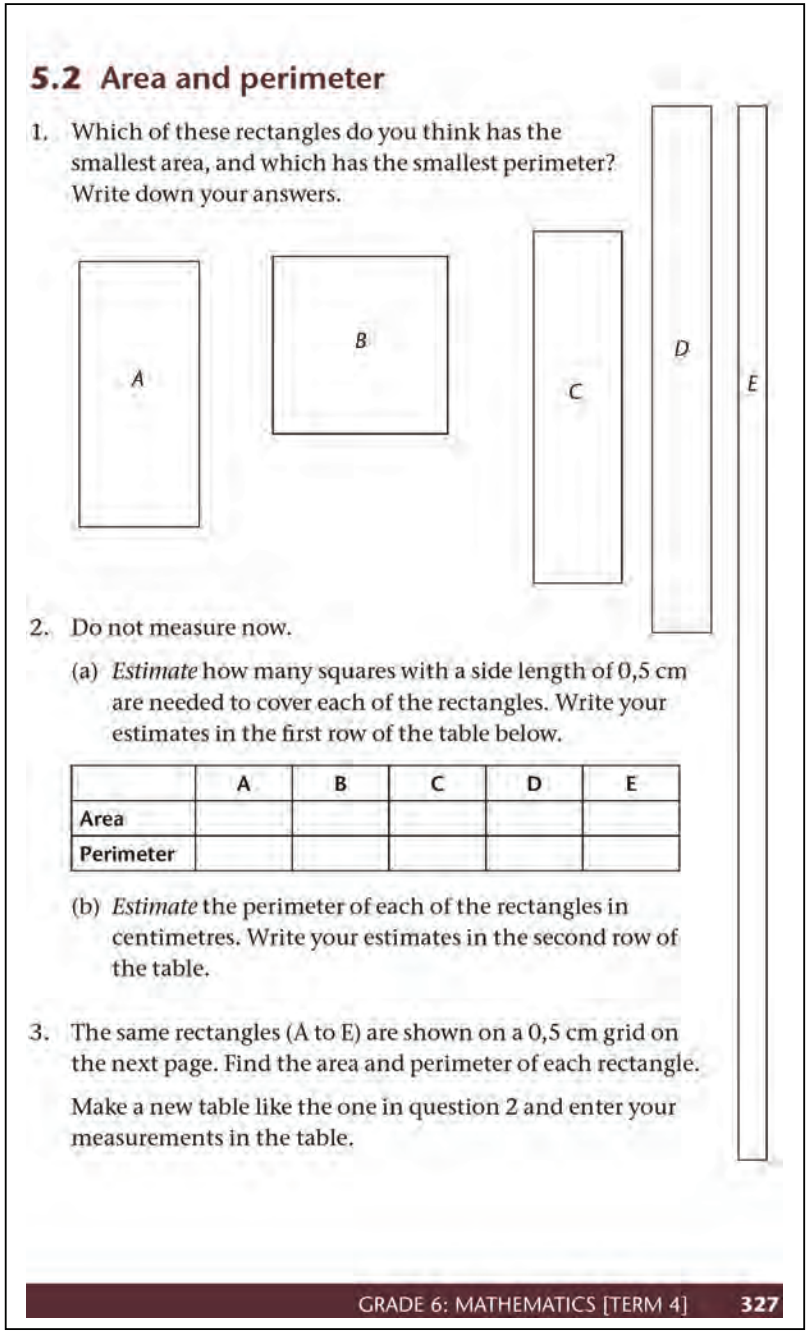
Some ideas on how you may assist learners who still confuse area and perimeter are given on the next page.

Answers

1. Learners will be able to assess their own predictions when they do question 3.
2. (a)–(b) Learners' estimates will differ.
3. See Learner Book page 328 for figures.

   Note: The area is measured in 0,5 cm *grid squares*; the perimeter in centimetres.

	A	B	C	D	E
Area	36 squares	36 squares	36 squares	36 squares	36 squares
Perimeter	13 cm	12 cm	15 cm	20 cm	37 cm



5.2 Area and perimeter

1. Which of these rectangles do you think has the smallest area, and which has the smallest perimeter? Write down your answers.

2. Do not measure now.

   (a) *Estimate* how many squares with a side length of 0,5 cm are needed to cover each of the rectangles. Write your estimates in the first row of the table below.

	A	B	C	D	E
Area					
Perimeter					

   (b) *Estimate* the perimeter of each of the rectangles in centimetres. Write your estimates in the second row of the table.

3. The same rectangles (A to E) are shown on a 0,5 cm grid on the next page. Find the area and perimeter of each rectangle.

   Make a new table like the one in question 2 and enter your measurements in the table.

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Note that figures A, B, C, D and E on Learner Book page 328 are part of question 3 on Learner Book page 327.

Answers

4. Only the following rectangles are possible with squares of 1 cm by 1 cm:

   4 cm by 6 cm with perimeter 20 cm
   2 cm by 12 cm with perimeter 28 cm
   3 cm by 8 cm with perimeter 22 cm
   1 cm by 24 cm with perimeter 50 cm

Teaching guidelines

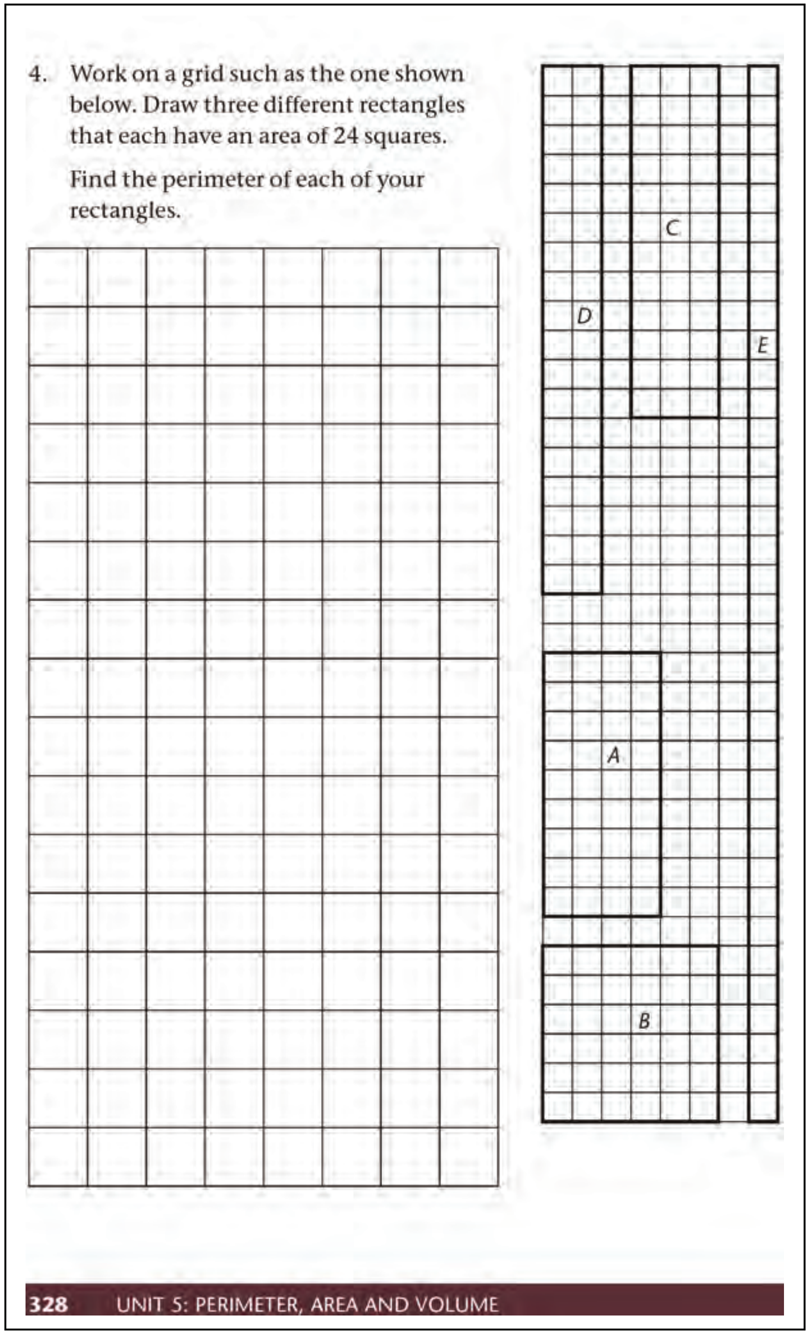
You can assist learners who still confuse perimeter and area in the following way:

If they need to find a perimeter, ask them to use one finger to trace around the outside of the shape. If they need to find the area, ask them to cover the shape with a flat hand, to remind them about what they are calculating.

If they continue to confuse perimeter and area, they will have a lot of difficulty with Mathematics later on. If you have learners who continue to mix up the two concepts, take some string or a shoelace and tie the ends to make a loop. Put the loop on the table so that it forms a shape roughly like a circle. Ask learners what the **perimeter** of the shape is. The answer is the length of that string. Ask them to show you the **area** of the shape. They should move their hands to indicate all the table space inside the string loop.

Now begin to pull the loop into a long oval shape. Ask them: *"Is the **perimeter** now different?"* (No, it is still the length of the string.) *"Is the **area** now different?"* (Yes, there is now less area inside the string loop.) Stretch the string loop some more, until it is a long narrow shape and the sides touch each other. Ask: *"Has the **perimeter** changed?"* (No, it is still the length of the string.) *"What has happened to the **area** inside the string?"* (The area has shrunk to almost nothing!) So with a string loop, you can change the area without changing the perimeter.

Also ask them what shape of that same loop will give them the largest possible area. If you leave them to play with the string, they may get the answer by themselves.



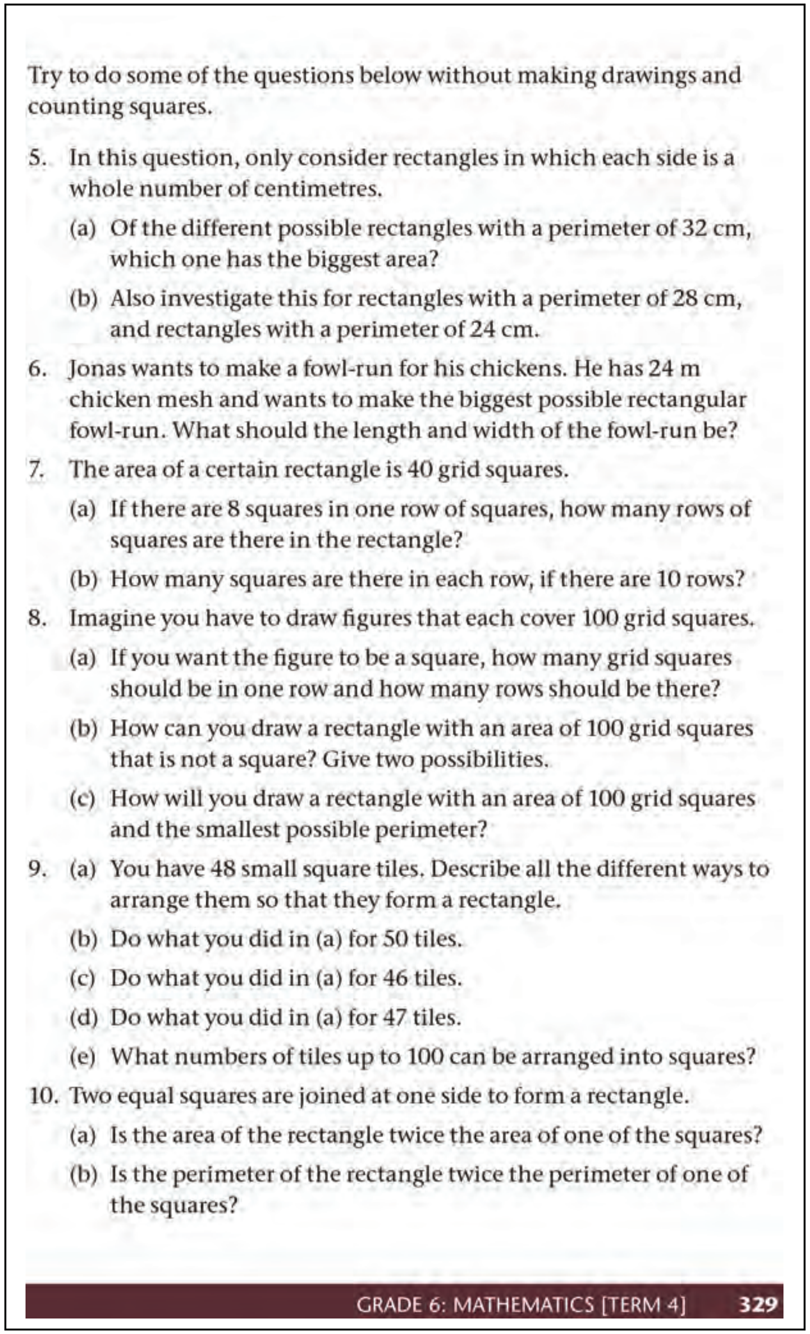
4. Work on a grid such as the one shown below. Draw three different rectangles that each have an area of 24 squares.

   Find the perimeter of each of your rectangles.

328 UNIT 5: PERIMETER, AREA AND VOLUME

Answers

5. (a) A square with side lengths 8 cm each (area: 64 cm squares).
   (b) A square with side lengths 7 cm each (area: 49 cm squares) and a square with side lengths 6 cm each (area: 36 cm squares).
6. It should be 6 m long and 6 m wide.
7. (a) 5 rows
   (b) 4 squares
8. (a) 10 squares in a row and 10 rows
   (b) 50 rows of 2 squares; 25 rows of 4 squares; 20 rows of 5 squares
   (c) The square in (a) with 10 rows of 10 squares each has a perimeter of 40.
9. (a) 48 tiles in 1 row; 24 tiles each in 2 rows; 16 tiles each in 3 rows; 12 tiles each in 4 rows; 8 tiles each in 6 rows; 6 tiles each in 8 rows; 4 tiles each in 12 rows; 3 tiles each in 16 rows; 2 tiles each in 24 rows; 1 tile in 48 rows
   (b) 50 tiles in 1 row; 25 tiles each in 2 rows; 10 tiles each in 5 rows; 5 tiles each in 10 rows; 2 tiles each in 25 rows; 1 tile in 50 rows
   (c) 46 tiles in 1 row; 23 tiles each in 2 rows; 2 tiles each in 23 rows; 1 tile in 46 rows
   (d) 47 tiles in 1 row; 1 tile in 47 rows
   (e) 1; 4; 9; 16; 25; 36; 49; 64; 81; 100
10. (a) Yes
    (b) No



Try to do some of the questions below without making drawings and counting squares.

5. In this question, only consider rectangles in which each side is a whole number of centimetres.
   (a) Of the different possible rectangles with a perimeter of 32 cm, which one has the biggest area?
   (b) Also investigate this for rectangles with a perimeter of 28 cm, and rectangles with a perimeter of 24 cm.
6. Jonas wants to make a fowl-run for his chickens. He has 24 m chicken mesh and wants to make the biggest possible rectangular fowl-run. What should the length and width of the fowl-run be?
7. The area of a certain rectangle is 40 grid squares.
   (a) If there are 8 squares in one row of squares, how many rows of squares are there in the rectangle?
   (b) How many squares are there in each row, if there are 10 rows?
8. Imagine you have to draw figures that each cover 100 grid squares.
   (a) If you want the figure to be a square, how many grid squares should be in one row and how many rows should be there?
   (b) How can you draw a rectangle with an area of 100 grid squares that is not a square? Give two possibilities.
   (c) How will you draw a rectangle with an area of 100 grid squares and the smallest possible perimeter?
9. (a) You have 48 small square tiles. Describe all the different ways to arrange them so that they form a rectangle.
   (b) Do what you did in (a) for 50 tiles.
   (c) Do what you did in (a) for 46 tiles.
   (d) Do what you did in (a) for 47 tiles.
   (e) What numbers of tiles up to 100 can be arranged into squares?
10. Two equal squares are joined at one side to form a rectangle.
    (a) Is the area of the rectangle twice the area of one of the squares?
    (b) Is the perimeter of the rectangle twice the perimeter of one of the squares?

5.3 Volume and capacity

Mathematical notes

The volume of an object can be understood as the number of small, identical cubes that can be stacked tightly together to form the object.

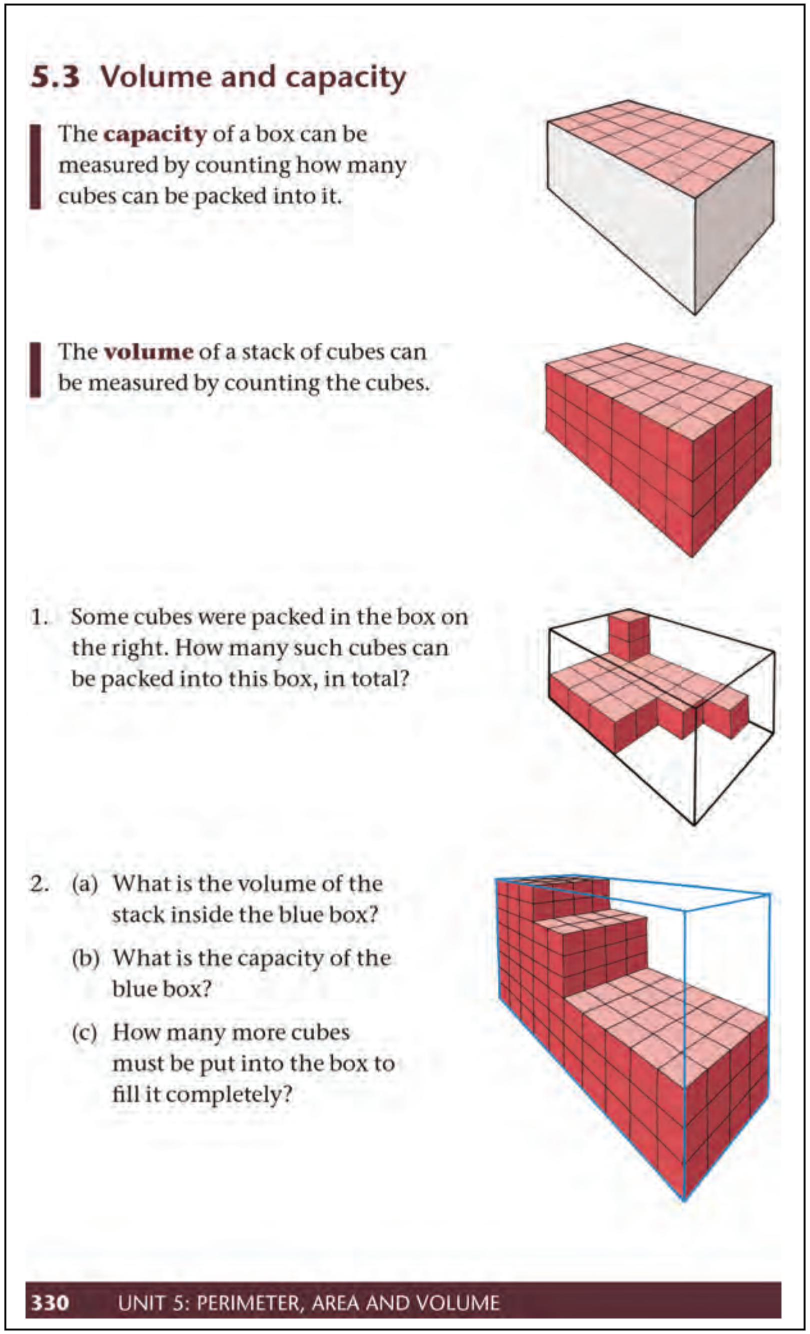
Questions 1 and 2 invite learners to imagine that they are working with actual cubes, packing them into a box. While working physically with actual boxes and cubes will make it easier for learners to find the answers, the experience of just using their imagination will strengthen their concept of volume and capacity.

Notes on questions

Learners may do question 2(c) before 2(b).

Answers

1. 72 cubes (3 layers; 24 cubes in each layer)
2. (a) 96 + 48 + 60 = 204 cubes
   (b) 320 cubes
   (c) 116 cubes



5.3 Volume and capacity

The **capacity** of a box can be measured by counting how many cubes can be packed into it.

The **volume** of a stack of cubes can be measured by counting the cubes.

1. Some cubes were packed in the box on the right. How many such cubes can be packed into this box, in total?

2. (a) What is the volume of the stack inside the blue box?
   (b) What is the capacity of the blue box?
   (c) How many more cubes must be put into the box to fill it completely?

330 UNIT 5: PERIMETER, AREA AND VOLUME

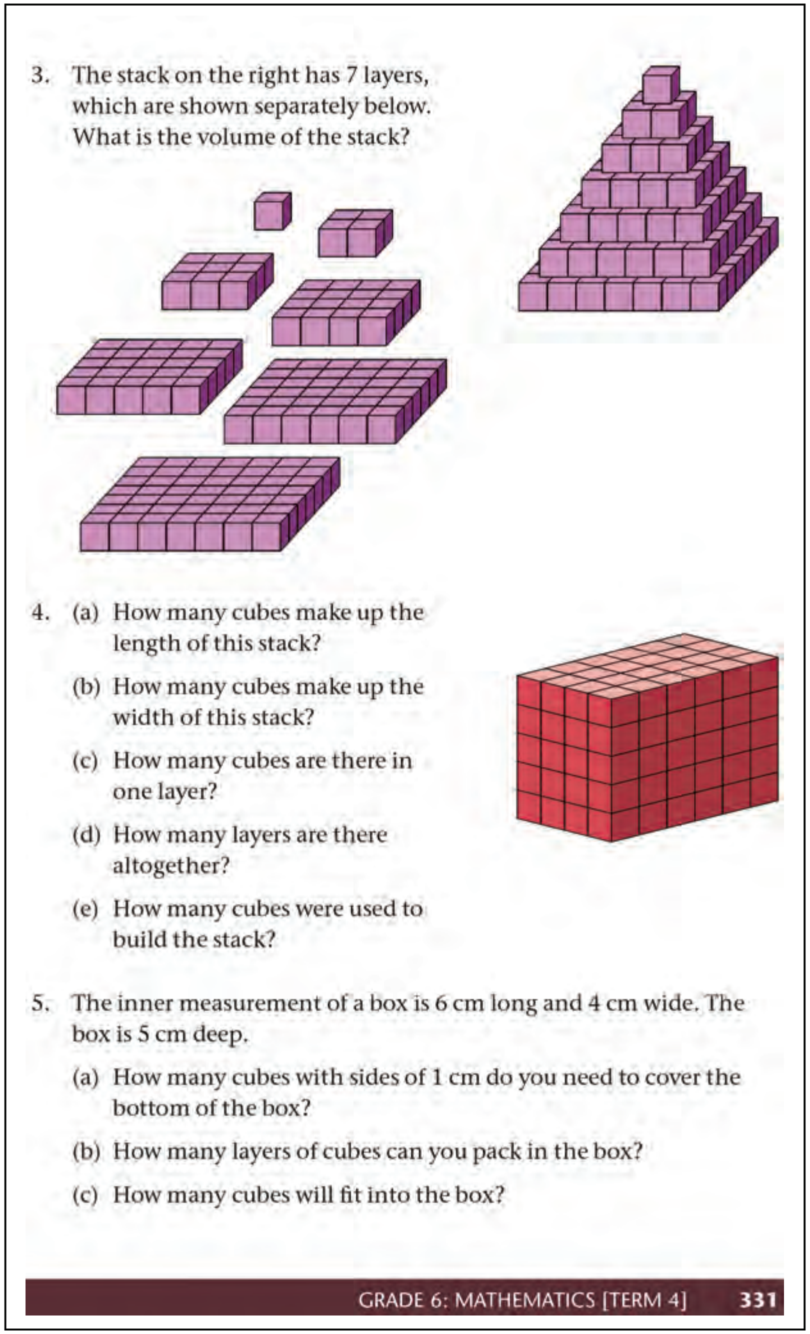
Teaching guidelines

Don’t use a formula for the volume/capacity of a cube when learners are working with questions 4 and 5. The idea of formulas for perimeter and area of rectangles and volume of rectangular prisms should be delayed until learners have developed strong concepts of perimeter, area and volume/capacity with respect to a variety of figures and shapes.

If the idea of formulas is introduced too early, the misconception that only rectangles have perimeter and area and only rectangular prisms have volume/capacity may be encouraged.

Answers

3. 49 + 36 + 25 + 16 + 9 + 4 + 1 = 140 cubes
4. (a) 6 (b) 4 (c) 24 (d) 5 (e) 120
5. (a) 24 (b) 5 (c) 120



3. The stack on the right has 7 layers, which are shown separately below. What is the volume of the stack?

4. (a) How many cubes make up the length of this stack?
   (b) How many cubes make up the width of this stack?
   (c) How many cubes are there in one layer?
   (d) How many layers are there altogether?
   (e) How many cubes were used to build the stack?

5. The inner measurement of a box is 6 cm long and 4 cm wide. The box is 5 cm deep.
   (a) How many cubes with sides of 1 cm do you need to cover the bottom of the box?
   (b) How many layers of cubes can you pack in the box?
   (c) How many cubes will fit into the box?

Grade 6 Term 4 Unit 6 The history of measurement

Learner Book Overview		
This unit has just one section	**Content**	**Pages in Learner Book**
The history of measurement	Getting to know some ways in which people measured and recorded measurements in the past Using own body parts (hands, feet, forearms) to measure the classroom and objects in the classroom Realising the benefits of standardised units of measurement	332 to 335

CAPS time allocation	1 hour
CAPS page references	29 and 283

Mathematical background

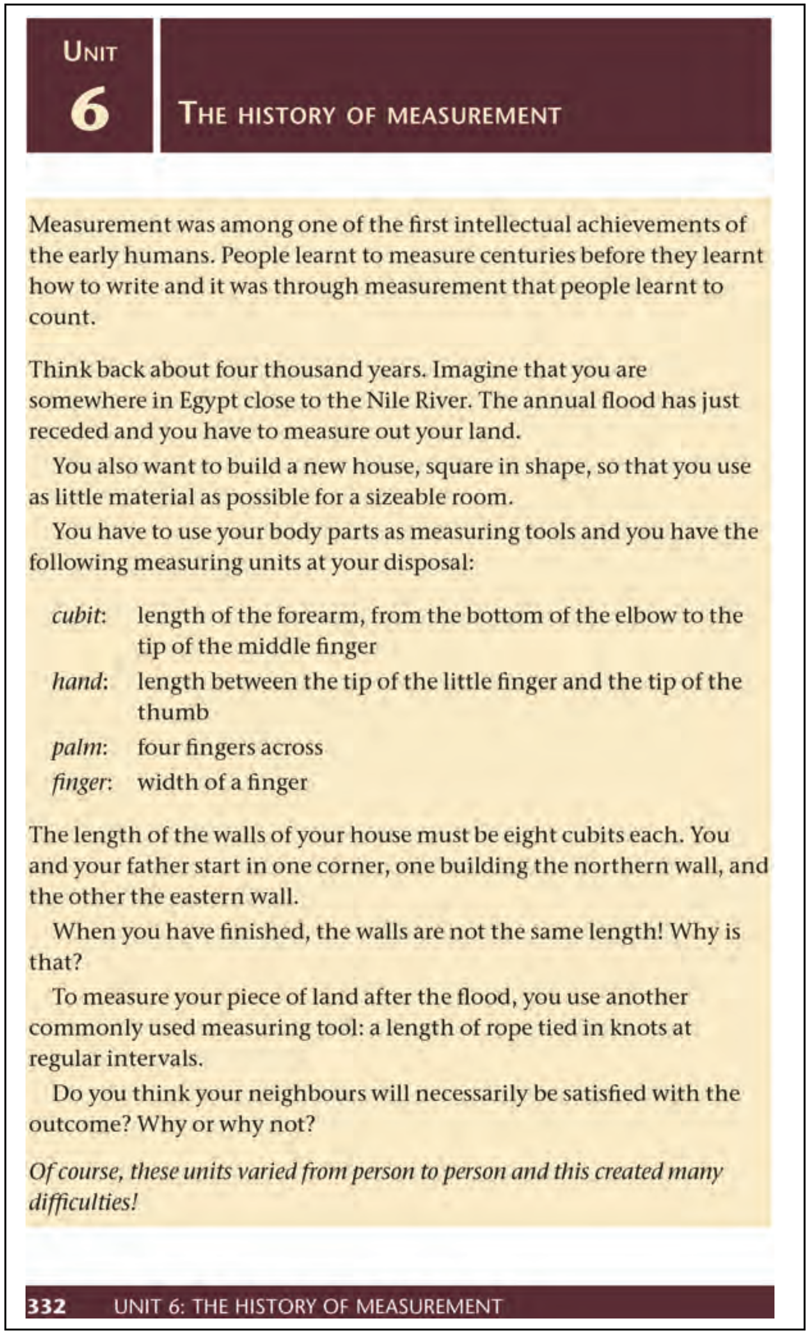
A key element of the history of measurement is the gradual development of standardisation, from a situation where many different units and systems of measurement were used in different parts of the world to the modern situation where the metric system is used almost universally.

In this short unit learners get to know some of the ways in which the ancient Egyptians measured and recorded measurements. Encourage them to do a bit of research into how their ancestors and/or people of other cultures used to measure and record measurements in the recent and/or distant past.

Teaching guidelines

The topic requires you to do some “storytelling”. You may study the shaded passage and prepare to present this to the class in an interesting way that will hold their attention.

The history may be enlivened by asking learners to perform actual measurements with their fingers, hands, forearms and feet. For example, you may ask all learners to measure the width of their desktop with their palms (four fingers across) and to compare answers. The different answers for the same distance create a situation where you can impress upon learners that there is a problem: different people state different lengths for the same distance. This practical problem can be used as a context for introducing the idea of **standardisation of units**.



UNIT
6

THE HISTORY OF MEASUREMENT

Measurement was among one of the first intellectual achievements of the early humans. People learnt to measure centuries before they learnt how to write and it was through measurement that people learnt to count.

Think back about four thousand years. Imagine that you are somewhere in Egypt close to the Nile River. The annual flood has just receded and you have to measure out your land.

You also want to build a new house, square in shape, so that you use as little material as possible for a sizeable room.

You have to use your body parts as measuring tools and you have the following measuring units at your disposal:

cubit: length of the forearm, from the bottom of the elbow to the tip of the middle finger
hand: length between the tip of the little finger and the tip of the thumb
palm: four fingers across
finger: width of a finger

The length of the walls of your house must be eight cubits each. You and your father start in one corner, one building the northern wall, and the other the eastern wall.

When you have finished, the walls are not the same length! Why is that?

To measure your piece of land after the flood, you use another commonly used measuring tool: a length of rope tied in knots at regular intervals.

Do you think your neighbours will necessarily be satisfied with the outcome? Why or why not?

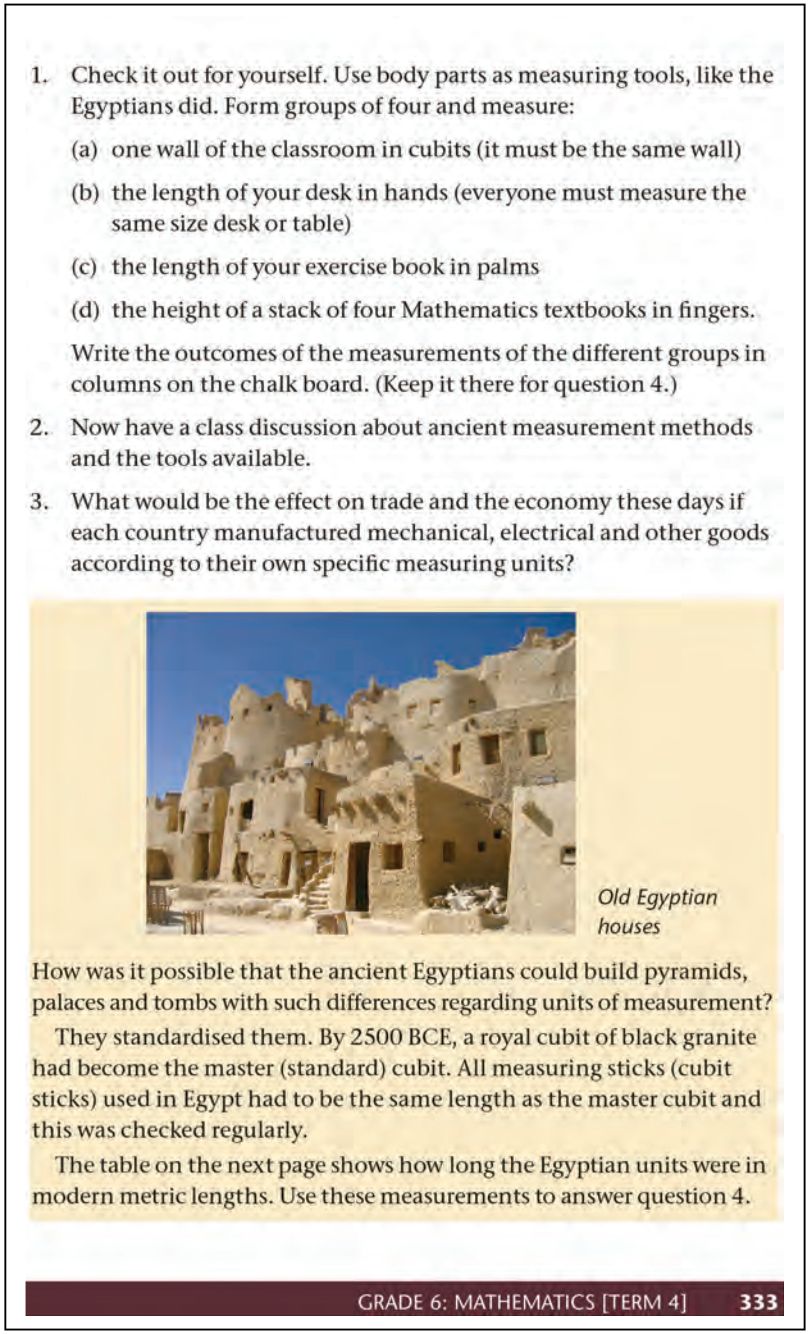
Of course, these units varied from person to person and this created many difficulties!

332 UNIT 6: THE HISTORY OF MEASUREMENT

Answers

1. Practical group activity
2. Class discussion
3. Class discussion, for example: If every country used its own measuring units, every other country would need tables with conversion factors. People in South Africa who ordered goods from other countries could find that, for example, parts do not fit into local machines, or they could find that the supplier had misunderstood the units we use in South Africa.

   In fact, there are two different measuring systems in use: America uses the old British system called the Imperial system (foot, pound, second) and most other countries use the metric system (metre, kilogram, second). Sometimes there is confusion and people make mistakes.



