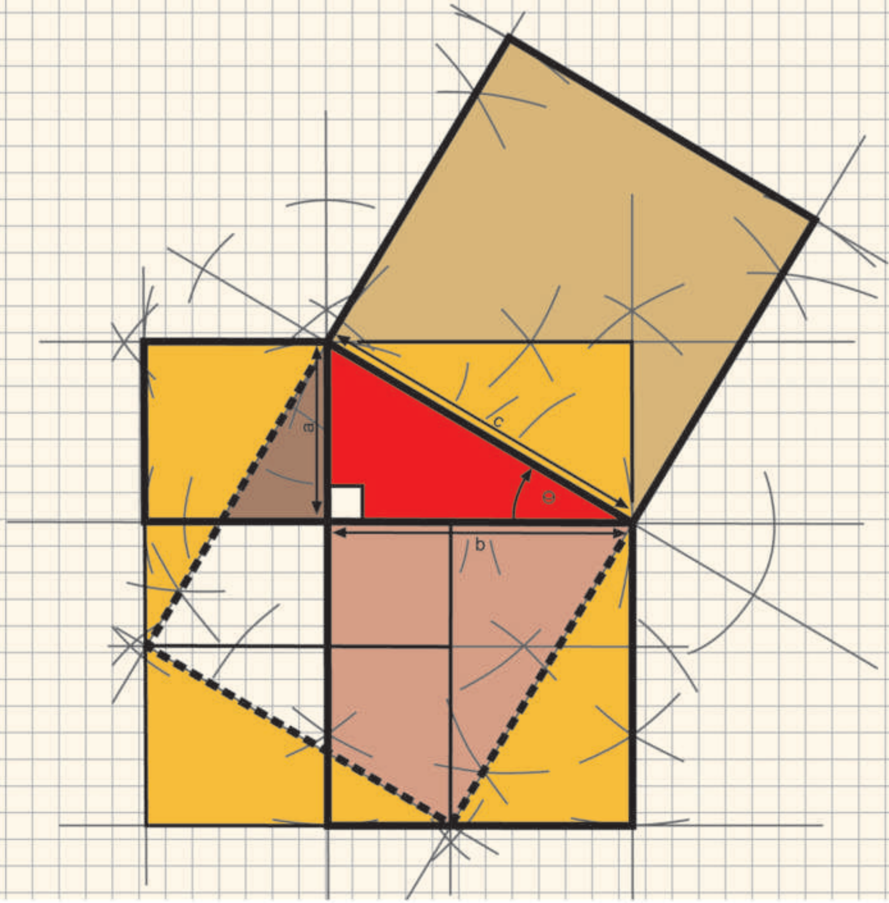
MATHEMATICS

GRADE 7

REVISED EDITION

TEACHER GUIDE



basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

sasol

Mathematics

Grade 7

Teacher Guide

Developed and funded as an ongoing project by the Sasol Inzalo Foundation in partnership with the Ukuqonda Institute

Published by **The Ukuqonda Institute** (http://www.ukuqonda.org.za)

Registered as a Title 21 Company, Reg. No. 2006/026363/08

Public Benefit Organisation, PBO No. 930035134

Mathematics Teacher Guide Grade 7

ISBN: 978-1-4315-2874-5

This Teacher Guide and its accompanying Learner Book (ISBN: 978-1-4315-2873-8) were developed with the participation of the Department of Basic Education (DBE) of South Africa with funding from the Sasol Inzalo Foundation (SaIF).

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Layout and typesetting: Lebone Publishing Services

Subject advisors from the DBE who contributed by means of review: The publisher thanks those subject advisors of the DBE who reviewed this book series on four occasions in 2013-2014, as well as in October 2017. The authors changed the text so as to align with the reviewers' requests/suggestions for improvements, as far as possible, and believe that the books improved as a result of that.

The reviewers were: [names to be supplied]

Printed by: [printer name and address]

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Grade 7 Term 1 Chapter 1 Working with whole numbers

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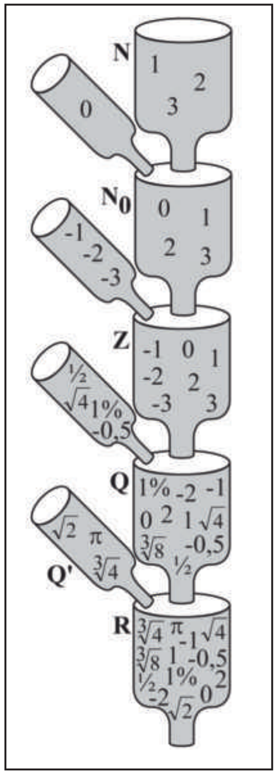
If natural numbers are added, the answer is also a natural number. We say: The set of natural numbers is closed under addition.



Mathematical background

- The illustration on the right shows how the set of real numbers has developed from the set of natural numbers. As additional sets of numbers are added to the natural numbers, the advantages of each new set of numbers increase and its disadvantages decrease.
- The set of **natural numbers** is defined by the set $N = \{1; 2; 3; 4; 5; ...\}$.
  - Advantages: N is closed under addition and multiplication; N contains the identity element for multiplication (1).
  - Disadvantages: N is not closed under subtraction or division; N does not contain the identity element for addition (0).
- The set of **whole numbers** $N_0 = \{0; 1; 2; 3; 4; 5; ...\}$ is formed by adding 0 to the set of natural numbers.
  - Additional advantage: N_0 contains the identity element for addition (0).
- The set of **integers** $Z = \{...; -3; -2; -1; 0; 1; 2; 3; ...\}$ is formed by adding the set $\{...; -4; -3; -2; -1\}$ to the set of whole numbers.
  - Additional advantage: Z is closed under subtraction.
- The set of **rational numbers** Q is formed by adding common fractions (and therefore mixed numbers), decimal fractions (and therefore decimal numbers), percentages and ratios to the set of integers.
  - Additional advantage: Q is closed under division (but division by 0 is undefined).
- The set of **real numbers** R is formed by adding the set of **irrational numbers** Q' (surds and numbers like π) to the set of rational numbers. This chapter focuses on features of and basic operations on whole numbers.

1.1 Revision

BUILD NUMBERS UP AND BREAK NUMBERS DOWN

Background information

There is a difference between the place value of a digit in a number and the value of that digit in the number.

- The **place value of a digit** in a number is determined by the **position of the digit** in that number. For example, in the 5-digit number 12 345:
  - the place value of the digit 1 is ten thousands
  - the place value of the digit 2 is thousands
  - the place value of the digit 3 is hundreds
  - the place value of the digit 4 is tens
  - the place value of the digit 5 is units

  Expressed in terms of the **place value of its digits**:

  12 345 = 1 ten thousand + 2 thousands + 3 hundreds + 4 tens + 5 units
- The **value of a digit** in a number is equal to the **product of that digit and its place value**. For example, in the 5-digit number 12 345:
  - the value of the digit 1 is 1 × 10 000 = 10 000
  - the value of the digit 2 is 2 × 1 000 = 2 000
  - the value of the digit 3 is 3 × 100 = 300
  - the value of the digit 4 is 4 × 10 = 40
  - the value of the digit 5 is 5 × 1 = 5

  Expressed in terms of the **values of its digits**:

  12 345 = 10 000 + 2 000 + 300 + 40 + 5

Teaching guidelines

Illustrate the difference between the place value of a digit in a number and the value of that digit in the number:

- Expand a number using the place value of its digits: 27 = 2 tens + 7 units
- Expand the same number using the values of its digits: 27 = 20 + 7

Discuss the notes provided on LB page 1.

CHAPTER 1
Working with whole numbers

1.1 Revision

Do not use a calculator at all in section 1.1.

BUILD NUMBERS UP AND BREAK NUMBERS DOWN

1. Write each of the following sums as a single number:
   (a) 4 000 + 800 + 60 + 5
   (b) 8 000 + 300 + 7
   (c) 40 000 + 9 000 + 200 + 3
   (d) 800 000 + 70 000 + 3 000 + 900 + 2
   (e) 8 thousands + 7 hundreds + 8 units
   (f) 4 hundred thousands + 8 ten thousands + 4 hundreds + 9 tens

The word **sum** is used to indicate two or more numbers that have to be added.

The answer obtained when the numbers are added, is also called the **sum**. We say: 20 is the sum of 15 and 5.

2. What is the sum of 8 000 and 24?
3. Write each of the numbers below as a sum of units, tens, hundreds, thousands, ten thousands and hundred thousands, like the numbers that were given in question 1(e) and (f).
   (a) 8 706 (b) 449 203
   (c) 83 490 (d) 873 092

When a number is written as a sum of units, tens, hundreds, thousands etc., it is called the **expanded notation**.

4. Arrange the numbers in question 3 from smallest to biggest.
5. Write the numbers in expanded notation (for example, 791 = 700 + 90 + 1).
   (a) 493 020 (b) 409 302 (c) 490 032 (d) 400 932
6. Arrange the numbers in question 5 from biggest to smallest.
7. Write each sum as a single number.
   (a) 600 000 + 40 000 + 27 000 + 100 + 20 + 34
   (b) 320 000 + 40 000 + 8 000 + 670 + 10 + 5
   (c) 500 000 + 280 000 + 7 000 + 300 + 170 + 38
   (d) 4 hundred thousands + 18 ten thousands + 4 hundreds + 29 tens + 5 units

An important note

Learners easily lose sight of what written numbers actually mean when they just say the digits in their minds, for example "three", "five", "four" and not the proper number name "three hundred and fifty-four". A simple and highly effective classroom activity to promote understanding of numbers in terms of their component parts is when the teacher says numbers out loud (using the proper number names) and learners write the numbers in symbols.

Answers

1. (a) 4 865 (b) 8 307 (c) 49 203
   (d) 873 902 (e) 8 708 (f) 480 490
2. 8 024
3. (a) 8 thousands + 7 hundreds + 6 units
   (b) 4 hundred thousands + 4 ten thousands + 9 thousands + 2 hundreds + 3 units
   (c) 8 ten thousands + 3 thousands + 4 hundreds + 9 tens
   (d) 8 hundred thousands + 7 ten thousands + 3 thousands + 9 tens + 2 units
4. 8 706; 83 490; 449 203; 873 092
5. (a) 400 000 + 90 000 + 3 000 + 20 (b) 400 000 + 9 000 + 300 + 2
   (c) 400 000 + 90 000 + 30 + 2 (d) 400 000 + 900 + 30 + 2
6. 493 020; 490 032; 409 302; 400 932
7. (a) 667 154 (b) 368 685 (c) 787 508 (d) 580 695
8. (a) 376 486 (b) 422 513 (c) 547 366 (d) 897 898 (e) 422 513
9. (a) 700 000 + 90 000 + 8 000 + 900 + 90 + 9 798 999
   (b) 900 000 + 60 000 + 9 000 + 800 + 70 + 9 969 879
   (c) 900 000 + 20 000 + 3 000 + 800 + 50 + 2 923 852
   (d) 1 000 000 + 200 000 + 70 000 + 4 000 + 300 + 80 + 4 1 274 384
10. (a) 475 385 (b) Yes (c) 46 027
11. (a) 6 (b) 30 (c) 210 (d) 420 (e) 840
12 (a) 10 000 (b) 5 × 200 (c) 50 × 20
   (d) 25 × 40 (e) 10 000 (f) 10 000
13. See LB page 2 alongside.
14. See LB page 3 on the next page.

8. Write each sum as a single number.
   (a) 300 000 + 70 000 + 6 000 + 400 + 80 + 6
   (b) 400 000 + 20 000 + 2 000 + 500 + 10 + 3
   (c) 500 000 + 40 000 + 7 000 + 300 + 60 + 6
   (d) 800 000 + 90 000 + 7 000 + 800 + 90 + 8
   (e) 300 000 + 110 000 + 12 000 + 400 + 110 + 3
9. In each case, add the two numbers. Write the answer in expanded form and also as a single number.
   (a) The number in 8(a) and the number in 8(b)
   (b) The number in 8(c) and the number in 8(b)
   (c) The number in 8(c) and the number in 8(a)
   (d) The number in 8(d) and the number in 8(a)
10. (a) Subtract the number in 8(b) from the number in 8(d).
   (b) Are the numbers in 8(b) and 8(e) the same?
   (c) Subtract the number in 8(a) from the number in 8(b).
11. Write each of the following products as a single number:
   (a) 2 × 3
   (b) 2 × 3 × 5
   (c) 2 × 3 × 5 × 7
   (d) 2 × 3 × 5 × 7 × 2
   (e) 2 × 3 × 5 × 7 × 2 × 2

The word **product** is used to indicate two or more numbers that have to be multiplied.

The answer obtained when numbers are multiplied, is also called the **product**. We say: 20 is the product of 2 and 10.

12. (a) What is the product of 20 and 500?
   (b) Write 1 000 as a product of 5 and another number.
   (c) Write 1 000 as a product of 50 and another number.
   (d) Write 1 000 as a product of 25 and another number.
   (e) What is the product of 2 500 and 4?
   (f) What is the product of 250 and 40?
13. In the table on the right, the number in each yellow cell is formed by adding the number in the red row above it to the number in the blue column to its left. Copy the table and fill the correct numbers in all the empty yellow cells.
14. The table on the next page is formed in the same way as the table on the right. Copy the table and fill in all the cells for which you know the answers immediately. Leave the other cells open for now.

+	2	3	4	5
10	12	13	14	15
20	22	23	24	25
30	32	33	34	35
40	42	43	44	45
50	52	53	54	55
60	62	63	64	65
70	72	73	74	75

2 MATHEMATICS GRADE 7: TERM 1

MULTIPLES

Background information

A **multiple** is a number made by multiplying together two other numbers. For example, if 2 × 3 = 6 then:

- 6 is a **multiple**
- 6 is a **common multiple** of 2 and 3.

A **number** is a multiple of any of its factors. For example, the factors of 12 are 1, 2, 3, 4, 6 and 12. This means that 12 is a multiple of 1, 2, 3, 4, 6 and 12 because:

- 12 = 1 × 12
- 12 = 2 × 6
- 12 = 3 × 4

To find the **multiples of a number**, multiply that number by the set of natural numbers {1; 2; 3; 4; 5; 6; 7; 8; 9; 10; ...}. For example:

- The multiples of 1 are {1; 2; 3; 4; 5; 6; 7; 8; 9; 10; ...}
- The multiples of 2 are {2; 4; 6; 8; 10; 12; 14; 16; 18; 20; ...}
- The multiples of 3 are {3; 6; 9; 12; 15; 18; 21; 24; 27; 30; ...}
- The multiples of 4 are {4; 8; 12; 16; 20; 24; 28; 32; 36; 40; ...} and so on.

Teaching guidelines

Start counting at 0.

- Count forwards in 1s. The answers 1, 2, 3, ... are multiples of 1.
  All are products of 1 and another number: 1 = 1 × 1; 2 = 1 × 2; 3 = 1 × 3; ...
- Count forwards in 2s. The answers 2, 4, 6, ... are multiples of 2.
  All are products of 2 and another number: 2 = 2 × 1; 4 = 2 × 2; 6 = 2 × 3; ...
- Count forwards in 3. The answers 3, 6, 9, ... are multiples of 3.
  All are products of 3 and another number: 3 = 3 × 1; 6 = 3 × 2; 9 = 3 × 3; ...
- Count forwards in 4. The answers 4, 8, 12, ... are multiples of 4.
  All are products of 4 and another number: 4 = 4 × 1; 8 = 4 × 2; 12 = 4 × 3; ...

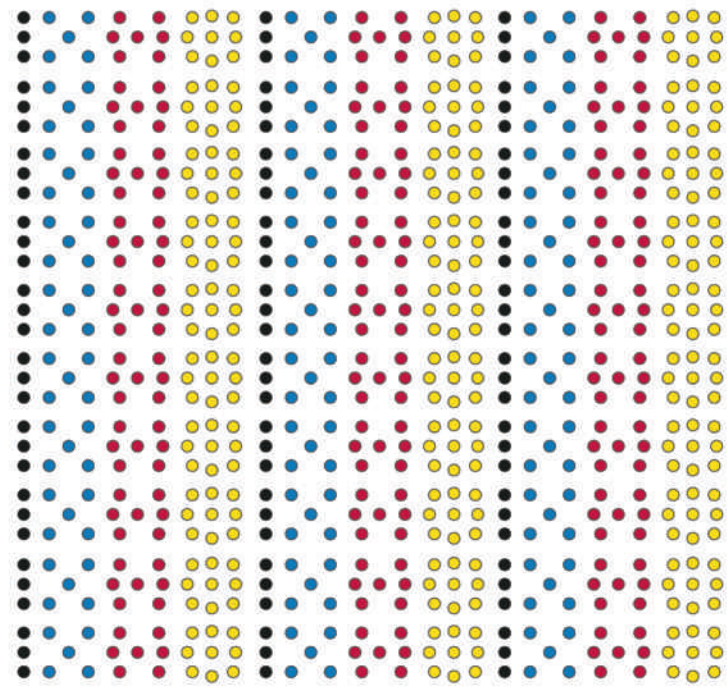
Answers

1. (a) Count the groups of blue and multiply the answer with 5.
   (b) 30 × 5 = 150

+	8	5	4	9	7	3	6	18	36	57
7	15	12	11	16	14	10	13	25	43	64
3	11	8	7	12	10	6	9	21	39	60
9	17	14	13	18	16	12	15	27	45	66
5	13	10	9	14	12	8	11	23	41	62
8	16	13	12	17	15	11	14	26	44	65
6	14	11	10	15	13	9	12	24	42	63
4	12	9	8	13	11	7	10	22	40	61

MULTIPLES

1. In the arrangement below, the blue dots are in groups like this: 
   The red dots are in groups like this: 
   (a) How would you go about finding the number of blue dots below, if you do not want to count them one by one?
   (b) Implement your plan, to find out how many blue dots there are.

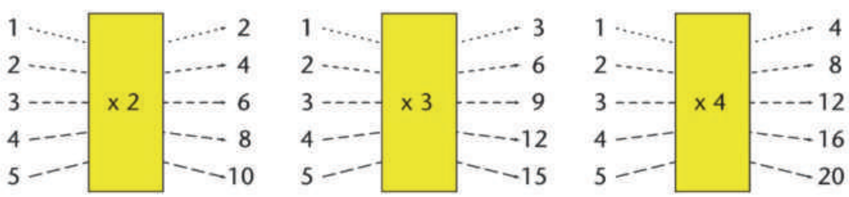


Answers

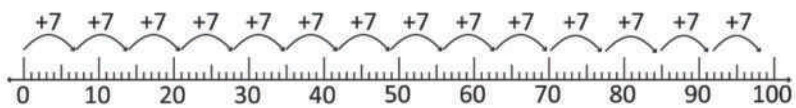
2. Accept any appropriate answer.
3. See LB page 4 alongside.
4. Accept any appropriate answer describing the method used, such as:
   3 × 10 = 30 groups of red dots   30 × 7 = 210 red dots
5. (a) See LB page 4 alongside.   (b) 35, 70 and 105

Additional notes on multiples

Flow diagrams can be used to find the multiples of a chosen number:



Number lines can be used to find multiples of a chosen number:



Number blocks can be used to identify patterns formed by some multiples:

Pattern formed by multiples of 9

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Suppose you want to know how many black dots there are in the arrangement on page 3. One way is to **count in groups** of three. When you do this, you may have to point with your finger or pencil to keep track.

The counting will go like this: *three, six, nine, twelve, fifteen, eighteen . . .*

Another way to find out how many black dots there are is to **analyse** the arrangement and **do some calculations**. In the arrangement, there are ten rows of threes from the top to the bottom, and three columns of threes from left to right, just as in the table alongside.

3	3	3
3	3	3
3	3	3
3	3	3
3	3	3
3	3	3
3	3	3
3	3	3
3	3	3
3	3	3

One way to calculate the total number of black dots is to do 3 × 10 = 30 for the dots in each column, and then 30 + 30 + 30 = 90. Another way is to add up each row (3 + 3 + 3 = 9) and then multiply by 10: 10 × 9 = 90. A third way is to notice that there are 3 × 10 = 30 groups of three, so the total is 3 × 30 = 90.

2. When you determined the number of blue dots in question 1(b), did you count in fives, or did you analyse and calculate, or did you use some other method? Now use a different method to determine the number of blue dots and check whether you get the same answer as before. Describe the method that you used.

3. The numbers that you get when you count in fives are called **multiples** of five. Copy the table below and draw circles around all the multiples of 5.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120

4. How many red dots are there in the arrangement on page 3? Describe the method that you use to find this out.

5. (a) Underline all the multiples of 7 in the table you drew for question 3.
   (b) Which multiples of 5 in the table are also multiples of 7?

A number that is a multiple of 5, and also a multiple of 7, is called a **common multiple** of 5 and 7.

Answers

6. Accept any appropriate answer describing the method used, such as:

   3 × 10 = 30 groups of yellow dots   30 × 9 = 270 yellow dots

7. (a) Refer to question 3 on LB page 4.

   (b) 63

8. (a) The number in the red row is multiplied by the number in the blue column.

   (b) See LB page 5 alongside.

9. See LB page 5 alongside.

6. How many yellow dots are there in the arrangement on page 3? Describe the method that you use to find this out.

7. (a) Cross out all the multiples of 9 in the table you drew for question 3.

   (b) Which numbers in the table in question 3 are common multiples of 7 and 9?

8. (a) Look at the numbers in the yellow cells of the table below. How are these numbers formed from the numbers in the red row and the numbers in the blue column?

   (b) Copy the table below and fill in all the cells for which you know the answers immediately. Leave the other cells open for now.

×	8	5	4	9	7	3	6	2	10	20
7	56	35	28	63	49	21	42	14	70	140
3	24	15	12	27	21	9	18	6	30	60
9	72	45	36	81	63	27	54	18	90	180
5	40	25	20	45	35	15	30	10	50	100
8	64	40	32	72	56	24	48	16	80	160
6	48	30	24	54	42	18	36	12	60	120
4	32	20	16	36	28	12	24	8	40	80
2	16	10	8	18	14	6	12	4	20	40
10	80	50	40	90	70	30	60	20	100	200
20	160	100	80	180	140	60	120	40	200	400

9. Copy the table below and write down the first thirteen multiples of each of the numbers in the column on the left. The multiples of 4 are already written in, as an example.

1	2	3	4	5	6	7	8	9	10	11	12	13
2	4	6	8	10	12	14	16	18	20	22	24	26
3	6	9	12	15	18	21	24	27	30	33	36	39
4	8	12	16	20	24	28	32	36	40	44	48	52
5	10	15	20	25	30	35	40	45	50	55	60	65
6	12	18	24	30	36	42	48	54	60	66	72	78
7	14	21	28	35	42	49	56	63	70	77	84	91
8	16	24	32	40	48	56	64	72	80	88	96	104
9	18	27	36	45	54	63	72	81	90	99	108	117
10	20	30	40	50	60	70	80	90	100	110	120	130
11	22	33	44	55	66	77	88	99	110	121	132	143
12	24	36	48	60	72	84	96	108	120	132	144	156
13	26	39	52	65	78	91	104	117	130	143	156	169

Answers

10. See answers on LB page 6 alongside.
11. Refer to question 8 on the previous page.
12. 270 yellow dots → 270 × 10 = 2 700 black spots

MULTIPLES OF 10, 100, 1 000 AND 10 000

Background information

The powers of 10 are $10^1 = 10$, $10^2 = 100$, $10^3 = 1\ 000$, $10^4 = 10\ 000$ and so on.

- Multiplication by 10 moves the position of each digit in a number one place value to the left.
- Multiplication by 100 moves the position of each digit in a number two place values to the left because $100 = 10 \times 10 = 10^2$.
- Multiplication by 1 000 moves the position of each digit in a number three place values to the left because $1\ 000 = 10 \times 10 \times 10 = 10^3$.
- Multiplication by 10 000 moves the position of each digit in a number four place values to the left because $10\ 000 = 10 \times 10 \times 10 \times 10 = 10^4$.

Teaching guidelines

Learners should understand the effect of multiplication by powers of 10 on the place value of the digits as well as the values of the digits in a number.

- Every time a number is multiplied by 10, its digits move one **place value position** to the left.
- The **value of each digit** increases tenfold.

Answers

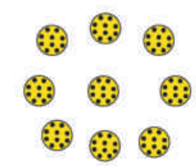
1. 10 × 10 × 10 = 1 000 or 20 × 50 = 1 000 or 5 × 20 × 10 = 1 000
   Accept any appropriate answer.
2. Accept any appropriate answer.
   1 000 × number of learners in the class
3. (a) 10 × 1 000 = 10 000
   (b) 10 000 × number of learners in the class
4. 10 × 10 000 = 100 000
5. (a) 10 × 100 000 = 1 000 000 (b) 100 × 100 000 = 10 000 000

10. Copy and complete this table. For some cells, you may find your table of multiples on the previous page helpful.

×	6	2	7	9	4	5	3	8	10	50
8	48	16	56	72	32	40	24	64	80	400
6	36	12	42	54	24	30	18	48	60	300
7	42	14	49	63	28	35	21	56	70	350
9	54	18	63	81	36	45	27	72	90	450
5	30	10	35	45	20	25	15	40	50	250
3	18	6	21	27	12	15	9	24	30	150
4	24	8	28	36	16	20	12	32	40	200
2	12	4	14	18	8	10	6	16	20	100

11. Go back to the table you drew for question 8(b). If you can easily fill in the numbers in some of the open spaces now, do it.

12. Suppose there are 10 small black spots on each of the yellow dots in the arrangement on page 3. How many small black spots would there be on all the yellow dots together, in the arrangement on page 3?



MULTIPLES OF 10, 100, 1 000 AND 10 000

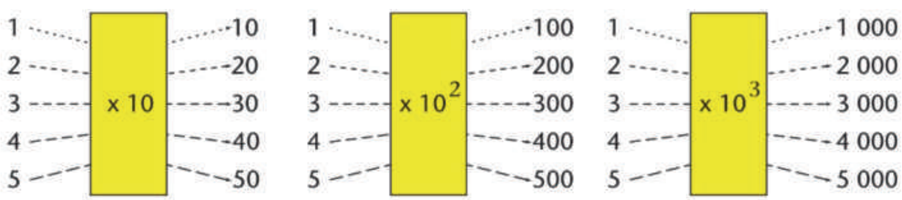
1. How many spotted yellow dots are there on page 7? Explain what you did to find out.
2. How many learners are there in your class? Suppose each learner in the class has a book like this. How many spotted yellow dots are there on the same page (that is, on page 7) of all these books together?
3. Each yellow dot has 10 small black spots, as you can see on this enlarged picture.
   
   (a) How many small black spots are there on page 7?
   (b) How many small black spots are there on page 7 in all the books in your class?
4. Here is a very big enlargement of one of the black spots on the yellow dots. There are 10 very small white spots on each small black spot. How many very small white spots are there on all the black spots on page 7?
   
5. (a) How many very small white spots are there on 10 pages like page 7?
   (b) How many very small white spots are there on 100 pages like page 7?

Answers

6. (a) 7 684
   (b) 70 000 + 6 000 + 800 + 40; 76 840
   (c) 700 000 + 60 000 + 8 000 + 400; 768 400
7. (a) 700 + 40 + 6
   (b) 7 000 + 400 + 60
   (c) 70 000 + 4 000 + 600
   (d) 700 000 + 40 000 + 6 000
   (e) 7 000 000 + 400 000 + 60 000

Additional notes on multiplication by powers of 10

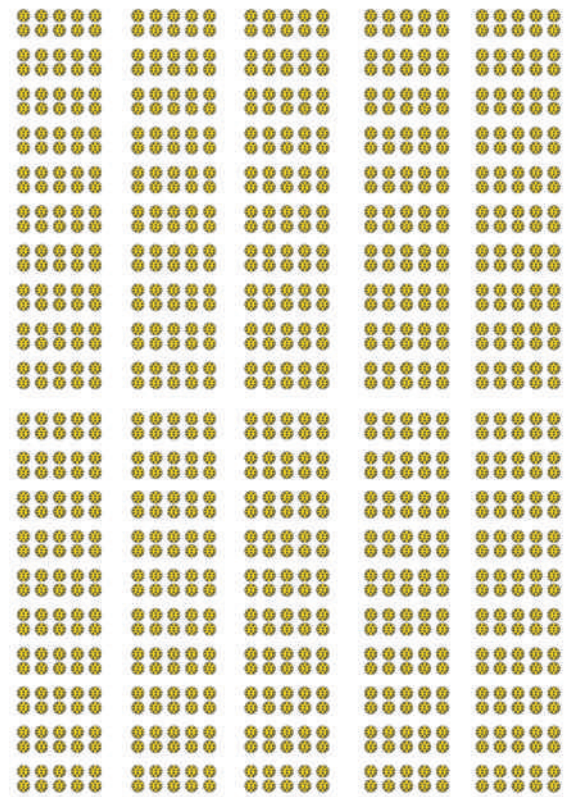
Flow diagrams can be used to illustrate the effect of multiplication by 10, 100, 1 000 and 10 000 on a number:



By using larger numbers as input values, learners should realise that, multiplication by:

- 10 moves each digit one place value position to the left; 10 increases the value of each digit tenfold
- 100 moves each digit two place value positions to the left; 100 increases the value of each digit hundredfold
- 1 000 moves each digit three place value positions to the left; 1 000 increases the value of each digit thousandfold
- 10 000 moves each digit four place value positions to the left; 10 000 increases the value of each digit ten thousandfold.

10 tens are a **hundred**: 10 × 10 = 100
10 hundreds are a **thousand**: 10 × 100 = 1 000
10 thousands are a **ten thousand**: 10 × 1 000 = 10 000
10 ten thousands are a **hundred thousand**: 10 × 10 000 = 100 000
10 hundred thousands are a **million**: 10 × 100 000 = 1 000 000



6. (a) Write 7 000 + 600 + 80 + 4 as a single number.
   (b) Write 10 times the number in (a) in expanded notation and as a single number.
   (c) Write 100 times the number in (a) in expanded notation and as a single number.
7. Write each of the following numbers in expanded notation:
   (a) 746 (b) 7 460 (c) 74 600
   (d) 746 000 (e) 7 460 000

Answers

8. (a) 10 × 1 000 (b) 100 × 100
   (c) 10 × 10 000 (d) 1 000 × 100
   (e) 1 000 × 1 000
9. See LB page 8 alongside.
10. See LB page 8 alongside.

8. (a) Write 10 000 as a product of 10 and one other number.
   (b) Write 10 000 as a product of 100 and one other number.
   (c) Write 100 000 as a product of 10 and one other number.
   (d) Write 100 000 as a product of 1 000 and one other number.
   (e) Write 1 000 000 as a product of 1 000 and one other number.

9. Copy the table below and fill in all the cells for which you know the answers immediately. Leave the other cells open for now.

×	10	20	30	40	50	60	70	80	90	100
2	20	40	60	80	100	120	140	160	180	200
3	30	60	90	120	150	180	210	240	270	300
4	40	80	120	160	200	240	280	320	360	400
5	50	100	150	200	250	300	350	400	450	500
6	60	120	180	240	300	360	420	480	540	600
7	70	140	210	280	350	420	490	560	630	700
8	80	160	240	320	400	480	560	640	720	800
9	90	180	270	360	450	540	630	720	810	900
10	100	200	300	400	500	600	700	800	900	1 000
11	110	220	330	440	550	660	770	880	990	1 100
12	120	240	360	480	600	720	840	960	1 080	1 200

10. Copy the table below and fill in all the cells in the table for which you know the answers immediately. Leave the other cells open for now.

×	100	200	300	400	500	600
2	200	400	600	800	1 000	1 200
3	300	600	900	1 200	1 500	1 800
4	400	800	1 200	1 600	2 000	2 400
5	500	1 000	1 500	2 000	2 500	3 000
6	600	1 200	1 800	2 400	3 000	3 600
7	700	1 400	2 100	2 800	3 500	4 200
8	800	1 600	2 400	3 200	4 000	4 800
9	900	1 800	2 700	3 600	4 500	5 400
10	1 000	2 000	3 000	4 000	5 000	6 000
11	1 100	2 200	3 300	4 400	5 500	6 600
12	1 200	2 400	3 600	4 800	6 000	7 200

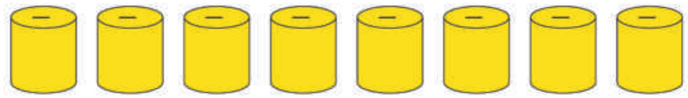
Answers

11. (a) 24 (learners' estimate)
    (b) 10; 20; 30; 40; 50; 60; 70; 80; 90; 100; 110; 120; 130; 140; 150; 160; 170; 180; 190; 200; 210; 220; 230; 240 ∴ 24 multiples
12. (a) 24
    100; 200; 300; 400; 500; 600; 700; 800; 900; 1 000; 1 100; 1 200; 1 300; 1 400; 1 500; 1 600; 1 700; 1 800; 1 900; 2 000; 2 100; 2 200; 2 300; 2 400
    (b) 9
    250; 500; 750; 1 000; 1 250; 1 500; 1 750; 2 000; 2 250
    (c) 4
    500; 1 000; 1 500; 2 000
    (d) 1
    2 000
13. (a) 30; 60; 90; 120; 150; 180; 210; 240
    (b) 300; 600; 900; 1 200; 1 500; 1 800; 2 100; 2 400
    Total value: R2 400
14. (a) 8 × 3 × R10 = R240 8 × 3 × R20 = R480 or 30 + 60 + 300 + 600
    8 × 3 × R100 = R2 400 8 × 3 × R200 = R4 800 = 990 × 8 = R7 920
    R240 + R480 + R2 400 + R4 800 = R7 920
    (b) Learners' own answers.
15. (a) Learners' own answers.
    (b) Answer depends on learners' own experiences and views, for example:
    Multiples of 50 have a repetitive pattern and you get to multiples of 100 at every second step.
16. Answer depends on learners' own experience and findings, for example:
    To count in forties is to count in double twenties, which has an easy pattern, because it is repeated over an interval of two decades. There is no easy pattern when counting in 70s. Counting in 90s is like counting in nines, like counting backwards 9; 8; 7; 6; ...

11. How many multiples of 10 are smaller than 250?
    (a) Estimate.
    (b) Check your estimate by writing down the multiples.

12. In each case, first estimate, then check by writing all the multiples down and counting them.
    (a) How many multiples of 100 are smaller than 2 500?
    (b) How many multiples of 250 are smaller than 2 500?
    (c) How many numbers smaller than 2 500 are multiples of both 100 and 250?
    (d) How many numbers smaller than 2 500 are multiples of both 250 and 400?

13. In each of the tins below, there are three R10 notes, three R20 notes, three R100 notes and three R200 notes.



Zain wants to know what the total value of all the R10 notes in all the tins is. He decides to find this out by counting in 30s, so he says: *thirty*, *sixty*, *ninety* . . . and so on while he points at one tin after another.

    (a) Complete what Zain started to do.
    (b) Count in 300s to find the total value of all the R100 notes in all the tins.

14. (a) How much money is there in total in the eight yellow tins in question 13?
    (b) Join with two classmates and tell them how you worked to find the total amount of money.

15. (a) Investigate what is easiest for you, to count in twenties or in thirties or in fifties, up to 500.
    (b) Many people find it easier to count in fifties than in thirties. Why do you think this is so?

16. What do you expect to be the most difficult, to count in forties or in seventies or in nineties? Investigate this and write a short report.

Here is some advice that can make it easier to count in certain counting units, for example in seventies.

It feels easier to count in fifties than in seventies because you get to multiples of 100 at every second step:

fifty, **hundred**, one hundred and fifty, **two hundred**, two hundred and fifty, **300**, 350, **400**, 450, **500** ... and so on.

When you count in seventies, this does not happen:

seventy, one hundred and forty, two hundred and ten, two hundred and eighty ...

CHAPTER 1: WORKING WITH WHOLE NUMBERS 9

Note on question 17

Make sure that learners understand how to use multiples of 100 as "stepping stones" when they count in multiples of 40, 60, 70, 80 and 90.

Answers

17. (a) 40 → 80 + 20 → 100 + 20 → 120 → 160 → 200 → 240 → 280 + 20 → 300 + 20 → 320 → 360 → 400 → 440 → 480 + 20 → 500 + 20 → 520 ... 920; 960; 1 000
    (b) 80; + 20 → 100 + 60 → 160; 240; 320; 400; 480; 560; 640; 720; 800; 880; 960; 1 040; 1 120; 1 200; 1 280; 1 360; 1 440; 1 520; 1 600
    (c) 90; 180; 270; 360; 450; 540; 630; 720; 810; 900; 990; 1 080; 1 170; 1 260; 1 350; 1 440; 1 530; 1 620; 1 710; 1 800
    (d) 700; 1 400; 2 100; 2 800; 3 500; 4 200; 4 900; 5 600; 6 300; 7 000
18. See LB page 10 alongside.

Additional notes on counting in nineties and eighties

An alternate method for using "stepping stones" is the following:

- To count in nineties, add 100 and subtract 10 to get the next number.
- To count in eighties, add 100 and subtract 20 to get the next number.

DOUBLING AND HALVING

Background information

Doubling means multiplying a number by 2 to get the next number.

Teaching guidelines

Start with any number and double it six times.

Answers

1. (a) 64; 128; 256; 512; 1 024; 2 048; 4 096; 8 192
   (b) 48; 96; 192; 384; 768; 1 536; 3 072; 6 144
   (c) 80; 160; 320; 640; 1 280; 2 560; 5 120; 10 240
   (d) 25; 30; 35; 40; 45; 50; 55; 60
   (e) 96; 192; 384; 768; 1 536; 3 072; 6 144; 12 288
2. Pattern (d)
   The number patterns in (d) are multiples of 5.

It may help you to cross over the multiples of 100 in two steps each time, like this:

70 + *30* → **(100)** + *40* → **140** + *60* → **(200)** + *10* → **210** + *70* → **280** ...

30 + 40 = 70 *60 + 10 = 70*

In this way, you make the multiples of 100 act as "stepping stones" for your counting.

17. (a) Count in forties up to 1 000. Try to use multiples of 100 as stepping stones. You can write the numbers while you count.
    (b) Write down the first twenty multiples of 80.
    (c) Write down the first twenty multiples of 90.
    (d) Write down the first ten multiples of 700.

18. Copy and complete this table.

×	60	50	70	90	40	20	30	80
8	480	400	560	720	320	160	240	640
6	360	300	420	540	240	120	180	480
7	420	350	490	630	280	140	210	560
9	540	450	630	810	360	180	270	720
5	300	250	350	450	200	100	150	400
3	180	150	210	270	120	60	90	240
4	240	200	280	360	160	80	120	320
2	120	100	140	180	80	40	60	160
70	4 200	3 500	4 900	6 300	2 800	1 400	2 100	5 600
30	1 800	1 500	2 100	2 700	1 200	600	900	2 400
60	3 600	3 000	4 200	5 400	2 400	1 200	1 800	4 800
80	4 800	4 000	5 600	7 200	3 200	1 600	2 400	6 400
40	2 400	2 000	2 800	3 600	1 600	800	1 200	3 200
90	5 400	4 500	6 300	8 100	3 600	1 800	2 700	7 200
50	3 000	2 500	3 500	4 500	2 000	1 000	1 500	4 000
20	1 200	1 000	1 400	1 800	800	400	600	1 600

DOUBLING AND HALVING

1. Write the next eight numbers in each pattern:
   (a) 1 2 4 8 16 32
   (b) 3 6 12 24
   (c) 5 10 20 40
   (d) 5 10 15 20
   (e) 6 12 24 48
2. Which pattern or patterns in question 1 are *not* formed by **repeated doubling**?

Note on question 4

To calculate **13 × 8**, start with 1 × 8 and use doubling as follows:

- If the first factor is doubled, the answer will double:
  - $\boxed{1} \times 8 = \boxed{8}$
  - $2 \times 8 = 16$
  - $\boxed{4} \times 8 = \boxed{32}$
  - $\boxed{8} \times 8 = \boxed{64}$
- Find the sum of the first factors that add up to 13: 8 + 4 + 1 = 13
- Add the corresponding multiples of 8: 64 + 32 + 8 = 104
- Write down the answer: 13 × 8 = 104

To calculate **37 × 9**, start with 1 × 9 and use doubling as follows:

- If the first factor is doubled, the answer will double:
  - $\boxed{1} \times 9 = \boxed{9}$
  - $2 \times 9 = 18$
  - $\boxed{4} \times 9 = \boxed{36}$
  - $8 \times 9 = 72$
  - $16 \times 9 = 144$
  - $\boxed{32} \times 9 = \boxed{288}$
- Find the sum of the first factors that add up to 37: 32 + 4 + 1 = 37
- Add the corresponding multiples of 9: 288 + 36 + 9 = 333
- Write down the answer: 37 × 9 = 333

Answers

3. See LB page 11 alongside.
4. (a) 42 84 168 336 672   672 + 84 + 21 = 777
   (b) 82 164 328 656   656 + 41 = 697

Background information

Halving means dividing a number by 2 to get the next number.

Teaching guidelines

Start with any large, even number and halve it repeatedly as far as possible.

The pattern 3 6 12 24 48 ... may be called the **repeated doubling pattern** that starts with 3.

3. Copy the table. Write the first nine terms of the repeated doubling patterns that start with the numbers in the left column of the table. The pattern for 13 has been completed as an example.

2	4	8	16	32	64	128	256	512
3	6	12	24	48	96	192	384	768
4	8	16	32	64	128	256	512	1 024
5	10	20	40	80	160	320	640	1 280
6	12	24	48	96	192	384	768	1 536
7	14	28	56	112	224	448	896	1 792
8	16	32	64	128	256	512	1 024	2 048
9	18	36	72	144	288	576	1 152	2 304
10	20	40	80	160	320	640	1 280	2 560
11	22	44	88	176	352	704	1 408	2 816
12	24	48	96	192	384	768	1 536	3 072
13	26	52	104	208	416	832	1 664	3 328
14	28	56	112	224	448	896	1 792	3 584
15	30	60	120	240	480	960	1 920	3 840
16	32	64	128	256	512	1 024	2 048	4 096
17	34	68	136	272	544	1 088	2 176	4 352
18	36	72	144	288	576	1 152	2 304	4 608
19	38	76	152	304	608	1 216	2 432	4 864

Doubling can be used to do multiplication.

For example, 29 × 8 can be calculated as follows:

8 doubled is 16, so 16 = 2 × 8 (step 1)
16 doubled is 32, so 32 = 4 × 8 (step 2)
32 doubled is 64, so 64 = 8 × 8 (step 3)
64 doubled is 128, so 128 = 16 × 8 (step 4). Doubling again will go past 29 × 8.
16 × 8 + 8 × 8 + 4 × 8 = (16 + 8 + 4) × 8 = 28 × 8.
So 28 × 8 = 128 + 64 + 32 which is 224. So 29 × 8 = 224 + 8 = 232.

4. Work as in the above example to calculate each of the following. Write only what you need to write.
   (a) 37 × 21   (b) 17 × 41

Note on question 5

To calculate **25 × 38**, start with 100 × 38 and use halving as follows:

- If the first factor is halved, the answer will be halved:
  - 100 × 38 = 3 800
  - 50 × 38 = 1 900
  - 25 × 38 = 950

To calculate **37 × 44**, start with 100 × 44 and use halving as follows:

- If the first factor is halved, the answer will be halved:
  - 100 × 44 = 4 400
  - 50 × 44 = 2 200
  - 25 × 44 = 1 100
  - 10 × 44 = 440
  - 2 × 44 = 88

Note that half of 25 does not yield a whole number, in this case explore multiples.
Half of 10 is 5, but 5 is not useful as it takes us to beyond 37.
In this case we double 44 which takes us to 37.

- Add the corresponding multiples of 44: 1 100 + 440 + 88 = 1 628
- Write down the answer: 37 × 44 = 1 628

Answers

5. (a) 64; 32; 16; 8; 4; 2; 1
   (b) 4 000; 2 000; 1 000; 500; 250
6. (a) 40 × 78 = 3 120; 20 × 78 = 1 560
   (b) 10 × 78 = 780; 5 × 78 = 390; 2 × 78 = 156
   1 560 + 780 + 390 + 156 = 2 886
7. See LB page 12 alongside. The school can buy 74 books. The books will cost R4 958.

USING MULTIPLICATION TO DO DIVISION

Teaching guidelines

To calculate **1 845 ÷ 15**, start with 100 × 15 and find "easy" multiples:

- 100 × 15 = 1 500 — 1 845 – 1 500 = 345
- 10 × 15 = 150
- 20 × 15 = 300 — 345 – 300 = 45
- 3 × 15 = 45 — 45 – 45 = 0

Write the answer: 1 845 ÷ 15 = 100 + 20 + 3 = 123

5. Continue each repeated halving pattern as far as you can:
   (a) 1 024 512 256 128 (b) 64 000 32 000 16 000 8 000

Halving can also be used to do multiplication.
For example, 37 × 28 can be calculated as follows:
100 × 28 = 2 800. Half of that is 50 × 28, which is half of 2 800, that is 1 400.
Half of 50 × 28 is half of 1 400, so 25 × 28 is 700.
10 × 28 = 280, so 25 × 28 + 10 × 28 = 980, so 35 × 28 = 980.
2 × 28 = 2 × 25 + 2 × 3 = 56, so 37 × 28 is 980 + 56 = 1 036.

6. 80 × 78 = 6 240. Use this information to work out each of the following:
   (a) 20 × 78 (b) 37 × 78

If chickens cost R27 each, how many chickens can you buy with R2 400? A way to use halving to work this out is shown below.

100 chickens cost 100 × 27 = R2 700. That is more than R2 400. Fifty chickens cost half as much, that is R1 350.
So I can buy 50 chickens and even more.
Half of 50 is 25, and half of R1 350 is R675.
So 75 chickens cost R1 350 + R675, which is R2 025. So there is R375 left.
Ten chickens cost R270, so 85 chickens cost R2 025 + R270 = R2 295. There is R105 left.
Three chickens cost 3 × R25 + 3 × R2 = R81.
I can buy 88 chickens and that will cost R2 376.

	Total cost	Thinking
100	R2 700	
50	R1 350	half of R2 700
25	R675	half of R1 350
75	R2 025	50 + 25 chickens
10	R270	10 × R27
85	R2 295	75 + 10 chickens
3	R81	3 × R27
88	R2 376	85 + 3 chickens

7. Use halving as in the above example to work out how many books at R67 each a school can buy with R5 000. Copy and use table on left to show your calculations.

The school can buy 74 books.

The books will cost R4 958.

	Total cost	Thinking
100	R6 700	
50	R3 350	half of R6 700
25	R1 675	half of R3 350
75	R5 025	50 + 25 books
74	R4 958	one book less

USING MULTIPLICATION TO DO DIVISION

1. R7 500 must be shared between 27 netball players. The money is in R10 notes, and no small change is available.
   (a) How much money will be used to give each player R100?

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Answers

1. (a) R2 700
   (b) Yes (R5 400)
   (c) No (R300 × 27 = R8 100)
   (d) 27 × R200 = R5 400   R7 500 – R5 400 = R2 100
   (e) Yes (27 × R50 = R1 350)
   (f) R270 per player
2. 100 × 17 m = 1 700 m
   200 × 17 m = 3 400 m   4 580 m – 3 400 m = 1 180 m
   50 × 17 m = 850 m   1 180 – 850 m = 330 m
   19 × 17 m = 323 m
   200 + 50 + 19 = 269 pieces
3. 20 × R26 = R520
   60 × R26 = R1 560   R1 800 – R1 560 = R240
   9 × R26 = R234   R240 – R234 = R6
   60 + 9 = 69 chickens

1.2 Ordering and comparing whole numbers

Teaching guidelines

- **Order numbers** by arranging them from smallest to biggest (ascending order), or from biggest to smallest (descending order).
- **Compare numbers** by their digit values from the left. For example:
  - 5 687 shows 5 thousands, 6 hundreds, 8 tens and 7 units
  - 5 678 shows 5 thousands, 6 hundreds, 7 tens and 8 units

  Thousands and hundreds are the same, but 8 tens is more than 7 tens, therefore 5 687 > 5 678 or 5 678 < 5 687.

Answers

1. Eleven days, 13 hours, 2 820 seconds
2. Two hundred and thirty-four million, five hundred thousand, three hundred and twenty
3. See LB page 13 alongside.

(b) Do you think there is enough money to give each player R200?
(c) Do you think there is enough money to give each player R300?
(d) How much of the R7 500 will be left over, if each player is given R200?
(e) Is there enough money left to give each player R50 more, in other words a total of R250 each?
(f) What is the highest amount that can be given to each player, so that less than R270 is left over? Remember that you cannot split up the R10 notes.

2. Work like you did in question 1 to solve this problem:
   There is 4 580 m of string on a big roll. How many pieces of 17 m each can be cut from this roll?
   Hint: You may start by asking yourself how much string will be used if you cut off 100 pieces of 17 m each.
3. Work like you did in questions 1 and 2 to solve this problem:
   A shop owner has R1 800 available with which he can buy chickens from a farmer. The farmer wants R26 for each chicken. How many chickens can the shop owner buy?

What you actually did in questions 1, 2 and 3 was to calculate 7 500 ÷ 27, 4 580 ÷ 17 and 1 800 ÷ 26. You solved division problems. Yet most of the work was to do multiplication, and a little bit of subtraction.

When you had to calculate 1 800 ÷ 26 in question 3, you may have asked yourself:

With what must I multiply 26, to get as close to 1 800 as possible?

Division is called the **inverse** of multiplication.
Multiplication is called the **inverse** of division.
Multiplication and division are **inverse operations**.

1.2 Ordering and comparing whole numbers

HOW FAR CAN YOU COUNT, AND HOW FAR IS FAR?

1. How long will it take to count to a million? Let us say it takes one second to count each number. Find out how long is one million seconds. Work in your exercise book. Give your final answer in days, hours and seconds.
2. Write 234 500 320 in words.
3. In each case write one of the symbols > or < to indicate which number is the smaller of the two.

(a)	876 243	>	876 234	(b)	534 616	<	543 016
(c)	701 021	>	698 769	(d)	103 232	>	99 878

Note on question 4

To compare whole numbers, position them on a number line.

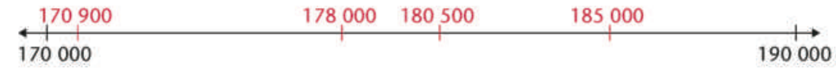
- For any two numbers on a number line, the left hand side number is smaller than the right hand side number.
- For any two numbers on a number line, the right hand side number is bigger than the left hand side number.

Answers

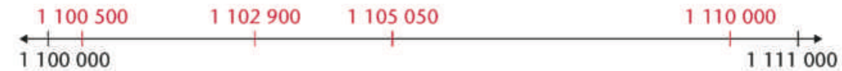
4. See LB page 14 alongside.
5. Uranus
6. Mars is farther away from the sun. It is about twice as far away from the sun than Venus is.
7. Mercury, Mars, Venus, Earth, Neptune, Uranus, Saturn, Jupiter.

4. Copy the number lines. In each case, place the numbers on the number line as carefully as you can.

   (a) 185 000; 178 000; 170 900; 180 500



   (b) 1 110 000; 1 102 900; 1 100 500; 1 105 050



The first row in the table shows the average distances of the planets from the sun. These distances are given in **millions of kilometres**.

One million kilometres is **1 000 000 km**.

The distances from the sun are called average distances, because the planets are not always the same distance from the sun. Their orbits are not circles.

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
Distance from the sun	58 million km	108 million km	150 million km	228 million km	778 million km	1 427 million km	2 870 million km	4 497 million km
Equatorial diameter	4 880 km	12 102 km	12 756 km	6 794 km	142 800 km	120 000 km	52 400 km	49 500 km

5. Which planet is the second farthest planet from the sun?
6. How does Mars' distance from the sun compare to that of Venus? Give two possible answers.
7. Arrange the planets from the smallest to the biggest.

Sometimes we do not need to know the exact number or exact amount. We say a loaf of bread costs about R10, or a bag of mealie meal costs about R20. The loaf of bread may cost R8 or R12 but it is close to R10. The mealie meal may cost R18 or R21 but it is close to R20.

When you read in a newspaper that there were 15 000 spectators at a soccer game, you know that that is not the actual number. In the language of mathematics we call this process **rounding off** or **rounding**.

ROUNDING TO 5s, 10s, 100s AND 1 000s

Background information

- **Estimation** is used to guess a number of objects without counting them. For example, a reporter estimates that there are 15 000 spectators at a soccer match.
- **Rounding** is used to make a real value easier to understand and more convenient to work with. For example, if 14 567 tickets for a soccer match are sold, the club may round off this number to the nearest 100 or 1 000.

Teaching guidelines

Rule for rounding off to the **nearest 5**: If a number:

- ends in 1 or 2, or 6 or 7, **round down** to the closest multiple of 5
- ends in 3 or 4, or 8 or 9, **round up** to the closest multiple of 5.

Rule for rounding off to the **nearest 10**: If a number:

- ends in 1, 2, 3 or 4, **round down** to the previous multiple of 10
- ends in 5, 6, 7, 8 or 9, **round up** to the next multiple of 10.

Rule for rounding off to the **nearest 100**: If the last two digits of the number:

- are less than 50, **round down** to the previous multiple of 100
- are equal to 50 or more, **round up** to the next multiple of 100.

Rule for rounding off to the **nearest 1 000**: If the last three digits of the number:

- are less than 500, **round down** to the previous multiple of 1 000
- are equal to 500 or more, **round up** to the next multiple of 1 000.

Rule for rounding off to the **nearest 10 000**: If the last four digits of the number:

- are less than 5 000, **round down** to the previous multiple of 10 000
- are equal to 5 000 or more, **round up** to the next multiple of 10 000.

Answers

1. (a) 610 (b) 90 (c) 455 (d) 1 330
2. See LB page 15 alongside.
3. See LB page 15 alongside.
4. See LB page 16 on the following page.

ROUNDING TO 5s, 10s, 100s AND 1 000s

To round off to the **nearest 5**, we round numbers that end in 1 or 2, or 6 or 7 **down** to the closest multiple of 5. We round numbers that end in 3 or 4, or 8 or 9 **up** to the closest multiple of 5.

For example, 233 is rounded down to 230, 234 is rounded up to 235, 237 is rounded down to 235 and 238 is rounded up to 240.

1. Round the following numbers to the nearest 5 by checking the **unit value**:

   (a) 612 (b) 87 (c) 454 (d) 1 328

To round off to the **nearest 10**, we round numbers that end in 1, 2, 3 or 4 **down** to the closest multiple of 10 (or decade). We round numbers that end in 5, 6, 7, 8 or 9 **up** to the closest multiple of 10.

For example, if you want to round off 534 to the nearest 10, you have to look at the units digit. The units digit is 4 and it is closer to 0 than to 10. The rounded off number will be 530.

2. Round the following numbers to the nearest 10 by checking the **unit value**:

   (a) 12 10 (b) 87 90 (c) 454 450 (d) 1 325 1 330

When **rounding to the nearest 100**, we look at the last **two digits** of the number. If the number is less than 50 we **round down** to the lower 100. If the number is 50 or more we **round up** to the higher 100.

3. Copy and complete the table.

	Round to the nearest 5	Round to the nearest 10	Round to the nearest 100
681	680	680	700
5 639	5 640	5 640	5 600
5 361	5 360	5 360	5 400
12 458	12 460	12 460	12 500

CHAPTER 1: WORKING WITH WHOLE NUMBERS 15

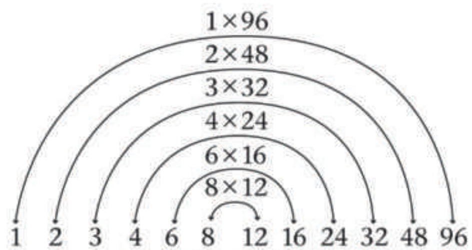
1.3 Factors, prime numbers and common multiples

DIFFERENT WAYS TO PRODUCE THE SAME NUMBER

Background information

A **factor** of a number can be divided into that number without leaving a remainder. For example, 1 is a factor of 96 because 96 ÷ 1 = 96, 2 is a factor of 96 because 96 ÷ 2 = 48, 3 is a factor of 96 because 96 ÷ 3 = 32, and so on.

The structure below shows all the factors of 96:



factors of 96

Any of the products above can be broken down into more factors. For example:

- 3 × 32 can be broken down into 3 × 2 × 16 or 3 × 4 × 8 or 3 × 2 × 2 × 8, etc.
- 4 × 24 can be broken down into 4 × 2 × 12 or 4 × 3 × 8 or 4 × 4 × 6, etc.

Teaching guidelines

The number 96 can be produced by multiplying 2, 6 and 8.

- We can write 96 = 2 × 6 × 8.
- We can say "96 is the product of 2, 6 and 8" or "96 can be expressed as the product 2 × 6 × 8".
- All products of 96 above can be broken down into 2 × 2 × 2 × 2 × 2 × 3. We say that 96 is written as a product of as many factors as possible (excluding 1).

Answers

The following calculation plans are not necessarily the only answers.

1. 8 × 10; 2 × 40
2. 1 × 2 × 5 × 8 = 80 (1 × 2) × (5 × 8) = 80
3. 2 × 15; 3 × 10; 5 × 6
4. (a) Yes 2 × 3 × 5

When **rounding to the nearest 1 000**, we look at the hundreds. Is the hundreds value less than, equal to or greater than 500? If less than 500, round down (the thousands value stays the same), if equal to 500 round up, and if greater than 500 round up too.

When **rounding to the nearest 10 000**, we look at the thousands. Is the thousands value less than, equal to or greater than 5 000? If less than 5 000, round down (the ten thousands value stays the same), if equal to 5 000 or greater than 5 000 round up.

4. Copy and then complete the table.

	Round to the nearest 1 000	Round to the nearest 10 000
142 389	142 000	140 000
343 621	344 000	340 000
356 552	357 000	360 000
100 489	100 000	100 000

1.3 Factors, prime numbers and common multiples

DIFFERENT WAYS TO PRODUCE THE SAME NUMBER

The number 80 can be produced by multiplying 4 and 20: 4 × 20 = 80.
The number 80 can also be produced by multiplying 5 and 16: 5 × 16 = 80.

1. In what other ways can 80 be produced by multiplying two numbers?

The number 80 can also be produced by multiplying 2, 10 and 4:
2 × 10 = 20 and 20 × 4 = 80 or 10 × 4 = 40 and 40 × 2 = 80.

We can use brackets to describe what calculation is done first. So instead of writing "2 × 10 = 20 and 20 × 4 = 80", we may write (2 × 10) × 4. Instead of writing "10 × 4 = 40 and 40 × 2", we may write 2 × (10 × 4).

2. Show how the number 80 can be produced by multiplying four numbers. Describe how you do it in two ways: without using brackets and by using brackets.
3. Show three different ways in which the number 30 can be produced by multiplying two numbers.
4. (a) Can the number 30 be produced by multiplying three whole numbers? Which three whole numbers?

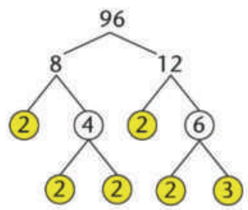
Note on question 9

To get as many factors as possible, excluding 1, each number must be factorised completely. The following methods show that 96 = 2 × 2 × 2 × 2 × 2 × 3:

Factor ladder

2	96	(96 is divisible by 2)
2	48	(48 is divisible by 2)
2	24	(24 is divisible by 2)
2	12	(12 is divisible by 2)
2	6	(6 is divisible by 2)
3	3	(3 is divisible by 3)
	1	(Stop when this is 1)

Factor tree



Answers

4. (b) No
5. See LB page 17 alongside.
6. 1 × 48; 2 × 24; 3 × 16; 4 × 12; 6 × 8
7. (a) 2 × 3 × 8 (b) 3 × 5 × 5
8. (a) Yes. 2 × 3 × 6 (b) Yes. 1 × 2 × 2 × 3 × 3
9. (a) 2 × 2 × 3 × 5 × 5 (b) 2 × 5 × 31 (c) 2 × 2 × 2 × 2 × 2 × 2 × 5
   (d) 2 × 3 × 5 × 11 (e) 2 × 2 × 5 × 17 (f) 2 × 5 × 5 × 7

PRIME NUMBERS

Background information

A **prime number** has only two different factors, 1 and itself.

Numbers with more than two factors are called **composite numbers**.

Teaching guidelines

Find numbers which can have only two different factors, 1 and itself.

Answers

1. (a) 2 × 2 × 3 × 3 (b) none (c) 2 × 19
   (d) 3 × 13 (e) 2 × 2 × 2 × 5 (f) none
   (g) 2 × 3 × 7 (h) none (i) 2 × 2 × 11
   (j) 3 × 3 × 5 (k) 2 × 23 (l) none
   (m) 2 × 2 × 2 × 2 × 3 (n) 7 × 7

Misconceptions

Some learners may think that 1 is a prime number. Although it can be written as a product of two factors (1 × 1), the two factors are not different, therefore 1 is not a prime number.

(b) Can the number 30 be produced by multiplying four whole numbers that do not include the number 1? If you answered "yes", which four numbers?

The number 105 can be produced by multiplying 3, 5 and 7, hence we can write 105 = 3 × 5 × 7. Mathematicians often describe this by saying "105 is the **product** of 3, 5 and 7" or "105 can be **expressed as the product** 3 × 5 × 7".

5. Express each of the following numbers as a product of three numbers.
   (a) 248 2 × 4 × 31 (b) 375 3 × 5 × 25

The whole numbers that are multiplied to form a number are called **factors** of the number. For example, 6 and 8 are factors of 48 because 6 × 8 = 48.

But 6 and 8 are not the only numbers that are factors of 48. 2 is also a factor of 48 because 48 = 2 × 24. And 24 is a factor of 48. The numbers 3 and 16 are also factors of 48 because 48 = 3 × 16.

6. Describe all the different ways in which 48 can be expressed as a product of two factors.

The number 36 can be formed by 2 × 2 × 3 × 3. Because 2 is used twice, it is called a **repeated factor** of 36. The number 3 is also a repeated factor of 36.

7. (a) Express 48 as a product of three factors.
   (b) Express 75 as a product of three factors.
8. (a) Can 36 be expressed as a product of three factors? How?
   (b) Can 36 be expressed as a product of five factors? How?
9. Express each of the following numbers as a product of as many factors as possible, including repeated factors. Do not use 1 as a factor.
   (a) 300 (b) 310 (c) 320
   (d) 330 (e) 340 (f) 350

PRIME NUMBERS

1. Express each of the following numbers as a product of as many factors as possible, including repeated factors. Do not use 1 as a factor.
   (a) 36 (b) 37 (c) 38
   (d) 39 (e) 40 (f) 41
   (g) 42 (h) 43 (i) 44
   (j) 45 (k) 46 (l) 47
   (m) 48 (n) 49

CHAPTER 1: WORKING WITH WHOLE NUMBERS 17

Answers

2. 37; 41; 43; 47
3. (a) 37; 41; 43; 47
   (b) 23 and 29
   (c) Yes
4. See LB page 18 alongside.
5. (a) 8 (2 × 2 × 2)
   (b) 30 (2 × 3 × 5)
6. Yes, he is correct.

2. Which of the numbers in question 1 cannot be expressed as a product of two whole numbers, except as the product *1 × the number itself*?

A number that cannot be expressed as a product of two whole numbers, except as the product *1 × the number itself*, is called a **prime number**.

3. (a) Which of the numbers in question 1 are prime numbers?
   (b) Which numbers between 20 and 30 are prime numbers?
   (c) Are 11 and 17 prime numbers?

Eratosthenes, a Greek mathematician who lived a long time ago, designed a method to find the prime numbers. The process is called "the sieve of Eratosthenes".

4. Copy the table on the right.
   Follow the steps to find all the prime numbers up to 100.

   Step 1: Cross out 1.

   Step 2: Circle 2, and then cross out all the multiples of 2.

   Step 3: Circle 3, then cross out all the multiples of 3.

   Step 4: Find the next number that has not been crossed out, encircle it, and cross out all its multiples.

   Continue like this.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

5. (a) What is the smallest number that can be formed as a product of three prime numbers, if the same factor may be repeated?
   (b) What is the smallest number that can be formed as a product of three prime numbers, if no repeated factors are allowed?

6. Manare did a lot of work, and found out that 840 can be formed as the product of 2, 2, 2, 3, 5 and 7. Check whether Manare is correct.

We can say that Manare **found the prime factors** of 840, or Manare **factorised 840 completely**.

We can write:

2 × 2 → 4 × 2 → 8 × 3 → 24 × 5 → 120 × 7 = 840.

Answers

7. (a) 825 (b) 315 (c) 2 002
8. (a) Agree. An even number is always a multiple of 2.
   (b) Agree. Half the number is obtained by dividing by 2. If the answer is even, it means you can divide by 2 again.
   (c) Disagree. The product of any odd prime numbers, other than 3, is odd (e.g. 5 × 7).
   (d) Agree. All numbers that end in 0 or 5 are multiples of 5.
9. See LB page 19 alongside.
10. Any three from the following list: 809; 811; 821; 823; 827; 829; 839.

HIGHEST COMMON FACTOR AND LOWEST COMMON MULTIPLE

Background information

- When a number is a factor of two or more other numbers, it is called a **common factor** of the other numbers. For example, 5 is a common factor of 110 and 105 because:
  - 110 = 2 × **5** × 11
  - 105 = 3 × **5** × 7.
- The biggest number that is a common factor of two or more numbers is called the **highest common factor** (**HCF**) of the numbers. For example, the HCF of 96 and 120 is 24 because 24 = 2 × 2 × 2 × 3 and:
  - 96 = **2** × **2** × **2** × 2 × 2 × **3**
  - 120 = **2** × **2** × **2** × **3** × 5.

Teaching guidelines

Learners should have a clear understanding of the difference between the concepts of common factors and highest common factors.

Answers

1. (a) 195 = 3 × 5 × 13 and 385 = 5 × 7 × 11
   (b) No
   (c) Yes

---

7. The prime factors of some numbers are given below. What are the numbers?
   (a) 3, 5, 5 and 11 (b) 3, 3, 5 and 7 (c) 2, 7, 11 and 13
8. Investigate which of the following statements you agree with. Give reasons for your agreement or disagreement in each case.
   (a) If a number is even, 2 is one of its prime factors.
   (b) If half an even number is also even, 2 is a repeated prime factor.
   (c) If a number is odd, 3 is one of its prime factors.
   (d) If a number ends in 0 or 5, then 5 is one of its prime factors.

Here is a method to find the prime factors of a number:

If the number is even, divide it by 2. If the answer is even, divide by 2 again. Continue like this as long as it is possible. If the answer is odd, divide by 3, if it is possible. Continue to divide by 3 as long as it is possible. Then switch to 5. Continue like this by each time trying to divide by the next prime number.

9. Find all the prime factors of each of the following numbers. Write only your answers below.
   (a) 588 2, 2, 3, 7 and 7 (b) 825 3, 5, 5 and 11
   (c) 729 3, 3, 3, 3, 3 and 3 (d) 999 3, 3, 3 and 37
   (e) 538 2 and 269 (f) 113 113
10. Find at least three prime numbers between 800 and 850.

HIGHEST COMMON FACTOR AND LOWEST COMMON MULTIPLE

1. (a) Factorise 195 and 385 completely.
   (b) Is 7 a factor of both 195 and 385?
   (c) Is 5 a factor of both 195 and 385?

> When a number is a factor of two or more other numbers, it is called a **common factor** of the other numbers. For example, the number 5 is a common factor of 195 and 385.

The factors of a certain number are 2; 2; 5; 7; 7; 11 and 17. The factors of another number are 2; 3; 3; 7; 7; 11; 13 and 23. The common prime factors of these two numbers are 2; 7; 7 and 11.

> The biggest number that is a factor of two or more numbers is called the **highest common factor (HCF)** of the numbers.

Background information

- When a number is a multiple of two or more other numbers, it is called a **common multiple** of the other numbers. For example, 36 is one of the common multiples of 6 and 9 because the first ten multiples of:
  - 6 are 6; 12; **18**; 24; 30; **36**; 42; 48; 54 and 60
  - 9 are 9; **18**; 27; **36**; 45; 54; 63; 72; 81; and 90.
- The smallest number that is a common multiple of two or more numbers is called the **lowest common multiple** (**LCM**) of the numbers. For example, the LCM of 6 and 9 is 18, the smallest of the multiples of 6 and 9 above.

Teaching guidelines

Discuss the difference between a common multiple and the lowest common multiple.

Answers

2. (a) 2 × 7 × 7 × 11 = 1 078 (b) 8 (c) 4 (d) 4 (e) 14
3. For example: 70 105 140 175 210 245 350 3 500
4. See LB page 20 alongside.
5. See LB page 20 alongside.
6. (a) HCF = 1 LCM = 35 (b) HCF = 1 LCM = 210
   (c) HCF = 10 LCM = 60 (d) HCF = 10 LCM = 100
   (e) HCF = 1 LCM = 72 (f) HCF = 1 LCM = 600
   (g) HCF = 4 LCM = 24 (h) HCF = 2 LCM = 90

1.4 Properties of operations

ORDER OF OPERATIONS AND THE ASSOCIATIVE PROPERTY

Background information

A **mathematical convention** is an agreement made by mathematicians to be followed all over the world to prevent different interpretations or understandings of the same numerical expression. For example:

- if subtraction is performed from the left: 200 – 100 – 30 = 100 – 30 = 70
- if subtraction is performed from the right: 200 – 100 – 30 = 200 – 70 = 130.

---

2. Find the HCF of the two numbers in each of the following cases.
   (a) 2 × 2 × 5 × 7 × 7 × 11 × 17 and 2 × 3 × 3 × 7 × 7 × 11 × 13 × 23
   (b) 24 and 40 (c) 8 and 12
   (d) 12 and 20 (e) 210 and 56
3. Write five different numbers, all different from 35, that have 35 as a highest common factor.
4. Copy pattern A and pattern B. Write the next seven numbers in each pattern:

A:	12	24	36	48	60	72	84	96	108	120	132
B:	15	30	45	60	75	90	105	120	135	150	165

The numbers in pattern A are called the **multiples** of 12. The numbers in pattern B are called the multiples of 15. The numbers, for example 60 and 120, that occur in both patterns, are called the **common multiples** of 12 and 15. The smallest of these numbers, namely 60, is called the **lowest common multiple** (LCM) of 12 and 15.

5. Continue writing multiples of 18 and 24 until you find the LCM:
   18 36 54 72
   24 48 72
6. Find the HCF and LCM of the given numbers in each case below:
   (a) 5 and 7 (b) 15 and 14
   (c) 20 and 30 (d) 10 and 100
   (e) 8 and 9 (f) 25 and 24
   (g) 8 and 12 (h) 10 and 18

1.4 Properties of operations

ORDER OF OPERATIONS AND THE ASSOCIATIVE PROPERTY

Suppose you want to tell another person to do some calculations. You may do this by writing instructions. For example, you may write the instruction 200 – 130 – 30. This may be called a **numerical expression**.

Suppose you have given the instruction 200 – 130 – 30 to two people, whom we will call Ben and Sara.

This is what Ben does: 200 – 130 = 70 and 70 – 30 = 40.

This is what Sara does: 130 – 30 = 100 and 200 – 100 = 100.

To prevent such different interpretations or understandings of the same numerical expression, mathematicians have made the following agreement, and this is followed all over the world:

Note

An important mathematical convention:

Numerical expressions that involve only addition and subtraction should be simplified by performing the operations from left to right, unless otherwise indicated in some way.

Teaching guidelines

- Demonstrate the necessity for the mathematical convention above by using different orders of operations to simplify mathematical expressions.
- Demonstrate the associative properties for addition and multiplication:
  - Three or more numbers can be added in any order.
  - Three or more numbers can be multiplied in any order.

Answers

1. Ben
2. See LB page 21 alongside.
3. See LB page 21 alongside.
4. Yes, it is associative. Example: 2 × 3 × 5 × 10 = 300; 10 × 5 × 3 × 2 = 300
5. 27 44 34 59 66 77 12
6. (a) 100 + 100 + 138 = 338 (b) 100 + 100 + 100 + 127 = 427

THE COMMUTATIVE PROPERTY OF ADDITION AND MULTIPLICATION

Background information

If a numerical expression involves:

- **only addition**, calculations can be performed in any order
- **only multiplication**, calculations can be performed in any order.

This is the commutative property of addition and multiplication.

Teaching guidelines

Demonstrate the commutative properties of addition and multiplication.

Answers

1. (a) R5 000 (b) R5 000 (c) R50 (d) 2c
2. See LB page 21 alongside.

In a numerical expression that involves **addition and subtraction only**, the operations should be performed **from left to right, unless otherwise indicated** in some way.

An agreement like this is called a **mathematical convention**.

1. Who followed this convention, Ben or Sara?
2. Follow the above convention and calculate each of the following:
   (a) 8 000 + 6 000 – 3 000 = 11 000
   (b) 8 000 – 3 000 + 6 000 = 11 000
   (c) 8 000 + 3 000 – 6 000 = 5 000
3. Follow the above convention and calculate each of the following:
   (a) R25 000 + R30 000 + R13 000 + R6 000 = R74 000
   (b) R13 000 + R6 000 + R30 000 + R25 000 = R74 000
   (c) R30 000 + R25 000 + R6 000 + R13 000 = R74 000

In question 3, all your answers should be the same. When three or more numbers are added, the order in which you perform the calculations makes no difference. This is called the **associative property of addition**. We also say that: **addition is associative**.

4. Investigate whether multiplication is associative. Use the numbers 2, 3, 5 and 10.
5. What must be added to each of the following numbers to get 100?
   73 56 66 41 34 23 88
6. Calculate each of the following. Note that you can make the work simple by being smart in deciding which additions to do first.
   (a) 73 + 54 + 27 + 46 + 138 (b) 34 + 88 + 41 + 66 + 59 + 12 + 127

THE COMMUTATIVE PROPERTY OF ADDITION AND MULTIPLICATION

1. (a) What is the total cost of 20 chairs at R250 each?
   (b) What is the total cost of 250 exercise books at R20 each?
   (c) R5 000 was paid for 100 towels. What is the price for one towel?
   (d) R100 was paid for 5 000 beads. What is the price for one bead?
2. Which of the following calculations will produce the same answer? Copy the calculations and mark those that will produce the same answers with a ✓ and those that won't with a ✗.
   (a) 20 × 250 and 250 × 20 ✓ (b) 5 000 ÷ 100 and 100 ÷ 5 000 ✗
   (c) 730 + 270 and 270 + 730 ✓ (d) 730 – 270 and 270 – 730 ✗

Answers

3. Accept any appropriate examples.

 (a) No (b) Yes (c) No

MORE CONVENTIONS AND THE DISTRIBUTIVE PROPERTY

Background information

- The **distributive property** allows the decomposition of a product into partial products that can be more easily calculated. For example, 7 × 648, which means 7 × (600 + 40 + 8), can be decomposed into: 7 × 600 + 7 × 40 + 7 × 8. A person who knows basic multiplication facts (tables) can easily calculate these partial products and add them up to evaluate 7 × 648. The column format is a convenient way of making the transition from 7 × (600 + 40 + 8) to 7 × 600 + 7 × 40 + 7 × 8.
- The distributive property enables us to **distribute multiplication over addition and subtraction**. For example:

  7 × 123 = 7 × (100 + 20 + 3) = 7 × 100 + 7 × 20 + 7 × 3
  = 700 + 140 + 21 = 861

  7 × 99 = 7 × (100 – 1) = 7 × 100 – 7 × 1 = 700 – 7 = 693

More mathematical conventions

- **Multiplication and division** should be done before addition and subtraction, unless otherwise stated.
- **Brackets** are used to specify that the operations within the brackets should be done first.

Misconceptions

"The distributive property **requires** that 'brackets be removed' before a product expression is evaluated." Although it is true that, for example, 7 × (25 + 75) = 7 × 25 + 7 × 75, there is no "requirement" that the form on the right should be executed to evaluate the expression. In fact, it is much easier to apply the operations specified in the form on the left, i.e. 25 + 75 = 100 and 7 × 100 = 700, than to apply the operations specified on the right, namely 7 × 25 = 175 and 7 × 75 = ... etc. It is important that learners realise that the distributive property is not a "law" that requires that something be done, it is a proposition that states that two different sets of calculations will consistently produce the same result, hence any of the two sets can be used.

25 + 75 and 75 + 25 have the same answer. The same is true for any other two numbers. We say: addition is **commutative**; the numbers can be swopped around.

3. Demonstrate each of your answers with two different examples.
   (a) Is subtraction commutative?
   (b) Is multiplication commutative?
   (c) Is division commutative?

MORE CONVENTIONS AND THE DISTRIBUTIVE PROPERTY

1. Do the following:
   (a) Multiply 5 by 3, then add the answer to 20. 35
   (b) Add 5 to 20, then multiply the answer by 5. 125

Mathematicians have agreed that **unless otherwise indicated, multiplication and division should be done before addition and subtraction**. According to this convention, the expression 20 + 5 × 3 should be taken to mean "multiply 5 by 3, then add the answer to 20" and not "add 5 to 20, then multiply the answer by 3".