1. Check it out for yourself. Use body parts as measuring tools, like the Egyptians did. Form groups of four and measure:
   (a) one wall of the classroom in cubits (it must be the same wall)
   (b) the length of your desk in hands (everyone must measure the same size desk or table)
   (c) the length of your exercise book in palms
   (d) the height of a stack of four Mathematics textbooks in fingers.

   Write the outcomes of the measurements of the different groups in columns on the chalk board. (Keep it there for question 4.)
2. Now have a class discussion about ancient measurement methods and the tools available.
3. What would be the effect on trade and the economy these days if each country manufactured mechanical, electrical and other goods according to their own specific measuring units?

Old Egyptian houses

How was it possible that the ancient Egyptians could build pyramids, palaces and tombs with such differences regarding units of measurement?

They standardised them. By 2500 BCE, a royal cubit of black granite had become the master (standard) cubit. All measuring sticks (cubit sticks) used in Egypt had to be the same length as the master cubit and this was checked regularly.

The table on the next page shows how long the Egyptian units were in modern metric lengths. Use these measurements to answer question 4.

Mathematical notes

The traditional British system (Imperial system) of measurement was used in South Africa till around 1960. In this system the following units were used for measuring length:

The **inch**: this red strip is exactly one inch long.

1 inch = 2,54 cm

For shorter lengths, fraction parts of an inch are used in the British system:

Quarter inches

Sixteenths of an inch

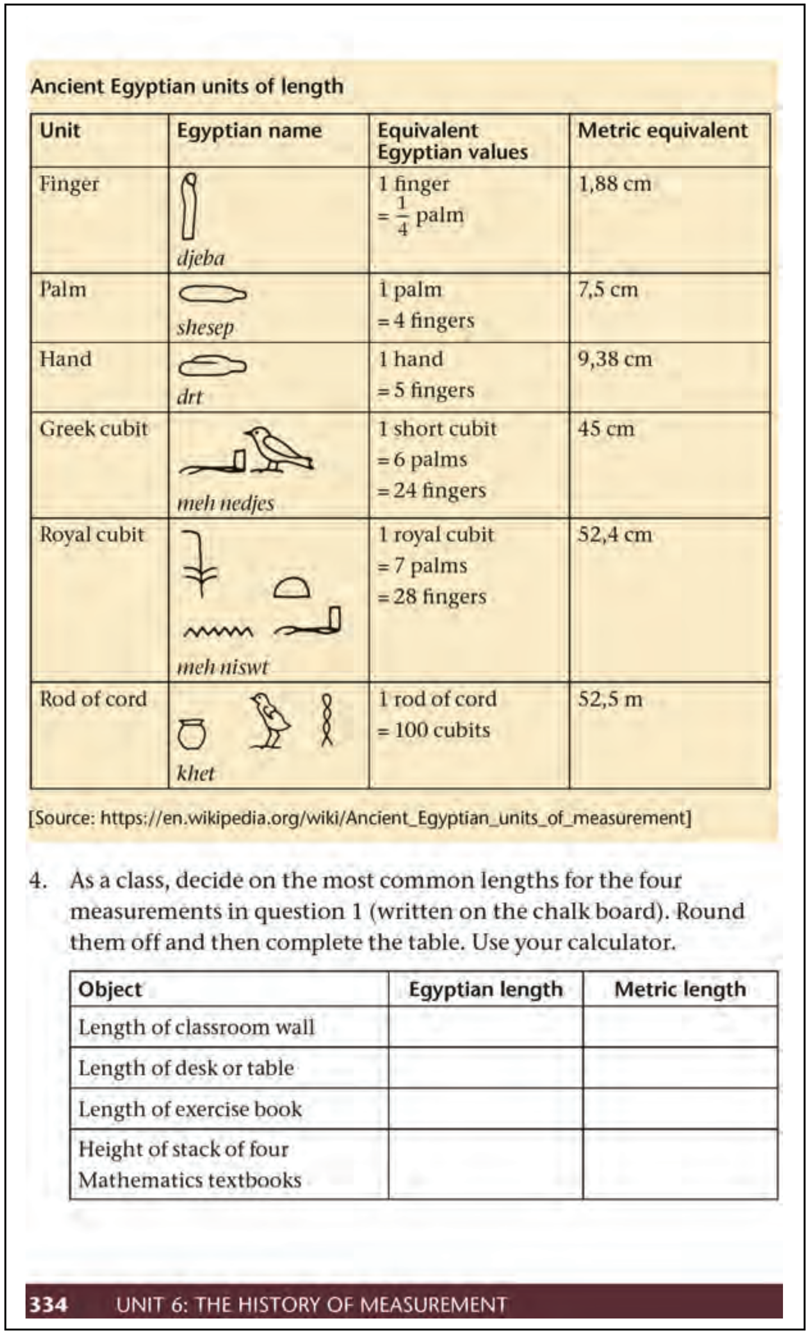
Thirty-secondths and sixty-fourths of an inch were used for more accurate measurements.

Larger units for measuring lengths in the British system are the:

- foot: 12 inches
- yard: 3 feet
- mile: 1 760 yards

Answers

4. Learners' answers will differ.



Ancient Egyptian units of length

Unit	Egyptian name	Equivalent Egyptian values	Metric equivalent
Finger	*djeba*	1 finger = $\frac{1}{4}$ palm	1,88 cm
Palm	*shesep*	1 palm = 4 fingers	7,5 cm
Hand	*drt*	1 hand = 5 fingers	9,38 cm
Greek cubit	*meh nedjes*	1 short cubit = 6 palms = 24 fingers	45 cm
Royal cubit	*meh niswt*	1 royal cubit = 7 palms = 28 fingers	52,4 cm
Rod of cord	*khet*	1 rod of cord = 100 cubits	52,5 m

[Source: https://en.wikipedia.org/wiki/Ancient_Egyptian_units_of_measurement]

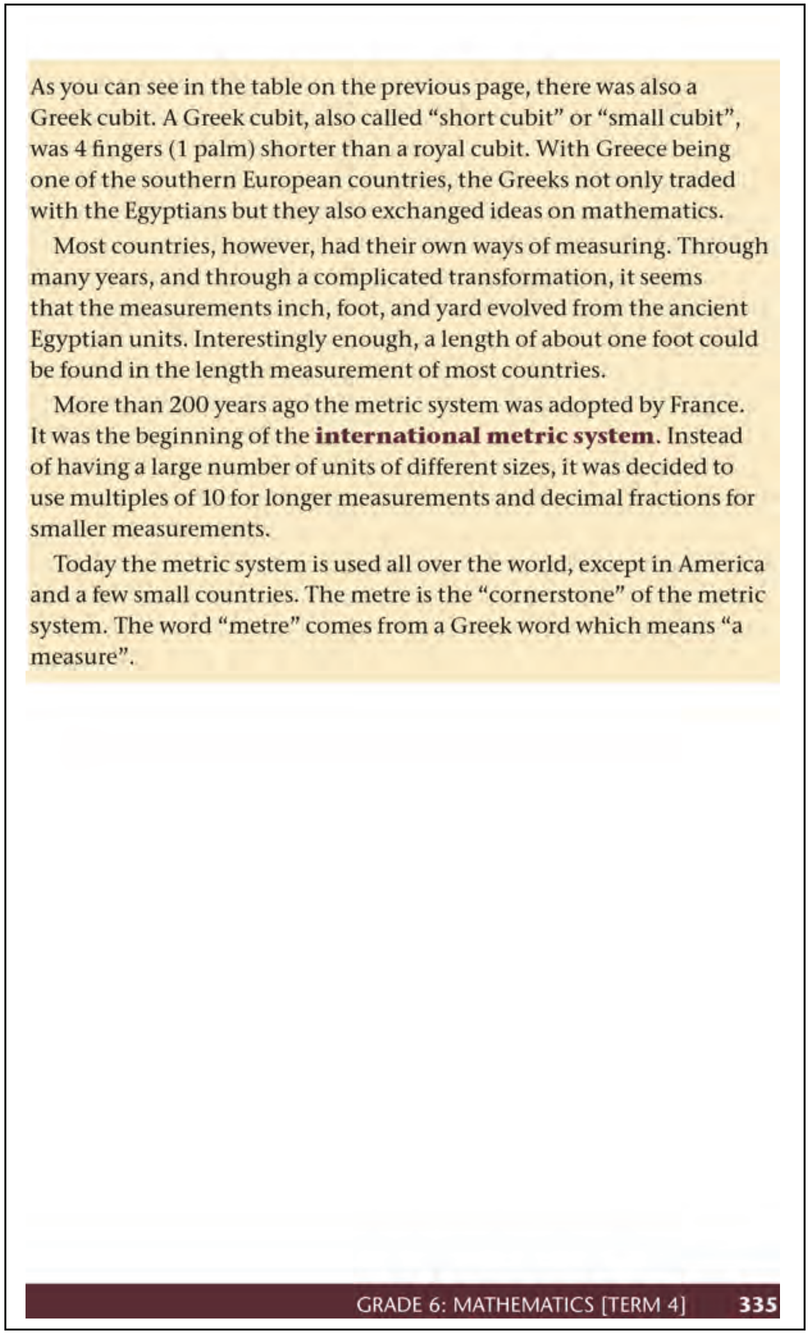
4. As a class, decide on the most common lengths for the four measurements in question 1 (written on the chalk board). Round them off and then complete the table. Use your calculator.

Object	Egyptian length	Metric length
Length of classroom wall		
Length of desk or table		
Length of exercise book		
Height of stack of four Mathematics textbooks		

Mathematical notes

The forearm (cubit) was used in many parts of the world as a unit for measuring length. Here are the words for cubit in some other languages:

- Zulu: yingalo
- Sotho: setsoe
- Shona: mita
- Swahili: dhiraa moja
- Chichewa: mkono
- Malay: hasta



As you can see in the table on the previous page, there was also a Greek cubit. A Greek cubit, also called "short cubit" or "small cubit", was 4 fingers (1 palm) shorter than a royal cubit. With Greece being one of the southern European countries, the Greeks not only traded with the Egyptians but they also exchanged ideas on mathematics.

Most countries, however, had their own ways of measuring. Through many years, and through a complicated transformation, it seems that the measurements inch, foot, and yard evolved from the ancient Egyptian units. Interestingly enough, a length of about one foot could be found in the length measurement of most countries.

More than 200 years ago the metric system was adopted by France. It was the beginning of the **international metric system**. Instead of having a large number of units of different sizes, it was decided to use multiples of 10 for longer measurements and decimal fractions for smaller measurements.

Today the metric system is used all over the world, except in America and a few small countries. The metre is the "cornerstone" of the metric system. The word "metre" comes from a Greek word which means "a measure".

Grade 6 Term 4 Unit 7 Whole numbers: Division

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
7.1 Explore division with bigger numbers	Solving sharing and grouping problems by estimation and multiplication	336
7.2 Two methods of division	Revision of division methods, and practice	337 to 338
7.3 Apply and use your knowledge	Word problems involving sharing and grouping, and multi-step problems	338 to 339
7.4 Ratio	Division in calculations involving ratio in a variety of contexts	339 to 341
7.5 More practice	A variety of problems requiring division and multiplication	341 to 342
7.6 Rate	A project that involves a constant rate in a mechanical context	343

CAPS time allocation	7 hours
CAPS page references	13 to 15 and 284 to 285

Mathematical background

Division is the inverse of multiplication.

- To ask *"How much is 12 000 ÷ 60?"* is the same as to ask *"By what number must 60 be multiplied to get 12 000"*, or *"60 × ? = 12 000"*.
- It does not matter how we keep track of progress while we divide or how we document our work when we divide – division with larger numbers is usually performed by adding up multiples of the divisor until the question *"By what number must the divisor be multiplied to get the total?"* is answered.

Division is used to solve four kinds of problems:

- Calculating the size of each part if a given quantity is divided into a given number of equal parts ("sharing").
- Calculating the number of parts if a given quantity is divided into equal parts of a given size ("grouping").
- Calculating one quantity if two quantities are in a given ratio and one of the quantities is given.
- Calculating a rate, or dividing by a rate.

7.1 Explore division with bigger numbers

Notes on questions

Questions 1 and 2 encourage understanding of division as the inverse of multiplication, and the use of multiplication facts to perform division. Questions 3 to 5 focus on estimating and checking as a division strategy.

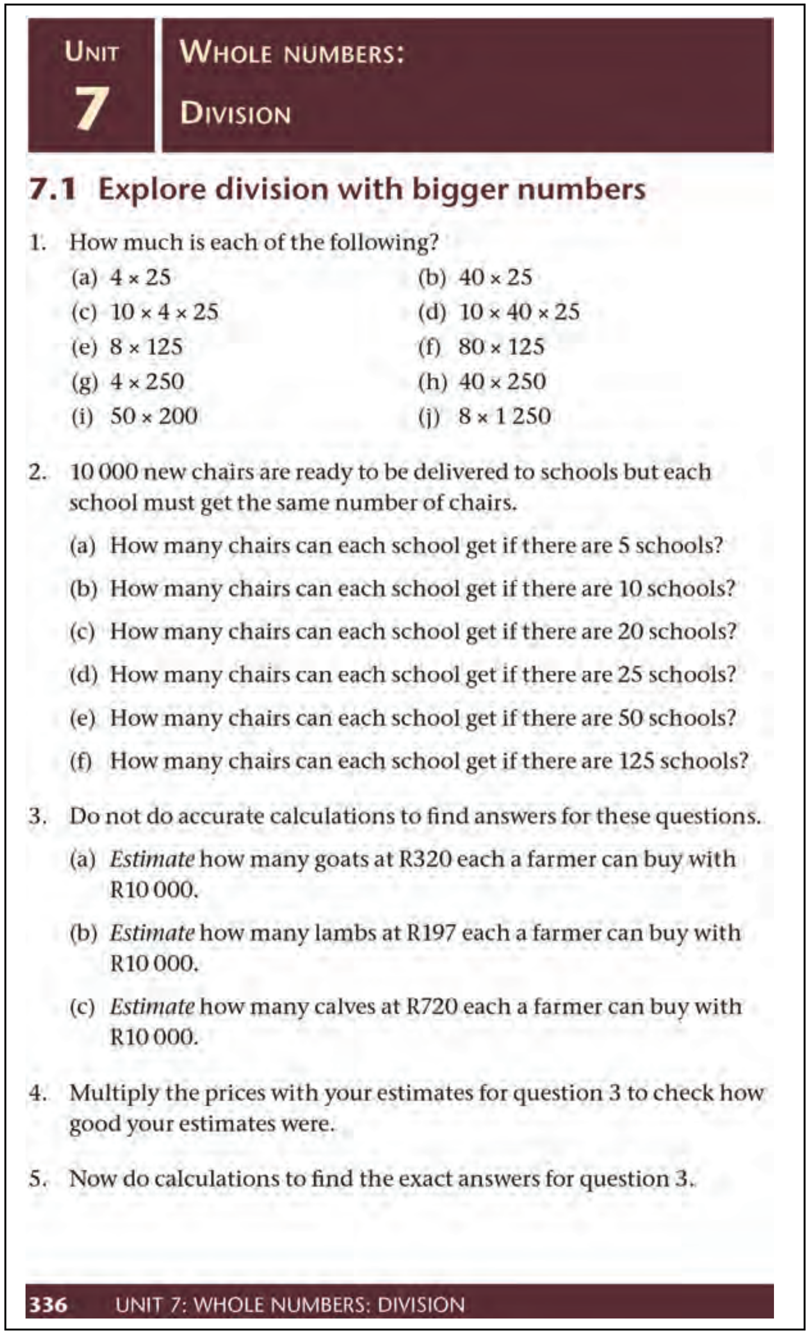
Teaching guidelines

Let learners do question 1, then take feedback and make sure all learners have the correct answers written in their books.

Let learners then do question 2. Tell them they may find some of the answers in question 1 useful when they do question 2. You may use this example: If there are 8 schools, then the answer for question 1(j) indicates that each school will receive 1 250 chairs.

Answers

1. (a) 100 (b) 1 000 (c) 1 000 (d) 10 000
   (e) 1 000 (f) 10 000 (g) 1 000 (h) 10 000
   (i) 10 000 (j) 10 000
2. (a) 2 000 (b) 1 000 (c) 500
   (d) 400 (e) 200 (f) 80
3. Learners' estimates will differ, e.g.
   (a) 30 (b) 50 (c) 10
4. Learners' estimates will differ, e.g.
   (a) 320 × 30 = 9 600
   (b) 197 × 50 = 9 850
   (c) 720 × 10 = 7 200
5. (a) 31 (b) 50 (c) 13



UNIT 7 WHOLE NUMBERS: DIVISION

7.1 Explore division with bigger numbers

1. How much is each of the following?
   (a) 4 × 25
   (b) 40 × 25
   (c) 10 × 4 × 25
   (d) 10 × 40 × 25
   (e) 8 × 125
   (f) 80 × 125
   (g) 4 × 250
   (h) 40 × 250
   (i) 50 × 200
   (j) 8 × 1 250

2. 10 000 new chairs are ready to be delivered to schools but each school must get the same number of chairs.
   (a) How many chairs can each school get if there are 5 schools?
   (b) How many chairs can each school get if there are 10 schools?
   (c) How many chairs can each school get if there are 20 schools?
   (d) How many chairs can each school get if there are 25 schools?
   (e) How many chairs can each school get if there are 50 schools?
   (f) How many chairs can each school get if there are 125 schools?

3. Do not do accurate calculations to find answers for these questions.
   (a) *Estimate* how many goats at R320 each a farmer can buy with R10 000.
   (b) *Estimate* how many lambs at R197 each a farmer can buy with R10 000.
   (c) *Estimate* how many calves at R720 each a farmer can buy with R10 000.

4. Multiply the prices with your estimates for question 3 to check how good your estimates were.

5. Now do calculations to find the exact answers for question 3.

7.2 Two methods of division

Teaching guidelines

Let learners calculate 2 784 ÷ 47 any way they prefer before informing them of the content in the two shaded passages.

Then explain, with reference to 7 283 ÷ 183, how division can be done by building the given total up with multiples of the divisor. The two shaded passages are reminders of two ways in which one can keep track of the building-up process:

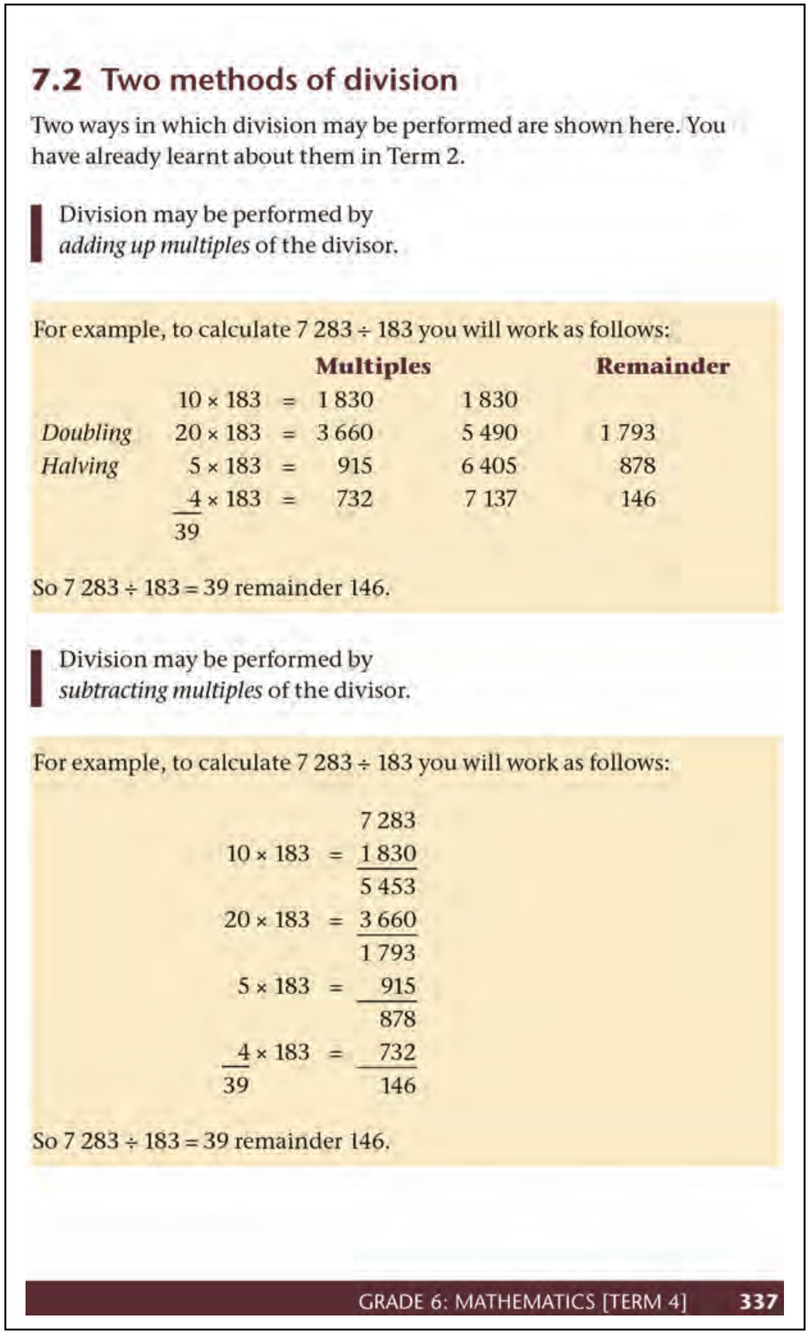
- Adding up the multiples (10 × 183, 20 × 183, etc.) as shown in the second column from the right in the first shaded passage to get an idea of how much should still be added. (The "remainders" in the first shaded passage need not be calculated.)

1 830
5 490
6 405
7 137
↓
7 283

- Subtracting the multiples one by one from the given total as shown in the second shaded passage.

Ask learners to explain to each other in small groups how they have kept track of their work when they calculated 2 784 ÷ 47. Ask them to precisely show whether they added up multiples of 47 as in the first shaded passage, whether they subtracted multiples of 47 as in the second shaded passage, or used a different method.

The purpose of this activity is not to impose any specific method of division on learners, but to provide each learner with opportunities to clarify his/her own way of doing division in his/her own mind.



7.2 Two methods of division

Two ways in which division may be performed are shown here. You have already learnt about them in Term 2.

Division may be performed by *adding up multiples* of the divisor.

For example, to calculate 7 283 ÷ 183 you will work as follows:

	Multiples				**Remainder**
	10 × 183	=	1 830	1 830	
Doubling	20 × 183	=	3 660	5 490	1 793
Halving	5 × 183	=	915	6 405	878
	4 × 183	=	732	7 137	146
	39				

So 7 283 ÷ 183 = 39 remainder 146.

Division may be performed by *subtracting multiples* of the divisor.

For example, to calculate 7 283 ÷ 183 you will work as follows:

		7 283
10 × 183	=	1 830
		5 453
20 × 183	=	3 660
		1 793
5 × 183	=	915
		878
4 × 183	=	732
39		146

So 7 283 ÷ 183 = 39 remainder 146.

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Answers

1. (a) 34 remainder 18 (b) 29
2. (a) 34 (b) 246 remainder 16

7.3 Apply and use your knowledge

Teaching guidelines

Remind learners regularly that they need to read the questions carefully, and try to understand the situation described in each question. Advise them to make a rough estimate of the answer in each case, and use a calculator to check how good their estimate is. If they cannot make and check an estimate, then they probably do not understand the situation.

Notes on questions

Question 1 requires grouping: learners have to determine how many parts of 237 each can be obtained from a total of 2 844.

Question 2 requires sharing: 6 104 is to be subdivided into 872 equal portions; learners have to calculate the size of each portion.

Questions 3, 4 and 5 all require grouping: in each case the number of equal parts have to be worked out.

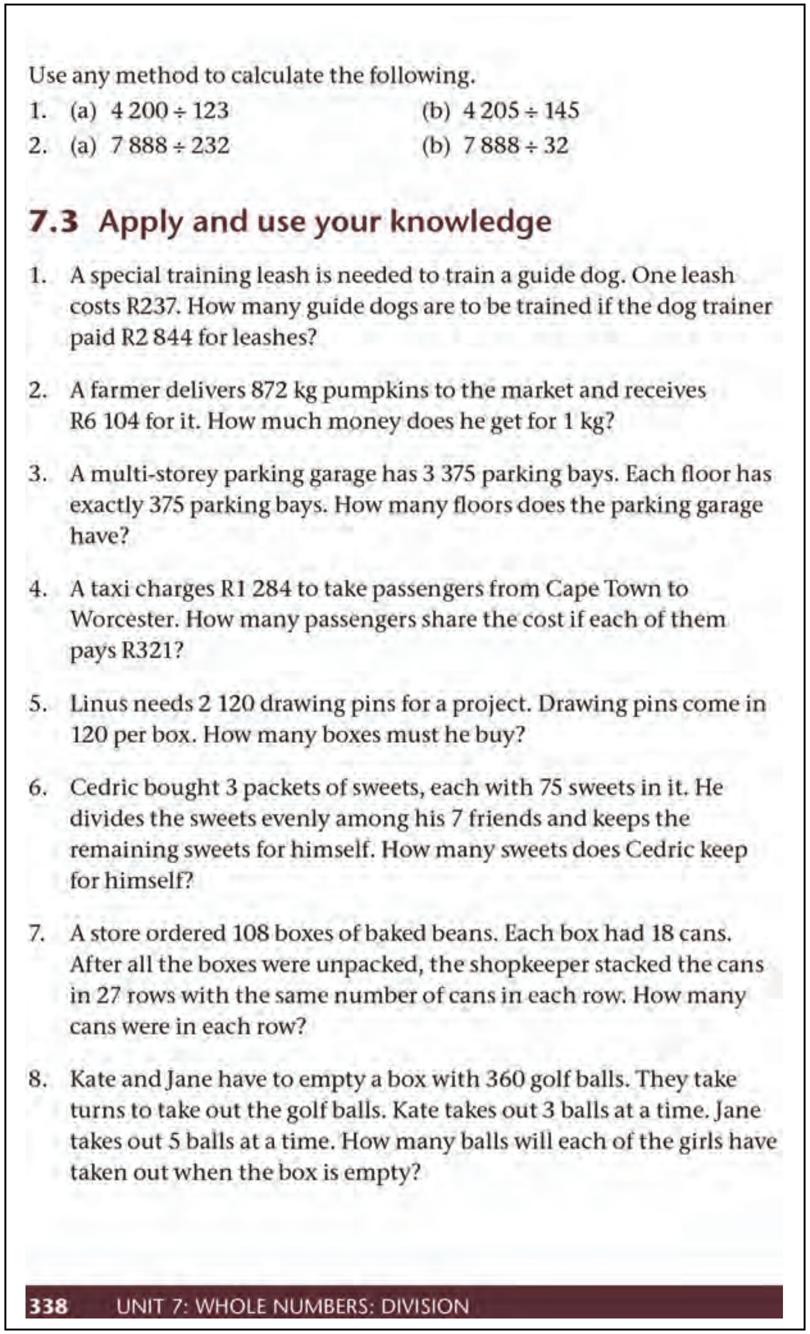
Question 6 is a two-step problem: multiplication followed by division as sharing.

Question 7 is also a two-step problem: multiplication followed by division as grouping.

Question 8 is meant as a challenge.

Answers

1. 12 dogs
2. R7 per kg
3. 9 floors
4. 4 passengers
5. 18 boxes
6. 3 × 75 ÷ 7 = 225 ÷ 7 = 32 remainder 1; Cedric keeps only 1 sweet for himself.
7. 108 × 18 ÷ 27 = 72 cans in each row
8. Number of golf balls taken out each time = 5 + 3 = 8
   Number of times golf balls are taken out = 360 ÷ 8 = 45
   Kate will have taken out 3 × 45 = 135 balls and
   Jane will have taken out 5 × 45 = 225 balls when the box is empty.



Use any method to calculate the following.

1. (a) 4 200 ÷ 123 (b) 4 205 ÷ 145
2. (a) 7 888 ÷ 232 (b) 7 888 ÷ 32

7.3 Apply and use your knowledge

1. A special training leash is needed to train a guide dog. One leash costs R237. How many guide dogs are to be trained if the dog trainer paid R2 844 for leashes?
2. A farmer delivers 872 kg pumpkins to the market and receives R6 104 for it. How much money does he get for 1 kg?
3. A multi-storey parking garage has 3 375 parking bays. Each floor has exactly 375 parking bays. How many floors does the parking garage have?
4. A taxi charges R1 284 to take passengers from Cape Town to Worcester. How many passengers share the cost if each of them pays R321?
5. Linus needs 2 120 drawing pins for a project. Drawing pins come in 120 per box. How many boxes must he buy?
6. Cedric bought 3 packets of sweets, each with 75 sweets in it. He divides the sweets evenly among his 7 friends and keeps the remaining sweets for himself. How many sweets does Cedric keep for himself?
7. A store ordered 108 boxes of baked beans. Each box had 18 cans. After all the boxes were unpacked, the shopkeeper stacked the cans in 27 rows with the same number of cans in each row. How many cans were in each row?
8. Kate and Jane have to empty a box with 360 golf balls. They take turns to take out the golf balls. Kate takes out 3 balls at a time. Jane takes out 5 balls at a time. How many balls will each of the girls have taken out when the box is empty?

Notes on questions

Question 9 is a two-step problem: subtraction followed by division as equal sharing. Question 10 is a two-step problem: division as grouping followed by multiplication. Questions 11 and 12 are examples of grouping.

Answers

9. (a) (3 485 – 21) ÷ 130 = 26 remainder 84. He can put 26 oranges in a pocket.
   (b) 84 oranges are left over.
10. 1 089 ÷ 99 = 11, then 11 × 3 = 33 CDs
11. 2 years and 4 months
12. 68 towels

7.4 Ratio

Teaching guidelines

The story in italics and question 1 provide learners with an opportunity to use their knowledge of equal sharing to develop an idea of ratio. There is no need to introduce the word ratio at this stage – it is important that the concept of ratio first develops in learners' minds. Questions 2 to 5 will provide other opportunities for this.

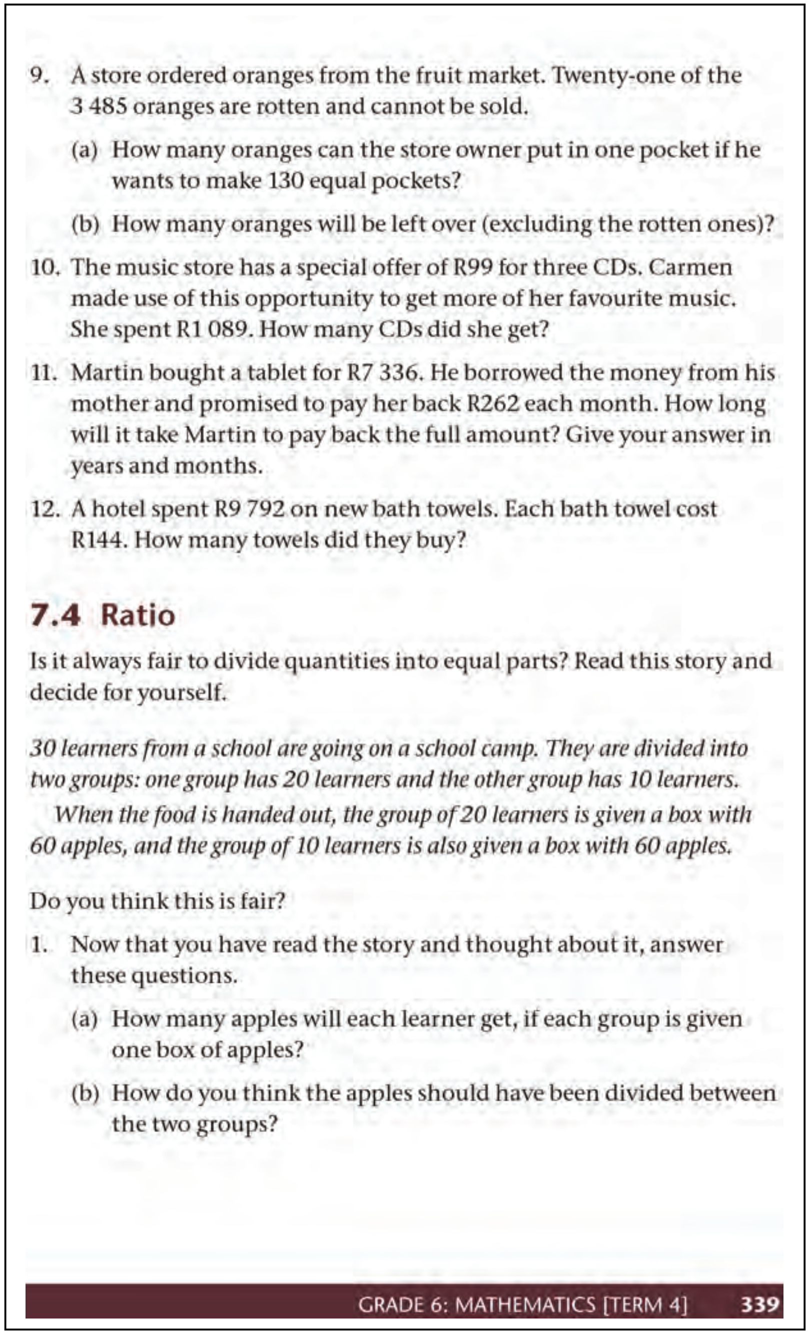
Read the story with the class and allow some discussion. You could ask learners to decide whether they would want to be in the group of 20 learners or in the group of 10 learners. You may write the following on the board to provide focus for the discussion, and to guide learners' thinking when they do question 1.

10 learners	20 learners
? apples	? apples

Learners must do questions 1(a) and (b) individually. Once they have written down answers for question 1(b), allow them to join other learners to compare and discuss their answers.

Answers

1. (a) In the group of 20 learners each learner gets 3 apples and in the group of 10 learners each learner gets 6 apples.
   (b) Learners' own ideas. Some learners may suggest that 80 apples should be given to the group of 20 learners and 40 apples to the group of 10 learners, i.e. 4 apples to each learner.



9. A store ordered oranges from the fruit market. Twenty-one of the 3 485 oranges are rotten and cannot be sold.
   (a) How many oranges can the store owner put in one pocket if he wants to make 130 equal pockets?
   (b) How many oranges will be left over (excluding the rotten ones)?
10. The music store has a special offer of R99 for three CDs. Carmen made use of this opportunity to get more of her favourite music. She spent R1 089. How many CDs did she get?
11. Martin bought a tablet for R7 336. He borrowed the money from his mother and promised to pay her back R262 each month. How long will it take Martin to pay back the full amount? Give your answer in years and months.
12. A hotel spent R9 792 on new bath towels. Each bath towel cost R144. How many towels did they buy?

7.4 Ratio

Is it always fair to divide quantities into equal parts? Read this story and decide for yourself.

30 learners from a school are going on a school camp. They are divided into two groups: one group has 20 learners and the other group has 10 learners.

When the food is handed out, the group of 20 learners is given a box with 60 apples, and the group of 10 learners is also given a box with 60 apples.

Do you think this is fair?

1. Now that you have read the story and thought about it, answer these questions.
   (a) How many apples will each learner get, if each group is given one box of apples?
   (b) How do you think the apples should have been divided between the two groups?

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Notes on questions

Question 2(a) is a straightforward multiplication question:
One row has 7 green beads, so 80 rows have 7 × 80 = 560 green beads.
One row has 3 yellow beads, so 80 rows have 3 × 80 = 240 yellow beads.

Question 2(b) is slightly more difficult. Since there are 10 beads in one row, a diagram with 9 000 beads will have 9 000 ÷ 10 = 900 rows. The number of green beads will hence be 7 × 900 = 6 300. Once learners have finished with question 2(b), ask them how many yellow beads there will be in a diagram with 9 000 beads (it is 9 000 –6 300 = 2 700).

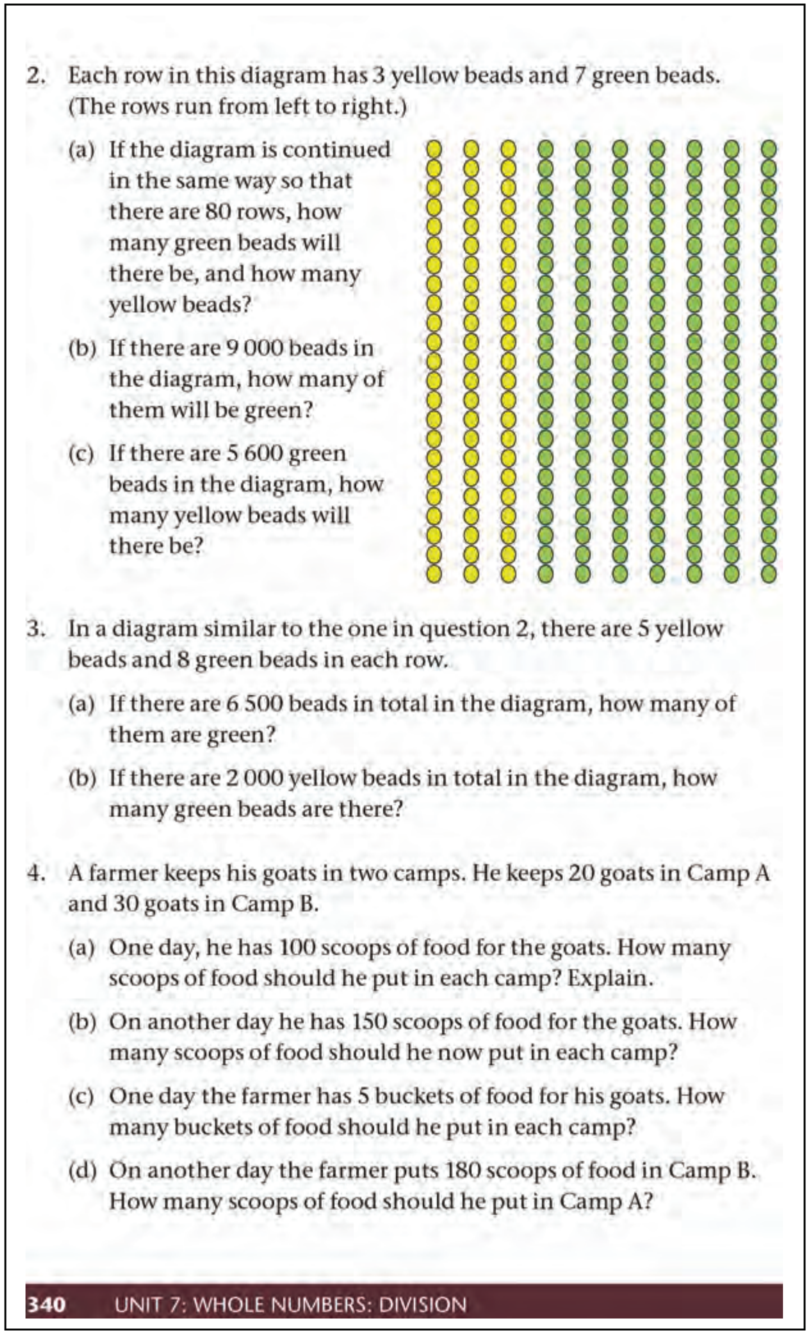
Question 2(c) is again slightly different to question 2(b). It is not the total number of beads in the diagram that is given, but the total number of green beads. The number of rows can be calculated as 5 600 ÷ 7 = 800 rows, from which the number of yellow beads can be calculated as 3 × 800 = 2 400.

As the above shows, learners do not need any knowledge of ratio to answer questions 2(b) and (c). The answers could be represented on the board as below, and learners may be asked to insert all the numbers for some more rows, for example a row with180 yellow beads and a row with 350 green beads. Doing this will provide experience with variable quantities that are in a constant ratio to each other.

1 row	3 yellow beads	7 green beads
80 rows	240 yellow beads	560 green beads
800 rows	2 400 yellow beads	5 600 green beads
900 rows	2 700 yellow beads	6 300 green beads

Answers

2. (a) 560 green beads and 240 yellow beads
   (b) 6 300 green beads
   (c) 2 400 yellow beads
3. (a) 4 000 green beads
   (b) 3 200 green beads
4. (a) 40 scoops in Camp A and 60 scoops in Camp B
   There are 50 goats in total, that means 2 scoops for every goat.
   (b) 60 scoops in Camp A and 90 scoops in Camp B
   (c) 2 buckets in Camp A and 3 buckets in Camp B
   (d) 120 scoops



2. Each row in this diagram has 3 yellow beads and 7 green beads. (The rows run from left to right.)
   (a) If the diagram is continued in the same way so that there are 80 rows, how many green beads will there be, and how many yellow beads?
   (b) If there are 9 000 beads in the diagram, how many of them will be green?
   (c) If there are 5 600 green beads in the diagram, how many yellow beads will there be?
3. In a diagram similar to the one in question 2, there are 5 yellow beads and 8 green beads in each row.
   (a) If there are 6 500 beads in total in the diagram, how many of them are green?
   (b) If there are 2 000 yellow beads in total in the diagram, how many green beads are there?
4. A farmer keeps his goats in two camps. He keeps 20 goats in Camp A and 30 goats in Camp B.
   (a) One day, he has 100 scoops of food for the goats. How many scoops of food should he put in each camp? Explain.
   (b) On another day he has 150 scoops of food for the goats. How many scoops of food should he now put in each camp?
   (c) One day the farmer has 5 buckets of food for his goats. How many buckets of food should he put in each camp?
   (d) On another day the farmer puts 180 scoops of food in Camp B. How many scoops of food should he put in Camp A?

Answers

5. (a) 8 cups (b) 14 cups (c) 50 cups
   (d) 175 cups (e) 30 cups (f) 600 kg

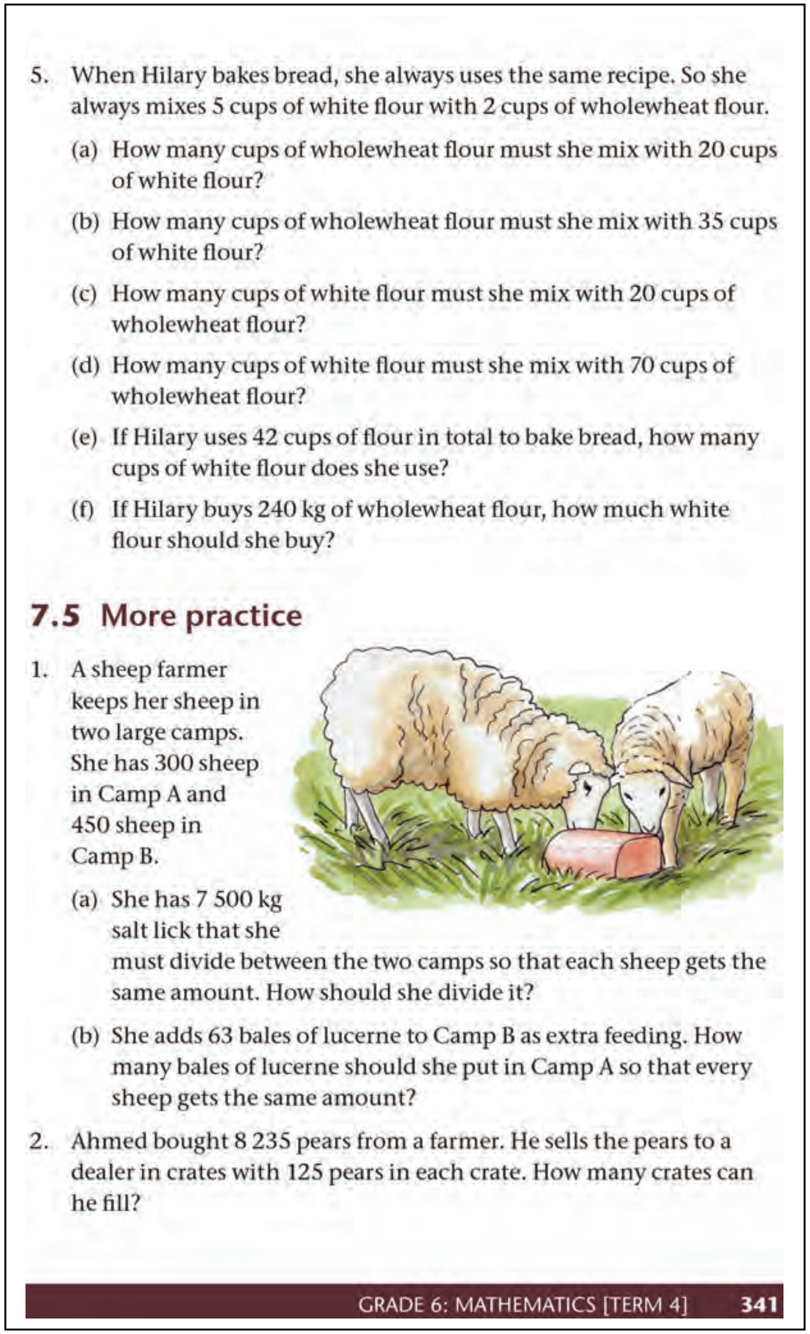
7.5 More practice

Teaching guidelines

This section includes a variety of problems and is intended for practice. Unless there is time to do all the questions, it may be a good idea to allow learners to choose which problems they would like to do. Challenge them to do as many as they can.

Answers

1. (a) Camp A: 3 000 kg
   Camp B: 4 500 kg
   (b) 42 bales
2. 65 crates



5. When Hilary bakes bread, she always uses the same recipe. So she always mixes 5 cups of white flour with 2 cups of wholewheat flour.
   (a) How many cups of wholewheat flour must she mix with 20 cups of white flour?
   (b) How many cups of wholewheat flour must she mix with 35 cups of white flour?
   (c) How many cups of white flour must she mix with 20 cups of wholewheat flour?
   (d) How many cups of white flour must she mix with 70 cups of wholewheat flour?
   (e) If Hilary uses 42 cups of flour in total to bake bread, how many cups of white flour does she use?
   (f) If Hilary buys 240 kg of wholewheat flour, how much white flour should she buy?

7.5 More practice

1. A sheep farmer keeps her sheep in two large camps. She has 300 sheep in Camp A and 450 sheep in Camp B.
   (a) She has 7 500 kg salt lick that she must divide between the two camps so that each sheep gets the same amount. How should she divide it?
   (b) She adds 63 bales of lucerne to Camp B as extra feeding. How many bales of lucerne should she put in Camp A so that every sheep gets the same amount?
2. Ahmed bought 8 235 pears from a farmer. He sells the pears to a dealer in crates with 125 pears in each crate. How many crates can he fill?

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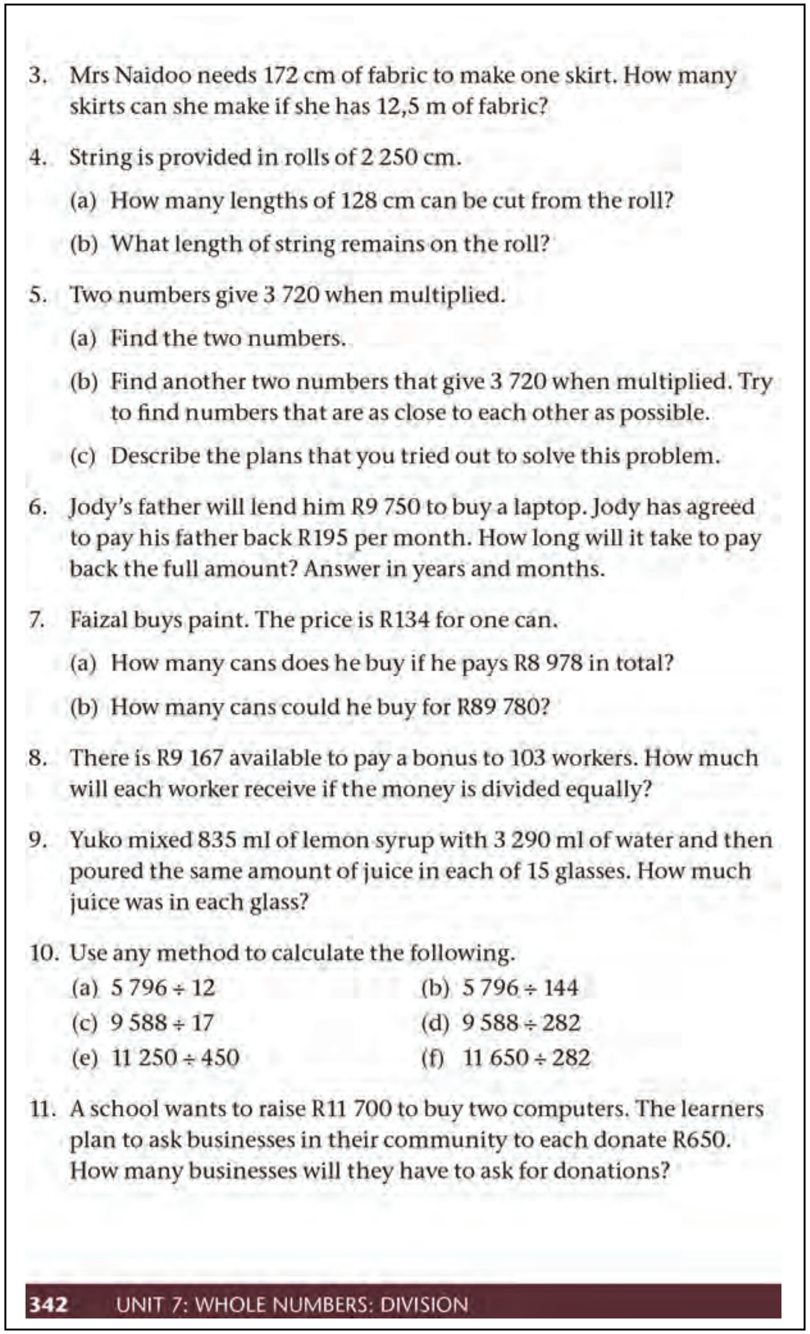
Answers

3. 7 skirts (12,5 m = 1 250 cm)
4. (a) 17
   (b) 74 cm
5. (a) and (b) There are many possibilities, e.g.

372 × 10	744 × 5	248 × 15	930 × 4
120 × 31	465 × 8	155 × 24	1 860 × 2
620 × 6	93 × 40	310 × 12	

   60 × 62 (numbers closest to each other)
   (c) Learners describe their plans.
6. 4 years and 2 months
7. (a) 67 cans
   (b) 670 cans
8. R89
9. 275 ml juice
10. (a) 483 (b) 40 remainder 36 (c) 564
    (d) 34 (e) 25 (f) 41 remainder 88
11. 18 businesses



3. Mrs Naidoo needs 172 cm of fabric to make one skirt. How many skirts can she make if she has 12,5 m of fabric?
4. String is provided in rolls of 2 250 cm.
   (a) How many lengths of 128 cm can be cut from the roll?
   (b) What length of string remains on the roll?
5. Two numbers give 3 720 when multiplied.
   (a) Find the two numbers.
   (b) Find another two numbers that give 3 720 when multiplied. Try to find numbers that are as close to each other as possible.
   (c) Describe the plans that you tried out to solve this problem.
6. Jody's father will lend him R9 750 to buy a laptop. Jody has agreed to pay his father back R195 per month. How long will it take to pay back the full amount? Answer in years and months.
7. Faizal buys paint. The price is R134 for one can.
   (a) How many cans does he buy if he pays R8 978 in total?
   (b) How many cans could he buy for R89 780?
8. There is R9 167 available to pay a bonus to 103 workers. How much will each worker receive if the money is divided equally?
9. Yuko mixed 835 ml of lemon syrup with 3 290 ml of water and then poured the same amount of juice in each of 15 glasses. How much juice was in each glass?
10. Use any method to calculate the following.
    (a) 5 796 ÷ 12 (b) 5 796 ÷ 144
    (c) 9 588 ÷ 17 (d) 9 588 ÷ 282
    (e) 11 250 ÷ 450 (f) 11 650 ÷ 282
11. A school wants to raise R11 700 to buy two computers. The learners plan to ask businesses in their community to each donate R650. How many businesses will they have to ask for donations?

7.6 Rate

Mathematical notes

This section allows learners to engage with a constant rate in a mechanical context. Since the cogwheel has 18 notches, the locomotive will move forward by 18 notches when the wheels turn around once. Learners calculate the rate of *18 notches forward movement per full turn of the wheels* in question 2.

Learners' understanding of the inverse relationship between multiplication and division will be strengthened by doing questions 3(a) and (b):

- In question 3(a) learners multiply the rate by 6 (the number of wheel turns) to determine the distance moved by the locomotive.
- In question 3(b) learners divide the number of wheel turns (54) by the rate to determine the distance.

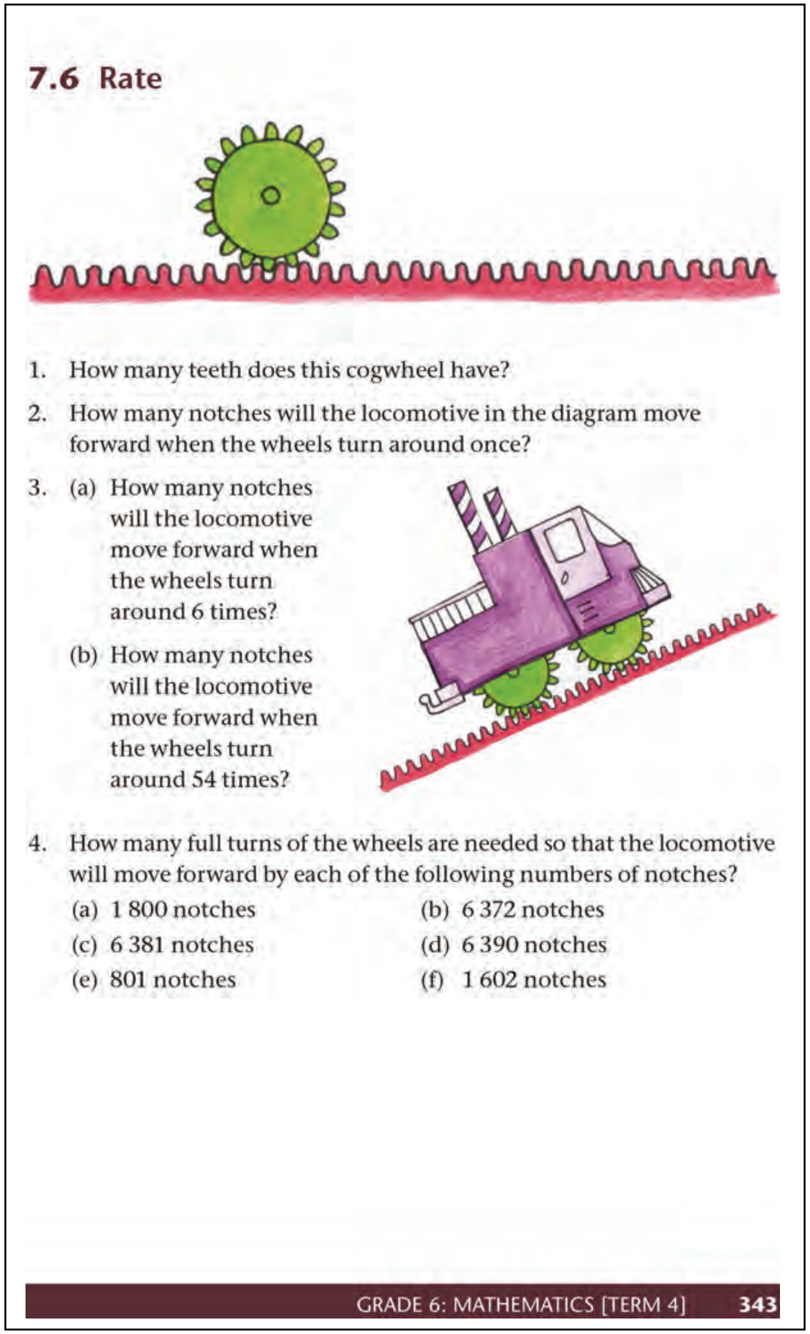
In question 4 learners calculate the number of wheel turns required to move the locomotive by a given distance, by dividing the distance by the rate.

Teaching guidelines

The whole of this section may be given as a project if there is not enough time available in class.

Answers

1. 18
2. 18
3. (a) 108
   (b) 972
4. (a) 100
   (b) 354
   (c) 354 and a half
   (d) 355
   (e) 44 full turns + 8 notches (8 eighteenths or 4 ninths of a turn)
   (f) 89 full turns + 16 notches (16 eighteenths or 8 ninths of a turn)



7.6 Rate

1. How many teeth does this cogwheel have?
2. How many notches will the locomotive in the diagram move forward when the wheels turn around once?
3. (a) How many notches will the locomotive move forward when the wheels turn around 6 times?
   (b) How many notches will the locomotive move forward when the wheels turn around 54 times?
4. How many full turns of the wheels are needed so that the locomotive will move forward by each of the following numbers of notches?

(a) 1 800 notches
(b) 6 372 notches
(c) 6 381 notches
(d) 6 390 notches
(e) 801 notches
(f) 1 602 notches

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Grade 6 Term 4 Unit 8 Number sentences

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
8.1 Statements of equivalence	Using number sentences to state that different calculation plans will produce the same result	344
8.2 Substitution, trial and improvement	Solving open number sentences by searching systematically for the number that will make the sentence true	345 to 346
8.3 Use number sentences when needed	Using number sentences as a tool to interpret situations and establish what calculations are needed	347 to 349

CAPS time allocation	3 hours
CAPS page references	20 and 286 to 287

Mathematical background

Number sentences serve three important purposes in mathematics apart from their use to state number facts such as $7 \times 9 = 63$:

- They can be used to state that different calculation plans are equivalent and they produce the same result:

$$7 \times (60 + 8) = 7 \times 60 + 7 \times 8$$

- They can be used to represent properties of operations that hold for all numbers, for example:

🟥 × (🟦 + 🟩) = 🟥 × 🟦 + 🟥 × 🟩

number A × (number B + number C) = number A × number B + number A × number C

- They can be used to describe real situations in mathematical terms, for example:
  Themba had R1 200. He bought three chickens for the same price each. Bertha had R576. She sold five chickens for the same price each that Themba had paid for a chicken. They now have the same amounts in cash. What was the price of a chicken?

1 200 – 3 × *price of a chicken* = 576 + 5 × *price of a chicken*

Open sentences like the above can be solved by trial and improvement, for example:

Thinking	Try any number	Try another number	A number in between	Stay close	Go closer to 80	Go even closer to 80
□	100	50	80	70	75	78
1 200 – 3 × □	900	1 050	960	990	975	966
576 + 5 × □	1 076	826	976	926	951	966
Difference	176	–(224)	16	–(64)	–(24)	0
Observation	576 + 5 × □ bigger	1 200 – 3 × □ bigger	Pretty close!	Difference now bigger	Not close enough	!

Note that successful trial and improvement requires acquiring certain thinking strategies, as indicated in the “Thinking” and “Observation” rows in the above table.

8.1 Statements of equivalence

Mathematical notes

The two sets of calculations in the shaded passage illustrate a certain equivalence that exists for any two numbers, namely:

number A × number A – number B × number B
= (number A + number B) × (number A – number B)

Stated differently:

the difference between the squares of two numbers
= the sum of the two numbers × the difference between the two numbers

This can also be represented by □ × □ − △ × △ = (□ + △) × (□ − △), which is represented as $x^2 - y^2 = (x - y)(x + y)$ in Grade 9 and higher.

The calculations that learners will do in questions 1 and 2 will show the truth of the above statement.

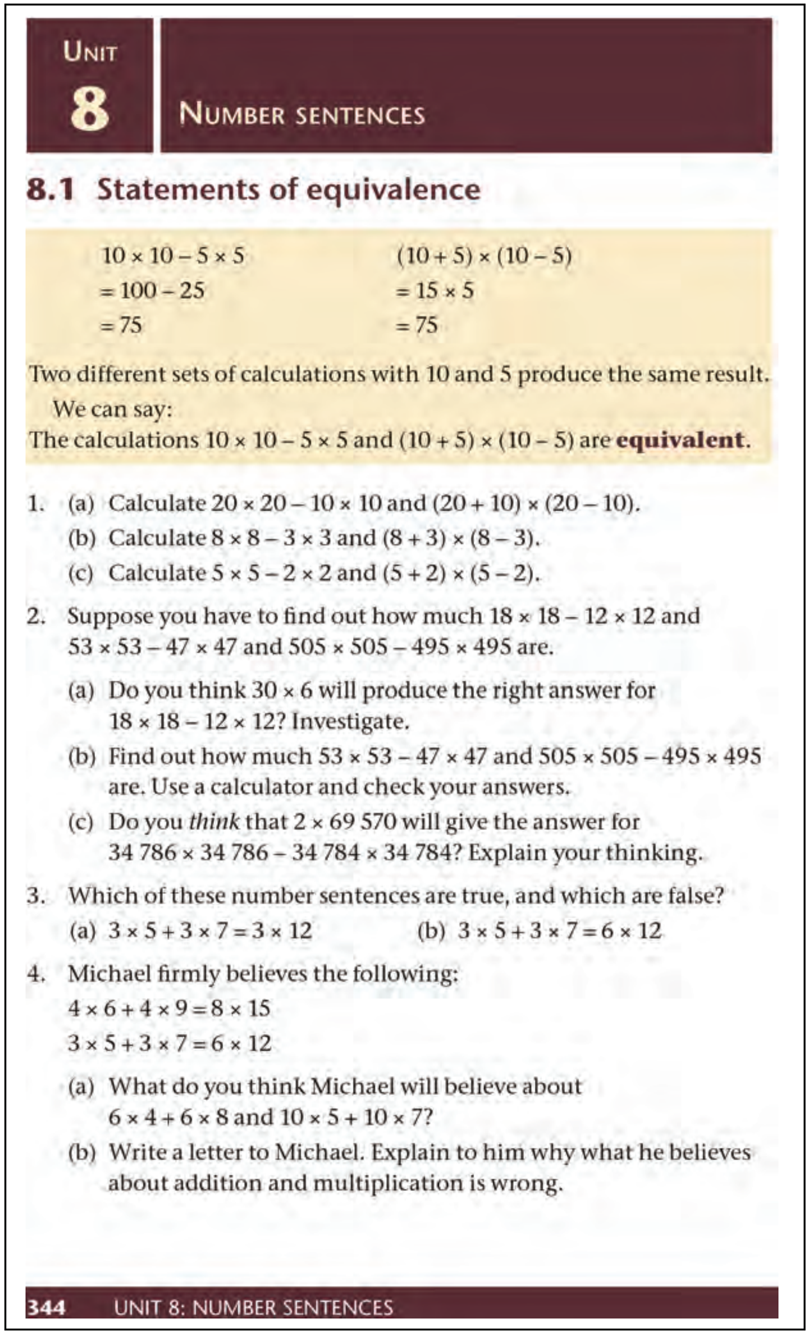
The purpose of Section 8.1 is to consolidate the concept of statements of equivalence by engaging learners with some true statements of equivalence based on the above general truth, and some false statements of equivalence in questions 3 and 4.

Teaching guidelines

With a view to persuade learners to take note of the shaded passage, ask them to do calculations to check whether it is true that 10 × 10 – 5 × 5 and (10 + 5) × (10 – 5) produce the same answer. In question 4 another equivalence is suggested, but investigation shows that pattern is false.

Answers

1. (a) Both are 300. (b) Both are 55. (c) Both are 21.
2. (a) Yes. Investigation confirms they are both180.
   (b) 53 × 53 – 47 × 47 = 600; 505 × 505 – 495 × 495 = 10 000
   (c) Learners' own answers. Example:
   The numbers 34 786 and 34 784 are exactly 2 units apart,
   and 34 786 + 34 784 = 69 570,
   so I do think 2 × 69 570 = 34 786 × 34 786 – 34 784 × 34 784,
   as this conforms to our pattern.
3. (a) True (b) False
4. (a) Michael will believe that 6 × 4 + 6 × 8 = 12 × 12 and 10 × 5 + 10 × 7 = 20 × 12.
   (b) Take in learners' letters to check their thinking.



UNIT 8 NUMBER SENTENCES

8.1 Statements of equivalence

10 × 10 – 5 × 5	(10 + 5) × (10 – 5)
= 100 – 25	= 15 × 5
= 75	= 75

Two different sets of calculations with 10 and 5 produce the same result.

We can say:
The calculations 10 × 10 – 5 × 5 and (10 + 5) × (10 – 5) are **equivalent**.

1. (a) Calculate 20 × 20 – 10 × 10 and (20 + 10) × (20 – 10).
   (b) Calculate 8 × 8 – 3 × 3 and (8 + 3) × (8 – 3).
   (c) Calculate 5 × 5 – 2 × 2 and (5 + 2) × (5 – 2).
2. Suppose you have to find out how much 18 × 18 – 12 × 12 and 53 × 53 – 47 × 47 and 505 × 505 – 495 × 495 are.
   (a) Do you think 30 × 6 will produce the right answer for 18 × 18 – 12 × 12? Investigate.
   (b) Find out how much 53 × 53 – 47 × 47 and 505 × 505 – 495 × 495 are. Use a calculator and check your answers.
   (c) Do you *think* that 2 × 69 570 will give the answer for 34 786 × 34 786 – 34 784 × 34 784? Explain your thinking.
3. Which of these number sentences are true, and which are false?
   (a) 3 × 5 + 3 × 7 = 3 × 12 (b) 3 × 5 + 3 × 7 = 6 × 12
4. Michael firmly believes the following:
   4 × 6 + 4 × 9 = 8 × 15
   3 × 5 + 3 × 7 = 6 × 12
   (a) What do you think Michael will believe about 6 × 4 + 6 × 8 and 10 × 5 + 10 × 7?
   (b) Write a letter to Michael. Explain to him why what he believes about addition and multiplication is wrong.

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8.2 Substitution, trial and improvement

Mathematical notes

Solving number sentences by trial and improvement is a very valuable learning experience, for at least four reasons:

- It provides learners with opportunities to develop strong understanding of what is meant by the solution of an open number sentence. This is the number that makes the sentence true.
- It provides learners with an experience of the so-called "numerical solution" of equations, which is very important in modern mathematical practice.
- It provides computation practice in a meaningful mathematical context.
- It provides learners with experience in working with an independent **variable** (the different numbers they substitute for the unknown number) and **dependent variables** (the results produced by applying the calculation plans). These are the basic concepts in algebra.

Notes on questions

Note that a numerical search for the solution of the open number sentence

5 × *the number* + 4 = 64 – 3 × *the number*

is started in the shaded passage. Learners have to continue this search in question 1 until they find the number ($7\frac{1}{2}$) that makes the sentence true.

Teaching guidelines

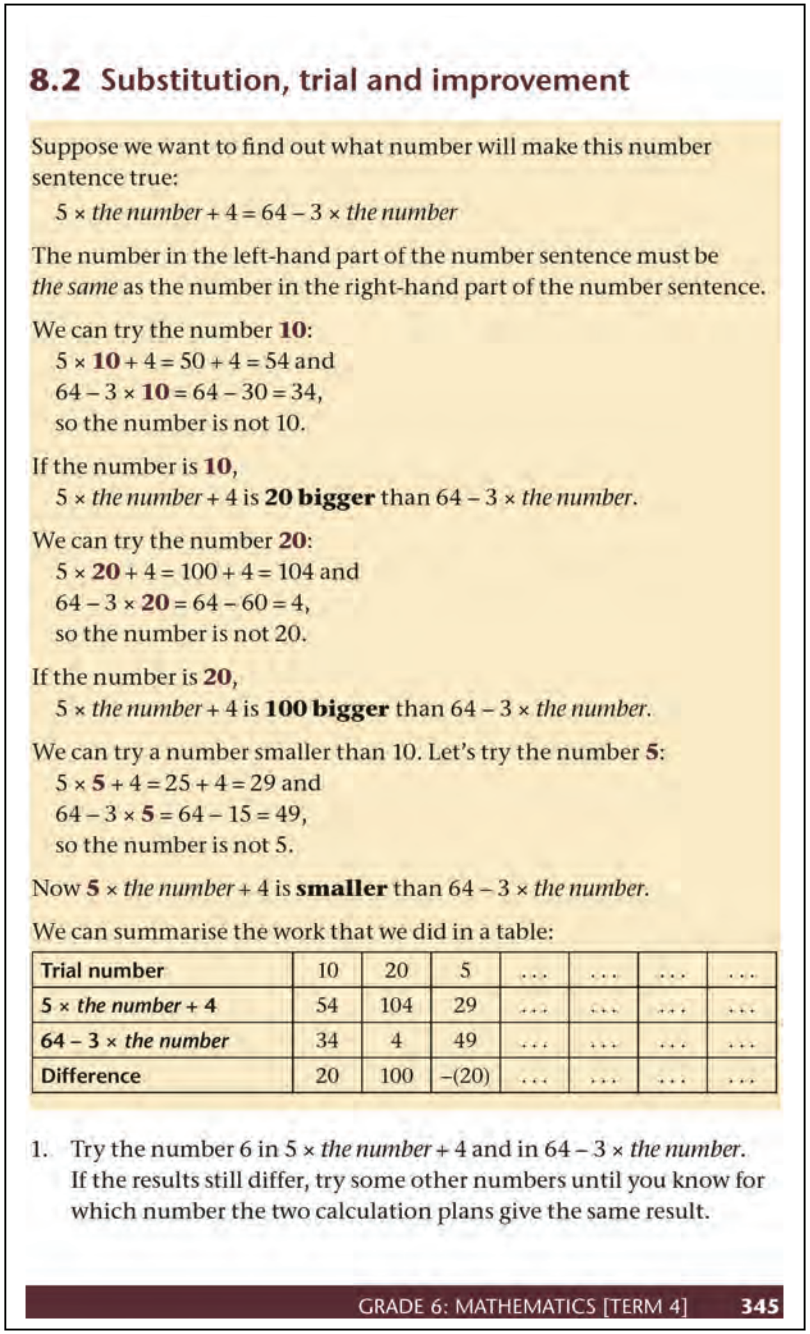
It is important for you to demonstrate solution by trial and improvement. Explain the reasons for the numbers that you choose when demonstrating the method, for example as in the "Mathematical background" on the opening page of this unit.