2. Follow the above convention and calculate each of the following:

(a) 500 + 20 × 10	(b) 500 – 20 × 10	(c) 500 + 20 – 10
(d) 500 – 20 + 10	(e) 500 + 200 ÷ 5	(f) 500 – 200 ÷ 5

If some of your answers are the same, you have made mistakes.

The above convention creates a problem. How can one describe the calculations in question 1(b) with a numerical expression, without using words?

To solve this problem, mathematicians have agreed to use brackets in numerical expressions. **Brackets are used to specify that the operations within the brackets should be done first.** Hence the numerical expression for 1(b) above is (20 + 5) × 5, and the answer is 125.

If there are **no brackets** in a numerical expression, it **means that multiplication and division should be done first, and addition and subtraction only later**.

If you wish to specify that addition or subtraction should be done first, that part of the expression should be enclosed in brackets.

Examples
The expression 12 + 3 × 5 means "multiply 3 by 5, then add 12". It *does not* mean "add 12 and 3, then multiply by 5".
If you wish to say "add 5 and 12, then multiply by 3", the numerical expression should be 3 × (5 + 12) or (5 + 12) × 3. They mean the same.

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Teaching guidelines

- Illustrate how the distributive property enables us to distribute multiplication over addition by calculating both sides of the following:
  $8 \times 234 = 8 \times (200 + 30 + 4) = 8 \times 200 + 8 \times 30 + 8 \times 4$
- Illustrate how the distributive property enables us to distribute multiplication over subtraction by calculating both sides of the following:
  $8 \times 394 = 8 \times (400 - 6) = 8 \times 400 - 8 \times 6$
- Discuss the mathematical conventions listed above.

Answers

1. See LB page 22 on previous page.

2.
(a) 700	(b) 300	(c) 510
(d) 490	(e) 500 + 40 = 540	(f) 500 – 40 = 460

3.
(a) 800	(b) 5 300	(c) 50 030
(d) 53 000	(e) 200	(f) 4 700
(g) 49 970	(h) 47 000	(i) 500 ÷ 20 = 25
(j) 10 + 15 = 25	(k) 600 ÷ 50 = 12	(l) 30 + 20 = 50

4. (a) 5 000 (b) 5 000 (c) 2 000 (d) 2 000

5. See LB page 23 alongside.

6.
A: 1 015	B: 1 003	C: 1 015	D: 605	E: 615
F: 1 015	G: 1 015	H: 1 015	I: 1 150	

7. (a) 4; 7; 2
   (b) Sample answer (example of numbers chosen above): (2 + 4) × 7 = 42
   (c) (4 × 7) + (2 × 7) = 28 + 14 = 42
   (d) Yes

3. Keep the various mathematical conventions about numerical expressions in mind when you calculate each of the following:

(a) 500 + 30 × 10	(b) (500 + 30) × 10
(c) 100 × 500 + 30	(d) 100 × (500 + 30)
(e) 500 – 30 × 10	(f) (500 – 30) × 10
(g) 100 × 500 – 30	(h) 100 × (500 – 30)
(i) (200 + 300) ÷ 20	(j) 200 ÷ 20 + 300 ÷ 20
(k) 600 ÷ (20 + 30)	(l) 600 ÷ 20 + 600 ÷ 30

4. Calculate the following:

(a) 50 × (70 + 30)	(b) 50 × 70 + 50 × 30
(c) 50 × (70 – 30)	(d) 50 × 70 – 50 × 30

Your answers for 4(a) and 4(b) should be the same.
Your answers for 4(c) and 4(d) should also be the same.

5. Do not do calculations A to I below. Just answer these questions about them. You will check your answers later.
   (a) Will A and B have the same answers? No
   (b) Will G and H have the same answers? Yes
   (c) Will A and D have the same answers? No
   (d) Will A and G have the same answers? Yes
   (e) Will A and F have the same answers? Yes
   (f) Will D and E have the same answers? No

A: 5 × (200 + 3)	B: 5 × 200 + 3
C: 5 × 200 + 5 × 3	D: 5 + 200 × 3
E: (5 + 200) × 3	F: (200 + 3) × 5
G: 5 × 203	H: 5 × 100 + 5 × 103
I: 5 × 300 – 5 × 70	

6. Now do calculations A to I. Then check the answers you gave in question 5.

7. (a) Choose three different numbers between 3 and 11, and write them down like this:
   Your first number: ..4.. Your second number: ..7.. Your third number: ..2..
   (b) Add your first number to your third number. Multiply the answer by your second number.
   (c) Multiply your first number by your second number. Also multiply your third number by your second number. Add the two answers.
   (d) If you worked correctly, you should get the same answers in (b) and (c). Do you think you will get the same result with numbers between 10 and 100, or any other numbers?

CHAPTER 1: WORKING WITH WHOLE NUMBERS 23

Answers

8. (a) 100 × (50 + 10) = 100 × 50 + 100 × 10 = 5 000 + 1 000 = 6 000
   (b) Accept any appropriate answers.
9. Yes. 100 × (50 – 10) = 100 × 40 = 4 000
   100 × 50 – 100 × 10 = 5 000 – 1 000 = 4 000
10. Yes, it is.

1.5 Basic operations

Background information

The CAPS (page 40) requires that "...the properties of numbers should provide the motivation for why and how operations with numbers work". To be aware of the role of the distributive, associative and commutative properties in addition, subtraction, multiplication and division, a person needs to interpret numbers in terms of their parts, as if they were written in expanded notation. For instance, addition in columns of, for example, 354 + 235 is based on the associative and commutative properties of addition. Instead of adding up 300 + 50 + 4 + 200 + 30 + 5 from left to right, the addition is performed by calculating (4 + 5) + (50 + 30) + (300 + 200).

By writing the numbers below each other and thinking of columns, is to "automatically" apply the associative and commutative properties: (300 + 50 + 4) + (200 + 30 + 5) is replaced by (4 + 5) + (50 + 30) + (300 + 200) by writing it in columns and working from right to left:

```
3 5 4
2 3 5
-----
```

Important note

Teachers are strongly advised to read the learner text on the following Learner Book pages before learners engage with these pages in class:

- Addition in columns: pages 24–26
- Subtraction in columns: pages 26–27
- Multiplication in columns (long multiplication): pages 28–29
- Division in columns (long division): pages 29–32

Many learners need support to make sense of very large numbers, and the CAPS requires engagement with whole numbers with up to nine digits. The activities on LB pages 24–32 provide learners with opportunities to imagine very large numbers of objects.

The fact that your answers for calculations like those in 7(b) and 7(c) are equal, for any numbers that you may choose, is called the **distributive property of multiplication over addition**.

It may be described as follows:
first number × second number + first number × third number
= first number × (second number + third number).
This can be described by saying that **multiplication distributes over addition**.

8. Check whether the distributive property is true for the following sets of numbers:
   (a) 100, 50 and 10
   (b) any three numbers of your own choice (you may use a calculator to do this)
9. Use the numbers in question 8(a) to investigate whether multiplication also distributes over subtraction.

It is quite fortunate that multiplication distributes over addition, because it makes it easier to multiply.

For example, 8 × 238 can be calculated by calculating 8 × 200, 8 × 30 and 8 × 8, and adding the answers: 8 × 238 = 8 × 200 + 8 × 30 + 8 × 8 = 1 600 + 240 + 64 = 1 904.

10. Check whether 8 × 238 is actually 1 904 by calculating 238 + 238 + 238 + 238 + 238 + 238 + 238 + 238, or by using a calculator.

1.5 Basic operations

A METHOD OF ADDITION

To add two numbers, the one may be written below the other.

For example, to calculate 378 539 + 46 285 the one number may be written below the other so that the units are below the units, the tens below the tens, and so on.

```
378 539
 46 285
```

Writing the numbers like this has the advantage that:

- the units parts (9 and 5) of the two numbers are now in the same column,
- the tens parts (30 and 80) are in the same column,
- the hundreds parts (500 and 200) are in the same column, and so on.

This makes it possible to work with each kind of part separately.

We only write this:	In your mind you can see this:					
378 539	300 000	70 000	8 000	500	30	9
46 285		40 000	6 000	200	80	5

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A METHOD OF ADDITION

Teaching guidelines

Use the following example to illustrate various layouts of addition in columns. It is important that learners notice the similarity among the sets of framed digits in the different methods.

- **Add in expanded form:**

  573 489 = 500 000 + 70 000 + 3 000 + 400 + 80 + 9

  379 692 = 300 000 + 70 000 + 9 000 + 600 + 90 + 2

  Sum = 800 000 + [1]40 000 + [1]2 000 + [1] 000 + [1]70 + [1]1

  = 940 000 + 13 000 + 181

  = 953 000 + 181

  = 953 181

- **Add the same numbers in columns without "carries":**

  573 489
  \+ 379 692
  [1]1 (add units)
  [1]70 (add 10s)
  [1] 000 (add 100s)
  [1]2 000 (add 1000s)
  [1]40 000 (add 10 000s)
  800 000 (add 100 000s)
  953 181

- **Add the same numbers in columns using "carries":**

  [1] [1] [1] [1] [1]
  5 7 3 4 8 9
  \+ 3 7 9 6 9 2
  9 5 3 1 8 1

Answers

1. (a) 325 623 (b) 959 374 (c) 394 054

   Total budget: R607 940

The numbers in each column can be added to get a new set of numbers:

378 539	300 000	70 000	8 000	500	30	9
46 285		40 000	6 000	200	80	5
14						14
110					110	
700				700		
14 000			14 000			
110 000		110 000				
300 000	300 000					
424 824						

It is easy to add the new set of numbers to get the answer.

Note that you can do the above steps in any order. Instead of starting with the units parts as shown above, you can start with the hundred thousands, or any other parts.

Starting with the units parts has an advantage though as it makes it possible to do more of the work mentally and to write less, as shown below:

378 539
46 285
424 824

To achieve this, only the units digit 4 of the 14 is written in the first step. The 10 of the 14 is remembered and added to the 30 and 80 of the tens column, to get 120.

We say the 10 is **carried** from the units column to the tens column. The same is done when the tens parts are added to get 120: only the digit "2" is written (in the tens column, so it means 20), and the 100 is carried to the next step.

1. Calculate each of the following:
   (a) 237 847 + 87 776 (b) 567 298 + 392 076 (c) 28 387 + 365 667

A municipal manager is working on the municipal budget for a year. He has to try to keep the total expenditure on new office equipment below R800 000. He still has to budget for new computers that are badly needed, but this is what he has written so far:

74 new office chairs	R 54 020
42 new computer screens	R 100 800
12 new printers	R 141 600
18 new tea trolleys	R 25 740
8 new carpets for senior staff offices	R 144 000
108 small plastic filing cabinets	R 52 380
new table for the boardroom	R 48 000
18 new chairs for the boardroom	R 41 400
	R 607 940

Answers

2. R242 400
3. R143 420
4. R607 940
5. (a) 25 841 (b) 537 888

METHODS OF SUBTRACTION

Teaching guidelines

There are various methods to subtract numbers. For example:

- **adding on:** To find the difference between two numbers, start at the smaller number and add on up to the larger number. This resembles skipping forward from the smaller number to the larger number on a number line. The total skipped forward then equals the difference between the two numbers, i.e. to determine 26 – 14: start at 14, skip 6 up to 20 and 6 more up to get to 26, total skipped forward is 6 + 6 = 12 and therefore the difference is 26 – 14 = 12.
- **using compensation:** To calculate 26 – 14, compensation can be used in one of two ways:
  - Create an "easier" number **to subtract from** by adding 4 to both numbers: 26 – 14 = (26 + 4) – (14 + 4) = 30 – 18 = 12
  - Create an "easier" number **to subtract from** by adding 6 to both numbers: 26 – 14 = (26 + 6) – (14 + 6) = 32 – 20 = 12
- **using expanded form:** Refer to the teaching guidelines at the bottom of this page.

Discuss the methods listed. Start with 2-digit numbers and extend to larger numbers.

Answers

1. (a) 3 242 (b) 41 347
   (c) 88 544 (d) 386 573
2. Answers are the same as in question 1.

Teaching guidelines (continued)

Subtracting in columns resembles writing numbers in expanded form. Use the method shown on the following page to explain the process. Start with numbers which will not involve any "borrowing" and end with numbers such as those on the following page.

2. How much has the municipal manager budgeted for printers and computer screens together?
3. How much, in total, has the municipal manager budgeted for chairs and tables?
4. Work out the total cost of all the items the municipal manager has budgeted for.
5. Calculate.
   (a) 23 809 + 2 009 + 23 (b) 320 293 + 16 923 + 349 + 200 323

METHODS OF SUBTRACTION

There are many ways to subtract one number from another. For example, R835 234 – R687 885 can be calculated by "filling up" from R687 885 to R835 234:

687 885 + 15 → 687 900 + 100 → 688 000 + 12 000 → 700 000 + 135 234 → 835 234

The difference between R687 885 and R835 234 can now be calculated by adding up the numbers that had to be added to 687 885 to get 835 234.

So R835 234 – R687 885 = R147 349.

15
100
12 000
135 234
147 349

Another easy way to subtract is to **round off and compensate**. For example, to calculate R3 224 – R1 885, the R1 885 may be rounded up to R2 000. The calculation can proceed as follows:

- Rounding R1 885 up to R2 000 can be done in two steps: 1 885 + 15 = 1 900, and 1 900 + 100 = 2 000. In total, 115 was added.
- 115 can now be added to 3 224 too: 3 224 + 115 = 3 339.

Instead of calculating R3 224 – R1 885, which is a bit difficult, R3 339 – R2 000 may be calculated. This is easy: R3 339 – R2 000 = R1 339.

This means that R3 224 – R1 885 = R1 339, because R3 224 – R1 885 = (R3 224 + R115) – (R1 885 + R115).

To do question 1, you may use any one of the above two methods, or any other method you may know and prefer. Do not use a calculator, because the purpose of this work is for you to come to understand how subtraction may be done. What you will learn here, will later help you to understand **algebra**.

1. Calculate each of the following:
   (a) 6 234 – 2 992 (b) 76 214 – 34 867
   (c) 134 372 – 45 828 (d) 623 341 – 236 768
2. Check each of your answers in question 1 by doing addition, or by doing subtraction with a different method than the method you have already used.

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- **Subtract without using "borrows":**

Step 1: Write both numbers in expanded form:

573 489 = 500 000 + 70 000 + 3 000 + 400 + 80 + 9

379 692 = 300 000 + 70 000 + 9 000 + 600 + 90 + 2

Step 2: Rearrange the numbers in row 1 so that subtraction is possible:

- 400 – 100 = 300; 100 + 80 = 180:
  573 489 = 500 000 + 70 000 + 3 000 + 300 + 180 + 9
- 3 000 – 1 000 = 2 000; 1 000 + 300 = 1 300:
  573 489 = 500 000 + 70 000 + 2 000 + 1 300 + 180 + 9
- 70 000 – 10 000 = 60 000; 10 000 + 2 000 = 12 000:
  573 489 = 500 000 + 60 000 + 12 000 + 1 300 + 180 + 9
- 500 000 – 100 000 = 400 000; 100 000 + 60 000 = 160 000:
  573 489 = 400 000 + 160 000 + 12 000 + 1 300 + 180 + 9

Step 3: Subtract in columns:

573 489 = 400 000 + 160 000 + 12 000 + 1 300 + 180 + 9

379 692 = 300 000 + 70 000 + 9 000 + 600 + 90 + 2

Difference = 100 000 + 90 000 + 3 000 + 700 + 90 + 7 = 193 797

- **Subtract the same numbers using "borrows":**

```
 4 16
   6 12
     2 13
       3 18
  5 7 3 4 8 9
– 3 7 9 6 9 2
  1 9 3 7 9 7
```

Answers

3. 10 000 was taken from the 30 000 in the ten thousands column, and added to the 4 000 that remained in the thousands column. 100 000 was taken from the 800 000 in the hundred thousands column, and added to the 20 000 that remained in the ten thousands column.
4. (a) R15 335 (b) R33 511
5. (a) 100 000; 97 316 (b) 400 000; 416 455
6. (a) 40 000; 42 359 (b) 160 000; 165 476
7. R800 000 – R607 940 = R192 060
8. (a) 370 035 (b) 370 034 (c) 200 541 (d) 200 541

Another method of subtraction is to think of the numbers in **expanded notation**. For example, to calculate R835 234 – R687 885, which was already done in a different way on the previous page, we could work like this:

We may write this:	In your mind you can see this:					
835 234	800 000	30 000	5 000	200	30	4
687 885	600 000	80 000	7 000	800	80	5

Unfortunately, it is not possible to subtract in the columns now. However, the parts of the bigger number can be rearranged to make the subtraction in each column possible:

835 234	700 000	120 000	14 000	1100	120	14
687 885	600 000	80 000	7 000	800	80	5
	100 000	40 000	7 000	300	40	9

The answer is now clearly visible; it is 147 349.

The rearrangement, also called "borrowing", was done like this:
10 was taken from the 30 in the tens column, and added to the 4 in the units column.
100 was taken from the 200 in the hundreds column, and added to the 20 that remained in the tens column. 1 000 was taken from the 5 000 in the thousands column, and added to the 100 that remained in the hundreds column.

3. Describe the other rearrangements that were made in the above work.

It is not practical to write the expanded notation and the rearrangements each time you do a subtraction. However, with some practice you can learn to do it all in your mind without writing it down. Some people make small marks above the digits of the bigger number, or even change the digits, to keep track of the rearrangements they make in their minds.

```
 835 234
 687 885
 147 349
```

4. Calculate the difference between the two car prices in each case.
   (a) R73 463 and R88 798 (b) R63 378 and R96 889
5. In each case, first estimate the answer to the nearest 100 000, then calculate.
   (a) 238 769 – 141 453 (b) 856 333 – 439 878
6. In each case, first estimate the answer to the nearest 10 000, then calculate.
   (a) 88 023 – 45 664 (b) 342 029 – 176 553
7. Look again at the municipal budget on page 25. How much money does the municipal manager have left to buy new computers?
8. Calculate.
   (a) 670 034 – 299 999 (b) 670 034 – 300 000
   (c) 376 539 – 175 998 (d) 376 541 – 176 000

CHAPTER 1: WORKING WITH WHOLE NUMBERS 27

A METHOD OF MULTIPLICATION

Teaching guidelines

Different methods can be used to multiply numbers:

- multiplying **a large number by a 1-digit number**:
  - Expand the large number into parts, multiply each part by the 1-digit number and add the final products. For example,
    $7 \times 234 = 7 \times (200 + 30 + 4) = 7 \times 200 + 7 \times 30 + 7 \times 4$
    $= 1\,400 + 210 + 28 = 1\,638$

  It is convenient to write the work in columns:
  - Multiply in columns without using "carries":

```
      2 3 4
  ×       7
  ---------
        2 8   (7 × 4)
      2 1 0   (7 × 30)
    1 4 0 0   (7 × 200)
    -------
    1 6 3 8
```

  - Multiply in columns using "carries":

```
  Th H  T  U
     2  2
     2  3  4
  ×        7
  ----------
  1  6  3  8
```

7 × 4 = 28; write 8 and carry 2.
7 × 3 + 2 carries = 23; write 3 and carry 2.
7 × 2 + 2 carries = 16; write 16.

- multiplying **a large number by a 2-digit number**:
  - Expand the 2-digit number into tens and units and multiply the large number by the two parts separately. Refer to question 4 on LB page 28.
  - Multiply in columns without using "carries". Refer to question 5 on LB page 28 alongside.
  - Multiply using "carries": Refer to question 6 on LB page 29.

Answers

1. (a) 6 288 (b) 31 077 (c) 47 160 (d) 471 600
2. Answers are the same as in question 1.
3. 53 448
4. 378 × 30 = 11 340; 378 × 6 = 2 268; 11 340 + 2 268 = 13 608
5. (a) 6 × 40 = 240 (b) 70 × 8 = 560 (c) 70 × 300 = 21 000

A METHOD OF MULTIPLICATION

6 × R3 258 can be calculated in parts, as shown below.

6 × R3 000 = R18 000
6 × R200 = R1 200
6 × R50 = R300
6 × R8 = R48

```
      3 2 5 8
  ×         6
  -----------
          4 8
        3 0 0
      1 2 0 0
    1 8 0 0 0
  -----------
    1 9 5 4 8
```

The four partial products can now be added to get the answer, which is R19 548. It is convenient to write the work in vertical columns for units, tens, hundreds and so on, as shown on the right above.

In fact, if you are willing to do some hard thinking you can produce the answer with even less writing. You can achieve this by working from right to left to calculate the partial products, and by "carrying" parts of the partial answers to the next column, as you can do when working from right to left in columns. It works like this:

```
      3 2 5 8
  ×         6
  -----------
    1 9 5 4 8
```

When 6 × 8 = 48 is calculated, only the "8" is written down, in the units column. The "4" that represents 40 is not written. It is kept "on hold" in your mind.

When 6 × 50 = 300 is calculated, the 40 from the previous step is added to 300 to get 340. Again, only the "4" that represents 40 is written. The 300 is kept on hold or "carried" to add to the answer of the next step. The work continues like this.

1. Calculate each of the following. Do not use a calculator.
   (a) 8 × 786 (b) 9 × 3 453 (c) 60 × 786 (d) 60 × 7 860
2. You may use a calculator to check your answers for question 1. Repeat the work if your answers are not correct, so that you can learn where you make mistakes. Then put your calculator away again.
3. Use your answers for questions 1(a) and (c) to find out how much 68 × 786 is.

To calculate 36 × 378, the work can be broken up in two parts, namely 30 × 378 and 6 × 378.

4. Calculate 36 × 378.

A complete write-up of calculating 76 × 2 348 in columns is shown on the right.

```
          2 3 4 8
  ×           7 6
  ---------------
A             4 8
B           2 4 0
C         1 8 0 0
D       1 2 0 0 0
E           5 6 0
F         2 8 0 0
G       2 1 0 0 0
H     1 4 0 0 0 0
  ---------------
      1 7 8 4 4 8
```

5. (a) Explain how the 240 in row B was obtained.
   (b) Explain how the 560 in row E was obtained.
   (c) Explain how the 21 000 in row G was obtained.

Answers

6. (a) 39 114 (b) 252 361
7. (a) 222 592 (b) 101 244 (c) 126 836 (d) 568 230
8. Answers are the same as in question 7.
9. See LB page 29 alongside.
10. (a) 139 042 (b) 634 149 (c) 135 108 (d) 495 320

A PROCESS CALLED LONG DIVISION

Teaching guidelines

Division is used for different purposes:

- **Grouping** is used to find out **how many parts there are**. For example, to find the answer to question 1, the total amount of R920 must be divided into groups of R37 each, each enough to buy one of 24 chickens. The remainder of R32 is not enough to buy another chicken.
- **Sharing** is used to find out **how big each part is**. For example, to find the answer to question 2, the total amount of R880 must be divided equally among 34 learners. After each learner has received R25, the remainder of R30 cannot be shared out in full rand among the 34 learners any further.

More examples of grouping and sharing

Do the following examples require grouping or sharing?

- Your school bought 30 computers for its five Grade 7 classes. How many computers will each classroom get? Answer: Sharing
- Winnie sells homemade ginger beer per glass. If she has 4 litres of ginger beer and each glass holds 225 ml, how many full glasses can she sell? Answer: Grouping

Answers

1. 24
2. R25
3. 27
4. 180 g
5. 84

A short write-up of calculating 76 × 2 348 in columns is shown on the right.

		2	3	4	8
			×	7	6
	1	4	0	8	8
1	6	4	3	6	0
1	7	8	4	4	8

You may try to do the calculations in question 6 in this way. If you find it difficult, you may first write some of them up completely, and then try again to write less when you multiply.

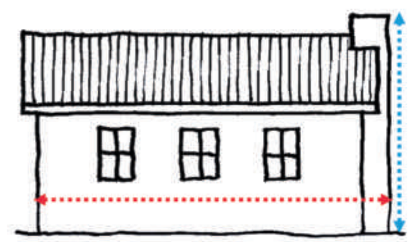
6. Calculate each of the following.
   (a) 53 × 738 (b) 73 × 3 457
7. Calculate.
   (a) 64 × 3 478 (b) 78 × 1 298 (c) 37 × 3 428 (d) 78 × 7 285
8. Use a calculator to check your answers for question 7. Redo the questions that you had wrong, so that you can learn to work correctly.
9. Use your correct answers for question 7 to give the answers to the following, without doing any calculations:
   (a) 101 244 ÷ 1 298 = 78 (b) 568 230 ÷ 7 285 = 78
10. Calculate, without using a calculator.
   (a) 3 659 × 38 (b) 27 × 23 487 (c) 486 × 278 (d) 2 135 × 232

A PROCESS CALLED LONG DIVISION

You may use a calculator to do questions 1 to 6.

1. You want to buy some live chickens at R37 each and you have R920 available. How many live chickens can you buy in total?
2. R880 is to be shared equally among 34 learners? How many full rands can each learner get?
3. You want to buy live chickens at R47 each. You have R1 280 available. How many live chickens can you buy?
4. 42 equal bags of rice weigh a total of 7 560 g. How much does one bag weigh?
5. The number 26 was multiplied by a secret number and the answer was 2 184. What was the secret number?

This is an accurate sketch of the back of a house. The red line on the sketch is 70 mm long and it shows the width of the house. The blue line on the sketch indicates the height of the chimney. **Do not measure the blue line now.**



Teaching guidelines (continued)

Estimation can be used to find the multiple of a number which is closest to a given number. For example:

- Use doubling to estimate the multiple of 75 closest to 3 050, start with:
  - 10 × 75 = 750; 3 050 – 750 = 2 300 left over; estimate too low
  - 20 × 75 = 1 500; 3 050 – 1 500 = 1 500 left over; estimate too low
  - 40 × 75 = 3 000; 3 050 – 3 000 = 50 left over; very close to real answer

  The multiple of 75 closest to 3 050 is 3 000.
- Use doubling to estimate the multiple of 74 closest to 4 080:
  - 10 × 74 = 740; 4 080 – 740 = 3 340 left over; estimate too low
  - 20 × 74 = 1 480; 4 080 – 1 480 = 2 600 left over; estimate too low
  - 40 × 74 = 2 960; 4 080 – 2 960 = 1 120 left over; estimate too low
  - 50 × 74 = 2 960 + 740 = 3 700; 4 080 – 3 700 = 380 left over; close to real answer. Note that doubling 40 would be too much, but adding 10 × 74 takes us to the closest multiple of 74.

  The multiple of 74 closest to 4 080 is 3 700.

Answers

6. (a) Find out how many lengths of the red line are equal to the actual width of the house.
   5 600 ÷ 70 = 80 so the house is 80 times bigger than the sketch.
   (b) 3 360 ÷ 80 = 42 the blue line should be 42 mm long.
   (c) It is 42 mm.
7. (a) Grouping
   (b) Sharing
8. (a) 50 × 74
   (b) 740 2 220 3 700 5 180 6 660
   (c) 250 × 38
   (d) 1 140 1 900 3 800 5 700 7 600 9 500 11 400
   (e) 30 × 287
   (f) 2 870 5 740 8 610 11 480 14 350 17 220 20 090
   (g) 240
9. Yes. The T-shirts will cost 115 × R67 = R7 705
10. (a) R6 700 (b) R1 800 (c) R460

The width of the actual house is 5 600 mm, and the height of the chimney is 3 360 mm.

6. (a) How many times is the house bigger than the sketch? Describe what you can do to find this out.
   (b) Calculate how long the blue line on the sketch should be.
   (c) Now measure the blue line to check your answer for (b).

Division is used for different purposes:

> In question 1 you knew that the amount is split into equal parts. You had to **find out how many parts there are** (how many chickens). This is called **grouping**.
>
> In question 2 you knew that the amount was split into 34 equal parts. You needed to **find out how big each part is** (how much money each learner will get). This is called **sharing**.

7. (a) What does question 3 require, sharing or grouping?
   (b) What does question 4 require, sharing or grouping?

In question 6 division was done for a different purpose than sharing or grouping.

Put your calculator away now. It is very important to be able to solve division problems by using your own mind. The activities that follow will help you to do this better than before. While you work on these activities, you will often have to **estimate** the product of two numbers. If you can estimate products well, division becomes easier to do. Hence, to start, do question 8, which will provide you with the opportunity to practise your product estimation skills.

8. (a) What do you think is closest to 4 080: 10 × 74 or 30 × 74 or 50 × 74 or 70 × 74 or 90 × 74?
   (b) Calculate some of the products to check your answer.
   (c) What do you think is closest to 9 238: 30 × 38 or 50 × 38 or 100 × 38 or 150 × 38 or 200 × 38 or 250 × 38 or 300 × 38?
   (d) Calculate some of the products to check your answer.
   (e) What do you think is closest to 9 746: 10 × 287 or 20 × 287 or 30 × 287 or 40 × 287 or 50 × 287 or 60 × 287 or 70 × 287?
   (f) Calculate some of the products to check your answer.
   (g) By what multiple of 10 should you multiply 27 to get as close to 6 487 as possible?
9. A principal wants to buy T-shirts for the 115 Grade 7 learners in the school. The T-shirts cost R67 each, and an amount of R8 500 is available. Do you think there is enough money to buy T-shirts for all the learners? Explain your answer.
10. (a) How much will 100 of the T-shirts cost?
    (b) How much money will be left if 100 T-shirts are bought?
    (c) How much money will be left if 20 more T-shirts are bought?

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Teaching guidelines (continued)

- During division:
  - the number that gets divided is called the **dividend**
  - the number that does the division is called the **divisor**.
- **Long division** involves subtraction of multiples of the divisor from the dividend until the remainder, if any, is smaller than the divisor. For example:
  - To divide 6 342 by 8, subtract multiples of 8 from 6 342:

    Use halving, starting from 1 000:
    $8 \times 1\,000 = 8\,000 \rightarrow 8 \times 500 = 4\,000 \rightarrow 8 \times 250 = 2\,000$

    Use doubling, starting from 10:
    $8 \times 10 = 80 \rightarrow 8 \times 20 = 160 \rightarrow 8 \times 40 = 320$

```
  6 342
– 4 000     (8 × 500 = 4 000)
  2 342
– 2 000     (8 × 250 = 2 000)
    342
–   320     (8 ×  40 =   320)
     22
–    16     (8 ×   2 =    16)
      6
```

    Answer: 6 342 ÷ 8 = 792 remainder 6

  - To divide 20 880 by 36, subtract multiples of 36 from 20 880:

    Use halving, starting from 1 000:
    $36 \times 1\,000 = 36\,000 \rightarrow 36 \times 500 = 18\,000$

    Use doubling, starting from 10:
    $36 \times 10 = 360 \rightarrow 36 \times 20 = 720 \rightarrow 36 \times 40 = 1\,440 \rightarrow 36 \times 80 = 2\,880$

```
  20 880
– 18 000    (36 × 500 = 18 000)
   2 880
–  2 880    (36 ×  80 =  2 880)
       0
```

    Answer: 20 880 ÷ 36 = 580

- Explain the correct layout for long division by working through the five steps on LB page 31 alongside.

The principal wants to work out exactly how many T-shirts, at R67 each, she can buy with R8 500. Her thinking and writing are described below.

Step 1

What she writes: 67 | 8 500

What she thinks: *I want to find out how many chunks of 67 there are in 8 500.*

Step 2

What she writes:	What she thinks:
100	*I think there are at least 100 chunks of 67 in 8 500.*
67 | 8 500	
6 700	*100 × 67 = 6 700. I need to know how much is left over.*
1 800	*I want to find out how many chunks of 67 there are in 1 800.*

Step 3 (She has to rub out the one "0" of the 100 on top, to make space.)

What she writes:	What she thinks:
120	*I think there are at least 20 chunks of 67 in 1 800.*
67 | 8 500	
6 700	
1 800	
1 340	*20 × 67 = 1 340. I need to know how much is left over.*
460	*I want to find out how many chunks of 67 there are in 460.*

Step 4 (She rubs out another "0".)

What she writes:	What she thinks:
125	*I think there are at least five chunks of 67 in 460.*
67 | 8 500	
6 700	
1 800	
1 340	
460	
335	*5 × 67 = 335. I need to know how much is left over.*
125	*I want to find out how many chunks of 67 there are in 125.*

Step 5 (She rubs out the "5".)

What she writes:	What she thinks:
126	*I think there is only one more chunk of 67 in 125.*
67 | 8 500	
6 700	
1 800	
1 340	
460	
335	
125	
67	*I wonder how much money will be left over.*
58	*So, we can buy 126 T-shirts and R58 will remain.*

CHAPTER 1: WORKING WITH WHOLE NUMBERS 31

Answers

Learners must not use a calculator.

11. (a) R3 995 ÷ 85 = R47
    Selina paid R47 per chicken.
    (b) 4 850 ÷ 78 = 62 with 14 remaining
    He can buy 62 goats.
12. (a) 150 with remainder 34 (b) 136 with remainder 3
    (c) 26 with remainder 36 (d) 347 with remainder 19
13. (a) 388 boxes with 13 chocolates remaining
    (b) 2 013 cartons

1.6 Problem solving

RATE AND RATIO

Background information

- A **ratio** compares the sizes of two (or more) quantities of the **same** kind. For example, if mortar for building a brick wall is made by mixing two parts of cement to seven parts of sand, the ratio of cement to sand is 2 to 7 (or 2 : 7), as long as the parts are measured in the same unit like buckets or spades full.
- A **rate** compares quantities of **different** kinds. For example, if a driver of a car takes three hours to travel 300 km, the car is travelling at a rate of 300 km per three hours or 100 km per hour.

Teaching guidelines

The concepts of **ratio** and **rate** (LB pages 32–34) are extremely important. A useful way to clarify the difference between rate and ratio is to consider a situation where two objects move at different constant speeds, say 120 km/h and 40 km/h. Each of the speeds is a rate, and the two speeds are in the ratio 3 : 1. In this case, a ratio is a relationship between two rates.

Answers

1. 688 000 ÷ 85 000 ≈ 8 days
2. 8 × 24 = 192 cm
3. 95 × 4 = 380 km

Do not use a calculator in the questions that follow. The purpose of this work is for you to develop a good understanding of how division can be done. Check all your answers by doing multiplication.

11. (a) Selina bought 85 chickens, all at the same price. She paid R3 995 in total. What did each of the chickens cost? Your first step can be to work out how much Selina would have paid if she paid R10 per chicken, but you can start with a bigger step if you wish.
    (b) Anton has R4 850. He wants to buy some young goats. The goats cost R78 each. How many goats can he buy?
12. Calculate the following without using a calculator:
    (a) 7 234 ÷ 48 (b) 3 267 ÷ 24
    (c) 9 500 ÷ 364 (d) 8 347 ÷ 24
13. (a) A chocolate factory made 9 325 chocolates of a very special kind one day. The chocolates were packed in small, decorated boxes, with 24 chocolates per box. How many boxes were filled?
    (b) A farmer sells eggs packed in cartons to the local supermarkets. There are 36 eggs in one carton. One month, the farmer sold 72 468 eggs to the supermarkets. How many cartons is this?

1.6 Problem solving

RATE AND RATIO

You may use a calculator for doing the work in this section.

1. The people in a village get their water from a nearby dam. On a certain day the dam contains 688 000 litres of water. The village people use about 85 000 litres of water each day. For how many days will the water in the dam last, if no rains fall?

Instead of saying "85 000 litres each day" or "8 cm each hour", people often say "**at a rate of** 85 000 litres **per day**" or "**at a rate of** 8 cm **per hour**".

2. During a period of very heavy rain, the water level in a certain river increases at a rate of 8 cm each hour. If it continues like this, by how much will the water level increase in 24 hours?
3. A woman is driving from Johannesburg to Durban. Her distance from Durban decreases at a rate of about 95 km per hour. How far does she travel, approximately, in four hours?

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Teaching guidelines (continued)

In question 5 three coloured patterns are shown.

- In pattern A, the ratio of yellow beads to red beads is 4 to 5. This is written as 4 : 5.
- In pattern B, the ratio between yellow beads and red beads is 3 : 6.
- In pattern C, the ratio between yellow beads and red beads is 2 : 7.

In question 6, machine A produces one tin for every three tins that machine B produces. We say that:

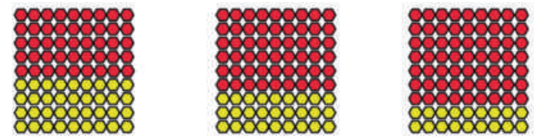
- the ratio between production speeds of machines A and B is 1 : 3
- the ratio between production speeds of machines B and A is 3 : 1.

Answers

4. 860 000 + (35 000 × 20) = 860 000 + 700 000 = 1 560 000 people
5. Pattern B: There are six red beads for every three yellow beads.
   Pattern C: There are seven red beads for every two yellow beads.
6. (a) See LB page 33 alongside.
   (b) 2 400 ÷ 800 = 3 times
   (c) 90 tins
   (d) 600 tins
   (e) Three tins

4. The number of unemployed people in a certain province increases at a rate of approximately 35 000 people per year. If there were 860 000 unemployed people in the year 2000, how many unemployed people will there be, approximately, in the year 2020?

5. In pattern A below, there are five red beads for every four yellow beads. Describe patterns B and C in the same way.



Pattern A *Pattern B* *Pattern C*

In a certain food factory, two machines are used to produce tins of baked beans. Machine A produces at a rate of 800 tins per hour, and machine B produces at a rate of 2 400 tins per hour.

6. (a) Copy and complete the following table, to show how many tins of beans will be produced by the two machines, in different periods of time.

Number of hours	1	2	3	5	8
Number of tins produced by machine A	800	1 600	2 400	4 000	6 400
Number of tins produced by machine B	2 400	4 800	7 200	12 000	19 200

(b) How much faster is machine B than machine A?
(c) How many tins will be produced by machine B in the time that it takes machine A to produce 30 tins?
(d) How many tins will be produced by machine B in the time that it takes machine A to produce 200 tins?
(e) How many tins will be produced by machine B in the time that it takes machine A to produce one tin?

The patterns in question 5 can be described like this:

In pattern A, the **ratio** of yellow beads to red beads is 4 to 5. This is written as 4: 5.

In pattern B, the ratio between yellow beads and red beads is 3: 6, and in pattern C the ratio is 2: 7. In question 6, machine A produces one tin for every three tins that machine B produces. This can be described by saying that the ratio between the production speeds of machines A and B is 1: 3.

CHAPTER 1: WORKING WITH WHOLE NUMBERS 33

Answers

7. (a) 15 km
   (b) See LB page 34 alongside.
   (c) 1,5 km
   (d) 2 : 3
8. R240 ÷ R8 = 30, so the R240 is 30 portions of R8 each.
   Sally gets R5 from each portion so she gets R30 × R5 = R150.
   David gets 30 × R3 = R90.
9. (a) Person 1: R3 600 Person 2: R10 800
   (b) Person 1: R6 000 Person 2: R8 400

FINANCIAL MATHEMATICS

Teaching guidelines

- **Loan** is the money that one borrows from a person or institution.
- **Interest** is the money one has to pay for using another person's money.
- **Interest rate** shows how much interest you have to pay for every R100 borrowed from another person or financial institution. For example, if you borrow R10 000 from a bank at 15% interest, it means that, apart from paying the R10 000 back to the bank, you have to pay 15 hundredths of R10 000 for the privilege of using the money that actually belongs to the bank:

  R10 000 ÷ 100 = R100

  R100 × 15 = R1 500

You will have to pay R1 500 interest to the bank.

In total, you will have to pay R10 000 + R1 500 = R11 500 back to the bank.

Answers

1. 12 000 ÷ 100 = 120; 120 × 15 = 1 800
2. (a) 8 000 ÷ 100 = 80; 80 × 12 = 960
   (b) 24 000 ÷ 100 = 240; 240 × 18 = 4 320
3. (a) R6 000 ÷ 100 = R60; 9 × R60 = R540
   (b) R21 000 ÷ 100 × 11 = R2 310; 3 × R2 310 = R6 930
   (c) R45 000 ÷ 100 × 12 = R5 400; 10 × R5 400 = R54 000

7. Two huge trucks are travelling very slowly on a highway. Truck A covers 20 km per hour, and truck B covers 30 km per hour. Both trucks keep these speeds all the time.
   (a) What distance will truck B cover in the same time that truck A covers 10 km?
   (b) In the table below, the distances that truck A covers in certain periods of time are given. Copy and complete the table to show the distances covered by truck B, in the same periods of time.

Distance covered by truck A	10 km	18 km	50 km	100 km	30 km
Distance covered by truck B	15 km	27 km	75 km	150 km	45 km

   (c) What distance will truck B cover in the same time that truck A covers 1 km?
   (d) What is the ratio between the speed at which truck A travels and the speed at which truck B travels?
8. R240 will be divided between David and Sally in the ratio 3 : 5. This means Sally gets R5 for every R3 David gets. How much will David and Sally each get in total?
9. How much will each person get, if R14 400 is shared between two people in each of the following ways?
   (a) In the ratio 1 : 3 (b) In the ratio 5 : 7

FINANCIAL MATHEMATICS

A man borrows R12 000 from a bank for one year. He has to pay 15% interest to the bank. This means that, apart from paying the R12 000 back to the bank after a year, he has to pay 15 hundredths of R12 000 for the privilege of using the money that actually belongs to the bank.

One hundredth of R12 000 can be calculated by dividing R12 000 by 100. This amount can then be multiplied by 15 to get 15 hundredths of R12 000.

15% is read as **15 per cent**, and it is just a different way to say **15 hundredths**.

When a person borrows money it is called a **loan**. The money paid for using another person's money is called **interest**.

Do not use a calculator when you do the following questions.

1. Calculate 12 000 ÷ 100, then multiply the answer by 15.
2. Calculate:
   (a) 12% of R8 000 (b) 18% of R24 000
3. In each case below, calculate how much interest must be paid.
   (a) An amount of R6 000 is borrowed for one year at 9% interest.
   (b) An amount of R21 000 is borrowed for three years at 11% interest per year.
   (c) An amount of R45 000 is borrowed for ten years at 12% interest per year.

Teaching guidelines (continued)

- A **profit** is made when something is sold for more than what it originally cost. For example, if you buy a bag of potatoes for R59 and sell the potatoes separately for R75 in total, you have made a profit of R75 – R59 = R16.
- A **loss** is made when something is sold for less than what it originally cost. For example, if you buy a mobile phone for R699 and sell it for R549, you have made a loss of R699 – R549 = R150.
- A **discount** is an amount which is taken off the price of something and is usually stated as a percentage. For example:

  Calculate 20% discount on R1 500.

  R1 500 ÷ 100 = R15

  R15 × 20 = R300

  The discount will be R300.

Answers

4. (a) R52 000 – R40 000 = R12 000
   (b) 1 hundredth of R100 000 = R1 000; 28 hundredths (or 28%) = R28 000
   (c) 1 hundredth of R120 000 = R1 200; 30 hundredths (or 30%) is R36 000
5. (a) R20
   (b) 20
   (c) R400
6. 1 hundredth of R4 000 = R40; 20 hundredths (or 20%) = R800. The discount is R800.
   The customer paid R4 000 – R800 = R3 200.

A car dealer buys a car for R60 000 and sells it for R75 000. The difference of R15 000 is called the **profit**. In this case, the profit is a quarter of R60 000, which is the same as 25 hundredths or 25%. This can be described by saying "the car dealer made a profit of 25%".

4. Calculate the amount of profit in each of the following cases. The information is about a car dealer who buys and sells used vehicles.
   (a) A car is bought for R40 000 and sold for R52 000.
   (b) A small truck is bought for R100 000 and sold at a profit of 28%.
   (c) A bakkie is bought for R120 000 and sold at a profit of 30%.

A shop owner bought a stove for R2 000 and sold it for R1 600. The shop owner did not make a profit, he sold the stove at a **loss** of R400.

5. (a) How much is one hundredth of R2 000?
   (b) How many hundredths of R2 000 is R400?
   (c) How much is 20% of R2 000?

Notice that by doing question 5(b) you have worked out at what percentage loss the shop owner sold the stove.

6. The shop owner also sold a fridge that normally sells for R4 000 at a **discount** of 20%. This means the customer paid 20% less than the normal price. Calculate the discount in rands and the amount that the customer paid for the fridge.

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
2.1 Quick squares and cubes	Squares and cubes recalled from prior knowledge	36 to 38
2.2 The exponential notation	The meaning of the concepts exponential notation, power, base and exponent/index	38 to 41
2.3 Squares and cubes	Calculating squares and cubes	41 to 42
2.4 The square root and the cube root	Calculating square roots and cube roots	43 to 45
2.5 Comparing numbers in exponential form	Random numbers in exponential form arranged in ascending and descending order	46 to 47
2.6 Calculations	Performing calculations with exponents, square roots and cube roots	48 to 50

CAPS time allocation	9 hours
CAPS content specification	Pages 43 to 44

Mathematical background

This chapter addresses:

- squares and cubes
- square roots and cube roots
- the exponential notation.

The work on squares and cubes, square roots and cube roots is valuable for at least two reasons:

- it contributes to fluency with smaller numbers, and
- it serves as an introduction to the exponential notation.

The **exponential notation** is a shorthand notation for repeated multiplication with the same number. For example, the exponent **5** in 2^5, indicates the number of occurrences of the factor 2 in the product $2 \times 2 \times 2 \times 2 \times 2$.

Squaring a number is similar to the way you would find the area of a square when its side length is given.

Square rooting a number is similar to the way you would find the side length of a square when its area is given.

Squaring and square rooting are inverse operations. For example, the square of 5 is 25 ($5^2 = 25$), therefore the square root of 25 is 5 ($\sqrt{25} = 5$).

Cubing a number is similar to the way you would find the volume of a cube when its side length is given.

Cube rooting a number is similar to the way you would find the side length of a cube when its volume is given.

Cubing and cube rooting are inverse operations. For example, the cube of 5 is 125 ($5^3 = 125$), therefore the cube root of125 is 5 ($\sqrt[3]{125} = 5$).

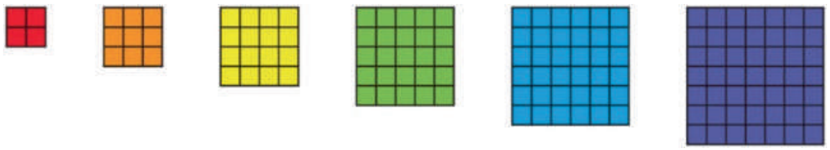
2.1 Quick squares and cubes

AGAIN AND AGAIN

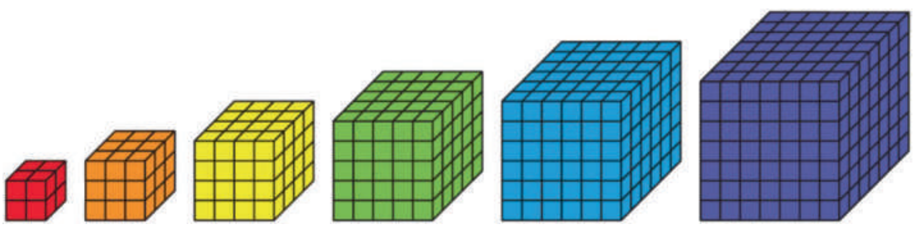
Teaching guidelines

The following strategy can be used to introduce squares and cubes:

- Learners investigate similarities between the drawings (squares) below and find a quick way to count the number of small squares in each drawing (multiply the number of squares along any edge by itself). Instead of saying "six times six", say "six squared" and write 6^2.



- Learners investigate similarities between the drawings (cubes) below and find a quick way to count the number of small cubes in each drawing (multiply the number of cubes along any edge by itself and by itself again). Instead of saying "six times six times six", say "six cubed" and write 6^3.



Answers

1. (a) 4 (b) 9 (c) 16 (d) 25 (e) 36 (f) 49
   (g) 64 (h) 81 (i) 100 (j) 121 (k) 144 (l) 1
2. See the answers on LB page 36 alongside.
3. (a) 5 (b) 10 (c) 8 (d) 6
4. (a) 100 + 25 + 4 = 129 (b) 5 × 100 + 70 + 3 = 573
   (c) 700 + 30 + 6 = 736 (d) 200 + 90 + 6 = 296
5. (a) 8 (b) 27 (c) 64 (d) 125 (e) 216
   (f) 343 (g) 512 (h) 729 (i) 1 000 (j) 1 331
   (k) 1 728 (l) 2 197 (m) 1

CHAPTER 2 Exponents

2.1 Quick squares and cubes

AGAIN AND AGAIN

1. How much is each of the following?
   (a) 2 × 2 (b) 3 × 3 (c) 4 × 4 (d) 5 × 5 (e) 6 × 6 (f) 7 × 7
   (g) 8 × 8 (h) 9 × 9 (i) 10 × 10 (j) 11 × 11 (k) 12 × 12 (l) 1 × 1

Instead of saying "ten times ten", we may say "ten squared" and we may write 10^2.

2. Copy and complete these tables.

2 × 2	5 × 5	10 × 10	12 × 12	4 × 4	8 × 8
2^2	5^2	10^2	12^2	4^2	8^2
2 squared	5 squared	10 squared	12 squared	4 squared	8 squared
4	25	100	144	16	64

11 × 11	7 × 7	1 × 1	9 × 9	3 × 3	6 × 6
11^2	7^2	1^2	9^2	3^2	6^2
11 squared	7 squared	1 squared	9 squared	3 squared	6 squared
121	49	1	81	9	36

3. 8 squared is 64, and 9 squared is 81.
   (a) What number squared is 25? (b) What number squared is 100?
   (c) What number squared is 64? (d) What number squared is 36?
4. Calculate:
   (a) $10^2 + 5^2 + 2^2$ (b) $5 \times 10^2 + 7 \times 10 + 3$
   (c) $7 \times 10^2 + 3 \times 10 + 6$ (d) $2 \times 10^2 + 9 \times 10 + 6$
5. How much is each of the following?
   (a) 2 × 2 × 2 (b) 3 × 3 × 3 (c) 4 × 4 × 4 (d) 5 × 5 × 5 (e) 6 × 6 × 6
   (f) 7 × 7 × 7 (g) 8 × 8 × 8 (h) 9 × 9 × 9 (i) 10 × 10 × 10
   (j) 11 × 11 × 11 (k) 12 × 12 × 12 (l) 13 × 13 × 13 (m) 1 × 1 × 1

Notes on quick squares and cubes

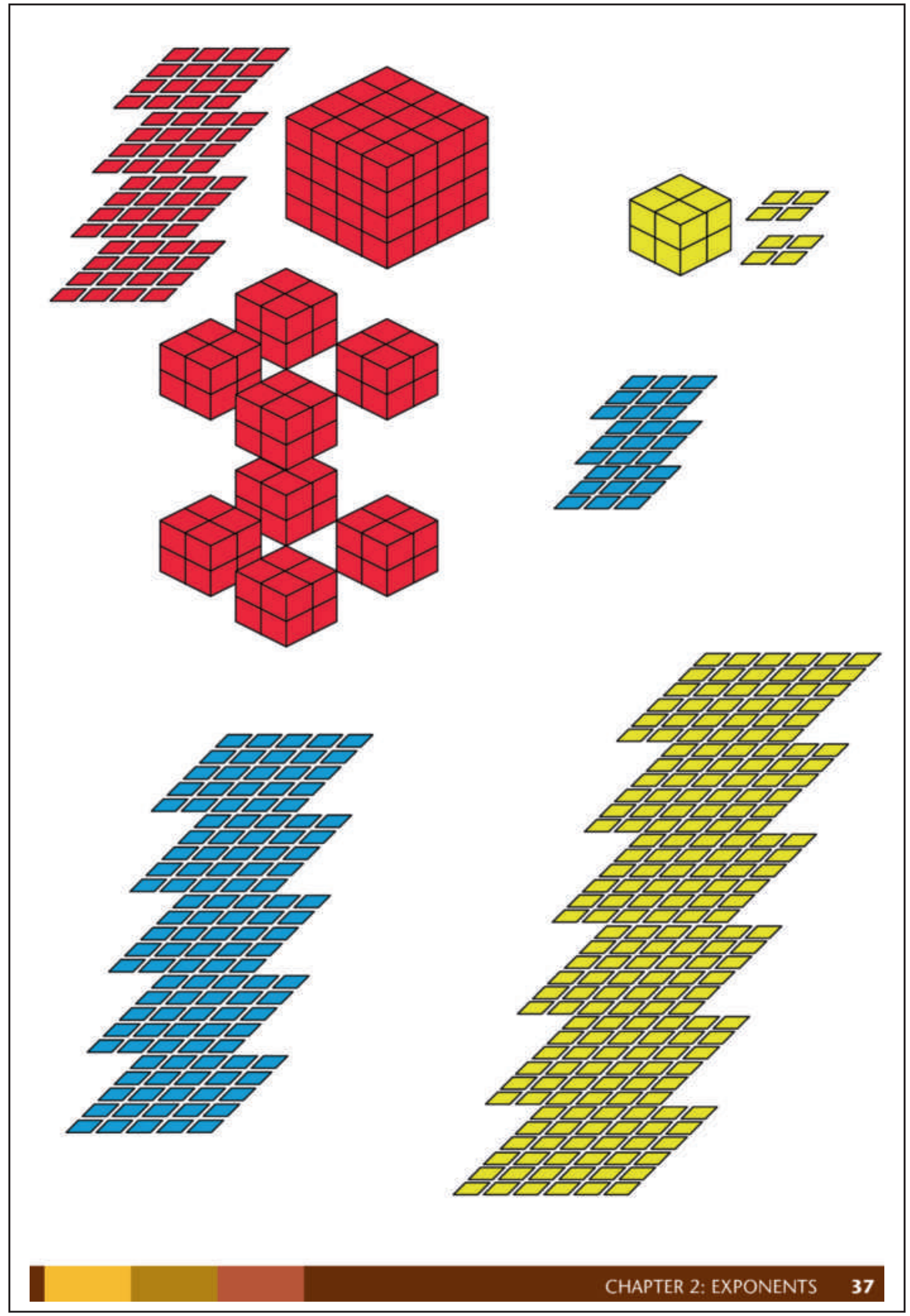
1. Learners have calculated squares in the Intermediate Phase. Section 2.1 revises squares and cubes of small whole numbers, as required by the CAPS (page 43). The pictures on LB page 37 highlight the geometric origin of the terms **square** and **cube**, and the intention is definitely not that all learners should count the tiles in the pictures one by one. Rather, they should be challenged to find a quick way to do it.
2. Almost all the exercises in this chapter can be completed without a calculator, because most of them involve squares and square roots up to 12^2, and cubes and cube roots up to 6^3. The CAPS (page 43) regards this number range as **mental calculations**. Learners are given plenty of practice exercises to allow them to recall these values quickly and easily.

Misconceptions

When first explaining powers and roots, try to avoid using 1 or 2 as the base. This is because if you use an example such as $2^2 = 4$, learners may think that the power of 2 requires them to double the base rather than multiply the base by itself.

Notes on question 4

In question 4(a) (LB page 36) learners have to calculate $10^2 + 5^2 + 2^2$, where 10^2 means 10×10, 5^2 means 5×5 and 2^2 means 2×2. According to BODMAS, multiplication should be done before addition, which implies that, in this case, 10^2, 5^2 and 2^2 should be calculated before any addition is done.



Answers

6. See LB page 38 alongside.
7. (a) 3 (b) 10 (c) 2 (d) 1 (e) 6 (f) 7
8. (a) 3 000 + 700 + 50 + 6 = 3 756 (b) 7 000 + 700 + 70 + 7 = 7 777
   (c) 8 000 + 100 + 40 + 2 = 8 142 (d) 4 000 + 300 + 40 + 9 = 4 349
   (e) 10 × 100 = 1 000 (f) 100 × 100 = 10 000
9. Numbers 8 and 4: the square of 8 = 64 and the cube of 4 = 64.
10. $3^2 + 4^2 = 5^2$; also $6^2 + 8^2 = 10^2$; $5^2 + 12^2 = 13^2$; ...

2.2 The exponential notation

REPEATED MULTIPLICATION WITH THE SAME NUMBER

Teaching guidelines

Find a shorthand notation for 4 × 4 × 4 without applying multiplication (4^3). This shorthand notation for repeated multiplication with the same number is known as **exponential notation**, with the exponent 3 indicating the number of occurrences of the factor 4 in the product 4 × 4 × 4.

Find a shorthand notation for 4 + 4 + 4 without performing addition (3 × 4). This shorthand notation for repeated addition with the same number is **multiplication**, with the 3 indicating the number of 4s to be added together.

Discuss the concepts **power**, **base** and **exponent** of a number written in exponential form (see LB page 40).

Mathematical content

- A **power** of a number is any number that can be expressed as a product of one repeated factor of that number.
- The repeated factor in a power is called the **base**.
- The number of repetitions is called the **exponent** or **index**.

Notes on the exponential notation

1. Learners need to understand that exponents enable us to write repeated multiplication in a shorthand way, called the exponential form. Ensure that learners can distinguish between the two concepts – i.e. repeated multiplication and repeated addition.

Instead of saying "10 times 10 times 10", we may say "10 cubed" and we may write 10^3.

6. Copy and complete the tables.

4 × 4 × 4	7 × 7 × 7	11 × 11 × 11	2 × 2 × 2	6 × 6 × 6	10 × 10 × 10
4^3	7^3	11^3	2^3	6^3	10^3
4 cubed	7 cubed	11 cubed	2 cubed	6 cubed	10 cubed
64	343	1 331	8	216	1 000

8 × 8 × 8	12 × 12 × 12	1 × 1 × 1	9 × 9 × 9	3 × 3 × 3	5 × 5 × 5
8^3	12^3	1^3	9^3	3^3	5^3
8 cubed	12 cubed	1 cubed	9 cubed	3 cubed	5 cubed
512	1 728	1	729	27	125

7. 5 cubed is 125, and 9 cubed is 729.
   (a) What number cubed is 27? (b) What number cubed is 1 000?
   (c) What number cubed is 8? (d) What number cubed is 1?
   (e) What number cubed is 216? (f) What number cubed is 343?
8. Calculate:
   (a) $3 \times 10^3 + 7 \times 10^2 + 5 \times 10 + 6$ (b) $7 \times 10^3 + 7 \times 10^2 + 7 \times 10 + 7$
   (c) $8 \times 10^3 + 1 \times 10^2 + 4 \times 10 + 2$ (d) $4 \times 10^3 + 3 \times 10^2 + 4 \times 10 + 9$
   (e) 10×10^2 (f) $10^2 \times 10^2$
9. Can you think of two numbers, so that the square of the one number is equal to the cube of the other number?
10. Can you think of two numbers, so that when you add their squares, you get the square of another number?

2.2 The exponential notation

REPEATED MULTIPLICATION WITH THE SAME NUMBER

1. Express each number below as a product of prime factors.
   Example: 250 = 2 × 5 × 5 × 5
   5 is a **repeated factor** of 250. It is repeated three times.
   (a) 35 5 × 7 (b) 70 2 × 5 × 7
   (c) 140 2 × 2 × 5 × 7 (d) 280 2 × 2 × 2 × 5 × 7
   (e) 81 3 × 3 × 3 × 3 (f) 625 5 × 5 × 5 × 5

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2. It is important for learners to be able to use a few different ways to express exponential statements in words. For example, learners need to understand that 5^2 can be described in words as "five squared" or "five to the power of two". The following statements: "a number, when squared, is nine" and "a number to the power of two is nine" and "what is the square root of nine?" all have the same meaning.

Misconceptions

Sometimes an expression like 2^3 is interpreted as 2 × 3, which may be described as "multiplying the base with the exponent", instead of 2 × 2 × 2. To help learners to sharply distinguish between powers like 2^3 and products like 2 × 3 it may help to let them explore expressions like 2 × 2 × 2 and 2 + 2 + 2 from time to time, and to write these in shorter forms. It is critical that learners understand that the exponential notation and the multiplication notation are both short hand but different forms to indicate repetition of multiplication and addition respectively.

Answers

1. See LB page 38 on previous page.
2. 2 is repeated two times as a factor of 140
   2 is repeated three times as a factor of 280
   3 is repeated four times as a factor of 81
   5 is repeated four times as a factor of 625
3. (a) 125 = 5 × 5 × 5 (b) 64 = 2 × 2 × 2 × 2 × 2 × 2
   (c) 100 = 10 × 10 (d) 1 000 = 10 × 10 × 10
4. (a) 16 (b) 32 (c) 64 (d) 128 (e) 256
   (f) 512 (g) 1 024 (h) 2 048 (i) 4 096 (j) 8 192
5. See LB page 39 alongside.
6. (a) eighth power of 15 (b) twelfth power of 12
7. (a) $125 = 5^3$ (b) $64 = 2^6$
   (c) $100 = 10^2$ (d) $1\ 000 = 10^3$

---

2. Which numbers in question 1 have repeated factors? In each case, state what number is repeated as a factor and how many times it is repeated.

> A number that can be expressed as a product of one repeated factor is called a **power** of that number.

Examples:
32 is a power of 2, because 32 = 2 × 2 × 2 × 2 × 2
100 000 is a power of 10, because 10 × 10 × 10 × 10 × 10 = 100 000

3. Express each number as a power of 2, 3, 5 or 10.
   (a) 125 (b) 64
   (c) 100 (d) 1 000
4. Calculate each of the following. You can use each answer to get the next answer.
   (a) 2 × 2 × 2 × 2 (b) 2 × 2 × 2 × 2 × 2
   (c) 2 × 2 × 2 × 2 × 2 × 2 (d) 2 × 2 × 2 × 2 × 2 × 2 × 2
   (e) 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 (f) 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2
   (g) 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2
   (h) 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2
   (i) 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2
   (j) 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2

Because the factor 2 is repeated five times, 32 is called the **fifth power of 2**, or **2 to the power 5**.
Similarly, 125 is the third power of 5.
125 can also be called "5 to the power 3" or "5 cubed".

5. The seventh power of 2 is shown in question 4(d).
   What power of 2 is shown in each of the following parts of question 4?
   (a) 4(j) thirteenth power (b) 4(i) twelfth power
   (c) 4(h) eleventh power (d) 4(f) ninth power
6. What power of what number is shown in each case below?
   (a) 15 × 15 × 15 × 15 × 15 × 15 × 15 × 15
   (b) 12 × 12 × 12 × 12 × 12 × 12 × 12 × 12 × 12 × 12 × 12 × 12

> Instead of writing "5 to the power 6" we may write 5^6.
> This is called the **exponential notation**.
> 5^6 means 5 × 5 × 5 × 5 × 5 × 5.
> 5 × 6 means 6 + 6 + 6 + 6 + 6.

7. Write each of the numbers in question 3 in exponential notation.

8. (a) 2^4 (b) 2^5 (c) 2^6 (d) 2^7 (e) 2^8
   (f) 2^9 (g) 2^{10} (h) 2^{11} (i) 2^{12} (j) 2^{13}
9. See LB page 40 alongside.
10. (a) $5^3 = 125$ (b) $10^4 = 10\ 000$ (c) $20^3 = 8\ 000$
11. See LB page 40 alongside.

POWERS OF DIFFERENT NUMBERS

Teaching guidelines

Learners should discover that all differences between:

- consecutive powers of 2 are equal to (2 – 1) times a power of 2:
  $2^2 - 2^1 = 1 \times 2$; $2^3 - 2^2 = 1 \times 2^2$; $2^4 - 2^3 = 1 \times 2^3$; $2^5 - 2^4 = 1 \times 2^4$
- consecutive powers of 3 are equal to (3 – 1) times a power of 3:
  $3^2 - 3^1 = 2 \times 3$; $3^3 - 3^2 = 2 \times 3^2$; $3^4 - 3^3 = 2 \times 3^3$; $3^5 - 3^4 = 2 \times 3^4$
- consecutive powers of 4 are equal to (4 – 1) times a power of 4:
  $4^2 - 4^1 = 3 \times 4$; $4^3 - 4^2 = 3 \times 4^2$; $4^4 - 4^3 = 3 \times 4^3$; $4^5 - 4^4 = 3 \times 4^4$
- consecutive powers of 5 are equal to (5 – 1) times a power of 5:
  $5^2 - 5^1 = 4 \times 5$; $5^3 - 5^2 = 4 \times 5^2$; $5^4 - 5^3 = 4 \times 5^3$; $5^5 - 5^4 = 4 \times 5^4$

Notes on question 2

Display the calculation $3^5 - 3^4$. Revise that 3^5 means 3 × 3 × 3 × 3 × 3 and 3^4 means 3 × 3 × 3 × 3, which implies that, according to BODMAS, 3^5 and 3^4 must be calculated before subtraction is performed.

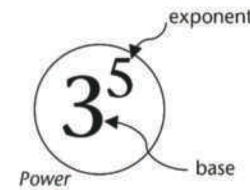
Answers

1. See LB page 40 alongside.
2. (a) 2 4 8 16 32 64 128
   (b) They are the powers of 2 again.
3. No. 9 – 3 = 6 which is not a power of 3.

8. Write each of the numbers in question 4 in exponential notation.

9. In each case write the number in exponential notation.
   (a) The fifth power of 5 5^5 (b) The sixth power of 5 5^6
   (c) The third power of 4 4^3 (d) 6 to the power 4 6^4
   (e) 4 to the power 6 4^6 (f) 5 to the power 15 5^{15}

3^5 means 3 × 3 × 3 × 3 × 3.
The repeating factor in a power is called the **base**.
The number of repetitions is called the **exponent** or **index**.
3^1 means 3. The base is 3 but there is no repetition.
Any number raised to the power 1 equals the number itself.



10. In each case below some information about a number is given. Each number can be expressed as a power. What is the number in each case?
    (a) The base is 5 and the index is 3.
    (b) The base is 10 and the exponent is 4.
    (c) The base is 20 and the exponent is 3.

11. Calculate each of the following:
    (a) 5 × 5 × 5 125 (b) 5 × 5 × 5 × 5 × 5 3 125
    (c) 5 + 5 + 5 15 (d) 5 + 5 + 5 + 5 + 5 25
    (e) 5 × 3 15 (f) 5^3 125

POWERS OF DIFFERENT NUMBERS

1. Copy and complete this table of powers of 2.

Exponent	1	2	3	4	5	6	7	8	9
Power of 2	2	4	8	16	32	64	128	256	512

Exponent	10	11	12	13	14
Power of 2	1 024	2 048	4 096	8 192	16 384

2. (a) Calculate each of the following:
   $2^2 - 2^1$ $2^3 - 2^2$ $2^4 - 2^3$ $2^5 - 2^4$
   $2^6 - 2^5$ $2^7 - 2^6$ $2^8 - 2^7$
   (b) Describe what you notice about the differences between consecutive powers of 2.

Numbers that follow on each other in a pattern are called **consecutive numbers**.

3. Suppose you calculate the differences between consecutive powers of 3. Do you think these differences will be the consecutive powers of 3 again?

4. See LB page 41 alongside.
5. (a) 6 18 54 162 486 1 458 4 374
   (b) Sample answer: They are not powers of 3, but double powers of 3.
   (c) 3 9 27 81 243 729 2 187
   (d) The answers in (c) are all powers of 3.
6. (a) Learners' own answers, for example multiples of powers of 4.
   (b) See LB page 41 alongside.

   Example of an answer: "I calculated the differences between consecutive powers of 4 and tried to write the answers in terms of powers of 4, for example a product of a power of 4 and another number." "I tried to find a pattern:

   16 – 4 = 12 = 3 × 4
   64 – 16 = 48 = 3 × 16
   256 – 64 = 192 = 3 × 64
   1 024 – 256 = 768 = 3 × 256 etc.
   All answers are (4 – 1) times a power of 4."
7. See LB page 41 alongside.

   100 – 10 = 90
   1 000 – 100 = 900
   10 000 – 1 000 = 9 000
   100 000 – 10 000 = 90 000
   1 000 000 – 100 000 = 900 000
   10 000 000 – 1 000 000 = 9 000 000
   The difference between consecutive powers of 10 is always (10 – 1) times a power of 10.

2.3 Squares and cubes

CALCULATING SQUARES AND CUBES

Teaching guidelines

Learners should be able to use different ways to express exponential statements in words. For example:

- 3^2 can be read as "three squared" or "three to the power two"
- 3^3 can be read as "three cubed" or "three to the power three".

Revise the meaning of the following concepts:

- A **square** is any number which is a product of two identical factors.
- A **cube** is any number which is a product of three identical factors.

4. Copy and complete this table of powers of 3.

Exponent	1	2	3	4	5	6	7	8	9
Power of 3	3	9	27	81	243	729	2 187	6 561	19 683

Exponent	10	11	12	13	14
Power of 3	59 049	177 147	531 441	1 594 323	4 782 969

5. (a) Calculate each of the following:
   $3^2 - 3^1$ $3^3 - 3^2$ $3^4 - 3^3$ $3^5 - 3^4$ $3^6 - 3^5$ $3^7 - 3^6$ $3^8 - 3^7$
   (b) How do these numbers differ from what you expected when you answered question 3?
   (c) Divide each of your answers in 5(a) by 2.
   (d) If you observe anything interesting, describe it.
6. In questions 1 to 5 you have investigated the differences between consecutive powers of 2 and 3. You have observed certain interesting things about these differences. You will now investigate, in the same way, the differences between consecutive powers of 4.
   (a) Before you investigate, think a bit. What do you expect to find?
   (b) Copy the table below and do your investigation. Write a short report on what you find.

Exponent	1	2	3	4	5	6	7	8
Power of 4	4	16	64	256	1 024	4 096	16 384	65 536

7. Do what you did in question 6, but now for powers of 10.

Exponent	1	2	3	4	5	6	7
Power of 10	10	100	1 000	10 000	100 000	1 000 000	10 000 000

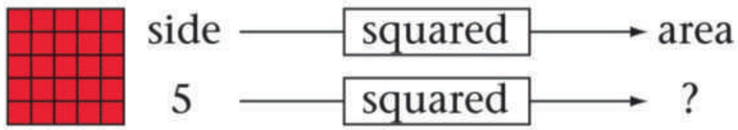
2.3 Squares and cubes

The number 9 is called the **square** of 3 because 3 × 3 = 9. The number 3, called the base, is multiplied by itself. **3^2** is read as **three squared** or **three to the power 2**.

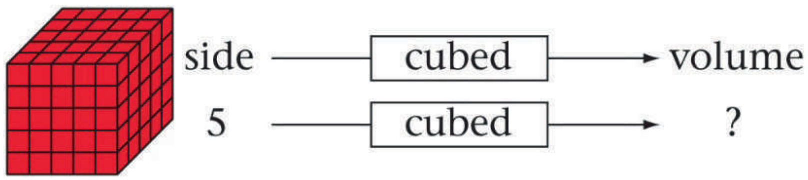
The number 27 is called the **cube** of 3 because 3 × 3 × 3 = 27. The base, the number 3, is multiplied by itself and again by itself. **3^3** is read as **three cubed** or **three to the power 3**.

Use flow diagrams to illustrate the process of squaring and cubing:

- To square a number means to multiply it by itself. This resembles finding the area of a square when the length of any side is given.



- To cube a number means to multiply it by itself and again by itself. This resembles finding the volume of a cube when the length of any edge is given.



Notes on question 5

To find numbers which are both squares and cubes, write down a product of six identical factors and find the answer. For example:

- $3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$ is both a square and a cube because $(3 \times 3 \times 3) \times (3 \times 3 \times 3) = 27 \times 27 = 27^2$ and $(3 \times 3) \times (3 \times 3) \times (3 \times 3) = 9 \times 9 \times 9 = 9^3$

Answers

1. See LB page 42 alongside.
2. (a) $1 \times 1 \times 1 = 1$ $\quad 2 \times 2 \times 2 = 8$ $\quad 3 \times 3 \times 3 = 27$
   (b) $5 \times 5 \times 5 = 125$ $\quad 10 \times 10 \times 10 = 1\ 000$ $\quad 4 \times 4 \times 4 = 64$
3. (a) Set B: {1; 8; 27; 64; 125; 216; 343; 512}
   (b) Set B: {1 000; 8 000; 27 000; 64 000; 125 000}
4. (a)

1	4	9	16	25	36	49	64
81	100	121	144	169	196	225	

   (b) The last digit is always either a 0, 1, 4, 5, 6 or a 9.
   (c) There are many possible answers. Examples are 20, 11, 14, 35, 26 and 39.
5. (a) both (b) both (c) square
   (d) cube (e) cube (f) square
   (g) neither (h) cube (i) square

CALCULATING SQUARES AND CUBES

Squaring the number 2 means that we must multiply 2 by itself. It means we have to calculate 2×2, which has a value of 4, and we write $2 \times 2 = 4$.

1. In (a) to (f) below, the numbers in set B are found by squaring each number in set A. Copy the table and write down the numbers that belong to set B in each case.

	Set A	Set B
(a)	{1; 2; 3; 4; 5; 6; 7; 8}	{1; 4; 9; 16; 25; 36; 49; 64}
(b)	{1; 3; 5; 7; 9; 11; 13}	{1; 9; 25; 49; 81; 121; 169}
(c)	{10; 20; 30; 40; 50}	{100; 400; 900; 1 600; 2 500}
(d)	{2; 4; 6; 8; 10; 12; 14}	{4; 16; 36; 64; 100; 144; 196}
(e)	{5; 10; 15; 20; 25}	{25; 100; 225; 400; 625}
(f)	{15; 12; 9; 6; 3}	{225; 144; 81; 36; 9}

Cubing the number 2 means that we must multiply 2 by itself, and again. It means we have to calculate $2 \times 2 \times 2$, which has a value of 8, and we write $2 \times 2 \times 2 = 8$.

2. (a) Cube 1. Also cube 2 and 3.
   (b) Cube 5. Also cube 10 and 4.
3. In (a) and (b) below, the numbers in set B are found by cubing each number in set A. Write down the numbers that belong to set B in each case.
   (a) **Set A:** {1; 2; 3; 4; 5; 6; 7; 8} (b) **Set A:** {10; 20; 30; 40; 50}
   Set B: **Set B:**
4. (a) Write down the squares of the first 15 natural numbers.
   (b) What do you observe about the last digit of each square number?
   (c) Give an example of a number that ends in one of the digits you have written above that is not a square.

The number 64 can be written both as a square and a cube.

$64 = 8^2$ and $64 = 4^3$

The number 17 is neither a square nor a cube.

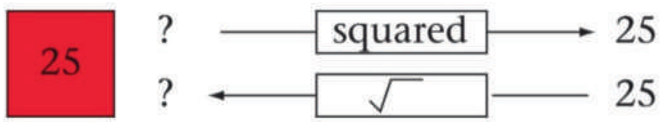
5. Are the following numbers squares, cubes, both or neither? Just write *square, cube, both* or *neither*. Compare your answers with the answers of two classmates.
   (a) 64 (b) 1 (c) 121
   (d) 1 000 (e) 512 (f) 400
   (g) 65 (h) 216 (i) 169

2.4 The square root and the cube root

DETERMINING WHAT NUMBER WAS SQUARED

Teaching guidelines

The first flow diagram below can be used to determine what number, when squared, equals 25. This resembles finding the side length of a square when its area is given. The answer is 5, because 5 × 5 = 25.



By changing the direction of the first flow diagram and changing the operation to $\sqrt{\ }$ (the symbol for square rooting), it can be shown that the question "What number, when squared, equals 25?" means the same as the question "What is the square root of 25?" This shows that square rooting is the inverse operation of squaring.

Note on notation for square roots

The symbol for square roots is $\sqrt{\ }$, not $\sqrt[2]{\ }$.

The symbols for other roots are $\sqrt[3]{\ }$, $\sqrt[4]{\ }$, $\sqrt[5]{\ }$, and so on.

Note to question 7

Squaring and square rooting are inverse operations.