Answers

1. Trial and improvement will lead learners to find that *the number* which will give the same solution lies between 7 and 8. Trying for $7\frac{1}{2}$ as being *the number* would then give the same result.

Trial number	10	20	5	6	7	8	$7\frac{1}{2}$
5 × *the number* + 4	54	104	29	34	39	44	$41\frac{1}{2}$
64 – 3 × *the number*	34	4	49	46	43	40	$41\frac{1}{2}$
Difference	20	100	–(20)	–(12)	–(4)	4	0

The two calculation plans give the same result (i.e. $41\frac{1}{2}$) for the number $7\frac{1}{2}$.



8.2 Substitution, trial and improvement

Suppose we want to find out what number will make this number sentence true:

5 × *the number* + 4 = 64 – 3 × *the number*

The number in the left-hand part of the number sentence must be *the same* as the number in the right-hand part of the number sentence.

We can try the number **10**:
5 × **10** + 4 = 50 + 4 = 54 and
64 – 3 × **10** = 64 – 30 = 34,
so the number is not 10.

If the number is **10**,
5 × *the number* + 4 is **20 bigger** than 64 – 3 × *the number*.

We can try the number **20**:
5 × **20** + 4 = 100 + 4 = 104 and
64 – 3 × **20** = 64 – 60 = 4,
so the number is not 20.

If the number is **20**,
5 × *the number* + 4 is **100 bigger** than 64 – 3 × *the number*.

We can try a number smaller than 10. Let's try the number **5**:
5 × **5** + 4 = 25 + 4 = 29 and
64 – 3 × **5** = 64 – 15 = 49,
so the number is not 5.

Now **5** × *the number* + 4 is **smaller** than 64 – 3 × *the number*.

We can summarise the work that we did in a table:

Trial number	10	20	5	...	...	...	...
5 × *the number* + 4	54	104	29	...	...	...	...
64 – 3 × *the number*	34	4	49	...	...	...	...
Difference	20	100	–(20)	...	...	...	...

1. Try the number 6 in 5 × *the number* + 4 and in 64 – 3 × *the number*. If the results still differ, try some other numbers until you know for which number the two calculation plans give the same result.

Notes on questions

Questions 2, 4 and 6 require learners to solve open number sentences.

Question 3 relates to question 2(c). Learners can consult the work they did when they engaged with question 2(c) to find some of the answers for question 3, for example:

□	10	100	50	5	20	30	25
10 × □ + 1 500	1 600	2 500	2 000	1 550	1 700	1 800	1 750
20 × □ + 1 250	1 450	3 250	2 250	1 350	1 650	1 850	1 750
Difference	150	–(750)	–(250)	200	50	–(50)	0

The above results show that:

10 × □ + 1 500 > 20 × □ + 1 250 for □ = 10; 5 and 20 (question 3(a)),

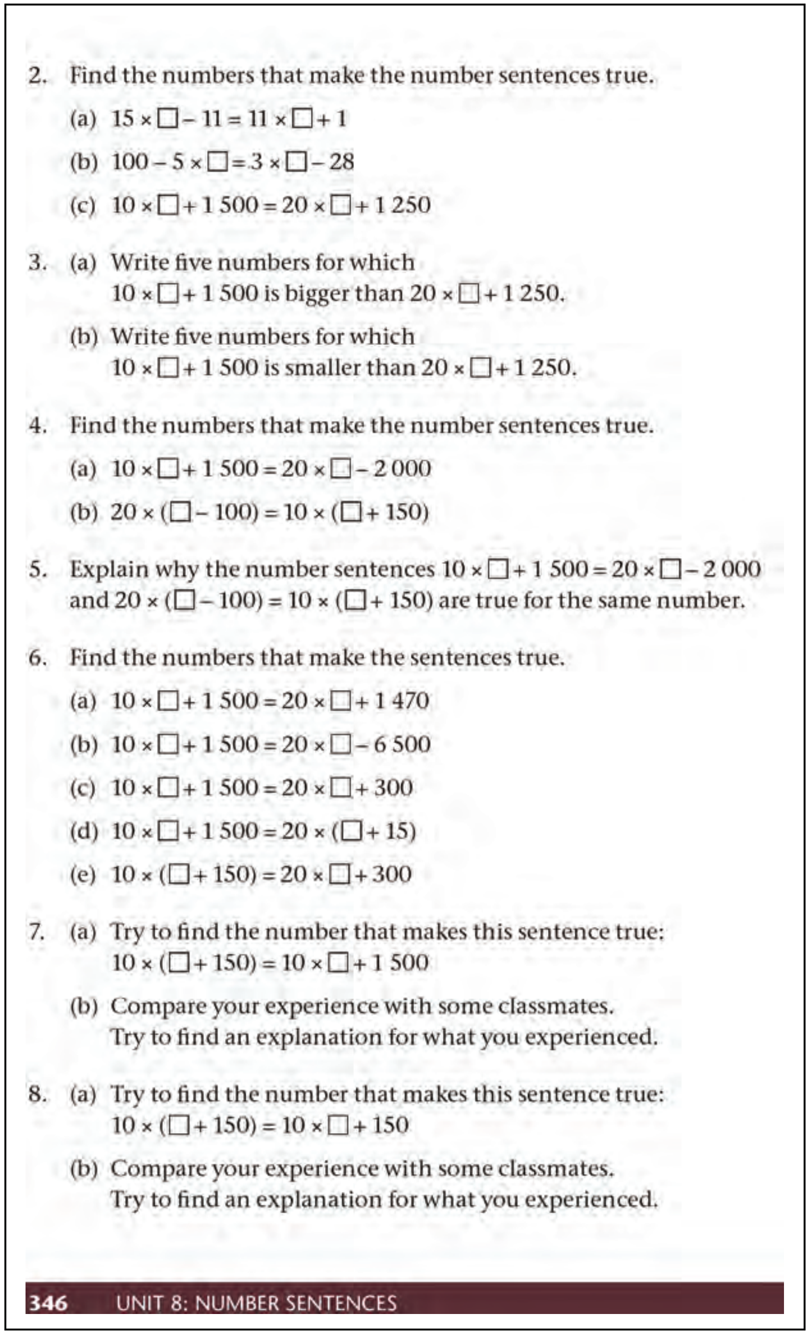
10 × □ + 1 500 = 20 × □ + 1 250 for □ = 25 (question 2(c)), and

10 × □ + 1 500 < 20 × □ + 1 250 for □ = 100; 50 and 30 (question 3(b)).

Answers

2. (a) 3 (b) 16 (c) 25
3. (a) Any five numbers smaller than 25.
   (b) Any five numbers bigger than 25.
4. (a) 350 (b) 350
5. 10 × (□ + 150) and 10 × □ + 1 500 are equivalent calculation plans because of the distributive property of multiplication and addition or subtraction.

   Similarly, 20 × (□ – 100) and 20 × □ – 2 000 are equivalent calculation plans as they produce the same results for all values of □.
6. (a) 3 (b) 800 (c) 120 (d) 120 (e) 120
7. (a) Any number will make the sentence true.
   (b) The two calculation plans are equivalent (the distributive property) hence they produce the same results.
8. (a) If we try different numbers, the difference between the answers remains the same. No number makes the sentence true.
   (b) The calculation plan on the left can be replaced with 10 × □ + 1 500 (as was already apparent in question 7). Clearly, 10 × □ + 1 500 is 1 350 more than 10 × □ + 150, irrespective of the value assigned to □.



2. Find the numbers that make the number sentences true.
   (a) 15 × □ – 11 = 11 × □ + 1
   (b) 100 – 5 × □ = 3 × □ – 28
   (c) 10 × □ + 1 500 = 20 × □ + 1 250
3. (a) Write five numbers for which 10 × □ + 1 500 is bigger than 20 × □ + 1 250.
   (b) Write five numbers for which 10 × □ + 1 500 is smaller than 20 × □ + 1 250.
4. Find the numbers that make the number sentences true.
   (a) 10 × □ + 1 500 = 20 × □ – 2 000
   (b) 20 × (□ – 100) = 10 × (□ + 150)
5. Explain why the number sentences 10 × □ + 1 500 = 20 × □ – 2 000 and 20 × (□ – 100) = 10 × (□ + 150) are true for the same number.
6. Find the numbers that make the sentences true.
   (a) 10 × □ + 1 500 = 20 × □ + 1 470
   (b) 10 × □ + 1 500 = 20 × □ – 6 500
   (c) 10 × □ + 1 500 = 20 × □ + 300
   (d) 10 × □ + 1 500 = 20 × (□ + 15)
   (e) 10 × (□ + 150) = 20 × □ + 300
7. (a) Try to find the number that makes this sentence true: 10 × (□ + 150) = 10 × □ + 1 500
   (b) Compare your experience with some classmates. Try to find an explanation for what you experienced.
8. (a) Try to find the number that makes this sentence true: 10 × (□ + 150) = 10 × □ + 150
   (b) Compare your experience with some classmates. Try to find an explanation for what you experienced.

346 UNIT 8: NUMBER SENTENCES

8.3 Use number sentences when needed

Teaching guidelines

Ensure that learners interpret the phrases "the morning of 1 September" and "at the end of the day" correctly: the stock at the end of the day on 1 September would be the same as the stock on the morning of 2 September.

Learners may need to be reminded that "2,4 million" is 2 400 000. Conduct a whole-class discussion to make sure that learners understand the context before they engage with question 1. A representation like the following would be quite useful, but don't write it on the board before learners have had some time to apply their minds to the text that describes the situation:

	1 Sept	2 Sept	3 Sept	4 Sept	5 Sept	6 Sept
2 400 000	+ 128 000	+ 128 000	+ 128 000	+ 128 000	+ 128 000	+ 128 000

Encourage learners to **use calculators** for this section.

To find the total number of bricks at the end of day on 2 September (question 1), learners just need to add 2 × 128 000 (the production on 1 and 2 September) to 2,4 million:

2 400 000 + 2 × 128 000 = 2 656 000.

Once learners have competed question 1 successfully and demonstrate at least good progress with question 2, you may add a third row to the above representation:

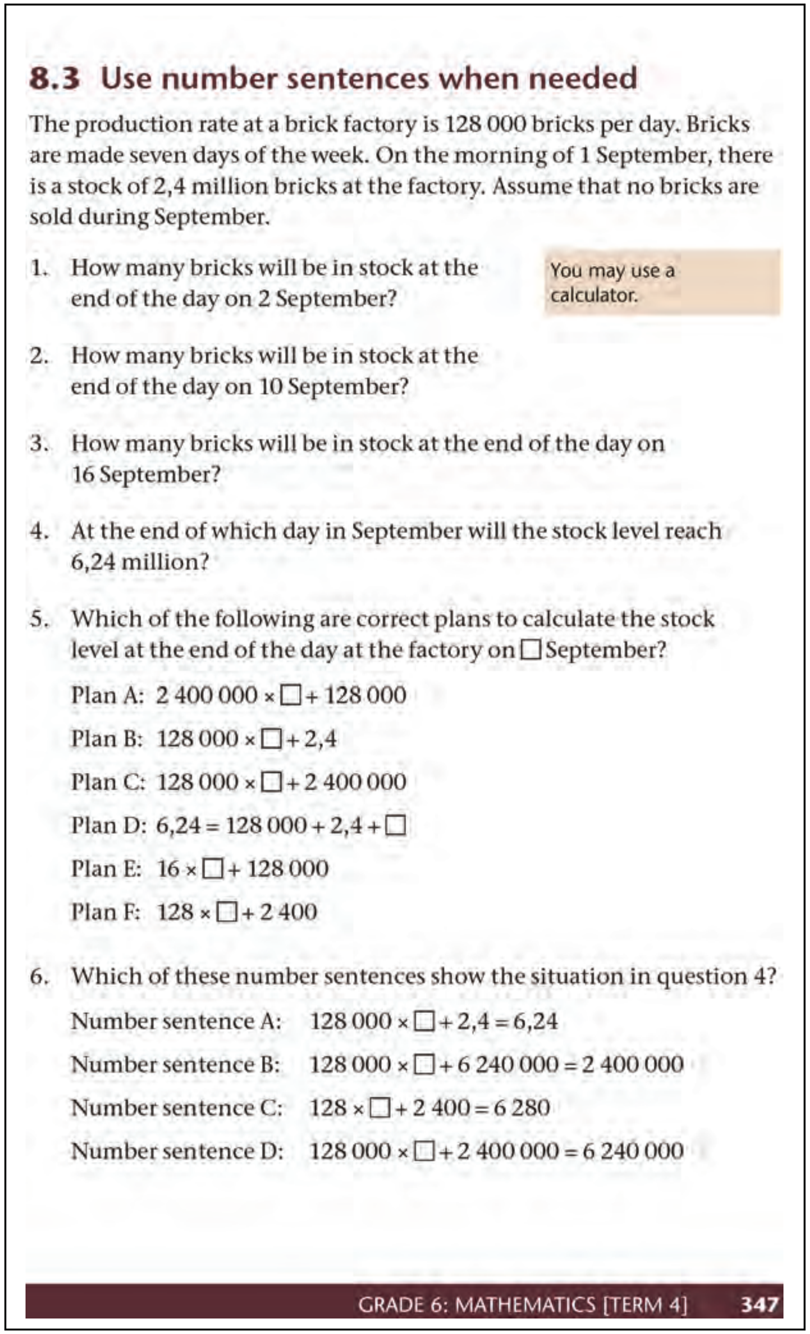
	1 Sept	2 Sept	3 Sept	4 Sept	5 Sept	6 Sept
2 400 000	+ 128 000	+ 128 000	+ 128 000	+ 128 000	+ 128 000	+ 128 000
2 400 000	2 528 000	2 656 000	2 784 000	2 912 000		

Learners may continue to add repeatedly to determine the stock levels on 10 September, or they may use the calculation plan 128 000 × 10 + 2 400 000. When they have completed question 2, demonstrate on the board that they could have used the calculation plan, and encourage them to use a similar calculation plan for question 3.

If some learners still use repeated addition of 128 000 for question 4, allow them to do so but let them solve the open sentence 128 000 × □ + 2 400 000 = 6 240 000 afterwards to emphasise that the situation can be shown in this way too.

Answers

1. 2 656 000 bricks
2. 3 680 000 bricks
3. 4 448 000 bricks
4. 30 September
5. Plan C
6. Number sentence D



8.3 Use number sentences when needed

The production rate at a brick factory is 128 000 bricks per day. Bricks are made seven days of the week. On the morning of 1 September, there is a stock of 2,4 million bricks at the factory. Assume that no bricks are sold during September.

You may use a calculator.

1. How many bricks will be in stock at the end of the day on 2 September?
2. How many bricks will be in stock at the end of the day on 10 September?
3. How many bricks will be in stock at the end of the day on 16 September?
4. At the end of which day in September will the stock level reach 6,24 million?
5. Which of the following are correct plans to calculate the stock level at the end of the day at the factory on □ September?

   Plan A: 2 400 000 × □ + 128 000

   Plan B: 128 000 × □ + 2,4

   Plan C: 128 000 × □ + 2 400 000

   Plan D: 6,24 = 128 000 + 2,4 + □

   Plan E: 16 × □ + 128 000

   Plan F: 128 × □ + 2 400
6. Which of these number sentences show the situation in question 4?

   Number sentence A: 128 000 × □ + 2,4 = 6,24

   Number sentence B: 128 000 × □ + 6 240 000 = 2 400 000

   Number sentence C: 128 × □ + 2 400 = 6 280

   Number sentence D: 128 000 × □ + 2 400 000 = 6 240 000

Teaching guidelines

If learners did not use the calculation plan 128 000 × ☐ + 2 400 000 for 21 and 28 September in question 7(a), encourage them to do so.

Answers

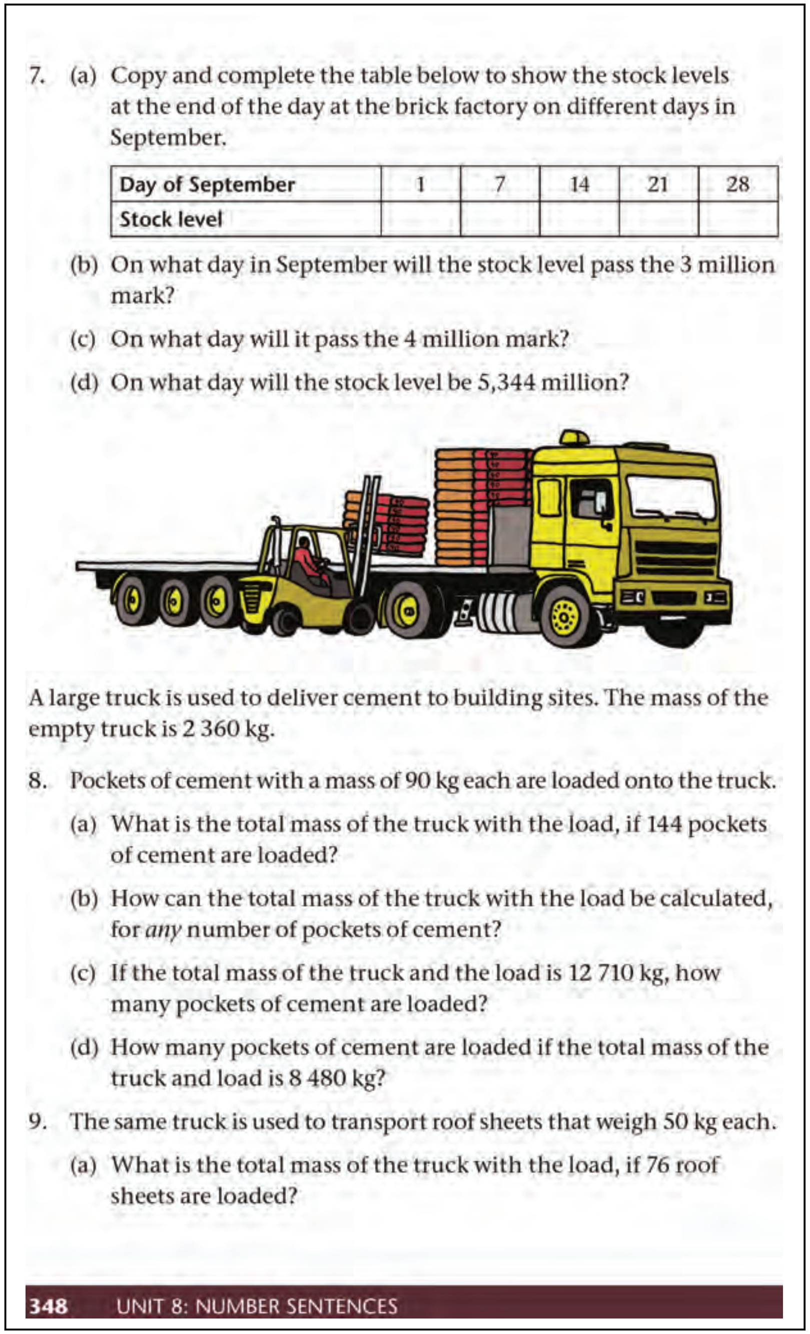
7. (a)

Day of September	1	7	14	21	28
Stock level	2 528 000	3 296 000	4 192 000	5 088 000	5 984 000

(b) Day 5
(c) Day 13
(d) Day 23

8. (a) 5 320 kg
(b) 2 360 kg + "the number" of pockets of cement × 90 kg
(c) 115 pockets
(d) 68 pockets

9. (a) 6 160 kg



7. (a) Copy and complete the table below to show the stock levels at the end of the day at the brick factory on different days in September.

Day of September	1	7	14	21	28
Stock level					

(b) On what day in September will the stock level pass the 3 million mark?

(c) On what day will it pass the 4 million mark?

(d) On what day will the stock level be 5,344 million?

A large truck is used to deliver cement to building sites. The mass of the empty truck is 2 360 kg.

8. Pockets of cement with a mass of 90 kg each are loaded onto the truck.

(a) What is the total mass of the truck with the load, if 144 pockets of cement are loaded?

(b) How can the total mass of the truck with the load be calculated, for *any* number of pockets of cement?

(c) If the total mass of the truck and the load is 12 710 kg, how many pockets of cement are loaded?

(d) How many pockets of cement are loaded if the total mass of the truck and load is 8 480 kg?

9. The same truck is used to transport roof sheets that weigh 50 kg each.

(a) What is the total mass of the truck with the load, if 76 roof sheets are loaded?

348 UNIT 8: NUMBER SENTENCES

Mathematical notes

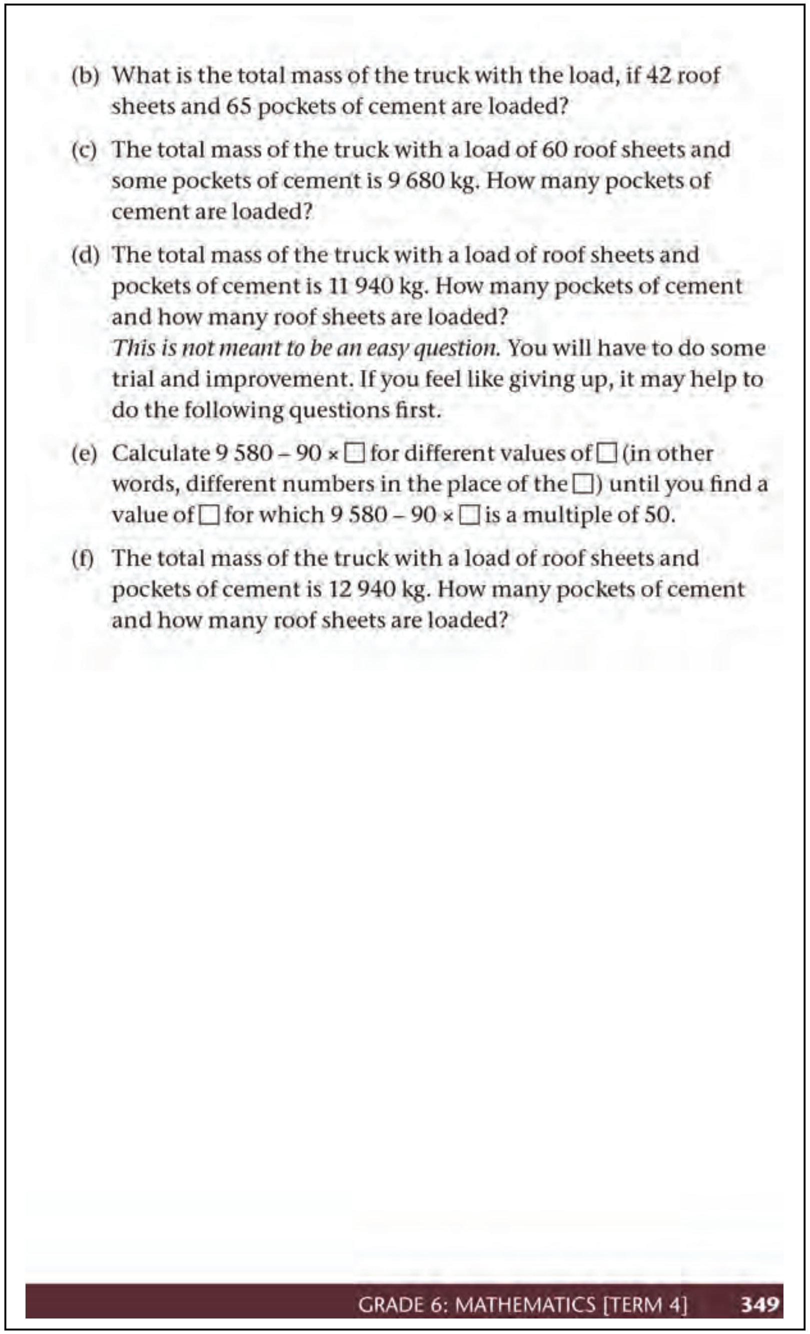
In some situations only whole numbers are acceptable answers, as in: *“How many pockets of cement and how many roof sheets are loaded?”*

Teaching guidelines

The aim is for learners to inspect and experience how numbers interrelate in a practical scenario, possibly using number sentences and/or tables of calculations.

Answers

9. (b) 10 310 kg
   (c) 48 pockets
   (d) 2 pockets of cement and 188 roof sheets
   (e) 9 850 and 10 300
   (f) 2 pockets of cement and 208 roof sheets



(b) What is the total mass of the truck with the load, if 42 roof sheets and 65 pockets of cement are loaded?

(c) The total mass of the truck with a load of 60 roof sheets and some pockets of cement is 9 680 kg. How many pockets of cement are loaded?

(d) The total mass of the truck with a load of roof sheets and pockets of cement is 11 940 kg. How many pockets of cement and how many roof sheets are loaded?
This is not meant to be an easy question. You will have to do some trial and improvement. If you feel like giving up, it may help to do the following questions first.

(e) Calculate 9 580 – 90 × □ for different values of □ (in other words, different numbers in the place of the □) until you find a value of □ for which 9 580 – 90 × □ is a multiple of 50.

(f) The total mass of the truck with a load of roof sheets and pockets of cement is 12 940 kg. How many pockets of cement and how many roof sheets are loaded?

Grade 6 Term 4 Unit 9 Transformations

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
9.1 Making larger copies of figures	An additional basic transformation	350 to 354
9.2 Enlargements and reductions	A more formal look at the additional basic transformation	355 to 356
9.3 Increasing the length of two sides only	A look at stretching in one direction	357

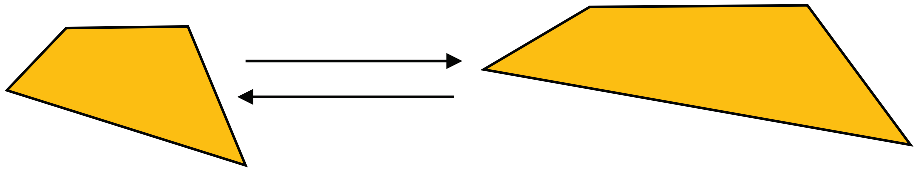
CAPS time allocation	3 hours
CAPS page references	23 and 288

Mathematical background

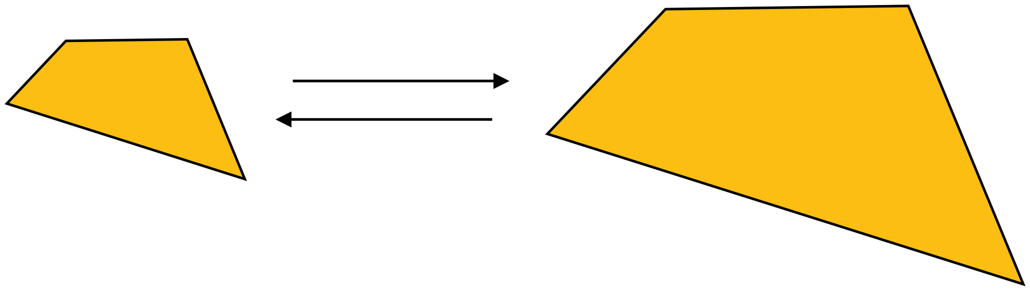
Previous work on transformations did not involve any changes in the shape or size of shapes, only different ways of moving shapes around.

Another kind of transformation involves changing the size of the shape. The shape can either be scaled up in size or scaled down in size. Such transformations occur in two basic ways:

- The shape is "stretched" or "squashed" along one direction only (expanded or contracted along one direction); the transformed shape is no longer the same shape as the original – the angle sizes have changed.



- Each length (distance between two points) of the shape is changed by the same scale factor, while all the angles remain unchanged. The enlarged or reduced shape has exactly the same shape as the original; it is just larger or smaller. These are called enlargements and reductions respectively.
  Enlargements and reductions are used in a variety of practical contexts, for example building plans and simple maps. Most photocopy machines can be used to make enlargements and reductions.



The second situation is really just two stretches (or two squashes) along directions that are at right angles to each other. The combined effect of the two identical changes results in the overall shape being preserved, along with all the angle sizes.

9.1 Making larger copies of figures

Mathematical notes

Enlarging a figure means making each distance longer by the same factor (e.g. twice as long, one and a half times as long, etc.) without changing the shape (i.e. keeping the angles unchanged). This may also be called **scaling**.

The easiest way to see how this occurs is to draw the same shape on different sheets of square grid paper, with the grids on each sheet made up of different sized squares (as shown on page 350). The shape is stretched by the same factor in every direction.

Teaching guidelines

Allow learners to spend some time individually on question 1 so that they can form their own opinions. Then allow discussion in groups or conduct a whole-class discussion.

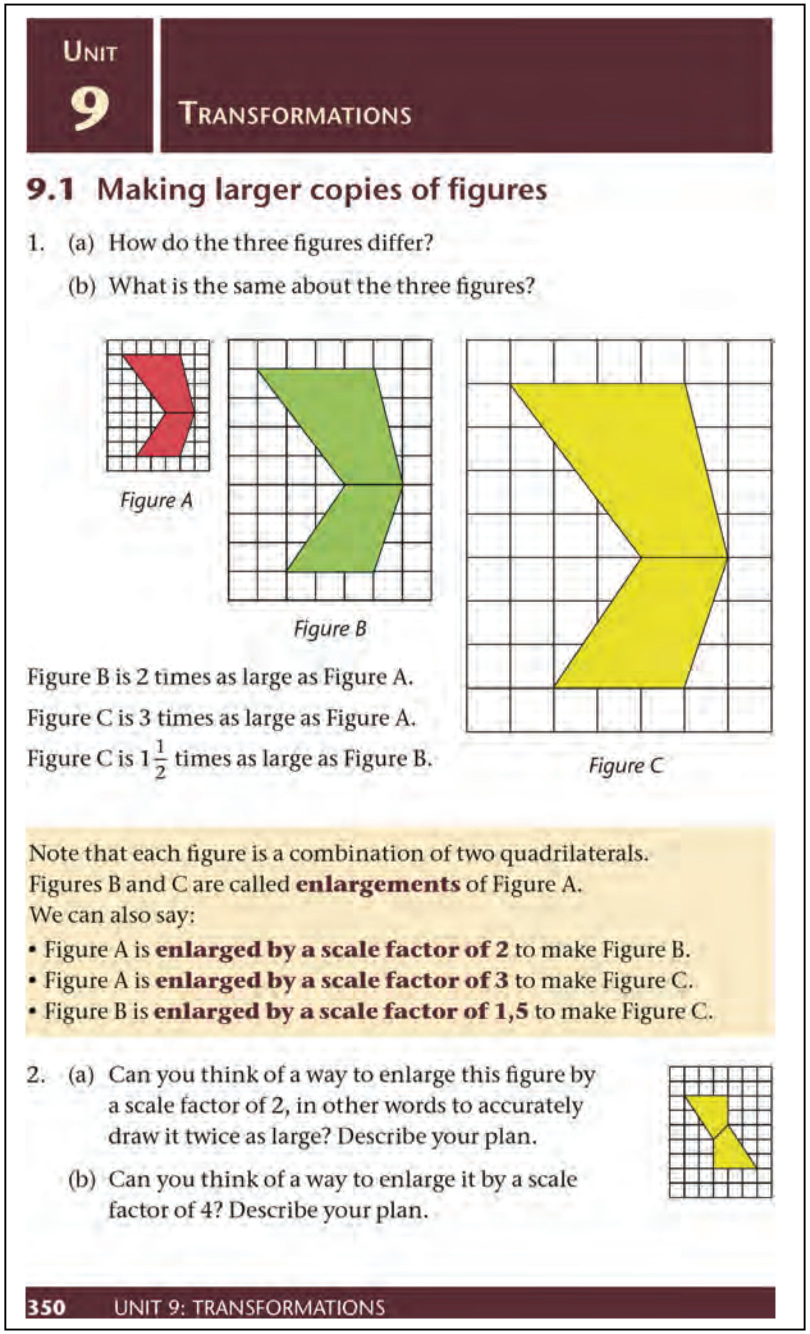
Possible misconceptions

Many learners may confuse scaling up and down (multiplication or division) with additive increase or decrease. Saying that each side of a shape is enlarged by a factor of 2 is not the same as saying that each side has been made 2 units longer. Enlargements and reductions are *multiplicative* changes, while increasing each side by the same *amount* is an *additive* change.

The shaded passage on page 351, questions 4 to 7 on page 354 and the whole of Section 9.3 are designed to develop understanding of the difference between scaling multiplicatively and extending or reducing additively.

Answers

1. (a) They are different in size and colour.
   (b) They have the same shape.
2. (a)–(b) Learners may come up with the idea of redrawing the shape on a grid with larger divisions. If they do not, bring it to their attention.



UNIT 9 TRANSFORMATIONS

9.1 Making larger copies of figures

1. (a) How do the three figures differ?
   (b) What is the same about the three figures?

Figure A

Figure B

Figure C

Figure B is 2 times as large as Figure A.

Figure C is 3 times as large as Figure A.

Figure C is $1\frac{1}{2}$ times as large as Figure B.

Note that each figure is a combination of two quadrilaterals.
Figures B and C are called **enlargements** of Figure A.
We can also say:

- Figure A is **enlarged by a scale factor of 2** to make Figure B.
- Figure A is **enlarged by a scale factor of 3** to make Figure C.
- Figure B is **enlarged by a scale factor of 1,5** to make Figure C.

2. (a) Can you think of a way to enlarge this figure by a scale factor of 2, in other words to accurately draw it twice as large? Describe your plan.
   (b) Can you think of a way to enlarge it by a scale factor of 4? Describe your plan.