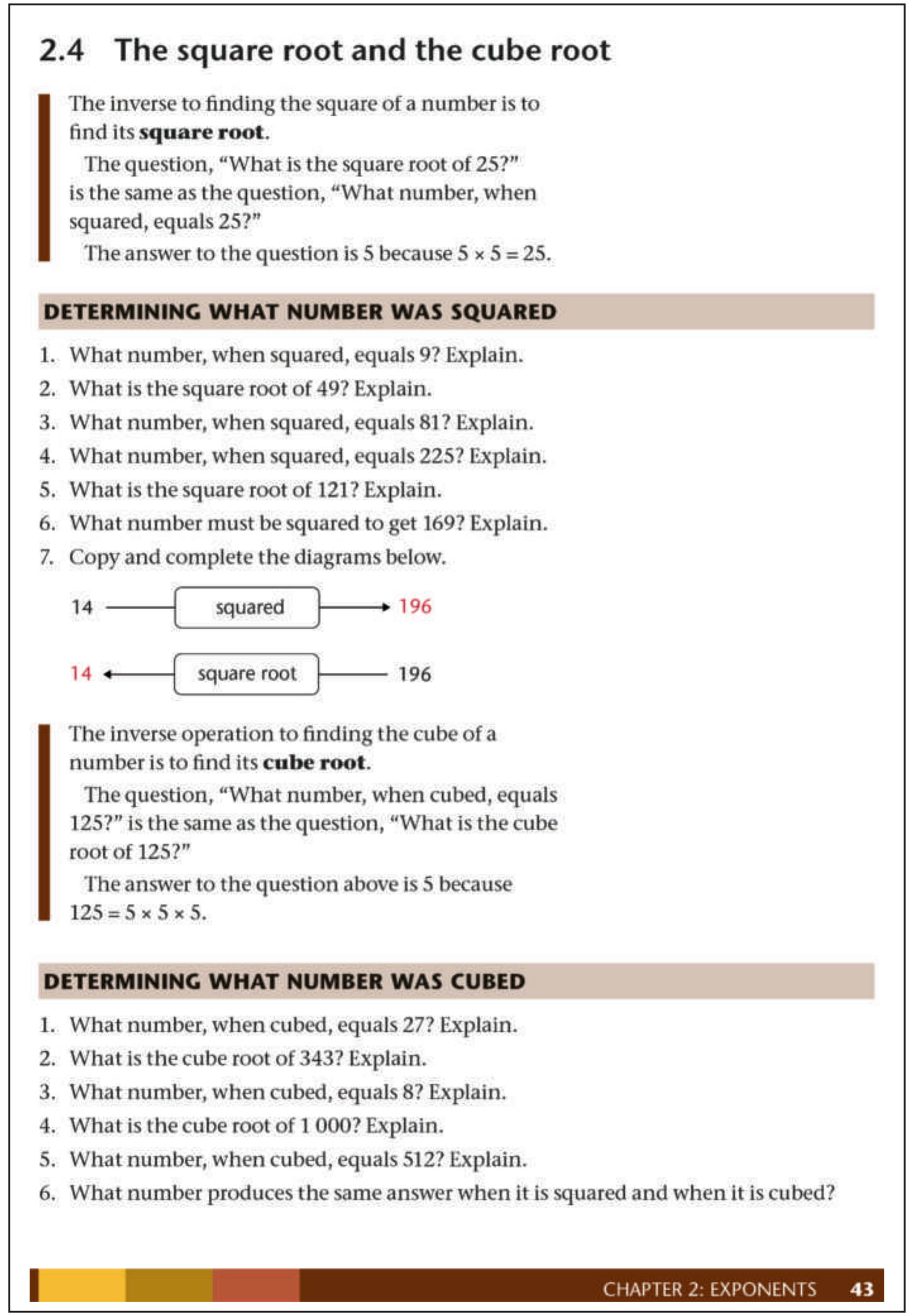
Answers

1. 3 because 3 × 3 = 9
2. 7 because 7 × 7 = 49
3. 9 because 9 × 9 = 81
4. 15 because 15 × 15 = 225
5. 11 because 11 × 11 = 121
6. 13 because 13 × 13 = 169
7. See LB page 43 alongside.

DETERMINING WHAT NUMBER WAS CUBED

Teaching guidelines

The first flow diagram below can be used to determine what number, when cubed, equals 125. This resembles finding the side length of a cube when its



2.4 The square root and the cube root

The inverse to finding the square of a number is to find its **square root**.

The question, "What is the square root of 25?" is the same as the question, "What number, when squared, equals 25?"

The answer to the question is 5 because 5 × 5 = 25.

DETERMINING WHAT NUMBER WAS SQUARED

1. What number, when squared, equals 9? Explain.
2. What is the square root of 49? Explain.
3. What number, when squared, equals 81? Explain.
4. What number, when squared, equals 225? Explain.
5. What is the square root of 121? Explain.
6. What number must be squared to get 169? Explain.
7. Copy and complete the diagrams below.

14 → squared → 196

14 ← square root ← 196

The inverse operation to finding the cube of a number is to find its **cube root**.

The question, "What number, when cubed, equals 125?" is the same as the question, "What is the cube root of 125?"

The answer to the question above is 5 because 125 = 5 × 5 × 5.

DETERMINING WHAT NUMBER WAS CUBED

1. What number, when cubed, equals 27? Explain.
2. What is the cube root of 343? Explain.
3. What number, when cubed, equals 8? Explain.
4. What is the cube root of 1 000? Explain.
5. What number, when cubed, equals 512? Explain.
6. What number produces the same answer when it is squared and when it is cubed?

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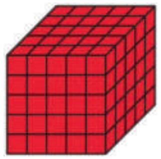
volume is given. The answer is 5, because 5 × 5 × 5 = 125.



? ——— cubed ———→ 125

? ←——— $\sqrt[3]{\ }$ ——— 125

By changing the direction of the first flow diagram and changing the operation to $\sqrt[3]{\ }$ (the symbol for cube rooting), it can be shown that the question "What number, when cubed, equals 125?" means the same as the question "What is the cube root of 125?" This illustrates that cube rooting is the inverse operation of cubing.

Note on question 7

Cubing and cube rooting are inverse operations.

Answers

1. 3 because 3 × 3 × 3 = 27
2. 7 because 7 × 7 × 7 = 343
3. 2 because 2 × 2 × 2 = 8
4. 10 because 10 × 10 × 10 = 1 000
5. 8 because 8 × 8 × 8 = 512
6. 1 and 0; because 1 × 1 = 1 and 1 × 1 × 1 = 1, and 0 × 0 = 0 and 0 × 0 × 0 = 0
7. See LB page 44 alongside.

CALCULATING SQUARE ROOTS AND CUBE ROOTS

Teaching guidelines

Learners need to understand that roots are the inverses of powers. So, square rooting is the inverse operation to squaring, and cube rooting is the inverse operation to cubing. For example:

- If $5^2 = 25$, then (by definition) $\sqrt{25} = 5$ (without a 2 outside the root sign).
- If $5^3 = 125$, then (by definition) $\sqrt[3]{125} = 5$ (with a 3 outside the root sign).

7. Copy and complete the diagrams below.

10 ——— cubed ———→ 1 000

10 ←——— cube root ——— 1 000

CALCULATING SQUARE ROOTS AND CUBE ROOTS

1. Copy and complete the table. The first one has been done for you.

	Number	Cube root	Check your answer
(a)	8	2	2 × 2 × 2 = 8
(b)	27	3	3 × 3 × 3 = 27
(c)	64	4	4 × 4 × 4 = 64
(d)	125	5	5 × 5 × 5 = 125
(e)	216	6	6 × 6 × 6 = 216
(f)	1 331	11	11 × 11 × 11 = 1 331
(g)	1 000	10	10 × 10 × 10 = 1 000
(h)	512	8	8 × 8 × 8 = 512
(i)	8 000	20	20 × 20 × 20 = 8 000

2. Copy and complete the table. The first one has been done for you.

	Number	Square root	Check your answer
(a)	9	3	3 × 3 = 9
(b)	1 600	40	40 × 40 = 1 600
(c)	144	12	12 × 12 = 144
(d)	196	14	14 × 14 = 196
(e)	625	25	25 × 25 = 625
(f)	900	30	30 × 30 = 900
(g)	16	4	4 × 4 = 16
(h)	400	20	20 × 20 = 400
(i)	121	11	11 × 11 = 121

Answers

1. See LB page 44 on previous page.
2. See LB page 44 on previous page.
3. (a) $\sqrt{169}$ (b) $\sqrt[3]{343}$ (c) $\sqrt{2\,500}$

   (d) $\sqrt[3]{729}$ (e) 25^3 (f) 25^2
4. See LB page 45 alongside.
5. See LB page 45 alongside.

The symbol $\sqrt{25}$ can be used to indicate the square root of 25. So we can write $\sqrt{25} = 5$.

The symbol $\sqrt[3]{125}$ can be used to indicate the cube root of 125. So we can write $\sqrt[3]{125} = 5$.

3. What mathematical symbol can be used to indicate each of the following?
   (a) The square root of 169 (b) The cube root of 343
   (c) The square root of 2 500 (d) The cube root of 729
   (e) The cube of 25 (f) The square of 25

By agreement amongst mathematicians, the symbol $\sqrt{}$ means the square root of the number that is written inside the symbol. So we normally write $\sqrt{4}$ instead of $\sqrt[2]{4}$.

For the cube root, however, the number 3 outside of the root sign $\sqrt[3]{}$ must be written in order to distinguish the cube root from the square root.

4. Copy the table. Find the values of each of the following. The first one has been done for you. Check your answers.

		Value	Check your answer
(a)	$\sqrt{64}$	8	$8 \times 8 = 64$
(b)	$\sqrt{49}$	7	$7 \times 7 = 49$
(c)	$\sqrt{36}$	6	$6 \times 6 = 36$
(d)	$\sqrt{784}$	28	$28 \times 28 = 784$
(e)	$\sqrt{2\,025}$	45	$45 \times 45 = 2\,025$
(f)	$\sqrt{324}$	18	$18 \times 18 = 324$

5. Copy the table. Find the values of each of the following. The first one has been done for you. Check your answers.

		Value	Check your answer
(a)	$\sqrt[3]{8}$	2	$2 \times 2 \times 2 = 8$
(b)	$\sqrt[3]{64}$	4	$4 \times 4 \times 4 = 64$
(c)	$\sqrt[3]{512}$	8	$8 \times 8 \times 8 = 512$
(d)	$\sqrt[3]{1}$	1	$1 \times 1 \times 1 = 1$
(e)	$\sqrt[3]{216}$	6	$6 \times 6 \times 6 = 216$
(f)	$\sqrt[3]{125}$	5	$5 \times 5 \times 5 = 125$

2.5 Comparing numbers in exponential form

BIGGER, SMALLER OR EQUAL?

Teaching guidelines

- The symbol < is used to indicate that the number on the left-hand side of the symbol is smaller than the number on the right-hand side: $3 < 5$.
- The symbol > is used to indicate that the number on the left-hand side of the symbol is bigger than the number on the right-hand side: $5 > 3$.
- The symbol = is used to indicate that the number on the left-hand side of the symbol has the same value as the number on the right-hand side: $2^3 = 8$.

An important note

Comparing numbers in exponential form (CAPS page 43), for example 2^6 and 5^2, helps learners to think about the distinction between the roles of the base and the exponent.

Misconceptions

Some learners think that powers like 2^6 and 6^2 have the same value because they interpret powers as multiplication of the base by the exponent.

- 2^6 means the product of 6 factors of 2: $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$
- 6^2 means the product of 2 factors of 6: $6 \times 6 = 36$

Answers

1. (a) $2^5 = 32$ $5^2 = 25$; 2^5 is bigger than 5^2
   (b) $3^4 = 81$ $4^3 = 64$; 3^4 is bigger than 4^3
   (c) $2^3 = 8$ $6^1 = 6$; 2^3 is bigger than 6^1

2. (a) = $\sqrt[3]{64} = \sqrt[3]{4 \times 4 \times 4} = 4$; $\sqrt{16} = \sqrt{4 \times 4} = 4$ (b) > $3^3 = 27$; $4^2 = 16$
   (c) = $6 \times 6 = 36$ (d) < $5 < 10$ (e) > $27 > 6$
   (f) < $16 < 81$ (g) < $8 < 9$ (h) = $1 = 1$
   (i) < $9 < 27$ (j) < $100 < 225$

3. 100^1 is bigger, because $1^{100} = 1$ and $100^1 = 100$
4. 4^2 ($4^2 = 16$; $2^4 = 16$)
5. Yes. (For example: $36 - 25 = 11$; $25 - 16 = 9$; $16 - 9 = 7$; $9 - 4 = 5$; $4 - 1 = 3$)

2.5 Comparing numbers in exponential form

BIGGER, SMALLER OR EQUAL?

1. Which is bigger?
   (a) 2^5 or 5^2
   (b) 3^4 or 4^3
   (c) 2^3 or 6^1

We can use mathematical symbols to indicate whether a number is bigger, smaller or has the same value as another number.

We use the symbol > to indicate that the number on the left-hand side of the symbol is bigger than the one on the right-hand side. **The number 5 is bigger than 3 and we express this in mathematical language as 5 > 3.**

The symbol < is used to indicate that the number on the left-hand side of the symbol is smaller than the number on the right-hand side. **The number 3 is smaller than 5 and we express this mathematically as 3 < 5.**

When numbers have the same value, we use the equal sign (=). **The numbers 2^3 and 8 have the same value and we write this as $2^3 = 8$.**

2. Use the symbols =, < or > to make the following true. Check your answers.

(a) $\sqrt[3]{64}$ = $\sqrt{16}$	(b) 3^3 > 4^2
(c) 6 = $\sqrt{36}$	(d) $\sqrt[3]{125}$ < $\sqrt{100}$
(e) 3^3 > $\sqrt[3]{216}$	(f) 2^4 < 3^4
(g) 2^3 < 3^2	(h) $\sqrt[3]{1}$ = $\sqrt{1}$
(i) 9 < 3^3	(j) 100 < 15^2

3. Which is bigger, 1^{100} or 100^1? Explain.
4. What is the biggest number you can make with the symbols 4 and 2?
5. Two whole numbers that follow on each other, like 4 and 5, are called consecutive numbers. Is the difference between the squares of two consecutive whole numbers always an odd number?

BE SMART WHEN DOING CALCULATIONS

Teaching guidelines

Learners are familiar with the distributive law, which enables us to spread multiplication over addition and subtraction. For example:

- $7 \times 15 = 7 \times (10 + 5) = 7 \times 10 + 7 \times 5 = 70 + 35 = 105$
- $7 \times 15 = 7 \times (20 - 5) = 7 \times 20 - 7 \times 5 = 140 - 35 = 105$

Knowledge of squares can be used in combination with the distributive law to do certain calculations much quicker. For example:

- $11 \times 14 = \mathbf{11} \times (\mathbf{11} + 3) = \mathbf{11 \times 11} + 11 \times 3 = \mathbf{121} + 33 = 154$
- $12 \times 19 = \mathbf{12} \times (\mathbf{12} + 7) = \mathbf{12 \times 12} + 12 \times 7 = \mathbf{144} + 84 = 228$

Answers

1. 209   2. 208   3. 270   4. 216

ARRANGING NUMBERS IN ASCENDING AND DESCENDING ORDER

Answers

1. (a) $\sqrt[3]{64} = 4;\ 3^2 = 9;\ \sqrt{64} = 8;\ \sqrt{36} = 6 \rightarrow \sqrt[3]{64};\ \sqrt{36};\ \sqrt{64};\ 3^2$

   (b) $\sqrt{225} = 15;\ \sqrt[3]{729} = 9;\ \sqrt[3]{1\,000} = 10;\ 2^2 = 4 \rightarrow 2^2;\ \sqrt[3]{729};\ \sqrt[3]{1\,000};\ \sqrt{225}$

   (c) $\sqrt[3]{1} = 1;\ 0;\ 100;\ 1\,000 \rightarrow 0;\ \sqrt[3]{1};\ 100;\ 10^3$

   (d) $1^2 = 1;\ 2^3 = 8;\ 4^2 = 16;\ 5^2 = 25 \rightarrow 1^2;\ 2^3;\ 4^2;\ 5^2$

2. (a) $\sqrt[3]{216} = 6;\ \sqrt[3]{10^3} = 10;\ 2^5 = 32;\ 20 \rightarrow 2^5;\ 20;\ \sqrt[3]{10^3};\ \sqrt[3]{216}$

   (b) $10^3 = 1\,000;\ \sqrt[3]{20^3} = 20;\ \sqrt{144} = 12;\ 12^2 = 144 \rightarrow 10^3;\ 12^2;\ \sqrt[3]{20^3};\ \sqrt{144}$

   (c) $\sqrt{121} = 11;\ \sqrt[3]{125} = 5;\ 11^2 = 121;\ 5^3 = 125 \rightarrow 5^3;\ 11^2;\ \sqrt{121};\ \sqrt[3]{125}$

   (d) $1^5 = 1;\ 2^4 = 16;\ 7^2 = 49;\ 6^3 = 216;\ 5^3 = 125 \rightarrow 6^3;\ 5^3;\ 7^2;\ 2^4;\ 1^5$

BE SMART WHEN DOING CALCULATIONS

Our knowledge of squares can help us to do some calculations much quicker. Suppose you want to calculate 11×12.

11^2 has a value of 121. We know that $11 \times 11 = 121$.
11×12 means that there are 12 elevens in total.
So $11 \times 12 = 11 \times 11 + 11$
$= 121 + 11$
$= 132$

Suppose you want to calculate 11×17.
$11 \times 17 = 17$ elevens in total $= 11$ elevens $+ 6$ elevens
Well, we know that $11 \times 11 = 121$
So $11 \times 17 = 11 \times 11 + 6 \times 11$
$= 121 + 66$
$= 187$

Now do the following calculations in your exercise book, using your knowledge of square numbers.

1. 11×19 2. 13×16 3. 15×18 4. 12×18

ARRANGING NUMBERS IN ASCENDING AND DESCENDING ORDER

The numbers 1, 4, 9, 16, 25, ... are arranged from the smallest to the biggest number. We say that the numbers 1, 4, 9, 16, 25, ... are arranged in **ascending order**.

The numbers 25, 16, 9, 4, 1, ... are arranged from the biggest to the smallest number. We say that the numbers 25, 16, 9, 4, 1, ... are arranged in **descending order**.

1. In questions (a) to (d), arrange the numbers in ascending order:
   (a) $\sqrt[3]{64};\ 3^2;\ \sqrt{64};\ \sqrt{36}$
   (b) $\sqrt{225};\ \sqrt[3]{729};\ \sqrt[3]{1\,000};\ 2^2$
   (c) $\sqrt[3]{1};\ 0;\ 100;\ 10^3$
   (d) $1^2;\ 2^3;\ 4^2;\ 5^2$
2. In questions (a) to (d), arrange the numbers in descending order:
   (a) $\sqrt[3]{216};\ \sqrt[3]{10^3};\ 2^5;\ 20$
   (b) $10^3;\ \sqrt[3]{20^3};\ \sqrt{144};\ 12^2$
   (c) $\sqrt{121};\ \sqrt[3]{125};\ 11^2;\ 5^3$
   (d) $1^5;\ 2^4;\ 7^2;\ 6^3;\ 5^3$

2.6 Calculations

THE ORDER OF OPERATIONS/WRITING NUMERICAL EXPRESSIONS IN WORDS

Teaching guidelines

An **acronym** is a pronounceable abbreviation and is often written with capital letters to indicate that it is not a **real** word. The acronym **BODMAS** reminds us of the order in which certain operations have to be done during calculations.

B: Brackets	Step 1: Do brackets.
O: Of	Step 2: Do "of" (which means multiplication).
D: Division M: Multiplication	Step 3: Do all multiplications and divisions, working from left to right.
A: Addition S: Subtraction	Step 4: Do all additions and subtractions, working from left to right.

- $5^2 \times 8 + 12 = 200 + 12 = 212$: Multiply 5 by itself and then by 8; then add 12.
- $5^2 \times (8 + 12) = 25 \times 20 = 500$: Add 8 and 12; then multiply by 25 (5 squared).

Misconceptions

1. That **exponents distribute**, for example that $(2 + 3)^2 = 2^2 + 3^2$. Help learners by asking them to calculate 5^2 as well as $2^2 + 3^2$.
2. Multiplying bases and adding/multiplying exponents, for example $3^4 5^2 = 15^6$ or $3^4 3^2 = 9^6$ or 9^8. Emphasis is often placed on the terminology **base**, **power** and **exponent** while the meaning of exponential notation (shorthand for repeated multiplication) is not prioritised.

Answers

1. (a) Multiply 2 with itself and then with 5; then add 3.
   (b) Add 2 and 3, and square the answer; multiply it by 5 twice (or by 25).
   (c) Add 36 and 64; then take the square root of the answer. Add this to 3 multiplied by itself two times (3 × 3 × 3).
   (d) Take the square root of 16 and the square root of 9 and add them.
   (e) Multiply 10 by 10 and 10 again. Multiply 9 by 9 and 9 again. Subtract your second answer from your first answer.
   (f) 18 divided by the square root of 9, all squared.
   (g) 26 minus the square root of 4, all divided by 6.

2.6 Calculations

THE ORDER OF OPERATIONS

When a numerical expression includes more than one operation, for example both multiplication and addition, what you do first makes a difference.

If there are no brackets in a numerical expression, it means that **multiplication and division must be done first, and addition and subtraction only later**. For example, the expression $12 + 3 \times 5$ means "multiply 3 by 5; then add 12". It does *not* mean "add 12 and 3; then multiply by 5".

If you wish to specify that addition **should be done first**, that part of the expression should be **put in brackets**. For example, if you wish to say "add 5 and 12; then multiply by 3", the numerical expression should be $3 \times (5 + 12)$ or $(5 + 12) \times 3$.

Here is another example: The expression $10 - 6 \div 3$ means "divide 6 by 3; then subtract the answer from 10". It does *not* mean "subtract 6 from 10; then divide by 3". If you wish to specify that subtraction should be done first, that part of the expression should be put in brackets. The numerical expression $(10 - 6) \div 3$ means "subtract 6 from 10; then divide the answer by 3".

It is important to know the **correct order** in which operations in a numerical expression should be done.

WRITING NUMERICAL EXPRESSIONS IN WORDS

1. Write each of the following numerical expressions in words:
   (a) $5 \times 2^2 + 3$ (b) $5^2 \times (2 + 3)^2$ (c) $\sqrt{36 + 64} + 3^3$
   (d) $\sqrt{16} + \sqrt{9}$ (e) $10^3 - 9^3$ (f) $(18 \div \sqrt{9})^2$
   (g) $\dfrac{26 - \sqrt{4}}{6}$

CALCULATIONS WITH EXPONENTS

Do these calculations without using a calculator.

1. Calculate:
   (a) $2^4 + 1^4$ (b) $(2 + 1)^4$ (c) $2^3 + 3^3 + 4^3$
   (d) $2^3 + 5^3 \times 3$ (e) $12^2 \div 2^3$ (f) $\dfrac{12 + 2 \times 3^2}{4^2 - 1^3}$

CALCULATIONS WITH EXPONENTS

Teaching guidelines

Exponential notation represents repeated multiplication of the same factor.

- The **base** shows the repeated factor.
- The **exponent** or **index** shows the number of repetitions.

An important note

The acronym **BIDMAS** stands for **B**rackets; **I**ndices (which imply powers), **D**ivision/**M**ultiplication from left to right, and **A**ddition/**S**ubtraction from left to right. This indicates that powers should usually be worked out before division and multiplication (which includes "Of").

Misconception

Note that most learners will indicate that 2(b) is true and yet it is not. Make them aware of the convention. Do a few other examples to consolidate the convention.

Note on question 1(b)

The base should be simplified before the exponent is applied.

Answers (question 1 starting on LB page 48)

1. (a) $16 + 1 = 17$ (b) $3^4 = 81$
   (c) $8 + 27 + 64 = 99$ (d) $8 + (125 \times 3) = 8 + 375 = 383$
   (e) $144 \div 8 = 18$ (f) $\frac{12 + (2 \times 9)}{16 - 1} = \frac{30}{15} = 2$
2. (a) $8 + 4 = 12$; $2^5 = 32$; not equal
   (b) $8 \times 4 = 32$; $2^5 = 32$; equal
3. (a) $125 + 5 = 130$; $5^4 = 625$; not same value
   (b) $125 \times 5 = 625$; $5^4 = 625$; same value
4. (a) $16 \times 256 = 4\,096$; $8^4 = 4\,096$; same value
   (b) $512 \times 8 = 4\,096$; $8^4 = 4\,096$; same value
5. (a) $16 + 9 = 25$ (b) $144 + 25 = 169$
6. (a) $2^5 = 32$; $2^6 = 64$; $2^7 = 128$; $2^8 = 256$; $2^9 = 512$; $2^{10} = 1\,024$; $2^{11} = 2\,048$; $2^{12} = 4\,096$
   (b) The last digit cycles through the values 2; 4; 8 and 6.
   (c) (i) 6 (ii) 4

2. Do the calculations below and then say which expression has the same value as 2^5.
   (a) $2^3 + 2^2$ (b) $2^3 \times 2^2$
3. Do the calculations below and then say which expression has the same value as 5^4.
   (a) $5^3 + 5^1$ (b) $5^3 \times 5^1$
4. Which of the expressions below has the same value as 8^4?
   (a) $2^4 \times 4^4$ (b) $8^3 \times 8$
5. Calculate the following:
   (a) $4^2 + 3^2$ (b) $12^2 + 5^2$
6. (a) Continue this list to find the values of the "powers of 2" from 2^1 to 2^{12}:
   $2^1 = 2$; $2^2 = 4$; $2^3 = 8$; $2^4 = 16$;
   (b) Do you notice a pattern in the last digit of the numbers? Write down the pattern in your own words.
   (c) Use the pattern to predict the *last digit* of the following values. (You should not need to actually calculate the values in full.)
   (i) 2^{20} (ii) $2^{1\,002}$

CALCULATIONS INVOLVING SQUARE ROOTS AND CUBE ROOTS

1. Calculate each of the following without using a calculator:
   (a) $\sqrt{64} + \sqrt{36}$ $= 8 + 6 = 14$ (b) $\sqrt{9 + 16}$ $= \sqrt{25} = 5$
   (c) $\sqrt{25}$ $= 5$ (d) $\sqrt{100}$ $= 10$
   (e) $\sqrt{64 + 36}$ $= \sqrt{100} = 10$ (f) $\sqrt{9} + \sqrt{16}$ $= 3 + 4 = 7$
2. Say whether each of the following is true or false. Explain your answer.
   (*Note*: ≠ in question (d) means "is not equal to")
   (a) $\sqrt{64 + 36} = \sqrt{64} + \sqrt{36}$ (b) $\sqrt{16} + \sqrt{9} = \sqrt{16 + 9}$
   (c) $\sqrt{100} = \sqrt{64} + \sqrt{36}$ (d) $\sqrt{25} \neq \sqrt{9} + \sqrt{16}$
   (e) $\sqrt{9 \times 9} = 9$ (f) $\sqrt[3]{2 \times 2 \times 2} = 2$
   (g) $\sqrt{169} - \sqrt{25} = 8$ (h) $\sqrt{169 - 25} = 12$
3. Calculate each of the following without using a calculator:
   (a) $2 + \sqrt[3]{8} + (3 + 2)^2$ (b) $2 + \sqrt[3]{8} + 3^2 + 2^2$
   (c) $2 + \sqrt[3]{8} + 2^5 - 2^3$ (d) $\frac{5 + 4 \times (\sqrt{169} - 2^3)}{5}$
   (e) $(15 - \sqrt{25})^3$ (f) $\frac{28 - 24 \div \sqrt{4}}{(\sqrt[3]{27} + 1)^2}$

CHAPTER 2: EXPONENTS 49

CALCULATIONS INVOLVING SQUARE ROOTS AND CUBE ROOTS

Note on question 2(a)

The inside of the root should be simplified before finding the square root.

Answers (questions on LB page 49)

1. See LB page 49 on previous page.
2. (a) False. 10 does not equal 8 + 6 (b) False. 4 + 3 does not equal 5
   (c) False. 10 does not equal 8 + 6 (d) True. 5 does not equal 3 + 4
   (e) True. true by definition (f) True. true by definition
   (g) True. 13 – 5 = 8 (h) True. the square root of 144 is 12
3. (a) $2 + 2 + 25 = 29$ (b) $2 + 2 + 9 + 4 = 17$
   (c) $2 + 2 + 32 - 8 = 28$ (d) $\frac{5 + 4 \times (13 - 8)}{5} = \frac{5 + (4 \times 5)}{5} = \frac{5 + 20}{5} = \frac{25}{5} = 5$
   (e) $(15 - 3)^3 = 10^3 = 1\,000$ (f) $\frac{28 - (24 \div 2)}{(3 + 1)^2} = \frac{(28 - 12)}{16} = 16 \div 16 = 1$

WORKSHEET

Answers

1. $6 \times 6 \times 6 \times 6 \times 6 \times 6$
2. 14^9
3. $81; 32; 64; 10 \rightarrow 10; 2^5; 4^3; 3^4$
4. (a) False. 10 is not equal to 8 + 6
   (b) True. 5 + 3 = 8
5. (a) $27 \times 4 = 108$ (b) $12 + 9 = 21$
   (c) $121 + 25 - 12 = 134$ (d) $2^4 \div 2 = 16 \div 2 = 8$
   (e) $81 - (16 \times 3) = 81 - 48 = 33$ (f) $7 + 5 + 1 - 8 = 5$
   (g) $(3 + 8)^2 = 11^2 = 121$ (h) $(\sqrt{25} \div 5) \times 93 = 1 \times 93 = 93$
   (i) $\frac{81 + 144 + 125 + 650}{5 \times 100} = \frac{1\,000}{500} = 2$ (j) $\frac{216 - 169 + 2}{49 \times 1} = \frac{49}{49} = 1$

WORKSHEET

1. Write in expanded form:

   6^6

2. Write in exponential form:

   14 to the power 9

3. Rewrite the numbers from the smallest to the biggest: $3^4; 2^5; 4^3; 10$

4. Say whether each of the following is true or false. Explain your answer.

   (a) $\sqrt{64 + 36} = \sqrt{64} + \sqrt{36}$ (b) $\sqrt{25} + \sqrt{9} = \sqrt{59 + 5}$

5. Calculate:

   (a) $3^3 \times 2^2$ (b) $\sqrt{144} + \sqrt{81}$

   (c) $11^2 + 5^2 - \sqrt{144}$ (d) $(14 - 12)^4 \div \sqrt[3]{8}$

   (e) $9^2 - 4^2 \times 3$ (f) $7 + \sqrt[3]{125} + 1^5 - 2^3$

   (g) $(\sqrt[3]{27} + \sqrt{64})^2$ (h) $(\sqrt{16 + 9} \div 5^1) \times 93$

   (i) $\frac{9^2 + 12^2 + 5^3 + 650}{\sqrt[3]{125} \times 10^2}$ (j) $\frac{6^3 - (\sqrt{169})^2 + \sqrt[3]{8}}{7^2 \times 1^9}$

EXPONENTS

Grade 7 Term 1 Chapter 3 Geometry of straight lines

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
3.1 Line segments, lines and rays	Difference between line segments, lines and rays	51 to 53
3.2 Parallel and perpendicular lines	Informal definitions of parallel lines, comparing parallel lines to lines that meet, definition of perpendicular lines	54 to 56

CAPS time allocation	2 hours
CAPS content specification	Page 47

Resources

Learners need a ruler or straight edge; a sharp pencil and a set square (to draw a right angle).

At this stage, they have not used a protractor.

Mathematical background

- **Line segments, lines** and **rays** can be thought of as a collection of points that are so close together that there are no spaces in between.
- We make a pencil (or pen) mark to indicate the position of a **point**. We use letters like A, B, P, Q (all upper-case) to name a point.
- A **line segment** has a beginning and an end. When we name a line segment, we need only name the two endpoints. We can measure the length of a line segment.
- A **line** extends indefinitely in both directions and has no beginning or end. We cannot measure the length of a line. We cannot draw a line on paper, but we add arrows to a line segment to represent a line. We name a line by choosing any two points on the line and labelling them with chosen letters, for example line AB, or line BA, etc. Sometimes a line can be named by a single letter (lower case), for example p.
- A line that has a definite starting point but no definite endpoint is called a **ray** and is named by the starting point first and any other point on the ray, for example AB. We cannot measure the length of a ray.
- Learners should understand the difference between lines and line segments. In the work that learners will encounter in the future, a line segment or a ray is often simply called a line.

3.1 Line segments, lines and rays

LINE SEGMENTS

Teaching guidelines

Learners will measure the line segments of the quadrilateral in question 1 more accurately if they measure in millimetres.

Talk about the meaning of the word **segment** as part of something. A line segment is a set of points with a definite starting and end point.

Misconceptions

Learners often confuse lines with line segments and lines with rays. Let learners draw points on a sheet of paper and join them two by two with a ruler to create line segments.

Notes on the questions

We usually mention the endpoints of the line segments in alphabetical order, for example AB and not BA, although BA is not incorrect.

Answers

1. See LB page 51 alongside.

Additional questions

1. Let learners make at least five points and name them A, B, C, D and E.

   Learners join the points so that every point is joined with all the other points. Learners then measure and name the line segments.

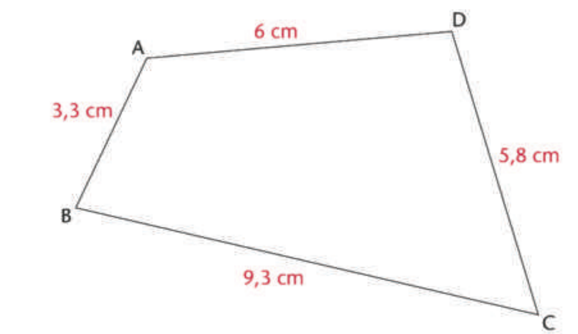
   Learners exchange their work with a partner who then tests the correctness of the measurements.

Chapter 3
Geometry of straight lines

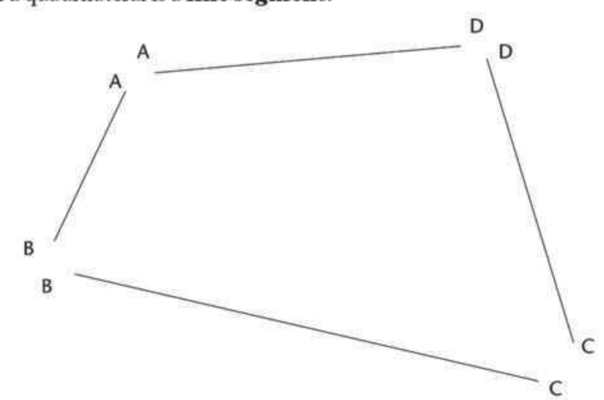
3.1 Line segments, lines and rays

LINE SEGMENTS

1. Measure each side of this quadrilateral. Copy the quadrilateral and write the measurements at each side.



Each side of a quadrilateral is a **line segment**.



Answers

2. See LB page 52 alongside.

LINES AND RAYS

Teaching guidelines

Talk about extending a line segment indefinitely in both directions which we show by an arrowhead at each end of the line segment. It is impossible to show the line completely for obvious reasons: it has no beginning and no end. Explain that this is the reason that we cannot measure the length of a line. Therefore, we show only a part of a line in a drawing, while the arrows indicate that it is a line.

Let learners think of a ray of light that starts at a source (for example, the sun) and travels on indefinitely in one direction. Explain that we can represent a ray on paper as beginning at a point and continuing indefinitely in a direction. A ray has a point at the beginning and an arrowhead on the other end.

Misconceptions

Learners often confuse lines and rays. You could use a physical explanation to help them remember, for example: represent a point by a closed fist; make a fist with each hand and extend your arms. Explain that the distance between the two fixed points (fists) represents a line segment. Now open one fist and point your fingers; this represents a ray. With both hands opened and fingers pointed in opposite directions we can represent the idea of a line.

Explain that a ray can point in any direction, but we mention the starting point of the line first. See examples, ray PQ and ray DC on LB page 52.

A line or a ray can be horizontal, vertical or slanted.

Answers

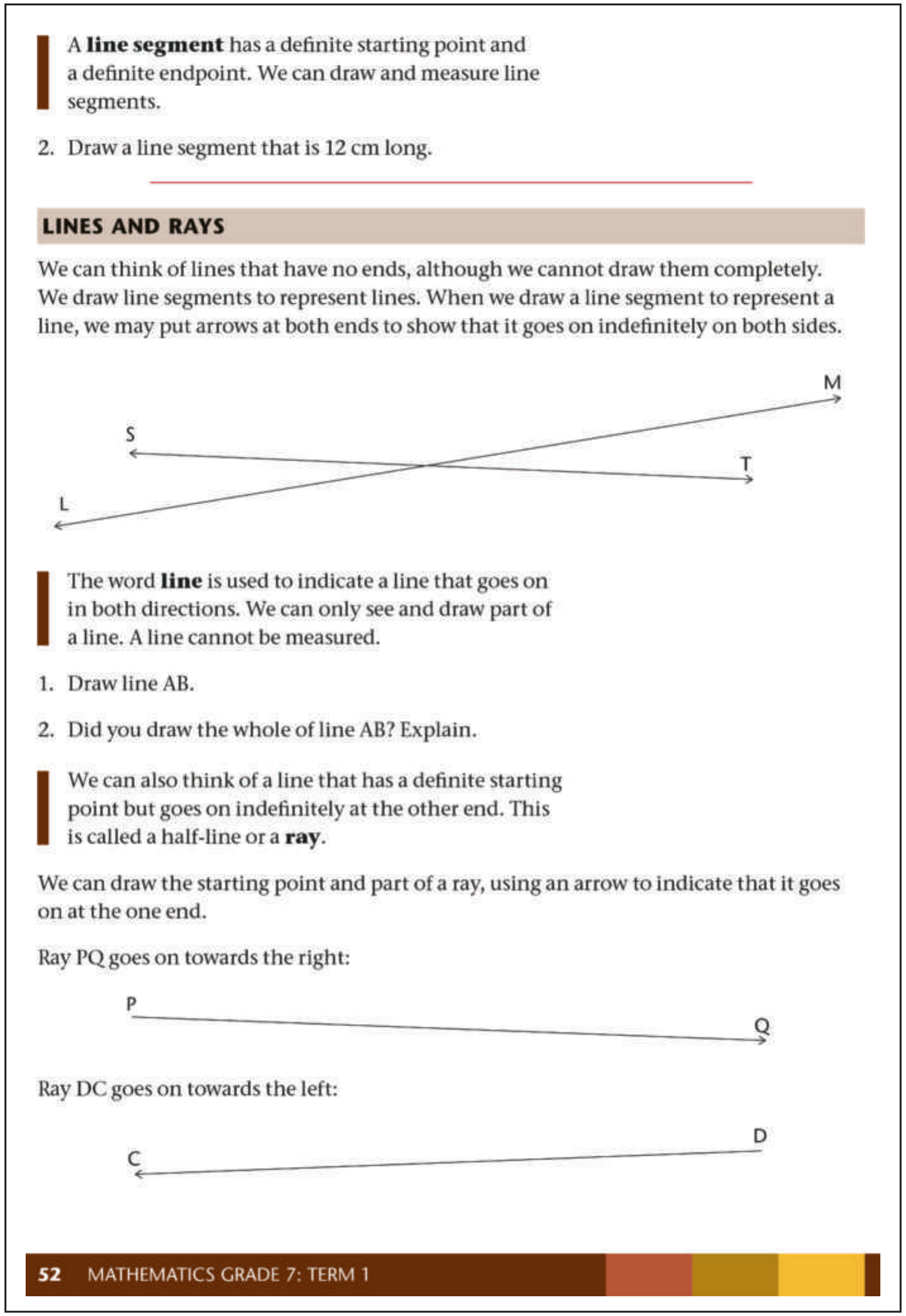
1. 

2. No. It is not possible to draw the whole line, because it has no definite starting point or endpoint.

Teaching guidelines

Circulate and make sure that all learners draw the rays correctly with the first letter as the starting point and the other letter at the arrowhead, for example ray EF with E at the start or the ray FT with F at the starting point.

Make sure that learners understand that lines could meet somewhere if they are extended (i.e. if they are not parallel).



A **line segment** has a definite starting point and a definite endpoint. We can draw and measure line segments.

2. Draw a line segment that is 12 cm long.

LINES AND RAYS

We can think of lines that have no ends, although we cannot draw them completely. We draw line segments to represent lines. When we draw a line segment to represent a line, we may put arrows at both ends to show that it goes on indefinitely on both sides.

M S T L

The word **line** is used to indicate a line that goes on in both directions. We can only see and draw part of a line. A line cannot be measured.

1. Draw line AB.
2. Did you draw the whole of line AB? Explain.

We can also think of a line that has a definite starting point but goes on indefinitely at the other end. This is called a half-line or a **ray**.

We can draw the starting point and part of a ray, using an arrow to indicate that it goes on at the one end.

Ray PQ goes on towards the right:

P Q

Ray DC goes on towards the left:

D C

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Line segments that do not cross each other (intersect) will not meet (as in question 5).

Rays that diverge (move away from each other) will not meet (as in question 7), but rays that converge (move towards each other) will meet if they are extended (as in question 8).

Question 9 could confuse learners. Point out that the rays cannot meet. Copy the drawing on the board and extend both rays JK and RS to show learners that even though RS is extended it passes the starting point J of the other ray.

Misconceptions

Learners may be confused when they answer questions like question 7. Make sure that they realise that these are rays and will not meet.

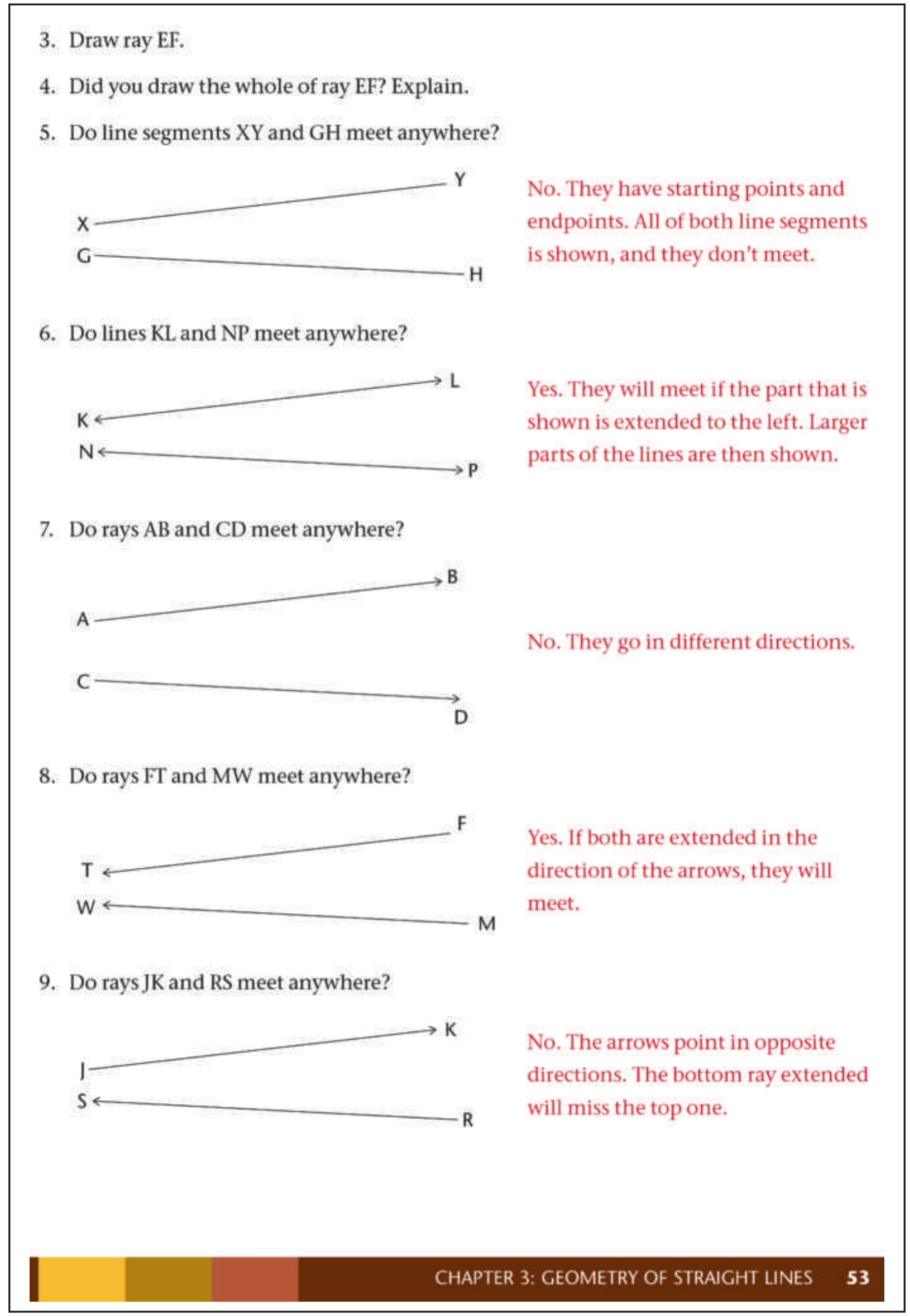
Notes on the questions

Remember that lines (or rays or line segments) will never cross or meet if they are parallel.

Answers

3.

4. No. You can only show the starting point of a ray. You can't show where it ends.
5. See LB page 53 alongside.
6. See LB page 53 alongside.
7. See LB page 53 alongside.
8. See LB page 53 alongside.
9. See LB page 53 alongside.



3. Draw ray EF.
4. Did you draw the whole of ray EF? Explain.
5. Do line segments XY and GH meet anywhere?

No. They have starting points and endpoints. All of both line segments is shown, and they don't meet.

6. Do lines KL and NP meet anywhere?

Yes. They will meet if the part that is shown is extended to the left. Larger parts of the lines are then shown.

7. Do rays AB and CD meet anywhere?

No. They go in different directions.

8. Do rays FT and MW meet anywhere?

Yes. If both are extended in the direction of the arrows, they will meet.

9. Do rays JK and RS meet anywhere?

No. The arrows point in opposite directions. The bottom ray extended will miss the top one.

CHAPTER 3: GEOMETRY OF STRAIGHT LINES 53

3.2 Parallel and perpendicular lines

PARALLEL LINES

Notes on the questions

This is an informal definition of parallel lines. Learners will learn the formal definition of parallel lines in future work. At this stage, they should realise that parallel lines are always the same distance apart.

Teaching guidelines

Talk about the fact that the distance between any two parallel lines always remains the same. Parallel lines are indicated by a set of arrows on each line that point in the same direction (see between C and E and between D and F on LB page 54). We write the names of the parallel lines as follows AG||BH. Explain that even if lines look parallel to the eye, we cannot assume that they are unless they are marked with the sets of arrows.

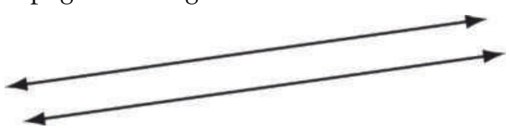
Give learners the opportunity to measure the lines to find out if they really are parallel.

Let learners think about using the measurements to not only find out if lines are parallel, but also to draw sets of parallel lines. Let them talk in pairs about using this method. They can do questions 2 and 3.

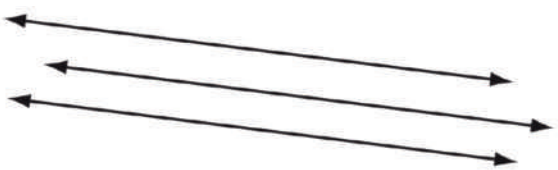
Answers

1. See LB page 54 alongside.

2.

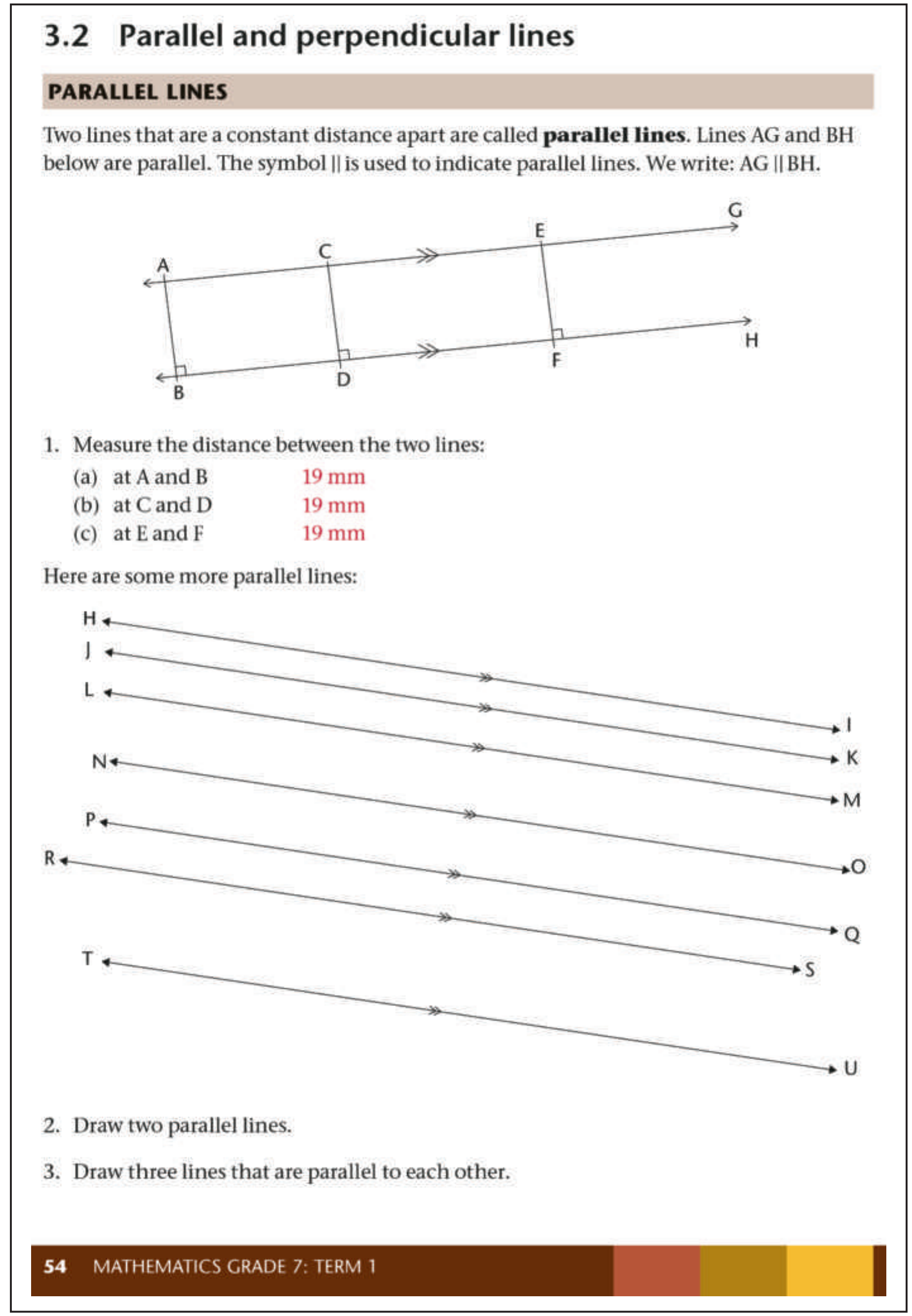


3.



Notes on the questions

These questions are designed to make learners think about the circumstances under which lines (or line segments and rays) are parallel and to understand the properties.



3.2 Parallel and perpendicular lines

PARALLEL LINES

Two lines that are a constant distance apart are called **parallel lines**. Lines AG and BH below are parallel. The symbol || is used to indicate parallel lines. We write: AG || BH.

1. Measure the distance between the two lines:
   (a) at A and B 19 mm
   (b) at C and D 19 mm
   (c) at E and F 19 mm

Here are some more parallel lines:

2. Draw two parallel lines.

3. Draw three lines that are parallel to each other.

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Teaching guidelines

Circulate and check that learners have the skills to manage the necessary measurements.

Talk through the answers to the questions with the learners when they have completed them. Allow learners to find ways of measuring the distance between two lines to determine whether they are parallel or not.

Let learners discuss in pairs or small groups what plan can be made to determine whether two line segments MN and AB are parallel or not.

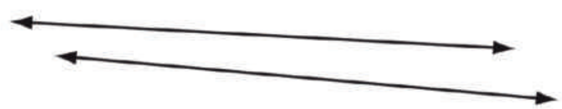
Misconceptions

Learners should realise that lines or line segments may not be parallel, even though they look like they are. The only way to be sure is to measure them or if the lines are marked to be parallel.

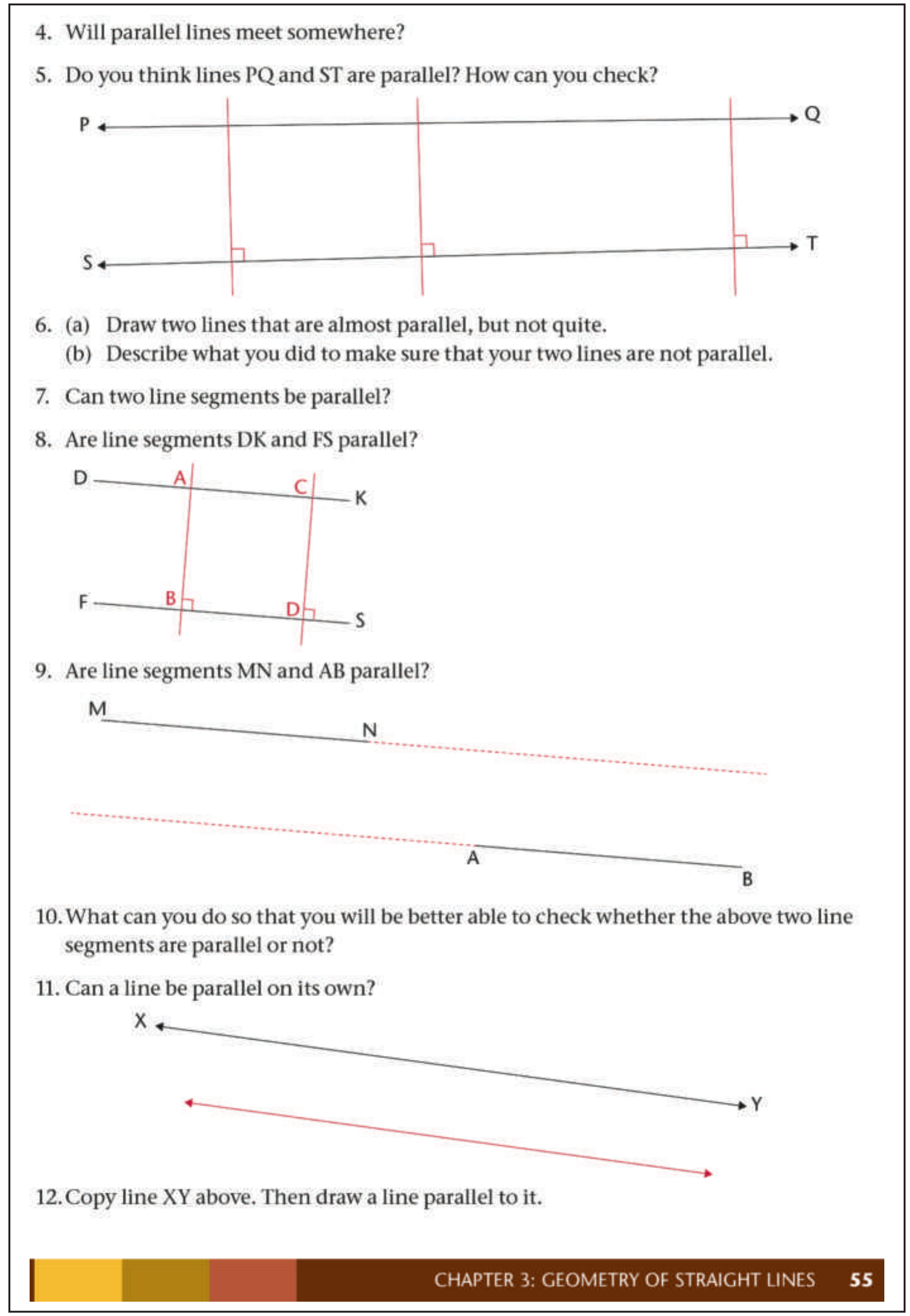
Answers

4. No. There is equal distance between them all the time. They will never meet.
5. See LB page 55 alongside.

   Draw a few lines to cut the lines at right angles so that you can measure the perpendicular distance between the lines at several places. If the lines are the same distance apart, they are parallel. PQ and ST are not parallel.
6. (a)



   (b) Learners' own descriptions. (For example: Draw one line. Place the ruler to be parallel with the first line. Then move it a little bit to be at a small angle with the first line.)
7. Yes
8. Yes, the length of AB = the length of CD.
   AB = CD = 2,4 cm (see the drawing on LB page 55).
9. Yes, it looks like it (see the drawing on LB page 55).
10. Extend segment MN to the right and extend segment AB to the left. Draw two lines perpendicular to the segments, such as those above in question 1, and measure the distance between the extensions of the segments. If the distances are the same, the segments are parallel.
11. No. A line can only be parallel to another line.
12. See the drawing on LB page 55.



4. Will parallel lines meet somewhere?
5. Do you think lines PQ and ST are parallel? How can you check?

6. (a) Draw two lines that are almost parallel, but not quite.
   (b) Describe what you did to make sure that your two lines are not parallel.
7. Can two line segments be parallel?
8. Are line segments DK and FS parallel?

9. Are line segments MN and AB parallel?

10. What can you do so that you will be better able to check whether the above two line segments are parallel or not?
11. Can a line be parallel on its own?

12. Copy line XY above. Then draw a line parallel to it.

PERPENDICULAR LINES

Notes on the questions

Learners should work with the symbol that is used to indicate perpendicular lines (lines at right angles to each other) to become familiarised with it (⊥).

Teaching guidelines

Talk about the fact that perpendicular lines make an angle of 90° with each other. We write AB⊥CD to indicate that AB is perpendicular to CD.

On a drawing, two perpendicular lines are indicated by a small square at the point where the lines cross each other (see LB page 56).

Sometimes lines in a drawing look as if they are perpendicular, but we only know they were meant to be perpendicular if they are marked with the small square where they intersect.

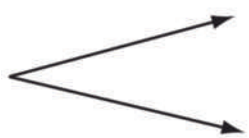
Let learners name the shape that is formed when two rays have the same starting point (an angle). Draw a variety of rays on the board that form acute angles, obtuse angles, perpendicular angles, etc.

Let learners discuss the answer to question 6.

Answers

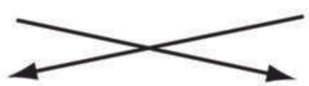
1. Four angles

2.

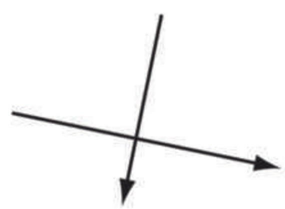


3.

4.



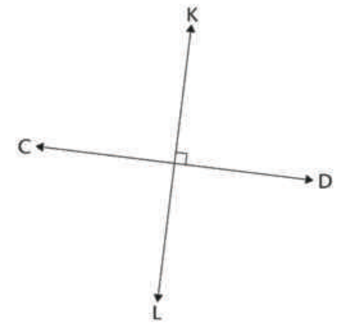
5.



6. See LB page 56 alongside.

PERPENDICULAR LINES

Lines CD and KL below are perpendicular to each other. The symbol ⊥ is used to indicate perpendicular lines. We write: CD ⊥ KL.



1. How many angles are formed at the point where the above two lines meet?

Two lines that form right angles are **perpendicular** to each other.

2. Draw two rays that have the same starting point.
3. Draw two rays that are perpendicular to each other and have the same starting point.
4. Draw two rays that meet, but not at their starting points.
5. Draw two rays that meet but not at their starting points, and that are perpendicular to each other.
6. Can you draw two rays that have the same starting point, and are parallel to each other?
   They will have to go in opposite directions. Below are ray AB and ray AC.

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
4.1 Angles revision	Vocabulary and the conventions for representing, describing and labelling angles	57 to 59
4.2 The degree: a unit for measuring angles	Recognising the need for a unit of measurement and developing an understanding of the degree as a suitable unit	59 to 60
4.3 Using the protractor	Explaining a protractor and how to use it to measure all types of angles	61 to 64
4.4 Using a protractor to construct angles	Constructing angles to a given size using a protractor	65 to 66
4.5 Parallel and perpendicular lines	Using a protractor to construct 90° angles (perpendicular lines) and measuring distances on perpendicular lines to construct parallel lines	66 to 68
4.6 Circles are very special figures	Understanding a circle as a curve of which the ends meet and every point on the curve is the same distance from a point in the centre of the curve	68 to 69
4.7 Using the compass	Learning how to use a compass and drawing circles using a compass	69 to 71
4.8 Using circles to draw other figures	Investigating patterns of circles and shapes that are formed by joining points of intersection	72 to 75
4.9 Parallel and perpendicular lines with circles	Investigating how parallel and perpendicular lines are formed when circles intersect	76

CAPS time allocation	10 hours
CAPS content specification	Page 45

Mathematical background

- An **angle** is formed when two lines, the sides of the angle, meet at a common point, the **vertex**.
- An angle measures the amount of turning about a point in degrees. For example, starting on BA and turning A about B to end at C, forms $A\hat{B}C$.
- The **label** of the vertex is always written in the middle, for example ∠ABC or angle ∠CBA means that B is the vertex. Another way of writing ∠ABC is $A\hat{B}C$ or simply $\hat{B}$. When there is more than one angle at a vertex, the two endpoints will have to be included, for example $A\hat{B}C$ or $C\hat{B}D$, to avoid confusion.
- Angles could have their sides slanted and learners should also be shown angles with sides that are not horizontal and/or vertical.
- In order to measure the size of an angle, the sides sometimes have to be lengthened, this is also called **producing the lines**.
- Parallel lines and perpendicular lines could also be slanted and learners should work with such lines oriented in different ways.

4.1 Angles revision

Background

There is more than one way to think about angles. For example, they can be thought of:

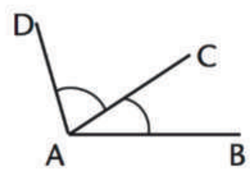
- As the space between two lines that cross or intersect, or meet at a vertex. Let learners think of examples from daily life where angles are encountered, for example traffic signs.
- As the amount of turning about a point, for example when a door is opened, each point on the surface of the door turns through the same angle.

Teaching guidelines

Work through the concept of an angle as an amount of turning about a point and discuss the words used to describe an angle (arms, vertex, etc.). Let learners think of standing in one spot and turning until they face in the same direction they were facing when they started the motion. Point out that they have turned through a **full turn**, or a **revolution**.

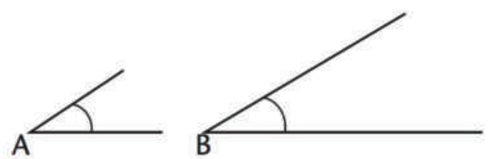
Use the activity to revise the angles they learnt about in previous grades. Let one or two learners stand in front of the class facing in a direction and let the rest of the class instruct the learner to turn through specific angles they know, for example, a right angle, a reflex angle, a straight angle, etc.

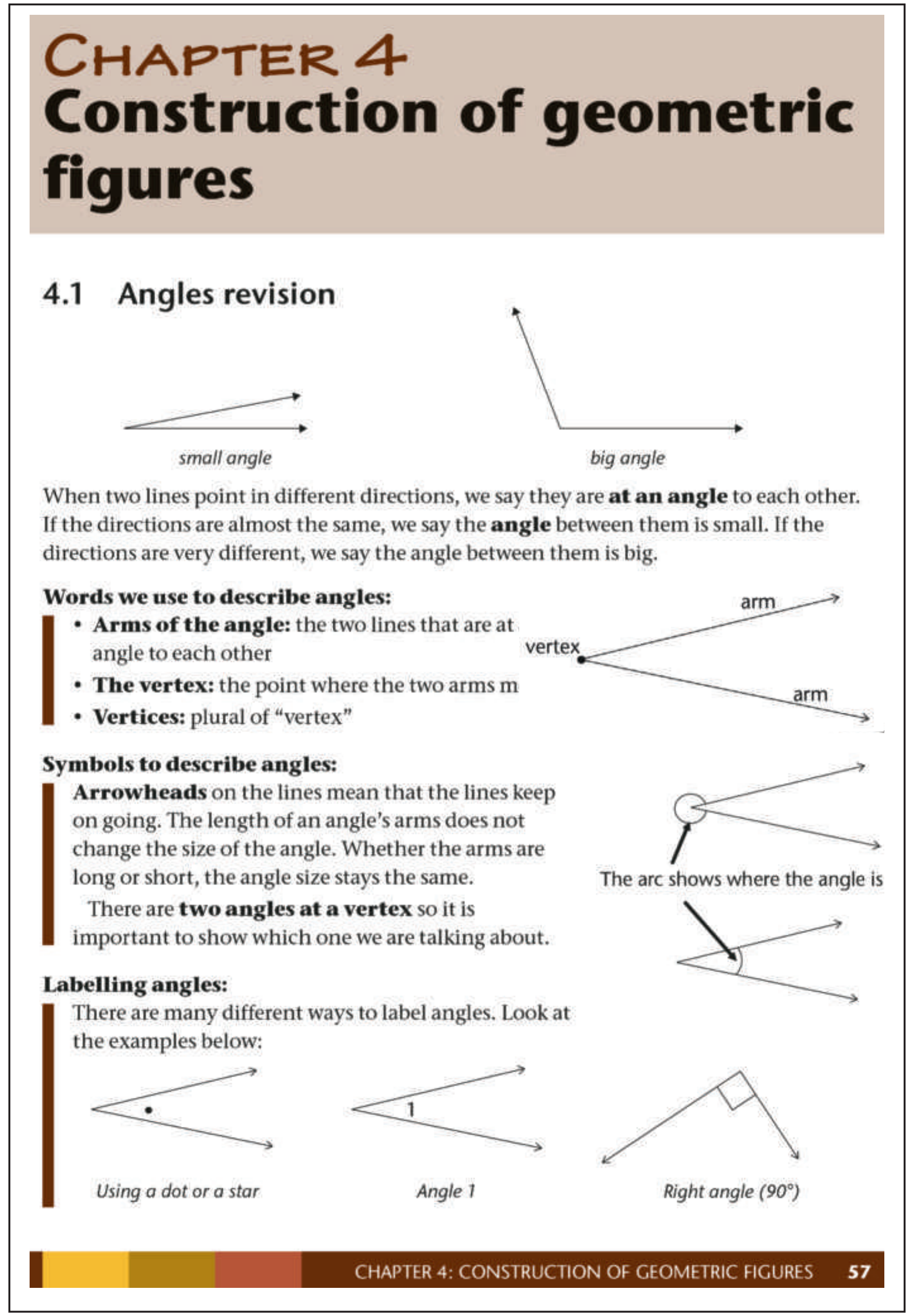
Show learners how to draw and label an angle on paper and how to write the name of the angle. Point out that using the "hat" label is convenient, but if there is more than one angle at a vertex, care should be taken to write the name of the angle in full. For example, at the vertex A in the drawing alongside, we find three different angles, $B\hat{A}C$, $C\hat{A}D$ and $B\hat{A}D$. In this case we avoid confusion by writing the name of the angle we mean to work with in full.



Misconceptions

Learners could think that an angle that takes up more space (B) is larger than the same size angle that takes up less space (A). Show learners that the amount of turn about the vertex is the same in both cases and explain that the length of the arms does not change the size of the angle.





CHAPTER 4
Construction of geometric figures

4.1 Angles revision

small angle *big angle*

When two lines point in different directions, we say they are **at an angle** to each other. If the directions are almost the same, we say the **angle** between them is small. If the directions are very different, we say the angle between them is big.

Words we use to describe angles:

- **Arms of the angle:** the two lines that are at angle to each other
- **The vertex:** the point where the two arms m
- **Vertices:** plural of "vertex"

arm vertex arm

Symbols to describe angles:

Arrowheads on the lines mean that the lines keep on going. The length of an angle's arms does not change the size of the angle. Whether the arms are long or short, the angle size stays the same.

There are **two angles at a vertex** so it is important to show which one we are talking about.

The arc shows where the angle is

Labelling angles:

There are many different ways to label angles. Look at the examples below:

Using a dot or a star *Angle 1* *Right angle (90°)*

CHAPTER 4: CONSTRUCTION OF GEOMETRIC FIGURES 57

REVISION: SEEING ANGLES AND DESCRIBING ANGLES

Teaching guidelines

Check that learners keep their rulers on the lines when making the sides of an angle longer (producing lines). Explain that being slightly off the line when you start can make a big difference when the line is lengthened and then measured.

Compare the learners' answers to questions 1 and 5. If the two lines in question 5 are moved towards each other, they will have no angle between them. As they are drawn on either side of a ruler, we would expect them to be parallel. Let learners investigate.

Use question 6 to re-enforce the concept of an angle as an amount of turning about a point. Draw a circle on the board and complete a clock face like the one on LB page 59. Use pencils (or any other suitable instrument) to show how the hands on the clock move and form angles.

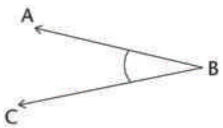
Misconceptions

Learners might think that the angles are the same for questions like 6 (c). Let them discuss in pairs the movement of the hour hand and the movement of the minute hand and how the corresponding angles compare in cases like these, for example quarter to twelve and 9 o'clock; 4 o'clock and twenty past 12, etc.

Answers

1. (a) Yes. No, they do not have to meet.
   (b) No
2. (d), (e), (c), (b), (a), (f)
3. You can place the corner of any square or rectangular object from everyday life on the angle, for example a sheet of paper, a book, ruler, small box or pencil sharpener.
4. They are the same size. Learners will use different methods, such as paper folding, tracing, or drawing and cutting out.
5. The lines are parallel. They have the same direction.
6. (a) The clock face is divided into 12 equal sections (30° each). Both angles are four sections big (or $4 \times 30° = 120°$).
   (b) The angle at 2 o'clock is half the size of the angle at 4 o'clock. At 4 o'clock the minute hand has moved twice as far away from the 12 than at 2 o'clock.
   (c) No, although the hour hand and the minute hand basically switch places, the hour hand is not pointing exactly at the 12, but to the right of the 12.

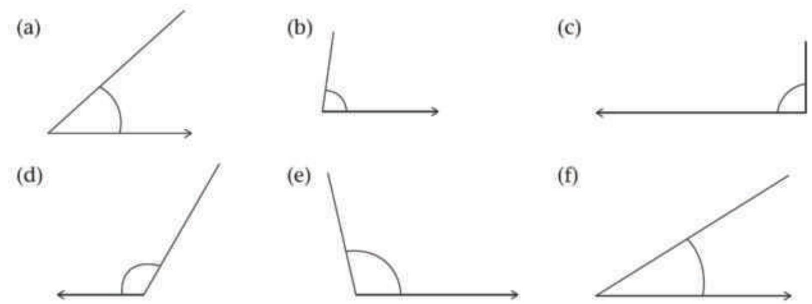
You can name the angle on the right in different ways: you can say $A\hat{B}C$ or $C\hat{B}A$ or just $\hat{B}$. The "hat" on the letter shows where the angle is.



REVISION: SEEING ANGLES AND DESCRIBING ANGLES

1. Look at the drawing on the right.
   (a) Are these lines at an angle to each other? Do the lines have to meet to be at an angle?
   (b) Copy the lines. Use a pencil and your ruler to draw the lines a bit longer so they meet. Did you change the angle between the lines when you extended them?

2. Arrange the angles from biggest to smallest. Just write the letters (a) to (f) in the correct order.



3. How can you check that an angle is a right angle without using any special mathematics equipment? (*Hint*: Think about where you can find right angles around you.)
4. Are these two angles the same size? Describe how you found your answer. (*Hint*: A piece of scrap paper may help!)

5. Two lines are drawn by holding down a ruler and drawing lines on both sides. What can you say about the two lines?

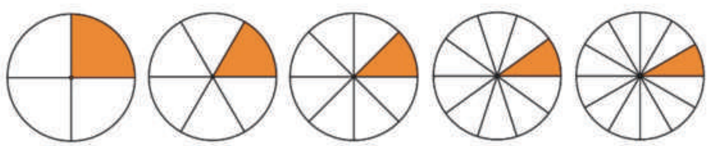
6. Look at the analogue clock face on the next page. The minute hand and the hour hand make an angle. Focus on the smaller angle for now.

7. Sample answer:
   Door: arms: door and doorframe; vertex: at the hinges.
   Laptop computer: arms: screen and keyboard; vertex: where they are joined.
   CD cover: arms: front part and tray; vertex: where they fit.
   Arm and body: arms: arm and body; vertex: shoulder.
   Your arm: arms: upper and lower arm; vertex: elbow.

4.2 The degree: a unit for measuring angles

Teaching guidelines

Discuss the need for a unit of measurement as discussed on LB page 59. Explain that, just as a unit to measure length has to be a length, so a unit to measure angles has to be an angle. Learners can imagine covering an angle with unit angles in the same way one covers an area with unit squares to determine the area. They can think of these unit angles as wedges. In each drawing below the wedges that fit around the point have the same size. The size of the angle of the wedge at the centre of the first drawing is a right angle. As the wedges that fit around a point become smaller, more of them are needed.



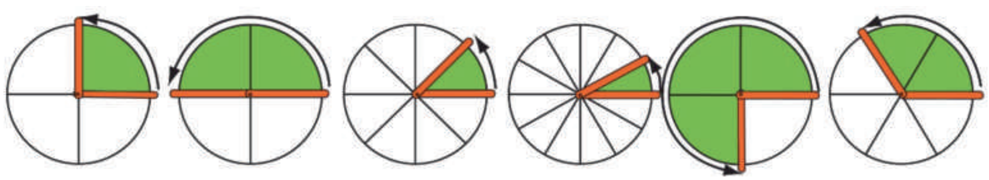
If we fit 360 equal wedges about a point, each tile or wedge has an angle of one degree. We can use degrees to measure the size of any angle.

SOME FAMILIAR ANGLES IN DEGREES

Teaching guidelines

Help learners to understand the angles as fractions of a revolution. For example, a quarter turn is $\frac{1}{4}$ of 360°. Draw a circle on the board to represent a revolution.

Draw lines on the circle to represent the fractions of the revolution similar to the ones shown below.



(a) Explain why the angle between the hands at 8 o' clock is the same size as the angle at 4 o' clock.
(b) Compare the angle at 2 o' clock with the angle at 4 o' clock. What do you notice? Why is this so?
(c) Is the angle at 3 o' clock the same as the angle at a quarter past 12? Explain.

7. When you open the cover of a hardcover book you can make different angles. Can you think of at least five other situations in everyday life where objects are turned through angles? Say what the arms and the vertices are in each of your examples.

4.2 The degree: a unit for measuring angles

Imagine if we didn't have units for measuring length. *How would tailors make clothes to the right size without a tape measure? How could an architect design a safe and beautiful house without a ruler? How could we lay out a professional soccer field without being able to measure accurately in metres?*

We need units and measuring instruments in many situations. You know that we use metres, centimetres, kilometres, millimetres, etc. for measuring lengths.

We should also have units for measuring angles. The units we use for measuring angles are very ancient. No one today is completely sure why, but our ancestors decided many thousands of years ago that a revolution should be divided into 360 equal parts. We call these parts degrees. The symbol for a degree is °.

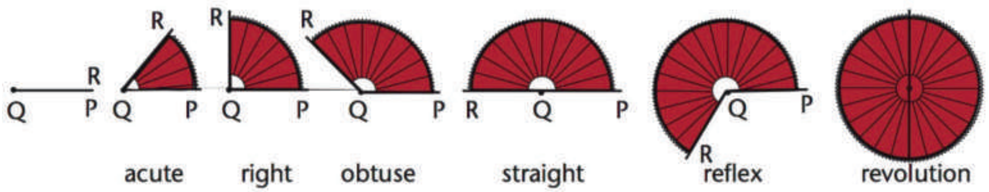
SOME FAMILIAR ANGLES IN DEGREES

1. Copy and complete the table by filling in the size of each angle described.

Angle (in words)	Angle (degrees)
right angle	90°
straight angle	180°
revolution	360°
half a right angle	45°
a third of a right angle	30°
a quarter of a right angle	22,5°
half a straight angle	90°
three quarters of a revolution	270°
a third of a revolution	120°

Answers

1. See LB page 59 on previous page.
2. (a) 360° (b) 30° (360 ÷ 12)
3. See LB page 60 alongside. If you can find a Chinese fan, it can add interest to show learners the fan opened in different positions to illustrate the types of angles they have to know, or use drawings like those below to remind learners of the angles they worked with in the previous grade. Show them two different positions to illustrate a reflex angle; one between 180° and 270° and one between 270° and 360°.