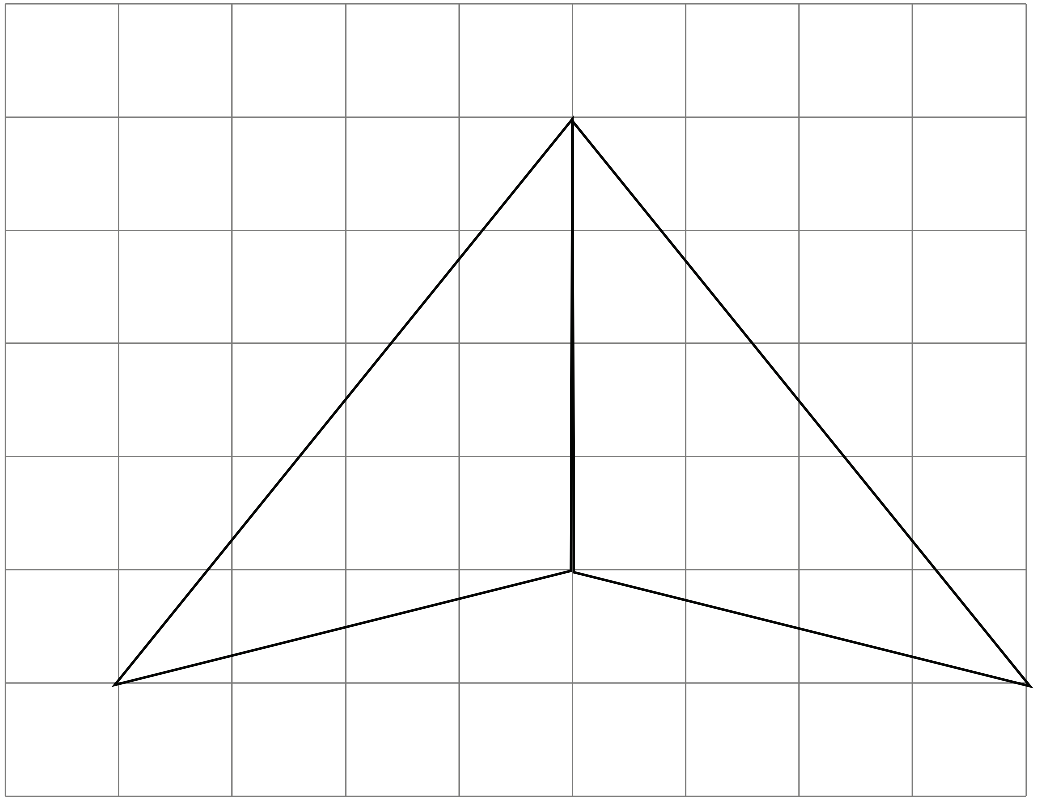
350 UNIT 9: TRANSFORMATIONS

Teaching guidelines

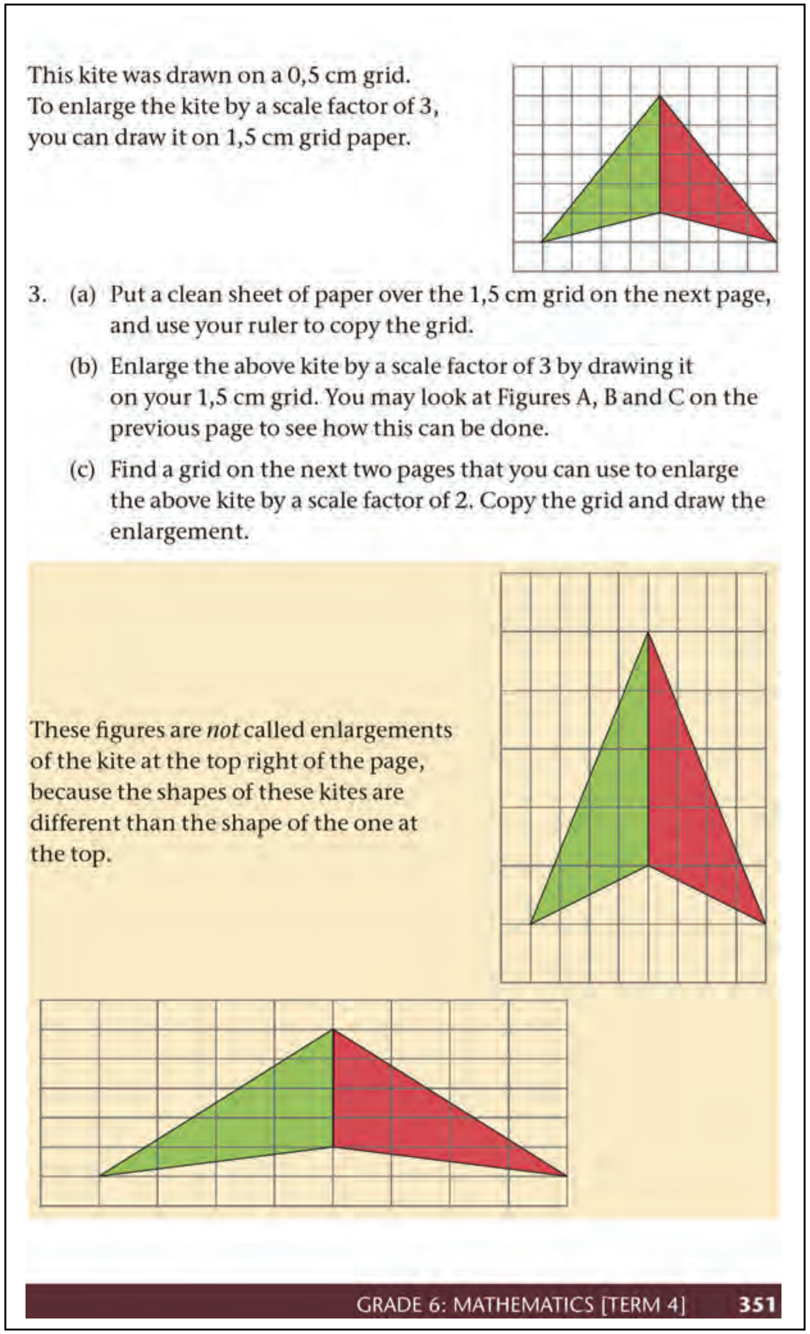
Explain to learners, with reference to the examples on pages 352 and 353, what is meant by grids with various cell sizes, for example 0,5 cm grids, 1 cm grids, 1,5 cm grids, etc.

Answers

3. (a)–(b)



(c) Answer on next page.



This kite was drawn on a 0,5 cm grid. To enlarge the kite by a scale factor of 3, you can draw it on 1,5 cm grid paper.

3. (a) Put a clean sheet of paper over the 1,5 cm grid on the next page, and use your ruler to copy the grid.
   (b) Enlarge the above kite by a scale factor of 3 by drawing it on your 1,5 cm grid. You may look at Figures A, B and C on the previous page to see how this can be done.
   (c) Find a grid on the next two pages that you can use to enlarge the above kite by a scale factor of 2. Copy the grid and draw the enlargement.

These figures are *not* called enlargements of the kite at the top right of the page, because the shapes of these kites are different than the shape of the one at the top.

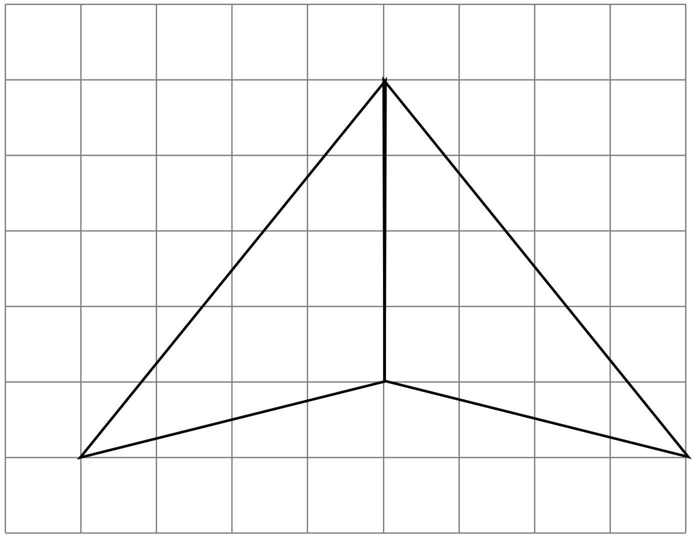
Teaching guidelines

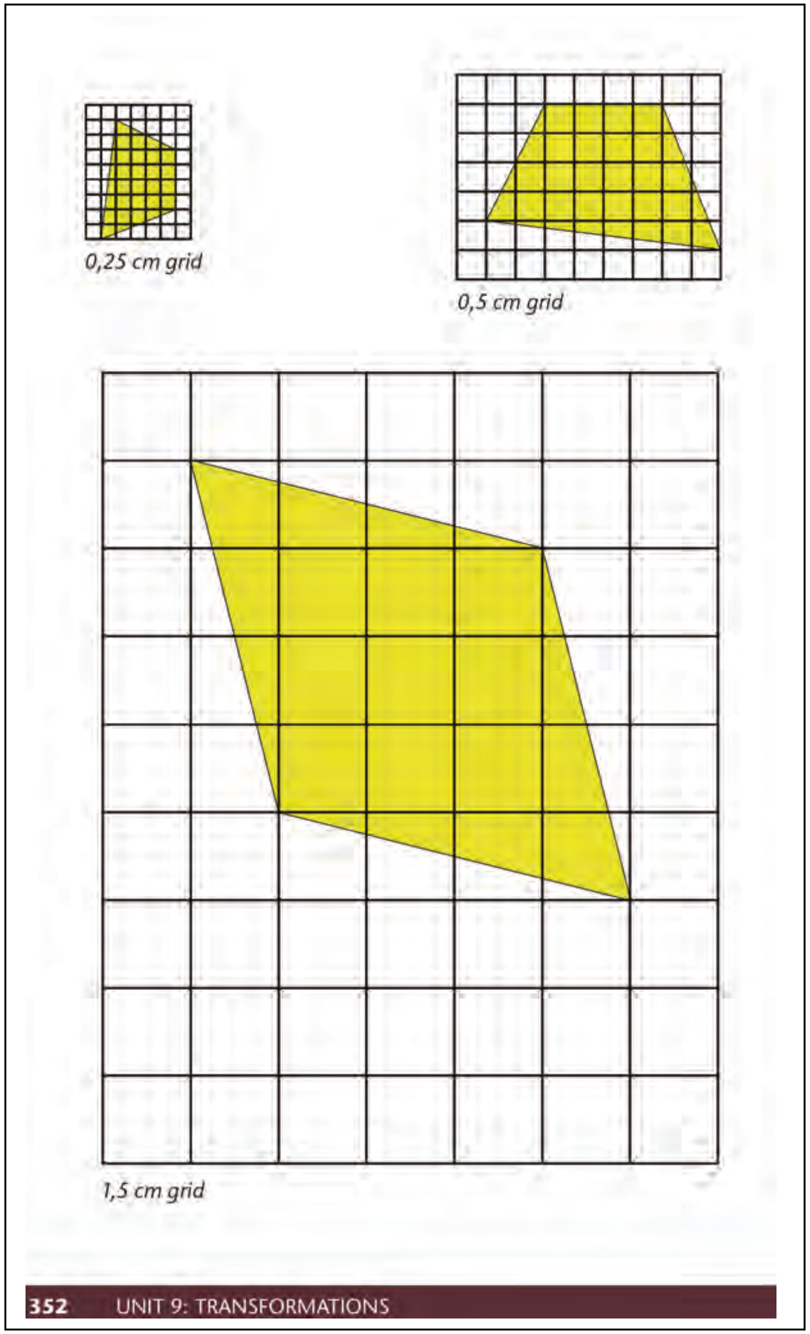
As mentioned on the previous page, the grids on pages 352 and 353 of the Learner Book are provided to demonstrate what is meant by grids of different cell sizes, and to provide templates that learners can copy by tracing.

You can also let learners measure the side lengths of the quadrilateral on the different grids and do calculations to verify that the lengths are decreased or increased by scale factors.

Answers *(continued)*

3. (c)



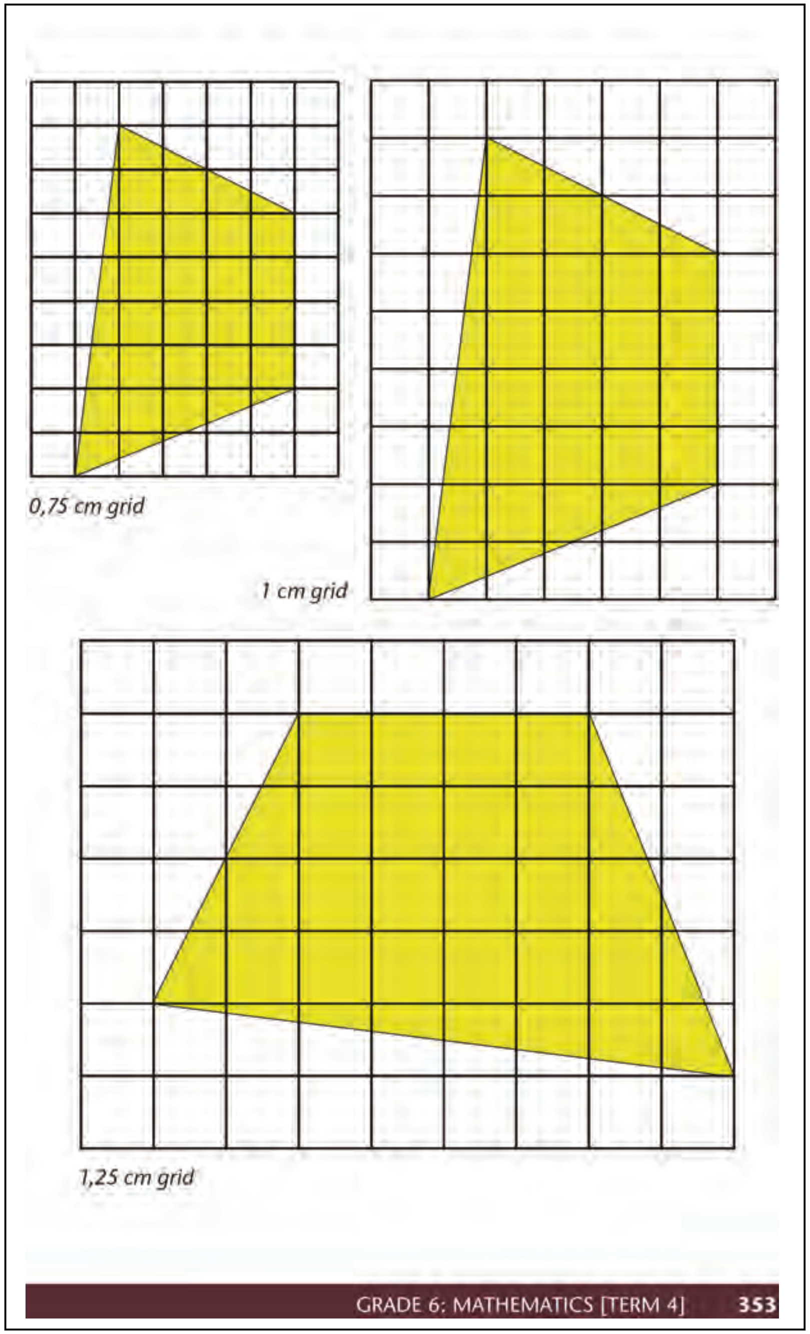


0,25 cm grid

0,5 cm grid

1,5 cm grid

352 UNIT 9: TRANSFORMATIONS



0,75 cm grid

1 cm grid

1,25 cm grid

GRADE 6: MATHEMATICS [TERM 4] 353

Teaching guidelines

Once learners have completed question 5, it may be useful to represent the answers as follows on the board:

5\. (a) $6 \longrightarrow \boxed{+3} \longrightarrow 9$ and $8 \longrightarrow \boxed{+3} \longrightarrow 11$

(b) $6 \longrightarrow \boxed{\times 1{,}5} \longrightarrow 9$ and $8 \longrightarrow \boxed{\times 1{,}5} \longrightarrow 12$

(c) $6 \longrightarrow \boxed{+8} \longrightarrow 14$ and $8 \longrightarrow \boxed{+8} \longrightarrow 16$

(d) $6 \longrightarrow \boxed{\times 2} \longrightarrow 12$ and $8 \longrightarrow \boxed{\times 2} \longrightarrow 16$

The changes in 5(a) and (c) cannot result from multiplying the length and the width by the same number, hence they are not examples of scaling.

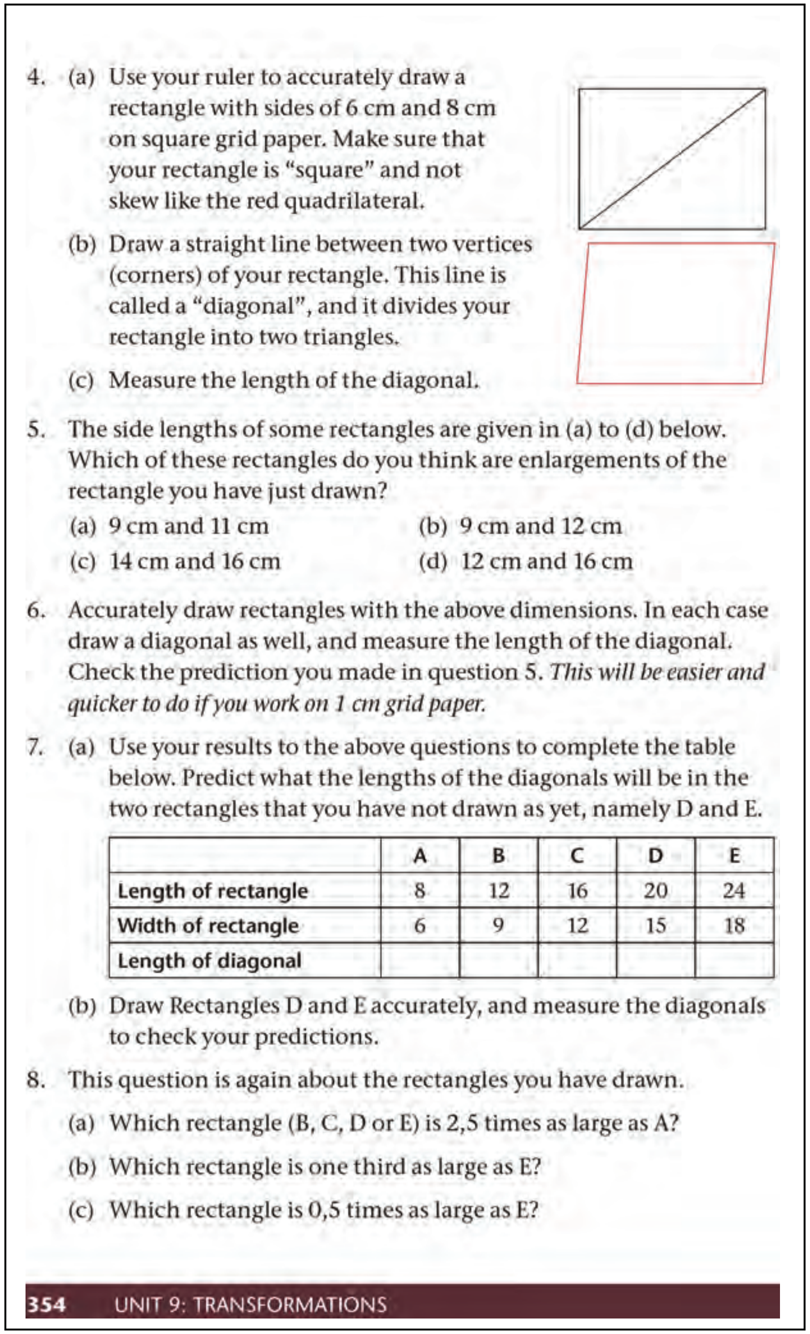
Answers

4. (a)–(c) Learners' own work
5. (b) and (d) are enlargements.
6. Learners' own work
7. (a) Learners predict the lengths of the diagonals and complete the tables.

	A	B	C	D	E
Length of rectangle	8	12	16	20	24
Width of rectangle	6	9	12	15	18
Length of diagonal	5,3	7,9	10,6	13,2	18,8

(b) Learners draw Rectangles D and E, using the measurements given, and measure the diagonals.

8. (a) D
   (b) A
   (c) B



4\. (a) Use your ruler to accurately draw a rectangle with sides of 6 cm and 8 cm on square grid paper. Make sure that your rectangle is "square" and not skew like the red quadrilateral.

(b) Draw a straight line between two vertices (corners) of your rectangle. This line is called a "diagonal", and it divides your rectangle into two triangles.

(c) Measure the length of the diagonal.

5\. The side lengths of some rectangles are given in (a) to (d) below. Which of these rectangles do you think are enlargements of the rectangle you have just drawn?

(a) 9 cm and 11 cm
(b) 9 cm and 12 cm
(c) 14 cm and 16 cm
(d) 12 cm and 16 cm

6\. Accurately draw rectangles with the above dimensions. In each case draw a diagonal as well, and measure the length of the diagonal. Check the prediction you made in question 5. *This will be easier and quicker to do if you work on 1 cm grid paper.*

7\. (a) Use your results to the above questions to complete the table below. Predict what the lengths of the diagonals will be in the two rectangles that you have not drawn as yet, namely D and E.

	A	B	C	D	E
Length of rectangle	8	12	16	20	24
Width of rectangle	6	9	12	15	18
Length of diagonal					

(b) Draw Rectangles D and E accurately, and measure the diagonals to check your predictions.

8\. This question is again about the rectangles you have drawn.

(a) Which rectangle (B, C, D or E) is 2,5 times as large as A?

(b) Which rectangle is one third as large as E?

(c) Which rectangle is 0,5 times as large as E?

354 UNIT 9: TRANSFORMATIONS

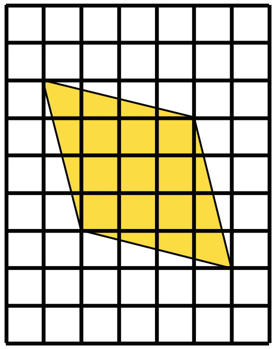
9.2 Enlargements and reductions

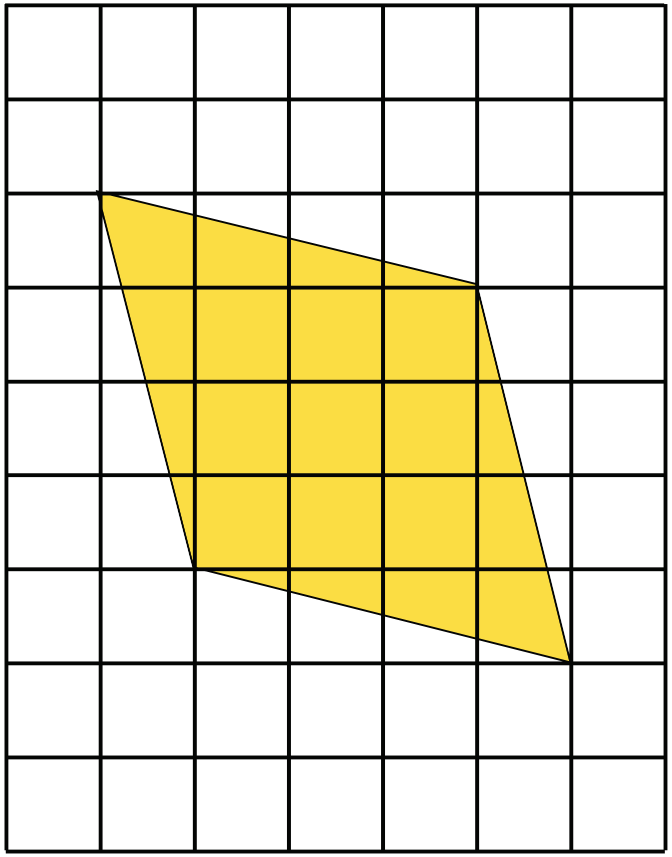
Mathematical notes

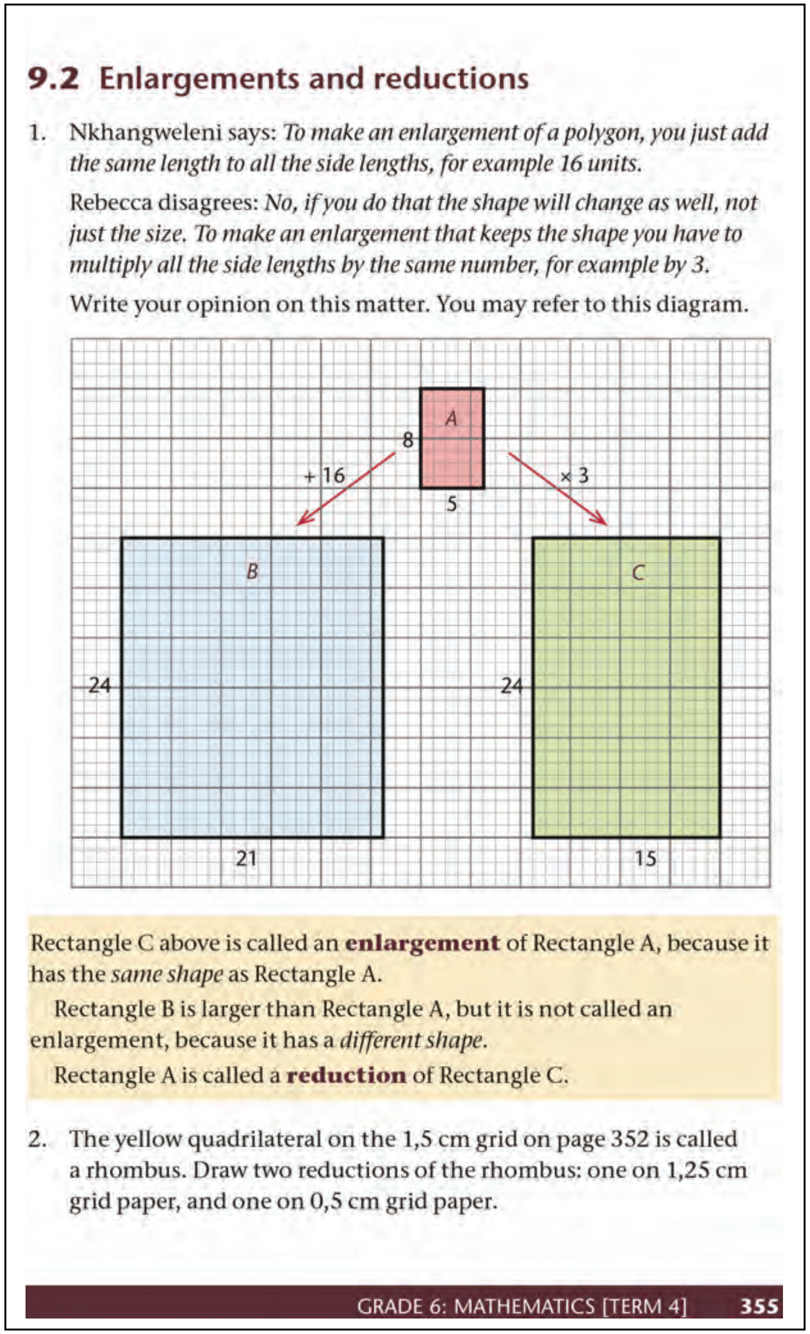
An enlargement involves a scale factor that is greater than 1. A reduction involves a scale factor that is between 0 and 1. So, a scale factor of $\frac{4}{5}$ produces a reduction, with each side 0,8 times the length of the original, while a scale factor of $\frac{5}{4}$ produces an enlargement with each of the sides 1,2 times the length of the original. A scale factor of 1 would leave the shape unchanged.

Answers

1. Learners' own work
2. 1,25 cm grid on the right;
   0,5 cm grid below







9.2 Enlargements and reductions

1. Nkhangweleni says: *To make an enlargement of a polygon, you just add the same length to all the side lengths, for example 16 units.*

   Rebecca disagrees: *No, if you do that the shape will change as well, not just the size. To make an enlargement that keeps the shape you have to multiply all the side lengths by the same number, for example by 3.*

   Write your opinion on this matter. You may refer to this diagram.

Rectangle C above is called an **enlargement** of Rectangle A, because it has the *same shape* as Rectangle A.

Rectangle B is larger than Rectangle A, but it is not called an enlargement, because it has a *different shape*.

Rectangle A is called a **reduction** of Rectangle C.

2. The yellow quadrilateral on the 1,5 cm grid on page 352 is called a rhombus. Draw two reductions of the rhombus: one on 1,25 cm grid paper, and one on 0,5 cm grid paper.

GRADE 6: MATHEMATICS [TERM 4] 355

Mathematical notes

A reduction by a factor of 3 (see shaded passage) can also be described as an enlargement by a factor of $\frac{1}{3}$.

Teaching guidelines

You may utilise the fact that a reduction by a factor of 3 can also be described as an enlargement by a factor of $\frac{1}{3}$ to reinforce the relationship between fractions and division.

To calculate the lengths (distances between points) when a shape is reduced by a factor of 3, you can divide the lengths of the original by 3, or you can calculate $\frac{1}{3}$ of each of the lengths.

Answers

3. (a) No
   (b) Yes
   (c) Yes
4. (a) 0,25 cm grid
   (b) 0,75 cm grid
   (c) 0,75 cm grid
   (d) 1,25 cm grid
   (e) 1 cm grid
5. See next page.
6. Yes, the length of each side is increased or decreased by the same scale factor.

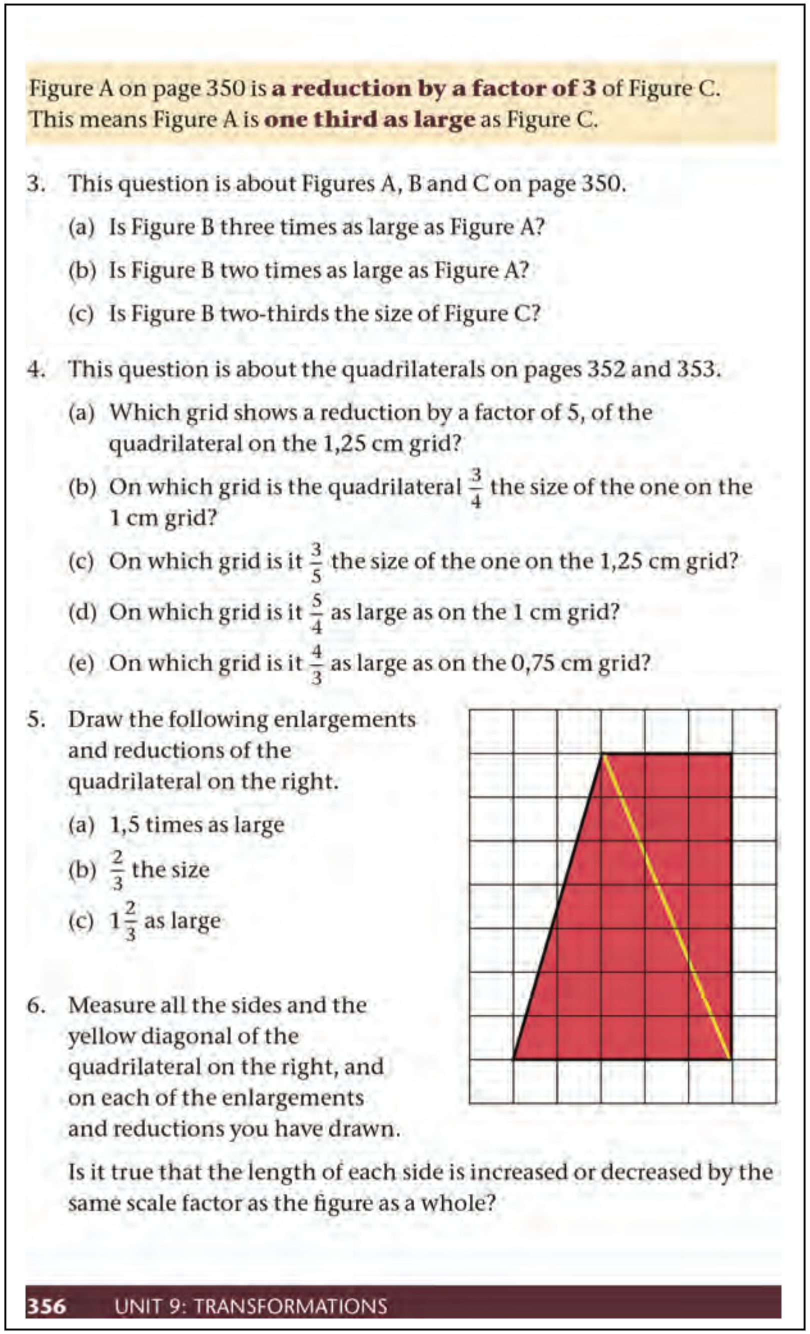


Figure A on page 350 is **a reduction by a factor of 3** of Figure C.
This means Figure A is **one third as large** as Figure C.

3. This question is about Figures A, B and C on page 350.
   (a) Is Figure B three times as large as Figure A?
   (b) Is Figure B two times as large as Figure A?
   (c) Is Figure B two-thirds the size of Figure C?

4. This question is about the quadrilaterals on pages 352 and 353.
   (a) Which grid shows a reduction by a factor of 5, of the quadrilateral on the 1,25 cm grid?
   (b) On which grid is the quadrilateral $\frac{3}{4}$ the size of the one on the 1 cm grid?
   (c) On which grid is it $\frac{3}{5}$ the size of the one on the 1,25 cm grid?
   (d) On which grid is it $\frac{5}{4}$ as large as on the 1 cm grid?
   (e) On which grid is it $\frac{4}{3}$ as large as on the 0,75 cm grid?

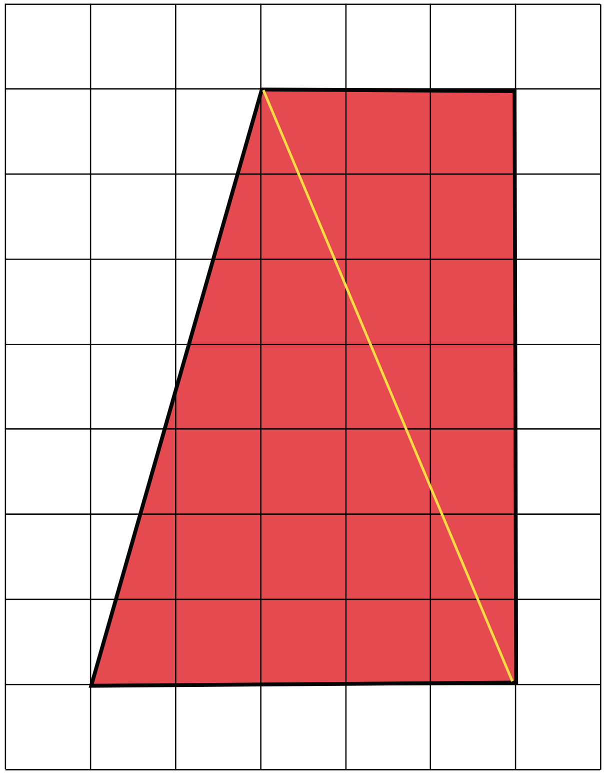
5. Draw the following enlargements and reductions of the quadrilateral on the right.
   (a) 1,5 times as large
   (b) $\frac{2}{3}$ the size
   (c) $1\frac{2}{3}$ as large

6. Measure all the sides and the yellow diagonal of the quadrilateral on the right, and on each of the enlargements and reductions you have drawn.