COMPARING ANGLES USING A4 PAPER

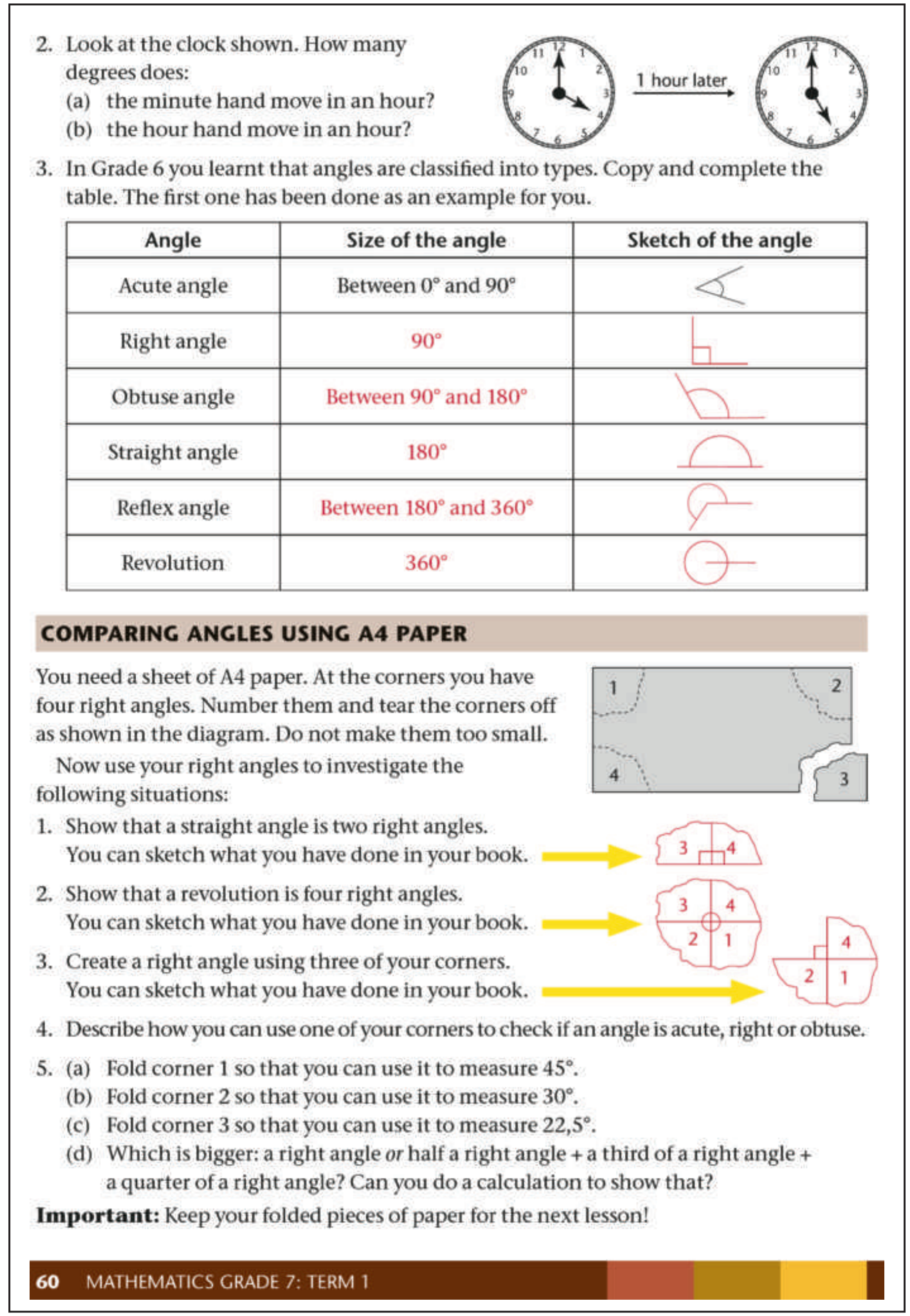
Teaching guidelines

Learners' understanding of a situation is increased when they have hands-on experiences. Some of the questions require learners to think laterally. Allow learners to experiment before giving solutions. Circulate amongst the learners to see if they grasp what they need to do.

Answers

1. See LB page 60 alongside.
2. See LB page 60 alongside.
3. See LB page 60 alongside.
4. The corner is a right angle (90°). Put the corner on an angle. If it matches the angle perfectly, the angle is a right angle. If the corner is bigger than the angle, then the angle is smaller than 90° and therefore acute. If the corner is smaller than the angle, then the angle is bigger than 90° and therefore obtuse.
5. (d) $\frac{1}{2} \times 90° + \frac{1}{3} \times 90° + \frac{1}{4} \times 90° = 45° + 30° + 22{,}5° = 97{,}5°$. So, $97{,}5° > 90°$.

 The second angle is bigger.



2. Look at the clock shown. How many degrees does:
 (a) the minute hand move in an hour?
 (b) the hour hand move in an hour?

1 hour later

3. In Grade 6 you learnt that angles are classified into types. Copy and complete the table. The first one has been done as an example for you.

Angle	Size of the angle	Sketch of the angle
Acute angle	Between 0° and 90°	
Right angle	90°	
Obtuse angle	Between 90° and 180°	
Straight angle	180°	
Reflex angle	Between 180° and 360°	
Revolution	360°	

COMPARING ANGLES USING A4 PAPER

You need a sheet of A4 paper. At the corners you have four right angles. Number them and tear the corners off as shown in the diagram. Do not make them too small.

Now use your right angles to investigate the following situations:

1. Show that a straight angle is two right angles.
 You can sketch what you have done in your book.
2. Show that a revolution is four right angles.
 You can sketch what you have done in your book.
3. Create a right angle using three of your corners.
 You can sketch what you have done in your book.
4. Describe how you can use one of your corners to check if an angle is acute, right or obtuse.
5. (a) Fold corner 1 so that you can use it to measure 45°.
 (b) Fold corner 2 so that you can use it to measure 30°.
 (c) Fold corner 3 so that you can use it to measure 22,5°.
 (d) Which is bigger: a right angle *or* half a right angle + a third of a right angle + a quarter of a right angle? Can you do a calculation to show that?

Important: Keep your folded pieces of paper for the next lesson!

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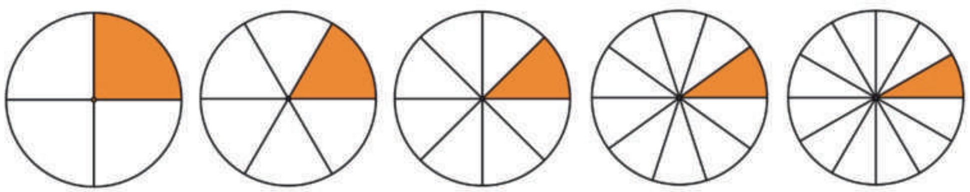
4.3 Using the protractor

Background

Many learners seem to struggle when using a protractor for the first time. For one thing, the shape is strange; furthermore, the measuring unit is so small that it would be near impossible to cut out a wedge or tile of one degree and to tile the angle to measure it. Also, there are no visible angles on the protractor and it has only two rows of small marks along the outer edge, which makes it confusing. Remind learners that angles are measured by the number of degrees of a circle that are between the arms of the angle, which explains why the protractor has the shape of a semi-circle. The protractors learners usually work with are semi-circles, but there are protractors that are a full circle.

We can help learners to understand by starting out with a protractor with a large unit angle. We soon notice that the unit needs to be smaller for the measurement to be more accurate.



(The Background section and Teaching guidelines are continued on TG page 69.)

MEASURING SOME FAMILIAR ANGLES

Teaching guidelines

Give learners the opportunity to work together and help each other learn how to handle the protractor.

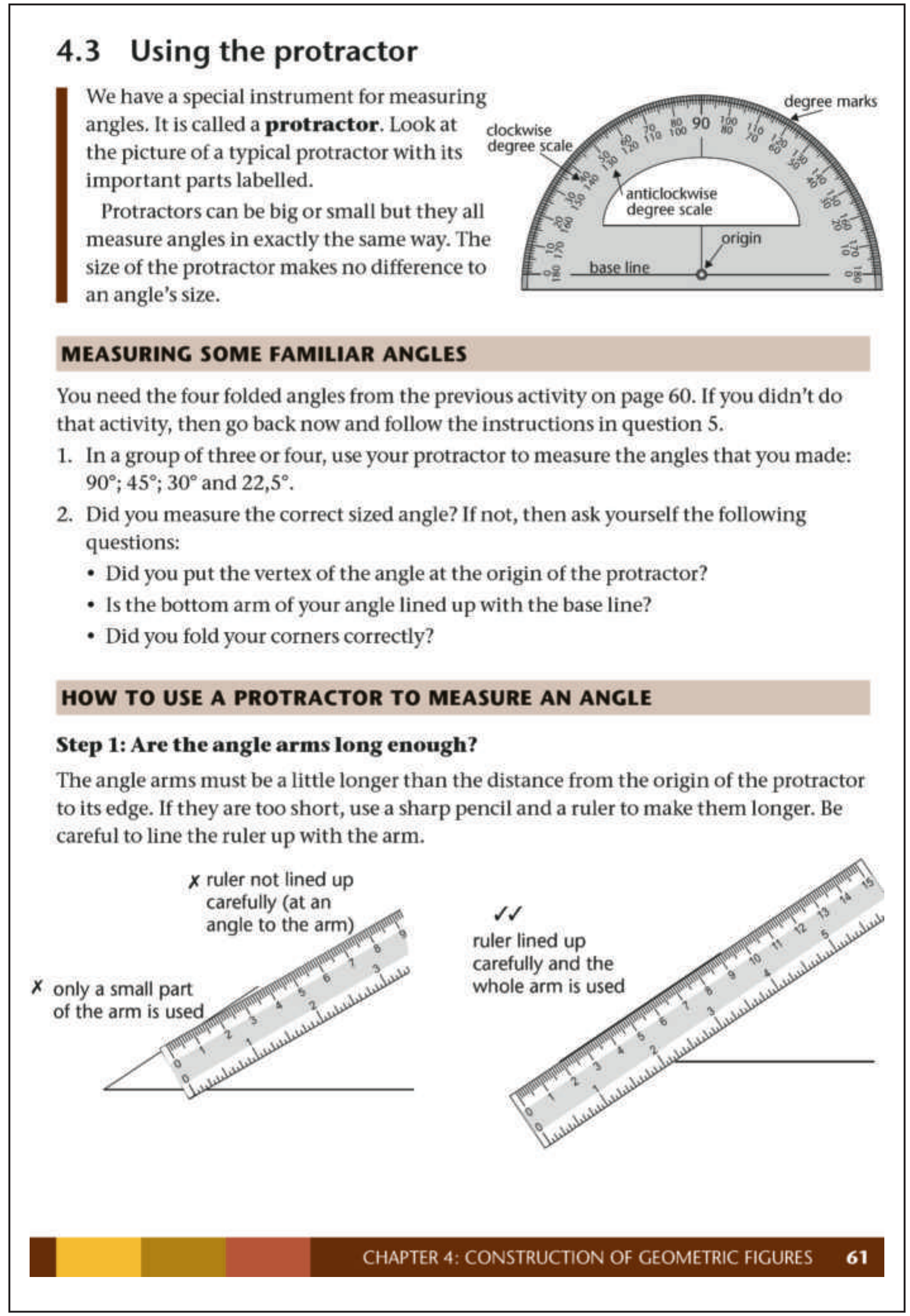
Let them discuss in their group the problems they experience and how to correct such problems or mistakes.

HOW TO USE A PROTRACTOR TO MEASURE AN ANGLE

Teaching guidelines

Work carefully through the instructions in the Learner Book, paying attention to the correct way to produce the arms: to line up the angle on the protractor.

Draw examples on the board and let learners draw and measure angles that have to be read in both a clockwise direction on the protractor and in an anti-clockwise direction.



4.3 Using the protractor

We have a special instrument for measuring angles. It is called a **protractor**. Look at the picture of a typical protractor with its important parts labelled.

Protractors can be big or small but they all measure angles in exactly the same way. The size of the protractor makes no difference to an angle's size.

MEASURING SOME FAMILIAR ANGLES

You need the four folded angles from the previous activity on page 60. If you didn't do that activity, then go back now and follow the instructions in question 5.

1. In a group of three or four, use your protractor to measure the angles that you made: 90°; 45°; 30° and 22,5°.
2. Did you measure the correct sized angle? If not, then ask yourself the following questions:
   - Did you put the vertex of the angle at the origin of the protractor?
   - Is the bottom arm of your angle lined up with the base line?
   - Did you fold your corners correctly?

HOW TO USE A PROTRACTOR TO MEASURE AN ANGLE

Step 1: Are the angle arms long enough?

The angle arms must be a little longer than the distance from the origin of the protractor to its edge. If they are too short, use a sharp pencil and a ruler to make them longer. Be careful to line the ruler up with the arm.

CHAPTER 4: CONSTRUCTION OF GEOMETRIC FIGURES 61

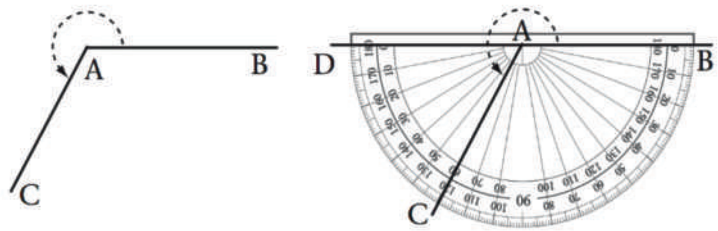
Let learners draw and measure angles and learn the range of the different types of angles, for example: obtuse angles are larger than 90° but smaller than 180°.

The examples in the Learner Book show angles with one side horizontal. Let learners draw and measure angles with one side vertical as well as angles with sides that are not horizontal or vertical. Illustrate a few such angles on the board.

Show them how to place the centre of the protractor on the vertex of the angle and then to pivot the protractor about that point until one of the sides lies on the side of the angle. Let learners discuss in pairs when to use the inner scale and when to use the outer scale. Draw some angles on the board and let learners decide which scale would be used to get the correct measurement of the angle.

Let learners work through the steps to measure the angles. Circulate to see if learners understand the instructions and can manage.

Suggest a plan to learners who struggle when they have to measure reflex angles. For example, get them to measure the size of reflex ∠BAC below:



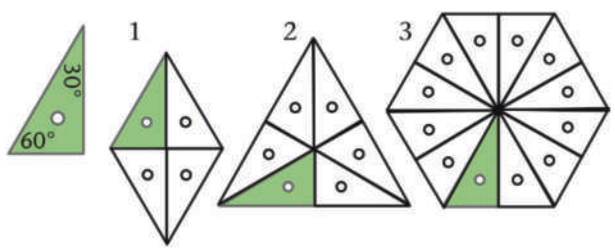
- Lengthen side BA to D.
- We know ∠BAD = 180°, so we need to measure ∠DAC and add it to 180°.
- We swing the protractor so that we can measure ∠DAC = 60°.
- We get ∠BAC = 180° + 60° = 240°.

Or: Measure the obtuse ∠BAC (120^o) and subtract it from 360°.

In either case, think about which scale to use.

Background and Teaching guidelines (continued from TG page 68)

Explain the idea of a measuring unit to learners by tiling about a point with the 60° set square. There are three possibilities to tile a revolution with the 60° set square, as shown in the drawings alongside.



Repeat the process with the 45° set square.

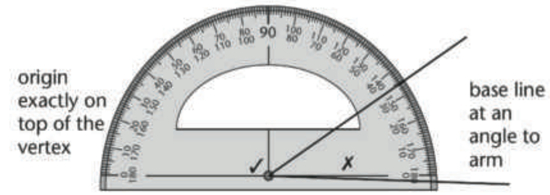
Now you are ready to start measuring your angle.

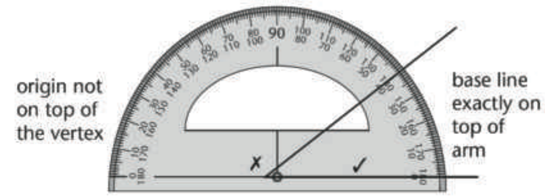
Step 2: Line up the angle and your protractor

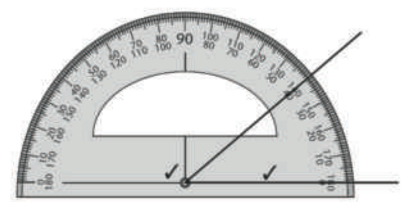
Place your protractor on top of the angle. Make sure of the following:

- the origin is exactly on the vertex of the angle, and
- the base line is exactly on top of one of the arms of the angle.

Keep adjusting the position of the protractor until the origin and the base line are exactly lined up.







origin exactly on top of the vertex and base line exactly on top of arm

Once your protractor is in the correct place, keep a finger on the protractor to stop it from moving. If it moves... start again! You are now ready to make a measurement.

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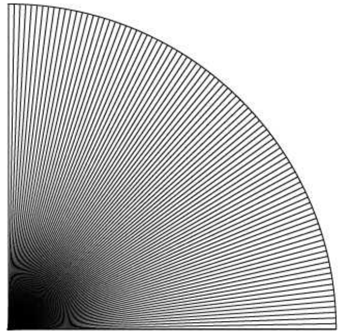
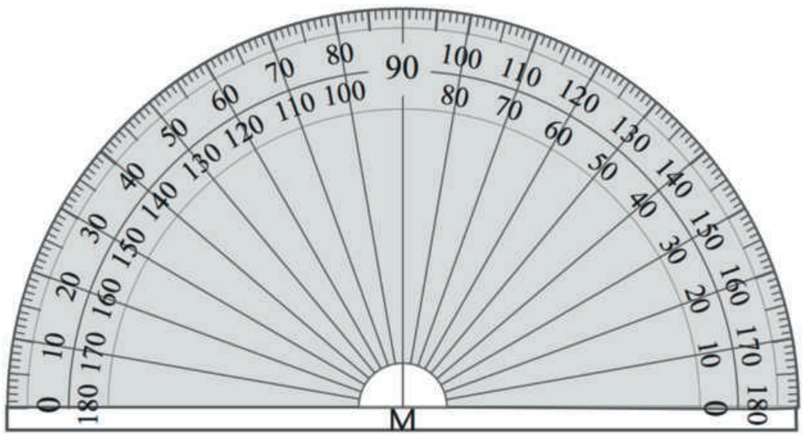
These tiles fit around the point but are only useful to measure certain angles. Refer to the circles on TG page 68.

A protractor represents 180 identical angles about the centre. Each small angle represents one degree. The drawing alongside shows half of a protractor with 90 tiles, each one degree. It is almost impossible to read accurately. This shows why a protractor has only the small marks at the outer edge or lines only at some angles.



The protractor shown here is commonly found in the school mathematical sets. Explain that the common vertex, M, of all the angles on the protractor is called the origin (or centre) of the protractor. Explain how the scales work.



PRACTISE MEASURING WITH A PROTRACTOR

Misconceptions

Learners use the wrong scale. They measure an acute angle using the wrong scale and the number of degrees they read off suggests an obtuse angle.

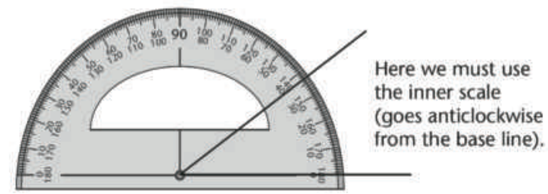
Circulate to make sure that learners can correct such mistakes and understand why it is not correct.

Answers

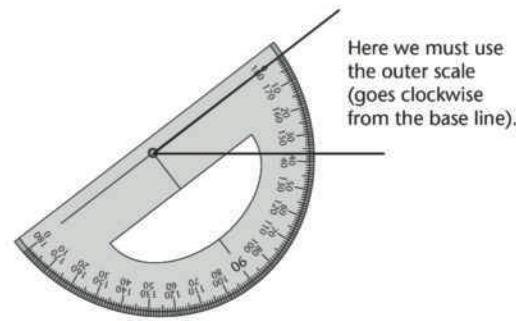
1. See LB page 64 on following page.

Step 3: Measure the angle

A protractor gives a clockwise degree scale and an anticlockwise degree scale. You choose the correct scale by finding the one that starts with 0° on the angle arm. Look at where the other angle arm passes under the degree scale. That is where your measurement is.



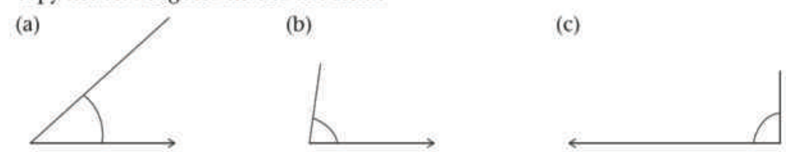
You can also place the protractor on the angle using the other arm. Then the correct position looks like this:



The angle in the pictures above is 37°. Do you agree? Do you see that there are two ways to measure an angle?

PRACTISE MEASURING WITH A PROTRACTOR

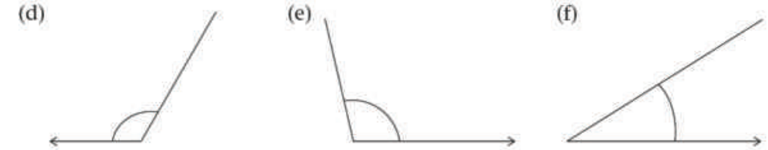
1. Measure the angles and copy and complete the table on the next page. If you need to, copy the drawing and extend the arms.



2. See LB page 64 alongside.
3. You can measure the angle that is on the other side and subtract its size from 360°. Or you can extend one of the arms, so that it cuts the angle into two parts. One part will be 180°. Measure the other (smaller) part of the angle and add it to 180° (see the Teaching guidelines on the previous page).

SOME THINGS TO THINK ABOUT

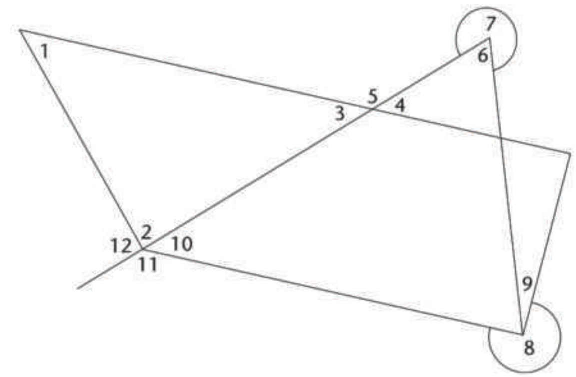
1. They are the same size (45°). We can work out why they should be the same as follows:
   angle 5 + angle 4 = straight angle = 180° and angle 5 + angle 3 = 180° and as angle 5 is common in the two statements, angle 3 = angle 4.
2. They add up to 360°. They form a revolution.
3. They add up to 180°. They form a straight angle.



Angle	(a)	(b)	(c)	(d)	(e)	(f)
Angle size in degrees	43°	83°	90°	120°	103°	32°

2. Copy the table below. Measure all the numbered angles in the following figure. Some angles can be measured directly, others not. Your protractor cannot measure reflex angles like angles 7 and 8. So you will have to make a plan!

Angle	Size
1	48°
2	88°
3	45°
4	45°
5	135°
6	65°
7	295°
8	268°
9	21°
10	44°
11	136°
12	92°



3. Write a short note for yourself about measuring reflex angles.

SOME THINGS TO THINK ABOUT

Look at your answers in question 2 above.

1. How do angles 3 and 4 compare?
2. What about angles 6 and 7?
3. What about angles 4 and 5?
4. There are some interesting ideas here. Try to do some further investigation and show your teacher what you discover.

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4.4 Using a protractor to construct angles

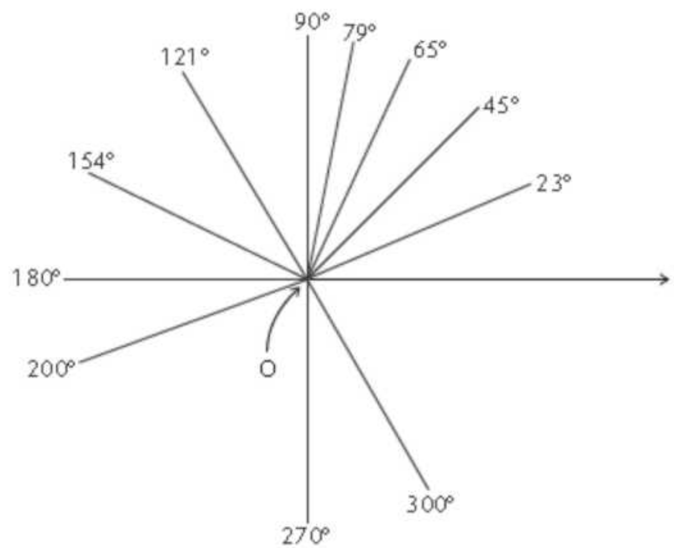
CONSTRUCTING ANGLES TO A GIVEN LINE

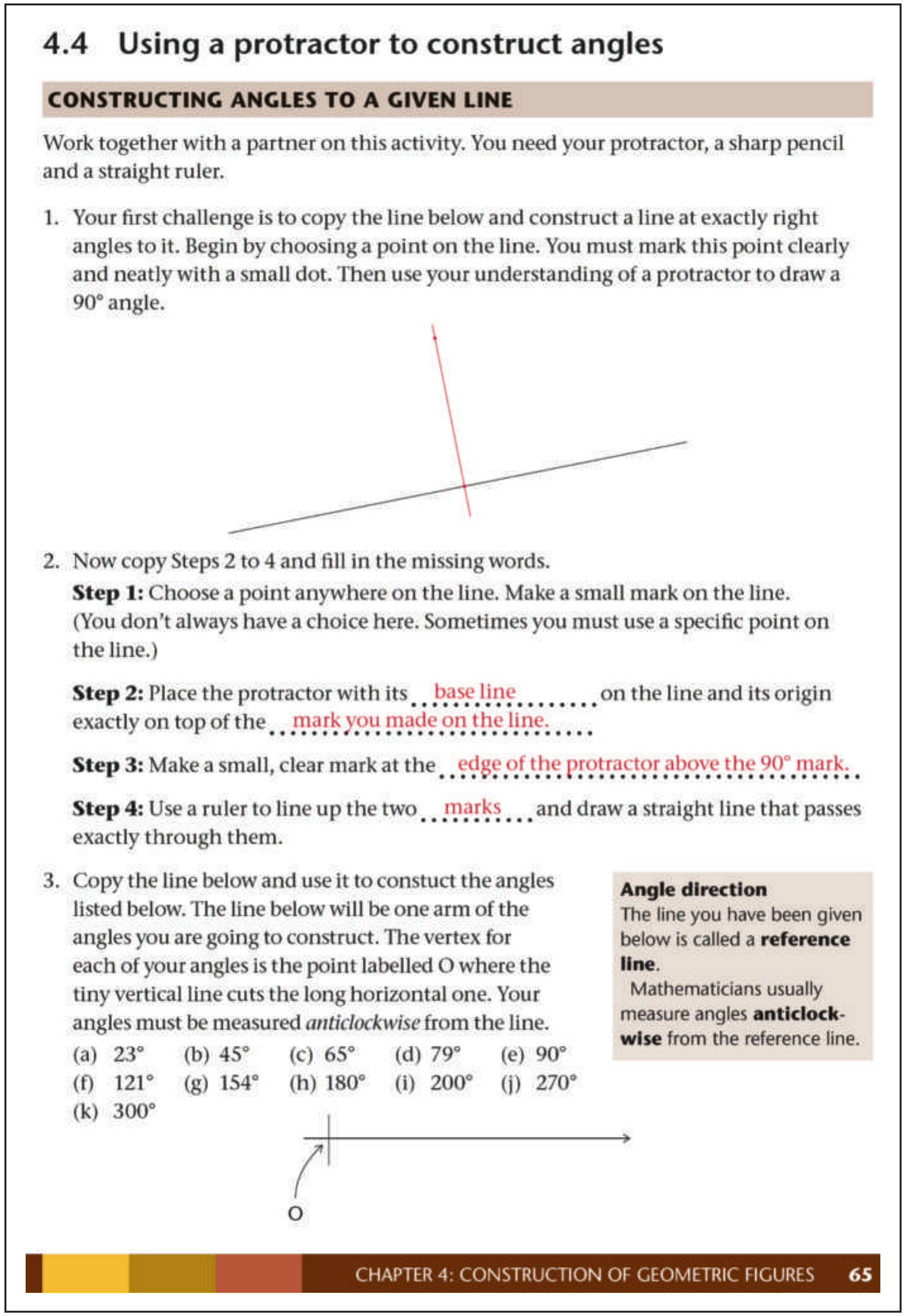
Teaching guidelines

Explain that measuring in an anti-clockwise direction on a reference line means using the inner scale on the protractor.

Answers

1. See LB page 65 alongside.
2. See LB page 65 alongside.
3.





4.4 Using a protractor to construct angles

CONSTRUCTING ANGLES TO A GIVEN LINE

Work together with a partner on this activity. You need your protractor, a sharp pencil and a straight ruler.

1. Your first challenge is to copy the line below and construct a line at exactly right angles to it. Begin by choosing a point on the line. You must mark this point clearly and neatly with a small dot. Then use your understanding of a protractor to draw a 90° angle.

2. Now copy Steps 2 to 4 and fill in the missing words.

Step 1: Choose a point anywhere on the line. Make a small mark on the line. (You don't always have a choice here. Sometimes you must use a specific point on the line.)

Step 2: Place the protractor with its ..base line.......... on the line and its origin exactly on top of the ..mark you made on the line.....

Step 3: Make a small, clear mark at the ..edge of the protractor above the 90° mark..

Step 4: Use a ruler to line up the two ..marks... and draw a straight line that passes exactly through them.

3. Copy the line below and use it to constuct the angles listed below. The line below will be one arm of the angles you are going to construct. The vertex for each of your angles is the point labelled O where the tiny vertical line cuts the long horizontal one. Your angles must be measured *anticlockwise* from the line.

(a) 23° (b) 45° (c) 65° (d) 79° (e) 90°
(f) 121° (g) 154° (h) 180° (i) 200° (j) 270°
(k) 300°

O

Angle direction
The line you have been given below is called a **reference line**.
Mathematicians usually measure angles **anticlockwise** from the reference line.

CHAPTER 4: CONSTRUCTION OF GEOMETRIC FIGURES 65

4. See LB page 66 alongside. It is an equilateral triangle
5. See LB page 66 alongside.

4.5 Parallel and perpendicular lines

Notes

Learners worked with parallel and perpendicular lines in the previous chapter. Work through the revision in the Learner Book and make sure that learners understand how to:

- recognise perpendicular and parallel lines
- write that lines are perpendicular or parallel
- show in a drawing that two lines are meant to be perpendicular or parallel.

CONSTRUCTING PERPENDICULAR AND PARALLEL LINES

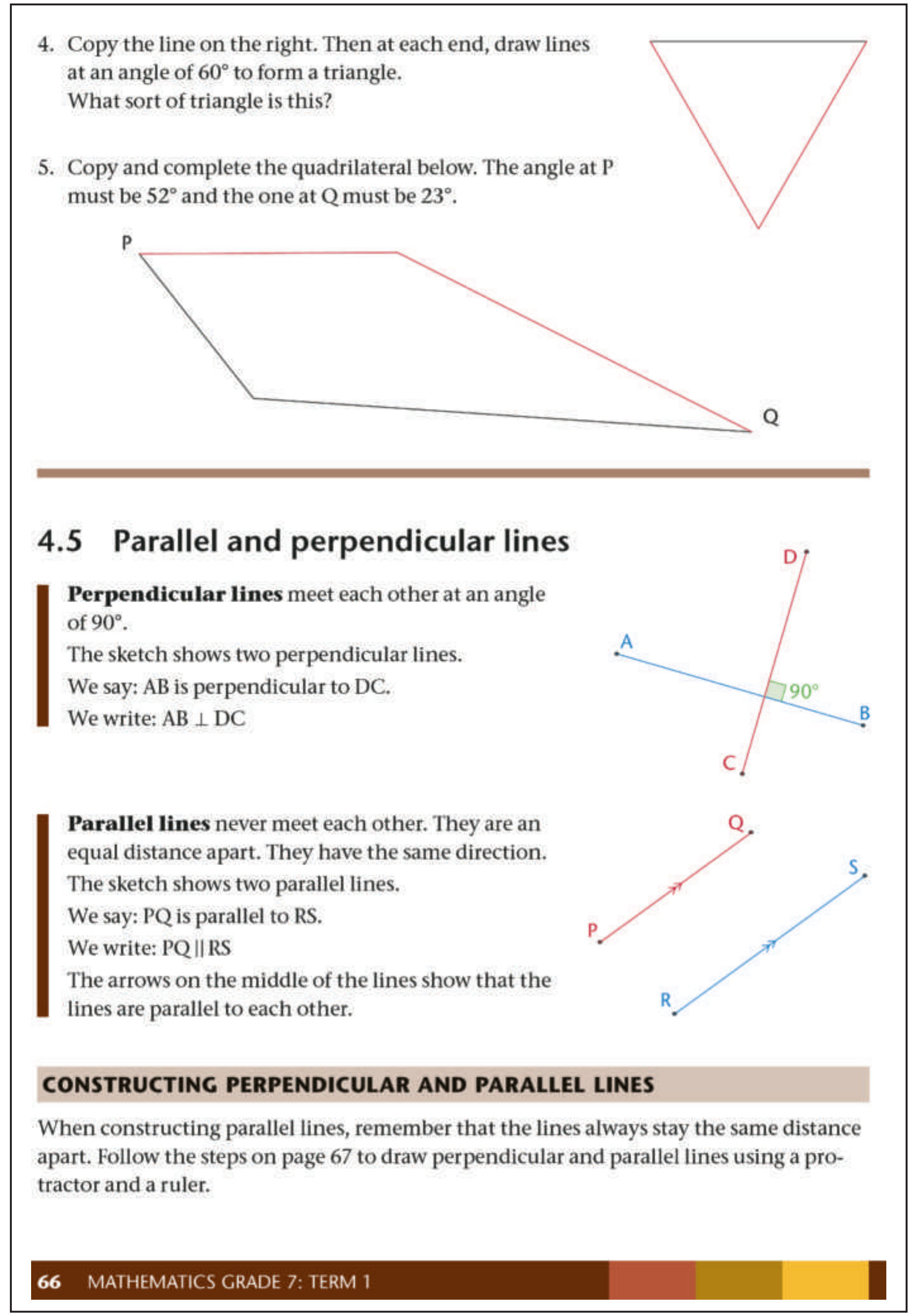
Background

In this section the emphasis is on using the protractor to construct perpendicular and parallel lines, but there are other ways of constructing these lines. Some of these methods are shown below the answers in this section.

Teaching guidelines

Work through the methods explained in the Learner Book so that learners practise using the protractor.

You could use the additional methods mentioned shown below to provide extra exercises for learners who need extension exercises.


4. Copy the line on the right. Then at each end, draw lines at an angle of 60° to form a triangle. What sort of triangle is this?

5. Copy and complete the quadrilateral below. The angle at P must be 52° and the one at Q must be 23°.

4.5 Parallel and perpendicular lines

Perpendicular lines meet each other at an angle of 90°.
The sketch shows two perpendicular lines.
We say: AB is perpendicular to DC.
We write: $AB \perp DC$

Parallel lines never meet each other. They are an equal distance apart. They have the same direction.
The sketch shows two parallel lines.
We say: PQ is parallel to RS.
We write: $PQ \parallel RS$
The arrows on the middle of the lines show that the lines are parallel to each other.

CONSTRUCTING PERPENDICULAR AND PARALLEL LINES

When constructing parallel lines, remember that the lines always stay the same distance apart. Follow the steps on page 67 to draw perpendicular and parallel lines using a protractor and a ruler.

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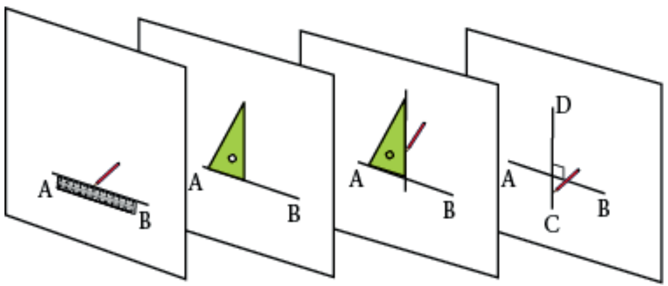
Answers

1. See steps 1–4 in the Learner Book. Step 4 (on LB page 68) shows the final construction drawing.
   Length of AC: 3,5 cm (Step 2).

Additional background (continued from TG page 73)

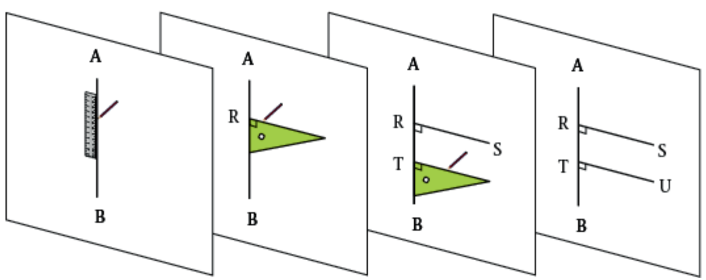
Follow the drawings below for an alternative way to draw a line perpendicular to a reference line:

- Draw reference line AB about 7 cm long and mark a point 4 cm from B.
- Place the edge of the set square on AB at the marked point.
- Draw a line along the upright edge of the set square.
- Remove the set square; lengthen the line.
- Mark the right angle where the lines cross with a small square.

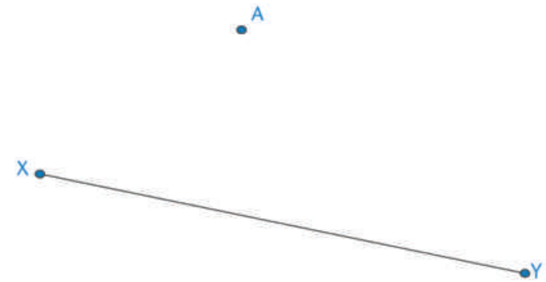


Follow the drawings below for an alternative way to draw parallel lines:

- Draw line AB about 7 cm long and mark a point R 4 cm from B.
- Place the short edge of the set square on AB at R and draw line RS perpendicular to AB.
- Move the set square to T and draw TU || RS.



1. We want to draw a line that is parallel to XY and that passes through point A.

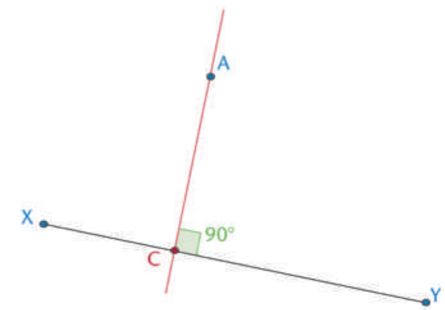


Step 1: Draw a perpendicular line between A and XY.

Copy the line XY above. Use your protractor to draw a line that goes through A and is at 90° to XY.

Label the point C where your new line touches XY.

Look at the sketch on the right if you get stuck.

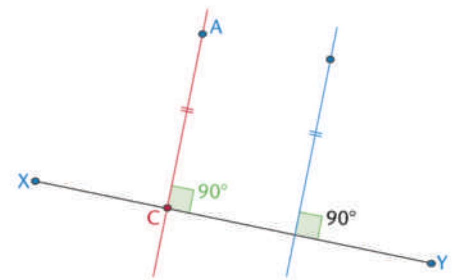


Step 2: Measure the perpendicular distance between the point and the line.

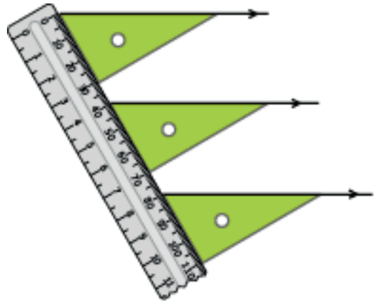
Write down the length of AC.

Step 3: Draw a point that is the same distance from the line.

Then draw another line that is perpendicular to line XY. Mark off the same length as AC on that line. The sketch below shows what you must do.



- Remove the set square. Lines can be drawn parallel to each other along a slanted reference line as illustrated in the drawing.



4.6 Circles are very special figures

Notes

It is useful for learners to draw circles as described on LB page 68. This method can be used when a gardener wants to make a circular flower bed. This would be done by knocking a stick or batten into the ground with a hammer; tying string around it and attaching another stick to the other end, pulling it taught and drawing the second stick along the ground to complete a circle.

A CIRCLE WITH STRING

Misconceptions

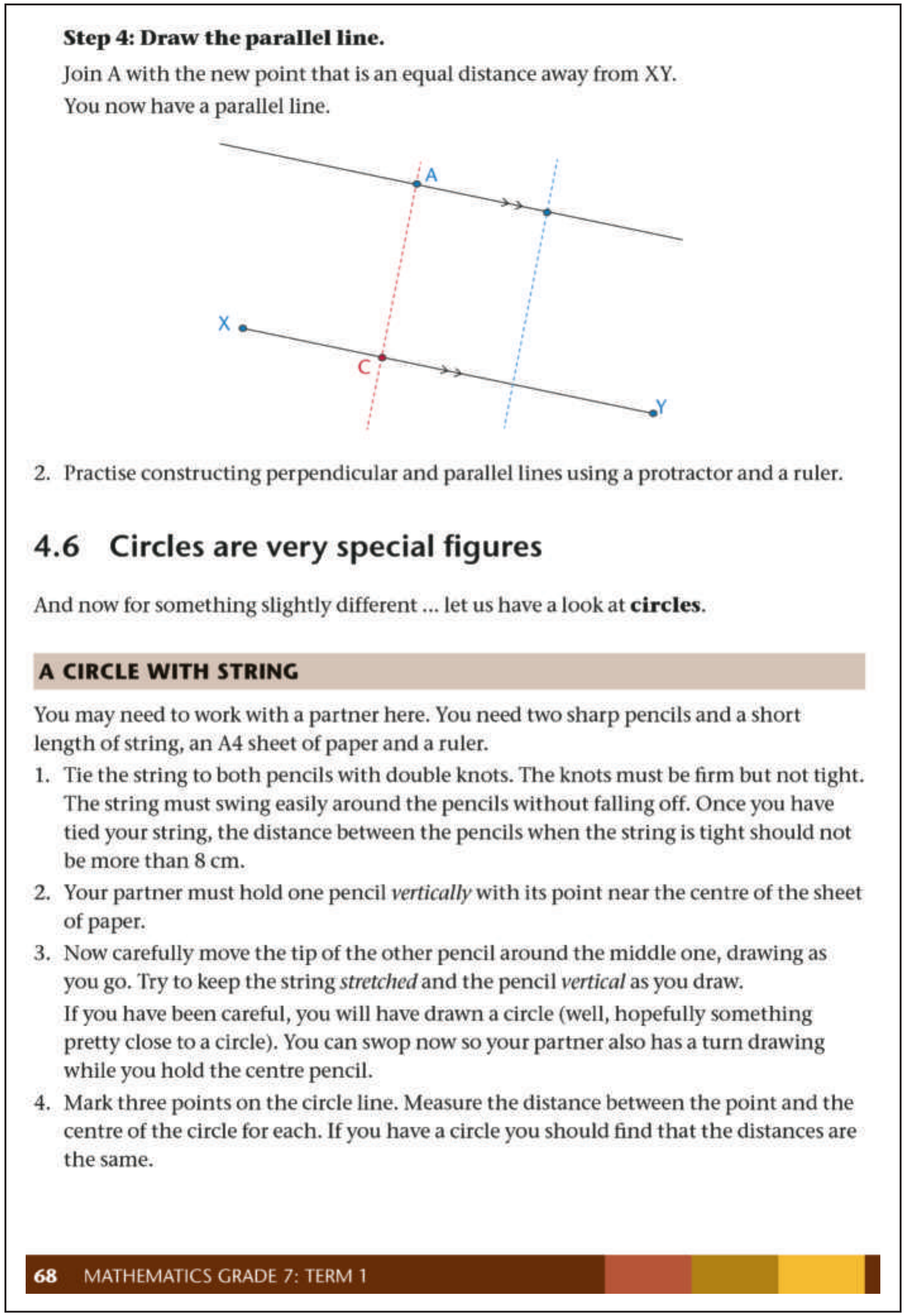
Learners need to be quite clear about the difference between the radius and the diameter of a circle. Learners often confuse the two. For example, if they want a circle that is two metres across, they must have a radius of one metre.

Learners sometimes do not realise that a diameter goes through the centre of the circle, so they would draw a chord and call it a diameter.

Teaching guidelines

Learners concentrate so much on managing the activity, that they might not realise the significance of the end result. Therefore, pay special attention to question 4 with a discussion about the fact that any point you choose on the circle that you drew, is the same distance from the centre of the circle as any other point.

Make sure learners understand the difference between the radius and the diameter. Make sure learners realise that a diameter is a line that runs through the centre of the circle, point to point. It may help to show two radii that are in opposite directions.



Step 4: Draw the parallel line.

Join A with the new point that is an equal distance away from XY.

You now have a parallel line.

2. Practise constructing perpendicular and parallel lines using a protractor and a ruler.

4.6 Circles are very special figures

And now for something slightly different ... let us have a look at **circles**.

A CIRCLE WITH STRING

You may need to work with a partner here. You need two sharp pencils and a short length of string, an A4 sheet of paper and a ruler.

1. Tie the string to both pencils with double knots. The knots must be firm but not tight. The string must swing easily around the pencils without falling off. Once you have tied your string, the distance between the pencils when the string is tight should not be more than 8 cm.
2. Your partner must hold one pencil *vertically* with its point near the centre of the sheet of paper.
3. Now carefully move the tip of the other pencil around the middle one, drawing as you go. Try to keep the string *stretched* and the pencil *vertical* as you draw.

   If you have been careful, you will have drawn a circle (well, hopefully something pretty close to a circle). You can swop now so your partner also has a turn drawing while you hold the centre pencil.
4. Mark three points on the circle line. Measure the distance between the point and the centre of the circle for each. If you have a circle you should find that the distances are the same.

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Discuss with learners that using the string-and-pencils method has its drawbacks.

Make sure that all learners are familiar with the words we use and their pronunciation.

Discuss the points in the Think about it box. The distance from the centre is not the same in all directions in any of the shapes mentioned.

4.7 Using the compass

Teaching guidelines

Learning to use a compass needs practice. Work through the description of the compass and demonstrate how to draw a circle using it. Let learners practise drawing circles and circular patterns to learn how to handle the compass.

Let learners practise measuring a radius accurately on a ruler. (See step 1 on LB page 70).

As well as drawing full circles, learners must also practise making small marks with the pencil of a compass, for example to mark off points on the circumference of a circle.

In conclusion of this section, work through the tips for drawing circles on LB pages 70 and 71.

Circles are special for many reasons. The most important reason is the following:

The distance from the centre of a circle to any point on the circle is the same.

This distance is called the **radius**.

We pronounce this: "ray-dee-us".

The plural of **radius** is **radii**.

We pronounce this: "ray-dee-eye".

Think about it

Can you think of any other figure where the distance between the centre and the edge is constant in all directions?

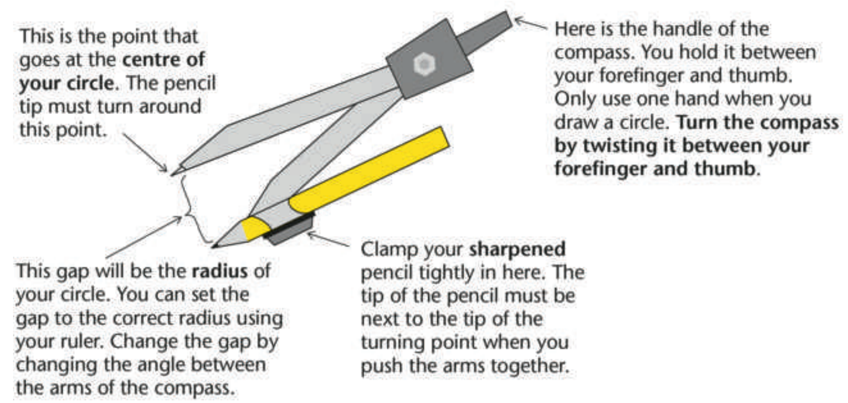
- A square?
- A hexagon?
- What about an oval shape (ellipse)?

Do some investigation to see what you can find.

Do you agree that the two pencils and string are not a good way to draw circles? The string is stretchy. It is difficult to change the radius. And, the drawing pencil can wander off course and make a spiral or a wobbly curve. We need something better.

4.7 Using the compass

We need a special instrument for drawing circles. It must have a pointy tip, like the centre pencil. It must also have a drawing tip, like the pencil you moved. If you can set the distance between these two tips, you can draw circles of any radius. This instrument is called a **pair of compasses**, or often just a **compass**.



CONSTRUCTING CIRCLES WITH A COMPASS

Teaching guidelines

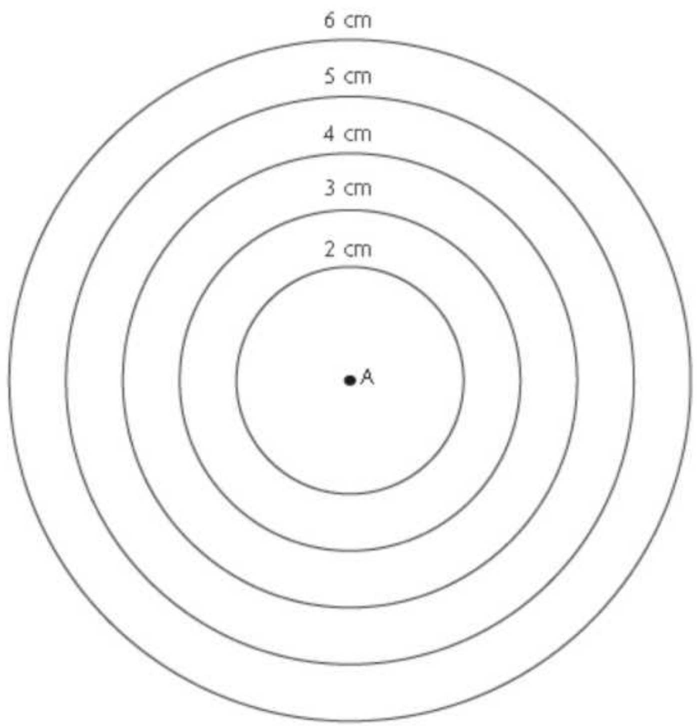
Guide learners through the steps, explained on LB pages 70 and 71, on how to draw a circle with the compass.

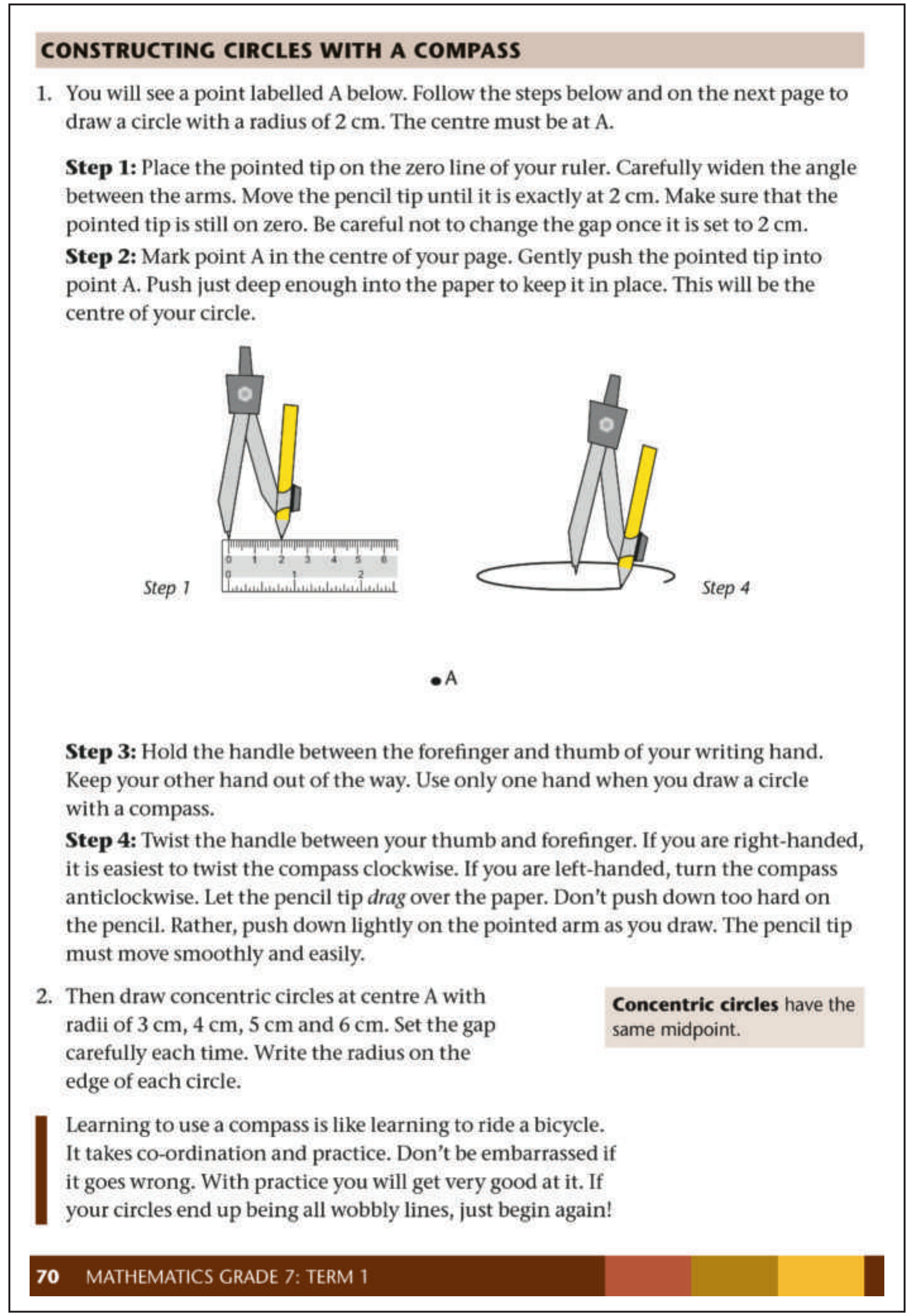
Discuss new words, like **concentric**. Draw a few sets of concentric circles so that learners can see how they come about. They have the same midpoint, but the radii are different.

Misconceptions

Learners confuse radius and diameter.

Answers





CONSTRUCTING CIRCLES WITH A COMPASS

1. You will see a point labelled A below. Follow the steps below and on the next page to draw a circle with a radius of 2 cm. The centre must be at A.

Step 1: Place the pointed tip on the zero line of your ruler. Carefully widen the angle between the arms. Move the pencil tip until it is exactly at 2 cm. Make sure the pointed tip is still on zero. Be careful not to change the gap once it is set to 2 cm.

Step 2: Mark point A in the centre of your page. Gently push the pointed tip into point A. Push just deep enough into the paper to keep it in place. This will be the centre of your circle.

Step 1 *Step 4*

A

Step 3: Hold the handle between the forefinger and thumb of your writing hand. Keep your other hand out of the way. Use only one hand when you draw a circle with a compass.

Step 4: Twist the handle between your thumb and forefinger. If you are right-handed, it is easiest to twist the compass clockwise. If you are left-handed, turn the compass anticlockwise. Let the pencil tip *drag* over the paper. Don't push down too hard on the pencil. Rather, push down lightly on the pointed arm as you draw. The pencil tip must move smoothly and easily.

2. Then draw concentric circles at centre A with radii of 3 cm, 4 cm, 5 cm and 6 cm. Set the gap carefully each time. Write the radius on the edge of each circle.

Concentric circles have the same midpoint.

Learning to use a compass is like learning to ride a bicycle. It takes co-ordination and practice. Don't be embarrassed if it goes wrong. With practice you will get very good at it. If your circles end up being all wobbly lines, just begin again!

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CIRCLES ON CIRCLES

Teaching guidelines

Guide weaker learners through the steps explained on LB page 71 to draw the six circles on the centre circle.

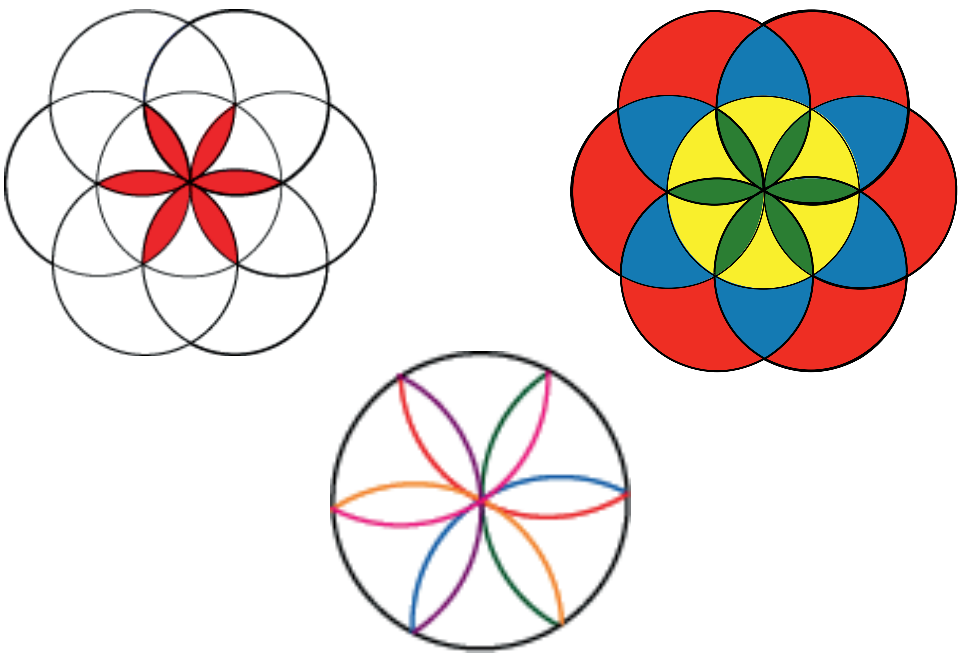
When learners are inexperienced, the patterns don't work out. Point out that they need to be precise when they place the compass point; the pencil has to be sharp and their touch light. If they press too hard on the compass, the pencil may move slightly outward, which will influence the circle drawn.

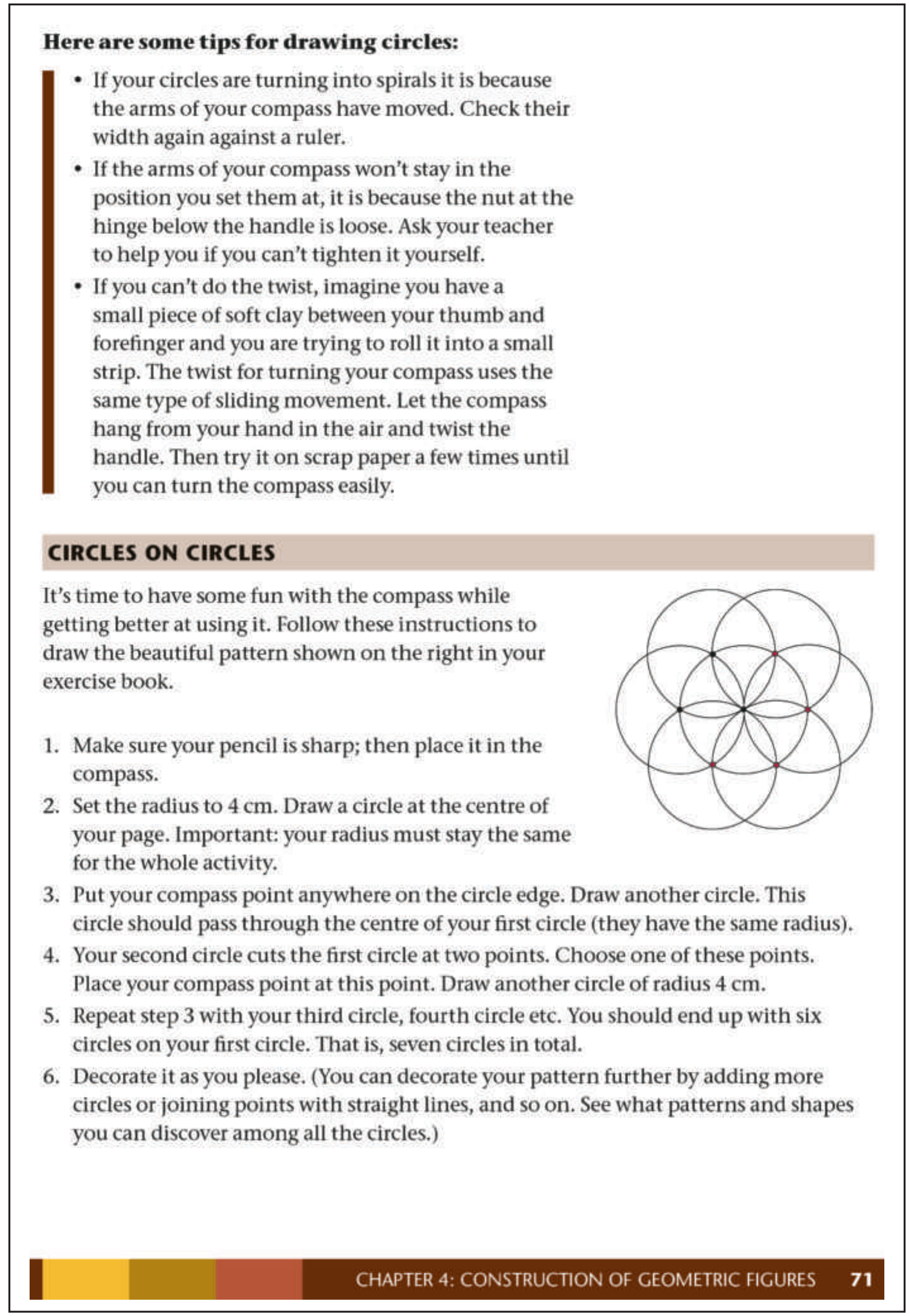
Notes

Suggest to learners that they use their patterns to decorate greeting cards (for example, a get-well card or a Mother's Day card, etc.), or they could decorate a cover of a book with repeated patterns of their own design. They could also use concentric circles in different colours as decoration. A few suggestions are shown in the answer section below.

Answers

1. to 6.





Here are some tips for drawing circles:

- If your circles are turning into spirals it is because the arms of your compass have moved. Check their width again against a ruler.
- If the arms of your compass won't stay in the position you set them at, it is because the nut at the hinge below the handle is loose. Ask your teacher to help you if you can't tighten it yourself.
- If you can't do the twist, imagine you have a small piece of soft clay between your thumb and forefinger and you are trying to roll it into a small strip. The twist for turning your compass uses the same type of sliding movement. Let the compass hang from your hand in the air and twist the handle. Then try it on scrap paper a few times until you can turn the compass easily.

CIRCLES ON CIRCLES

It's time to have some fun with the compass while getting better at using it. Follow these instructions to draw the beautiful pattern shown on the right in your exercise book.

1. Make sure your pencil is sharp; then place it in the compass.
2. Set the radius to 4 cm. Draw a circle at the centre of your page. Important: your radius must stay the same for the whole activity.
3. Put your compass point anywhere on the circle edge. Draw another circle. This circle should pass through the centre of your first circle (they have the same radius).
4. Your second circle cuts the first circle at two points. Choose one of these points. Place your compass point at this point. Draw another circle of radius 4 cm.
5. Repeat step 3 with your third circle, fourth circle etc. You should end up with six circles on your first circle. That is, seven circles in total.
6. Decorate it as you please. (You can decorate your pattern further by adding more circles or joining points with straight lines, and so on. See what patterns and shapes you can discover among all the circles.)

CHAPTER 4: CONSTRUCTION OF GEOMETRIC FIGURES 71

4.8 Using circles to draw other figures

Background

Engineers, architects, designers, sign writers, production designers and some artists use compasses and circles. Many logos and well-known symbols incorporate circles or parts of circles, for example the yin-yang logo, the Mercedes-Benz logo, etc.

GEOMETRIC FIGURES HIDING IN THE CIRCLES

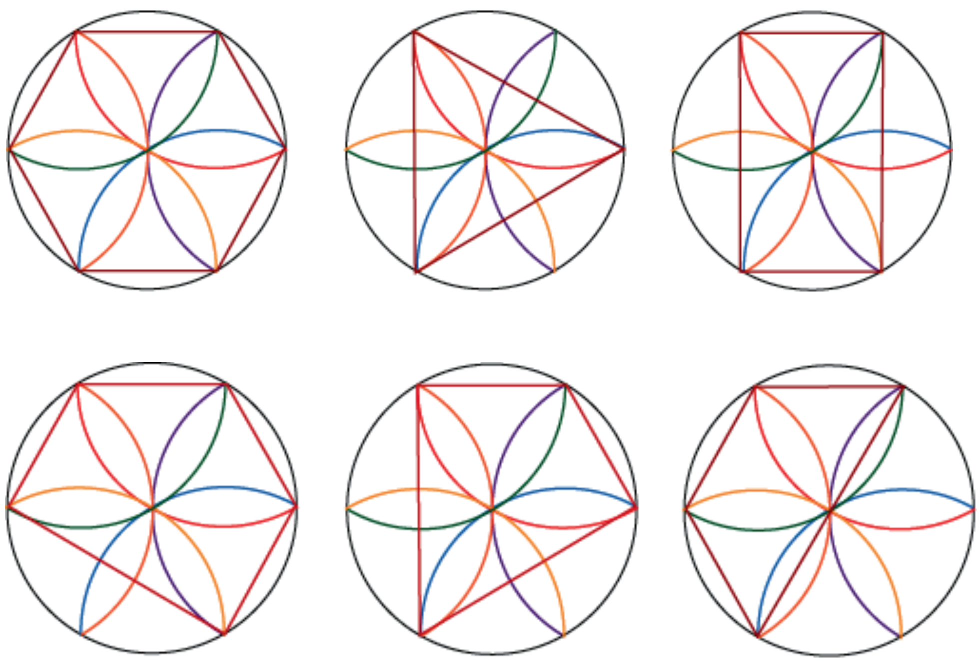
Teaching guidelines

Let learners draw the seven circles again. (They could need more than one set or you could hand out photocopies of a set.)

Set a challenge for them to find as many of the following different shapes as they can: triangles, quadrilaterals, pentagons and hexagons.

Answers

The drawings below show a few suggestions.

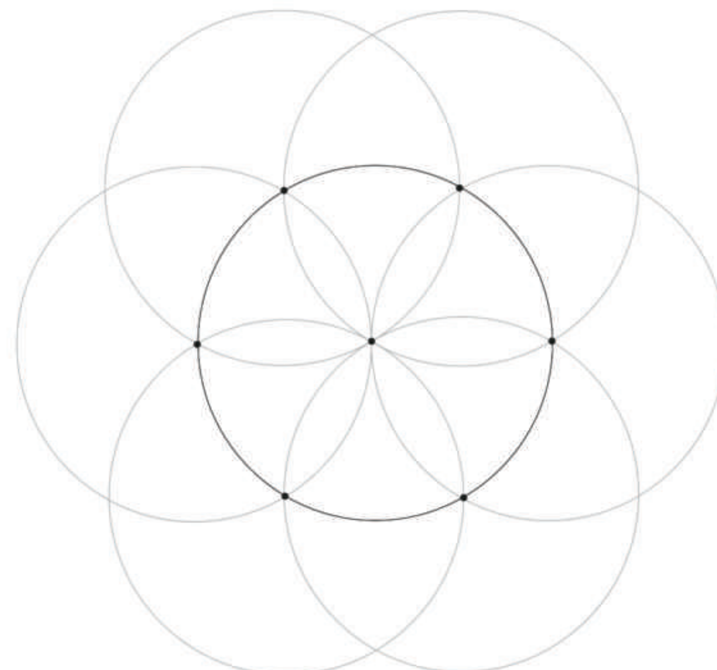


4.8 Using circles to draw other figures

GEOMETRIC FIGURES HIDING IN THE CIRCLES

Below is a set of seven circles like the one you drew. Sit with a partner and try to find hidden polygons.

You will find these polygons by joining the points where the circles cut each other. The points will be the vertices of the polygons. Look carefully. There are triangles, quadrilaterals, pentagons and hexagons. When you can see them, neatly and carefully rule in their sides with a pencil. If there is not enough space on the set of circles below, redraw the circles on a separate piece of paper and show the figures there. If you wish, you can measure the angles at each vertex and the lengths of the sides.



Arcs of circles

We do not have to draw whole circles to construct figures. We are only really interested in the points where the circles cross each other, so we could just draw arcs where they cross. Next year, you will use arcs in your geometric constructions.

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Arcs of circles

Notes

The word **circumference** is used when we find the distance around a circle, a sphere, a cylinder, or other such curved shape. The distance around a shape that has vertices, such as a rectangle, is called the **perimeter**. So perimeter and circumference refer to the same type of measurement: distance around the shape or around the base of an object.

An **arc** is part of the length of the circumference of a circle. It could be a small part or it could be all the way around, the full circle.

Radii of the same circle are all equal; therefore the straight distance between any point on the circle and the centre of the circle is the radius of the circle. Starting from any point on the circle and marking off six equal arcs, we should get the sides of a regular hexagon.

In practise though, learners sometimes find that their last arc does not go through the original point on the circumference. If this happens, help learners to find why and to improve the effort.

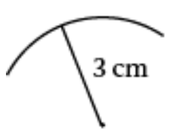
Teaching guidelines

Discuss the different drawings on LB page 73.

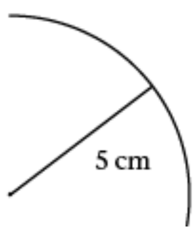
Learners should do the practical work on LB page 73, drawing the arcs. Circulate to make sure that learners can follow the instructions.

Answers

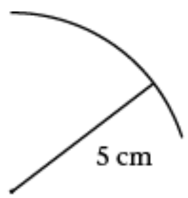
1.



2.



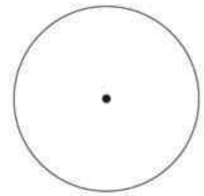
3.



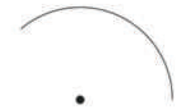
ENRICHMENT

It takes some skill to construct the hexagon using only the arcs as required in this section. Learners can work in pairs and try to give reasons why this works.

An **arc** is a small part of a circle. We use the term **circumference** when we refer to the distance around a circle or around any other curved shape.

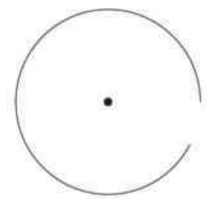


This is a full circle.

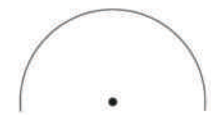


This is only part of a circle. A part of a circle like this is called an ***arc****.*

This arc is smaller.



This arc is almost a full circle.



This arc is just more than a semi-circle.

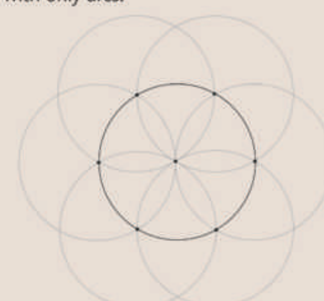
This arc is a quarter circle.

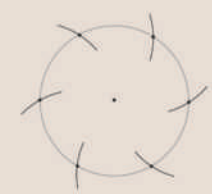
Do the following in your exercise book:

1. Draw an arc using a radius of 3 cm.
2. Draw an arc bigger than a quarter circle, using a radius of 5 cm.
3. Draw an arc smaller than a quarter circle, using a radius of 5 cm.

ENRICHMENT

Once you have finished the work in section 4.8, experiment with drawing only the arcs that you need in various constructions. Here is an example to show how to construct a regular hexagon with only arcs:





CHAPTER 4: CONSTRUCTION OF GEOMETRIC FIGURES 73

FAMILIAR FIGURES IN THE SEVEN-CIRCLE PATTERN

Notes

If learners have completed the activities from LB pages 72–73 they should not have a problem completing this activity on LB page 74.

Teaching guidelines

Let them measure the angles of the shapes as an extension. You could encourage learners to make a poster of their drawings and the results.

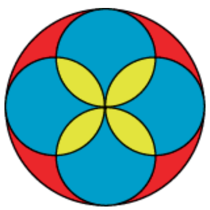
Answers

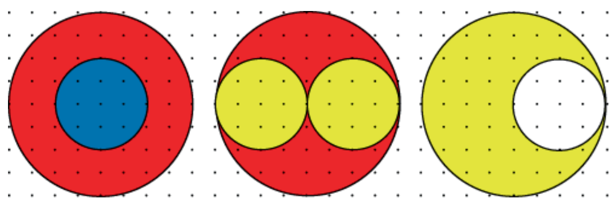
1. Learners draw the five seven-circle sets and connect the points as instructed.
2. See LB page 74 alongside.

Additional exercises

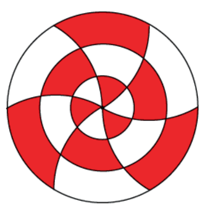
You could draw these designs on the board and challenge learners to copy them. They can then colour them in. The dotted background in the drawings can help them to decide what the radii should be.

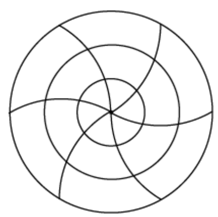
1. Learners can try to reproduce these designs and use them to make greeting cards, gift wrapping or decorative covers for books.

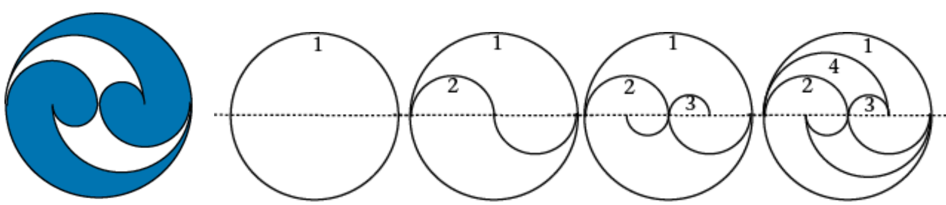




2. You could challenge stronger learners to try the following designs using their compasses. Construction methods are shown next to the drawings.

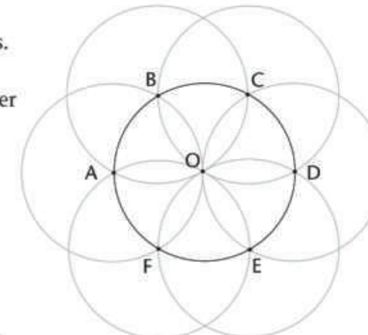






FAMILIAR FIGURES IN THE SEVEN-CIRCLE PATTERN

For this activity you need five seven-circle sets like the ones drawn in the previous two activities. Start by drawing these on blank pieces of paper. Don't make your radius bigger than 4 cm. Number your sets figure 2 to figure 6. Label each figure as shown on the right.



1. Copy the figure above right by following the instructions below.
   - Figure 1: Draw lines connecting AB, BC, CD, ... up to FA.
   - Figure 2: Draw lines connecting A, O and B.
   - Figure 3: Draw lines connecting B, F and D.
   - Figure 4: Draw lines connecting BC, CE, EF and FB.
   - Figure 5: Draw lines connecting CD, DE, EF and FC.
   - Figure 6: Draw lines connecting AB, BC, CE and EA.
2. Copy and complete the table below. It shows the name of each figure and its properties. Figure 1 (on the right) has been done as an example.

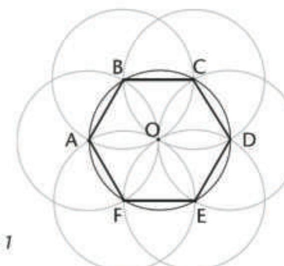


Figure 1

Figure	Name of figure	Properties
1	Regular hexagon	six-sided figure. All the sides are equal. All the interior angles are equal.
2	Equilateral triangle	3-sided figure. All three sides are equal. All three angles are equal (60°).
3	Equilateral triangle	3-sided figure. All three sides are equal. All three angles are equal (60°).
4	Rectangle	4-sided figure. Has two pairs of parallel sides: two long sides and two short sides. All four angles are right angles.
5	Trapezium	4-sided figure. Has two obtuse angles and two acute angles. One pair of sides is parallel.
6	Kite	4-sided figure. Adjacent sides are equal in length. One pair of angles is equal in size.

CONSTRUCT SOME MORE FIGURES

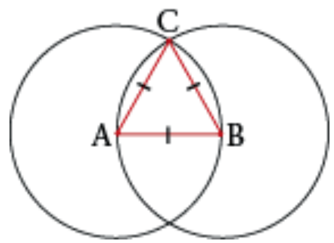
Teaching guidelines

Let learners work in pairs or small groups but the learners should each make their own drawing. When their drawings are complete, they can compare and discuss their findings.

The drawings learners make for question 2 will differ as learners choose a size for the angles of their drawings. Remind learners of the properties of the sides of a parallelogram.

Answers

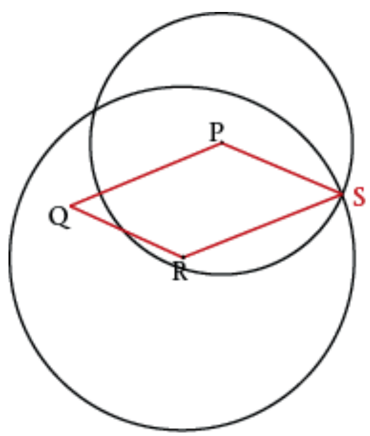
1. (a)–(g) Learners follow the instructions.

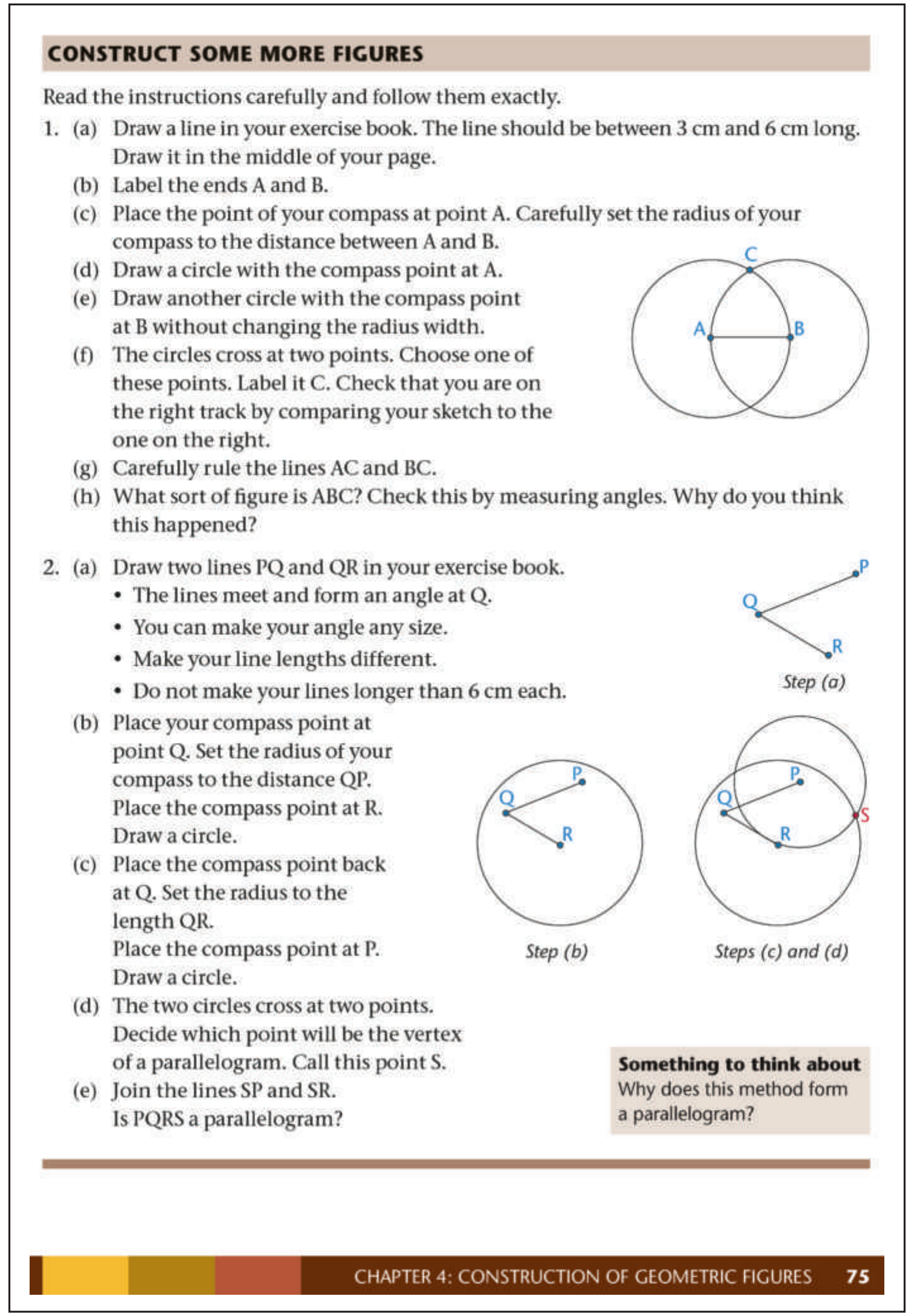


(h) ABC is an equilateral triangle (each angle = 60°). This is because the distance from the centre of a circle to its edge is the same in any direction ∴ AB = AC = BC.

2. (a)–(d) Learners follow the instructions.

(e) Yes





CONSTRUCT SOME MORE FIGURES

Read the instructions carefully and follow them exactly.

1. (a) Draw a line in your exercise book. The line should be between 3 cm and 6 cm long. Draw it in the middle of your page.
   (b) Label the ends A and B.
   (c) Place the point of your compass at point A. Carefully set the radius of your compass to the distance between A and B.
   (d) Draw a circle with the compass point at A.
   (e) Draw another circle with the compass point at B without changing the radius width.
   (f) The circles cross at two points. Choose one of these points. Label it C. Check that you are on the right track by comparing your sketch to the one on the right.
   (g) Carefully rule the lines AC and BC.
   (h) What sort of figure is ABC? Check this by measuring angles. Why do you think this happened?

2. (a) Draw two lines PQ and QR in your exercise book.
   - The lines meet and form an angle at Q.
   - You can make your angle any size.
   - Make your line lengths different.
   - Do not make your lines longer than 6 cm each.

   (b) Place your compass point at point Q. Set the radius of your compass to the distance QP. Place the compass point at R. Draw a circle.
   (c) Place the compass point back at Q. Set the radius to the length QR. Place the compass point at P. Draw a circle.
   (d) The two circles cross at two points. Decide which point will be the vertex of a parallelogram. Call this point S.
   (e) Join the lines SP and SR. Is PQRS a parallelogram?

Step (a)

Step (b)

Steps (c) and (d)

Something to think about
Why does this method form a parallelogram?

4.9 Parallel and perpendicular lines with circles

PARALLEL AND PERPENDICULAR

Notes

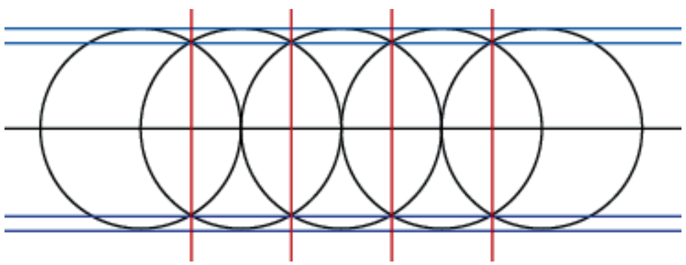
When learners are given practical tasks to engage in they learn better than by reading from textbooks or just listening to a teacher. The tasks in this section will give them an opportunity to think about what they have been taught and to increase their confidence in their ability to reason and debate with their peers and teacher.

Teaching guidelines

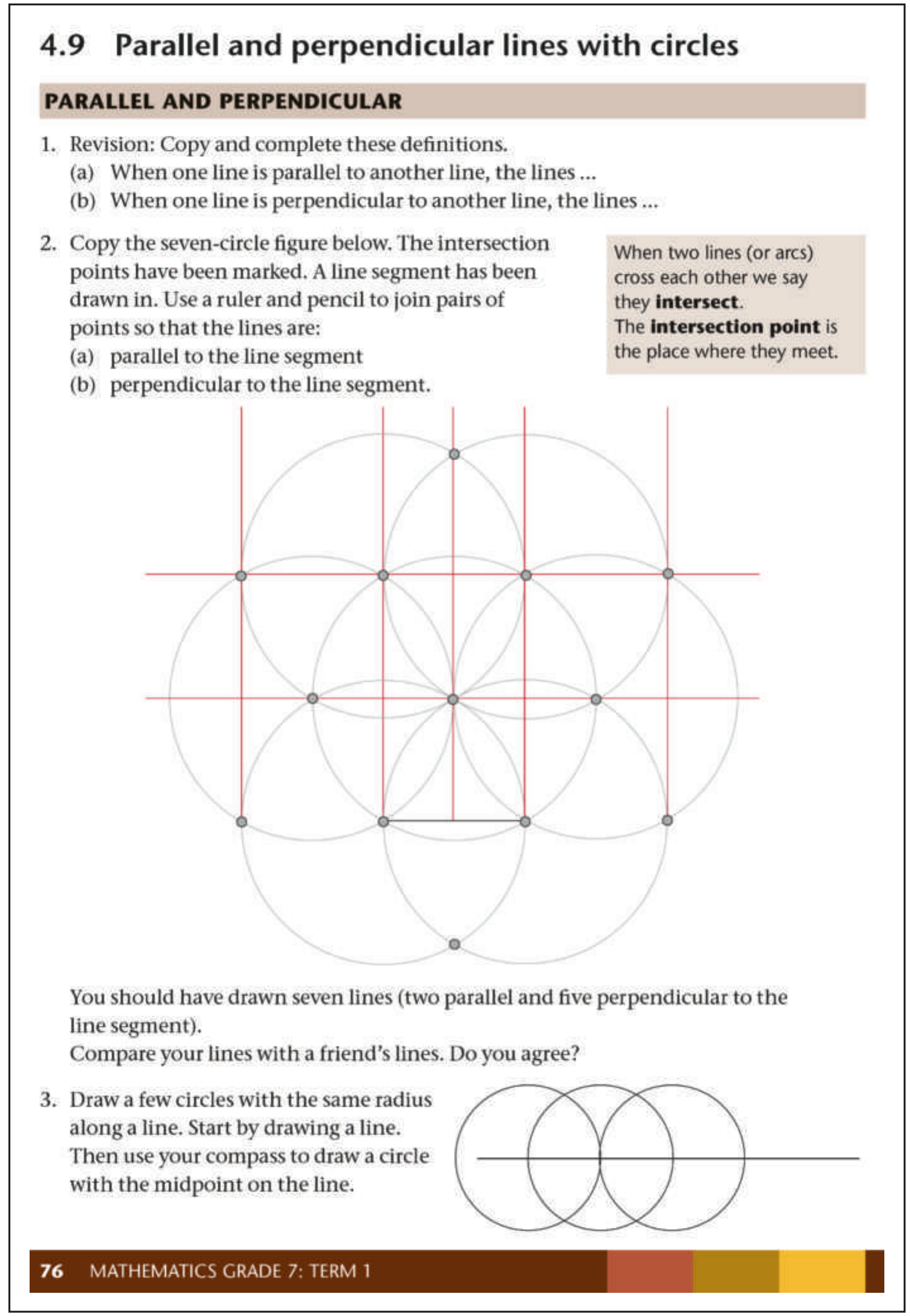
Revise the important properties of parallel lines and perpendicular lines. Introduce the word **intersection**.

Answers

1. (a) ... are always the same distance apart and will never meet.
   (b) ... meet at right angles.
2. See the answers on the drawing LB page 76.
3. A sample drawing is shown below.



   (a) Let learners discuss with a partner how to find the shape in the seven-circle figure.
   (b) Perpendicular lines are shown on the drawing in red.
   (c) Parallel lines are shown on the drawing in purple.



4.9 Parallel and perpendicular lines with circles

PARALLEL AND PERPENDICULAR

1. Revision: Copy and complete these definitions.
   (a) When one line is parallel to another line, the lines ...
   (b) When one line is perpendicular to another line, the lines ...

2. Copy the seven-circle figure below. The intersection points have been marked. A line segment has been drawn in. Use a ruler and pencil to join pairs of points so that the lines are:
   (a) parallel to the line segment
   (b) perpendicular to the line segment.

When two lines (or arcs) cross each other we say they **intersect**.
The **intersection point** is the place where they meet.

You should have drawn seven lines (two parallel and five perpendicular to the line segment).
Compare your lines with a friend's lines. Do you agree?

3. Draw a few circles with the same radius along a line. Start by drawing a line. Then use your compass to draw a circle with the midpoint on the line.

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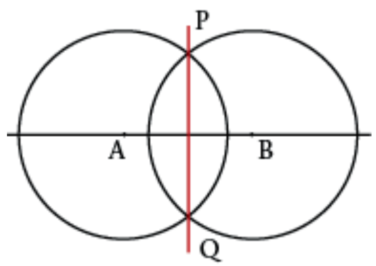
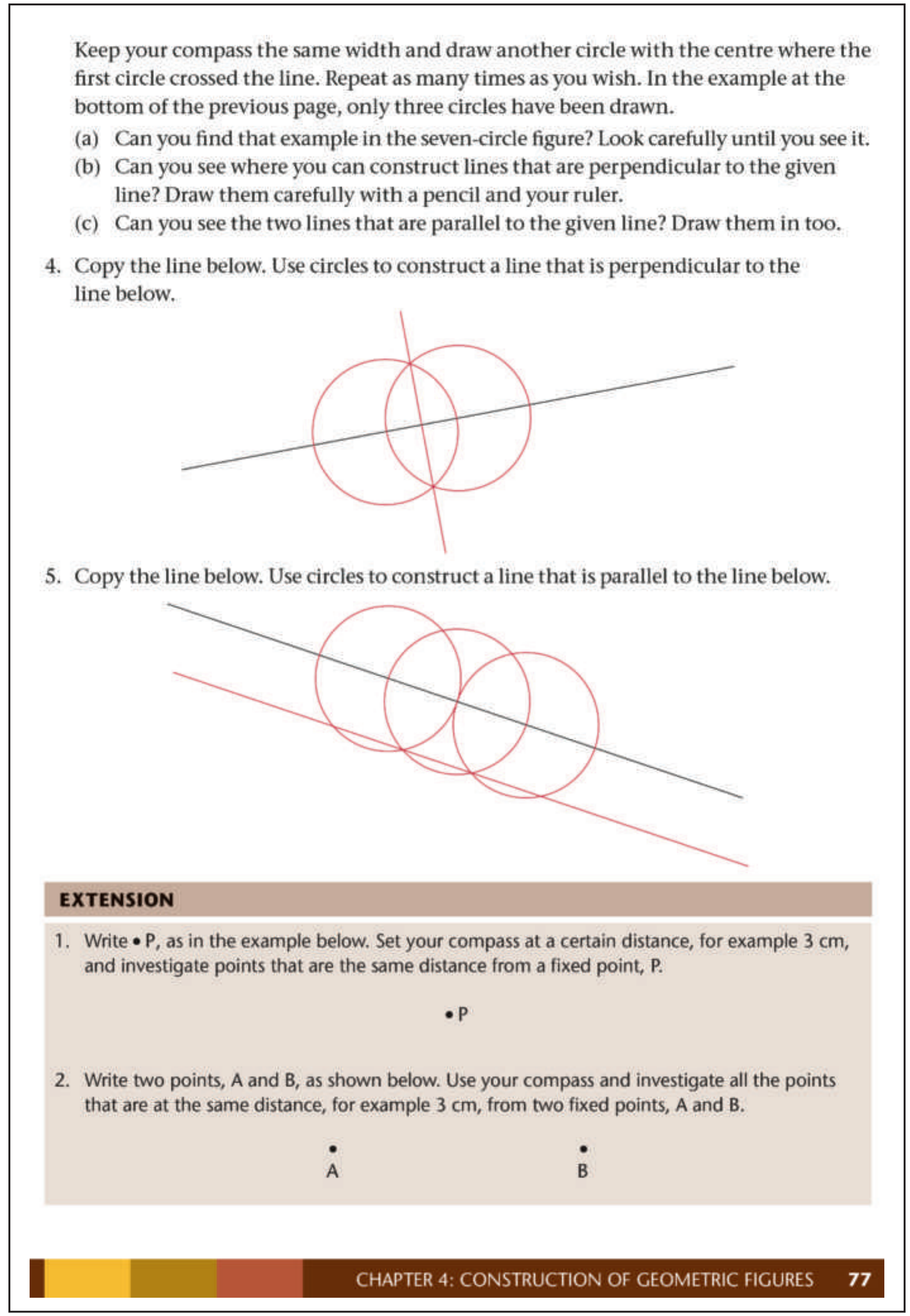
4. See LB page 77 alongside.
5. See LB page 77 alongside.

Extension

1. The points are on the circumference of a circle.
2. Learners set their compass with a radius 3 cm and draw a circle with midpoint A. With the same radius, they then draw a circle with midpoint B. The points where the two circles intersect, say P and Q, are both 3 cm from A and from B.

   This activity can be further extended by letting the learners connect P and Q (points of intersection of the circles). Ask them to investigate line segment PQ. It is perpendicular to AB (test by using a protractor) and any point on this line is equidistant from points A and B. The learners can test this by using their compasses or by having discussions with logical reasoning. They will learn about this when they learn to construct a perpendicular bisector of a line segment in Grade 8.





Keep your compass the same width and draw another circle with the centre where the first circle crossed the line. Repeat as many times as you wish. In the example at the bottom of the previous page, only three circles have been drawn.

(a) Can you find that example in the seven-circle figure? Look carefully until you see it.
(b) Can you see where you can construct lines that are perpendicular to the given line? Draw them carefully with a pencil and your ruler.
(c) Can you see the two lines that are parallel to the given line? Draw them in too.

4. Copy the line below. Use circles to construct a line that is perpendicular to the line below.

5. Copy the line below. Use circles to construct a line that is parallel to the line below.

EXTENSION

1. Write • P, as in the example below. Set your compass at a certain distance, for example 3 cm, and investigate points that are the same distance from a fixed point, P.

   • P

2. Write two points, A and B, as shown below. Use your compass and investigate all the points that are at the same distance, for example 3 cm, from two fixed points, A and B.

   • A   • B

CHAPTER 4: CONSTRUCTION OF GEOMETRIC FIGURES 77

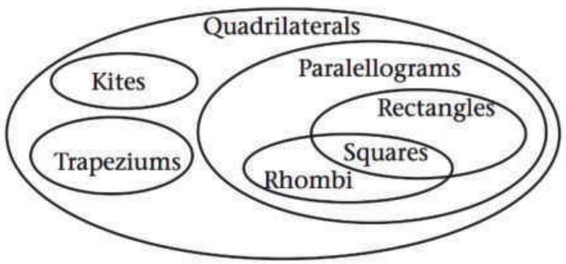
Grade 7 Term 1 Chapter 5 Geometry of 2D shapes

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
5.1 Triangles, quadrilaterals, circles and others	Distinguish between triangles, quadrilaterals and circles; use some properties to draw some of these shapes	Pages 78 to 79
5.2 Different types of triangles	Recognise, describe, sort, name and compare triangles according to their sides and use properties to find unknown values in equilateral triangles, isosceles triangles and right-angled triangles	Pages 80 to 83
5.3 Different types of quadrilaterals	Describe, sort, name and compare different quadrilaterals in terms of length of sides, parallel and perpendicular sides, size of angles (right angles or not); find unknown sides in quadrilaterals	Pages 84 to 88
5.4 Circles	Describe and name parts of a circle	Pages 88 to 89
5.5 Similar and congruent shapes	Recognise and describe similar and congruent figures by comparing shape and size	Pages 90 to 93

CAPS time allocation	10 hours
CAPS content specification	Pages 46 to 47

Mathematical background

- **Triangles** can be classified according to the properties of their sides or their angles, for example: a triangle of which all the sides are different lengths is called a scalene triangle; if two sides of a triangle have the same length, it is called an isosceles triangle and if all three sides have the same length, a triangle is equilateral. If a triangle is obtuse, one angle is greater than 90°; in an acute-angled triangle all angles are less than 90°; a right-angled triangle has one angle equal to 90°. Triangles can be classified using both sides and angles, for example: right-angled, isosceles, and so on.
- **Quadrilaterals** are classified as closed shapes with four straight sides. Additional properties allow us to classify quadrilaterals as shown in the Venn diagram on the right.



The question learners should learn to ask is: What are the properties that a shape must have to be classified as a particular shape? For example, rectangles need four right angles, therefore a square can be called a rectangle.

In Mathematics, **similarity** is a big idea that relates many areas, such as enlargement, scale factor, area growth, congruence, to name a few. Similar shapes have all their sides in the same ratio and corresponding angles are equal.

Shapes are **congruent** if their corresponding sides are equal (in the ratio 1 : 1) and their corresponding angles are equal. This means that all congruent shapes are also similar.

5.1 Triangles, quadrilaterals, circles and others

DECIDE WHICH IS WHICH AND DRAW SOME FIGURES

Notes on the tasks

In order to describe, sort, name and compare shapes, learners need to be able to communicate about the properties of various shapes, so they need the correct terminology, such as **obtuse** and **acute angles**. Learners can demonstrate their understanding of these words by drawing triangles and quadrilaterals with acute and obtuse angles. Their drawings will be a clear indication of their understanding of these and other terms, such as **opposite** and **adjacent** sides.

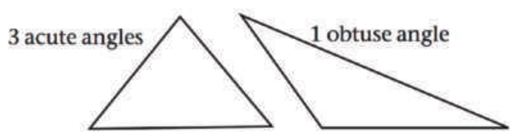
Teaching guidelines

Use the shapes on the LB page 79 on the next page as a first activity. Learners answer questions 1 to 3 using these shapes. This is a powerful way to help learners understand what is meant by the classifying of shapes, before any teaching of this topic is done. The page has a variety of closed shapes, including shapes with curved sides and straight sides, regular and irregular polygons, familiar shapes, which they should be able to name, and others that may be unfamiliar to them. Do not give them any guidance other than asking them to consider the shapes and to think about ways in which some of them are similar and some are different.

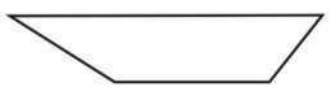
Talk about the meaning of the words **opposite** and **adjacent** (sides).

Answers

1. C and H
2. A; G; J; P; Q
3. D; F; I; O
4. Examples:

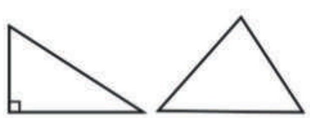


5. (a)



(b) No

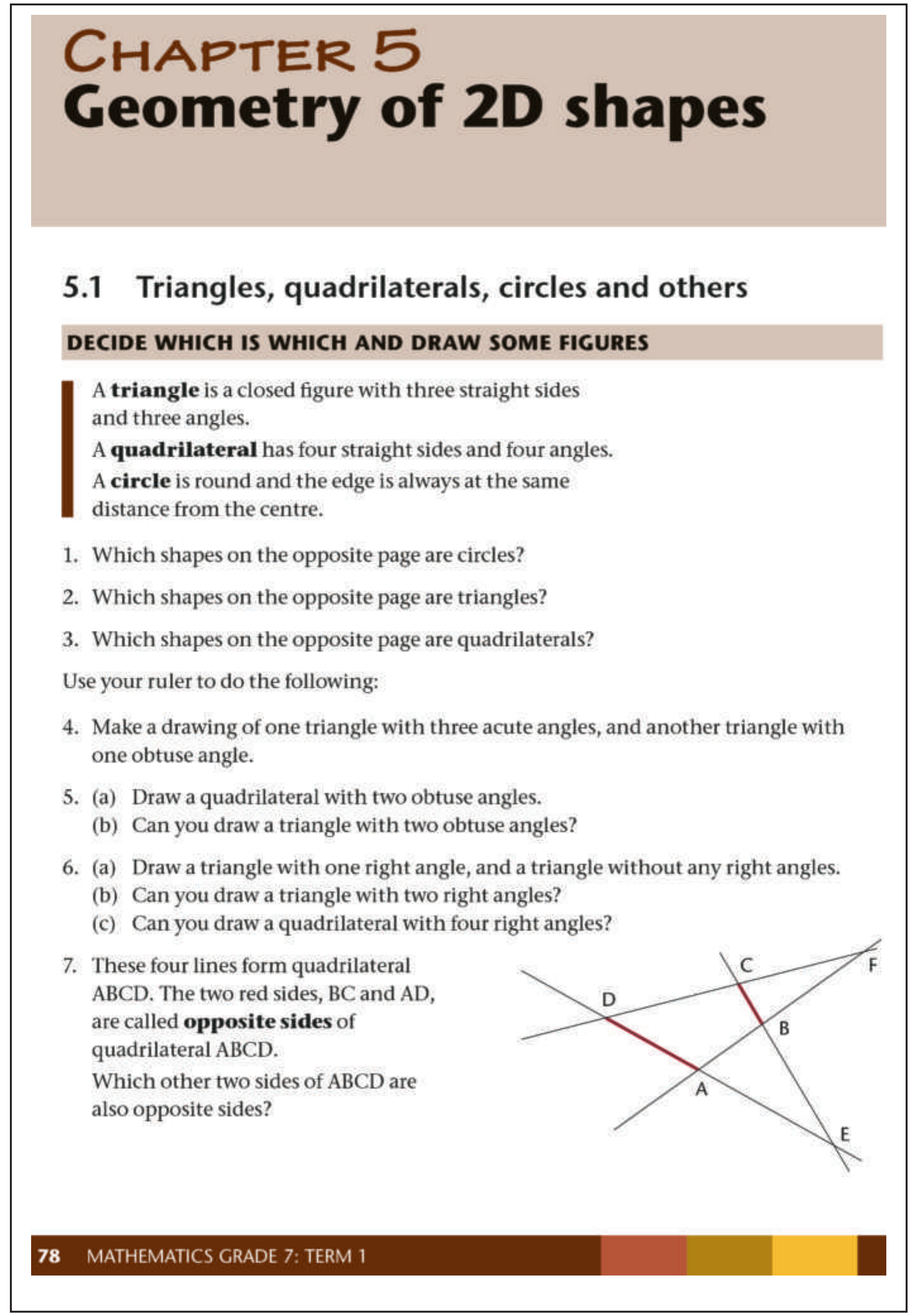
6. (a)



(b) No. The sum of the angles of a triangle is 180° and two right angles add up to 180°.

(c) Yes. A rectangle (or a square).

7. See LB page 78 alongside. AB and DC are also opposite sides.



CHAPTER 5
Geometry of 2D shapes

5.1 Triangles, quadrilaterals, circles and others

DECIDE WHICH IS WHICH AND DRAW SOME FIGURES

A **triangle** is a closed figure with three straight sides and three angles.

A **quadrilateral** has four straight sides and four angles.

A **circle** is round and the edge is always at the same distance from the centre.

1. Which shapes on the opposite page are circles?
2. Which shapes on the opposite page are triangles?
3. Which shapes on the opposite page are quadrilaterals?

Use your ruler to do the following:

4. Make a drawing of one triangle with three acute angles, and another triangle with one obtuse angle.
5. (a) Draw a quadrilateral with two obtuse angles.
   (b) Can you draw a triangle with two obtuse angles?
6. (a) Draw a triangle with one right angle, and a triangle without any right angles.
   (b) Can you draw a triangle with two right angles?
   (c) Can you draw a quadrilateral with four right angles?
7. These four lines form quadrilateral ABCD. The two red sides, BC and AD, are called **opposite sides** of quadrilateral ABCD.
   Which other two sides of ABCD are also opposite sides?

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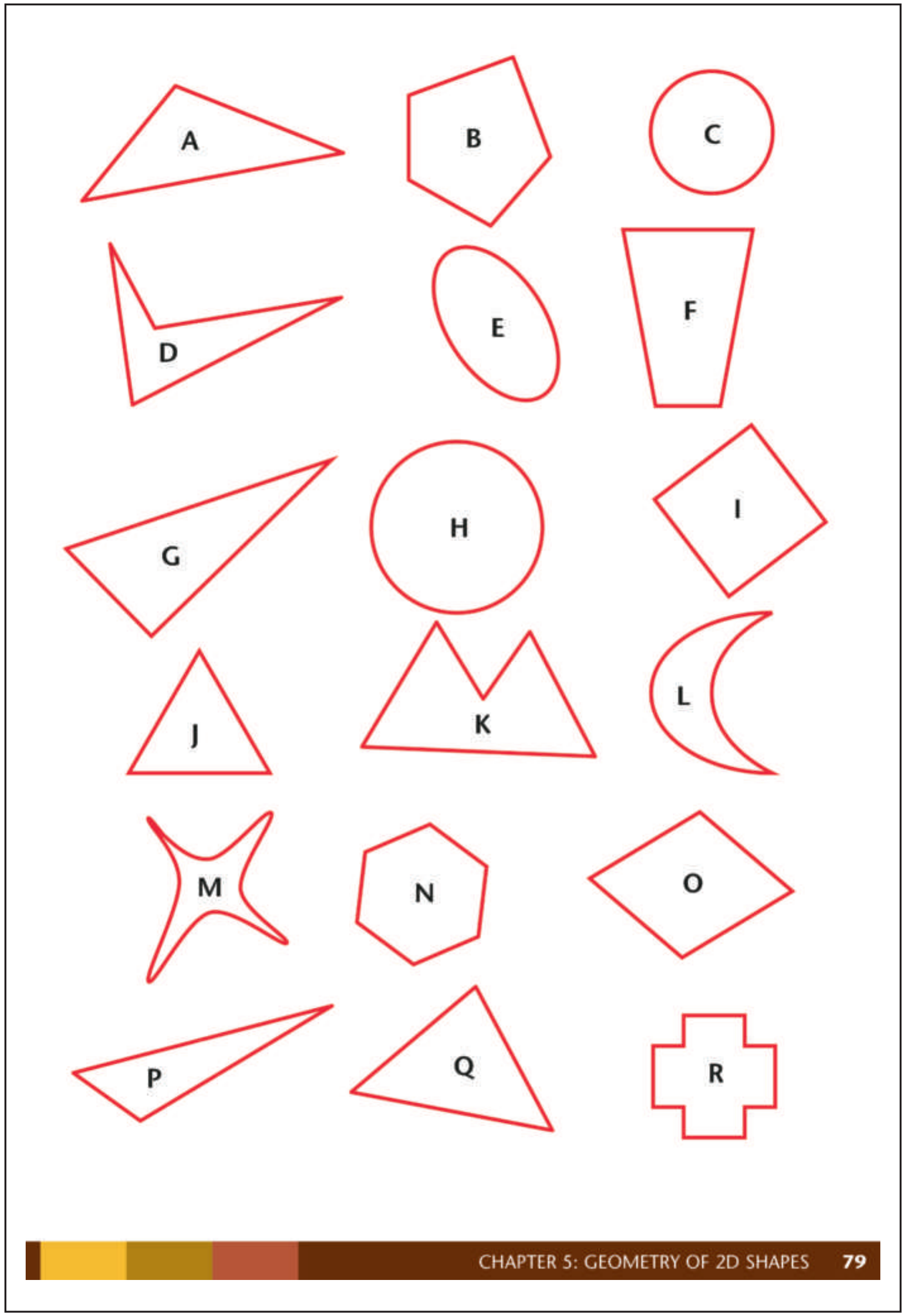
A
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R
CHAPTER 5: GEOMETRY OF 2D SHAPES 79

8. (a) AB and BC; BC and CD; CD and DA
   (b) DC
9. The first statement is correct – there are four sides in a quadrilateral, so every side has two adjacent sides and one opposite side. His second statement is not correct.
10. Yes. There are only three sides, so every side has two adjacent sides: one on either side.
11. (a) adjacent (b) opposite (c) adjacent
    (d) opposite (e) adjacent

5.2 Different types of triangles

EQUILATERAL, ISOSCELES, SCALENE AND RIGHT-ANGLED TRIANGLES

Background

Triangles can be classified according to their sides or their angles.

- Triangles with all three sides equal and all three angles equal are **equilateral**.
- **Isosceles** triangles have two sides equal. The two angles opposite the equal sides are also equal.
- **Scalene** triangles have three sides of different lengths and all the angles are all different sizes.
- The biggest angle in a triangle is opposite the longest side and the smallest angle is opposite the shortest side.

Triangles classified according to their angles are:

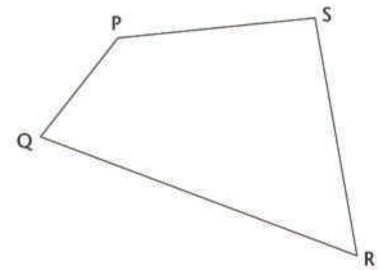
- acute-angled triangles where all the angles are less than 90°
- obtuse-angled triangles where one angle is greater than 90° (but smaller than 180°)
- right-angled triangles where one angle is a right angle.

Teaching guidelines

Explain the meaning of the words, **isosceles**, **equilateral** and **scalene**. Show learners how to identify these triangles and help them to classify triangles according to their sides.

Point out the example of triangles with acute angles, an obtuse angle and a right angle.

8. The lines DA and AB in the figure in question 7 are called **adjacent sides**. They meet at a point that is one of the vertices (corner points) of the quadrilateral.
   (a) Name another two adjacent sides in ABCD.
   (b) AB is adjacent to DA in the quadrilateral ABCD. Which other side of ABCD is also adjacent to DA?
9. William says:
   "Each side of a quadrilateral has two adjacent sides. Each side of a quadrilateral also has two opposite sides."
   Is William correct? Give reasons for your answer.
10. William also says:
    "In a triangle, each side is adjacent to all the other sides."
    Is this true? Give a reason for your answer.
11. In each case, say whether the two sides are opposite sides or adjacent sides of the quadrilateral PQRS.
    (a) QP and PS
    (b) QP and SR
    (c) PQ and RQ
    (d) PS and QR
    (e) SR and QR



5.2 Different types of triangles

EQUILATERAL, ISOSCELES, SCALENE AND RIGHT-ANGLED TRIANGLES

A triangle with two equal sides is called an **isosceles triangle**.
A triangle with three equal sides is called an **equilateral triangle**.
A triangle with a right angle is called a **right-angled triangle**.
A triangle with three sides with different lengths and no right angle is called a **scalene triangle**.

1. Measure every angle in each of the **isosceles triangles** given on the next page. Do you notice anything special? If you are not sure, draw more isosceles triangles in your exercise book.

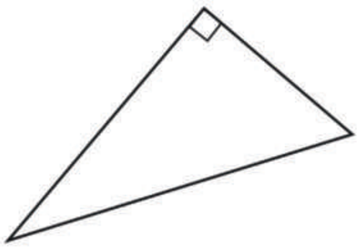
Explain that a triangle can be isosceles acute-angled or isosceles obtuse-angled or isosceles right-angled. Let them investigate the possible combinations.

They should find that if a triangle is equilateral it is also equi-angular and such an equilateral triangle cannot be obtuse-angled or right-angled.

Misconceptions

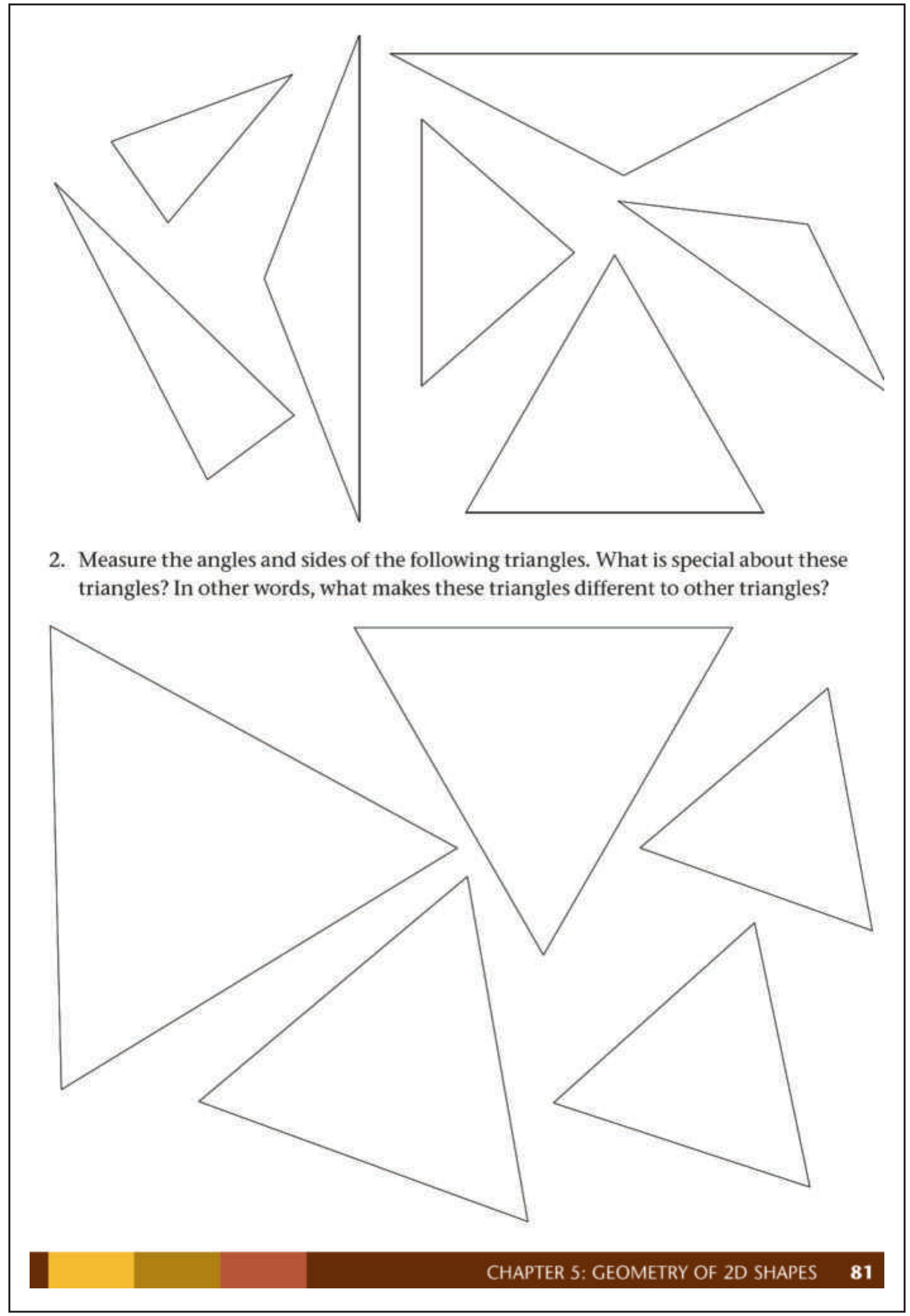
A common misconception learners have is that it is only possible to classify triangles in one way: either as scalene, right-angled or isosceles. Let them work with examples that are scalene and right-angled or right-angled and isosceles, or, using angles: scalene, acute-angled or scalene, obtuse-angled, etc.

Another misconception they have has to do with the orientation of a shape. Learners do not recognise triangles as right-angled if the orientation is such that the right-angled sides are not vertical and horizontal, for example:



Answers

1. Two of the angles in every isosceles triangle are equal.
2. All of the angles are 60° and all three of the sides in every triangle are equal.



2. Measure the angles and sides of the following triangles. What is special about these triangles? In other words, what makes these triangles different to other triangles?

CHAPTER 5: GEOMETRY OF 2D SHAPES 81

Answers

3. (a) Every triangle has one 90° angle. The sum of the other two angles is 90°.
   (b) The longest side is always opposite the 90° angle, the biggest angle in the triangle.

COMPARING AND DESCRIBING TRIANGLES

Notes on the questions

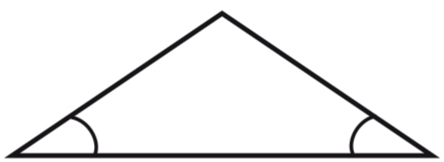
If there are no markings on a geometrical drawing of a shape, you cannot assume facts about the shape, for example, even though a triangle seems to be isosceles, you can only know it is if there are markings indicating the equal sides.

Teaching guidelines

Explain to learners that the markings on a drawing give information about the shapes. This makes it possible to compare drawings and to describe what type of drawing it is.

Demonstrate the following using the drawings in question 1on LB page 82:

- Equal sides are marked in the same way, normally with a short line (or two short lines close together), as in drawing A or drawing C.
- A right angle is indicated with a small square, as in drawing B.
- Angles that are equal are marked in the same way, for example:

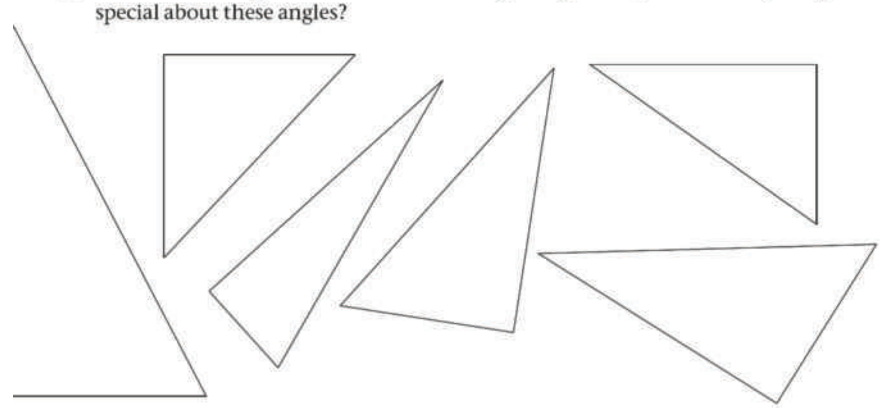


Answers

1. (a) C; isosceles
   (b) A; equilateral
   (c) B; right-angled

- These triangles are called **equilateral triangles**.

3. (a) Measure each angle in each of the following triangles. Do you notice anything special about these angles?



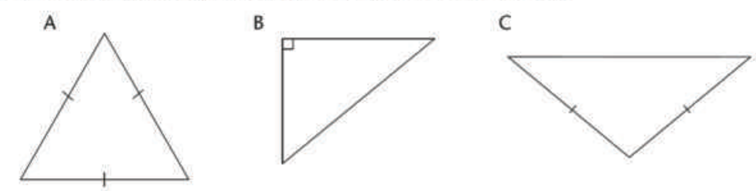
   (b) Identify the longest side in each of the triangles. If you are not sure which one is the longest side, measure the sides. What do you notice about the longest side in each of these triangles?

- These triangles are called **right-angled triangles**.

COMPARING AND DESCRIBING TRIANGLES

When two or more sides of a shape are equal in length, we show this using short lines on the equal sides.

1. Use the following triangles to answer the questions that follow.



   (a) Which triangle has only two sides that are equal?
   What is this type of triangle called?
   (b) Which triangle has all three sides equal?
   What is this type of triangle called?
   (c) Which triangle has an angle equal to 90°?
   What is this type of triangle called?

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Answers

2. See LB page 83 alongside.

FINDING UNKNOWN SIDES IN TRIANGLES

Teaching guidelines

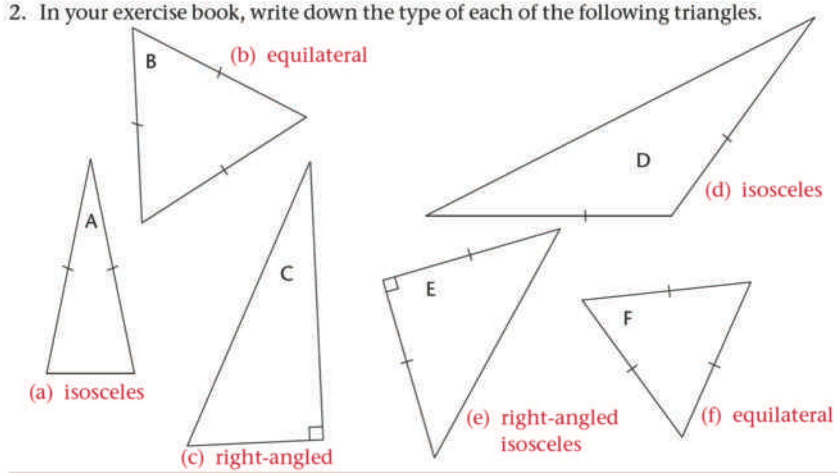
Learners first identify the type of triangle by interpreting the markings (for example question 2). Then they use their knowledge of that type of triangle to get more information, for example: two sides are marked the same in the second triangle in question 1, therefore they are equal and therefore the sides are both 70 mm.

Further information that can be deduced in this case is that $\hat{D} = \hat{F}$.

Answers

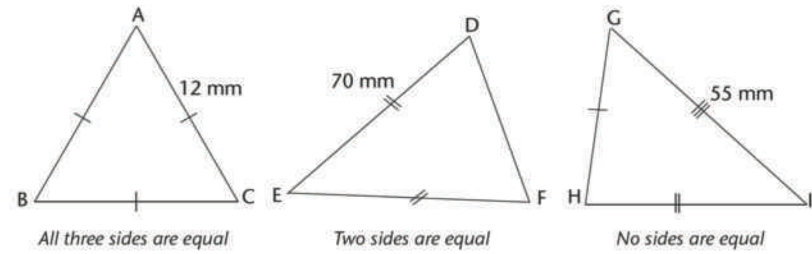
1. (a) △ABC: equilateral   △DEF: isosceles   △GHI: scalene
   (b) See LB page 83 alongside.
   (c) No. There are no equal sides, and no relationships between the sides are known.
2. (a) (Right-angled) isosceles; two equal sides
   (b) JK and KL; marked on the figure
   (c) 6 cm; it is the same as KL
   (d) $K\hat{J}L$ and $J\hat{L}K$; $J\hat{K}L$ is 90° so it can only be these other two angles.
   (e) 45°; by measuring both angles or only one.

2. In your exercise book, write down the type of each of the following triangles.



FINDING UNKNOWN SIDES IN TRIANGLES

1. (a) Name each type of triangle below.



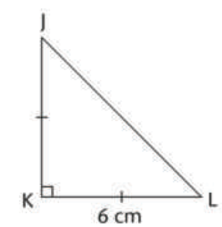
(b) Use the given information to determine the length of the following sides:

AB: 12 mm   BC: 12 mm   EF: 70 mm

(c) Can you determine the lengths of GH and HI? Explain your answer.

2. The square in the corner of ΔJKL shows that it is a right angle. Give a reason for each of your answers below.

(a) Is this triangle scalene, isosceles, or equilateral?
(b) Name the two sides of the triangle that are equal.
(c) What is the length of JK?
(d) Name two equal angles in this triangle.
(e) What is the size of $\hat{J}$ and $\hat{L}$?



CHAPTER 5: GEOMETRY OF 2D SHAPES 83

5.3 Different types of quadrilaterals

INVESTIGATING QUADRILATERALS

Notes on the questions

In this context, learners need to understand the meaning of the words **adjacent** and **opposite**.

Please refer to the additional notes (on TG page 94) which cover the way in which learning takes place.

Teaching guidelines

If possible, have a collection of the quadrilaterals in the groups 1 to 6 on LB pages 84 to 86 ready for each learner (either to cut out or already cut out). Otherwise let learners make the necessary measurements on the quadrilaterals in the book.

Let learners sort the shapes without any guidance, the only instruction should be to work in pairs (small groups or even alone) and arrange the shapes into groups in any way they want to. Learners should then discuss in their groups why they have grouped them in that way. Learners have been working with these groups of shapes in the earlier grades, so they might have been exposed to this kind of activity before. This will then be re-enforcement for those learners.

Encourage learners to make different groups with different rules, for example these all have right angles, or these all have at least on pair of sides parallel, etc.

Let them write up their reasons for the groupings and discuss the interesting ones with the class.

Then let learners answer question 1. It is useful if they draw up a table to organise their answers, for example:

	Group					
Property	1	2	3	4	5	6
Both pairs of opposite sides parallel	✓		✓	✓		✓
Only some adjacent sides equal		✓				

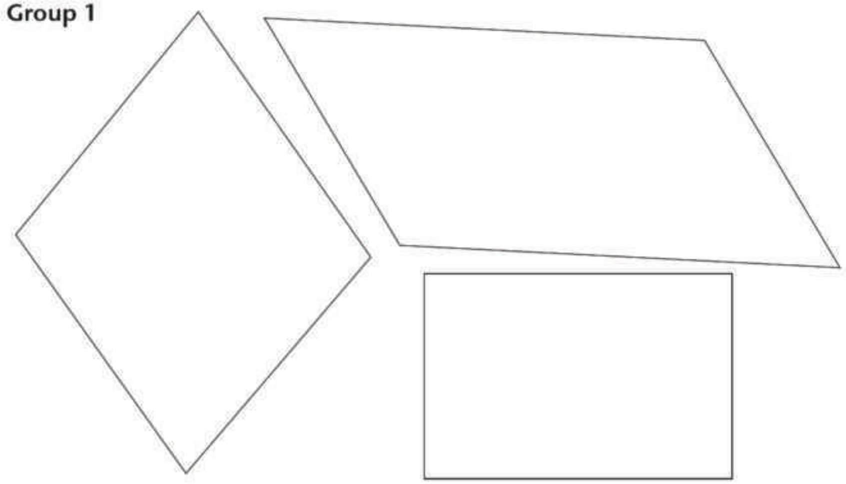
They can fill in the names of the groups in their tables when they complete the rest of the questions. The table may help them to compare the properties of the shapes and make deductions about the groups.

5.3 Different types of quadrilaterals

INVESTIGATING QUADRILATERALS

1. The two pages that follow show different groups of quadrilaterals.
   (a) In which groups are both pairs of opposite sides parallel?
   (b) In which groups are only some adjacent sides equal?
   (c) In which groups are all four angles equal?
   (d) In which groups are all the sides in each quadrilateral equal?
   (e) In which groups are all four sides equal?
   (f) In which groups is each side perpendicular to the sides adjacent to it?
   (g) In which groups are opposite sides equal?
   (h) In which groups is at least one pair of adjacent sides equal?
   (i) In which groups is at least one pair of opposite sides parallel?
   (j) In which groups are all the angles right angles?
2. The figures in group 1 are called **parallelograms**.
   (a) What do you observe about the opposite sides of parallelograms?
   (b) What do you observe about the angles of parallelograms?
3. The figures in group 2 are called **kites**.
   (a) What do you observe about the sides of kites?
   (b) What else do you observe about the kites?

Group 1



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Misconceptions

Learners cannot identify opposite sides or adjacent sides.

Learners may be uncertain how to find the properties of quadrilaterals. (The Learner Book guides learners step-by-step.)

Answers

1. (a) 1; 3; 4 and 6 (b) 2
   (c) 4 and 6 (d) 3 and 6
   (e) 3 and 6 (f) 4 and 6
   (g) 1; 3; 4; 6 (h) 2; 3; 6
   (i) 1; 3; 4; 5; 6 (j) 4 and 6
2. (a) They are equal and parallel.
   (b) Opposite angles are equal.
3. (a) Two pairs of adjacent sides are equal.
   (b) One pair of opposite angles is equal.

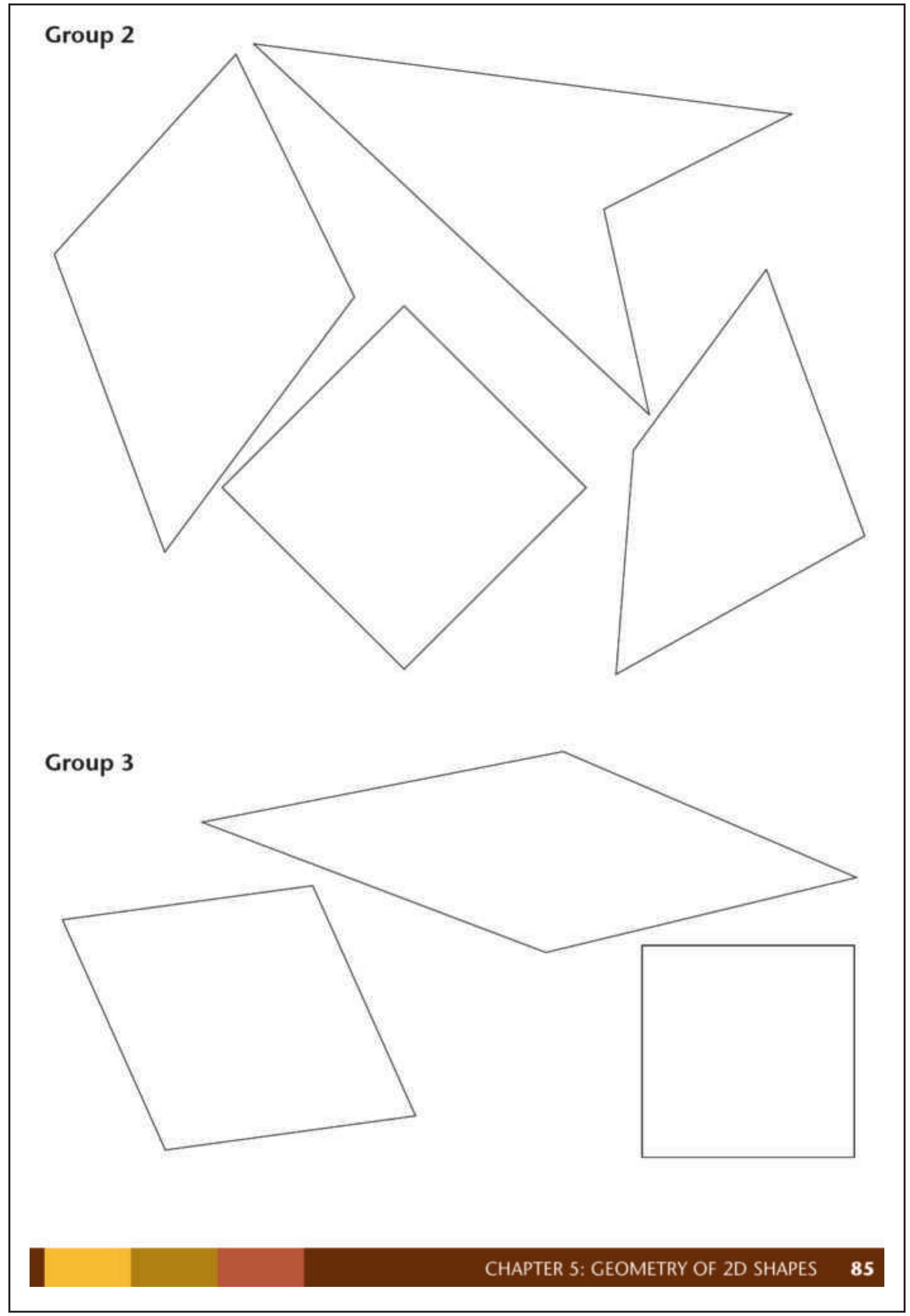
Teaching guidelines

Ask questions to make learners think about the connections between the groups of shapes, for example:

- Could a square be considered a rhombus?
- Could a rectangle be considered a parallelogram?

Show learners what the reasoning should be. What does the shape need to have to belong to the group? For example, what does a square need to have to be considered a rhombus? (All four sides have to be the same length).

It may be that the shape has more than the minimum required properties. As in the case of the square being a rhombus, it has to have all its sides equal in length, which it has; it has the extra property that all its angles are also equal; this is what makes it a special rhombus.



Answers

4. (a) All sides are equal; opposite sides are parallel.
   (b) Opposite angles are equal.
5. (a) They are equal and parallel.
   (b) All angles are right angles (90°).
   (c) They are not equal, unless the rectangle is a square (a special type of rectangle).
6. One pair of opposite sides is parallel.
7. (a) All sides are equal and opposite sides are parallel.
   (b) All the angles are right angles.

Additional notes for teachers (refer to TG page 92)

Young minds typically do not make deductive connections, for example, if asked: “Is a square a rectangle?” most learners will say no, because the shapes look different. Thinking on this level depends on the visual stimulus and leads to sorting shapes because of what they look like. If learners are able to operate on the next cognitive level, they are able to apply deductive reasoning and make connections between different sets of information.

Sorting quadrilaterals according to their properties is an important activity which helps learners move cognitively from one thinking level to another.

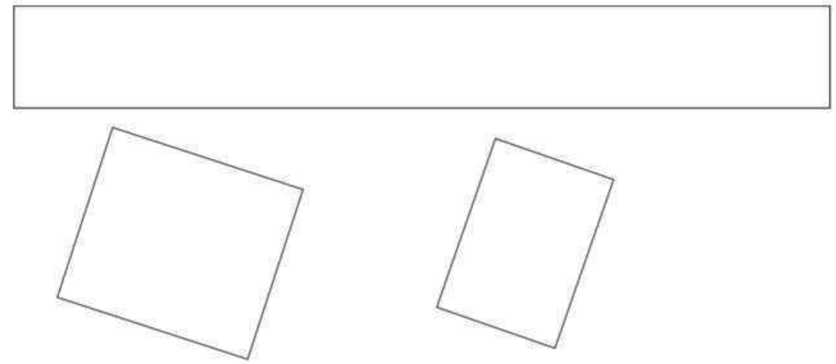
Thinking on the next level applies the mind to the available information and learners can then answer the question above (Is a square a rectangle?) and similar questions by comparing properties: “ What properties must a square have to be classified a rectangle?” This is the first step towards classifying.

We want learners to be able to analyse and make deductions and connections in order to solve problems. They will usually not move from one thinking level to the next spontaneously. Teachers have to create the activities that enable learners to do that.

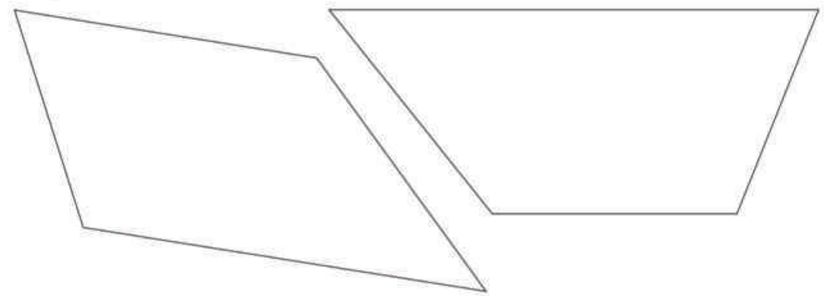
Working with 2D shapes like quadrilaterals lends itself to creating activities of sorting mentioned above.

Making sure that learners have a sound understanding of basic concepts and developing new ideas on these concepts may lead to a deeper understanding of geometry.

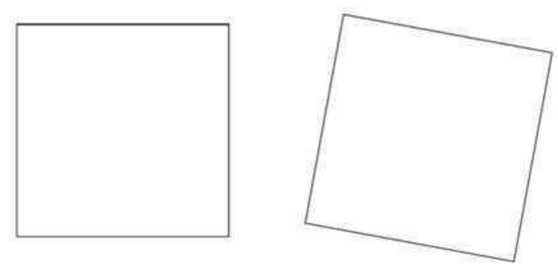
Group 4



Group 5



Group 6



4. The figures in group 3 are called **rhombi**.
   (a) What do you observe about the sides of rhombi?
   (b) What else do you observe about the rhombi?

Note:
One **rhombus**; two or more **rhombi**.

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COMPARING AND DESCRIBING SHAPES

Teaching guidelines

Remind learners to interpret and use the information given by the markings on the drawings.

Talk through the meaning of the markings on each drawing.

Answers

1. See LB page 87 alongside.
2. Group A: All of the sides in each figure are equal.
   Group B: At least one pair of opposite sides is parallel.
   At least one pair of opposite sides is equal.
3. Group A: One is a triangle (three sides); the other two are quadrilaterals. The triangle has no parallel sides, the others have. In the rhombus, all angles are not equal.
   Group B: The trapezium has only one pair of sides parallel and only one pair of opposite sides equal. In the rectangle, all angles are equal; the rectangle has no acute angles and no obtuse angles.

FINDING UNKNOWN SIDES IN QUADRILATERALS

Notes on the questions

Learners apply reasoning using the properties of shapes.

Teaching guidelines

Question 4 on LB page 88 enables you to check whether your learners know and use the conventions of naming quadrilaterals (starting at a vertex and moving in one direction).

Check their use of the conventions to indicate equal and parallel sides and right angles.

Answers

1. (a) Parallelogram, because two opposite sides are equal and the other two opposite sides are also equal.
   (b) CD (AB = CD is given)
   (c) 2,5 m (AD = BC; AD = 2,5 m)

---

5. The figures in group 4 are called **rectangles**.
   (a) What do you observe about the opposite sides of rectangles?
   (b) What do you observe about the angles of rectangles?
   (c) What do you observe about the adjacent sides of rectangles?
6. The figures in group 5 are called **trapeziums**.
   (a) What do you observe about the opposite sides of trapeziums?

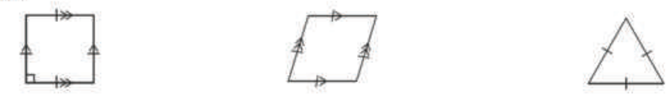
The arrows show which sides are parallel to each other.

7. The figures in group 6 are called **squares**.
   (a) What do you observe about the sides of squares?
   (b) What do you observe about the angles of squares?

COMPARING AND DESCRIBING SHAPES

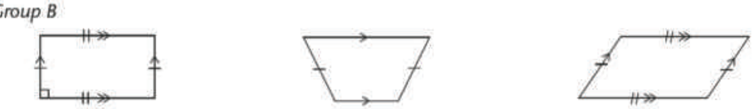
1. Name each shape in each group.

Group A



(a) square (b) rhombus (c) equilateral triangle

Group B



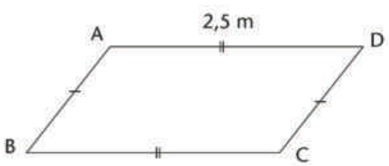
(a) rectangle (b) trapezium (c) parallelogram

2. In what way(s) are the figures in each group the same?
3. In what way(s) does one of the figures in each group differ from the other two figures in the group?

FINDING UNKNOWN SIDES IN QUADRILATERALS

Use what you know about the sides and angles of quadrilaterals to answer the following questions. **Give reasons for your answers.**

1. (a) What type of quadrilateral is ABCD?
   (b) Name a side equal to AB.
   (c) What is the length of BC?



2. (a) Kite, because two adjacent sides are equal and the other two adjacent sides are also equal.
   (b) See LB page 88 alongside.
3. (a) Trapezium, because one pair of sides is parallel.
   (b) 24 mm (LM = JK; LM = 24 mm)
4. See LB page 88 alongside.

5.4 Circles

Notes on the questions

Learners worked with circles in the previous chapter. In this chapter they learn the terminology associated with circles.

Teaching guidelines

Learners will be familiar with the term **radius**, but you will now explain the meaning of **diameter**.

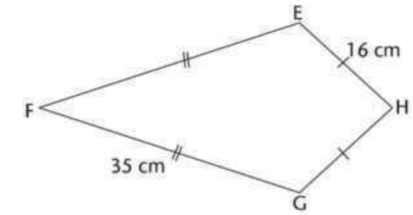
Misconceptions

Learners confuse radius and diameter.

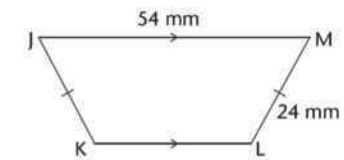
Answers

1. (a) See LB page 88 alongside.
   (b) See LB page 88 alongside.

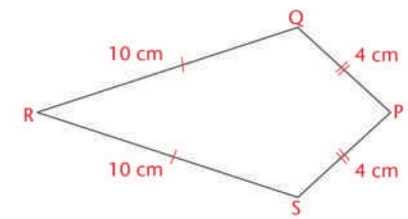
2. (a) What type of quadrilateral is EFGH?
   (b) What are the lengths of the sides EF and GH?
   35 cm (FG = EF)
   16 cm (EH = GH)



3. (a) What type of quadrilateral is JKLM?
   (b) What is the length of JK?

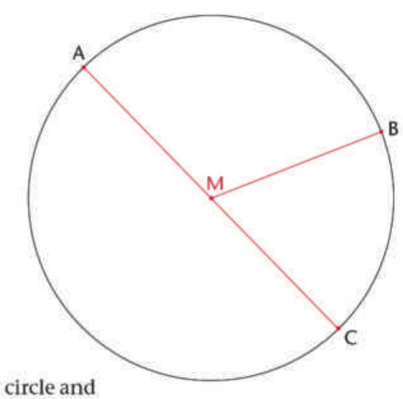


4. Figure PQRS is a kite with PQ = 4 cm and QR = 10 cm. Copy and complete the following drawing by:
   (a) labelling the vertices of the kite
   (b) showing on the drawing which sides are equal
   (c) labelling the length of each side.



5.4 Circles

1. (a) Copy the circle on the right. Make a dot in the middle of the circle. Write the letter M next to the dot. If your dot is in the middle of the circle, it is called the **midpoint** or **centre**.
   (b) Draw lines MA, MB and MC from M to the red points A, B and C.



The three red points are on the circle with midpoint M.

A straight line, such as AC, drawn across a circle and passing through its midpoint is called the **diameter** of the circle.

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Answers

2. Learners' own measurements. They may differ slightly.

Teaching guidelines

Draw a circle on the board like the one in the book. Draw the parts of the circle on the drawing one by one: the radius; the chord; the diameter.

Introduce the words **segments** and **sector**.

Explain how to recognise a sector – i.e. it is the area between two radii and an arc. A sector always has a vertex at the centre of the circle; like a slice of a pie or a cake.

Explain how to recognise a segment – i.e. it is the area between a single line drawn through the circle (a chord) and the arc it forms.

Note that:

- a semi-circle can be seen as both a segment and a sector
- a diameter can also be seen as a chord of a circle.

Misconceptions

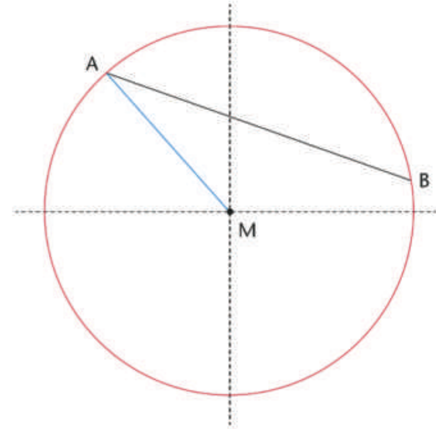
Learners confuse segments and sectors.

2. Measure MA, MB and MC.
   If MA, MB and MC are equal in length, you have chosen the midpoint well.
   If they are not equal, you may wish to improve your sketch of a circle and its parts.

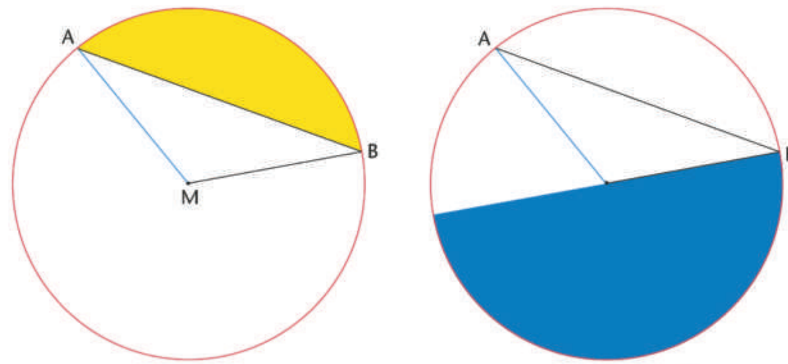
A straight line from the midpoint of a circle to a point on the circle is called a **radius** of the circle.

The blue line, MA, is a **radius**. Any straight line from the centre to the circle is a radius.

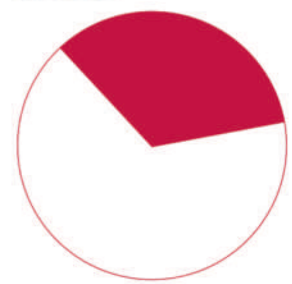
The black line AB joins two points on the circle. We call this line a **chord** of the circle.



In the following two diagrams, the coloured sections are **segments** of a circle. A segment is the area between a chord and an arc.



In the circle on the right, the red section is called a **sector** of a circle. As you can see, a sector is the region between two radii and an arc.



CHAPTER 5: GEOMETRY OF 2D SHAPES 89

5.5 Similar and congruent shapes

Notes and background information

If two geometrical shapes have the same shape, they are called similar. They do not have to be the same size to be similar. This means that one shape can be matched with another by uniformly enlarging or shrinking (scaling) it and with possible additional transformations, such as reflection, rotation or translation. This means that a shape that is similar to another shape, can be rescaled and repositioned so that it fits exactly on the other. For example, all circles are similar to each other, all squares are similar to each other, and all equilateral triangles are similar to each other, but not all rectangles are similar to each other, and not all isosceles triangles are similar to each other.

The reason why all squares are similar is because their angles are equal and their corresponding sides are in proportion. This applies to all equilateral triangles as well. On the other hand, not all rectangles are similar as the angles of all rectangles are equal but two rectangles will only be similar if their corresponding sides are in the same ratio (sides are in proportion).

Congruent shapes have the same size and shape if their corresponding angles and sides are equal.

All congruent shapes are also similar.

The choice of figures on LB pages 90 and 91 forces learners to consider shape and size when asked to identify differences in three groups of figures. This should help familiarise them with the concepts of **similarity** and **congruence** when these two concepts are formally presented after the initial activity.

Teaching guidelines

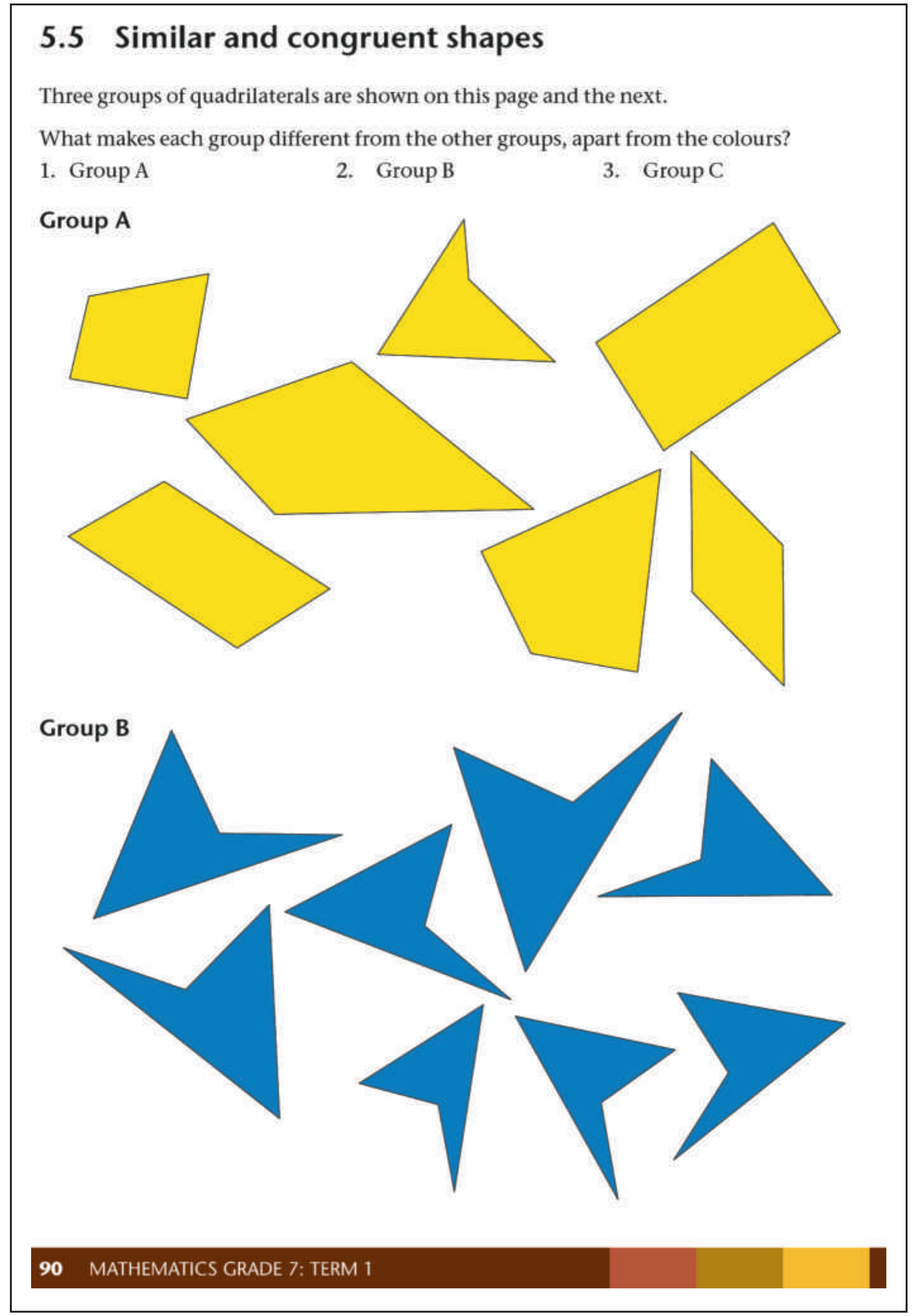
Let learners investigate the elements in the groups by measuring and comparing the properties. In the first group, they should find that the angles differ from shape to shape.

In the second group, the measurement of the angles in the shapes should be the same, but the lengths of the sides differ. Suggest to learners that they try calculating the ratios between the sides.

The shapes in the third group all have the same size and are therefore congruent.

Misconceptions

Learners often think that similar shapes cannot be congruent and don't realise that all congruent shapes are also similar.



5.5 Similar and congruent shapes

Three groups of quadrilaterals are shown on this page and the next.

What makes each group different from the other groups, apart from the colours?

1. Group A 2. Group B 3. Group C

Group A

Group B

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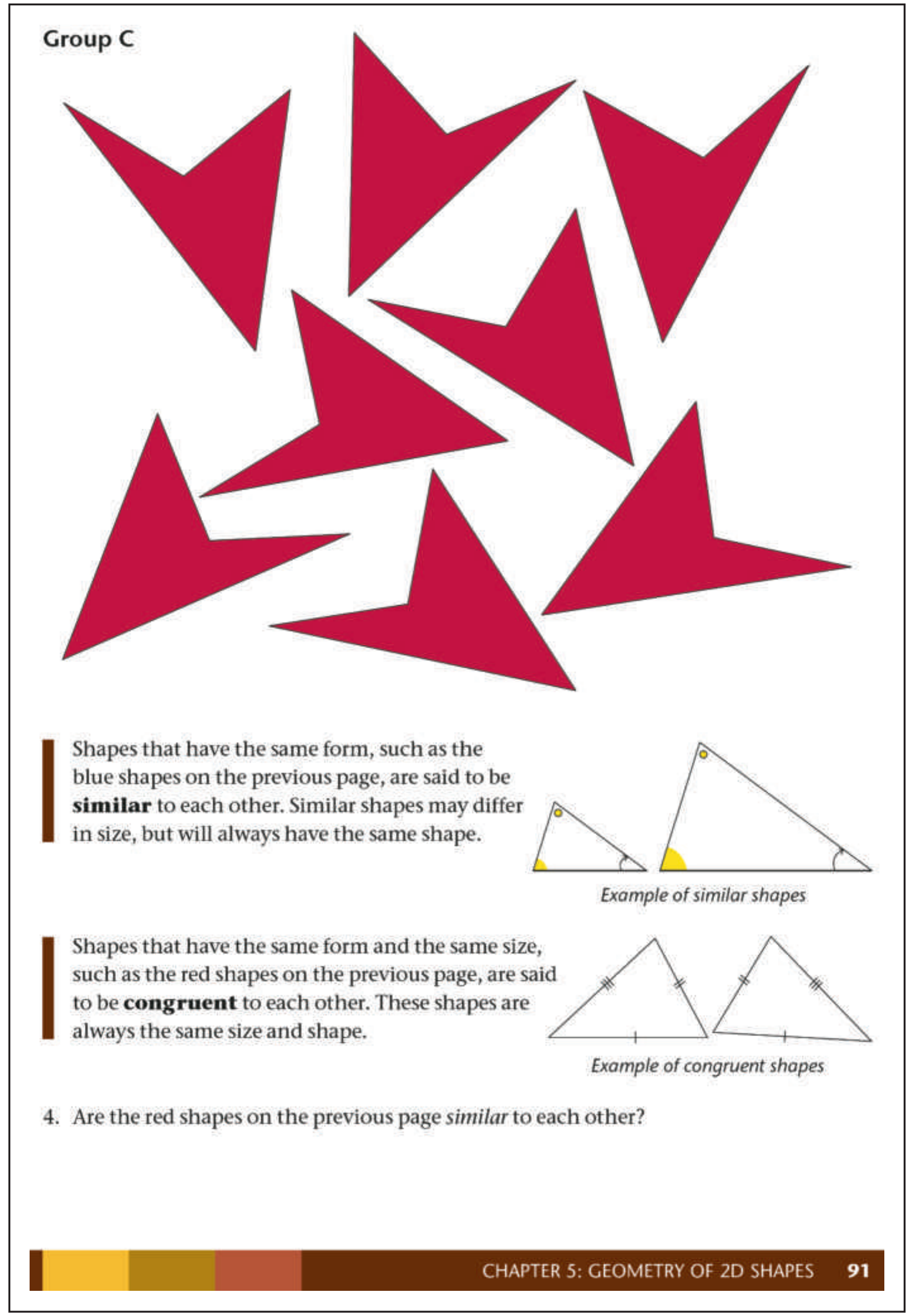
Answers

1. Group A: All of the quadrilaterals have different properties. None of them have the same shape and/or size. They do not belong to one class of quadrilaterals.
2. Group B: All of them have the same shape but have different sizes.
3. Group C: All have the same shape and size.
4. Yes

Additional notes on similarity and congruence

Learners can develop the concept of congruence by understanding the properties of shapes. The concept of similarity can be developed from understanding proportional reasoning applied to shapes.



Group C

Shapes that have the same form, such as the blue shapes on the previous page, are said to be **similar** to each other. Similar shapes may differ in size, but will always have the same shape.

Example of similar shapes

Shapes that have the same form and the same size, such as the red shapes on the previous page, are said to be **congruent** to each other. These shapes are always the same size and shape.

Example of congruent shapes

4. Are the red shapes on the previous page *similar* to each other?

CHAPTER 5: GEOMETRY OF 2D SHAPES 91

Teaching guidelines

Point out that the makings on a drawing apply to that drawing only. For example, in Group D, the first shape and the third shape (both squares), show the same markings on the sides. It is obvious that the side length of the first square is less than that of the second square; therefore, the markings simply indicate that the sides in a shape all have the same length.