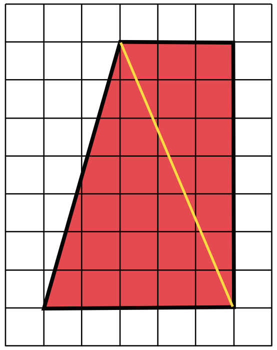
   Is it true that the length of each side is increased or decreased by the same scale factor as the figure as a whole?

Answers *(continued)*

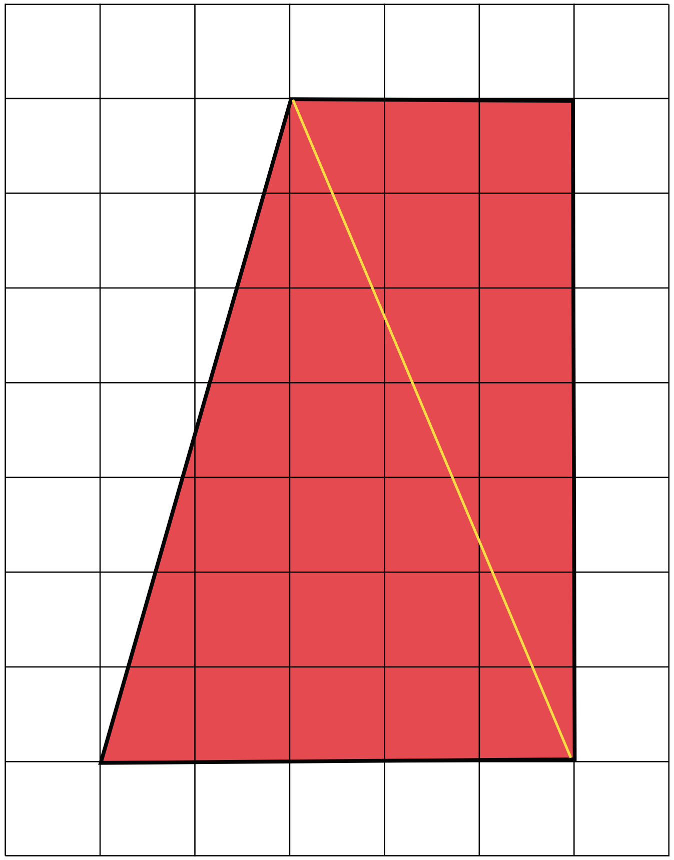
5. (a)



(b)



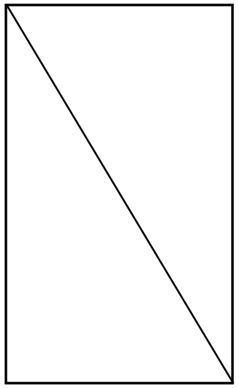
(c)



9.3 Increasing the lengths of two sides only

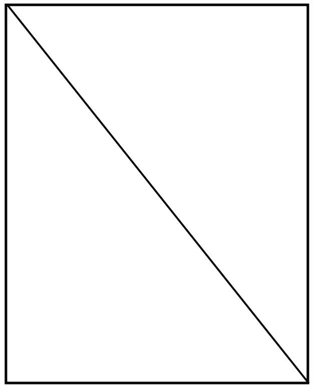
Answers

1. (a)



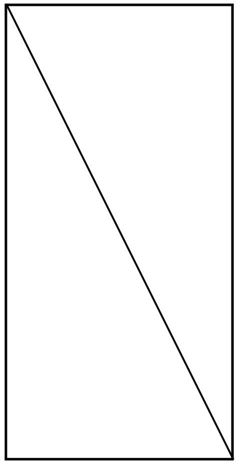
Diagonal is 5,8 cm long.

(b)



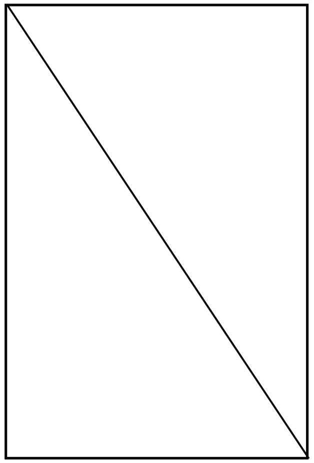
Diagonal is 6,4 cm long.

(c)



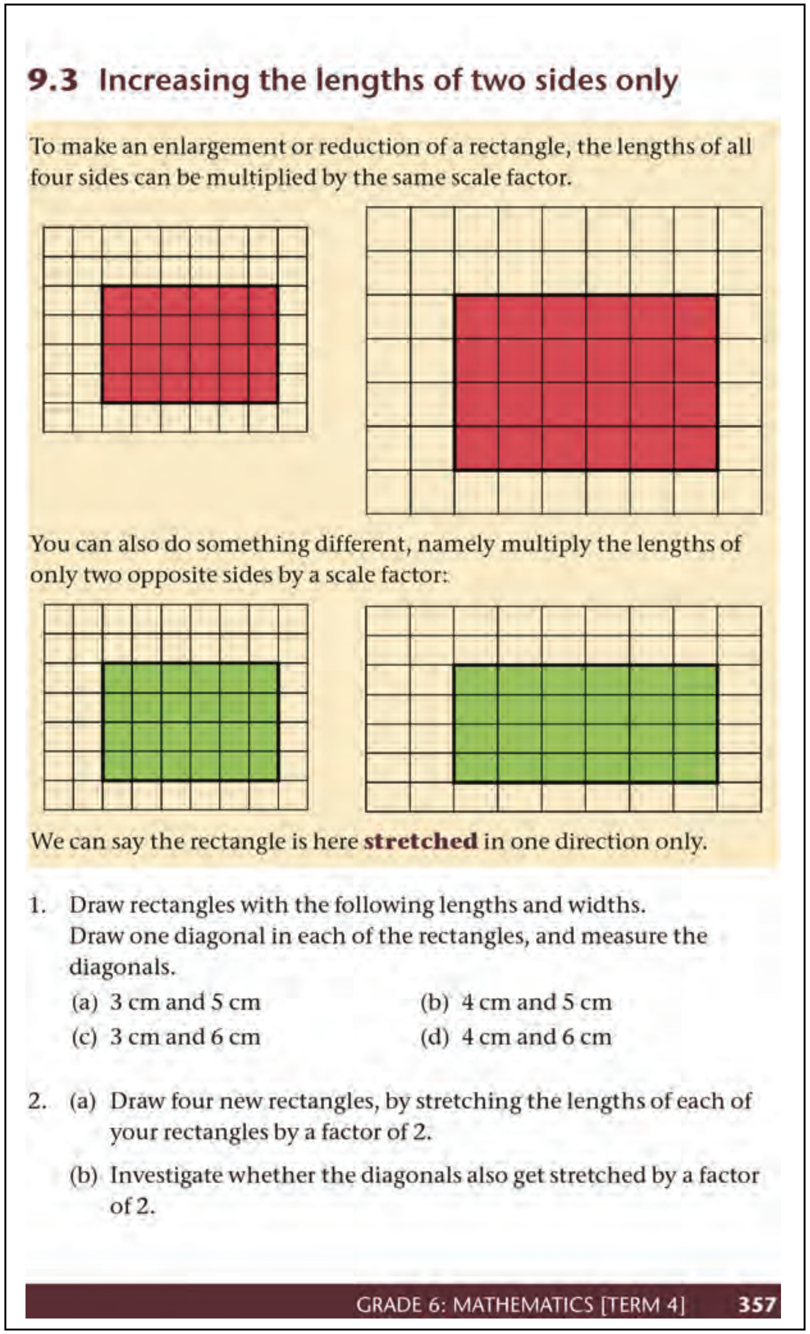
Diagonal is 6,7 cm long.

(d)



Diagonal is 7,2 cm long.

2. See next page.



9.3 Increasing the lengths of two sides only

To make an enlargement or reduction of a rectangle, the lengths of all four sides can be multiplied by the same scale factor.

You can also do something different, namely multiply the lengths of only two opposite sides by a scale factor:

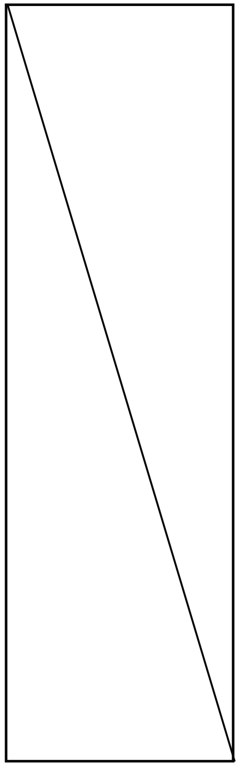
We can say the rectangle is here **stretched** in one direction only.

1. Draw rectangles with the following lengths and widths. Draw one diagonal in each of the rectangles, and measure the diagonals.
   (a) 3 cm and 5 cm
   (b) 4 cm and 5 cm
   (c) 3 cm and 6 cm
   (d) 4 cm and 6 cm

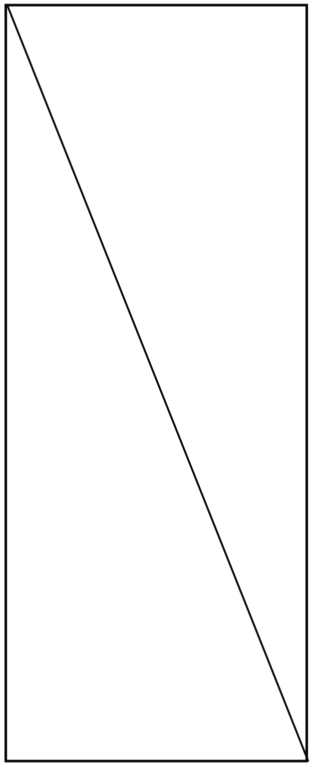
2. (a) Draw four new rectangles, by stretching the lengths of each of your rectangles by a factor of 2.
   (b) Investigate whether the diagonals also get stretched by a factor of 2.

Answers *(continued)*

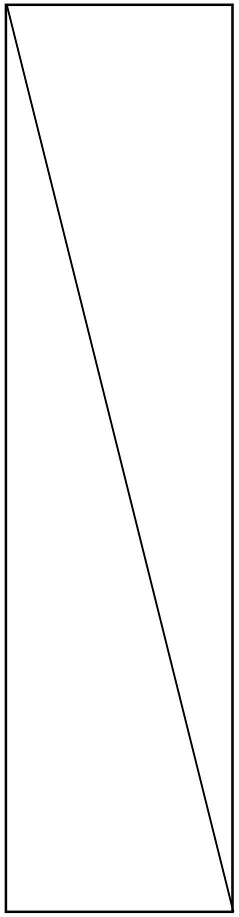
2. (a)



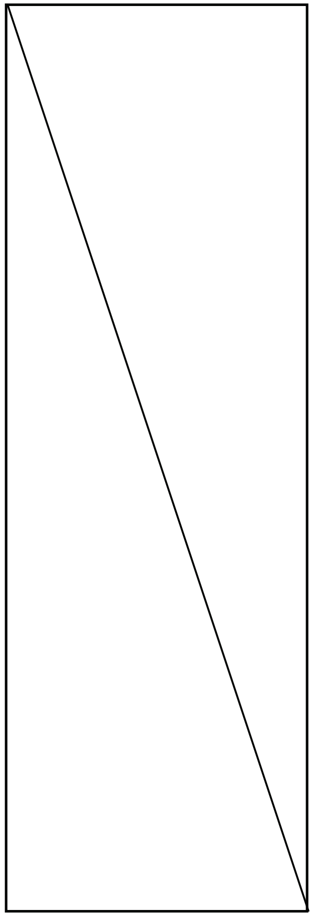
Diagonal is 10,4 cm long.
In the original it was 5,8 cm long.



Diagonal is 7,2 cm long.
In the original it was 6,4 cm long.



Diagonal is 12,4 cm long.
In the original it was 6,7 cm long.



Diagonal is 12,7 cm long.
In the original it was 7,2 cm long.

(b) No, the diagonals don't get stretched by a factor of 2.

Grade 6 Term 4 Unit 10 Position and movement

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
10.1 Locate positions on a grid	Giving locations (“addresses”) of objects that are laid out on a grid	358 to 359
10.2 Giving directions using a map	Directions to get from one location to another	360 to 362

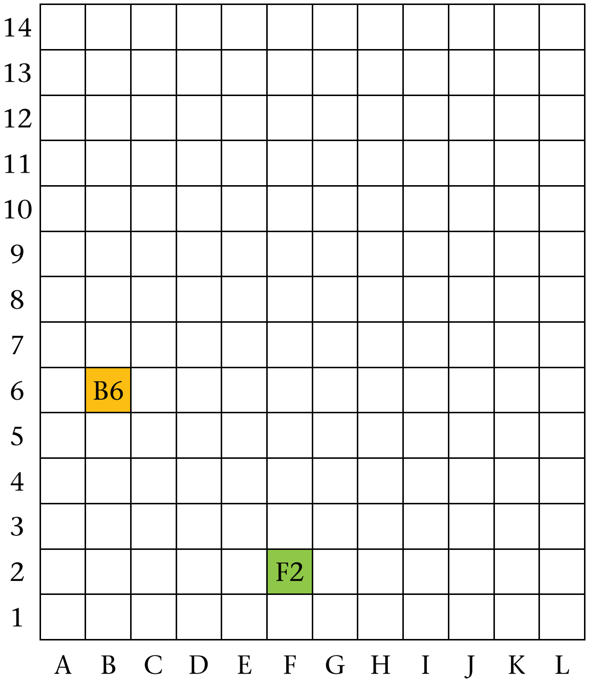
CAPS time allocation	2 hours
CAPS page references	24 and 288

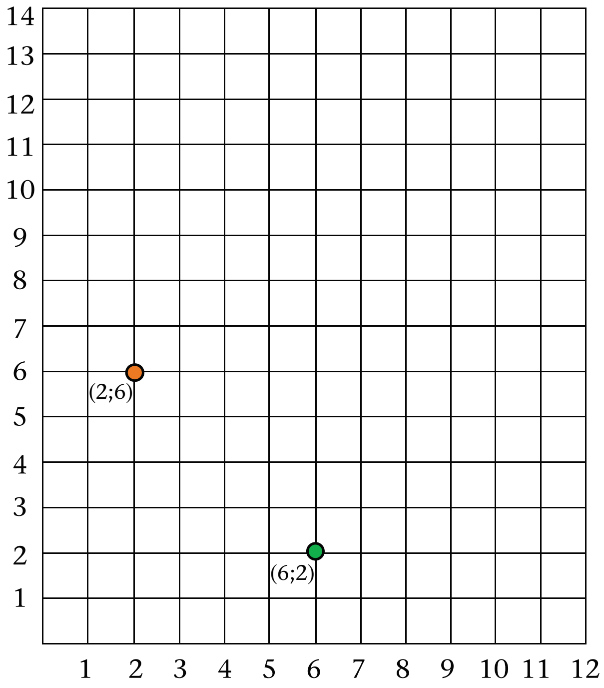
Mathematical background

Square grids are used in Mathematics as well as Geography to represent positions and movements. Each cell on a square grid has an “address” that is specified in terms of its position in relation to the so-called “axes”, as demonstrated below.

The grid on the left shows **alpha-numeric addresses**, which are used in Social Sciences (Geography) and Intermediate Phase Mathematics.

The grid on the right shows **Cartesian coordinate addresses**, such as used in Mathematics from Grade 7 onwards.





Resources

Square grid paper

10.1 Locate positions on a grid

Mathematical notes

This section asks learners to find the locations ("addresses") of objects according to a grid, and to use grid locations to determine what is located at those positions. These two actions are opposites of each other.

Teaching guidelines

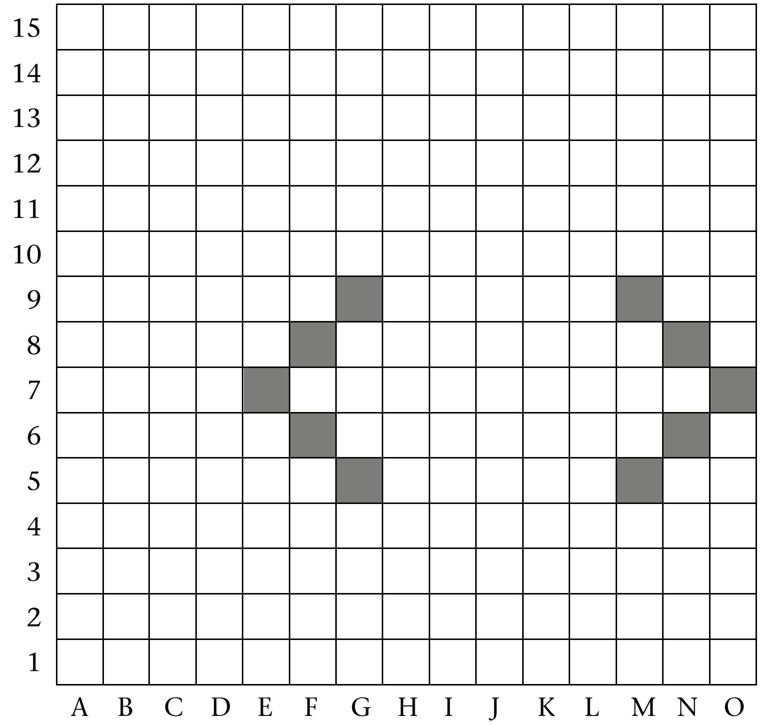
Although many learners may be able to engage effectively with the questions straightaway, you may draw a grid on the board and do some examples. Point out to learners that they work with grids and grid references in Geography too. Preferably, you should let learners take out their Social Sciences textbooks and look up some maps, so that they can see the correspondence with the work in this section.

To save classroom time, you could photocopy the grid for question 4 on page 448 in the Addendum.

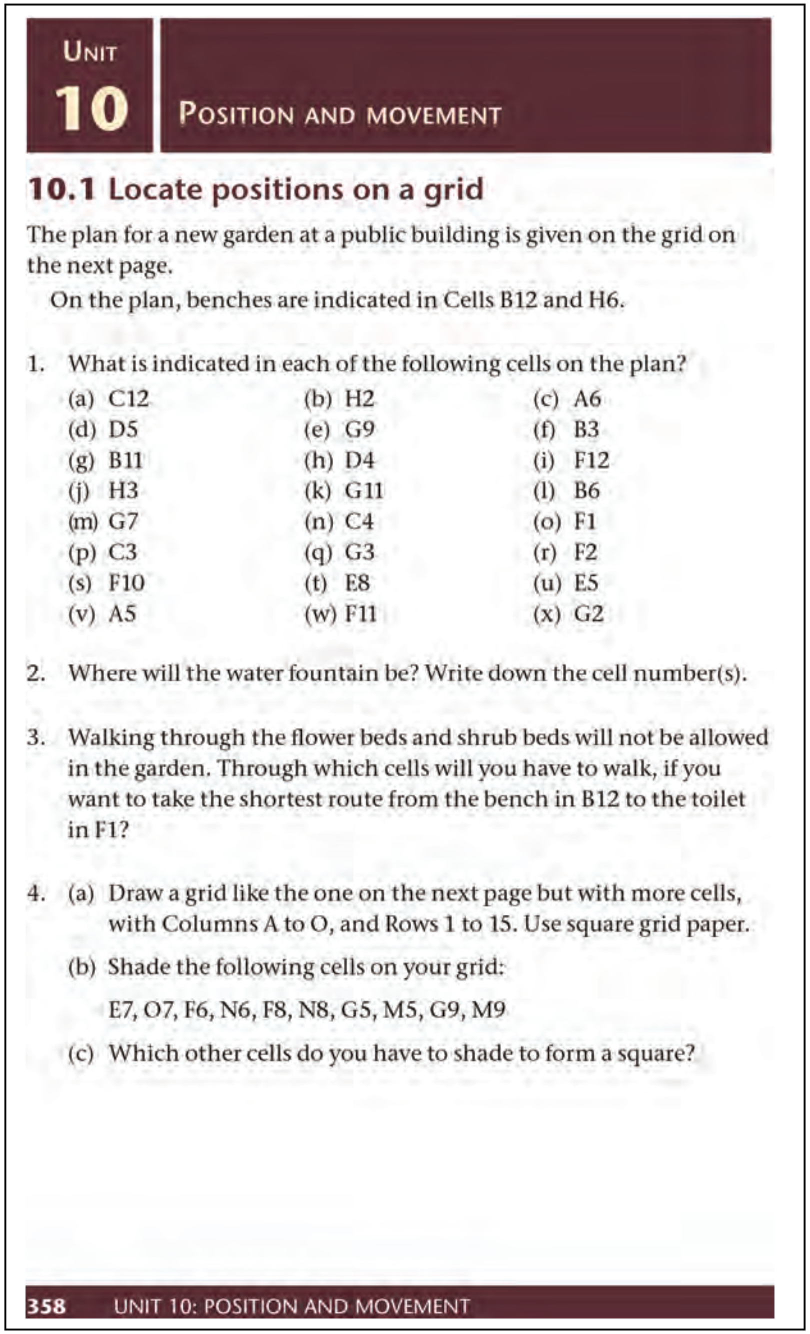
Answers

1.–3. See next page. (Questions 1 to 3 are repeated on the next page for your convenience.)

4. (a)–(b)



(c) H4 and H10; I3 and I11; J2 and J12; K3 and K11; L4 and L10



UNIT **10** POSITION AND MOVEMENT

10.1 Locate positions on a grid

The plan for a new garden at a public building is given on the grid on the next page.

On the plan, benches are indicated in Cells B12 and H6.

1. What is indicated in each of the following cells on the plan?
   (a) C12
   (b) H2
   (c) A6
   (d) D5
   (e) G9
   (f) B3
   (g) B11
   (h) D4
   (i) F12
   (j) H3
   (k) G11
   (l) B6
   (m) G7
   (n) C4
   (o) F1
   (p) C3
   (q) G3
   (r) F2
   (s) F10
   (t) E8
   (u) E5
   (v) A5
   (w) F11
   (x) G2

2. Where will the water fountain be? Write down the cell number(s).

3. Walking through the flower beds and shrub beds will not be allowed in the garden. Through which cells will you have to walk, if you want to take the shortest route from the bench in B12 to the toilet in F1?

4. (a) Draw a grid like the one on the next page but with more cells, with Columns A to O, and Rows 1 to 15. Use square grid paper.
   (b) Shade the following cells on your grid:
   E7, O7, F6, N6, F8, N8, G5, M5, G9, M9
   (c) Which other cells do you have to shade to form a square?

Questions (repeated for your convenience)

1. What is indicated in each of the following cells on the plan?

(a) C12	(b) H2	(c) A6
(d) D5	(e) G9	(f) B3
(g) B11	(h) D4	(i) F12
(j) H3	(k) G11	(l) B6
(m) G7	(n) C4	(o) F1
(p) C3	(q) G3	(r) F2
(s) F10	(t) E8	(u) E5
(v) A5	(w) F11	(x) G2

2. Where will the water fountain be? Write down the cell number(s).

3. Walking through the flower beds and shrub beds will not be allowed in the garden. Through which cells will you have to walk, if you want to take the shortest route from the bench in B12 to the toilet in F1?

Answers to questions 1 to 3

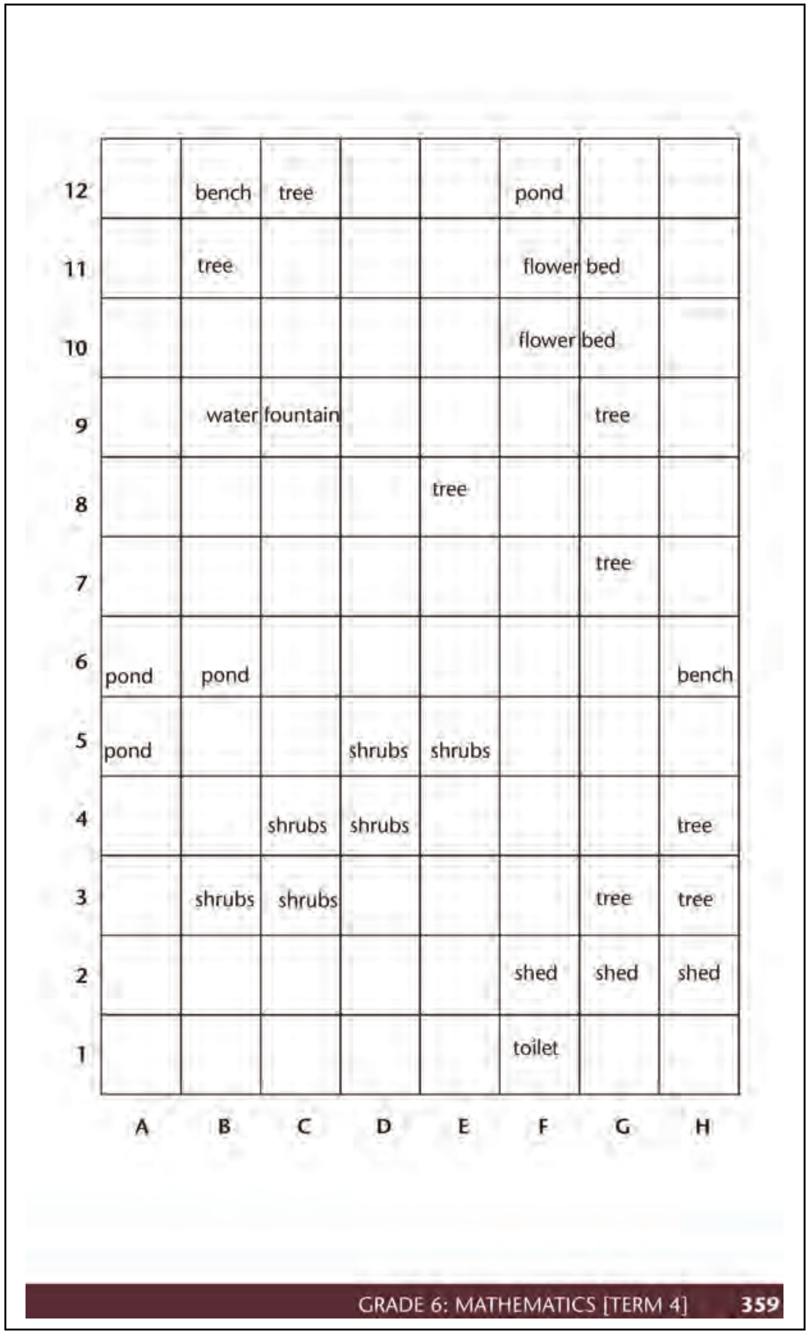
1.

(a) Tree	(b) Shed	(c) Pond	(d) Shrubs
(e) Tree	(f) Shrubs	(g) Tree	(h) Shrubs
(i) Pond	(j) Tree	(k) Flower bed	(l) Pond
(m) Tree	(n) Shrubs	(o) Toilet	(p) Shrubs
(q) Tree	(r) Shed	(s) Flowerbed	(t) Tree
(u) Shrubs	(v) Pond	(w) Flowerbed	(x) Shed

2. B9 and C9

3. Check learners' suggestions, e.g. A12, A11, A10, A9, A8, A7, B7, C7, C6, D6, E6, F6, F5, F4, F3, E3, E2, E1, F1 (toilet)

4. (a)–(c) Learners' own practical work (see previous page).



12		bench	tree			pond		
11		tree				flower	bed	
10						flower	bed	
9		water	fountain				tree	
8					tree			
7							tree	
6	pond	pond						bench
5	pond			shrubs	shrubs			
4			shrubs	shrubs				tree
3		shrubs	shrubs				tree	tree
2						shed	shed	shed
1						toilet		
	A	B	C	D	E	F	G	H

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10.2 Giving directions using a map

Mathematical notes

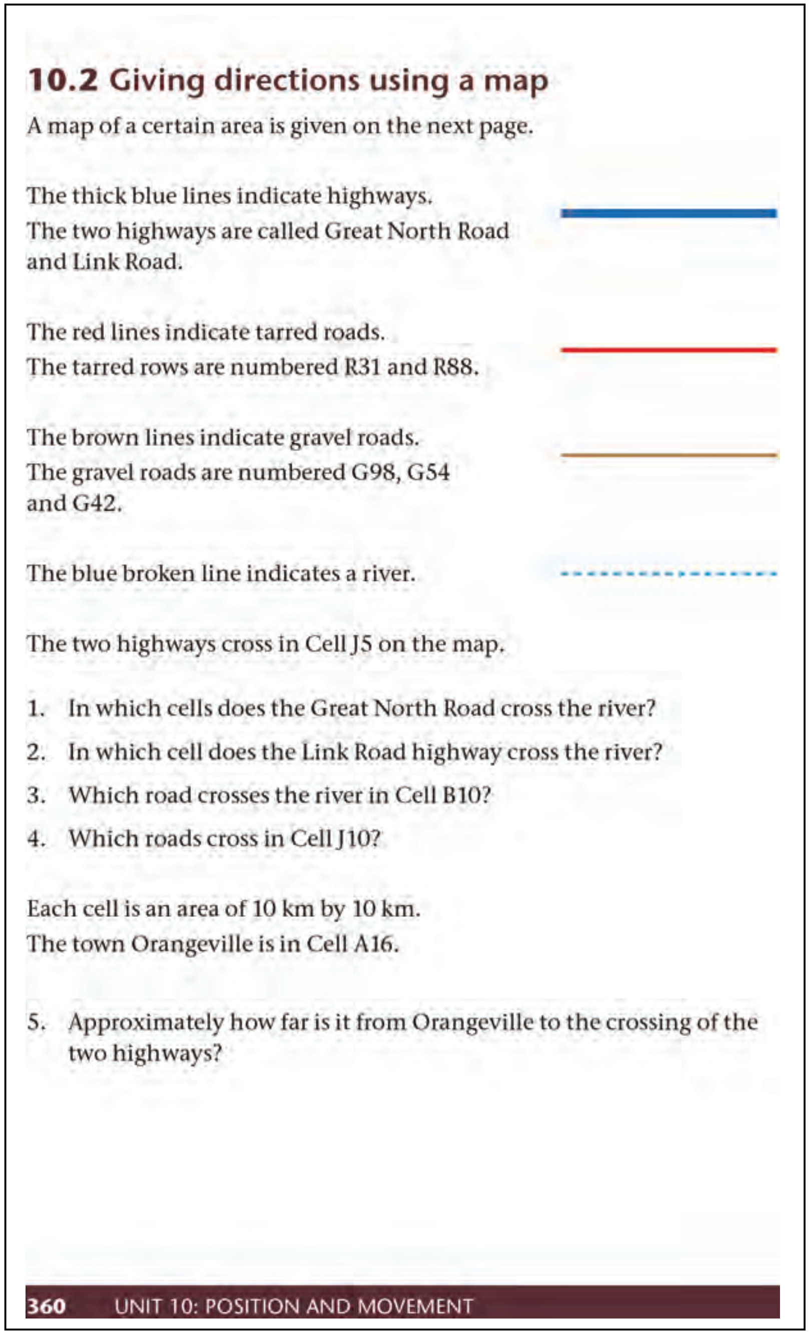
Map reading is an important basic life skill everyone should develop.

Teaching guidelines

This section emphasises giving clear instructions or descriptions on how to get from one place to another. If learners are vague or unfocused in their descriptions, encourage them to try to be clearer. It may help if pairs of learners "instruct" each other to follow their directions to see how well they describe the route.

Answers

1. F12 and H8
2. H4
3. G42
4. G42 and G98
5. Approximately 120 km to 130 km



10.2 Giving directions using a map

A map of a certain area is given on the next page.

The thick blue lines indicate highways. ▬

The two highways are called Great North Road and Link Road.

The red lines indicate tarred roads. ▬

The tarred rows are numbered R31 and R88.

The brown lines indicate gravel roads. ▬

The gravel roads are numbered G98, G54 and G42.

The blue broken line indicates a river. - - - - - - - -

The two highways cross in Cell J5 on the map.

1. In which cells does the Great North Road cross the river?
2. In which cell does the Link Road highway cross the river?
3. Which road crosses the river in Cell B10?
4. Which roads cross in Cell J10?

Each cell is an area of 10 km by 10 km.

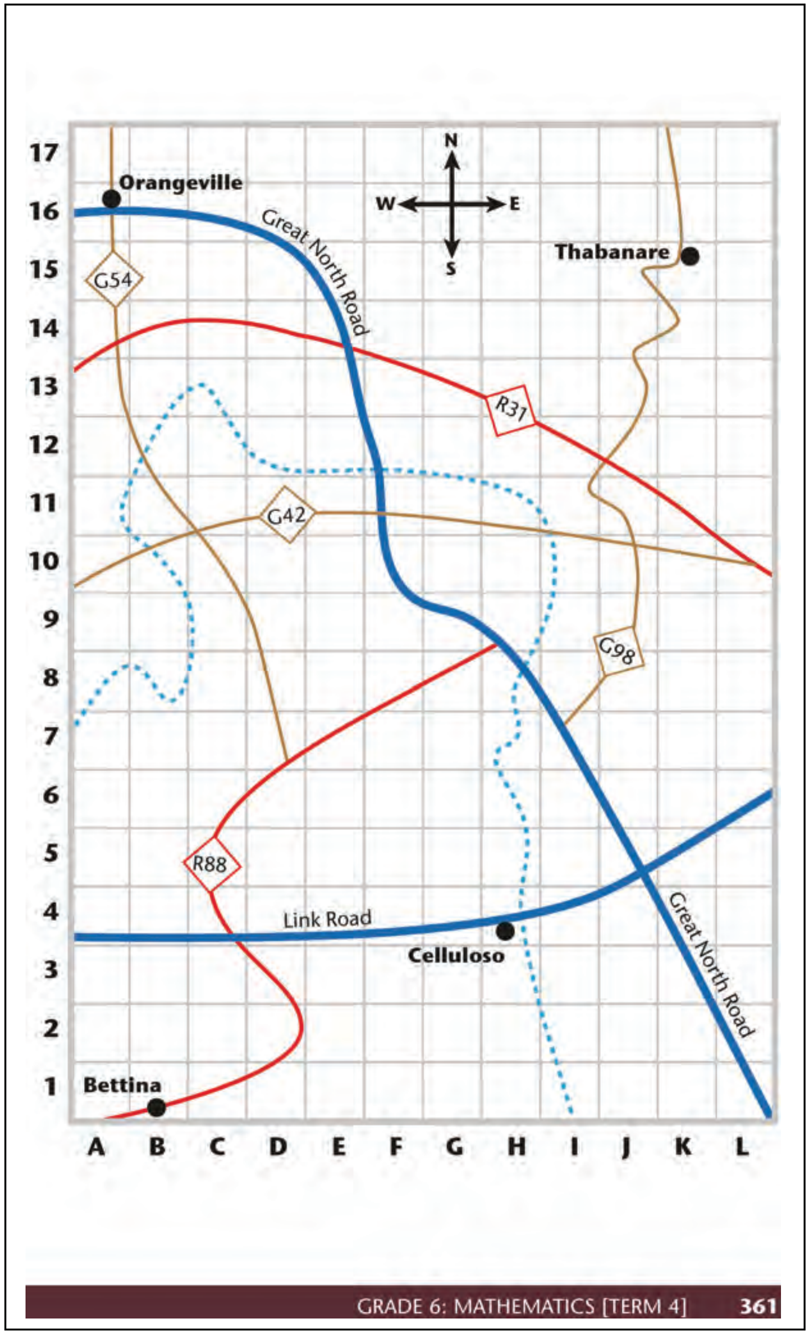
The town Orangeville is in Cell A16.