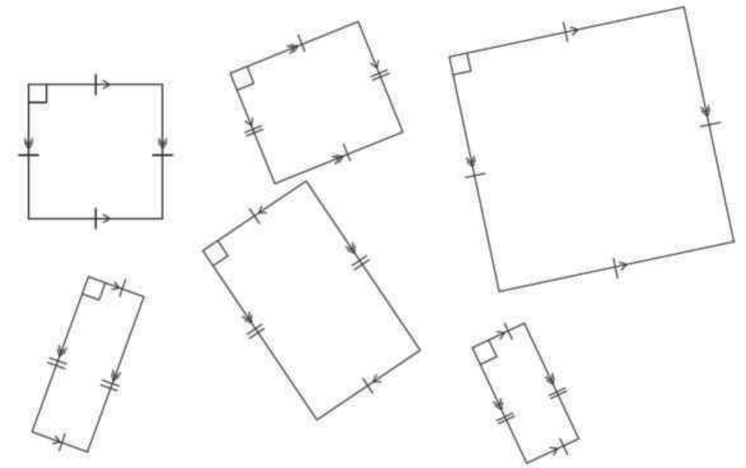
In Group E, the markings are meant to show equality. The sides in each shape that are marked with similar marks have the same length, and the angles marked with a small open circle have the same size. In the same way, the angles marked the same in group F are equal, but the side lengths of the triangles are obviously not equal.

Answers

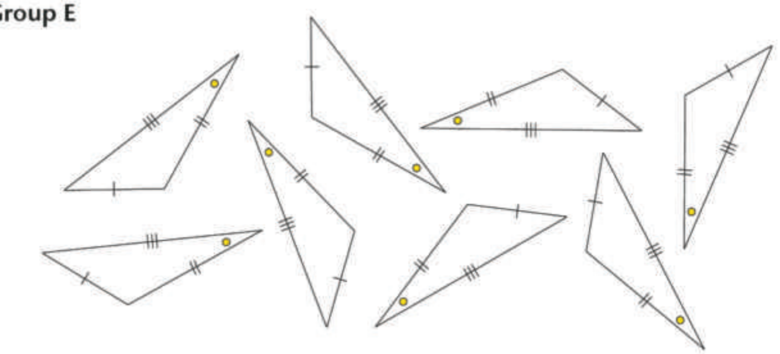
5. See LB page 92 alongside.

5. Look at groups D, E, F, and G on this page and the next. In each case, say whether the shapes are similar and congruent, similar but not congruent, or neither similar nor congruent.
   - (a) Group D neither similar nor congruent
   - (b) Group E similar and congruent
   - (c) Group F similar but not congruent
   - (d) Group G similar and congruent

Group D



Group E



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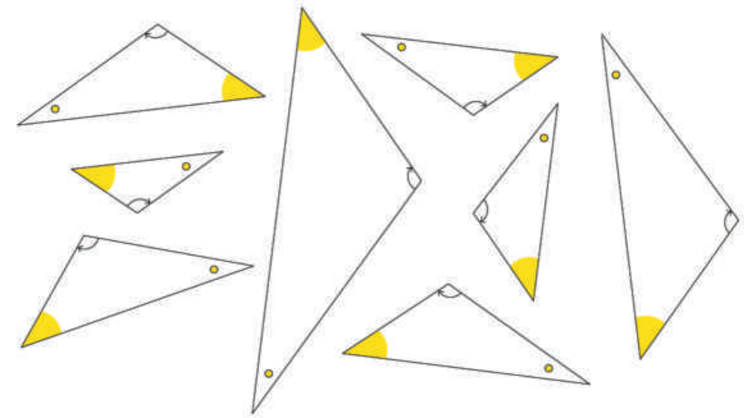
Answers (continued)

Group F



Group G

CHAPTER 5: GEOMETRY OF 2D SHAPES 93

Term 2

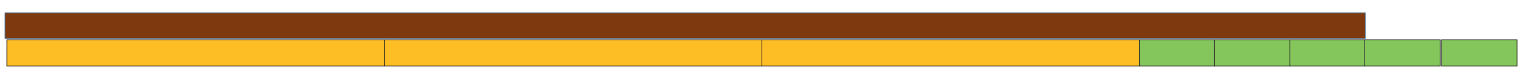
Grade 7 Term 2 Chapter 6 Fractions

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
6.1 Measuring accurately with parts of a unit	Describe the same part of a whole, collection, quantity or unit of measurement with different fractions; specify equivalent fractions	Pages 95 to 99
6.2 The fraction notation	Produce equivalent fractions with a formula	Pages 99 to 100
6.3 Adding fractions	Add and subtract fractions	Pages 100 to 101
6.4 Tenths and hundredths (percentages)	Introduce percentages	Pages 101 to 103
6.5 Thousandths, hundredths and tenths	Introduce fractions related to decimal fractions	Page 103
6.6 Fraction of a fraction	Determine a fraction of a fraction	Pages 104 to 105
6.7 Multiplying with fractions	Multiply fractions	Pages 106 to 109
6.8 Ordering and comparing fractions	Use equivalent fractions to order and compare fractions	Page 109

CAPS time allocation	5 hours
CAPS content specification	Pages 93 to 108

Mathematical background

It is widely assumed that fractions were invented in the context of accurate measurement, to facilitate the measurement of objects or parts of objects smaller than the commonly used unit of measurement. This is reflected in the Latin names of our current units of measurement, for example centimetres (hundredths of a metre) and millimetres (thousandths of a metre). The length of the brown strip measured with the yellow strip as a unit is three and three fifths of the yellow unit.



- Fractions are used as **measures** (refer to the example above).
- Fractions are used to describe **parts of whole objects**, for example “quarter of an apple” or “a half-loaf of bread”.
- Fractions are used to describe **parts of collections**, for example “three eighths of the learners in a school”.
- Fractions are used to describe **parts of non-physical quantities**, for example “63 hundredths of the available marks”, normally written in percentage notation.
- Fractions are used to explain decimals: 34, 56 is $30 + 4 + \frac{5}{10} + \frac{6}{100}$ or 3 tens + 4 units + 5 tenths + 6 hundredths.

A **fraction** is a number of equal parts of the same whole, collection, quantity or unit of measurement.

6.1 Measuring accurately with parts of a unit

A STRANGE MEASURING UNIT

Background information

Learners would read something like $\frac{5}{8}$ simply as "five over eight" without recognising it as **five eighths**, which is what it actually means. It is critical that learners learn to say the fraction names, because without that there is no guarantee that they have a sound understanding of fractions.

It is useful to distinguish the following **three phases** in the development of the concept of equivalent fractions in learners' minds:

- Awareness that the same part of a whole, collection, quantity or unit of measurement can be described with different fractions (section 6.1).
- The ability to specify equivalent fractions (latter part of section 6.1).
- Producing equivalent fractions with a formula (section 6.2).

In everyday life fractions are used in an approximate way (for example referring to a quarter of an apple is seldom exactly a quarter, and different apples may differ in size). In Mathematics fractions are used in an exact way. Since measurement is a very appropriate context for fractions, learners are introduced to a **new** unit – the greystick, which is used to measure different lengths. The context creates the *need* for fractions because it is not possible to measure accurately using only whole units. In order to measure accurately, the measuring unit must be subdivided into smaller fractional parts, like fifths or tenths.

When a whole is divided into equal parts, there are three quantities involved: the number of parts, the size of each part and the size of the whole.

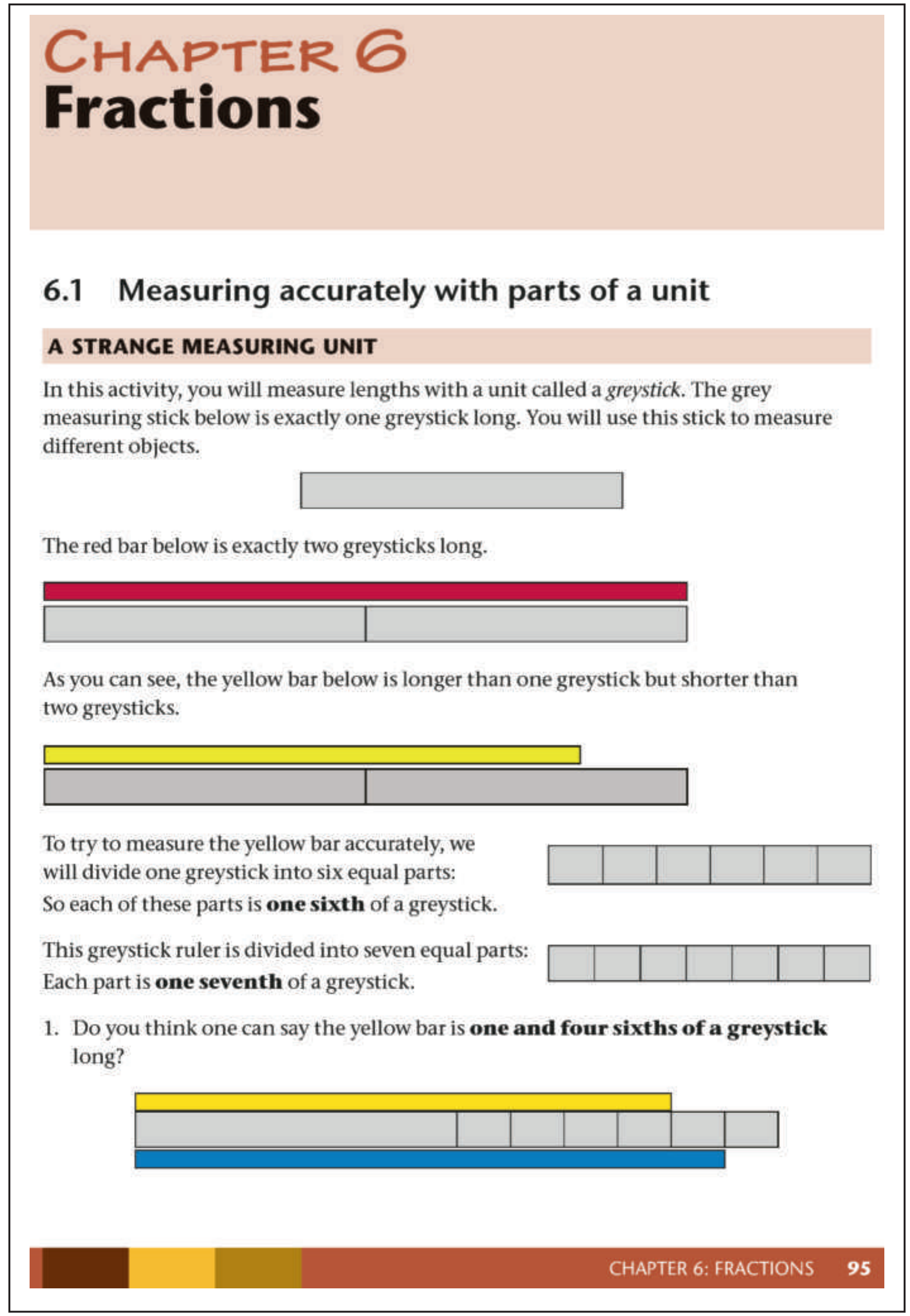
- Finding the size of each part when the size of the whole and the number of parts are given resembles **sharing** situations.
- Finding the number of equal parts when the size of the whole and the size of each part are given resembles **grouping** situations.

Teaching guidelines

Discuss the fact that, by dividing a given length into different equal parts, the same length can be described in different ways.

Answers

1. Yes



CHAPTER 6
Fractions

6.1 Measuring accurately with parts of a unit

A STRANGE MEASURING UNIT

In this activity, you will measure lengths with a unit called a *greystick*. The grey measuring stick below is exactly one greystick long. You will use this stick to measure different objects.

The red bar below is exactly two greysticks long.

As you can see, the yellow bar below is longer than one greystick but shorter than two greysticks.

To try to measure the yellow bar accurately, we will divide one greystick into six equal parts:
So each of these parts is **one sixth** of a greystick.

This greystick ruler is divided into seven equal parts:
Each part is **one seventh** of a greystick.

1. Do you think one can say the yellow bar is **one and four sixths of a greystick** long?

CHAPTER 6: FRACTIONS 95

Misconceptions

Learners see a fraction like $\frac{2}{5}$ as one whole number on top of another whole number, which is not true. When we say "two fifths" we use the whole number 2 to indicate a number of objects. However, the objects are no longer wholes in the normal sense of the word; the objects are now fifths. It is not the two in "two fifths" that constitutes the fraction; the two is still a whole number. What makes "two fifths" a fraction is the kind of object that is counted. The first sequence of questions with the heading **A strange measuring unit** in section 6.1 is designed to develop this basic understanding of fractions. This works best if the symbolic notation for fractions is not used at this stage, and if learners are required to say the fraction names out aloud. Many learners may find these questions very hard, simply because they do not have any concept of fractions yet. It is critical that they acquire this concept now.

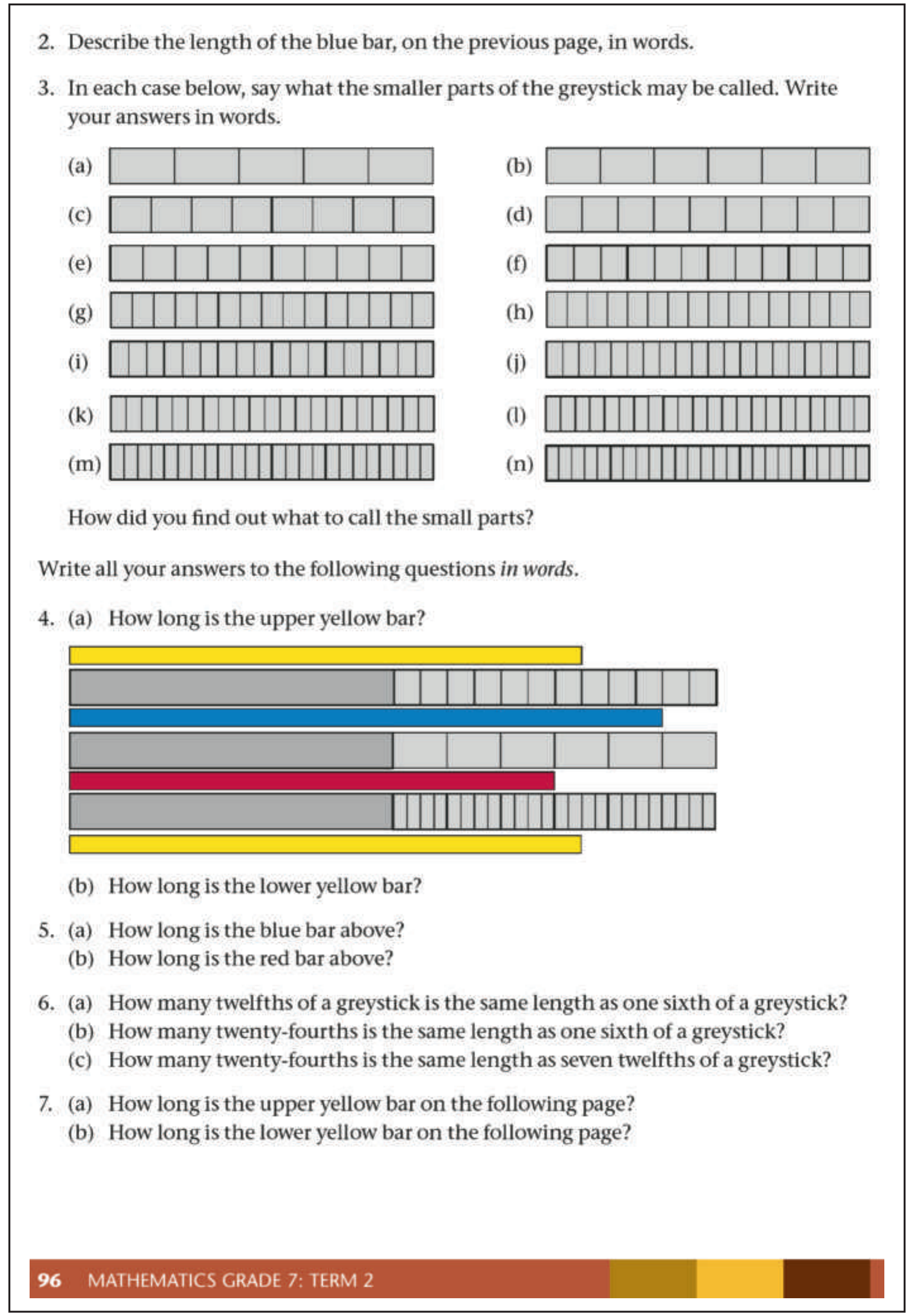
Note on question 4

Armed with the language of equal parts, learners can describe the length of the yellow bar as either "one and seven twelfths of a greystick" or as "one and fourteen twenty-fourths of a greystick". This lays the foundation for equivalent fractions.

Answers

2. one and five sixths of a greystick
3. (a) fifths (b) sixths (c) eighths
   (d) ninths (e) tenths (f) twelfths
   (g) fifteenths (h) sixteenths (i) eighteenths
   (j) twentieths (k) twenty-firsts (l) twenty-seconds
   (m) twenty-fourths (n) twenty-fifths

   By counting the small parts and using ordinal number names to describe or name the parts.
4. (a) one and seven twelfths (b) one and fourteen twenty-fourths
5. (a) one and ten twelfths of a greystick/one and five sixths of a greystick
   (b) one and three sixths/one and twelve twenty-fourths of a greystick
6. (a) two twelfths (b) four twenty-fourths
   (c) fourteen twenty-fourths
7. (a) one and eight tenths of a greystick
   (b) one and four fifths of a greystick



2. Describe the length of the blue bar, on the previous page, in words.
3. In each case below, say what the smaller parts of the greystick may be called. Write your answers in words.

   (a) (b) (c) (d) (e) (f) (g) (h) (i) (j) (k) (l) (m) (n)

   How did you find out what to call the small parts?

Write all your answers to the following questions *in words*.

4. (a) How long is the upper yellow bar?

   (b) How long is the lower yellow bar?
5. (a) How long is the blue bar above?
   (b) How long is the red bar above?
6. (a) How many twelfths of a greystick is the same length as one sixth of a greystick?
   (b) How many twenty-fourths is the same length as one sixth of a greystick?
   (c) How many twenty-fourths is the same length as seven twelfths of a greystick?
7. (a) How long is the upper yellow bar on the following page?
   (b) How long is the lower yellow bar on the following page?

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Answers

(c) one and seven tenths of a greystick/one and fourteen twentieths of a greystick

(d) one and three fifths of a greystick/one and twelve twentieths of a greystick

8. (a) three fifths

   (b) three fourths (or three quarters)

DESCRIBE THE SAME LENGTH IN DIFFERENT WAYS (EQUIVALENT FORMS)

Background information

The questions in this section are designed to develop the concept of equivalent fractions. This concept forms the basis for ordering and comparing fractions as required by CAPS (page 49). Learners cannot do any of the operations with fractions without understanding the concept of equivalent fractions. The CAPS requires that learners should "use knowledge of equivalent fractions to add and subtract common fractions". This is only possible if learners have a thorough grounding in equivalent fractions.

Many FET learners and even university students currently have the serious misconception that common fractions, decimal fractions and percentages are different kinds of numbers. This may be partly due to the misleading phrases **common fractions** and **decimal fractions**, which suggest different kinds of fractions. Not understanding the decimal and percentage notations in terms of tenths, hundredths, thousandths, etc. is possibly the bigger cause of the misconceptions. This is why the CAPS prioritises tenths and hundredths in the first sentence of the section on fractions on CAPS pages 49–50.

Teaching guidelines

Illustrate the concept of equivalent fractions by describing the same portion of a greystick using different equal parts.

Answers

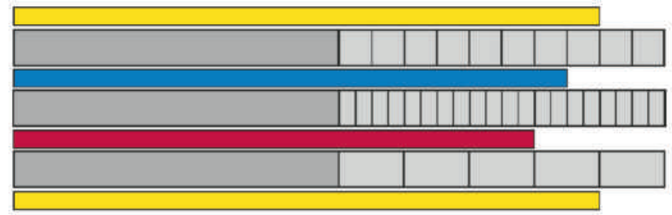
1. (a) one eighteenth (b) three eighteenths

   (c) six eighteenths (d) fifteen eighteenths

2. (a) Sample answer, based on the abovementioned greysticks: nine twelfths; twelve sixteenths; fifteen twentieths; eighteen twenty-fourths

   (b) eight twelfths and sixteen twenty-fourths

3. See LB page 98 on the following page.

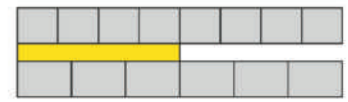


(c) How long is the blue bar?
(d) How long is the red bar?

8. (a) How many fifths of a greystick is the same length as 12 twentieths of a greystick?
   (b) How many fourths (or quarters) of a greystick is the same length as 15 twentieths of a greystick?

DESCRIBE THE SAME LENGTH IN DIFFERENT WAYS

Two fractions may describe the same length. You can see here that three sixths of a greystick is the same as four eighths of a greystick.



When two fractions describe the same portion we say they are **equivalent**.

1. (a) What can each small part on this greystick be called?

   (b) How many eighteenths is one sixth of the greystick?
   (c) How many eighteenths is one third of the greystick?
   (d) How many eighteenths is five sixths of the greystick?

2. (a) Write (in words) the names of four different fractions that are all equivalent to three quarters.
   (b) Which equivalents for two thirds can you find on the greysticks?

3. The information that two thirds is equivalent to four sixths, to six ninths and to eight twelfths is written in the second row of the table on the following page. Copy the table and complete the other rows of the table in the same way.

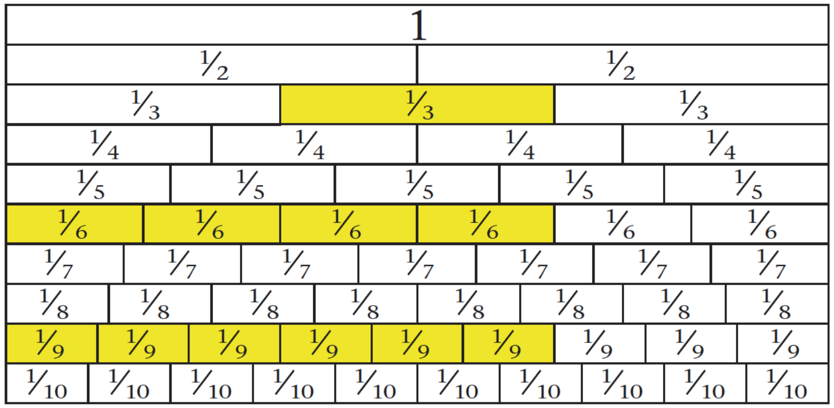
CHAPTER 6: FRACTIONS 97

Critical knowledge that must be understood by learners

- Equivalent fractions are different, but they represent the same quantities.
- Equivalent fractions are different ways to represent the same part of a whole, collection, quantity or measurement.

Additional information on equivalent fractions

A fraction wall can be used to investigate the concept of equivalent fractions.



Answers

4. See LB page 98 alongside.
5. See LB page 98 alongside.
6. See LB page 98 alongside.

thirds	fourths	fifths	sixths	eighths	ninths	tenths	twelfths	twentieths
1	–	–	2	–	3	–	4	–
2	–	–	4	–	6	–	8	–
–	3	–	–	6	–	–	9	15
–	–	1	–	–	–	2	–	4
–	–	2	–	–	–	4	–	8
–	–	3	–	–	–	6	–	12
–	–	4	–	–	–	8	–	16

4. Copy and complete this table in the same way as the table in question 3.

fifths	tenths	fifteenths	twentieths	twenty-fifths	fiftieths	hundredths
1	2	3	4	5	10	20
2	4	6	8	10	20	40
3	6	9	12	15	30	60
4	8	12	16	20	40	80
5	10	15	20	25	50	100
6	12	18	24	30	60	120
7	14	21	28	35	70	140

5. Copy the greysticks. Draw on them to show that three fifths and nine fifteenths are equivalent. Draw freehand; you need not measure and draw accurately.

6. Copy and complete these tables in the same way as the table in question 4.

eighths	sixteenths	twenty-fourths
1	2	3
2	4	6
3	6	9
4	8	12
5	10	15
6	12	18
7	14	21
8	16	24
9	18	27

twenty-fourths	sixths	twelfths	eighteenths
4	1	2	3
8	2	4	6
12	3	6	9
16	4	8	12
20	5	10	15
24	6	12	18
28	7	14	21
32	8	16	24
36	9	18	27

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Note on question 7

This question provides a lead into addition of fractions with the same denominator.

Answers

7. (a) eight twelfths
   (b) eight twelfths
   (c) fourteen twelfths
   (d) seven twelfths (one third = four twelfths; one quarter = three twelfths)

6.2 The fraction notation

Background information

This section lays the foundation for the **third phase** in the development of the concept of equivalent fractions, namely the production of equivalent fractions with a formula.

Teaching guidelines

Use a fraction wall to revise the concept of equivalent fractions, for example:

- $\frac{2}{3} = \frac{4}{6}$ on a fraction wall
- $\frac{6}{9} = \frac{2}{3}$ on a fraction wall

Answers

1. three eighths
2. three fifths
3. two fifths
4. Two tenths is blue and eight tenths is red.
5. (a) Four twentieths is blue; ten twentieths is red; five twentieths is white
   (b) Sample answer: one fifth is blue; one half is red; one quarter is white
6. four ninths
7. yellow: three twenty-fourths; blue: fifteen twenty-fourths; red: six twenty-fourths

7. (a) How much is five twelfths plus three twelfths?
   (b) How much is five twelfths plus one quarter?
   (c) How much is five twelfths plus three quarters?
   (d) How much is one third plus one quarter? It may help if you work with the equivalent fractions in twelfths.

6.2 The fraction notation

This strip is divided into eight equal parts. Five eighths of this strip is red.

1. What part of the strip is blue?
2. What part of this strip is yellow?

3. What part of the strip is red?
4. What part of this strip is coloured blue and what part is coloured red?

5. (a) What part of this strip is blue, what part is red and what part is white?

   (b) Express your answer differently with equivalent fractions.
6. A certain strip is not shown here. Two ninths of the strip is blue, and three ninths of the strip is green. The rest of the strip is red. What part of the strip is red?
7. What part of the strip below is yellow, what part is blue, and what part is red?

The number of parts in a fraction is called the **numerator** of the fraction. For example, the numerator in five sixths is five.

The type of part in a fraction is called the **denominator**. It is the name of the parts that are being referred to and it is determined by the size of the part. For example, sixths is the denominator in five sixths.

$^5/_6$ is a short way to write five sixths.
We may also write $\frac{5}{6}$.
Even when we write $^5/_6$ or $\frac{5}{6}$, we still say "five sixths".
$/_6$ and $\frac{\quad}{6}$ are short ways to write *sixths*.

To **enumerate** means "to find the number of".
To **denominate** means "to give a name to".

Teaching guidelines (continued)

Learners should have a clear understanding of the following:

- The number of parts in a fraction is called the **numerator** of the fraction. For example, in five sixths the numerator is five.
- The type of part in a fraction is called the **denominator**. It is the name of the parts that are being referred to and is determined by the size of that part. For example, in five sixths the denominator is sixths.
- The short way to write **five sixths** is $\mathbf{\frac{5}{6}}$ and to write **sixths** is $\mathbf{\frac{\ }{6}}$.

Answers

8. (a) $\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$; Yes (b) $\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$; Yes

(c) $\frac{3}{4} \times \frac{4}{4} = \frac{12}{16}$; Yes (d) $\frac{3}{4} \times \frac{6}{6} = \frac{18}{24}$; Yes

6.3 Adding fractions

BIGGER AND SMALLER PARTS

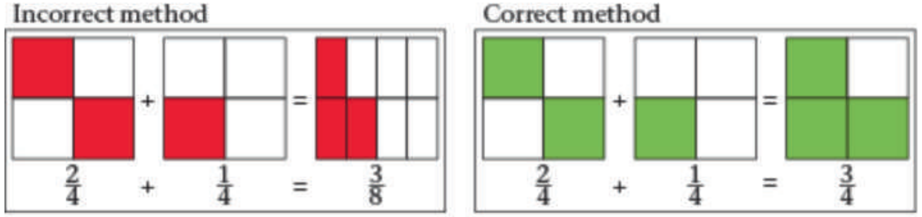
Background information

Misconceptions of fractions often lead to mistakes such as the following:

$\frac{2}{4} + \frac{1}{4} = \frac{3}{8}$

This problem can be addressed by using any of the following strategies:

- the correct fraction names: two quarters + one quarter = three quarters
- a diagram to illustrate the correct way to add fractions:



Remember: 2 quarters + 1 quarter = 3 quarters

Teaching guidelines

Illustrate the correct way to add fractions with equal denominators.

The numerator (number of parts) is written above the line of the fraction: $\frac{numerator}{...}$

The denominator is indicated by a number written below the line: $\frac{...}{denominator}$

8. Consider the fraction three quarters. It can be written as $\frac{3}{4}$.

(a) Multiply both the numerator and the denominator by two to form a new fraction. Is the new fraction equivalent to $\frac{3}{4}$? You may check on the diagram.

(b) Multiply both the numerator and the denominator by three to form a new fraction. Is the new fraction equivalent to $\frac{3}{4}$?

(c) Multiply both the numerator and the denominator by four to form a new fraction. Is the new fraction equivalent to $\frac{3}{4}$?

(d) Multiply both the numerator and the denominator by six to form a new fraction. Is the new fraction equivalent to $\frac{3}{4}$?

6.3 Adding fractions

BIGGER AND SMALLER PARTS

Gertie was asked to solve this problem:

A team of road-builders built $\frac{8}{12}$ km of road in one week, and $\frac{10}{12}$ km in the next week. What is the total length of road that they built in the two weeks?

She thought like this to solve the problem:

*$\frac{8}{12}$ is **eight twelfths** and $\frac{10}{12}$ is **ten twelfths**, so altogether it is **eighteen twelfths**.*

I can write $\frac{18}{12}$ or "18 twelfths".

*I can also say 12 twelfths of a kilometre is 1 kilometre, so **18 twelfths** is **1 km and 6 twelfths of a kilometre**.*

This I can write as $1\frac{6}{12}$. It is the same as $1\frac{1}{2}$ km.

Gertie was also asked the question: How much is $4\frac{5}{9} + 2\frac{7}{9}$?

She thought like this to answer it:

$4\frac{5}{9}$ is four wholes and five ninths, and $2\frac{7}{9}$ is two wholes and seven ninths.

So altogether it is six wholes and 12 ninths. But 12 ninths is nine ninths (one whole) and three ninths, so I can say it is seven wholes and three ninths.

I can write $7\frac{3}{9}$.

Note on question 4(c)

Learners should use equivalent fractions to convert both fractions to $\frac{}{35}$.

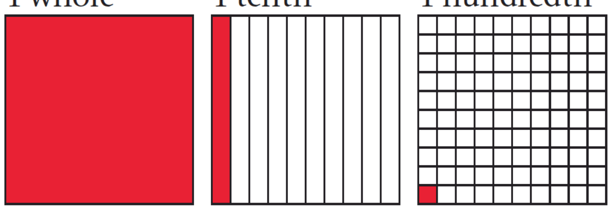
Answers

1. No, $7\frac{1}{3}$ is equivalent to $7\frac{3}{9}$, ($\frac{1}{3}$ is $\frac{3}{9}$ expressed in its simplest form).
2. $3\frac{2}{12}$ or $3\frac{1}{6}$
3. $5\frac{5}{8} - 3\frac{7}{8} = 4\frac{13}{8} - 3\frac{7}{8} = 1\frac{6}{8}$ or $1\frac{3}{4}$
4. (a) $3\frac{9}{7} - 3\frac{6}{7} = \frac{3}{7}$ (b) $3\frac{9}{7} = 4\frac{2}{7}$
   (c) $3\frac{30}{35} + 1\frac{28}{35} = 4\frac{58}{35} = 5\frac{23}{35}$ (d) $4\frac{15}{40} - 2\frac{32}{40} = 3\frac{55}{40} - 2\frac{32}{40} = 1\frac{23}{40}$
   (e) $1\frac{9}{30} - \frac{20}{30} = \frac{39}{30} - \frac{20}{30} = \frac{19}{30}$ (f) $3\frac{5}{10} - 1\frac{5}{10} = 2$
   (g) $\frac{25}{8} = 3\frac{1}{8}$ (h) $6\frac{8}{20} + 2\frac{5}{20} - \frac{10}{20} = 8\frac{3}{20}$
   (i) $\frac{65}{8} = 8\frac{1}{8}$ (j) $16\frac{32}{7} = 20\frac{4}{7}$
   (k) $5\frac{6}{7} - 2\frac{1}{3} = 5\frac{18}{21} - \frac{7}{21} = 3\frac{11}{21}$ (l) $\left(\frac{27}{10} + \frac{34}{10}\right) - \left(\frac{14}{10} + \frac{37}{10}\right) = \frac{61}{10} - \frac{51}{10} = \frac{10}{10} = 1$
5. $\frac{3}{4} + 2\frac{2}{4} + 3\frac{3}{4} + 3 + 1\frac{2}{4} = 9\frac{10}{4} = 11\frac{2}{4}$ pages or $11\frac{1}{2}$ pages

6.4 Tenths and hundredths (percentages)

Teaching guidelines

Introduce the concepts of tenths and hundredths.



Answers

1. (a) 100×3 biscuits = 300 biscuits (b) 500 sweets $\div$ 100 = 5 sweets
2. $10 \times 10 = 100$
3. (a) R2 $\div$ 100 = 2c (b) R2 $\div$ 5 = 40c (c) $\frac{25}{100}$ or $\frac{1}{4}$

1. Would Gertie be wrong if she said her answer was $7\frac{1}{3}$?
2. Senthereng has $4\frac{7}{12}$ bottles of cooking oil. He gives $1\frac{5}{12}$ bottles to his friend Willem. How much oil does Senthereng have left?
3. Margaret has $5\frac{5}{8}$ bottles of cooking oil. She gives $3\frac{7}{8}$ bottles to her friend Naledi. How much oil does Margaret have left?
4. Calculate each of the following:
   (a) $4\frac{2}{7} - 3\frac{6}{7}$ (b) $3\frac{6}{7} + \frac{3}{7}$ (c) $3\frac{6}{7} + 1\frac{4}{5}$
   (d) $4\frac{3}{8} - 2\frac{4}{5}$ (e) $1\frac{3}{10} - \frac{2}{3}$ (f) $3\frac{5}{10} - 1\frac{1}{2}$
   (g) $\frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8}$ (h) $6\frac{2}{5} + 2\frac{1}{4} - \frac{1}{2}$
   (i) $\frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8} + \frac{5}{8}$
   (j) $2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7} + 2\frac{4}{7}$
   (k) $(4\frac{2}{7} + 1\frac{4}{7}) - 2\frac{1}{3}$ (l) $(2\frac{7}{10} + 3\frac{2}{5}) - (1\frac{2}{5} + 3\frac{7}{10})$
5. Neo's report had five chapters. The first chapter was $\frac{3}{4}$ of a page, the second chapter was $2\frac{1}{2}$ pages, the third chapter was $3\frac{3}{4}$ pages, the fourth chapter was three pages and the fifth chapter was $1\frac{1}{2}$ pages. How many pages was Neo's report in total?

6.4 Tenths and hundredths (percentages)

1. (a) 100 children each get three biscuits. How many biscuits is this in total?
   (b) 500 sweets are shared equally between 100 children. How many sweets does each child get?
2. The picture below shows a strip of licorice. The very small pieces can easily be broken off on the thin lines. How many very small pieces are shown in the picture?

3. Gatsha runs a spaza shop. He sells strips of licorice like the above for R2 each.
   (a) What is the cost of one very small piece of licorice, when you buy from Gatsha?
   (b) Jonathan wants to buy one fifth of a strip of licorice. How much should he pay?
   (c) Batseba eats 25 very small pieces. What part of a whole strip of licorice is this?

Each small piece of the above strip is **one hundredth** of the whole strip.

CHAPTER 6: FRACTIONS 101

Answers

4. (a) Because the strip is divided into 100 pieces of equal size.
   (b) ten hundredths
5. (a) 40 hundredths
   (b) four tenths
   (c) 75 hundredths
   (d) three fifths
   (e) 80 hundredths
6. Gabieba: 27 hundredths — Miriam: 32 hundredths
   Sannie: 28 hundredths — Enoch: 20 hundredths
   Mpati: 30 hundredths — Mpho: 33 hundredths
7. Gabieba: 65c — Miriam: 77c
   Sannie: 67c — Enoch: 48c
   Mpati: 72c — Mpho: 79c
8. (a) $300c \div 100 = 3c$
   (b) $7 \times 3c = 21c$
   (c) $25 \times 3c = 75c$
   (d) $300 \div 4 = 75c$
   (e) $40 \times 3c = 120c$ or R1,20
   (f) $\frac{1}{5}$ of $300c = 60c$ so $\frac{2}{5} = 120c$ or R1,20
9. Because $\frac{40}{100}$ and $\frac{2}{5}$ are equivalent fractions.

Background information

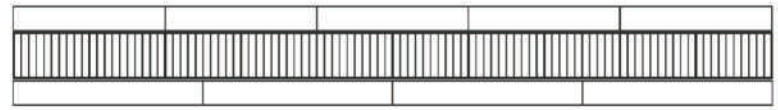
- **Per cent** indicates a special type of fraction in which the value given is a measure of the number of parts in every hundred parts of a whole, collection, quantity or measurement. This means that per cent is another word for hundredth.
- The **symbol** for per cent is %.

Teaching guidelines (continued)

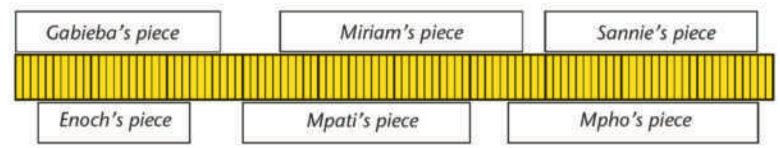
Introduce the concepts of per cent before learners start with question 10.

4. (a) Why can each small piece be called *one hundredth* of the whole strip?
   (b) How many hundredths is the same as one tenth of the strip?

Gatsha often sells parts of licorice strips to customers. He uses a "quarters marker" and a "fifths marker" to cut off the pieces correctly from full strips. His two markers are shown below, next to a full strip of licorice.



5. (a) How many hundredths is the same as two fifths of the whole strip?
   (b) How many tenths is the same as $\frac{2}{5}$ of the whole strip?
   (c) How many hundredths is the same as $\frac{3}{4}$ of the whole strip?
   (d) Freddie bought $\frac{60}{100}$ of a strip. How many fifths of a strip is this?
   (e) Jamey bought part of a strip for R1,60. What part of a strip did she buy?
6. Gatsha, the owner of the spaza shop, sold pieces of yellow licorice to different children. Their pieces are shown below. How much (what part of a whole strip) did each of them get?



7. The yellow licorice shown above costs R2,40 (240 cents) for a strip. How much does each of the children have to pay? Round off the amounts to the nearest cent.
8. (a) How much is $\frac{1}{100}$ of 300 cents? (b) How much is $\frac{7}{100}$ of 300 cents?
   (c) How much is $\frac{25}{100}$ of 300 cents? (d) How much is $\frac{1}{4}$ of 300 cents?
   (e) How much is $\frac{40}{100}$ of 300 cents? (f) How much is $\frac{2}{5}$ of 300 cents?
9. Explain why your answers for questions 8(e) and 8(f) are the same.

Another word for **hundredth** is **per cent**.
Instead of saying Miriam received **32 hundredths** of a licorice strip, we can say Miriam received **32 per cent** of a licorice strip. The symbol for per cent is %.

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Answers

10. (a) R400 (b) R384 (c) R680 (d) R1 920
11. (a) R40 (b) R38,40 (c) R68 (d) R192
12. (a) R75 (b) R72 (c) R127,50 (d) R360
13. R110 000 + (20% of R110 000) = R110 000 + R22 000 = R132 000
14. R125 000 – (30% of 125 000) = R125 000 – R37 500 = R87 500
15. Only numbers that are multiples of 2 and/or 5.

6.5 Thousandths, hundredths and tenths

MANY EQUAL PARTS

Background information

The purpose of this section is the introduction of fractions which play an important role in the structure of decimal fractions.

Teaching guidelines

Use a metre stick as a unit and introduce the concepts of tenths (decimetres), hundredths (centimetres) and thousandths (millimetres) of a metre.

Answers

1. 50 kg = 50 000 g; 50 000 g ÷ 1 000 = 50 g for each refugee
2. (a) R600 (b) R60 (c) R6 (d) R600
   (e) R600 (f) R420 (g) R420 (h) R42
3. (a) $\frac{74}{80}$ or $\frac{37}{40}$ (b) $6\frac{1}{10}$ (c) $\frac{37}{100}$ (d) 1 (e) $\frac{307}{1\,000}$ (f) $\frac{37}{100}$
4. (a) $\frac{374}{1\,000}$ or $\frac{187}{500}$ (b) $1\frac{4}{10}$ or $1\frac{2}{5}$ (c) $1\frac{5}{10}$ or $1\frac{1}{2}$ (d) $\frac{254}{1\,000}$ or $\frac{127}{500}$
5. (a) $\frac{100}{1\,000}+\frac{230}{1\,000}+\frac{346}{1\,000}=\frac{676}{1\,000}$; $\frac{600}{1\,000}+\frac{30}{1\,000}+\frac{46}{1\,000}=\frac{676}{1\,000}$; True.
   (b) LHS is the same as in the previous question but RHS is $\frac{726}{1\,000}$; Not true.
   (c) LHS = $\frac{676}{1\,000}$; RHS = $\frac{600}{1\,000}+\frac{70}{1\,000}+\frac{6}{1\,000}=\frac{676}{1\,000}$; True.
   (d) Both sides can be expressed as $\frac{600}{1\,000}+\frac{70}{1\,000}+\frac{6}{1\,000}$; True.

10. How much is 80% of each of the following?
    (a) R500 (b) R480 (c) R850 (d) R2 400
11. How much is 8% of each of the amounts in 10(a), (b), (c) and (d)?
12. How much is 15% of each of the amounts in 10(a), (b), (c) and (d)?
13. Building costs of houses increased by 20%. What is the new building cost for a house that previously cost R110 000 to build?
14. The value of a car decreases by 30% after one year. If the price of a new car is R125 000, what is the value of the car after one year?
15. Investigate which denominators of fractions can easily be converted to powers of 10.

6.5 Thousandths, hundredths and tenths

MANY EQUAL PARTS

1. In a camp for refugees, 50 kg of sugar must be shared equally between 1 000 refugees. How much sugar will each refugee get? Keep in mind that 1 kg is 1 000 g. You can give your answer in grams.
2. How much is each of the following?
   (a) one tenth of R6 000 (b) one hundredth of R6 000
   (c) one thousandth of R6 000 (d) ten hundredths of R6 000
   (e) 100 thousandths of R6 000 (f) seven hundredths of R6 000
   (g) 70 thousandths of R6 000 (h) seven thousandths of R6 000
3. Calculate.
   (a) $\frac{3}{10}+\frac{5}{8}$ (b) $3\frac{3}{10}+2\frac{4}{5}$ (c) $\frac{3}{10}+\frac{7}{100}$
   (d) $\frac{3}{10}+\frac{70}{100}$ (e) $\frac{3}{10}+\frac{7}{1\,000}$ (f) $\frac{3}{10}+\frac{70}{1\,000}$
4. Calculate.
   (a) $\frac{3}{10}+\frac{7}{100}+\frac{4}{1\,000}$ (b) $\frac{3}{10}+\frac{70}{100}+\frac{400}{1\,000}$
   (c) $\frac{6}{10}+\frac{20}{100}+\frac{700}{1\,000}$ (d) $\frac{2}{10}+\frac{5}{100}+\frac{4}{1\,000}$
5. In each case investigate whether the statement is true or not, and give reasons for your final decision.
   (a) $\frac{1}{10}+\frac{23}{100}+\frac{346}{1\,000}=\frac{6}{10}+\frac{3}{100}+\frac{46}{1\,000}$ (b) $\frac{1}{10}+\frac{23}{100}+\frac{346}{1\,000}=\frac{7}{10}+\frac{2}{100}+\frac{6}{1\,000}$
   (c) $\frac{1}{10}+\frac{23}{100}+\frac{346}{1\,000}=\frac{6}{10}+\frac{7}{100}+\frac{6}{1\,000}$ (d) $\frac{676}{1\,000}=\frac{6}{10}+\frac{7}{100}+\frac{6}{1\,000}$

CHAPTER 6: FRACTIONS 103

6.6 Fraction of a fraction

FORM PARTS OF PARTS

Background information

The focus in this section is the calculation of a fraction of a whole, collection, quantity or measurement.

Teaching guidelines

Learners should realise that, for example, to calculate:

- $\frac{1}{5}$ of R60 means divide R60 by 5
- $\frac{2}{5}$ of R60 means divide R60 by 5 and multiply the answer by 2
- $\frac{3}{5}$ of R60 means divide R60 by 5 and multiply the answer by 3, etc.

Remind learners that **of** as an operation means multiply.

Answers

1. (a) R12 (b) R36
2. One tenth of R80 = R8, so seven tenths of R80 = 7 × R8 = R56
3. (a) 8 pula
   (b) 8 euro
   (c) $4\frac{4}{5}$ or 4,8 pula (4 pula and 80 thebe)
4. Because 5 divides into 20 without a remainder.
5. (a) 90c (b) R3,50
   (c) R8,40 (d) R2,50
6. (a) 12 secret objects (b) 15 secret objects
7. (a) 12 (b) 25
8. (a) 15 twentieths (b) R3
   (c) 150 g (d) 150 g
9. (a) two fortieths (b) 14 fortieths
10. Yes

6.6 Fraction of a fraction

FORM PARTS OF PARTS

1. (a) How much is one fifth of R60?
   (b) How much is three fifths of R60?
2. How much is seven tenths of R80? (You may first work out how much one tenth of R80 is.)
3. In Britain the unit of currency is the pound sterling, in Western Europe it is the euro, and in Botswana it is the pula.
   (a) How much is two fifths of 20 pula?
   (b) How much is two fifths of 20 euro?
   (c) How much is two fifths of 12 pula?
4. Why was it so easy to calculate two fifths of 20, but difficult to calculate two fifths of 12?

There is a way to make it easy to calculate something like three fifths of R4. You just change the rands to cents!

5. Calculate each of the following. You may change the rands to cents to make it easier.
   (a) three eighths of R2,40 (b) seven twelfths of R6
   (c) two fifths of R21 (d) five sixths of R3
6. You will now do some calculations about secret objects.
   (a) How much is three tenths of 40 secret objects?
   (b) How much is three eighths of 40 secret objects?

The secret objects in question 6 are fiftieths of a rand.

7. (a) How many fiftieths is three tenths of 40 fiftieths?
   (b) How many fiftieths is five eighths of 40 fiftieths?
8. (a) How many twentieths of a kilogram is the same as $\frac{3}{4}$ of a kilogram?
   (b) How much is one fifth of 15 rands?
   (c) How much is one fifth of 15 twentieths of a kilogram?
   (d) So, how much is one fifth of $\frac{3}{4}$ of a kilogram?
9. (a) How much is $\frac{1}{12}$ of 24 fortieths of some secret object?
   (b) How much is $\frac{7}{12}$ of 24 fortieths of the secret object?
10. Do you agree that the answers for the previous question are two fortieths and 14 fortieths? If you disagree, explain why you disagree.

Note on question 12

To calculate $\frac{7}{12}$ of $\frac{3}{5}$, the $\frac{3}{5}$ must be divided by 12.

To do so, find an equivalent of $\frac{3}{5}$ which can be divided by 12: $\frac{3}{5} \times \frac{4}{4} = \frac{12}{20}$.

We know that one twelfth of $\frac{12}{20}$ is $\frac{1}{20}$.

Therefore, seven twelfths of $\frac{12}{20}$ is $7 \times \frac{1}{20} = \frac{7}{20}$.

So, $\frac{7}{12}$ of $\frac{3}{5} = \frac{7}{20}$.

Note that the answer to question 12 below is equivalent to $\frac{7}{20}$.

Answers

11. (a) 16
    (b) 48
    (c) 2
    (d) 48
    (e) Because $\frac{3}{5}$ and $\frac{24}{40}$ are equivalent fractions.

12. It is $\frac{14}{40}$ because $\frac{3}{5} = \frac{24}{40}$ [from question 11(e)], and $\frac{7}{12}$ of $\frac{3}{5}$ is therefore the same as $\frac{7}{12}$ of $\frac{24}{40}$, which was calculated in question 9(b) as $\frac{14}{40}$.

13. (a) $\frac{8}{12} = \frac{16}{24}$ $\frac{1}{8}$ of $\frac{16}{24}$ is $\frac{2}{24}$ $\frac{3}{8}$ of $\frac{16}{24}$ is $\frac{6}{24}$
    (b) $\frac{2}{3} = \frac{8}{12}$ so the answer is the same as in question 13(a), which is $\frac{6}{24}$.

14. (a) $\frac{5}{8} = \frac{20}{32}$ $\frac{3}{4}$ of $\frac{20}{32} = \frac{15}{32}$
    (b) $\frac{2}{3} = \frac{20}{30}$ $\frac{7}{10}$ of $\frac{20}{30} = \frac{14}{30}$
    (c) $\frac{1}{2} = \frac{3}{6}$ $\frac{2}{3}$ of $\frac{3}{6} = \frac{2}{6}$
    (d) $\frac{3}{5} = \frac{15}{25}$ $\frac{3}{5}$ of $\frac{15}{25} = \frac{9}{25}$

11. (a) How much is $\frac{1}{5}$ of 80?
    (b) How much is $\frac{3}{5}$ of 80?
    (c) How much is $\frac{1}{40}$ of 80?
    (d) How much is $\frac{24}{40}$ of 80?
    (e) Explain why $\frac{3}{5}$ of 80 is the same as $\frac{24}{40}$ of 80.

12. Look again at your answers for questions 9(b) and 11(e). How much is $\frac{7}{12}$ of $\frac{3}{5}$? Explain your answer.

The secret object in question 9 was an envelope with R160 in it.

After the work you did in questions 9, 10 and 11, you know that:

- $\frac{24}{40}$ and $\frac{3}{5}$ are just two ways to describe the same thing, and
- $\frac{7}{12}$ of $\frac{3}{5}$ is therefore the same as $\frac{7}{12}$ of $\frac{24}{40}$.

It is easy to calculate $\frac{7}{12}$ of $\frac{24}{40}$: one twelfth of 24 is 2, so seven twelfths of 24 is 14, so seven twelfths of 24 fortieths is 14 fortieths.

$\frac{3}{8}$ of $\frac{2}{3}$ can be calculated in the same way. But one eighth of $\frac{2}{3}$ is a slight problem, so it would be better to use some equivalent of $\frac{2}{3}$. The equivalent should be chosen so that it is easy to calculate one eighth of it; so it would be nice if the numerator could be eight.

$\frac{8}{12}$ is equivalent to $\frac{2}{3}$, so instead of calculating $\frac{3}{8}$ of $\frac{2}{3}$ we may calculate $\frac{3}{8}$ of $\frac{8}{12}$.

13. (a) Calculate $\frac{3}{8}$ of $\frac{8}{12}$.
    (b) So, how much is $\frac{3}{8}$ of $\frac{2}{3}$?

14. In each case replace the second fraction by a suitable equivalent, and then calculate.
    (a) How much is $\frac{3}{4}$ of $\frac{5}{8}$?
    (b) How much is $\frac{7}{10}$ of $\frac{2}{3}$?
    (c) How much is $\frac{2}{3}$ of $\frac{1}{2}$?
    (d) How much is $\frac{3}{5}$ of $\frac{3}{5}$?

6.7 Multiplying with fractions

PARTS OF RECTANGLES AND PARTS OF PARTS

Teaching guidelines

To divide a side of a rectangle into:

- halves, quarters, eighths, sixteenths and so on, halve the side repeatedly
- thirds, sixths and ninths, draw two lines to partition the whole side approximately in thirds
- fifths and tenths, draw a line that divides the side into two parts, the one part about one-and-a-half as long as the other one.

Diagrams used to explain multiplication of fractions should be drawn freehand and quickly, so that it does not take up much time. These diagrams are not normally used for making measurements, but are rather used to support thinking conceptually about fractions.

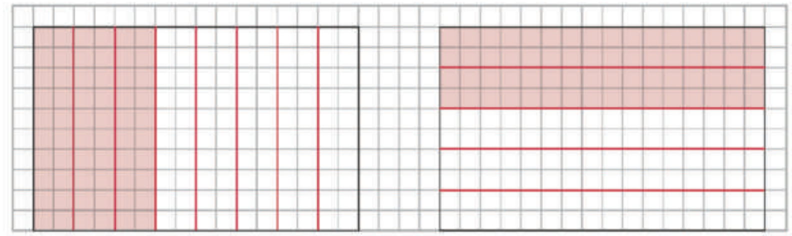
Answers

1. See LB page 106 alongside.
2. See LB page 106 alongside.
3. (a) five eighths
   (b) three fifths
   (c) 40
   (d) 15 fortieths

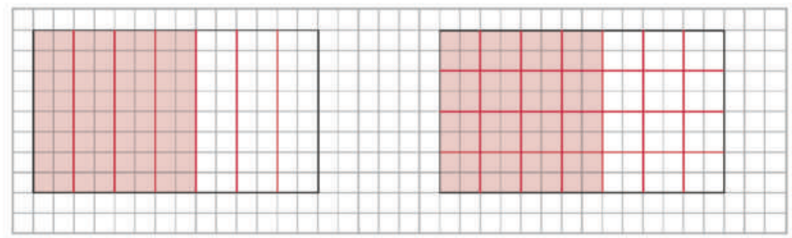
6.7 Multiplying with fractions

PARTS OF RECTANGLES AND PARTS OF PARTS

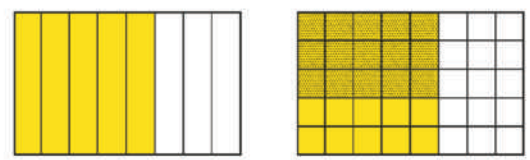
1. (a) Copy the rectangles below. Divide the rectangle on the left into eighths by drawing vertical lines. Lightly shade the left three eighths of the rectangle.
   (b) Divide the rectangle on the right into fifths drawing horizontal lines. Lightly shade the upper two fifths of the rectangle.



2. (a) Copy the rectangles below. Shade four sevenths of the rectangle on the left below.
   (b) Shade 16 twenty-eighths of the rectangle on the right below.



3. (a) What part of each big rectangle below is coloured yellow?



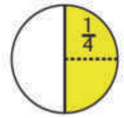
   (b) What part of the *yellow* part of the rectangle on the right is dotted?
   (c) Into how many squares is the whole rectangle on the right divided?
   (d) What part of the whole rectangle on the right is yellow *and* dotted?

Additional notes on multiplication with fractions

Circular diagrams can also be used to illustrate multiplication with basic fractions:

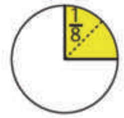
- $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

  Half of a semi-circle is a quarter of the circle.



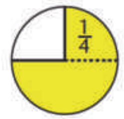
- $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

  Half of a quarter circle is one eighth of the circle.



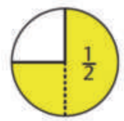
- $\frac{1}{3}$ of $\frac{3}{4} = \frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$

  A third of a three-quarter circle is a quarter of the circle.



- $\frac{2}{3}$ of $\frac{3}{4} = \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$

  Two-thirds of three-quarters of a circle is half of the circle.



Answers

4. (a) See LB page 107 alongside. $\frac{15}{32}$

   (b) See LB page 107 alongside. $\frac{8}{15}$

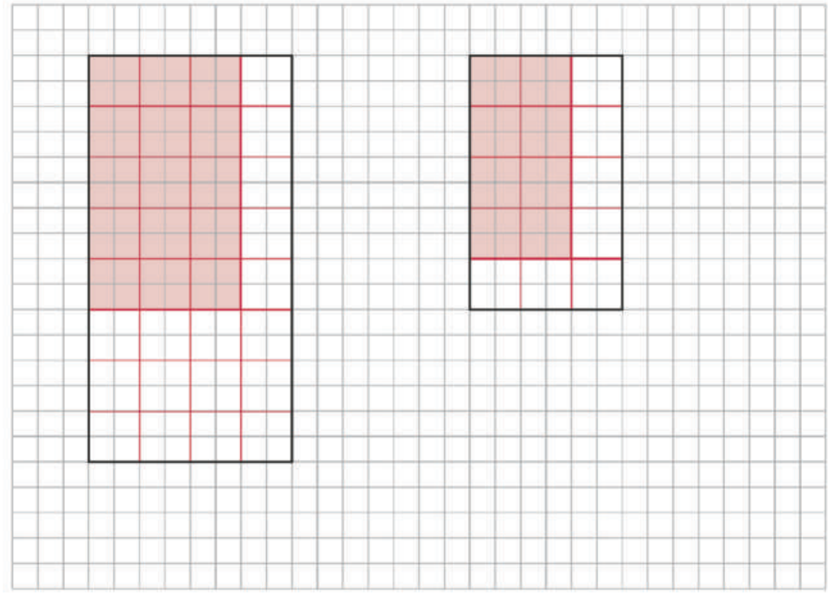
5. The answers are the same, so multiplying the numerators with each other and the denominators with each other seems to be a way of finding the answer to $\frac{3}{4}$ of $\frac{5}{8}$.

6. (a) $5 \times 8 = 40$ apples

   (b) $10 \times \frac{6}{4} = \frac{60}{4} = 15$ apples

4. On grid paper make diagrams to help you to figure out how much each of the following is:

   (a) $\frac{3}{4}$ of $\frac{5}{8}$ (b) $\frac{2}{3}$ of $\frac{4}{5}$



Here is something you can do with the fractions $\frac{3}{4}$ and $\frac{5}{8}$:
multiply the two numerators and make this the numerator of a new fraction.
Also multiply the two denominators, and make this the denominator of a new fraction
$\frac{3 \times 5}{4 \times 8} = \frac{15}{32}$.

5. Compare the above with what you did in question 14(a) of section 6.6 and in question 4(a) at the top of this page. What do you notice about $\frac{3}{4}$ of $\frac{5}{8}$ and $\frac{3 \times 5}{4 \times 8}$?

6. (a) Alan has five heaps of eight apples each. How many apples is that in total?
   (b) Sean has ten heaps of six quarter apples each. How many apples is that in total?

Instead of saying $\frac{5}{8}$ of R40 or $\frac{5}{8}$ of $\frac{2}{3}$ of a floor surface,
we may say $\frac{5}{8} \times$ R40 or $\frac{5}{8} \times \frac{2}{3}$ of a floor surface.

CHAPTER 6: FRACTIONS 107

Teaching guidelines (continued)

Learners should discover that two fractions can be multiplied by using the formula $\frac{\text{numerator} \times \text{numerator}}{\text{denominator} \times \text{denominator}}$.

Answers

7. (a) $\frac{15}{60}$ (b) $\frac{14}{40}$

8. (a) $\frac{3 \times 5}{10 \times 6} = \frac{15}{60}$ The answers are the same.

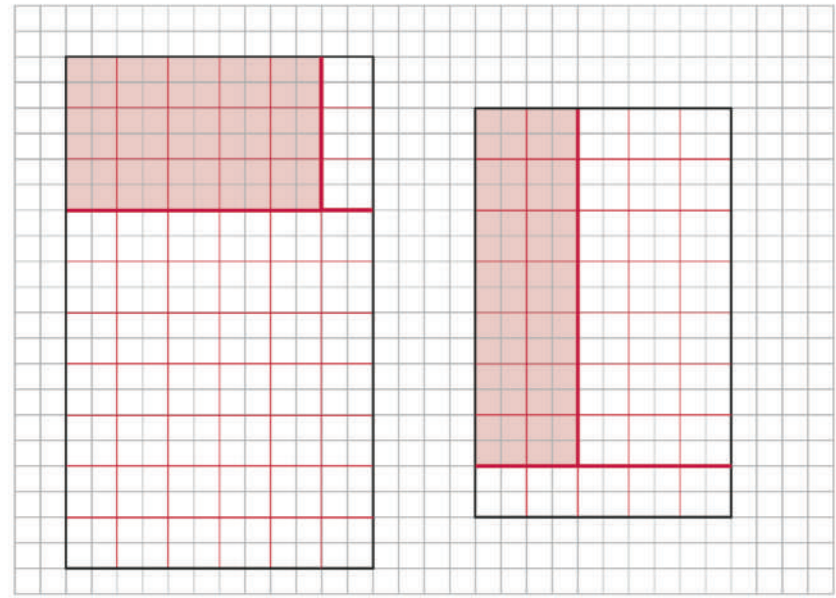
   (b) $\frac{2 \times 7}{5 \times 8} = \frac{14}{40}$ The answers are the same.

9. (a) $\frac{35}{72}$ (b) $\frac{6}{12}$

10. Yes, it does.

7. Use the diagrams below to figure out how much each of the following is:

   (a) $\frac{3}{10} \times \frac{5}{6}$ (b) $\frac{2}{5} \times \frac{7}{8}$



8. (a) Perform the calculations $\frac{\text{numerator} \times \text{numerator}}{\text{denominator} \times \text{denominator}}$ for $\frac{3}{10}$ and $\frac{5}{6}$ and compare the answer to your answer for question 7(a).

   (b) Do the same for $\frac{2}{5}$ and $\frac{7}{8}$.

9. Perform the calculations $\frac{\text{numerator} \times \text{numerator}}{\text{denominator} \times \text{denominator}}$ for

   (a) $\frac{5}{6}$ and $\frac{7}{12}$ (b) $\frac{3}{4}$ and $\frac{2}{3}$

10. Use the diagrams on the following page to check whether the formula $\frac{\text{numerator} \times \text{numerator}}{\text{denominator} \times \text{denominator}}$ produces the correct answers for $\frac{5}{6} \times \frac{7}{12}$ and $\frac{3}{4} \times \frac{2}{3}$.

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Answers

11. (a) $\frac{1}{6} \times R60 = R10$ (b) $\frac{4}{63} \times R63 = R4$ (c) $\frac{8}{15} \times R45 = R24$

12. (a) $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$ of and hour; $\frac{3}{8} \times 60$ minutes $= \frac{180}{8} = 22\frac{4}{8}$ or $22\frac{1}{2}$ minutes

(b) $\frac{3}{4} \times \frac{3}{8} = \frac{9}{32}$ kg

(c) $7 \times \frac{3}{4} + \frac{1}{2} \times \frac{3}{4} = \frac{21}{4} + \frac{3}{8} = \frac{42}{8} + \frac{3}{8} = \frac{45}{8} = 5\frac{5}{8}$ kg or $\frac{15}{2} \times \frac{3}{4} = \frac{45}{8} = 5\frac{5}{8}$ kg

6.8 Ordering and comparing fractions

Teaching guidelines

Fractions can only be compared properly if their denominators are the same. For example:

- To compare halves and quarters, convert all to quarters because the LCM of 2 and 4 is 4.
- To compare halves, quarters and eighths, convert all to eighths because the LCM of 2, 4 and 8 is 8.
- To compare sixteenths, eighths, twenty-fourths and twelfths, convert all to forty-eighths because the LCM of 16, 8, 24 and 12 is 48.

Fractions with the same denominator are in:

- ascending order if their numerators are in ascending order
- descending order if their numerators are in descending order.

Answers

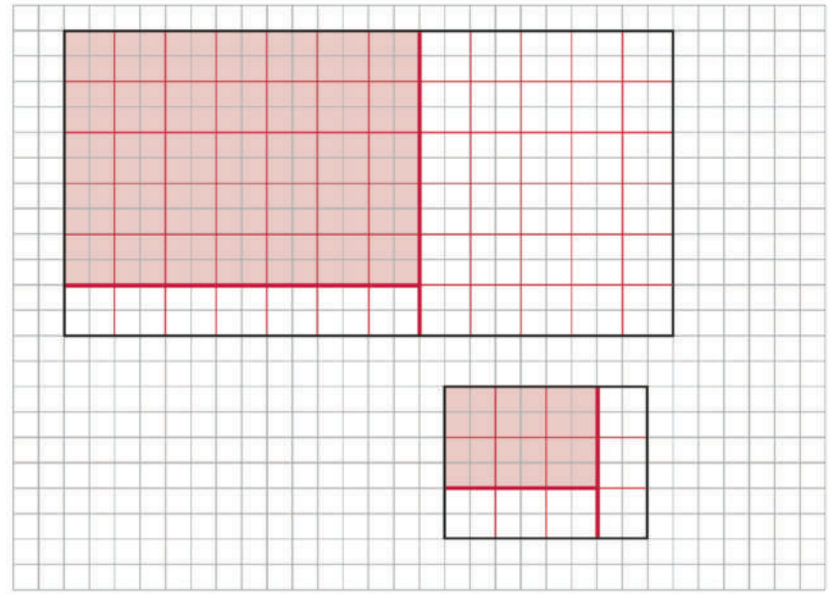
1. (a) $\frac{21}{48}; \frac{18}{48}; \frac{22}{48}; \frac{20}{48}; \frac{23}{48} \rightarrow \frac{3}{8}; \frac{5}{12}; \frac{7}{16}; \frac{11}{24}; \frac{23}{48}$

(b) $\frac{703}{1\,000}; \frac{650}{1\,000}; \frac{700}{1\,000}; \frac{730}{1\,000}; \frac{710}{1\,000} \rightarrow \frac{13}{20}; \frac{7}{10}; \frac{703}{1\,000}; \frac{71}{100}$; 73%

2. (a) $\frac{41}{60}; \frac{38}{60}; \frac{42}{60}; \frac{44}{60}; \frac{51}{60} \rightarrow \frac{17}{20}; \frac{11}{15}; \frac{7}{10}; \frac{41}{60}; \frac{19}{30}$

(b) Order according to what is needed to make 1;
order the denominators $\rightarrow \frac{23}{24}; \frac{19}{20}; \frac{7}{8}; \frac{5}{6}; \frac{2}{3}$

3. See LB page 109 alongside.



11. Calculate each of the following:
(a) $\frac{1}{2}$ of $\frac{1}{3}$ of R60 (b) $\frac{2}{7}$ of $\frac{2}{9}$ of R63 (c) $\frac{4}{3}$ of $\frac{2}{5}$ of R45

12. (a) John normally practises soccer for three quarters of an hour every day. Today he practised for only half his usual time. How long did he practise today?
(b) A bag of peanuts weighs $\frac{3}{8}$ of a kilogram. What does $\frac{3}{4}$ of a bag weigh?
(c) Calculate the mass of $7\frac{1}{2}$ packets of sugar if one packet has a mass of $\frac{3}{4}$ kg.

6.8 Ordering and comparing fractions

1. Order the following from the smallest to the biggest:
(a) $\frac{7}{16}; \frac{3}{8}; \frac{11}{24}; \frac{5}{12}; \frac{23}{48}$ (b) $\frac{703}{1\,000}; \frac{13}{20}; \frac{7}{10}$; 73%; $\frac{71}{100}$

2. Order the following from the biggest to the smallest:
(a) $\frac{41}{60}; \frac{19}{30}; \frac{7}{10}; \frac{11}{15}; \frac{17}{20}$ (b) $\frac{23}{24}; \frac{2}{3}; \frac{7}{8}; \frac{19}{20}; \frac{5}{6}$

3. Use the symbols =, > or < to make the following true:
(a) $\frac{7}{17}$ = $\frac{21}{51}$ (b) $\frac{1}{17}$ > $\frac{1}{19}$

CHAPTER 6: FRACTIONS 109

Answers

1. (a) $\frac{3}{20}+\frac{5}{20}=\frac{8}{20}$; eight twentieths (b) $\frac{5}{12}+\frac{11}{12}=\frac{16}{12}$; sixteen twelfths

   (c) $\frac{3}{2}+\frac{5}{4}=\frac{6}{4}+\frac{5}{4}=\frac{11}{4}=2\frac{3}{4}$ (d) $\frac{3}{5}+\frac{3}{10}=\frac{6}{10}+\frac{3}{10}=\frac{9}{10}$

   two wholes and three quarters nine tenths

2. See LB page 110 alongside.

3. (a) three tenths plus seven thirtieths = nine thirtieths + seven thirtieths $=16$ thirtieths $=\frac{16}{30}$

   (b) two fifths plus seven twelfths = 24 sixtieths + 35 sixtieths = 59 sixtieths $=\frac{59}{60}$

   (c) one hundredth plus seven tenths = one hundredth + 70 hundredths = 24 fortieths – 15 fortieths = 71 hundredths $=\frac{71}{100}$

   (d) three fifths minus three eighths = 24 fortieths – 15 fortieths = nine fortieths $=\frac{9}{40}$

   (e) two wholes and three tenths plus five wholes and nine tenths = seven wholes and 12 tenths = eight wholes and two tenths $=8\frac{2}{10}$

4. $\frac{12}{100}\times 5\,000=\frac{60\,000}{100}=600$; R5 000 + R600 = R5 600 per month

5. Increased by 10%: R7 500 + R750 = R8 250 per month

   Decreased by 10%: R8 250 – R825 = R7 425 per month

6. (a) $\frac{13}{20}-\frac{8}{20}=\frac{5}{20}=\frac{1}{4}$ (b) $3\frac{24}{100}-1\frac{20}{100}=2\frac{4}{100}=2\frac{1}{25}$

   (c) $5\frac{36}{44}-2\frac{11}{44}=3\frac{25}{44}$ (d) $\frac{14}{21}+\frac{12}{21}=\frac{26}{21}=1\frac{5}{21}$

7. (a) $\frac{9}{2}=4\frac{1}{2}$ (b) $\frac{30}{135}=\frac{2}{9}$ (c) $\frac{30}{32}=10$ (d) $\frac{6}{12}=\frac{1}{2}$

8. (a) $\frac{8}{3}\times\frac{8}{3}=\frac{64}{9}=7\frac{1}{9}$ (b) $\frac{42}{5}\times\frac{10}{3}=\frac{420}{15}=28$

   (c) $\left(\frac{2}{6}+\frac{3}{6}\right)\times\frac{6}{7}=\frac{5}{6}\times\frac{6}{7}=\frac{30}{42}=\frac{5}{7}$ (d) $\frac{6}{24}=\frac{1}{4}$

   (e) $\frac{5}{6}+\frac{2}{15}=\frac{25+4}{30}=\frac{29}{30}$ (f) $\frac{3}{4}-\frac{10}{30}=\frac{9}{12}-\frac{4}{12}=\frac{5}{12}$

WORKSHEET

1. Do the calculations given below. Rewrite each question in the common fraction notation. Then write the answer in words and in the common fraction notation.
   (a) three twentieths + five twentieths (b) five twelfths + 11 twelfths
   (c) three halves + five quarters (d) three fifths + three tenths

2. Complete the equivalent fractions.
   (a) $\frac{5}{7}=\frac{35}{49}$ (b) $\frac{9}{11}=\frac{27}{33}$ (c) $\frac{15}{10}=\frac{3}{2}$
   (d) $\frac{1}{9}=\frac{4}{36}$ (e) $\frac{45}{18}=\frac{5}{2}$ (f) $\frac{4}{5}=\frac{28}{35}$

3. Do the calculations given below. Rewrite each question in words. Then write the answer in words and in the common fraction notation.
   (a) $\frac{3}{10}+\frac{7}{30}$ (b) $\frac{2}{5}+\frac{7}{12}$
   (c) $\frac{1}{100}+\frac{7}{10}$ (d) $\frac{3}{5}-\frac{3}{8}$
   (e) $2\frac{3}{10}+5\frac{9}{10}$

4. Joe earns R5 000 per month. His salary increases by 12%. What is his new salary?

5. Ahmed earned R7 500 per month. At the end of a certain month, his employer raised his salary by 10%. However, one month later his employer had to decrease his salary again by 10%. What was Ahmed's salary then?

6. Calculate each of the following and simplify the answer to its lowest form:
   (a) $\frac{13}{20}-\frac{2}{5}$ (b) $3\frac{24}{100}-1\frac{2}{10}$
   (c) $5\frac{9}{11}-2\frac{1}{4}$ (d) $\frac{2}{3}+\frac{4}{7}$

7. Evaluate.
   (a) $\frac{1}{2}\times 9$ (b) $\frac{3}{5}\times\frac{10}{27}$ (c) $\frac{2}{3}\times 15$ (d) $\frac{2}{3}\times\frac{3}{4}$

8. Calculate.
   (a) $2\frac{2}{3}\times 2\frac{2}{3}$ (b) $8\frac{2}{5}\times 3\frac{1}{3}$
   (c) $(\frac{1}{3}+\frac{1}{2})\times\frac{6}{7}$ (d) $\frac{2}{3}\times\frac{1}{2}\times\frac{3}{4}$
   (e) $\frac{5}{6}+\frac{2}{3}\times\frac{1}{5}$ (f) $\frac{3}{4}-\frac{2}{5}\times\frac{5}{6}$

FRACTIONS

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
7.1 Other symbols for tenths and hundredths	Introducing the decimal notation for tenths, hundredths and thousandths	Pages 111 to 112
7.2 Percentages and decimal fractions	Work with notations for common fractions, percentages and decimals	Pages 112 to 114
7.3 Decimal measurements	Show decimal numbers on a number line	Pages 114 to 116
7.4 More decimal concepts	Skip count in decimals; place values and digit values in decimals	Pages 116 to 118
7.5 Ordering and comparing decimal fractions	Arrange decimals in ascending and descending order	Pages 118 to 119
7.6 Rounding off, saying it nearly but not quite	Round off to whole numbers, tenths, hundredths and thousandths	Pages 119 to 120
7.7 Addition and subtraction with decimal fractions	Add and subtract decimals using common fraction notation; use decimals in real-life situations	Pages 120 to 121
7.8 Multiplication and decimal fractions	Multiply and divide by 10, 100, 1 000, 0,1, 0,01 and 0,001; multiply decimals by whole numbers and decimals by decimals	Pages 122 to 124
7.9 Division and decimal fractions	Divide decimals by whole numbers	Pages 124 to 125

CAPS time allocation	5 hours
CAPS content specification	Pages 78 to 79

Mathematical background

Decimal notation is an alternative notation to common fractions and percentages.

- In the decimal notation, a limited range of fractional units is used to describe fractions, namely tenths, hundredths, thousandths, and so on. In this chapter, we use the context of measurement to follow on from the previous chapter.
- **Percentages** make up an even more limited system of describing quantities, in the sense that they allow hundredths only. In practice, however, percentages are extended to include fractional percentages, for example 25,5%. As the percentage is yet another notation for fractional parts, we include percentages in tasks throughout this chapter. On LB page 113, in question 6, learners express numbers in three notations. One of these is the common fraction notation, using hundredths.

The **calculator** is an efficient teaching aid for learners to explore and investigate decimals. Learners should know how to program their calculators to add or subtract a decimal number repeatedly. This could assist them in evaluating their ability to count forwards or backwards in decimals. It can also be used to explore the rules for multiplication and division by powers of 10 (10; 100; 1 000; 0,1; 0,01; 0,001) and to discover general rules like the following:

- If you multiply a number by a number less than 1, the answer will be less than the original number.
- When you divide a number by a number less than 1, the answer is more than the original number.

7.1 Other symbols for tenths and hundredths

TENTHS AND HUNDREDTHS AGAIN ...

Background information

In this section learners are introduced to the decimal notation for the common fractions one tenth (0,1) and one hundredth (0,01).

Teaching guidelines

- Illustrate that $\frac{1}{10}$ (common fraction notation) = 0,1 (decimal notation) therefore, $\frac{2}{10} = 0{,}2$; $\frac{3}{10} = 0{,}3$; etc.
- Illustrate that $\frac{1}{100}$ (common fraction notation) = 0,01 (decimal notation) therefore, $\frac{2}{100} = 0{,}02$; $\frac{3}{100} = 0{,}03$; etc.
- Recall that one tenth is equivalent to ten hundredths therefore, 0,1 = 0,10.

Notes on questions 3 and 4

Discuss why 0,3 + 0,05 = 0,35 (3 tenths + 5 hundredths = 30 hundredths + 5 hundredths = 35 hundredths = 0,35)

Answers

1. (a) 10 hundredths or $\frac{10}{100}$ or 1 tenth or $\frac{1}{10}$

   (b) Red: $\frac{1}{100}$ Blue: $\frac{30}{100}$ or $\frac{3}{10}$ Green: $\frac{2}{100}$ Not coloured: $\frac{57}{100}$
2. Yellow: 0,1 Red: 0,01 Blue: 0,3 Green: 0,02 Not coloured: 0,57
3. 3 tenths $\left(\frac{3}{10}\right)$ and 7 hundredths $\left(\frac{7}{100}\right)$ is not coloured. That is $\frac{37}{10}$ or 0,37.
4. $\frac{62}{100}$ 0,62
5. They are both right.

CHAPTER 7 The decimal notation for fractions

7.1 Other symbols for tenths and hundredths

TENTHS AND HUNDREDTHS AGAIN ...

1. (a) What part of the rectangle below is coloured yellow?

   (b) What part of the rectangle is red? What part is blue? What part is green, and what part is not coloured?

0,1 is another way to write $\frac{1}{10}$ and
0,01 is another way to write $\frac{1}{100}$.
0,1 and $\frac{1}{10}$ are different notations for the same number.
$\frac{1}{10}$ is called the **(common) fraction notation** and 0,1 is called the **decimal notation**.

2. Write the answers for 1(a) and (b) in decimal notation.
3. Three tenths and seven hundredths of a rectangle is coloured red, and two tenths and six hundredths of the rectangle is coloured brown. What part of the rectangle (how many tenths and how many hundredths) is not coloured? Write your answer in fraction notation and in decimal notation.
4. On Monday, Steve ate three tenths and seven hundredths of a strip of licorice. On Tuesday, Steve ate two tenths and five hundredths of a strip of licorice. How much licorice did he eat on Monday and Tuesday together? Write your answer in fraction notation and in decimal notation.
5. Lebogang's answer for question 4 is *five tenths and 12 hundredths*. Susan's answer is *six tenths and two hundredths*. Who is right, or are they both wrong?

Teaching guidelines (continued)

Discuss why 0,35 (35 hundredths) can be expressed as any of the following:

- 1 tenth + 25 hundredths (10 hundredths + 25 hundredths)
- 2 tenths + 15 hundredths (20 hundredths + 15 hundredths)
- 3 tenths + 5 hundredths (30 hundredths + 5 hundredths).

The normal way of expressing these quantities is the third option because, by international agreement, the number of hundredths should be kept below 10.

Answers

6. (a) 3,7 (b) 4,19 (c) 4,7 (d) 0,04

... AND THOUSANDTHS

Teaching guidelines

- Illustrate that $\frac{1}{1\,000}$ (common fraction notation) = 0,001 (decimal notation) therefore, $\frac{2}{1\,000} = 0{,}02$; $\frac{3}{1\,000} = 0{,}003$; etc.
- Recall that one tenth is equivalent to ten hundredths, which is equivalent to hundred thousandths, therefore, 0,1 = 0,10 = 0,100.

Answers

1. (a) 0,007 (b) 0,009 (c) 0,147 (d) 0,999
2. (a) 2,374 (b) 12,104 (c) 2,004
   (d) 67,123 (e) 34,061 (f) 654,003

7.2 Percentages and decimal fractions

HUNDREDTHS, PERCENTAGES AND DECIMALS

Background information

- **Per cent** is another word for hundredths. We do not say: "How many per cent of a drawing is blue?" We say: "What percentage of the drawing is blue?"
- Fractions written in words (for example, three tenths) can be expressed in decimal notation (0,3), percentage notation (30%) and common fraction notation ($\frac{3}{10}$).

The same quantity can be expressed in different ways in tenths and hundredths.

For example, *three tenths and 17 hundredths* can be expressed as *two tenths and 27 hundredths* or *four tenths and seven hundredths*.

All over the world, people have agreed to keep the number of hundredths in such statements below ten. This means that the normal way of expressing the above quantity is *four tenths and seven hundredths*.

Written in decimal notation, four tenths and seven hundredths is 0,47. This is read as *nought comma four seven* and NOT *nought comma forty-seven*.

6. What is the decimal notation for each of the following numbers?

   (a) $3\frac{7}{10}$ (b) $4\frac{19}{100}$ (c) $\frac{47}{10}$ (d) $\frac{4}{100}$

... AND THOUSANDTHS

0,001 is another way of writing $\frac{1}{1\,000}$.

1. What is the decimal notation for each of the following?

   (a) $\frac{7}{1\,000}$ (b) $\frac{9}{1\,000}$ (c) $\frac{147}{1\,000}$ (d) $\frac{999}{1\,000}$

2. Write the following numbers in the decimal notation:

   (a) $2 + \frac{3}{10} + \frac{7}{100} + \frac{4}{1\,000}$ (b) $12 + \frac{1}{10} + \frac{4}{1\,000}$

   (c) $2 + \frac{4}{1\,000}$ (d) $67\frac{123}{1\,000}$

   (e) $34\frac{61}{1\,000}$ (f) $654\frac{3}{1\,000}$

7.2 Percentages and decimal fractions

HUNDREDTHS, PERCENTAGES AND DECIMALS

1. The rectangle below is divided into small parts.

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Teaching guidelines

Recall the following:

- Per cent is another word for hundredths.
- The symbol for per cent is %.
- Six hundredths can be described as six per cent and written as 6%.

Discuss why 0,37 and 37% and $\frac{37}{100}$ are different symbols for the same thing: 37 hundredths.

Note on question 4

Calculating, for example, 13% of a whole, collection, quantity or measurement means dividing it by 100 and multiplying the answer by 13.

Note on question 6(f)

Seven eighths has to be written as hundredths, which is not possible. However, it can be written as thousandths and therefore as a percentage (87,5%). This is an opportunity to make learners aware of fractional percentages.

Answers

1. (a) 100 small parts in the rectangle; ten small parts in one tenth of the rectangle.
   (b) Blue: $\frac{30}{100}$ or $\frac{3}{10}$ Green: $\frac{2}{100}$ or $\frac{1}{50}$ Red: $\frac{1}{100}$
2. Two per cent is green; one per cent is red.
3. 30 per cent is blue; 57 per cent is white.
4. (a) R4 (b) R148 (c) R259
5. (a) $\frac{1}{4}$ or $\frac{25}{100}$;0,25 (b) $\frac{1}{4}$ or $\frac{25}{100}$; or 0,25 (c) 2 (d) 4; 4
6. (a) 0,3; 30%; $\frac{30}{100}$ (b) 0,07; 7%; $\frac{7}{100}$
   (c) 0,37; 37%; $\frac{37}{100}$ (d) 0,7; 70%; $\frac{70}{100}$
   (e) 0,75; 75%; $\frac{75}{100}$ (f) 0,875; 87,5%; ($\frac{875}{1\,000}$; not possible as hundredths)
7. (a) R60 + R14 = R74
   (b) R74

(a) How many of these small parts are there in the rectangle? And in one tenth of the rectangle?
(b) What part of the rectangle is blue? What part is green? What part is red?

Instead of *six hundredths*, you may say six *per cent*.
It means the same.
Ten per cent of the rectangle above is yellow.

2. Use the word "per cent" to say what part of the rectangle is green. What part is red?

3. What percentage of the rectangle is blue? What percentage is white?

We do not say: "How many per cent of the rectangle is green?" We say: "What percentage of the rectangle is green?"

The symbol % is used for "per cent".
Instead of writing "17 per cent", you may write 17%.
Per cent means *hundredths*. The symbol % is a bit like the symbol $\frac{\quad}{100}$.

4. (a) How much is 1% of R400? (In other words: How much is $\frac{1}{100}$ or 0,01 of R400?)
   (b) How much is 37% of R400?
   (c) How much is 37% of R700?

5. (a) 25 apples are shared equally between 100 people. What fraction of the apples does each person get? Write your answer as a common fraction and as a decimal fraction.
   (b) How much is 1% (one hundredth) of 25?
   (c) How much is 8% of 25?
   (d) How much is 8% of 50? And how much is 0,08 of 50?

0,37 and 37% and $\frac{37}{100}$ are different symbols for the same thing: *37 hundredths*.

6. Express each of the following in three ways:
   - in the *decimal notation*,
   - in the *percentage notation*, and
   - if possible, in the *common fraction notation, using hundredths*.

   (a) three tenths (b) seven hundredths
   (c) 37 hundredths (d) seven tenths
   (e) three quarters (f) seven eighths

7. (a) How much is three tenths of R200 and seven hundredths of R200 altogether?
   (b) How much is $\frac{37}{100}$ of R200?

Note on question 10

The floor is covered by 50 tiles. To write answers in common fraction notation, using hundredths, each count out of 50 has to be doubled.

Answers

7. (c) R74
   (d) R74

8. (a) 0,2; 20%; $\frac{20}{100}$ (b) 0,5; 50%; $\frac{50}{100}$
   (c) 0,25; 25%; $\frac{25}{100}$ (d) 0,75; 75%; $\frac{75}{100}$

9. (a) 25%
   (b) $\frac{75}{100}$ or $\frac{3}{4}$
   (c) 75%

10. See LB page 114 alongside.

Additional questions (mental mathematics)

Express in common fractions, decimal and percentage notations:

1. 3 eighths: $\frac{3}{8}$; 0,375; 37,5%
2. 2 fifths: $\frac{2}{5}$; 0,4; 40%
3. 1 tenth: $\frac{1}{10}$; 0,1; 10%
4. 2 thirds: $\frac{2}{3}$; 0,667; 66,7%
5. 5 eighths: $\frac{5}{8}$; 0,625; 62,5%
6. 4 fifths: $\frac{4}{5}$; 0,8; 80%
7. 9 tenths: $\frac{9}{10}$; 0,9; 90%
8. 1 third: $\frac{1}{3}$; 0,333; 33,3%
9. 7 tenths: $\frac{7}{10}$; 0,7; 70%
10. 1 quarter: $\frac{1}{4}$; 0,25; 25%
11. 7 eighths: $\frac{7}{8}$; 0,875; 87,5%
12. 3 fifths: $\frac{3}{5}$; 0,6; 60%
13. 3 tenths: $\frac{3}{10}$; 0,3; 30%
14. 3 quarters: $\frac{3}{4}$; 0,75; 75%
15. 1 eighth: $\frac{1}{8}$; 0,125; 12,5%
16. 1 fifth: $\frac{1}{5}$; 0,2; 20%

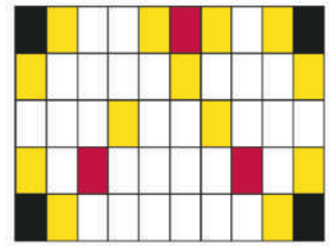
(c) How much is 0,37 of R200?
(d) And how much is 37% of R200?

8. Express each of the following in three ways:
   - in the *decimal notation*,
   - in the *percentage notation*, and
   - in the *common fraction notation, using hundredths*.

   (a) 20 hundredths (b) 50 hundredths
   (c) 25 hundredths (d) 75 hundredths

9. (a) Jan eats a quarter of a watermelon. What percentage of the watermelon is this?
   (b) Sibu drinks 75% of the milk in a bottle. What fraction of the milk is this?
   (c) Jeminah uses 0,75 (seven tenths and five hundredths) of the paint in a tin. What percentage of the paint does she use?

10. The floor of a large room is shown alongside. What part of the floor is covered in each of the four colours? Copy the table below. Express your answer in four ways:
    (a) in the *common fraction notation, using hundredths*,
    (b) in the *decimal notation*,
    (c) in the *percentage notation*, and
    (d) if possible, *in the common fraction notation, as tenths* **and** *hundredths* (for example $\frac{3}{10} + \frac{4}{100}$).



	(a)	(b)	(c)	(d)
white	$\frac{60}{100}$	0,6	60%	–
red	$\frac{6}{100}$	0,06	6%	–
yellow	$\frac{26}{100}$	0,26	26%	$\frac{2}{10} + \frac{6}{100}$
black	$\frac{8}{100}$	0,08	8%	–

7.3 Decimal measurements

MEASURING ON A NUMBER LINE

1. Read the lengths at the marked points (A to D) for the number lines on the next two pages. Give your answers as accurate as possible in decimal notation.

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7.3 Decimal measurements

MEASURING ON A NUMBER LINE

Background information

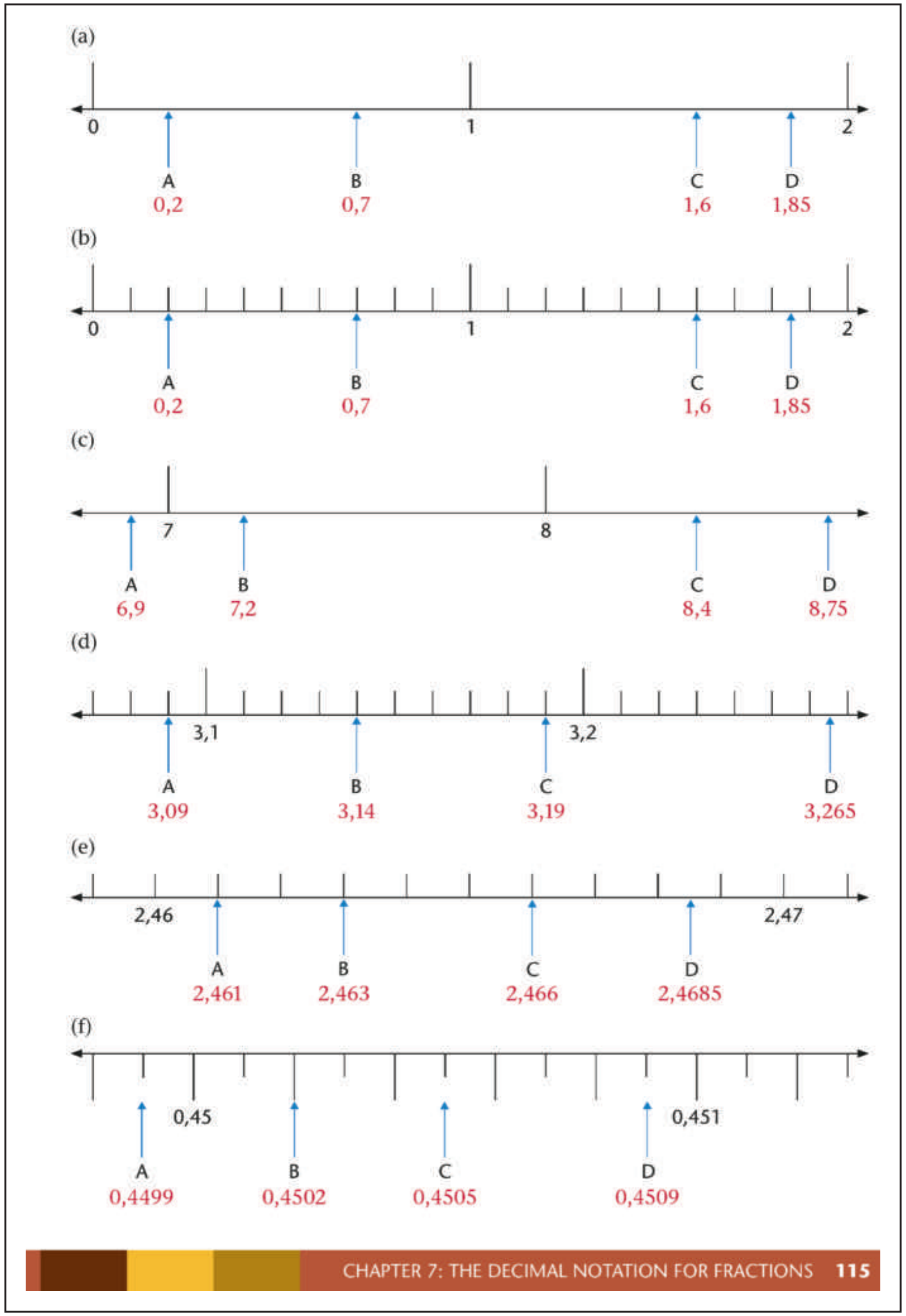
Measurement can never be exact – it is always an approximation. The smaller the divisions in which the unit is subdivided, the more exact the measurement. We always estimate the number of smallest units. When we say that a length is 2,86, we actually mean that it is somewhere between 2,85 and 2,87.

Teaching guidelines

The estimation tasks (for example, question 1(a)) help learners to acquire a feeling for the size of the number and its position on the number line. It is crucial that learners make sense of different scales in these tasks, and that you discuss the scales in class. Ask questions such as: "What did you have to consider before you decided what value the point represents, or where to place the number?"

Answers

1. (a) – (f) See the number lines on LB page 115 alongside.



Answers

1. (g) See the number line on LB page 116 alongside.
2. See LB page 116 alongside.
3. See LB page 116 alongside.

7.4 More decimal concepts

DECIMAL JUMPS

Background information

The calculator is an efficient teaching aid for learners to explore and investigate decimals. The suggestion to program a calculator as a **counting machine** on LB page 117 should be taken seriously. This is valuable in helping learners to "count forwards and backwards in decimal fractions to at least two decimal places" as stated by CAPS (page 51). They can count in groups and use the calculator to check their counting without a teacher being present.

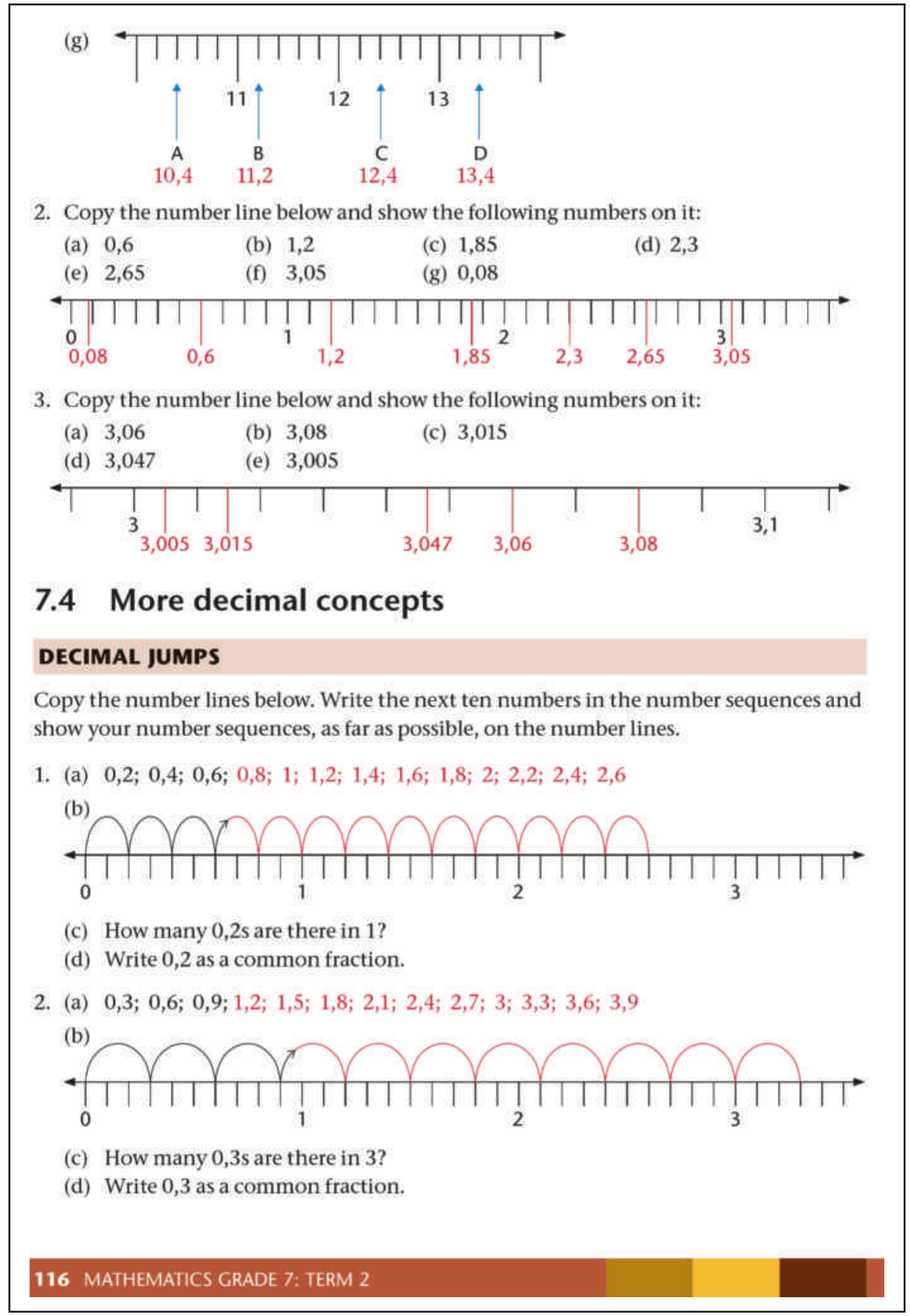
Teaching guidelines

Illustrate how to program a calculator to do the same operation over and over again. For example, to generate the following number sequences on a CASIO *fx-82ES PLUS*:

- 0,1; 0,2; 0,3; 0,4; 0,5; ...:
  Press 0,1 [+] 0,1 [=] [+] 0,1 [=] and then [=] repeatedly.
- 1,00; 0,98; 0,96; 0,94; 0,92; ...:
  Press 1,00 [−] 0,02 [=] [−] 0,02 [=] and then [=] repeatedly.

Answers

1. (a) and (b) See LB page 116 alongside.
   (c) 5
   (d) $\frac{1}{5}$
2. (a) and (b) See LB page 116 alongside.
   (c) 10
   (d) $\frac{3}{10}$


(g)

2. Copy the number line below and show the following numbers on it:
   (a) 0,6 (b) 1,2 (c) 1,85 (d) 2,3
   (e) 2,65 (f) 3,05 (g) 0,08

3. Copy the number line below and show the following numbers on it:
   (a) 3,06 (b) 3,08 (c) 3,015
   (d) 3,047 (e) 3,005

7.4 More decimal concepts

DECIMAL JUMPS

Copy the number lines below. Write the next ten numbers in the number sequences and show your number sequences, as far as possible, on the number lines.

1. (a) 0,2; 0,4; 0,6; 0,8; 1; 1,2; 1,4; 1,6; 1,8; 2; 2,2; 2,4; 2,6
   (b)

   (c) How many 0,2s are there in 1?
   (d) Write 0,2 as a common fraction.
2. (a) 0,3; 0,6; 0,9; 1,2; 1,5; 1,8; 2,1; 2,4; 2,7; 3; 3,3; 3,6; 3,9
   (b)

   (c) How many 0,3s are there in 3?
   (d) Write 0,3 as a common fraction.

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Answers

3. (a) to (b) see LB page 117 alongside.
   (c) 4
   (d) $\frac{1}{4}$
4. Learners check their answer with a calculator.
5. (a) See LB page 117 alongside.
   (b) see LB page 117 alongside
6. Learners check their answers with a calculator.

PLACE VALUE

Background information

Consider the decimal number 4,567:

- The **place values** of the digits are units, tenths, hundredths and thousandths respectively.
- The **values of the digits** are 4 × 1 = 4, 5 × 0,1 = 0,5, 6 × 0,01 = 0,06 and 7 × 0,001 = 0,007 respectively.

We can use either place values or digit values to write 4,567 in **expanded form**:

- **place values**: 4 units + 5 tenths + 6 hundredths + 7 thousandths
- **digit values**: 4 + 0,5 + 0,06 + 0,007

Teaching guidelines

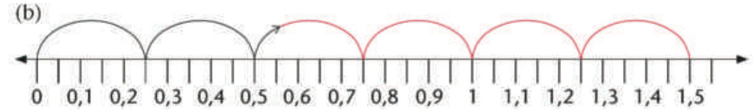
Revise the difference between the following concepts:

- the **place value of a digit** in a number
- the **value of that digit** in the number.

Answers

1.	(a) 2,57	(b) 2,507
	(c) 2,057	(d) 5,431
	(e) 5,143	(f) 5,314

3. (a) 0,25; 0,5; 0,75; 1; 1,25; 1,5; 1,75; 2; 2,25; 2,5; 2,75; 3
   (b)



   (c) How many 0,25s are there in 1?
   (d) Write 0,25 as a common fraction.

A calculator can be programmed to do the same operation over and over again.
For example, press 0,1 + = (do not press CLEAR or any other operation). Press the = key repeatedly and see what happens.
The calculator counts in 0,1s.

4. You can check your answers for questions 1 to 3 with a calculator. Program the calculator to help you.
5. Write the next five numbers in the number sequences:
   (a) 9,3; 9,2; 9,1; 9; 8,9; 8,8; 8,7; 8,6
   (b) 0,15; 0,14; 0,13; 0,12; 0,11; 0,1; 0,09; 0,08; 0,07
6. Check your answers with a calculator. Program the calculator to help you.

PLACE VALUE

1. Write each of the following as one number:

(a) 2 + 0,5 + 0,07	(b) 2 + 0,5 + 0,007
(c) 2 + 0,05 + 0,007	(d) 5 + 0,4 + 0,03 + 0,001
(e) 5 + 0,04 + 0,003 + 0,1	(f) 5 + 0,004 + 0,3 + 0,01

We can write 3,784 in expanded notation as 3,784 = 3 + 0,7 + 0,08 + 0,004.
We can also name these parts as follows:

- the 3 represents the **units**
- the 7 represents the **tenths**
- the 8 represents the **hundredths**
- the 4 represents the **thousandths**

We say: the **value** of the seven is seven tenths but the **place value** of the 7 is tenths, because any digit **in that place** will represent the number of tenths.

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Answers

2. (a) 0,04; hundredths (b) 0,008; thousandths
   (c) 0,9; tenths (d) 0,8; tenths
   (e) 0,04; hundredths (f) 0,007; thousandths

7.5 Ordering and comparing decimal fractions

FROM BIGGEST TO SMALLEST AND SMALLEST TO BIGGEST

Background information

- Decimal fractions are compared by looking at their number of tenths first, then at their hundredths, then at their thousandths, etc.
- The value of a decimal fraction does not change if zeros are written at the end because $\frac{1}{10}$, $\frac{10}{100}$ and $\frac{100}{1\,000}$ are equivalent and therefore, written in decimal notation: 0,1; 0,10 and 0,100 are also equivalent.

Teaching guidelines

Explain why 0,1; 0,10 and 0,100 are equivalent decimal fractions.

Discuss how to order decimal fractions. For example:

- To order up to **two-digit decimal fractions**, write zeros at the end of the fractions so that each fraction represents hundredths.
- To order up to **three-digit decimal fractions**, write zeros at the end of the fractions so that each fraction represents thousandths.

Answers

1. 0,8 0,75 0,62 0,55 0,5 0,465 0,4 0,15 0,05

   Learners' methods may differ. Sample explanation: I added zeros at the end of the numbers so that each number represents thousandths. That made it easier for me to compare the numbers.
2. (a) – (b) See LB page 118 alongside.

For example, in 2,536 the **value** of the three is 0,03, and its **place value** is hundredths, because the value of the **place where it stands** is hundredths.

2. Now write the value (in decimal fractions) and the place value of each of the underlined digits.
   (a) 2,345 (b) 4,678 (c) 1,953
   (d) 34,856 (e) 564,34 (f) 0,987

7.5 Ordering and comparing decimal fractions

FROM BIGGEST TO SMALLEST AND SMALLEST TO BIGGEST

1. Order the following numbers from biggest to smallest. Explain your method.
   0,8 0,05 0,5 0,15 0,465 0,55 0,75 0,4 0,62
2. Below are the results of some of the 2012 London Olympic events. Copy and complete the tables. In each case, order them from first to last place.
   (a) Women: Long jump – Final

Name	Country	Distance	Place
Anna Nazarova	RUS	6,77 m	5th
Brittney Reese	USA	7,12 m	1st
Elena Sokolova	RUS	7,07 m	2nd
Ineta Radevica	LAT	6,88 m	4th
Janay DeLoach	USA	6,89 m	3rd
Lyudmila Kolchanova	RUS	6,76 m	6th

   (b) Women: 400 m hurdles – Final

Name	Country	Time	Place
Georganne Moline	USA	53,92 s	5th
Kaliese Spencer	JAM	53,66 s	4th
Lashinda Demus	USA	52,77 s	2nd
Natalya Antyukh	RUS	52,70 s	1st
T'erea Brown	USA	55,07 s	6th
Zuzana Hejnová	CZE	53,38 s	3rd

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Answers

2. (c) – (d) See LB page 119 alongside.
3. Examples:
   (a) 3,6 or 3,55 or 3,679
   (b) 3,2 or 3,41 or 3,889
   (c) 3,15 or 3,11 or 3,195
4. Unlimited/infinite
5. See LB page 119 alongside.

7.6 Rounding off - saying it nearly but not quite

Background information

Decimal fractions can be rounded off to the nearest whole number or to one, two, three, etc. digits after the decimal comma.

Rule for rounding to the nearest whole number:

- If the 10ths digit is 5 or more, round up to the next whole number.
- If the 10ths digit is less than 5, round down to the previous whole number.

Rule for rounding to one decimal place (tenths):

- If the 100ths digit is 5 or more, round up to the next tenth.
- If the 100ths digit is less than 5, round down to the previous tenth.

Rule for rounding to two decimal places (hundredths):

- If the 1 000ths digit is 5 or more, round up to the next hundredth.
- If the 1 000ths digit is less than 5, round down to the previous hundredth.

Rule for rounding to three decimal places (thousandths):

- If the 10 000ths digit is 5 or more, round up to the next thousandth.
- If the 10 000ths digit is less than 5, round down to the previous thousandth.

A powerful calculator activity to help learners think about rounding off is to ask them to enter a number with as many digits as the calculator will display. For example, if the calculator can take eight digits, they enter 12345678. If they add 0,4, the number will stay the same, but if they add 0,6, the last digit will change to 9.

(c) Men: 110 m hurdles – Final

Name	Country	Time	Place
Aries Merritt	USA	12,92 s	1st
Hansle Parchment	JAM	13,12 s	3rd
Jason Richardson	USA	13,04 s	2nd
Lawrence Clarke	GBR	13,39 s	4th
Orlando Ortega	CUB	13,43 s	6th
Ryan Brathwaite	BAR	13,40 s	5th

(d) Men: Javelin – Final

Name	Country	Distance	Place
Andreas Thorkildsen	NOR	82,63 m	6th
Antti Ruuskanen	FIN	84,12 m	3rd
Keshorn Walcott	TRI	84,58 m	1st
Oleksandr Pyatnytsya	UKR	84,51 m	2nd
Tero Pitkämäki	FIN	82,80 m	5th
Vítezslav Veselý	CZE	83,34 m	4th

3. In each case, give a number that falls between the two numbers.
   (This means you may give *any* number that falls anywhere between the two numbers.)
   (a) 3,5 and 3,7 (b) 3,9 and 3,11 (c) 3,1 and 3,2
4. How many numbers are there between 3,1 and 3,2?
5. Copy and fill in <, > or =.
   (a) 0,4 < 0,52 (b) 0,4 > 0,32 (c) 2,61 < 2,7
   (d) 2,4 = 2,40 (e) 2,34 < 2,567 (f) 2,34 > 2,251

7.6 Rounding off

Just as whole numbers can be rounded off to the nearest 10, 100 or 1 000, decimal fractions can be rounded off to the nearest whole number or to one, two, three etc. digits after the comma. A decimal fraction is rounded off to the number whose value is closest to it. Therefore 13,24 rounded off to one decimal place is 13,2 and 13,26 rounded off to one decimal place is 13,3. A decimal ending in a 5 is an equal distance from the two numbers to which it can be rounded off. Such decimals are rounded off to the biggest number, so 13,15 rounded off to one decimal place becomes 13,2.

SAYING IT NEARLY BUT NOT QUITE - ROUNDING OFF

Teaching guidelines

Discuss the rules for rounding off decimal numbers to the nearest whole number, tenth, hundredth and thousandth.

Answers

1.	8	18	205	2	1	35	12	1
2.	7,7	18,9	21,5	0,6	0,9	1,4	3,8	
3.	3,43	54,12	4,81	3,76	4,26	10,22	9,37	300,00

ROUND OFF TO HELP YOU CALCULATE

Background information

When an amount of money is divided by any number, the quotient is always, when necessary, rounded off to two decimal places, that is, to the nearest cent.

Teaching guidelines

Discuss what to do if R7 is to be shared equally amongst eight people.

Answers

1. R44,65 ÷ 4 = 11,1625 ≈ R11,16
   Three brothers will get R11,16 and one brother will get R11,17.
2. 3,75 × 11,99 = 44,9625 ≈ R44,96
3. (a) 89 × 4 = 356 (b) 227 + 72 – 29 = 270

7.7 Addition and subtraction with decimal fractions

MENTAL CALCULATIONS

Teaching guidelines

Mental addition and subtraction with decimal fractions can be practised by using number chains or a calculator (refer to section 7.4).

Answers

1. See LB page 120 alongside.

SAYING IT NEARLY BUT NOT QUITE

1. Round each of the following numbers off to the nearest whole number:
   7,6 18,3 204,5 1,89 0,9 34,7 11,5 0,65
2. Round each of the following numbers off to one decimal place:
   7,68 18,93 21,47 0,643 0,938 1,44 3,81
3. Round each of the following numbers off to two decimal places:
   3,432 54,117 4,809 3,762 4,258 10,222 9,365 299,996

ROUND OFF TO HELP YOU CALCULATE

1. John and three of his brothers sell an old bicycle for R44,65. How can the brothers share the money fairly?
2. A man buys 3,75 m of wood at R11,99 per metre. What does the wood cost him?
3. Estimate the answers of each of the following by rounding off the numbers:
   (a) 89,3 × 3,8 (b) 227,3 + 71,8 – 28,6

7.7 Addition and subtraction with decimal fractions

MENTAL CALCULATIONS

1. Copy and complete the number chain.

34,123 → + 20 → 54,123 → + 400 → 454,123 → – 0,1 → 454,023
↓ – 2 ↓
422,011 ← – 30 ← 452,011 ← – 0,01 ← 452,021 ← – 0,002 ← 452,023
↓ – 200 ↓
222,011 → + 0,2 → 222,211 → + 0,02 → 222,231 → + 0,001 → 222,232
↓ – 0,01 ↓
222,489 ← + 0,007 ← 222,482 ← + 0,06 ← 222,422 ← + 0,2 ← 222,222

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Teaching guidelines (continued)

Explain how to add and subtract decimal fractions:

- Convert the decimal fractions to common fractions.
- Convert all common fractions to tenths, hundredths or thousandths.
- Add and subtract.
- Convert the answer to a decimal fraction.

Answers

2.	(a) 0,9	(b) 1,1	(c) 2,1	(d) 2,15	(e) 0,95
	(f) 0,32	(g) 2,9	(h) 2,99	(i) 1,9	

SOME REAL-LIFE PROBLEMS

Background information

Calculations with decimal fractions feature in a variety of real-life problems, for example, to determine:

- the winners of track and field contests in athletics
- the price of products sold in small quantities, like mincemeat
- the distance travelled by vehicles and their fuel consumption
- interest rates charged by banks and financial institutions.

Teaching guidelines

Discuss examples where decimals feature in real-life problems.

Answers

1. R435,60
2. 174,3 km
3. 13,2 seconds
4. Contestant A: 7,5 + 8 + 7,7 = 23,2 Contestant B: 8,5 + 8,9 + 8,7 = 26,1
   Contestant C: 7,9 + 8,1 + 7,8 = 23,8 Contestant D: 8,9 + 9 + 9,1 = 27
   D: 27 (first); B: 26,1 (second); C: 23,8 (third); A: 23,2 (fourth)
5. 14,80 mm –13,97 mm = 0,83 mm
6. 2,535 kg

When you add or subtract decimal fractions, you can change them to common fractions to make the calculation easier.

For example:

$0,4 + 0,5$
$= \frac{4}{10} + \frac{5}{10}$
$= \frac{9}{10}$
$= 0,9$

2. Calculate each of the following:

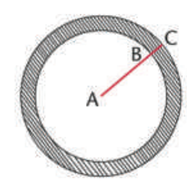
(a) $0,7 + 0,2$	(b) $0,7 + 0,4$	(c) $1,3 + 0,8$
(d) $1,35 + 0,8$	(e) $0,25 + 0,7$	(f) $0,25 + 0,07$
(g) $3 - 0,1$	(h) $3 - 0,01$	(i) $2,4 - 0,5$

SOME REAL-LIFE PROBLEMS

1. The owner of an internet café looks at her bank statement at the end of the day. She finds the following amounts paid into her account: R281,45; R39,81; R104,54 and R9,80. How much money was paid into her account on that day?
2. At the beginning of a journey the odometer in a car reads: 21 589,4. At the end of the journey the odometer reads: 21 763,7. What distance was travelled?
3. At an athletics competition, an athlete runs the 100 m race in 12,8 seconds. The announcer says that the athlete has broken the previous record by 0,4 seconds. What was the previous record?
4. In a surfing competition, five judges give each contestant a mark out of 10. The highest and the lowest marks are ignored and the other three marks are totalled. Work out each contestant's score and place the contestants in order from first to last.

A:	7,5	8	7	8,5	7,7	B:	8,5	8,5	9,1	8,9	8,7
C:	7,9	8,1	8,1	7,8	7,8	D:	8,9	8,7	9	9,3	9,1

5. A pipe is measured accurately. AC = 14,80 mm and AB = 13,97 mm.
   How thick is the pipe (BC)?



6. Mrs Mdlankomo buys three packets of mincemeat. The packets weigh 0,356 kg, 1,201 kg and 0,978 kg respectively. What do they weigh together?

7.8 Multiplication and decimal fractions

THE POWER OF TEN

Teaching guidelines

Learners formulate rules for multiplication and division by powers of ten.

Rules for multiplication of a number by:

- 10, 100 or 1 000: With the comma remaining fixed, move each digit one, two or three places respectively to the left (the number increases).
- 0,1, 0,01 or 0,001: With the comma remaining fixed, move each digit one, two or three places respectively to the right (the number decreases).

Multiplying a number by 0,1 is the same as dividing it by 10.

Rules for division of a number by:

- 10, 100 or 1 000: With the comma remaining fixed, move each digit one, two or three places respectively to the right (the number decreases).
- 0,1, 0,01 or 0,001: With the comma remaining fixed, move each digit one, two or three places respectively to the left (the number increases).

Dividing a number by 0,1 is the same as multiplying it by 10.

Answers

1. (a) See LB page 122 alongside.
   (b) No, it is not correct; it is true only if you multiply with whole numbers. Multiply by 10; 100 or 1 000: the value of each digit in the number becomes 10, 100 or 1 000 times bigger and each digit thus moves one, two or three places to the left. Multiply by 0,1; 0,01 or 0,001: each digit moves one, two or three places to the right. The comma remains fixed. To multiply a number by 0,1 is the same as dividing it by 10.
   (d) 5 30 4,2 67,5
2. (a) See LB page 122 alongside.
   (b) Divide by 10; 100 or 1 000: the value of each digit in the number becomes 10, 100 or 1 000 times smaller and so each digit moves one, two or three places to the left.
   (c) 0,05 0,003 0,042
3. See LB page 122 alongside.

7.8 Multiplication and decimal fractions

THE POWER OF TEN

1. (a) Copy and complete the multiplication table.

×	1 000	100	10	1	0,1	0,01	0,001
6	6 000	600	60	6	0,6	0,06	0,006
6,4	6 400	640	64	6,4	0,64	0,064	0,0064
0,5	500	50	5	0,5	0,05	0,005	0,0005
4,78	4 780	478	47,8	4,78	0,478	0,0478	0,00478
41,2	41 200	4 120	412	41,2	4,12	0,412	0,0412

(b) Is it correct to say that "multiplication makes bigger"? When does multiplication make bigger?
(c) Formulate rules for multiplying with 10; 100; 1 000; 0,1; 0,01 and 0,001. Can you explain the rules?
(d) Now use your rules to calculate each of the following:
0,5 × 10 0,3 × 100 0,42 × 10 0,675 × 100

2. (a) Copy and complete the division table.

÷	1	10	100	1 000
6	6	0,6	0,06	0,006
6,4	6,4	0,64	0,064	0,0064
0,5	0,5	0,05	0,005	0,0005
4,78	4,78	0,478	0,0478	0,00478
41,2	41,2	4,12	0,412	0,0412

(b) Formulate rules for dividing with 10; 100 and 1 000. Can you explain the rules?
(c) Now use your rules to calculate each of the following:
0,5 ÷ 10 0,3 ÷ 100 0,42 ÷ 10

3. Copy and complete the following statement:
Multiplying with 0,1 is the same as dividing by 10.

Now discuss it with a partner or explain to him or her why this is so.

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Answers

4. See LB page 123 alongside.

MULTIPLYING DECIMALS WITH WHOLE NUMBERS

Background information

To multiply a decimal number with a whole number, follow these steps:

- Write the decimal number in common fraction notation.
- Multiply the numerator by the whole number.
- Divide the answer by the denominator.

Teaching guidelines

Learners should be able to explain what is meant by multiplying a decimal by a whole number.

Answers

1. (a) $\frac{3}{10} \times 7 = \frac{21}{10} = 2{,}1$

   (b) $\frac{21}{100} \times 91 = \frac{1\,911}{100} = 19{,}11$

   (c) $8 \times \frac{4}{10} = \frac{32}{10} = 3{,}2$

2. (a) $4 \times 7 = 28 \rightarrow 2{,}8$

   (b) $55 \times 7 = 385 \rightarrow 3{,}85$

   (c) $12 \times 12 = 144 \rightarrow 1{,}44$

   (d) $601 \times 2 = 1\,202 \rightarrow 1{,}202$

3. Sample answer: Estimate the answer, ignore the comma, multiply and put the comma back, according to the estimated answer.

4. Copy and fill in the missing numbers:

1,23456 → × 10 → 12,3456 → × 10 → 123,456 → × 100 → 12 345,6

↓ × 100 ↓

1 234 560 → ÷ 10 → 123 456 → ÷ 10 → 12 345,6 → ÷ 10 → 1 234,56

↓ ÷ 10 ↓

123,456 → ÷ 100 → 1,23456 → ÷ 10 → 0,123456 → ×1 000 000 → 123 456

What does multiplying a decimal number with a whole number mean?

What does something like 4 × 0,5 mean?

What does something like 0,5 × 4 mean?

4 × 0,5 means four groups of $\frac{1}{2}$, which is $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$, which is 2.

0,5 × 4 means $\frac{1}{2}$ of 4, which is 2.

A real-life example where we would find this is:

$$\begin{aligned} 6 \times 0{,}42 \text{ kg} &= 6 \times \frac{42}{100} \\ &= (6 \times 42) \div 100 \\ &= 252 \div 100 \\ &= 2{,}52 \text{ kg} \end{aligned}$$

What really happens is that we convert 6 × 0,42 to the product of two whole numbers, do the calculation and then convert the answer to a decimal fraction again (÷ 100).

MULTIPLYING DECIMALS WITH WHOLE NUMBERS

1. Calculate each of the following. Use fraction notation to help you.
   (a) 0,3 × 7 (b) 0,21 × 91 (c) 8 × 0,4
2. Estimate the answers to each of the following and then calculate:
   (a) 0,4 × 7 (b) 0,55 × 7
   (c) 12 × 0,12 (d) 0,601 × 2
3. Make a rule for multiplying with decimals. Explain your rule to a partner.

MULTIPLYING DECIMALS WITH DECIMALS

Background information

To multiply a decimal number with a decimal number, follow these steps:

- Write the decimal numbers in common fraction notation.
- Multiply the numerators.
- Divide the answer by the product of the denominators.

Teaching guidelines

Learners should be able to explain what is meant by multiplying a decimal by a decimal.

Answers

1. (a) $0,6 \times 0,4 = \frac{6}{10} \times \frac{4}{10} = \frac{24}{100} = 0,24$

   (b) $0,06 \times 0,4 = \frac{6}{100} \times \frac{4}{10} = \frac{24}{1\,000} = 0,024$

   (c) $0,06 \times 0,04 = \frac{6}{100} \times \frac{4}{100} = \frac{24}{10\,000} = 0,0024$

2. (a) $0,4 \times 0,7$
   $= (4 \div 10) \times (7 \div 10)$
   $= (4 \times 7) \div (10 \times 10)$
   $= 28 \div 100$
   $= 0,28$

   (b) $0,4 \times 7$
   $= (4 \times 7) \div 10$
   $= 28 \div 10$
   $= 2,8$

   (c) $0,04 \times 0,7$
   $= (4 \div 100) \times (7 \div 10)$
   $= (4 \times 7) \div (100 \times 10)$
   $= 28 \div 1\,000$
   $= 0,028$

What does multiplying a decimal with a decimal mean?

For example, what does 0,32 × 0,87 mean?

If you buy 0,32 m of ribbon and each metre costs R0,87, you can write it as 0,32 × 0,87.

$0,32 \times 0,87 = \frac{32}{100} \times \frac{87}{100}$	[Write as common fractions]
$= \frac{32 \times 87}{10\,000}$	[Multiplication of two fractions]
$= \frac{2\,784}{10\,000}$	[The product of the whole numbers 32 × 87]
$= 0,2784$	[Convert to a decimal by dividing the product by 10 000]

The product of two decimals is thus converted to the product of whole numbers and then converted back to a decimal.

The product of two decimals and the product of two whole numbers with the same digits differ only in terms of the place value of the products, in other words the position of the decimal comma. It can also be determined by estimating and checking.

MULTIPLYING DECIMALS WITH DECIMALS

1. Calculate each of the following. Use fraction notation to help you.
   (a) 0,6 × 0,4 (b) 0,06 × 0,4 (c) 0,06 × 0,04

Mandla uses this method to multiply decimals with decimals:

$0,84 \times 0,6 = (84 \div 100) \times (6 \div 10)$
$= (84 \times 6) \div (100 \times 10)$
$= 504 \div 1\,000$
$= 0,504$

2. Calculate the following using Mandla's method:
   (a) 0,4 × 0,7 (b) 0,4 × 7 (c) 0,04 × 0,7

7.9 Division and decimal fractions

Look carefully at the following three methods of calculation:

1. 0,6 ÷ 2 = 0,3 [six tenths ÷ 2 = three tenths]
2. 12,4 ÷ 4 = 3,1 [(12 units + four tenths) ÷ 4]
   = (12 units ÷ 4) + (four tenths ÷ 4)
   = three units + one tenth
   = 3,1

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7.9 Division and decimal fractions

DIVIDING DECIMALS BY WHOLE NUMBERS

Background information

Division of decimals by whole numbers resembles **sharing** situations.

Various methods can be used to divide decimals by whole numbers (refer to LB pages 124 and 125). An alternative for the third method in the Learner Book is the following:

$2{,}8 \div 5$

$= \frac{28}{10} \div 5$ (28 is not divisible by 5)

$= \frac{280}{100} \div 5$ (280 is divisible by 5)

$= \frac{56}{100}$

$= 0{,}56$

Teaching guidelines

Discuss the methods for division listed on LB pages 124 and 125 as well as the method provided above.

Answers

1. See LB page 125 alongside.
2. (a) 8 hundredths ÷ 4 = 2 hundredths = 0,02
   (b) 144 tenths ÷ 12 = 12 tenths → 1,2
   (c) 84 tenths ÷ 7 = 12 tenths → 1,2
   (d) 45 tenths ÷ 15 = 3 tenths → 0,3
   (e) 1 655 thousandths ÷ 5 = 331 thousandths → 0,331
   (f) 225 hundredths ÷ 25 = 9 hundredths → 0,009
3. 9 990 ÷ 15 = 666 → One kilogram of bananas costs R6,66.
4. (a) 67 (b) 0,067
5. (a) 1,6 (b) 0,16
6. 20 × 0,45 = R9
7. 42,95 × 0,215 = 9,23425
   Depending on the shop, she would pay either R9,20 or R9,25.

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3. 2,8 ÷ 5 = 28 tenths ÷ 5
   = 25 tenths ÷ 5 and three tenths ÷ 5
   = five tenths and (three tenths ÷ 5) [three tenths cannot be divided by 5]
   = five tenths and (30 hundredths ÷ 5) [three tenths = 30 hundredths]
   = five tenths and six hundredths
   = 0,56

DIVIDING DECIMALS BY WHOLE NUMBERS

1. Complete.
   (a) 8,4 ÷ 2
   = (8 units + 4 tenths) ÷ 2
   = (8 units ÷ 2) + (4 tenths ÷ 2)
   = 4 units + 2 tenths
   = 4,2

   (b) 3,4 ÷ 4
   = (3 units + 4 tenths) ÷ 4
   = (32 tenths + 20 hundredths) ÷ 4
   = (32 tenths ÷ 4) + (20 hundredths ÷ 4)
   = 8 tenths + 5 hundredths
   = 0,85
2. Calculate each of the following in the shortest possible way:
   (a) 0,08 ÷ 4 (b) 14,4 ÷ 12
   (c) 8,4 ÷ 7 (d) 4,5 ÷ 15
   (e) 1,655 ÷ 5 (f) 0,225 ÷ 25
3. A grocer buys 15 kg of bananas for R99,90. What do the bananas cost per kilogram?
4. Given 26,8 ÷ 4 = 6,7. Write down the answers to the following without calculating:
   (a) 268 ÷ 4 (b) 0,268 ÷ 4
5. Given 128 ÷ 8 = 16. Write down the answers to the following without calculating:
   (a) 12,8 ÷ 8 (b) 1,28 ÷ 8
6. John buys 0,45 m of chain. The chain costs R20 per metre. What does John pay for the chain?
7. You may use a calculator for this question.
   Anna buys a packet of mincemeat. It weighs 0,215 kg. The price for the mincemeat is R42,95 per kilogram. What does she pay for her packet of mincemeat? (Give a sensible answer.)

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
8.1 Constant and variable quantities	The term **relationship** is explained in terms of variable quantities (input and output values) and represented in various ways, by a word formula and by means of a flow diagram	Pages 126 to 127
8.2 Different ways to describe relationships	Different ways of representing the same relationship; equivalent computational instructions (rules); ways of identifying the rule of a relationship	Pages 126 to 132

CAPS time allocation	3 hours
CAPS content specification	Pages 53

Mathematical background

The understanding of the use of letters to represent variables in generalised arithmetic lies at the heart of the ability to understand algebra. Therefore, this topic is very important in the development of learners' understanding.

Letters to represent numbers are used as unknown constants in equations; as variables in formulae which show the relationships between the variables; as variables in identities which are true for all values of the variables.

Working with patterns provides the opportunity to determine variable values as input values, output values and to determine relationships between variables.

- A relationship exists between variables if one variable is influenced by the other, for example, the cost of fuel for a car is influenced by the number of litres that are put into the tank.
- The relationship is usually given as a rule that we apply to each input value to get an output value. It works like a "machine" that we can represent in a flow diagram as follows: input → rule → output. A flow diagram is another way of describing a computational instruction; the rule of the relationship which will later become the formula.
- In a number pattern each output value is connected to a particular input value.
- Spider diagrams (which are actually combined flow diagrams) are helpful for learners to see a number pattern.
- If we know the input and output values of a pattern, we can find the rule.
- Simple number patterns grow if the input is multiplied (or divided) by a value to give the output, or if an input value is added to or subtracted from.
- Questions that can be asked include: Find the output if an input value is known. Find the rule, or find input value if the output value is known.
- Relationships can be represented in different ways, such as in words, flow diagrams, tables and graphs (which will be discussed in future work).
- The core algebraic concept of equivalence can be illustrated using patterns. Equivalent computational instructions (rules) are those that produce the same output values for the same input values. For example, "divide by 5 and multiply by 2" has the same effect as multiplying by 0,4 (or $\frac{2}{5}$).

8.1 Constant and variable quantities

LOOK FOR CONNECTIONS BETWEEN QUANTITIES

Teaching guidelines

Make sure that learners understand the difference between constant quantities and variable quantities. Ask questions such as: "Which quantities are involved in this situation?", "Which quantities change?", "How do they change?", etc.

The example of a relationship between variables is represented in various ways. The relationship between the number of red squares and the number of yellow squares in patterned arrangements is described by:

- a word formula (the number of yellow squares is equal to...)
- means of a flow diagram.

This situation gives an opportunity to clarify the terms **output number** and **input number** and to point out that every input number is linked to a specific output number.

Misconceptions

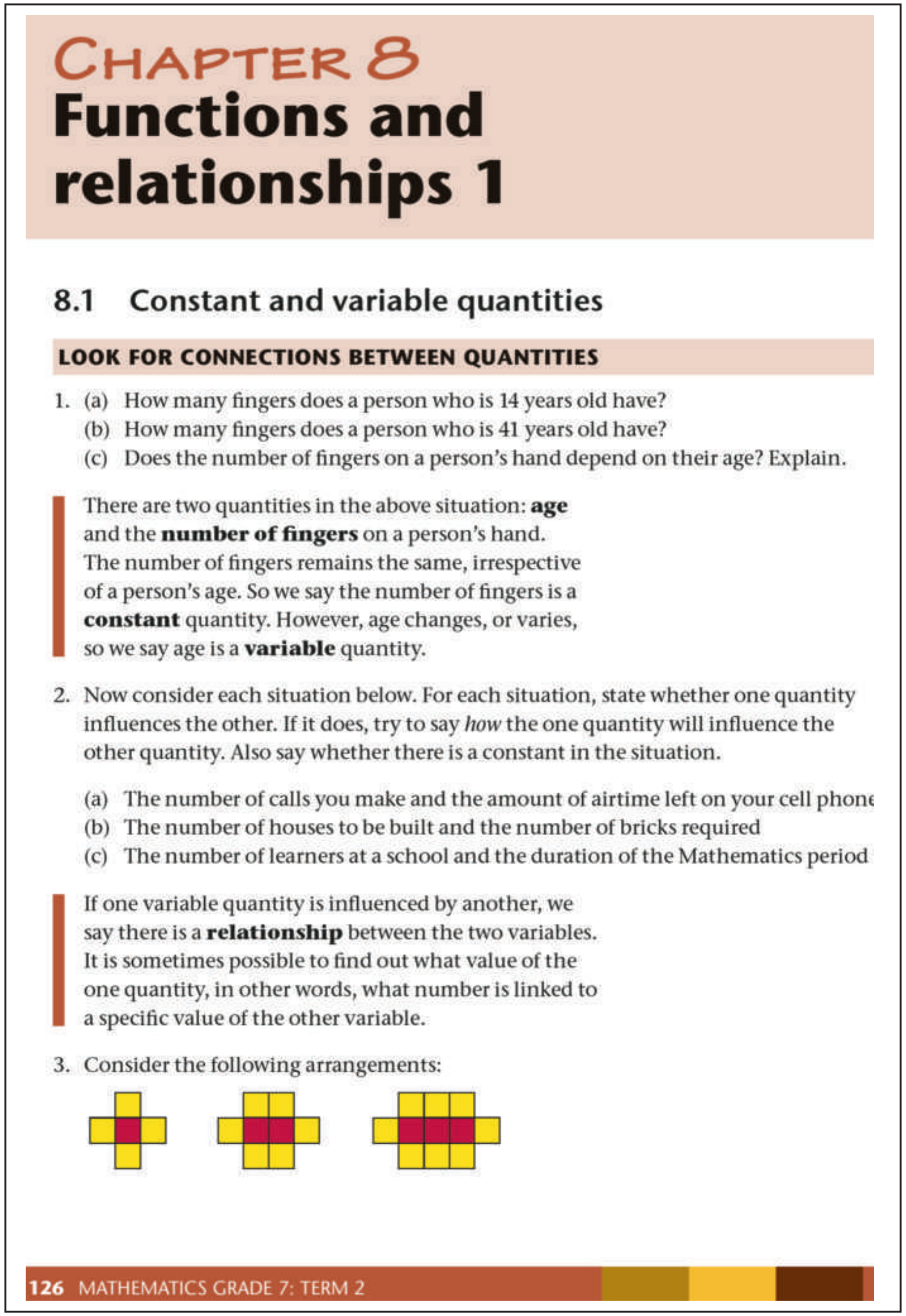
Learners often think of a letter symbol as an unknown constant and not as a variable quantity. Working with relationships between variable quantities is useful in that it provides learners with opportunities to understand that letter symbols can have different meanings in different mathematical contexts. (See Additional notes on TG page 142).

Notes on the questions

The term **relationship** is explained in terms of variable quantities from the start. Questions 1 and 2 are meant to stimulate some discussion about variable and constant quantities, and how one variable can affect another variable.

Answers

1. (a) ten fingers (b) ten fingers
   (c) No. A person is born with ten fingers and will always have ten fingers at any age.
2. (a) The number of calls you make will influence the amount of airtime left on your cell phone. For each phone call you make, a certain amount of airtime is used. The more calls you make, the less airtime you will have left over.
   (b) The number of houses to be built will influence the number of bricks required: i.e. the more houses to be built, the more bricks that will be required. The total number of bricks required varies on the number of houses to be built.



CHAPTER 8

Functions and relationships 1

8.1 Constant and variable quantities

LOOK FOR CONNECTIONS BETWEEN QUANTITIES

1. (a) How many fingers does a person who is 14 years old have?
   (b) How many fingers does a person who is 41 years old have?
   (c) Does the number of fingers on a person's hand depend on their age? Explain.

There are two quantities in the above situation: **age** and the **number of fingers** on a person's hand. The number of fingers remains the same, irrespective of a person's age. So we say the number of fingers is a **constant** quantity. However, age changes, or varies, so we say age is a **variable** quantity.

2. Now consider each situation below. For each situation, state whether one quantity influences the other. If it does, try to say *how* the one quantity will influence the other quantity. Also say whether there is a constant in the situation.
   (a) The number of calls you make and the amount of airtime left on your cell phone
   (b) The number of houses to be built and the number of bricks required
   (c) The number of learners at a school and the duration of the Mathematics period

If one variable quantity is influenced by another, we say there is a **relationship** between the two variables. It is sometimes possible to find out what value of the one quantity, in other words, what number is linked to a specific value of the other variable.

3. Consider the following arrangements:

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(c) The number of learners at a school varies and depends on a lot of different factors. The duration of the Mathematics period is constant and does not vary. The number of learners does not influence the duration of the Mathematics period, and the duration of the Mathematics period does not influence the number of learners at school.

3. (a) 4 (b) 6 (c) 8
(d) See LB page 127 alongside. (e) 22 (f) 44

8.2 Different ways to describe relationships

COMPLETE SOME FLOW DIAGRAMS AND TABLES OF VALUES

Teaching guidelines

A flow diagram (and a table) is limited in that not all possible input values can be shown. It is important to know what type of numbers the input values are, for example only integers, or fractions and whole numbers, etc.

A flow diagram usually shows the operator as well. Refer to the note in question 1.

In order to find input values when output values are known, learners have to invert the operator and apply it to the output value. Help learners who are unsure by asking leading questions such as: "How did we get the output value 12 from the input value 6?", "So what should we do with the output value 12 to get the input value 6?" Learners may find this easy, but may need more help with inverting the operations in more complicated rules, as in question 4.

Misconceptions

Learners often get the order of operations wrong when working from output values to find input values. Using flow diagrams "backwards" can help them understand the process.

Notes on the questions

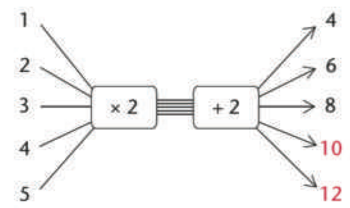
Different ways of representing the same relationship is an important topic in the Grade 7 CAPS, and hence in the work in this chapter. Each representation has strong points and weak points.

Answers

1. (a) See LB page 127 alongside. (b) Natural numbers or whole numbers.

(a) How many yellow squares are there if there is only one red square?
(b) How many yellow squares are there if there are two red squares?
(c) How many yellow squares are there if there are three red squares?
(d) Copy and complete the flow diagram below by filling in the missing numbers.

Can you see the connection between the arrangements above and the flow diagram? We can also describe the relationship between the red and yellow squares in words.



Input numbers
(Number of red squares)

Output numbers
(Number of yellow squares)

In words:
The number of yellow squares is found by multiplying the number of red squares by 2 and then adding 2 to the answer.

(e) How many yellow squares will there be if there are 10 red squares?
(f) How many yellow squares will be there if there are 21 red squares?

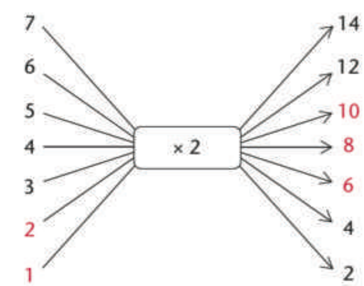
8.2 Different ways to describe relationships

COMPLETE SOME FLOW DIAGRAMS AND TABLES OF VALUES

A relationship between two quantities can be shown with a flow diagram. In a flow diagram we cannot show all the numbers, so we show only some.

1. Copy the flow diagram below and calculate the missing input and output numbers.

(a)



Each **input number** in a flow diagram has a corresponding **output number**. The first (top) input number corresponds to the first output number. The second input number corresponds to the second output number, and so on.
We say × 2 is the **operator**.

(b) What types of numbers are given as input numbers?

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Answers

1. (c) See LB page 128 alongside.
2. See LB page 128 alongside.
3. See LB page 128 alongside.

Teaching guidelines

This section also deals with recognising links between different representations of the same relationship. Discuss the fact that a relationship can be represented in different ways.

In question 4, the information about the relationship:

- is given in the spider (flow) diagram
- is also given as a verbal description of how the output numbers are produced (the rule of the relationship which will later become the formula) and
- is given as a table of values.

Discuss the strengths and weaknesses of each representation with learners. (Tables may not show the rule; both tables and spider diagrams do not show all values of the input or output values; all the representations strengthen the concept of the relationship between variables; and so on.)

Misconceptions

Learners may think that the only values the variables (input and output) can have are those shown in the flow diagram or table.

Answers

4. (a) See LB page 128 alongside.

Additional notes (Refer to TG page 140)

We use letters to represent unknown numbers in both equations and formulae, but there is a difference in their meaning.

When we use a letter in a formula, it could have many values. A letter used like this represents a variable, for example: $7 \times a + 3 = b$; the value of b depends on the value of a, and there can be many different combinations of a and b that will make the equation true.

On the other hand, a letter can also be used to represent an unknown value that makes an equation true, for example $7a + 3 = 14$. In this case the letter has only one value and is a placeholder for the solution.

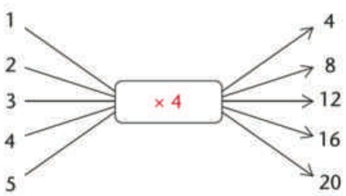
(c) In the previous flow diagram, the output number 14 corresponds to the input number 7. Copy and complete the following sentences in the same way:

In the relationship shown in the previous flow diagram, the output number 10 corresponds to the input number 5.

The input number 3 corresponds to the output number 6.

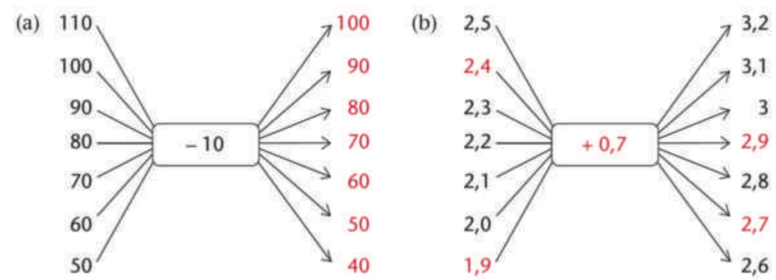
If more places are added to the flow diagram, the input number 20 will correspond to the output number 40.

2. Copy and complete this flow diagram by writing the appropriate operator, and then write the rule for completing this flow diagram in words.

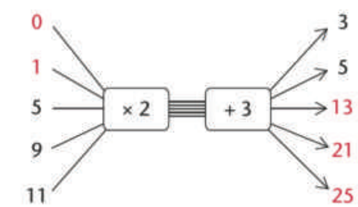


Multiply the input number by 4.

3. Copy and complete the flow diagrams below. You have to find out what the operator for (b) is and fill it in yourself.



4. Copy and complete the flow diagram:

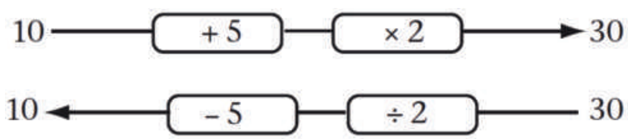


Teaching guidelines

In questions 5 and 7 the core algebraic concept of equivalence is addressed. Equivalent computational instructions (rules) are those that produce the same output values for the same input values. They look different, but produce the same result for the same input.

Show learners, for example, "divide by 5 and multiply by 2" has the same effect as multiplying by 0,4 (or $\frac{2}{5}$).

Show learners how to invert the operations in the rule and apply these to the output value to get the corresponding unknown input value. It can be done effectively by showing the "backwards" flow diagram. For example, if the rule was to add five and multiply by 2, the inverted rule is: divide by 2 and subtract 5.



Answers

5. See LB page 129 alongside.

A completed flow diagram shows two kinds of information:

- It shows what calculations are done to produce the output numbers.
- It shows which output number is connected to which input number.

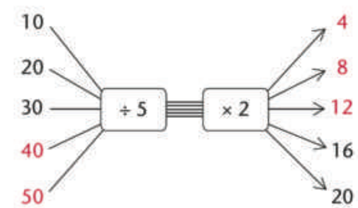
The flow diagram that you completed in question 4 shows the following information:

- Each input number is multiplied by 2 and then 3 is added to produce the output numbers.
- It shows which output number is connected to which input number.

The relationship between the input and output numbers can also be shown in a table:

Input numbers	0	1	5	9	11
Output numbers	3	5	13	21	25

5. (a) Copy and complete the flow diagram, then describe in words how the output numbers below can be calculated.

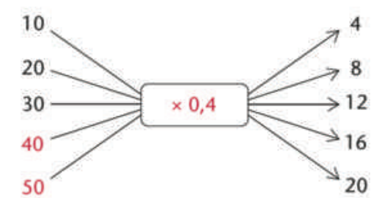


Each input number is divided by 5, then the answer is multiplied by 2 to produce the output number.

(b) Copy and complete the table below to show which output numbers are connected to which input numbers in the above flow diagram.

Input numbers	10	20	30	40	50
Output numbers	4	8	12	16	20

(c) Copy the flow diagram below and fill in the appropriate operator.



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Answers

5. (d) The operators are equivalent. The operator in 5(c) combines the two steps of the operator in 5(a), so they produce the same output numbers. They describe equivalent computation procedures:
(÷ 5 × 2 is the same as $\times \frac{2}{5}$ or × 0,4).

6. See LB page 130 alongside.

7. (a) See LB page 130 alongside.

(b) The operators represent equivalent computation procedures and produce the same output numbers. The operator in 6(b) combines the first two steps of the operator in 7(a) ($\times \frac{9}{5}$ is the same as × 1,8).

(c) See LB page 131 on following page.

Yes. The operators describe equivalent computation procedures; the calculations are done in a different order but produce the same output numbers.

Notes on the questions

Rules that have two or three steps can be used to highlight to learners the importance of the order of operations. For example, "multiply by 5 and then add 2" cannot be interchanged to give "add 2 and then multiply by 5", the answers are different, but "multiply by 5 and divide by 9" can be interchanged to "divide by 9 first and then multiply by 5".

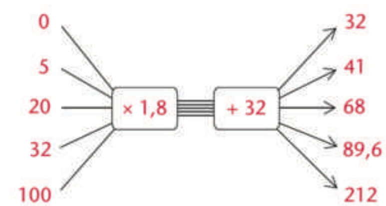
(d) The flow diagrams in question 5(a) and 5(c) have different operators, but they produce the same output values for the same input values. Explain.

6. The rule for converting temperature given in degrees Celsius to degrees Fahrenheit is given as: "Multiply the degrees Celsius by 1,8 and then add 32."

(a) Check whether the table below was completed correctly. If you find a mistake, note what it is and correct it.

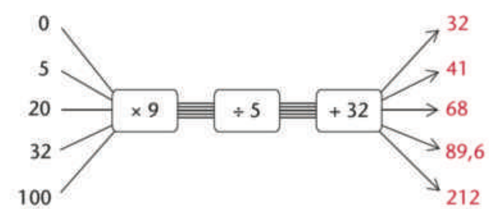
Temperature in degrees Celsius	0	5	20	32	100
Temperature in degrees Fahrenheit	32	41	68	89,6	212

(b) Copy and complete the flow diagram to represent the information in (a).



7. Another rule for converting temperature given in degrees Celsius to degrees Fahrenheit is given as: "Multiply the degrees Celsius by 9, then divide the answer by 5 and then add 32 to the answer."

(a) Copy and complete the flow diagram below.



(b) Why do you think the flow diagrams in questions 6(b) and 7(a) produce the same output numbers for the same input numbers, even though they have different operators?

(c) Copy and complete the flow diagram on the next page. Does this flow diagram give the same output values as the flow diagram in question 7(a)? Explain.

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Teaching guidelines

A table shows which output numbers correspond to certain input numbers. However, the rule that links the input numbers to the output numbers is not explicitly stated in a table. Learners often have difficulty identifying the rule when presented by a table as the only representation of the relationship.

By drawing a square on the board you can show learners that the area and the side length form a relationship. Input values: side length; output values: area.

For extension, you could reverse the relationship and make area the input and the side length the output. This involves finding the square root of numbers. For example: the area of a square is given as 4, 25, 49, 81, 121. Find the side length.

Misconceptions

Learners misinterpret the instructions to "multiply by itself" and to "cube".

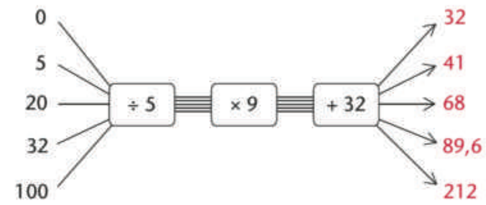
Notes on the questions

In questions 8 and 9, the output numbers are respectively the squares and the cubes of whole number patterns. These are examples of relationships where there is neither a constant difference nor a constant ratio between consecutive output numbers.

Learners often have difficulty identifying the rule when presented by a table as the only representation of the relationship.

Answers

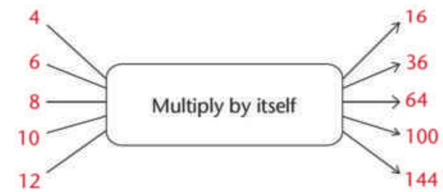
8. (a) See the completed table on LB page 131 alongside.
   (b) See the completed flow diagram on LB page 131 alongside.
9. (a) See the completed table on LB page 131 alongside.
   (b) Multiply each input number (stack number) by itself and again by itself; in other words, the input numbers must be cubed.



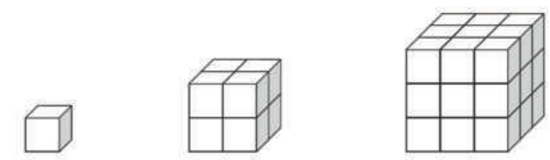
8. The rule for calculating the area of a square is: "Multiply the length of a side of the square by itself."
   (a) Copy and complete the table below.

Length of side	4	6	8	10	12
Area of square	16	36	64	100	144

   (b) Copy and complete the flow diagram to represent the information in the table.



9. (a) The pattern below shows stacks of building blocks. The number of blocks in each stack is dependent on the number of the stack.



Stack 1 *Stack 2* *Stack 3*

Copy and complete the table below to represent the relationship between the number of blocks and the number of the stack.

Stack number	1	2	3	4	5	6	7	8
Number of blocks	1	8	27	64	125	216	343	512

   (b) Describe in words how the output values can be calculated.

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Notes on the questions

The three questions on LB page 131 are designed to make learners aware of a way of identifying the rule of the relationship if it has not been given. Learners investigate the differences between consecutive output numbers in given flow diagrams and link that to the multiplicative operator of the flow diagram. This multiplicative operator is the quotient of the difference between two output numbers and the difference between the corresponding input numbers.

In question 1(a), the difference between consecutive output numbers is 1 and the difference between consecutive input numbers is also 1. (The difference between the last two output numbers is 2, but the input numbers also have a difference of 2.)

In questions 1(b) and (c), the difference between consecutive output numbers is 5 for a difference of 1 between the corresponding input numbers (or 10 if the input numbers have a difference of 2). In both cases, the input numbers were multiplied by 5 to get the output numbers.

For 1(d) and (f), the output numbers have a difference of 2 and the input number is multiplied by 2 to give the output number. This value represents the **rate** at which the output numbers change. This concept will be dealt with in more detail in later grades.

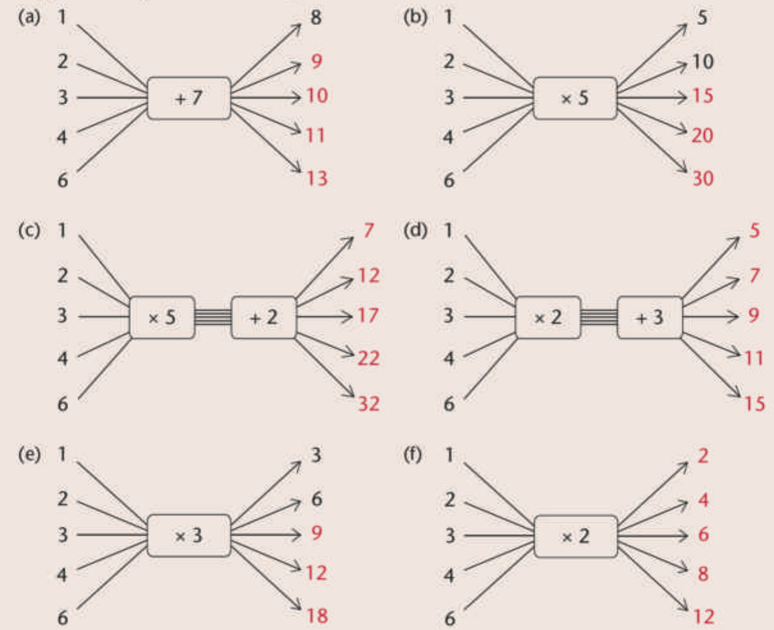
If this link is made, learners should recognise that they should find the differences between the output values in the table (for example in question 3 it is 7); this is the number by which every input number should be multiplied (the multiplicative operator in the flow diagram) as part of the rule that will produce the output number. It could be that something has to be added (or subtracted) to get the correct rule that will produce the output number. In question 3: for a difference of 1 between input numbers, there is a difference of 7 between output numbers. Therefore, every input number is multiplied by 7 and then 2 is added to get the correct value of the output number. The rule is: multiply the input number by 7 and add 2.

Answers

1. See the completed flow diagrams on LB page 132 alongside.
2. The difference between consecutive output numbers divided by the difference of consecutive input numbers is equal to the multiplicative operator.
3. The difference between the consecutive output numbers is 7 and the difference between the consecutive input numbers is 1. The multiplicative operator is 7 ÷ 1. If you multiply 7 by 1, you need to add 2 to get 9. The rule is therefore: multiply by 7 and add 2.

EXTENSION: LINKING FLOW DIAGRAMS, TABLES OF VALUES, AND RULES

1. Copy and complete the flow diagrams below.



2. Calculate the differences between the consecutive output numbers and compare them to the differences between the consecutive input numbers. Consider the operator of the flow diagram. What do you notice?
3. Determine the rule to calculate the missing output numbers in this relationship and copy and complete the table:

Input numbers	1	2	3	4	5	7	10
Output numbers	9	16	23	30	37	51	72

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Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
9.1 Perimeter of polygons	Definition of perimeter; measuring perimeter	Page 133
9.2 Perimeter formulae	Formulae for calculating perimeter and applying the formulae	Page 134
9.3 Area and square units	Definition of area of a shape; square units to measure area; converting units	Pages 135 to 136
9.4 Area of squares and rectangles	Investigating the area of squares and rectangles; formulae and applying the formulae for the areas of squares and rectangles; doubling sides and the effect it has on the area of squares	Pages 137 to 140
9.5 Area of triangles	Heights and bases of triangles; formulae for the area of a triangle; applying the formula	Pages 140 to 144

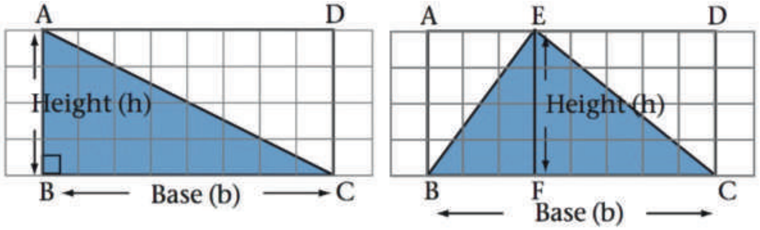
CAPS time allocation	7 hours
CAPS content specification	Pages 55 to 56

Mathematical background

The **perimeter** is the distance around the border of a shape. To find the perimeter of a shape, we will either be given the measurements or we can measure the sides. The perimeter can be found by adding the lengths of the sides or by using a formula. For example: perimeter of a rectangle = 2 × length + 2 × breadth or, perimeter of a square = 4 × side length.

The **area** of a shape is a measure of the surface it covers. The area is given in square units. For example, if the length and breadth is given in centimetres, the area will be square centimetres.

- The formula to find the area of a rectangle is given by the number of square centimetres along the length multiplied by the number of square centimetres along the breadth (or width). A = length × breadth = $A = l \times b$.
- The formula to find the area of a square is A = length × breadth, but because $l = b = s$, the side length, we write $A = s \times s$.



Some irregular shapes are composites (made up of) of squares and rectangles. Their areas can be determined by dividing them into squares and rectangles.

Any triangle can fit inside a rectangle that has the same base and height as the triangle. The first drawing on the right shows a right-angled triangle that fits in a rectangle with the same base and height. We can check by counting squares that the area of the triangle (16 squares) is half that of the rectangle (32 squares). Therefore, the area of triangle ABC is $A = \frac{1}{2} \times b \times h$.

The second drawing on the right is not a right-angled triangle. The altitude (height) divides it into two right-angled triangles, each in a rectangle, the one in rectangle ABFE (12 squares) and the other one in rectangle DCFE (20 squares). The area of the triangles EBF and ECF is half of the area of the rectangles ABFE and DCFE, so the sum of the areas is half of the area of the rectangle ABCD; so the area of the triangle EBC = $A = \frac{1}{2} \times b \times h$. The same argument could be applied for the area of an obtuse-angled triangle.