5. Approximately how far is it from Orangeville to the crossing of the two highways?

Answers

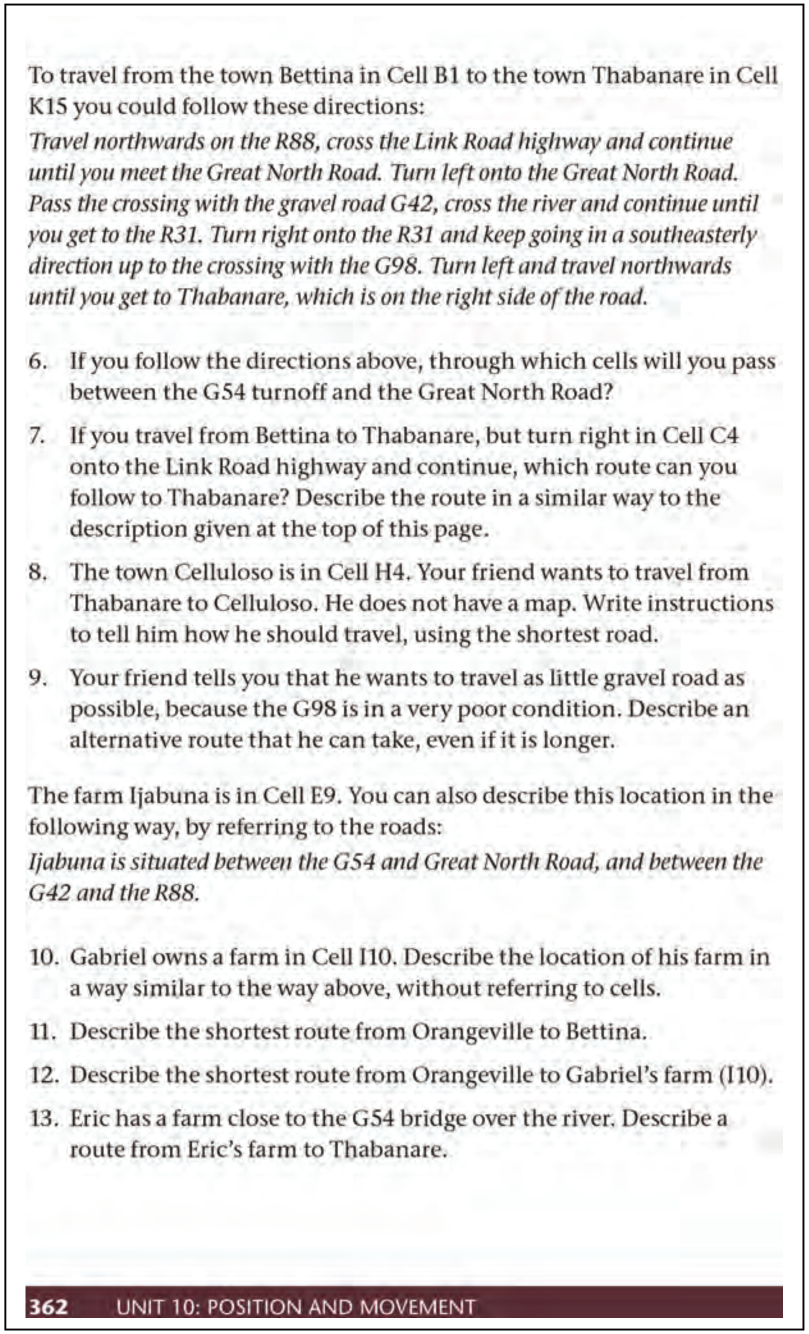
Please see the next page for the Learner Book questions *and* answers. For your convenience the answers are also given here, alongside the map.

6. D7; E7; F8; G8; H9
7. From Bettina continue north on the R88 until it crosses Link Road. Turn right onto Link Road. Continue on Link Road until it crosses Great North Road. Turn left onto Great North Road. Continue on Great North Road, then turn right onto the G98. Continue on the G98 up to Thabanare.
8. Learners' own work. Consider and discuss their options. Example: *From Thabanare travel south on the G98 until you reach Great North Road. Turn left onto Great North Road until your reach the crossing with Link Road. Turn right onto Link Road and continue to Celluloso.*
9. Learners' own work. Consider and discuss their options. Example: *From Thabanare travel south on the G98 to the crossing with the R31. Turn right onto the R31 and continue to the crossing with Great North Road. Turn left onto Great North Road and continue to the crossing with Link Road. Turn right onto Link Road and continue to Celluloso.*
10. Learners' own work. Consider and discuss their options. Example: *Gabriel's farm is between Great North Road and the G98, and south of the G42, just east of the river and west of the crossing between the G98 and the G42.*
11. Follow the G54 out of Orangeville, crossing the R31, the river and the G42, and continue to the T-junction with the R88. Turn right onto the R88, crossing Link Road and continue on the R88 up to Bettina.
12. Follow Great North Road south, cross the river and continue to the crossing with the G42. Turn left onto the G42 and follow the road until just over the river again. Look out for the farm road to your right.
13. Turn onto the G54 north and continue up to the crossing with the R31. Turn right onto the R31 and follow the road in the southeasterly direction, crossing Great North Road, and continue to the crossing with the G98. Turn left (in a northerly direction) and continue on the G98 up to Thabanare.



Answers *(as on previous page)*

6. D7; E7; F8; G8; H9
7. From Bettina continue north on the R88 until it crosses Link Road. Turn right onto Link Road. Continue on Link Road until it crosses Great North Road. Turn left onto Great North Road. Continue on Great North Road, then turn right onto the G98. Continue on the G98 up to Thabanare.
8. Learners' own work. Consider and discuss their options. Example: *From Thabanare travel south on the G98 until you reach Great North Road. Turn left onto Great North Road until your reach the crossing with Link Road. Turn right onto Link Road and continue to Celluloso.*
9. Learners' own work. Consider and discuss their options. Example: *From Thabanare travel south on the G98 to the crossing with the R31. Turn right onto the R31 and continue to the crossing with Great North Road. Turn left onto Great North Road and continue to the crossing with Link Road. Turn right onto Link Road and continue to Celluloso.*
10. Learners' own work. Consider and discuss their options. Example: *Gabriel's farm is between Great North Road and the G98, and south of the G42, just east of the river and west of the crossing between the G98 and the G42.*
11. Follow the G54 out of Orangeville, crossing the R31, the river and the G42, and continue to the T-junction with the R88. Turn right onto the R88, crossing Link Road and continue on the R88 up to Bettina.
12. Follow Great North Road south, cross the river and continue to the crossing with the G42. Turn left onto the G42 and follow the road until just over the river again. Look out for the farm road to your right.
13. Turn onto the G54 north and continue up to the crossing with the R31. Turn right onto the R31 and follow the road in the southeasterly direction, crossing Great North Road, and continue to the crossing with the G98. Turn left (in a northerly direction) and continue on the G98 up to Thabanare.



To travel from the town Bettina in Cell B1 to the town Thabanare in Cell K15 you could follow these directions:

Travel northwards on the R88, cross the Link Road highway and continue until you meet the Great North Road. Turn left onto the Great North Road. Pass the crossing with the gravel road G42, cross the river and continue until you get to the R31. Turn right onto the R31 and keep going in a southeasterly direction up to the crossing with the G98. Turn left and travel northwards until you get to Thabanare, which is on the right side of the road.

6. If you follow the directions above, through which cells will you pass between the G54 turnoff and the Great North Road?
7. If you travel from Bettina to Thabanare, but turn right in Cell C4 onto the Link Road highway and continue, which route can you follow to Thabanare? Describe the route in a similar way to the description given at the top of this page.
8. The town Celluloso is in Cell H4. Your friend wants to travel from Thabanare to Celluloso. He does not have a map. Write instructions to tell him how he should travel, using the shortest road.
9. Your friend tells you that he wants to travel as little gravel road as possible, because the G98 is in a very poor condition. Describe an alternative route that he can take, even if it is longer.

The farm Ijabuna is in Cell E9. You can also describe this location in the following way, by referring to the roads:

Ijabuna is situated between the G54 and Great North Road, and between the G42 and the R88.

10. Gabriel owns a farm in Cell I10. Describe the location of his farm in a way similar to the way above, without referring to cells.
11. Describe the shortest route from Orangeville to Bettina.
12. Describe the shortest route from Orangeville to Gabriel's farm (I10).
13. Eric has a farm close to the G54 bridge over the river. Describe a route from Eric's farm to Thabanare.

362 UNIT 10: POSITION AND MOVEMENT

Grade 6 Term 4 Unit 11 Probability

Learner Book Overview		
Sections in this unit	**Content**	**Pages in Learner Book**
11.1 Tossing a coin	Critical investigation of outcomes when tossing a coin	363
11.2 Spinner experiments	The same possible outcomes, but different areas give different probabilities	364 to 366
11.3 The Subtraction Game	Possible differences between the outcomes on two dice	367 to 368
11.4 The Addition Game	Possible sums of the outcomes on two dice	369

CAPS time allocation	2 hours
CAPS page references	31 and 289

Mathematical background

It sometimes makes sense to argue that one thing is more likely to happen than something else. For example, if you drop a glass on a cement floor, it is more likely to break than not to break. In such situations, there are valid grounds to believe that one possible outcome is more likely than another. There are also situations where no grounds exist to believe that one possible outcome is more likely than another. For example, when you roll a die, there are no grounds to believe that the outcome "4" (or any other of the six possible outcomes) is more likely to occur than any of the other five possible outcomes. Events like flipping a coin and rolling a die are called **random events**. A random event is an event with different possible outcomes, all of which are equally likely. The outcome of a random event such as rolling a die is **completely unpredictable**. The only thing that can be predicted with respect to a single repetition of the event is that one of the possible outcomes will happen.

Two serious misconceptions about random events are quite common:

- A statement like "The probability of getting a 4 when rolling a die is one sixth, or one out of six" is sometimes falsely interpreted to mean that a prediction about the outcome of a single repetition of an event can be made. Such a statement only provides information about what may be expected if the event is repeated many times, as explained below.
- The belief that in random events such as rolling a die, past events influence future events, leading to expectations such as "If I have rolled the die 50 times and 6 did not appear once, the chances are now very high that 6 will appear on the next roll." This is false.

Although the outcome of a single random event is unpredictable, some predictions can be made about the combined outcomes of many repetitions of a random event. For example, if an ordinary die is rolled many, many times, the number 4 (or any other number in the range 1, 2, 3, 4, 5, 6) can be expected to occur roughly one sixth of the time. Suppose another die, which is not marked 1, 2, 3, 4, 5, 6 on its six faces but red on one face, blue on two faces and yellow on three faces, is rolled many times: red can be safely predicted to land on top about one sixth of the time, blue to land on top roughly one third of the time and yellow to land on top roughly half of the time.

Random events ("probability") can be investigated **theoretically**, by arguing logically. For example, one may **argue** that if a die is rolled many times, roughly the same number of each of the six different possible outcomes may occur. Random events can also be investigated **empirically**, by **performing** the events repeatedly and **analysing the actual outcomes**. In this unit learners are engaged in both theoretical and empirical investigations of random events.

Resources

Coins, cardboard for making spinners, squared paper, dice

11.1 Tossing a coin

Teaching guidelines

Questions 1 and 2 are intended to help learners to distinguish between events where one may have reasonable grounds for predicting the outcomes, and random events, i.e. events where there are no reasonable grounds for predicting any specific outcome.

Assure learners that it is fine if they find question 2 difficult to answer. In question 3 they will do an investigation that will provide them with some understanding of the situation described in question 2.

When pairs of learners have finished question 3, write all their results on the board, as shown in the example on the right.

Heads	Tails
21	29
26	24
27	23
22	28
23	27
30	20
27	23

Put the following question to learners:

"Suppose we do this again, do you think you can say which of 'heads' or 'tails' will happen more often?"

Allow some class discussion. Learners should understand that there are no grounds on which they can make a prediction.

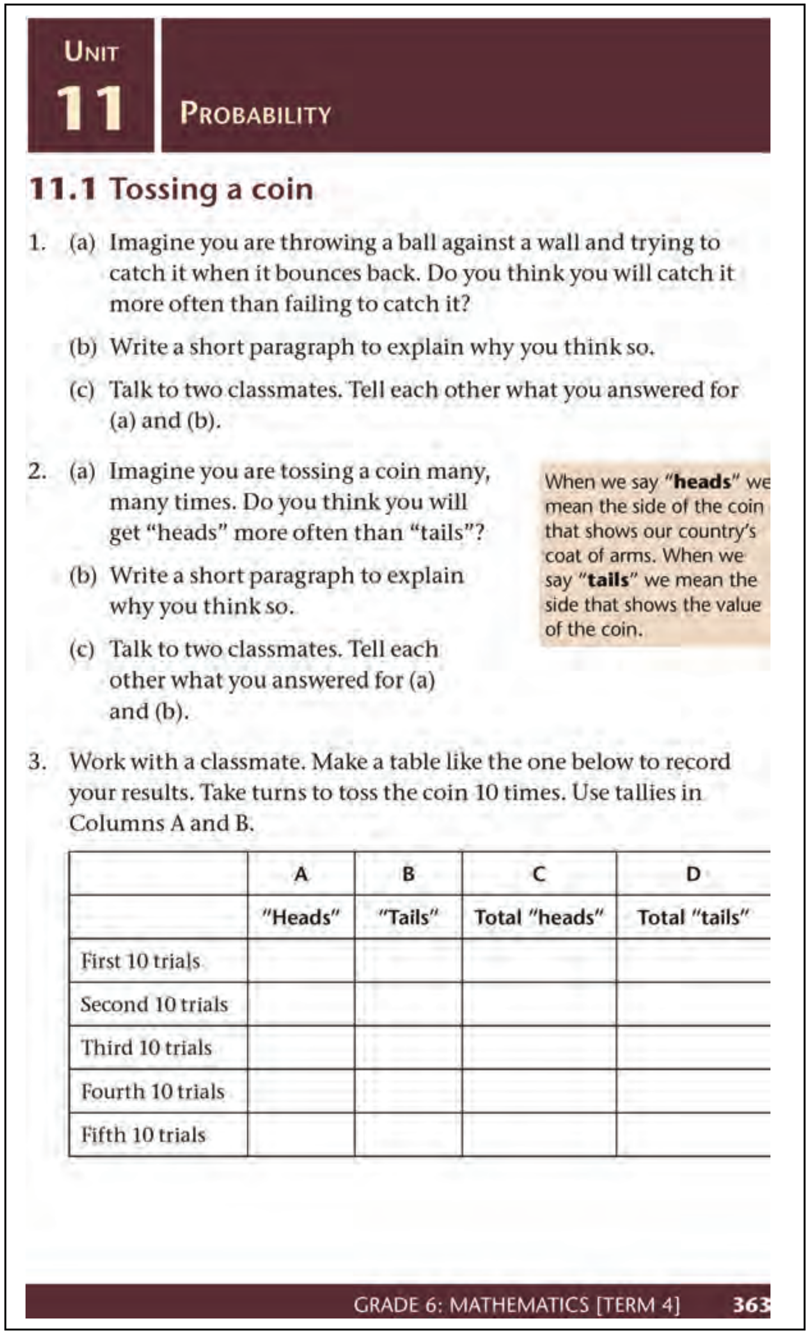
Put the following question to learners:

"What is different between the ball catching situation (question 1) and the coin tossing situation (question 2)?"

Again, allow some class discussion. Learners should come to understand that in the ball catching situation the person can influence the outcome, but not in the case of the coin-tossing situation.

Answers

1. (a) Learners have to state their opinion on the matter.
   (b) Learners may say things like "I am good at catching" or "I will throw the ball so that it is easy to catch". It does not really matter what they write. The purpose of the question is to make learners think of a situation where they believe they know what the outcomes will be.
2. (a) Hopefully learners will realise that they cannot answer the question.
   (b) Learners cannot really be expected to write "correct" reasons such as "heads and tails are equally likely", but some may write something to that effect though in a less formal way.
3. Learners' tables will differ.



UNIT 11

PROBABILITY

11.1 Tossing a coin

1. (a) Imagine you are throwing a ball against a wall and trying to catch it when it bounces back. Do you think you will catch it more often than failing to catch it?
   (b) Write a short paragraph to explain why you think so.
   (c) Talk to two classmates. Tell each other what you answered for (a) and (b).
2. (a) Imagine you are tossing a coin many, many times. Do you think you will get "heads" more often than "tails"?
   (b) Write a short paragraph to explain why you think so.
   (c) Talk to two classmates. Tell each other what you answered for (a) and (b).

When we say "**heads**" we mean the side of the coin that shows our country's coat of arms. When we say "**tails**" we mean the side that shows the value of the coin.

3. Work with a classmate. Make a table like the one below to record your results. Take turns to toss the coin 10 times. Use tallies in Columns A and B.

	A	B	C	D
	"Heads"	"Tails"	Total "heads"	Total "tails"
First 10 trials				
Second 10 trials				
Third 10 trials				
Fourth 10 trials				
Fifth 10 trials				

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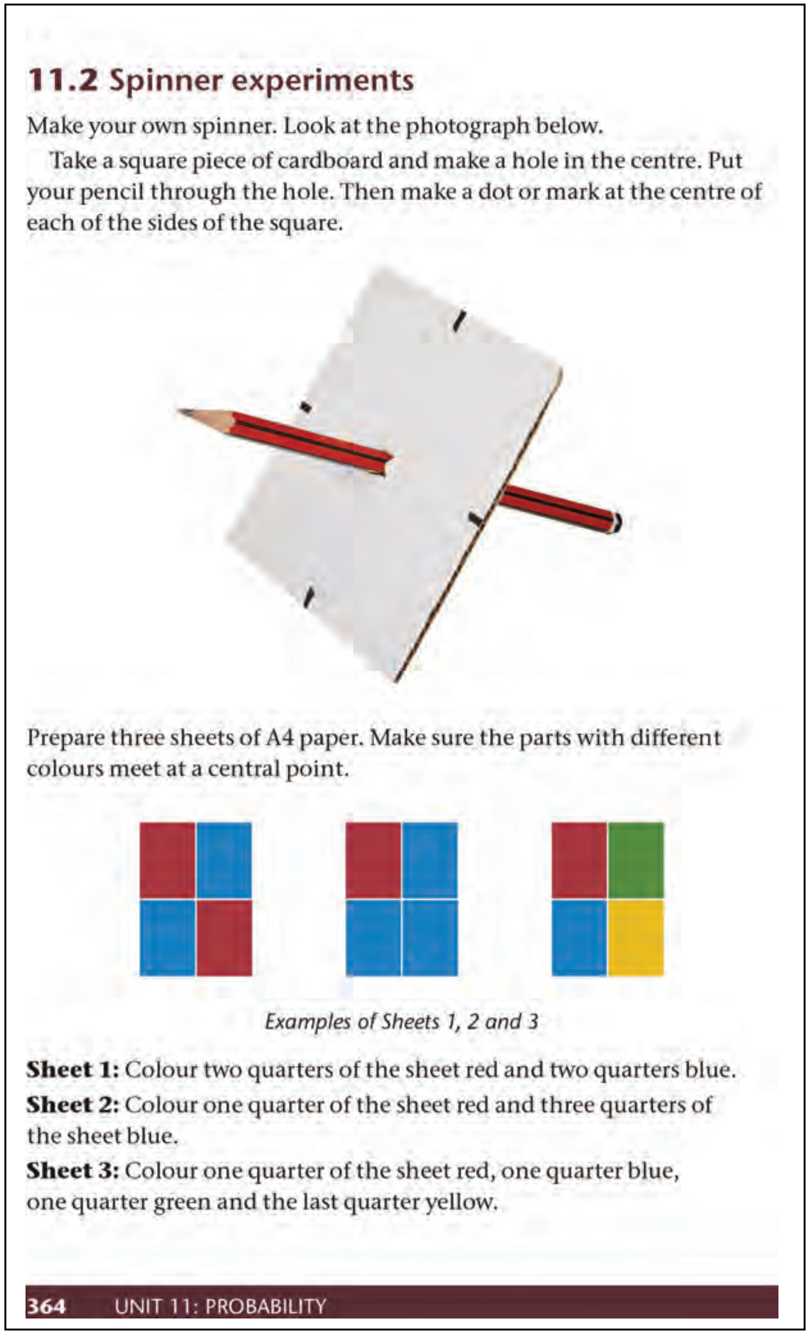
11.2 Spinner experiments

Mathematical notes

In the spinner experiment the possible outcomes are the colours that are used to shade the page. Colours with equal areas have an equal chance to occur.

Teaching guidelines

To save valuable classroom time, you may make spinners and coloured sheets beforehand. Learners can then begin with the actual experiment straightaway. Take care to cut the cardboard in squares. A useful way to locate the midpoint is to draw a cross by joining the midpoints of opposite sides.



11.2 Spinner experiments

Make your own spinner. Look at the photograph below.

Take a square piece of cardboard and make a hole in the centre. Put your pencil through the hole. Then make a dot or mark at the centre of each of the sides of the square.

Prepare three sheets of A4 paper. Make sure the parts with different colours meet at a central point.

Examples of Sheets 1, 2 and 3

Sheet 1: Colour two quarters of the sheet red and two quarters blue.

Sheet 2: Colour one quarter of the sheet red and three quarters of the sheet blue.

Sheet 3: Colour one quarter of the sheet red, one quarter blue, one quarter green and the last quarter yellow.

Notes on questions

One learner of the pair draws up a tally table to record the results, and the other one shades squares in a ten by ten block. The shaded squares help learners to understand that we cannot predict a specific outcome since there is no fixed pattern in the block.

Teaching guidelines

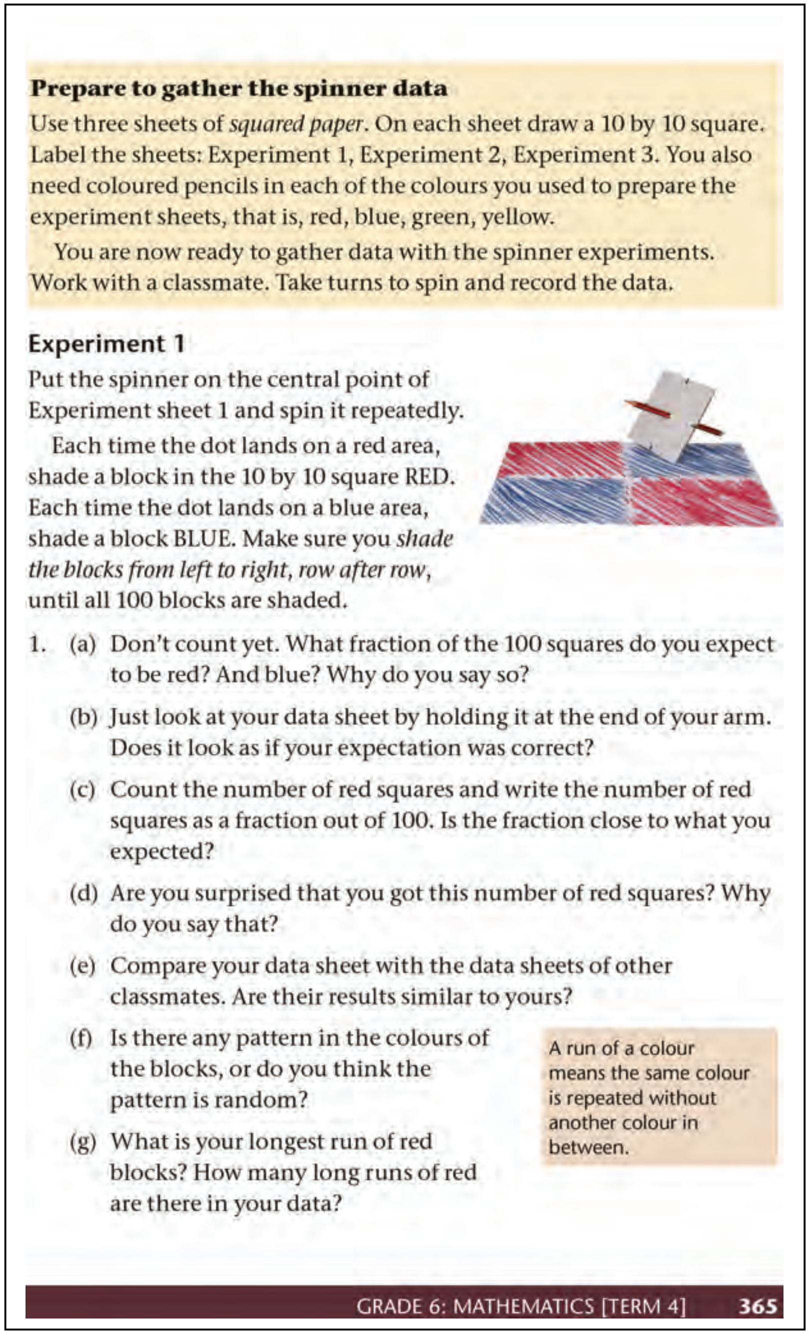
Let learners work in pairs so that each learner spins 50 times, and the pair has data of 100 spins.

You may let different groups do the different experiments to save time. The class discussion must then compare the results.

Keep the tally tables and shaded 10 by 10 squares to compare when all three experiments are done.

Answers: Experiment 1

1. (a) Learners' predictions may differ. We can predict that about one half of the 100 spins will land on red and the other half on blue.
   - (b) Answers will differ. The block of shaded squares is the picture of the random results. The tally table gives the number of red and blue without showing when learners got which colour.
   - (c) The number of red squares are the same number as in the tally table. Learners may be surprised if the total is not "close" to half of the hundred spins.
   - (d) Learners may have different opinions about results that are surprisingly low or surprisingly high.
   - (e) Different blocks will be coloured red and blue on the different data sheets. The results will be similar in the sense that there is no clear pattern.
   - (f) There are no fixed patterns in the blocks. The "pattern" is random.
   - (g) Answers may differ. It is possible to get many blocks with the same colour next to each other.



Prepare to gather the spinner data

Use three sheets of *squared paper*. On each sheet draw a 10 by 10 square. Label the sheets: Experiment 1, Experiment 2, Experiment 3. You also need coloured pencils in each of the colours you used to prepare the experiment sheets, that is, red, blue, green, yellow.

You are now ready to gather data with the spinner experiments. Work with a classmate. Take turns to spin and record the data.

Experiment 1

Put the spinner on the central point of Experiment sheet 1 and spin it repeatedly.

Each time the dot lands on a red area, shade a block in the 10 by 10 square RED. Each time the dot lands on a blue area, shade a block BLUE. Make sure you *shade the blocks from left to right, row after row,* until all 100 blocks are shaded.

1. (a) Don't count yet. What fraction of the 100 squares do you expect to be red? And blue? Why do you say so?
   - (b) Just look at your data sheet by holding it at the end of your arm. Does it look as if your expectation was correct?
   - (c) Count the number of red squares and write the number of red squares as a fraction out of 100. Is the fraction close to what you expected?
   - (d) Are you surprised that you got this number of red squares? Why do you say that?
   - (e) Compare your data sheet with the data sheets of other classmates. Are their results similar to yours?
   - (f) Is there any pattern in the colours of the blocks, or do you think the pattern is random?
   - (g) What is your longest run of red blocks? How many long runs of red are there in your data?

A run of a colour means the same colour is repeated without another colour in between.

Answers

1. (h)– (i) Answers may differ. It is possible to get many blocks with the same colour next to each other.
2. Yes, they have the same chance. The red and blue areas on which the spinner may land are equal in size.
3. We expect similar results, because the coin also has two equally likely outcomes.

Experiment 2

Teaching guidelines

Learners work in pairs. One keeps a tally table and the other shades the cells in a 10 by 10 square.

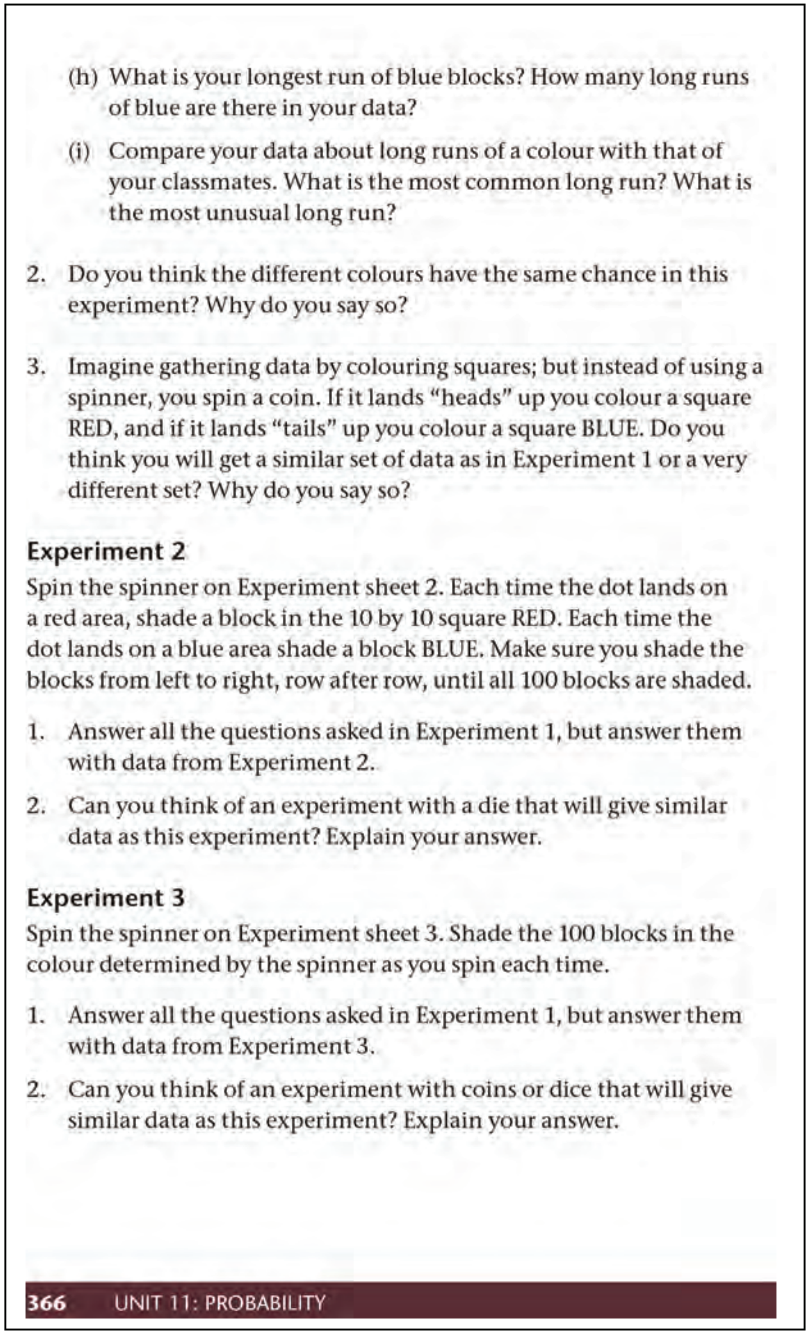
Answers

1. (a) Expect about 75% blue and about 25% red.
   (b)–(e) Answers will differ.
   (f) There is still no fixed pattern. More blue than red blocks are more obvious than in Experiment 1.
2. A possible die experiment with outcomes 3 to 1 is: Roll two dice and add the numbers. Shade red if the total is 7 or 10. Shade blue if you get any other total. Together a total of 7 or 10 has a 9 out of 36 chance (one quarter) to occur. It is possible to find other combinations of totals that have a one quarter probability.

Experiment 3

Answers

1. The colours all have an equal chance of occurring. We expect about 25% of the total results to be blue, 25% to be red, 25% to be green and 25% to be yellow. Actual results may differ somewhat from these expectations.
2. No, a coin has two outcomes and a die has six outcomes. The spinner in Experiment 3 has 4 outcomes.



(h) What is your longest run of blue blocks? How many long runs of blue are there in your data?

(i) Compare your data about long runs of a colour with that of your classmates. What is the most common long run? What is the most unusual long run?

2. Do you think the different colours have the same chance in this experiment? Why do you say so?
3. Imagine gathering data by colouring squares; but instead of using a spinner, you spin a coin. If it lands "heads" up you colour a square RED, and if it lands "tails" up you colour a square BLUE. Do you think you will get a similar set of data as in Experiment 1 or a very different set? Why do you say so?

Experiment 2

Spin the spinner on Experiment sheet 2. Each time the dot lands on a red area, shade a block in the 10 by 10 square RED. Each time the dot lands on a blue area shade a block BLUE. Make sure you shade the blocks from left to right, row after row, until all 100 blocks are shaded.

1. Answer all the questions asked in Experiment 1, but answer them with data from Experiment 2.
2. Can you think of an experiment with a die that will give similar data as this experiment? Explain your answer.

Experiment 3

Spin the spinner on Experiment sheet 3. Shade the 100 blocks in the colour determined by the spinner as you spin each time.

1. Answer all the questions asked in Experiment 1, but answer them with data from Experiment 3.
2. Can you think of an experiment with coins or dice that will give similar data as this experiment? Explain your answer.

11.3 The Subtraction Game

Mathematical notes

When we roll a die, there are six equally likely possible outcomes. When two dice are rolled simultaneously, there are 36 equally likely possible combined outcomes, corresponding to the cells in the table on the right.

There are six possible numerical differences between the outcomes on the two dice: 0; 1; 2; 3; 4 and 5. These differences are not equally likely when the two dice are rolled simultaneously. For example, a difference of 5 occurs in only **two** of the 36 equally likely outcomes (the yellow cells in the table on the right), while a difference of 1 occurs in **ten** of the 36 equally likely outcomes. We may thus expect (predict) that when two dice are rolled many times, a difference of 1 would appear about five times as often as a difference of 5.

Differences	0	1	2	3	4	5
Occurrences out of 36	6	10	8	6	4	2

Die A

Die B

	1	**2**	**3**	**4**	**5**	**6**
1						
2						
3						
4						
5						
6						

Die A

Die B

	1	**2**	**3**	**4**	**5**	**6**
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

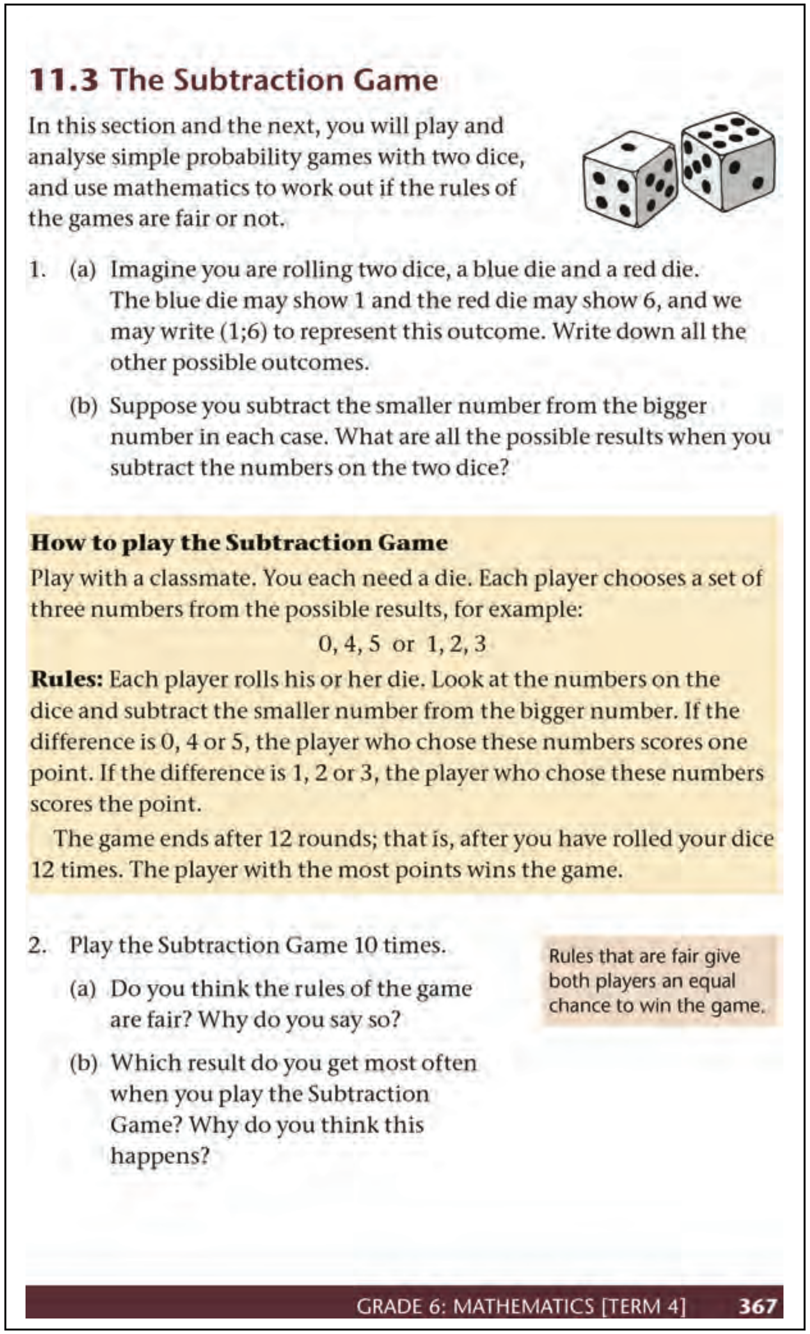
The above arguments demonstrate that although the outcomes of a random event can only be predicted to occur more or less equally often, certain combinations of outcomes can be predicted to occur more often than others.

Teaching guidelines

Let one learner take 0, 4 and 5 as his/her numbers, and the other learner 1, 2 and 3. They will soon notice that the one learner wins much more often than the other learner. Suggest to learners that this may relate to the numbers assigned to each player. Then let one learner take 0, 1 and 5 and the other learner 2, 3 and 4, and continue to play.

Answers

1. (a) (1; 5), (1; 4), (1; 3), (1; 2), (1; 1), (2; 6), (2; 5), (2;4), (2; 3), (2; 2), (2; 1), (3; 6), (3; 5), (3; 4), (3; 3), (3; 2), (3; 1), (4; 6), (4; 5), (4; 4), (4; 3), (4; 2), (4; 1), (5; 6), (5; 5), (5; 4), (5; 3), (5; 2), (5; 1), (6; 6), (6; 5), (6; 4), (6; 3), (6; 2), (6; 1)
   (b) 0; 1; 2; 3; 4; 5
2. (a) Opinions may differ. A game is not fair if the rules give one person a better chance to win than the other.
   (b) Answers will differ. 0 or 1 is likely to occur more often than the other differences.



11.3 The Subtraction Game

In this section and the next, you will play and analyse simple probability games with two dice, and use mathematics to work out if the rules of the games are fair or not.

1. (a) Imagine you are rolling two dice, a blue die and a red die. The blue die may show 1 and the red die may show 6, and we may write (1;6) to represent this outcome. Write down all the other possible outcomes.
   (b) Suppose you subtract the smaller number from the bigger number in each case. What are all the possible results when you subtract the numbers on the two dice?

How to play the Subtraction Game

Play with a classmate. You each need a die. Each player chooses a set of three numbers from the possible results, for example:

0, 4, 5 or 1, 2, 3

Rules: Each player rolls his or her die. Look at the numbers on the dice and subtract the smaller number from the bigger number. If the difference is 0, 4 or 5, the player who chose these numbers scores one point. If the difference is 1, 2 or 3, the player who chose these numbers scores the point.

The game ends after 12 rounds; that is, after you have rolled your dice 12 times. The player with the most points wins the game.

2. Play the Subtraction Game 10 times.
   (a) Do you think the rules of the game are fair? Why do you say so?
   (b) Which result do you get most often when you play the Subtraction Game? Why do you think this happens?

Rules that are fair give both players an equal chance to win the game.

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Note about question 3

To save classroom time, you may give each pair a copy of one of the tables on page 449 in the Addendum.

Answers

3. (a)–(b) See table below.

Player A's die

Player B's die

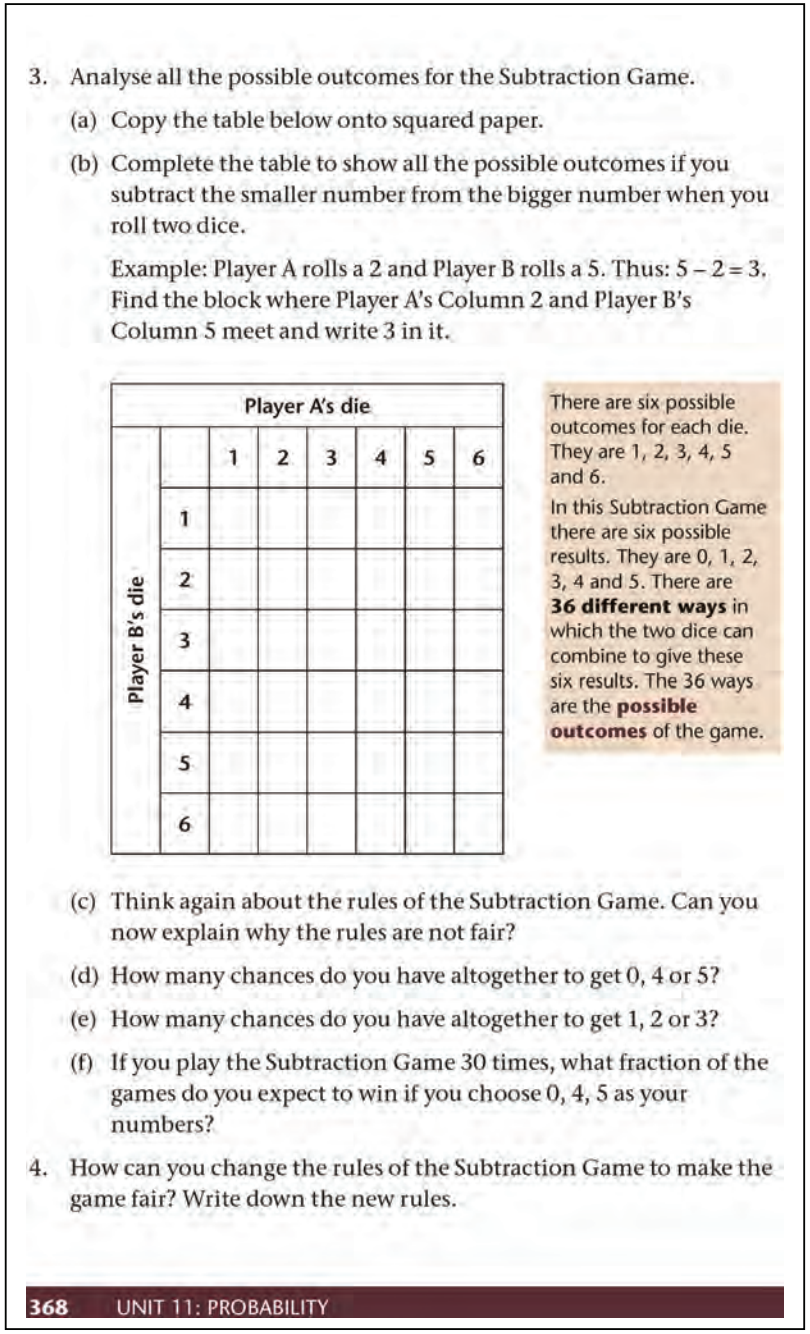
	1	**2**	**3**	**4**	**5**	**6**
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

(c) The five possible differences are not equally likely to be produced when the two dice are rolled simultaneously. For example, a difference of 5 occurs in only **two** of the 36 equally likely outcomes, while a difference of 1 occurs in **ten** of the 36 equally likely outcomes. We may thus expect (predict) that when two dice are rolled many times, a difference of 1 would appear about five times as often as a difference of 5. That may be a reason why the rules are not fair.

(d) 0 has 6 out of 36 chances to occur. 4 has 4 out of 36 chances to occur.
5 has 2 out of 36 chances to occur. Altogether, the player who chooses (0, 4, 5) has a 12 out of 36 chance to score a point.

(e) 1 has 10 out of 36 chances to occur; 2 has 8 out of 36 chances to occur;
3 has 6 out of 36 chances to occur. Altogether, the player who chooses (1, 2, 3) has a 24 out of 36 chance to score a point. This player has double the chance of the other player to score a point.

4. Rules are fair when both players have an equal chance to get the numbers they choose. The rules must not prescribe which sets of numbers the players must choose, because players who understand probability will choose numbers that have a better chance of winning.
If you want rules that prescribe groups of numbers, then (1, 3, 5) and (0, 2, 4) have the same chance of occurring if you play the Subtraction Game.



3. Analyse all the possible outcomes for the Subtraction Game.

(a) Copy the table below onto squared paper.

(b) Complete the table to show all the possible outcomes if you subtract the smaller number from the bigger number when you roll two dice.

Example: Player A rolls a 2 and Player B rolls a 5. Thus: 5 – 2 = 3. Find the block where Player A's Column 2 and Player B's Column 5 meet and write 3 in it.

Player A's die

Player B's die

	1	**2**	**3**	**4**	**5**	**6**
1						
2						
3						
4						
5						
6						

There are six possible outcomes for each die. They are 1, 2, 3, 4, 5 and 6.
In this Subtraction Game there are six possible results. They are 0, 1, 2, 3, 4 and 5. There are **36 different ways** in which the two dice can combine to give these six results. The 36 ways are the **possible outcomes** of the game.

(c) Think again about the rules of the Subtraction Game. Can you now explain why the rules are not fair?

(d) How many chances do you have altogether to get 0, 4 or 5?

(e) How many chances do you have altogether to get 1, 2 or 3?

(f) If you play the Subtraction Game 30 times, what fraction of the games do you expect to win if you choose 0, 4, 5 as your numbers?

4. How can you change the rules of the Subtraction Game to make the game fair? Write down the new rules.

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11.4 The Addition Game

Teaching guidelines

Begin by letting learners work in pairs to roll two dice and add the numbers. Ask them to figure out which totals are possible to get.

When learners play their addition games with their own rules, let them tally the results. Use the tallied results to discuss whether the game was fair.

To save classroom time, you may give each pair a copy of page 450 in the Addendum.

Answers

1. 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12

2. Answers will vary.

It would be reasonable to expect that each of 1, 2, 3, 4, 5 and 6 will occur at least once, but this is not guaranteed.

3.

Player A's die

Player B's die

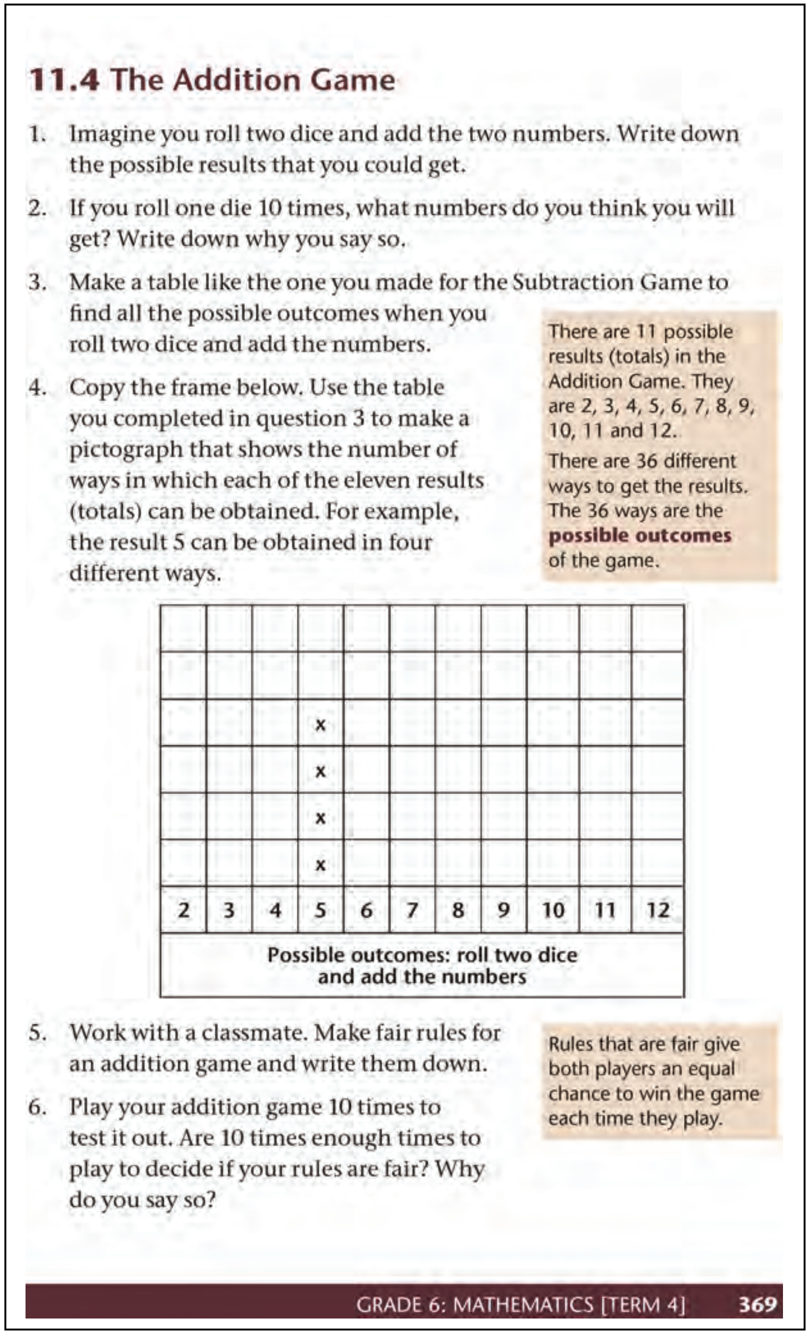
	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

4.

					x					
				x	x	x				
			x	x	x	x	x			
		x	x	x	x	x	x	x		
	x	x	x	x	x	x	x	x	x	
x	x	x	x	x	x	x	x	x	x	x
2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**

Possible outcomes: roll two dice and add the numbers

5. Fair rules will give both players an equal chance to win. Learners will make different rules. They must explain what the chances of the outcomes are.
   Example: A game for two players. If the sum is an even number, one player scores a point. If the sum is an odd number, the other player scores a point. The first player to reach a total of 10 wins. The rule is fair, because there are 18 out of 36 chances to get an even sum, and 18 out of 36 chances to get an odd sum.
6. Learners tally results while they play. Did results with greater chances to occur actually occur more frequently? If we play a small number of times, we can expect unusual results. We know the game is fair because we checked that the chances of scoring are the same for each player.



11.4 The Addition Game

1. Imagine you roll two dice and add the two numbers. Write down the possible results that you could get.
2. If you roll one die 10 times, what numbers do you think you will get? Write down why you say so.
3. Make a table like the one you made for the Subtraction Game to find all the possible outcomes when you roll two dice and add the numbers.
4. Copy the frame below. Use the table you completed in question 3 to make a pictograph that shows the number of ways in which each of the eleven results (totals) can be obtained. For example, the result 5 can be obtained in four different ways.

There are 11 possible results (totals) in the Addition Game. They are 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.

There are 36 different ways to get the results. The 36 ways are the **possible outcomes** of the game.

			x							
			x							
			x							
			x							
2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**

Possible outcomes: roll two dice and add the numbers

5. Work with a classmate. Make fair rules for an addition game and write them down.
6. Play your addition game 10 times to test it out. Are 10 times enough times to play to decide if your rules are fair? Why do you say so?

Rules that are fair give both players an equal chance to win the game each time they play.

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Addendum

General resources

Resources for specific activities

100

10000

200

90

80

9

300

70

60

400

50

40

5

Place value cards
for learners

(7 pages = 1 set)

500

30

20

4

600

10

3

2

1

700

1000

6

800

900

7

8

2000	6000
3000	7000
4000	8000
5000	9000

20000

30000

40000

50000

60000

70000

80000

90000

100000

200000

300000

400000

500000

600000

700000

800000

900000

1000000

1000

Place value cards
for teachers

(14 pages = 1 set)

100

10

200

20

300

30

400

40

500

50

600

60

700

70

800

80

900

90

2000

1

3000

2

4000

3

5000

4

60000	5
70000	6

8000	7
9000	8

10000

20000

30000

40000

50000

60000

70000

80000

90000

9

Square grid paper (1 cm × 1 cm)

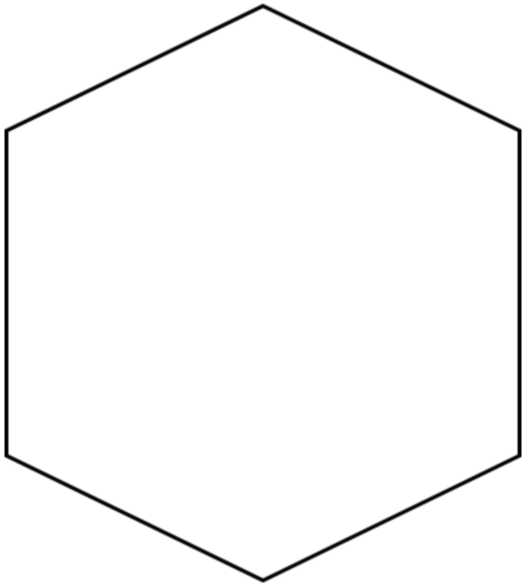
Graph paper

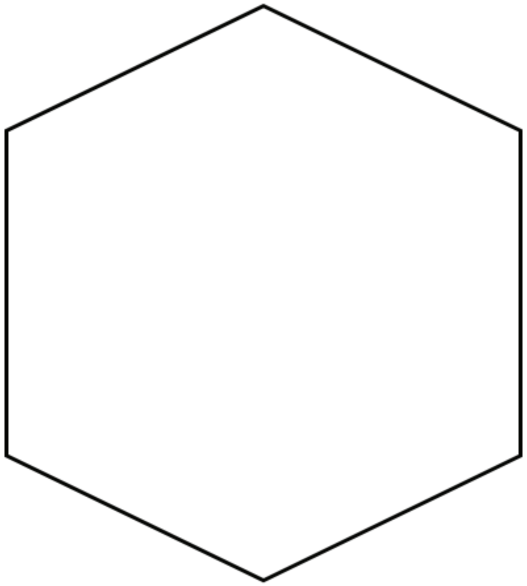
Graph paper / Square grid paper (0,5 cm × 0,5 cm)

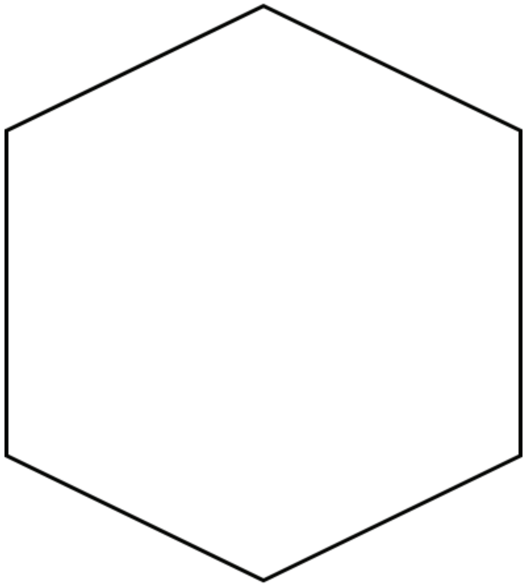
Square grid paper (1,25 mm × 1,25 mm)

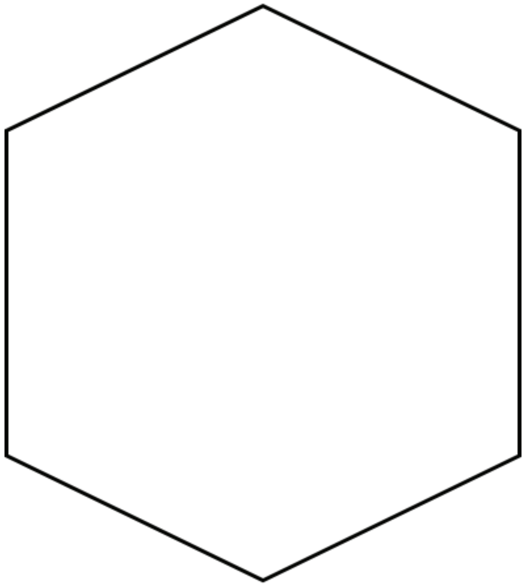
Dotted paper

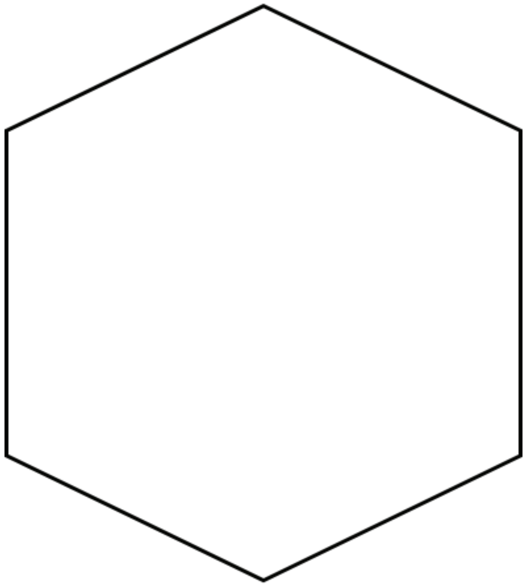
Term 3 Unit 5: Section 5.2, questions 3 to 10 (TG pp. 266–267; LB pp. 239–240)

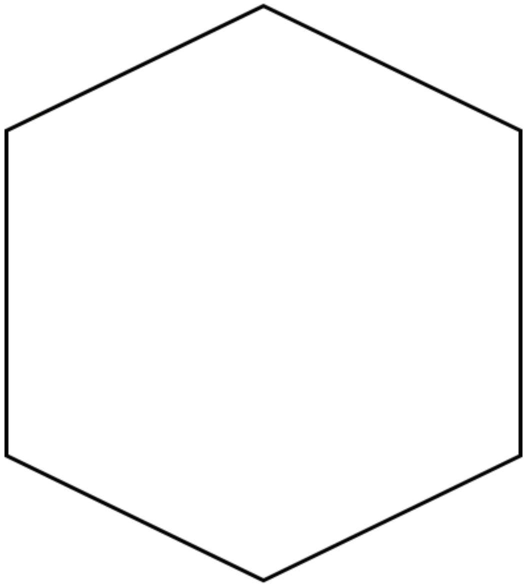




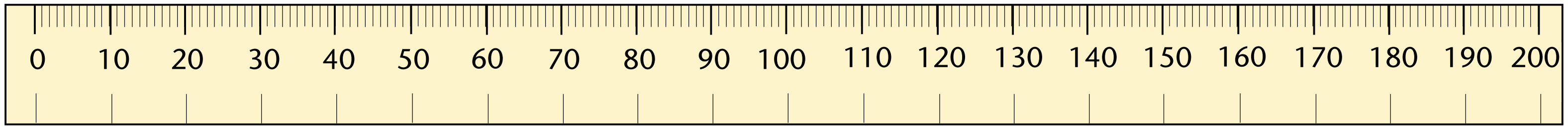


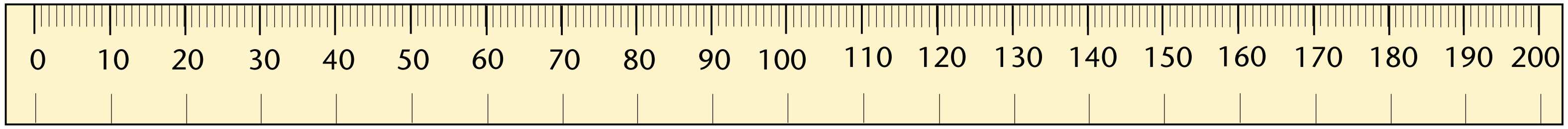


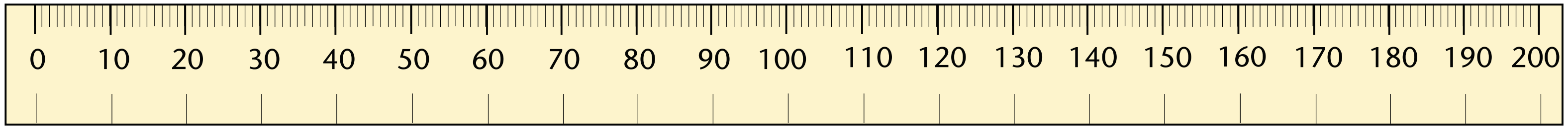


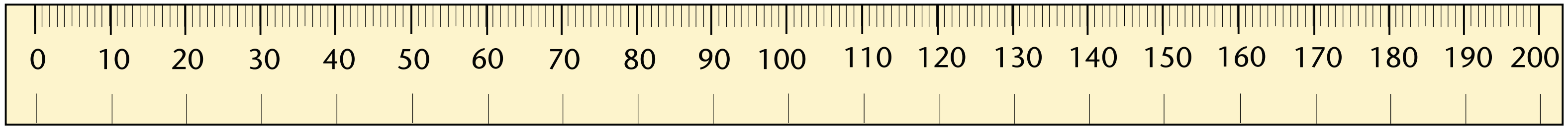


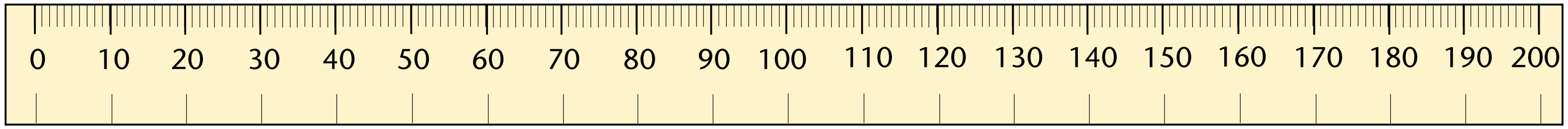
Term 4 Unit 5: Section 5.1 Millimetre measuring tapes (TG p. 364)

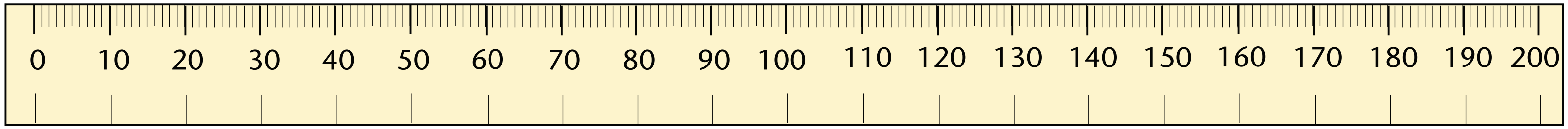


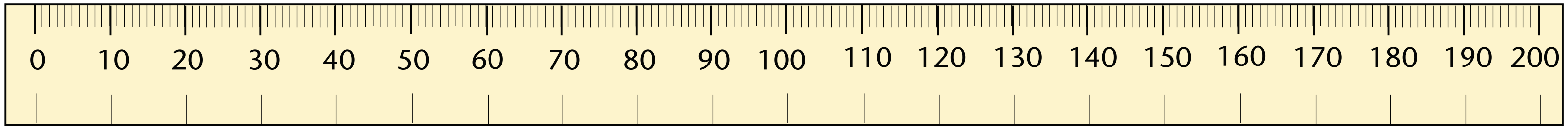




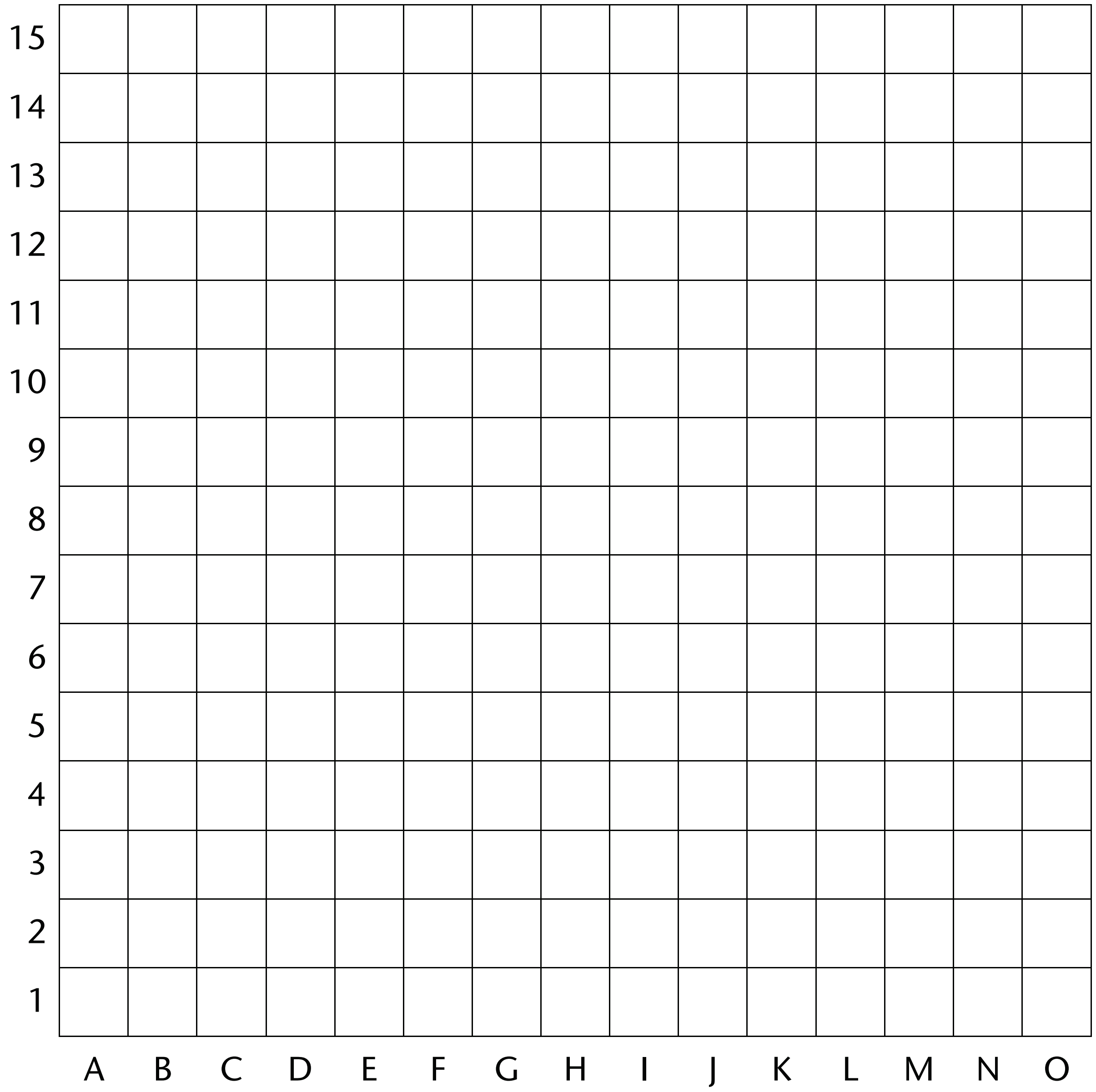








Term 4 Unit 10: Section 10.1, question 4 (TG p. 405; LB p. 358)



Term 4 Unit 11: Section 11.3, question 3 (TG p. 416; LB p. 368)

The Subtraction Game

Player A's die

Player B's die

	1	**2**	**3**	**4**	**5**	**6**
1						
2						
3						
4						
5						
6						

The Subtraction Game

Player A's die

Player B's die

	1	**2**	**3**	**4**	**5**	**6**
1						
2						
3						
4						
5						
6						

Term 4 Unit 11: Section 11.4, questions 3 and 4 (TG p. 417; LB p. 369)

The Addition Game

Player A's die

Player B's die

	1	**2**	**3**	**4**	**5**	**6**
1						
2						
3						
4						
5						
6						

2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**

Possible outcomes: roll two dice and add the numbers