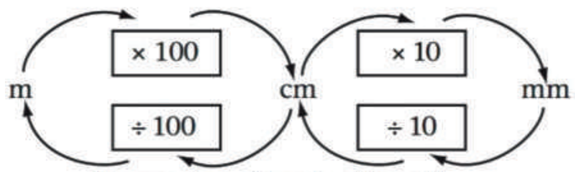
9.1 Perimeter of polygons

MEASURING PERIMETERS

Teaching guidelines

Ask learners to imagine walking around a shape on its outer line. This works for both regular and irregular objects. The idea here is of length (i.e. how long?).

We use the measurement units: millimetre, centimetre and metre. The following diagram is useful when converting between units of length.



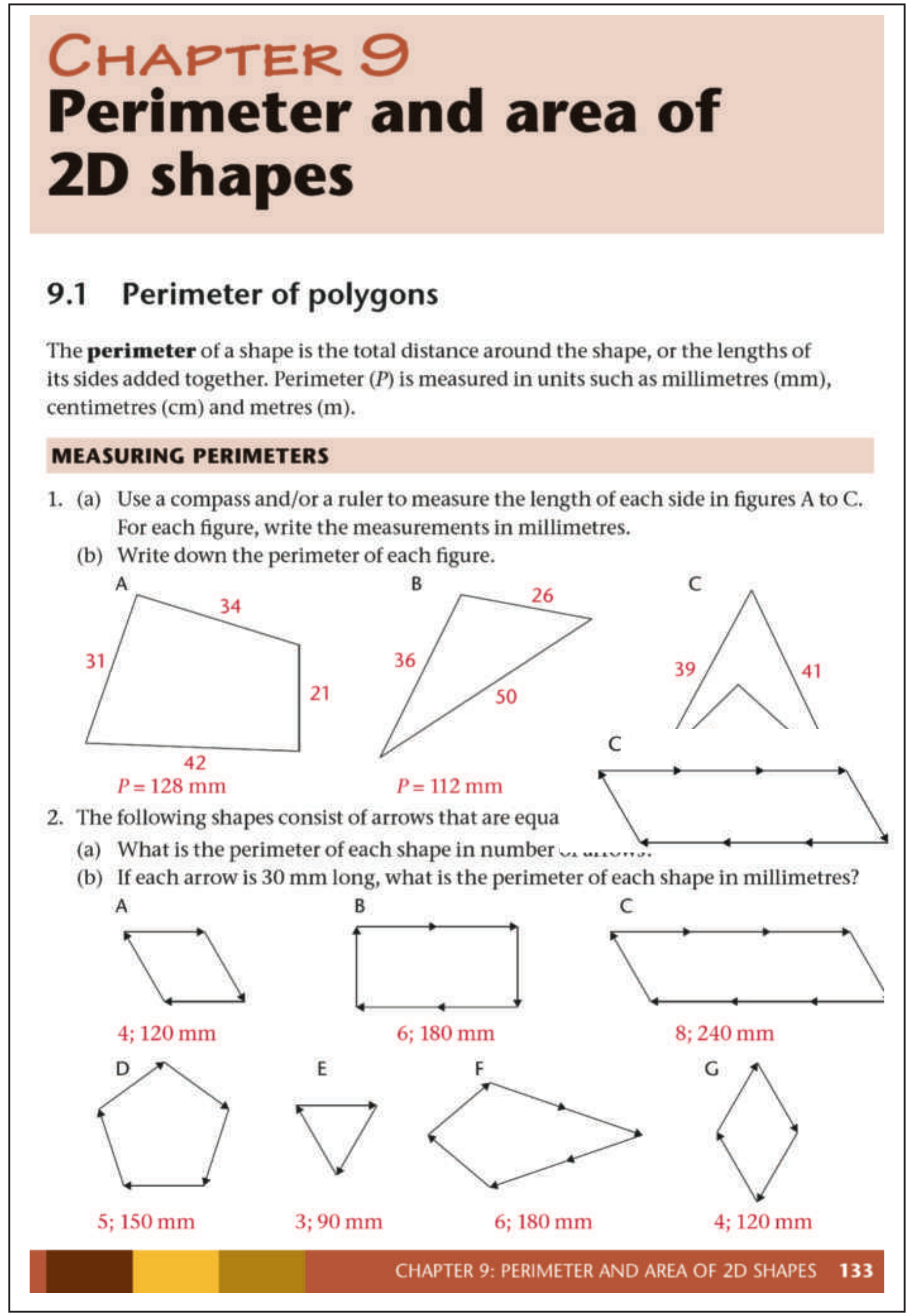
Conversion of units of length

Misconceptions

Learners add all sides whether they are in the same units or not. Help them to see how to change units when necessary.

Answers

1. See LB page 133 alongside.
2. (a) A: four arrows B: six arrows C: eight arrows
   D: five arrows E: three arrows F: six arrows
   G: four arrows
   (b) A: 120 mm B: 180 mm C: 240 mm
   D: 150 mm E: 90 mm F: 180 mm
   G: 120 mm


CHAPTER 9
Perimeter and area of 2D shapes

9.1 Perimeter of polygons

The **perimeter** of a shape is the total distance around the shape, or the lengths of its sides added together. Perimeter (P) is measured in units such as millimetres (mm), centimetres (cm) and metres (m).

MEASURING PERIMETERS

1. (a) Use a compass and/or a ruler to measure the length of each side in figures A to C. For each figure, write the measurements in millimetres.
   (b) Write down the perimeter of each figure.

2. The following shapes consist of arrows that are equa
   (a) What is the perimeter of each shape in number
   (b) If each arrow is 30 mm long, what is the perimeter of each shape in millimetres?

CHAPTER 9: PERIMETER AND AREA OF 2D SHAPES 133

9.2 Perimeter formulae

Teaching guidelines

Use the definition of perimeter to derive the formulae: adding all the sides and simplifying where necessary.

Explain that when we use letter symbols in multiplication, we can leave out the multiplication sign, $4 \times s = 4s$.

Remind learners how to simplify using brackets so that, $2l + 2b = 2(l + b)$.

Remind learners that letter symbols marked differently mean they may have different values, for example s_1, s_2 and s_3 may have the same value, but if they are marked differently it usually means that they have different values.

APPLYING PERIMETER FORMULAE

Teaching guidelines

Learners may not be sure how to find an unknown side if the perimeter is known. Explain the following:

- There are two quantities in the formula $P = 4 \times s$. If we know one of them, we can calculate the value of the other one. If we know the value of P, we should ask, "what value multiplied by 4 gives P?" to find the value of s.
- There are three quantities in the formula $P = 2 \times (l + b)$. If we know two of them, we can calculate the value of the third one. Learners may struggle to know how to find l or b (for example in question 7). Show learners how to reverse the operations that were performed to get P: perimeter $= 2 \times (l + b)$; therefore, $l + b = \frac{1}{2} \times$ perimeter; $l = \frac{1}{2} \times$ perimeter $- b$; etc.

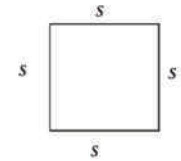
Answers

1. $4 \times 17{,}5 = 70$ cm
2. $3 \times 32 = 96$ cm (An equilateral triangle's sides are all equal in length.)
3. $4s = 7{,}2$ m; so, $s = 7{,}2 \div 4 = 1{,}8$ m
4. $6{,}4 = 2{,}5 + 2{,}5 + s_3$; $s_3 = 6{,}4 - 2{,}5 - 2{,}5 = 1{,}4$ cm
5. $P = 2(l + b) = 2 \times (40 + 25) = 2 \times 65 = 130$ cm
6. $P = 2(l + b) = 2 \times (2{,}4 + 4) = 2 \times 6{,}4 = 12{,}8$ m
7. $P = 2(l + b)$ So the length (l) of two sides $= 8{,}88 - (2 \times b) = 8{,}88 - 2{,}4 = 6{,}48$ m.
   Length of one side $= 6{,}48 \div 2 = 3{,}24$ m, so the rectangle is 3,24 m long.
8. See the completed tables on LB page 134 alongside.

9.2 Perimeter formulae

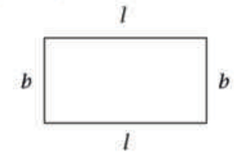
If the sides of a square are all s units long:

Perimeter of square $= s + s + s + s$
$= 4 \times s$
or $P = 4s$



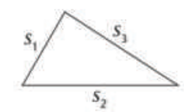
If the length of a rectangle is l units and the breadth (width) is b units:

Perimeter of rectangle $= l + l + b + b$
$= 2 \times l + 2 \times b$
$= 2l + 2b$
or $P = 2(l + b)$



A triangle has three sides, so:

Perimeter of triangle $= s_1 + s_2 + s_3$
or $P = s_1 + s_2 + s_3$



APPLYING PERIMETER FORMULAE

1. Calculate the perimeter of a square if the length of one of its sides is 17,5 cm.
2. One side of an equilateral triangle is 32 cm. Calculate the triangle's perimeter.
3. Calculate the length of one side of a square if the perimeter of the square is 7,2 m. (Hint: $4s = ?$ Therefore $s = ?$)
4. Two sides of a triangle are 2,5 cm each. Calculate the length of the third side if the triangle's perimeter is 6,4 cm.
5. A rectangle is 40 cm long and 25 cm wide. Calculate its perimeter.
6. Calculate the perimeter of a rectangle that is 2,4 m wide and 4 m long.
7. The perimeter of a rectangle is 8,88 m. How long is the rectangle if it is 1,2 m wide?
8. Do the necessary calculations and then copy and complete the tables. (All the measurements refer to rectangles.)

	Length	Breadth	Perimeter		Length	Breadth	Perimeter
(a)	74 mm	30 mm	208 mm	(b)	25 mm	20 mm	90 mm
(c)	2 cm	1,125 cm	6,25 cm	(d)	5,5 cm	5,5 cm	22 cm
(e)	7,5 m	3,8 m	22,6 m	(f)	3,5 m	2,5 m	12 m

9.3 Area and square units

SQUARE UNITS TO MEASURE AREA

Area is the size of a flat surface of a shape.

Teaching guidelines

Learners should imagine covering the surface with something such as paint or paper. We emphasise this idea of covering a shape when we do investigations in which learners count the square units that a shape covers.

Help them not to confuse the concepts of perimeter (circumference) and area. You can help them to develop the correct associations of the wording by asking questions such as: "How long is the perimeter?" and "How many units will cover the shape?"

Encourage learners to discuss their solution methods in class. This will help them to use and apply these concepts correctly.

Answers

1. See LB page 135 alongside.

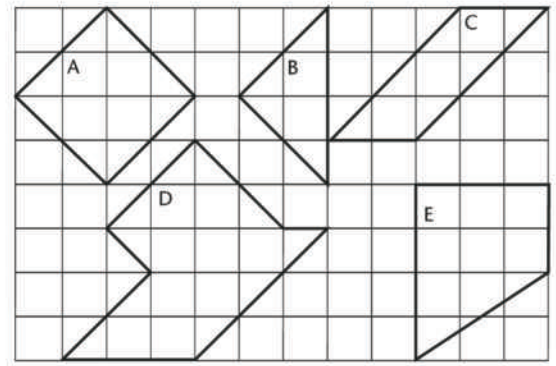
9.3 Area and square units

The **area** of a shape is the size of the flat surface surrounded by the border (perimeter) of the shape.

Usually, area (A) is measured in square units, such as square millimetres (mm^2), square centimetres (cm^2) and square metres (m^2).

SQUARE UNITS TO MEASURE AREA

1. In your book, write down the area of figures A to E below by counting the square units. (Remember to add halves or smaller parts of squares.)



A is 8 square units.

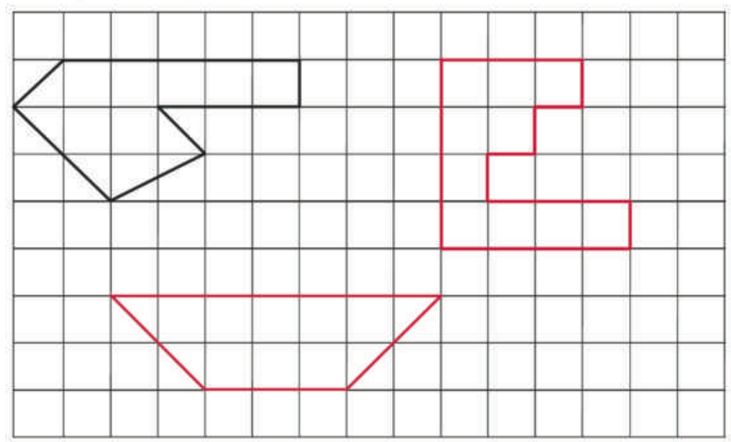
B is 4 square units.

C is 6 square units.

D is 14 square units.

E is 9 square units.

2. Each square in the grid below measures 1 cm^2 (1 cm × 1 cm).



Answers

2. (a) 10 cm^2

   (b) Learners make their own drawings. Two suggestions are given on LB page 135 on the previous page.

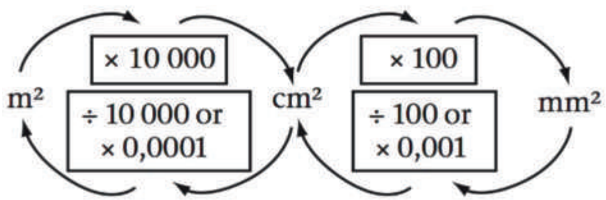
CONVERSION OF UNITS

Teaching guidelines

It is important for learners to use the correct units and to convert between units correctly.

We work with mm^2, cm^2 and m^2 as the preferred units. Verbally, you should say “square millimetre”, “square centimetre” and “square metre”.

The drawings below could help learners to remember how to convert between units.



Conversion of units of area

Misconceptions

Learners have difficulty deciding what the conversion factor should be because they do not have a concept of relative sizes, for example, they do not really know how long a metre is in comparison to a centimetre. This perception can be changed with a measuring tape by comparing a metre to a centimetre, etc.

Answers

1. See LB page 136 alongside.
2. See LB page 136 alongside.

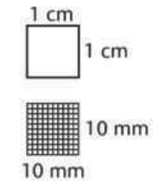
(a) What is the area of the shape drawn on the grid?

(b) Trace the grid, then draw two shapes of your own. The shapes should have the same area, but different perimeters.

CONVERSION OF UNITS

The figure on the right shows a square with sides of 1 cm.
The area of the square is one square centimetre (1 cm^2).

How many squares of 1 mm by 1 mm (1 mm^2) would fit into the 1 cm^2 square? ..100.. copy and complete: 1 cm^2 = ..100.. mm^2



To change cm^2 to mm^2:

1 cm^2 = 1 cm × 1 cm
= 10 mm × 10 mm
= 100 mm^2

Similarly, to change mm^2 to cm^2:

1 mm^2 = 1 mm × 1 mm
= 0,1 cm × 0,1 cm
= 0,01 cm^2

We can use the same method to convert between other square units too. Copy and complete:

From m^2 to cm^2:	**From cm^2 to m^2:**
1 m^2 = 1 m × 1 m	1 cm^2 = 1 cm × 1 cm
= ..100.. cm × ..100.. cm	= 0,01 m × 0,01 m
= ..10 000.. cm^2	= ..0,0001.. m^2

So, to convert between m^2, cm^2 and mm^2 you do the following:

- cm^2 to mm^2 → multiply by 100
- m^2 to cm^2 → multiply by 10 000
- mm^2 to cm^2 → divide by 100
- cm^2 to m^2 → divide by 10 000

Do the necessary calculations in your exercise book. Then copy the conversions and fill in your answers.

1. (a) 5 m^2 = ..50 000.. cm^2 (b) 5 cm^2 = ..500.. mm^2
   (c) 20 cm^2 = ..0,002.. m^2 (d) 20 mm^2 = ..0,2.. cm^2
2. (a) 25 m^2 = ..250 000.. cm^2 (b) 240 000 cm^2 = ..24.. m^2
   (c) 460,5 mm^2 = ..4,605.. cm^2 (d) 0,4 m^2 = ..4 000.. cm^2
   (e) 12 100 cm^2 = ..1,21.. m^2 (f) 2,295 cm^2 = ..229,5.. mm^2

9.4 Area of squares and rectangles

INVESTIGATING THE AREA OF SQUARES AND RECTANGLES

Teaching guidelines

Remind learners that the area of a rectangle or square is measured by the number of square units that it covers.

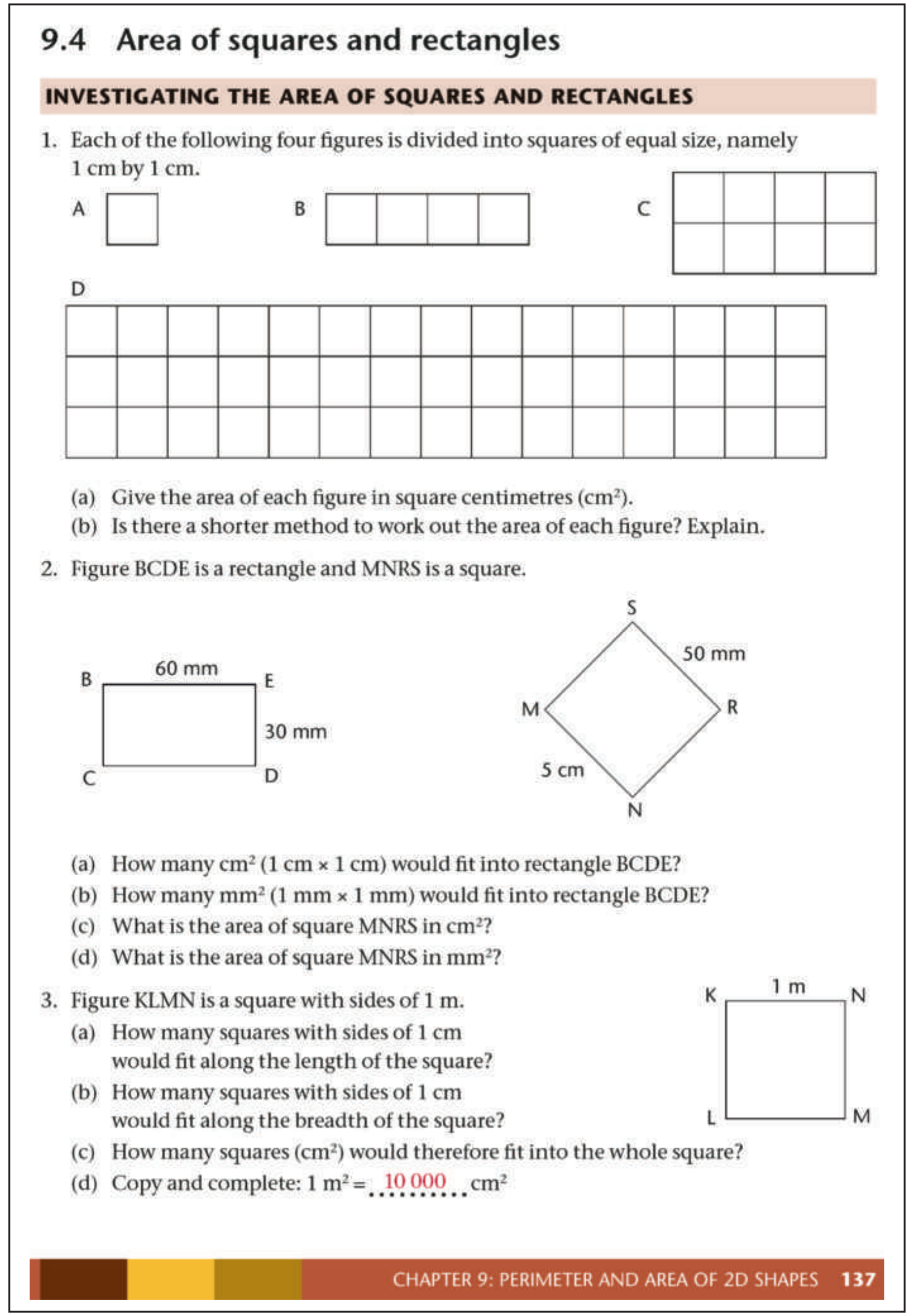
Let learners work with grid paper, draw shapes and find their areas.

Remind them that we work with square millimetres (mm^2), square centimetres (cm^2) and square metres (m^2).

Encourage learners to discuss their solution methods in class. This will help them to use and apply these concepts correctly.

Answers

1. (a) A: 1 cm^2
   B: 4 cm^2
   C: 8 cm^2
   D: 45 cm^2
   (b) Yes, multiply the number of rows by the number of columns.
2. (a) 18
   (b) 1 800
   (c) 25 cm^2
   (d) 2 500 mm^2
3. (a) 100
   (b) 100
   (c) 10 000
   (d) See LB page 137 alongside.



9.4 Area of squares and rectangles

INVESTIGATING THE AREA OF SQUARES AND RECTANGLES

1. Each of the following four figures is divided into squares of equal size, namely 1 cm by 1 cm.

   (a) Give the area of each figure in square centimetres (cm^2).
   (b) Is there a shorter method to work out the area of each figure? Explain.

2. Figure BCDE is a rectangle and MNRS is a square.

   (a) How many cm^2 (1 cm × 1 cm) would fit into rectangle BCDE?
   (b) How many mm^2 (1 mm × 1 mm) would fit into rectangle BCDE?
   (c) What is the area of square MNRS in cm^2?
   (d) What is the area of square MNRS in mm^2?

3. Figure KLMN is a square with sides of 1 m.

   (a) How many squares with sides of 1 cm would fit along the length of the square?
   (b) How many squares with sides of 1 cm would fit along the breadth of the square?
   (c) How many squares (cm^2) would therefore fit into the whole square?
   (d) Copy and complete: 1 m^2 = 10 000 cm^2

CHAPTER 9: PERIMETER AND AREA OF 2D SHAPES 137

FORMULAE: AREA OF RECTANGLES AND SQUARES

Teaching guidelines

Show learners that to calculate the area of a rectangle, we need to know how many square centimetres (or metres) are in a row along a side (the length) and how many of these rows fit along the breadth. The formula is $A = l \times b$.

The formula for the area of a square is $l \times l = l^2$ (or $s \times s = s^2$).

We can use the unit square centimetres for smaller areas and square metres for larger areas, such as the classroom floor, etc.

Work through the examples 1, 2 and 3 with learners.

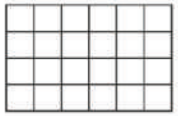
In example 3 the learners are required to use the known values of the area and the breadth to calculate the length. To do this, learners have to solve a simple equation. Some learners may have difficulty doing this. A strategy to help them understand is to reverse the operations; the question to ask is "What was done to the length to get the area?" the answer is "It was multiplied by the breadth." Therefore, if we divide both sides by the breadth, we should get the length.

A quick way of calculating the number of squares that would fit into a rectangle is to multiply the number of squares that would fit along its length by the number of squares that would fit along its breadth.

FORMULAE: AREA OF RECTANGLES AND SQUARES

In the rectangle on the right:

Number of squares = squares along the length × squares along the breadth
= 6 × 4
= 24



l = 6 squares

b = 4 squares

From this we can deduce the following:

Area of rectangle = length of rectangle × breadth of rectangle
$A = l \times b$

(where A is the area in square units, l is the length and b is the breadth)

Area of square = length of side × length of side
$A = l \times l$
$= l^2$

(where A is the area in square units, and l is the length of a side)

The units of the values used in the calculations must be the same. Remember:

- 1 m = 100 cm and 1 cm = 10 mm
- 1 cm^2 = 1 cm × 1 cm = 10 mm × 10 mm = 100 mm^2
- 1 m^2 = 1 m × 1 m = 100 cm × 100 cm = 10 000 cm^2
- 1 mm^2 = 1 mm × 1 mm = 0,1 cm × 0,1 cm = 0,01 cm^2
- 1 cm^2 = 1 cm × 1 cm = 0,01 m × 0,01 m = 0,0001 m^2

Examples

1. Calculate the area of a rectangle with a length of 50 mm and a breadth of 3 cm. Give the answer in cm^2.

   Solution:

   Area of rectangle $= l \times b$
   = (50 × 30) mm^2 or A = (5 × 3) cm^2
   = 1 500 mm^2 or = 15 cm^2

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APPLYING THE FORMULAE

Teaching guidelines

Show learners examples of shapes that are put together by combining squares and rectangles, and how to work out their areas by calculating the areas of the squares of rectangles separately and adding them. See question 4 on LB page 140.

Misconceptions

Learners use lengths that are not in the same units.

Answers

1. (a) $A = l \times b$
   $= 12 \times 9$
   $= 108 \text{ cm}^2$

   (b) $A = l^2$
   $= 110 \text{ mm} \times 110 \text{ mm}$
   $= 12\,100 \text{ mm}^2 = 121 \text{ cm}^2$

   (c) 2,5 cm = 25 mm $A = l \times b$
   $= 25 \times 105$
   $= 2\,625 \text{ mm}^2$

   (d) $P = 2(l + b)$ therefore $2 \times b = 24 - (2 \times 8) = 24 - 16 = 8$ cm
   Therefore, $b = 8 \div 2 = 4$ cm
   Area: $l \times b = 8 \times 4 = 32 \text{ cm}^2$

2. (a) $A = l \times b = 100 \text{ m} \times 69 \text{ m} = 6\,900 \text{ m}^2$

   (b) $6\,900 \times 45 =$ R310 500

   (c) Smaller. A hectare is $10\,000 \text{ m}^2$.
   The area of a rugby field (calculated in question (a) above) is $6\,900 \text{ m}^2$.

3. See the completed table on LB page 139 alongside.

2. Calculate the area of a square bathroom tile with a side of 150 mm.
   Solution:
   Area of square tile $= l \times l$
   $= (150 \times 150) \text{ mm}^2$
   $= 22\,500 \text{ mm}^2$
   The area is therefore $22\,500 \text{ mm}^2$ (or 225 cm^2).

3. Calculate the length of a rectangle if its area is 450 cm^2 and its width is 150 mm.
   Solution:
   Area of rectangle $= l \times b$
   $450 = l \times 15$
   $30 \times 15 = l \times 15$ or $450 \div 15 = l$
   $30 = l$ $30 = l$
   The length is therefore 30 cm (or 300 mm).

APPLYING THE FORMULAE

1. Calculate the area of each of the following shapes:
   (a) a rectangle with sides of 12 cm and 9 cm
   (b) a square with sides of 110 mm (answer in cm^2)
   (c) a rectangle with sides of 2,5 cm and 105 mm (answer in mm^2)
   (d) a rectangle with a length of 8 cm and a perimeter of 24 cm

2. A rugby field has a length of 100 m (goal post to goal post) and a breadth of 69 m.
   (a) What is the area of the field (excluding the area behind the goal posts)?
   (b) What would it cost to plant new grass on that area at a cost of R45/m^2?
   (c) Another unit for area is the hectare (ha). It is mainly used for measuring land. The size of 1 ha is the equivalent of 100 m × 100 m. Is a rugby field greater or smaller than 1 ha? Explain your answer.

3. Do the necessary calculations and then copy and complete the table below. (All the measurements refer to rectangles.)

	Length	Breadth	Area
(a)	15 m	8 m	120 m^2
(b)	120 mm	50 mm	60 cm^2
(c)	3,5 m	4,3 m	15,05 m^2
(d)	2,3 cm	1,2 cm	2,76 cm^2
(e)	5,2 m	460 cm	23,92 m^2

Answers

4. See LB page 140 alongside.
5. (a) $A = l \times b$
   $= 12 \times 8$
   $= 96\text{ m}^2$
   (b) Carrots: $\frac{1}{2}$ of $96\text{ m}^2 = 48\text{ m}^2$
   Tomatoes and potatoes: each $\frac{1}{4}$ of $96\text{ m}^2 = 24\text{ m}^2$
   (c) $P = 2(l + b) = 2 \times (12 + 8) = 2 \times 20 = 40$ m
   40 m × R38 = R1 520
6. (a) 5 m = 500 cm; 4 m = 400 cm
   Tiles along the length: 500 ÷ 40 = 12,5 tiles
   Tiles along the breadth: 400 ÷ 20 = 20 tiles
   He needs 20 rows of 12,5 tiles = 250 tiles
   (b) 250 ÷ 20 = 12,5
   He therefore needs to buy 13 boxes.

Notes on questions 7 and 8

In questions 7 and 8 learners investigate the different ways in which the perimeter and area of a square change when the length of the sides are repeatedly increased by 1 cm. The perimeter increases by 4 cm for every 1 cm increase in the length of the sides, i.e. the perimeter increases at a constant rate.

However, the area does not increase at a constant rate.

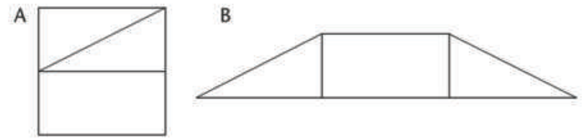
Answers

7. (a) See LB page 141.
   (b) See LB page 141.
8. (a)

Length of each side in cm	1	2	3	4	5	6
Area of square in cm^2	1	4	9	16	25	36
Perimeter in cm	4	8	12	16	20	24

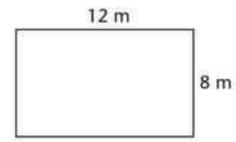
   (b) By 4 cm
   (c) The increase varies, and the increases are 3 cm^2, 5 cm^2, 7 cm^2, 9 cm^2, 11 cm^2.

4. Figure A is a square with sides of 20 mm. It is cut as shown in A and the parts are combined to form figure B. Calculate the area of figure B.

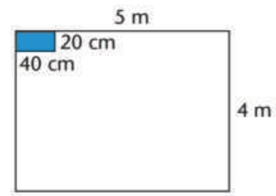


20 × 20 = 400 mm²
The area stays the same after rearrangement.

5. Margie plants a vegetable patch measuring 12 m × 8 m.



   (a) What is the area of the vegetable patch?
   (b) She plants carrots on half of the patch, and tomatoes and potatoes on a quarter of the patch each. Calculate the area covered by each type of vegetable?
   (c) How much will she pay to put fencing around the patch? The fencing costs R38/m.
6. Mr Allie has to tile a kitchen floor measuring 5 m × 4 m. The blue tiles he uses each measure 40 cm × 20 cm.



   (a) How many tiles does Mr Allie need?
   (b) The tiles are sold in boxes containing 20 tiles. How many boxes should he buy?

Copy the grid on page 141. The size of each square making up the grid is 1 cm × 1 cm.

7. (a) For each square drawn on the grid, label the lengths of its sides.
   (b) Write down the area of each square. (Write the answer inside the square.)
8. (a) Write the sidelengths and the areas of the squares in a table like this.

Length of each side in cm	1	2	3	4	5	6
Area of square in cm²	1	4	9	16	25	36
Perimeter in cm	4	8	12	16	20	24

   (b) By how much does the perimeter increase when the length of the sides increase by 1 cm?
   (c) By how much does the area increase when the length of the sides increase by 1 cm?

9.5 Area of triangles

HEIGHTS AND BASES OF A TRIANGLE

Background

The concept of the area of a triangle is extended to include all triangles, acute and obtuse. For this you need the perpendicular height of the triangle as measured from the base. The height (h) of a triangle is a perpendicular line segment drawn from a vertex to its opposite side. The opposite side, which forms a right angle with the height, is called the base (b) of the triangle. Any triangle has three possible bases (any one of the three sides can be seen as the base) and therefore three possible heights. This is illustrated on LB page 142.

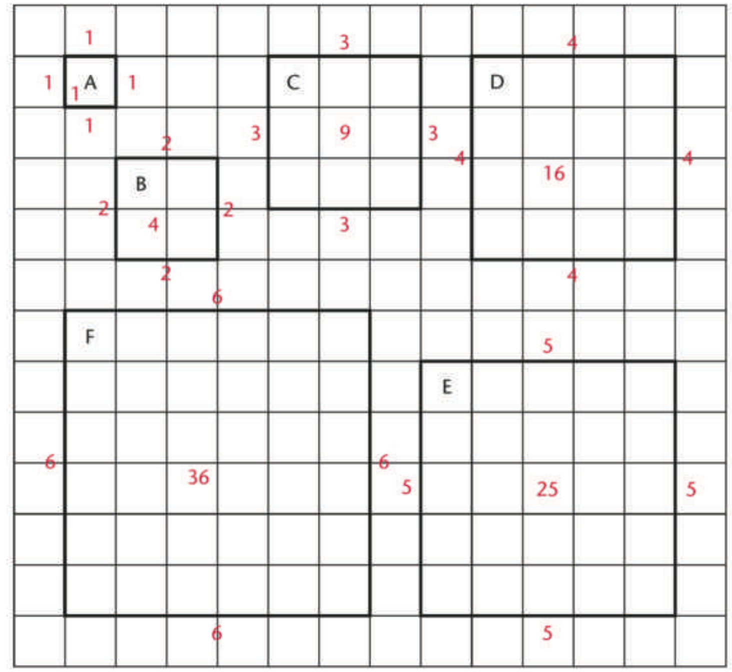
Every base has a height associated with that base. The height is always represented by the perpendicular line drawn from the vertex to whichever side is considered to be the base.

Teaching guidelines

Learners should draw sketches of a variety of obtuse and acute triangles, so that they can find appropriate heights and bases to calculate the areas of these triangles.

Misconceptions

Learners find it difficult to imagine a height of a triangle that is not vertical as in the second and third drawings on LB page 142. Pay special attention to rectifying this misconception.



9.5 Area of triangles

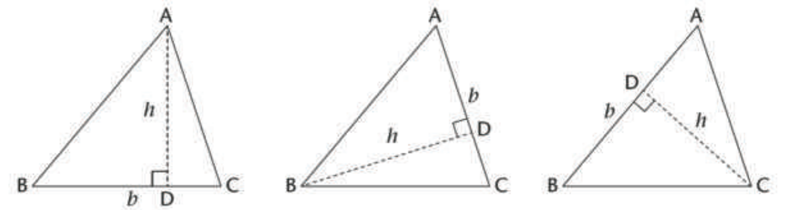
HEIGHTS AND BASES OF A TRIANGLE

The **height (*h*)** of a triangle is a perpendicular line segment drawn from a vertex to its opposite side. The opposite side, which forms a right angle with the height, is called the **base (*b*)** of the triangle. Any triangle has three heights and three bases.

CHAPTER 9: PERIMETER AND AREA OF 2D SHAPES **141**

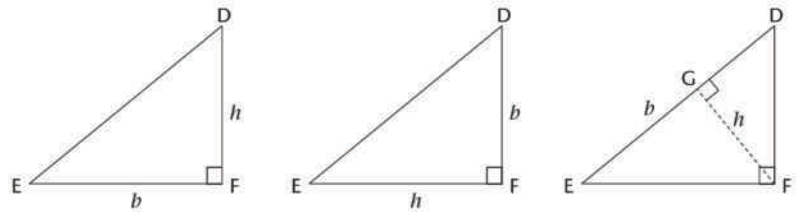
Answers

1. Answers are on the drawings on LB page 142 alongside.
2. Answers are on the drawings on LB page 142 alongside.



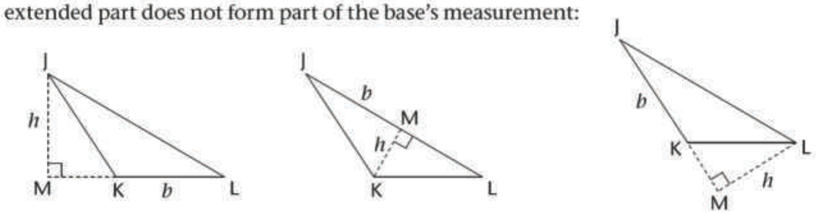
AD = height	BD = height	CD = height
BC = base	AC = base	AB = base

In a right-angled triangle, two sides are already at right angles:



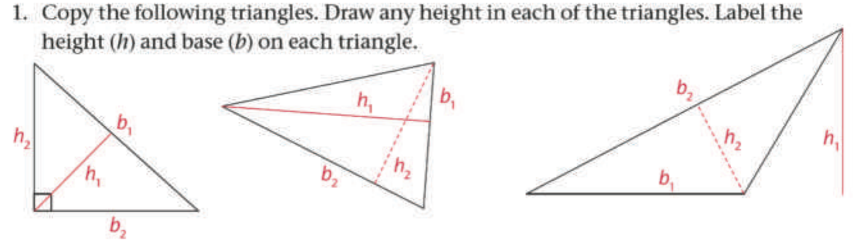
DF = height	EF = height	FG = height
EF = base	DF = base	DE = base

Sometimes a base must be extended outside of the triangle in order to draw the perpendicular height. This is shown in the first and third triangles below. Note that the extended part does not form part of the base's measurement:



JM = height	KM = height	LM = height
KL = base	JL = base	JK = base

1. Copy the following triangles. Draw any height in each of the triangles. Label the height (*h*) and base (*b*) on each triangle.



2. Label another set of heights and bases on each triangle of question 1.

FORMULA: AREA OF A TRIANGLE

Teaching guidelines

Use grid paper to explain how the area of a triangle is half of the area of the rectangle that lies between the same parallel lines and on the same base. Let learners draw a rectangle and a triangle that fits inside the rectangle.

Start with a right-angled triangle and let them count the blocks on the grid to convince themselves that the area of the triangle is half of the rectangle. Let them cut out the rectangle and then cut along the diagonal to get two triangles. Fitting them onto each other (rotate one of them) should prove that the two triangles have the same area = half of the rectangle. Then proceed to other acute-angled triangles inside the rectangle.

The formula to calculate the area of a triangle is therefore:

$A = \frac{1}{2} \times \text{base} \times \text{height} \ (= \frac{1}{2} bh)$

where the height is the perpendicular line drawn on the base to the opposite vertex of the triangle.

Let learners also use the formulae with obtuse-angled triangles. Show them how to determine the perpendicular height which will be outside the triangle in this case.

APPLYING THE AREA FORMULA

Teaching guidelines

In the case of question 3 on LB page 144, learners may need help to know what to do to find the unknown.

Misconceptions

In the case of a drawing like the second drawing in question 1, learners may use HG as the base and try to use EH as the height and then not use EF as the base.

Answers

1. (a) $\triangle ABC$: $A = \frac{1}{2}(b \times h)$

$= \frac{1}{2}(18 \times 6)$

$= \frac{1}{2} \times 108$

$= 54 \text{ cm}^2$

(b) $\triangle EFG$: $A = \frac{1}{2}(b \times h)$

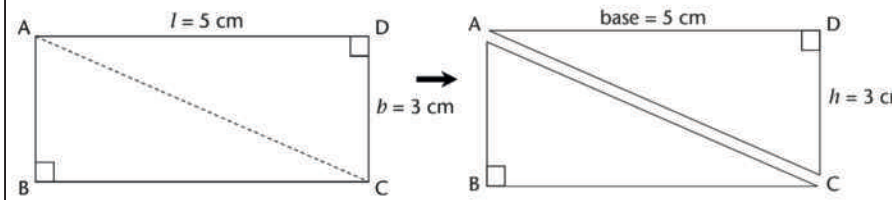
$= \frac{1}{2}(4 \times 16)$

$= \frac{1}{2} \times 64$

$= 32 \text{ cm}^2$

FORMULA: AREA OF A TRIANGLE

ABCD is a rectangle with length = 5 cm and breadth = 3 cm. When A and C are joined, it creates two triangles that are equal in area: ΔABC and ΔADC.



Area of rectangle = $l \times b$

Area of ΔABC (or ΔADC) = $\frac{1}{2}$ (Area of rectangle)

= $\frac{1}{2}(l \times b)$

In rectangle ABCD, AD is its length and CD is its breadth.

But look at ΔADC. Can you see that AD is a base and CD is its height?

So instead of saying:

Area of ΔADC or any other triangle = $\frac{1}{2}(l \times b)$

we say:

Area of a triangle = $\frac{1}{2}$ (base × height)

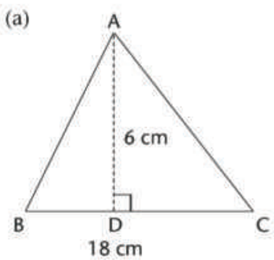
= $\frac{1}{2}(b \times h)$

In the formula for the area of a triangle, b means "base" and not "breadth", and h means perpendicular height.

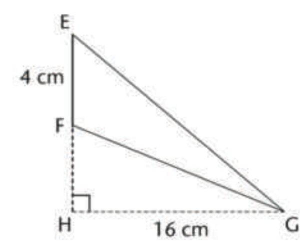
APPLYING THE AREA FORMULA

1. Use the formula to calculate the areas of the following triangles: ΔABC, ΔEFG, ΔJKL and ΔMNP.

(a)



(b)



Answers

1. (c) $\triangle JKL$: $A = \frac{1}{2}(b \times h)$

$= \frac{1}{2}(210 \times 400)$

$= \frac{1}{2} \times 84\ 000$

$= 42\ 000 \text{ mm}^2$

(d) $\triangle MNP$: $A = \frac{1}{2}(b \times h)$

$= \frac{1}{2}(10 \times 8{,}66)$

$= \frac{1}{2} \times 86{,}6$

$= 43{,}3 \text{ cm}^2$

2. (a) $QR = 20 - 10 = 10$ cm

Area of $\triangle PQR$: $\frac{1}{2}(10 \times 12)$

$= \frac{1}{2} \times 120$

$= 60 \text{ cm}^2$

(b)

Area of $\triangle PQR$: $\frac{1}{2}(9 \times 32)$

$= \frac{1}{2} \times 228$

$= 144 \text{ m}^2$

(c) $QR = 86{,}2 \div 2 = 43{,}1$ cm

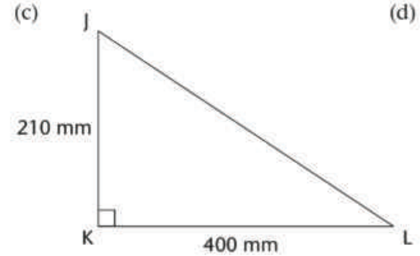
Area of $\triangle PQR = \frac{1}{2}(43{,}1 \times 35) = \frac{1}{2} \times 1\ 508{,}5 = 754{,}25 \text{ cm}^2$

3. $A = \frac{1}{2}(b \times h)$

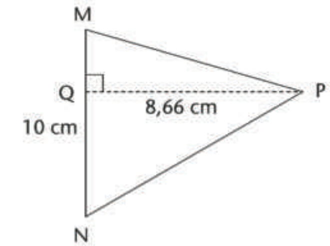
$42 = \frac{1}{2}(b \times 16)$

Base $= 42 \times 2 \div 16 = 5{,}25$ m

(c)

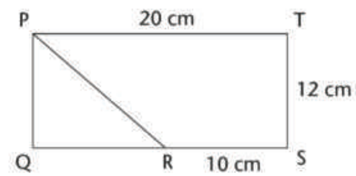


(d)

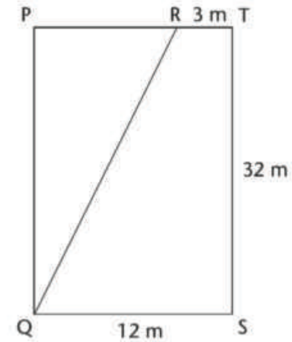


2. PQST is a rectangle in each case below. Calculate the area of ΔPQR each time.

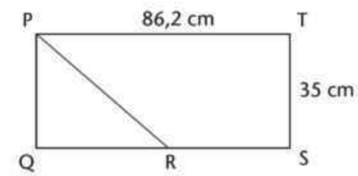
(a)



(b)



(c) R is the midpoint of QS.



3. In ΔABC, the area is 42 m^2, and the perpendicular height is 16 m. Find the length of the base.

144 MATHEMATICS GRADE 7: TERM 2

WORKSHEET

Teaching guidelines

This worksheet can be used as formative assessment or as a test.

Answers

1.

ABCD	EFGH	JKL
$P = 4 \times 7 = 28$ cm	$P = 2(8{,}2 + 5) = 26{,}4$ cm	$P = 24$ m (or 2 400 cm)
$A = 7 \times 7 = 49$ cm^2 (or 4 900 mm^2)	$A = 8{,}2 \times 5 = 41$ cm^2	$A = \frac{1}{2}(8{,}6 \times 5) = 43 \div 2 = 21{,}5$ m^2

2. (a) $P = 2(l + b) = 2(3 + 9) = 24$ cm

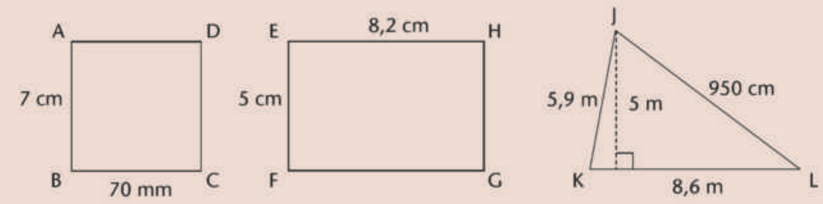
   (b) $A = l \times b = 3 \times 9 = 27$ cm^2

   (c) $A = \frac{1}{2}(b \times h) = \frac{1}{2}(4 \times 3) = 6$ cm^2

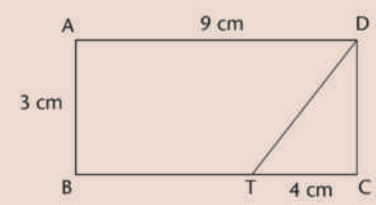
   (d) A = Area of ABCD – Area of $\triangle$DTC $= 27 - 6 = 21$ cm^2

WORKSHEET

1. Calculate the perimeter (P) and area (A) of the following figures:



2. Figure ABCD is a rectangle:
   AB = 3 cm, AD = 9 cm and TC = 4 cm.



   (a) Calculate the perimeter of ABCD.
   (b) Calculate the area of ABCD.
   (c) Calculate the area of ΔDTC.
   (d) Calculate the area of ABTD.

PERIMETER AND AREA OF 2D SHAPES

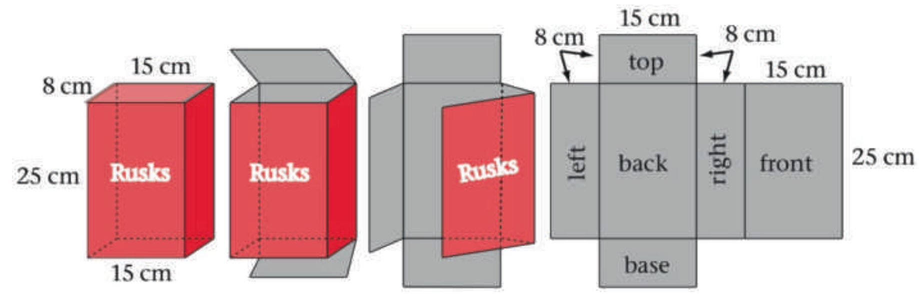
Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
10.1 Surface area of cubes and rectangular prisms	Defining surface area; using nets of rectangular prisms and cubes; working out surface areas	Pages 146 to 149
10.2 Volume of rectangular prisms and cubes	Defining volume as amount of space taken up; counting cubes to represent volume; driving and applying a formula to calculate volume	Pages 150 to 153
10.3 Converting between cubic units	Understanding cubic measurements and converting between cubic measurements	Pages 153 to 155
10.4 Volume and capacity	Understanding the difference between volume and capacity; converting between units and calculating capacity and volume	Pages 155 to 157

CAPS time allocation	8 hours
CAPS content specification	Page 57

Mathematical background

The surface area and volume of 3D objects are closely linked concepts. A 3D object has many surfaces but can only have one surface area, in other words, there is an outer surface that can be covered. A 2D shape only has area, but not surface area (per definition).

The **surface area** of an object is the sum of the areas of all the faces. We work with nets covering the whole outer surface. Using 2D nets bridges the gap between thinking about 3D objects and 2D shapes. **Nets** enable us to use the formulae for 2D shapes to find the surface area of a 3D object.



A 3D object is a **prism** if the side faces are perpendicular to the base. A prism consists of a base and a face opposite to it that has the same area. The side faces can be imagined to fold open so that it forms a rectangle with width equal to the height of the prism and length equal to the perimeter of the prism. Therefore, an alternative formula for the surface area is twice the area of the base plus the perimeter of the base times the height.

The **volume** of an object is a measure of the amount of space it takes up. The two most used units to measure volume are:

- The **cubic centimetre** (cm^3) – a cube with sides of 1 cm.
- The **cubic metre** (m^3) – a cube with sides of 1 m. This means the sides are 100 cm, therefore the cubic metre consists of 100 × 100 × 100 = 1 000 000 cm^2. In other words, there are 100 layers, each consisting of 10 000 cubes with volume 1 cm^3.

The **capacity** of a container is the amount it can hold. The volume is the amount that is in it. The standard unit of capacity is:

- The **litre** (ℓ) – 1 ℓ fills a space 10 cm long, 10 cm wide and 10 cm deep.

Capacities smaller than one litre can be measured in:

- The **millilitre** (ml) – 1 ml fills a space 1 cm long, 1 cm wide and 1 cm deep. 1 000 ml = 1 ℓ.

10.1 Surface area of cubes and rectangular prisms

INVESTIGATING SURFACE AREA

Teaching guidelines

Learners first work with the concept by calculating the areas of all the faces and adding them.

Let the learners spend plenty of time working with nets. They should construct them and cut them out. This will help them to see that the 3D objects are made up of 2D shapes such as squares, triangles and rectangles, which they are already familiar with. We don't want learners to memorise complicated formulae for the surface areas of 3D objects. They should learn to identify each surface and add the answers to get the total.

You may make wall charts of these nets, based on the learners' work.

Notes on the questions

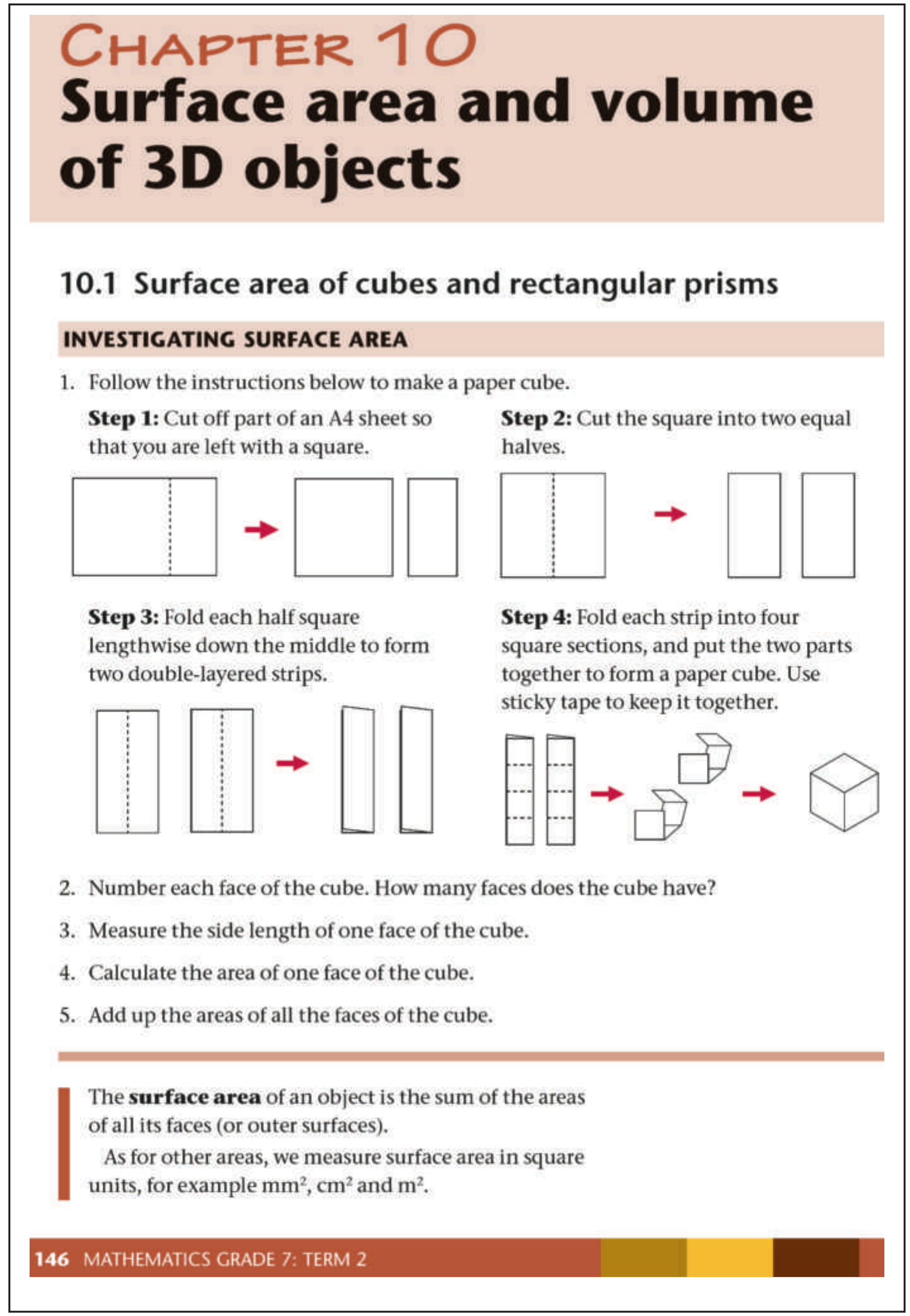
Learners work with square millimetres (mm^2), square centimetres (cm^2) and square metres (m^2).

Show learners that changing millimetres to centimetres before they multiply to find the area may give a more manageable answer.

The continuity in thinking from 2D to 3D allows learners to link the work that they are doing now to what they have already learnt, as well as work they will do in the future. When learners can comprehend the link between 2D shapes and 3D objects, they will easily understand what we mean by the surface area of any 3D object.

Answers

2. 6
3. 5,2 cm
4. 5,2 cm × 5,2 cm = 27,04 cm^2
5. 27,04 cm^2 × 6 = 162,24 cm^2



CHAPTER 10
Surface area and volume of 3D objects

10.1 Surface area of cubes and rectangular prisms

INVESTIGATING SURFACE AREA

1. Follow the instructions below to make a paper cube.

Step 1: Cut off part of an A4 sheet so that you are left with a square.

Step 2: Cut the square into two equal halves.

Step 3: Fold each half square lengthwise down the middle to form two double-layered strips.

Step 4: Fold each strip into four square sections, and put the two parts together to form a paper cube. Use sticky tape to keep it together.

2. Number each face of the cube. How many faces does the cube have?
3. Measure the side length of one face of the cube.
4. Calculate the area of one face of the cube.
5. Add up the areas of all the faces of the cube.

The **surface area** of an object is the sum of the areas of all its faces (or outer surfaces).

As for other areas, we measure surface area in square units, for example mm^2, cm^2 and m^2.

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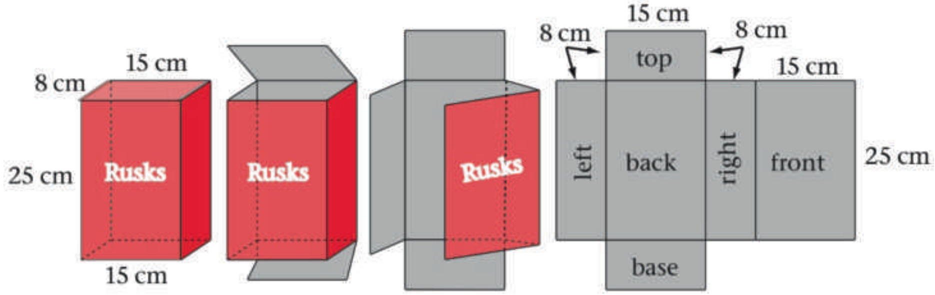
USING NETS OF RECTANGULAR PRISMS AND CUBES

Teaching guidelines

You could let learners calculate the areas of the faces of a rectangular prism and let them compare those answers with the calculation of the net.

Follow the steps on LB page 147 to create a net of a matchbox. Let learners complete the questions and evaluate the given formula.

You could also follow another approach. If possible, get every learner to bring an empty box to class. Get them to take the measurements and then cut the box open to get the net.



To find the surface area, learners have to find the area of the base and the top and add the area of the long rectangle, which is made up out of the left and right sides and the back and the front sides.

For enrichment: There may be learners who can extend their thinking to see that the surface area is the sum of the areas of the base and the top and the area of the long rectangle. The length of the long rectangle is the perimeter of the box around the base. The breadth (width) of this long rectangle is the same as the height of the rectangular prism. The other two faces, the base and the top, have the same area; therefore, the calculation consists of finding the area of two rectangles. In the example above:

Surface area of prism $= 2 \times 8 \times 15 + 2(8 + 15) \times 25$
$= 2lb + 2(l + b)h$

Learners discuss whether the following formula will also work:

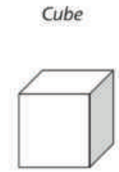
Surface area of prism = 2 × area of the base + perimeter of base × height

Answers

4. $2 \times (5 \times 4) + 2 \times (1 \times 5) + 2 \times (1 \times 4) = 2 \times 20 + 2 \times 5 + 2 \times 4$
   $= 40 + 10 + 8 = 58 \text{ cm}^2$
5. The formula is correct – learners used it in question 4.

A cube has six identical square faces. A die (plural: dice) is an example of a cube.

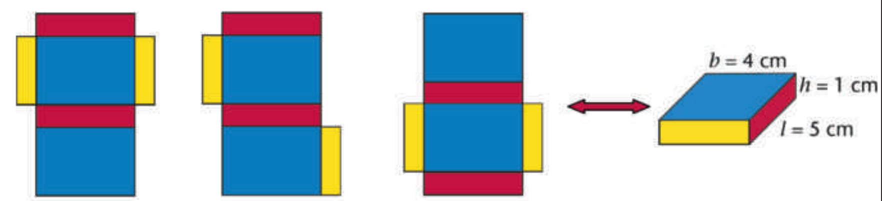
A rectangular prism also has six faces, but its faces can be squares and/or rectangles. A matchbox is an example of a rectangular prism.



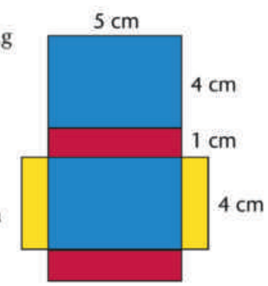
USING NETS OF RECTANGULAR PRISMS AND CUBES

It is sometimes easier to see all the faces of a rectangular prism or cube if we look at its net. A **net** of a prism is the figure obtained when cutting the prism along some of its edges, unfolding it and laying it flat.

1. Take a sheet of paper and wrap it around a matchbox so that it covers the whole box without going over the same place twice. Cut off extra bits of paper as necessary so that you have only the paper that covers each face of the matchbox.
2. Flatten the paper and draw lines where the paper has been folded. Your sheet might look like one of the following nets (there are also other possibilities):



3. Notice that there are six rectangles in the net, each matching a rectangular face of the matchbox. Point to the three pairs of identical rectangles in each net.
4. Use the measurements given to work out the surface area of the prism. (Add up the areas of each face.)
5. Explain to a classmate why you think the following formula is or is not correct:



Surface area of a rectangular prism $= 2(l \times b) + 2(l \times h) + 2(b \times h)$

CHAPTER 10: SURFACE AREA AND VOLUME OF 3D OBJECTS 147

Answers

6. (b) 1 cm^2; 6 cm^2
   (c) Correct – refer to questions (a) and (b)
   (d) $6l^2 = 6 \times (3 \times 3) = 54$ cm^2

WORKING OUT SURFACE AREAS

Teaching guidelines

Learners use the formula:

Surface area of a rectangular prism = $2lb + 2lh + 2bh$

Remind learners that the formula to find the surface area of a cube is = $6s^2$, where s is the side length of a cube.

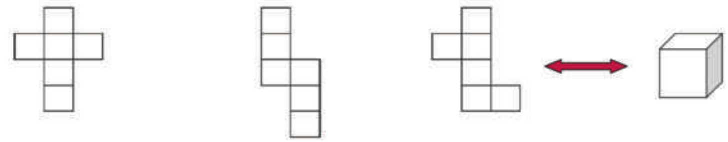
Misconceptions

Learners calculate only the surface areas of the faces they can see.

Answers

1. A: $2(2 \times 3) + 2(3 \times 8) + 2(2 \times 8)$
   $= 12 + 48 + 32 = 92$ cm^2
   B: $6(15 \times 15) = 6 \times 225$
   $= 1\,350$ cm^2
   C: $2(55 \times 15 + 15 \times 70 + 55 \times 70)$
   $= 2(852 + 1\,050 + 3\,850)$
   $= 2 \times 5\,752 = 11\,504$ mm^2
   D: $2(30 \times 20 + 20 \times 60 + 30 \times 60)$
   $= 2(600 + 1\,200 + 1\,800)$
   $= 2 \times 3\,600 = 7\,200$ mm^2
2. (a) Box A:
   $2(100 \times 20 + 20 \times 50 + 100 \times 50)$
   $2(2\,000 + 1\,000 + 5\,000)$
   $= 2 \times 8\,000 = 16\,000$ cm^2
   Box B:
   $2(2 \times 1{,}2 + 1{,}2 \times 0{,}6 + 0{,}6 \times 2)$
   $= 2(2{,}4 + 0{,}72 + 1{,}2)$
   $= 2 \times 4{,}32 = 8{,}64$ m^2
   (b) Box A = 16 000 cm^2 = 1,6 m^2
   Total surface area of box A + box B: 1,6 + 8,64 = 10,24 m^2
   Cost of paint: 10,24 × 1,34 = R13,72

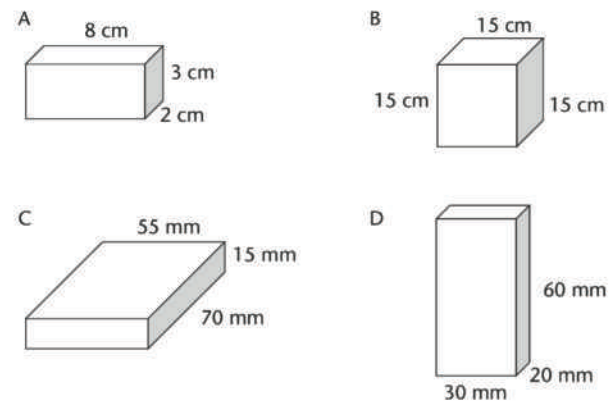
6. Here are three different nets of the same cube.



(a) Can you picture in your mind how the squares can fold up to make a cube?
(b) If the length of an edge of the cube is 1 cm, what is the area of one of its faces? What then is the area of all its six faces?
(c) Explain to a classmate why you think the following formula is or is not correct:
Surface area of a cube = $6(l \times l) = 6l^2$
(d) If the length of an edge of the cube above is 3 cm, what is the surface area of the cube?

WORKING OUT SURFACE AREAS

1. Work out the surface areas of the following rectangular prisms and cubes.



2. The two boxes on the following page are rectangular prisms. The boxes must be painted.
   (a) Calculate the total surface area of Box A and of Box B.
   (b) What will it cost to paint both boxes if the paint costs R1,34 per m^2?

Answers

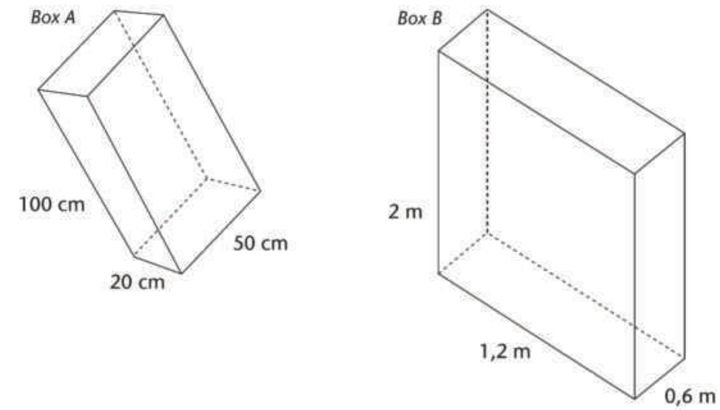
3. (0,9 × 0,3) + (0,6 × 0,3) + (1,5 × 0,3)
 + 2(0,4 × 0,3) + (0,8 × 0,3)
 + 2(0,9 × 0,4) + 2(1,5 × 0,4)
 = [0,27 + 0,18 + 0,45] + [2(0,12) + 0,24] + [2(0,36) + 2(0,6)]
 = 0,9 + (0,24 + 0,24) + (0,72 + 1,2)
 = 0,9 + 0,48 + 1,92 = 3,3 m^2

4. (a) Convert the dimensions of the holes from millimetres to metres.
 2[3 × 1,5 – (0,6 × 0,5 + 0,4 × 0,75 + 0,6 × 0,6)]
 = 2[4,5 – (0,3 + 0,3 + 0,36)]
 = 2(4,5 – 0,96)
 = 2(3,54)
 = 7,08 m^2
 (Alternative method: calculate the **area** of the holes in mm^2 and convert later).

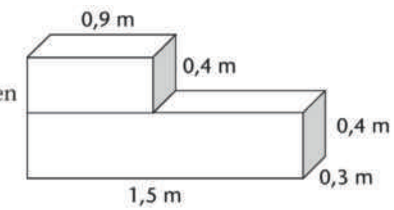
 (b) Side faces: 2(0,5 × 1,5) = 2 × 0,75 = 1,5 m^2
 Top face: 3 × 0,5 = 1,5 m^2

 (c) 7,08 + 1,5 + 1,5
 = 10,08 m^2

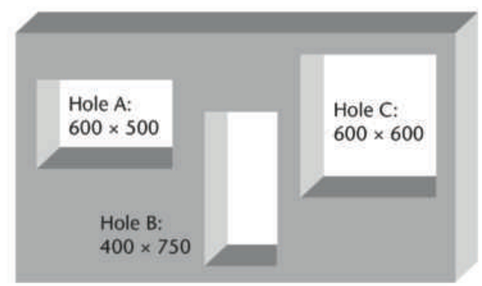
 (d) 10,08 × 2
 = R20,16



3. Two cartons, which are rectangular prisms, are glued together as shown. Calculate the surface area of this object. (Note which faces can be seen and which cannot be seen.)



4. This large plastic wall measures 3 m × 0,5 m × 1,5 m. It has to be painted for the Uyavula Literacy Project. The wall has three holes in it, labelled A, B and C, as shown. The holes go right through the wall. The measurements of the holes are in millimetres.



(a) Calculate the area of the front and back surfaces that must be painted.
(b) Calculate the area of the two side faces, as well as the top face.
(c) Calculate the total surface area of the wall, excluding the bottom and the inner surfaces where the holes are, because these will not be painted.
(d) What will it cost if the water-based paint costs R2,00 per m^2?

Remember from the previous chapter:
1 cm^2 = 100 mm^2
1 m^2 = 10 000 cm^2

10.2 Volume of rectangular prisms and cubes

Teaching guidelines

To teach volume of rectangular prisms and cubes, we focus again on the basic understanding of a solid.

The volume of an object is the amount of space it takes up.

CUBES TO MEASURE AMOUNT OF SPACE

Teaching guidelines

Volume is measured in cubic units. A 3D object is built out of smaller cubes of equal size. By counting the cubes, we can calculate the volume of the 3D object.

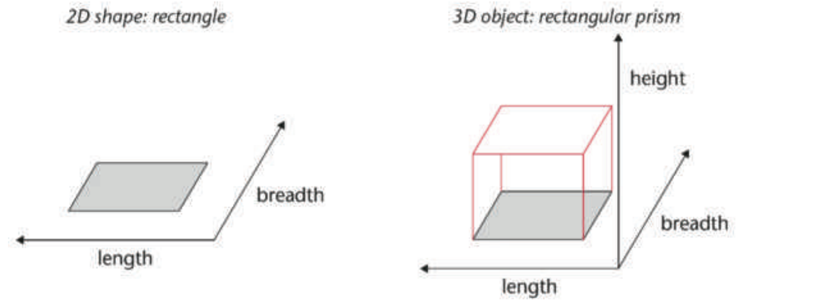
The volume of each object in question 1 can be given in terms of the number of building blocks, by counting, for example in C the first layer has 2 × 4 and there are three layers, so the volume is 24 cubic blocks.

Notes on the questions

All the blocks used in the stacks have to be of the same cubic measurement.

10.2 Volume of rectangular prisms and cubes

2D shapes are flat and have only two dimensions, namely length (l) and breadth (b). 3D objects have three dimensions, namely length (l), breadth (b) and height (h). You can think of a dimension as a direction in space. Look at these examples:



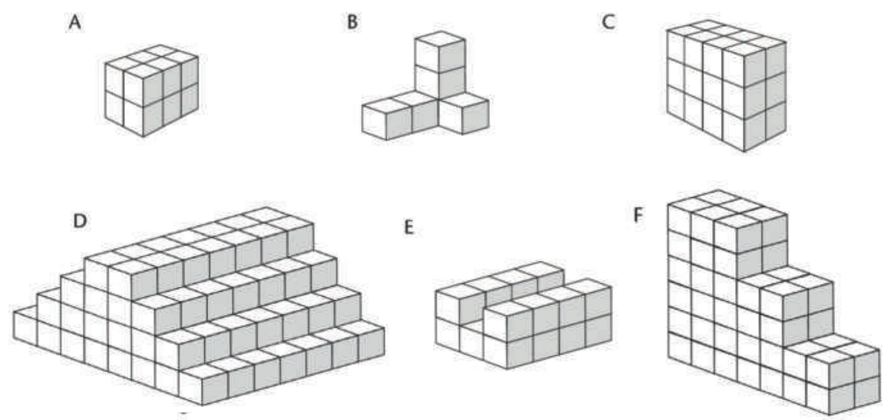
3D objects therefore take up space in a way that 2D shapes do not. We can measure the amount of space that 3D objects take up.

Every object in the real world is 3D. Even a sheet of paper is a 3D object. Its height is about 0,1 mm.

CUBES TO MEASURE AMOUNT OF SPACE

We can use cubes to measure the amount of space that an object takes up.

1. Identical toy building cubes were used to make the stacks shown below.



Answers

1. (a) B
   (b) D
   (c) B, A, E, C, F, D
   (Number of cubes per stack: A = 12; B = 6; C = 24; D = 140; E = 20; F = 60)

Teaching guidelines

A cubic centimetre is a cube with sides 1 cm.

The rectangular prism in question 2 is a regular 3D object. We can add the number of layers, each containing the same number of cubes, to calculate the volume. This shows us that to calculate the volume of a 3D object, we need the area of the base which we then multiply by height (volume = area of base × height). The height is equal to the number of layers used.

Notes on the questions

All the measurements have to be in the same unit: millimetres, centimetres or metres.

2. (a) $36 \div 4$ = nine layers   height = 9 cm
   (b) $36 \div 6$ = six layers   height = 6 cm
   (c) They both take up the same space.
   They both have the same volume.
   (d) $6\text{ cm}^3 \times 7 = 42\text{ cm}^3$
   (e) $48 \div 8 = 6$ cm

FORMULA TO CALCULATE VOLUME

Teaching guidelines

Building on the work done in the section above, learners derive the formula to calculate the volume of a rectangular prism:

Volume = area of base × height = lbh

This formula works for any prism. In the case of a cube, the formula is:

Volume = area of base × height = $s \times s \times s = s^3$ where s is the side length

The unit of the answer is written in the exponential form as it indicates that three measurements are multiplied, for example, length in centimetres and breadth in centimetres and height in centimetres gives us volume in cm^3.

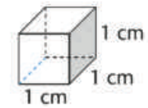
The same argument applies for any unit, m^3, km^3, mm^3, etc. All the measurements have to be in the same unit: millimetres, centimetres or metres.

(a) Which stack takes up the least space?
(b) Which stack takes up the most space?
(c) Order the stacks from the one that takes up the least space to the one that takes up the most space. (Write the letters of the stacks.)

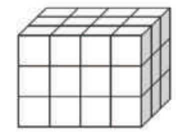
The space (in all directions) occupied by a 3D object is called its **volume**.

Cubes are the units we use to measure volume.

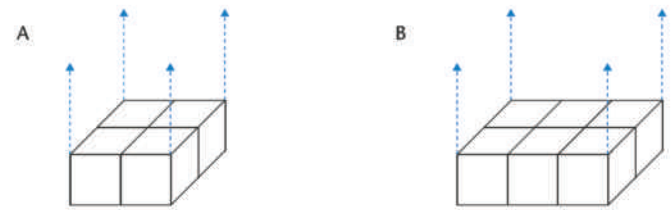
A cube with edges of 1 cm (that is, 1 cm × 1 cm × 1 cm) has a volume of one cubic centimetre (1 cm^3).



2. The figure on the right shows a rectangular prism made from 36 cubes, each with an edge length of 1 cm. The prism thus has a volume of 36 cubic centimetres (36 cm^3).



(a) The stack is taken apart and all 36 cubes are stacked again to make a new rectangular prism with a base of four cubes (see A below.) How many layers of cubes will the new prism be? What is the height of the new prism?

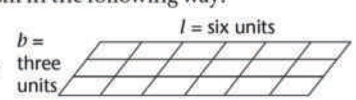


(b) Repeat (a), but this time make a prism with a base of six cubes (see B above).
(c) Which one of the rectangular prisms in questions (a) and (b) takes up the most space in all directions? (Which one has the greatest volume?)
(d) What will be the volume of the prism in question (b) if there are seven layers of cubes altogether?
(e) A prism is built with 48 cubes, each with an edge length of 1 cm. The base consists of eight layers. What is the height of the prism?

FORMULA TO CALCULATE VOLUME

You can think about the volume of a rectangular prism in the following way:

Step 1: Measure the area of the bottom face (also called the base) of a rectangular prism. For the prism given here: $A = l \times b = 6 \times 3 = 18$ square units.



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APPLYING THE FORMULAE

Teaching guidelines

Once learners have an understanding of how the formula was derived, they can calculate the volume of any rectangular prism or cube.

Learners may need help with question 5 where they have to use the formula backwards; in effect solve a simple equation.

Misconceptions

Learners do not understand that the measurements have to be in the same unit.

Answers

1. A: $V = l \times b \times h$
   $= 17 \times 5 \times 12$
   $= 1\ 020\ \text{m}^3$

   B: $V = l \times b \times h$
   $= 8 \times 3 \times 9$
   $= 216\ \text{cm}^3$

   C: $V = l^3$
   $= 5^3 = 125\ \text{cm}^3$

   D: $V = l^3$
   $= (1{,}5)^3 = 1{,}5 \times 1{,}5 \times 1{,}5 = 3{,}375\ \text{cm}^3$

2. (a) $V = l \times b \times h$
   $= 7 \times 6 \times 6 = 252\ \text{m}^3$

   (b) $V = l \times b \times h$
   $= 55 \times 10 \times 20 = 11\ 000\ \text{cm}^3$

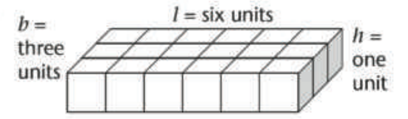
   (c) $48 \times 4 = 192\ \text{m}^3$

   (d) $16 \times 12 = 192\ \text{m}^3$

3. (a) $V = l^3$
   $= 7^3 = 7 \times 7 \times 7 = 343\ \text{cm}^3$

   (b) $V = l^3$
   $= 12^3 = 12 \times 12 \times 12 = 1\ 728\ \text{mm}^3$

Step 2: A layer of cubes, each one unit high, is placed on the flat base. The base now holds 18 cubes. It is $6 \times 3 \times 1$ cubic units.

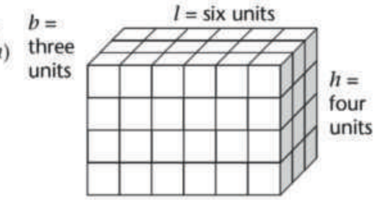


Step 3: Three more layers of cubes are added so that there are four layers altogether. The prism's height (h) is four units. The volume of the prism is:

$V = (6 \times 3) \times 4$

or $V = \text{Area of base} \times \text{number of layers}$

$= (l \times b) \times h$



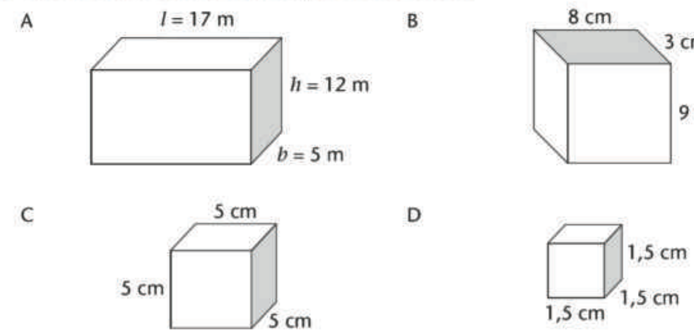
Therefore:

Volume of a rectangular prism $= \text{Area of base} \times \text{height}$
$= l \times b \times h$

Volume of a cube $= l \times l \times l$ (edges are all the same length)
$= l^3$

APPLYING THE FORMULAE

1. Calculate the volume of these prisms and cubes.



2. Calculate the volume of prisms with the following measurements:
   (a) $l = 7$ m, $b = 6$ m, $h = 6$ m
   (b) $l = 55$ cm, $b = 10$ cm, $h = 20$ cm
   (c) Surface of base $= 48\ \text{m}^2$, $h = 4$ m
   (d) Surface of base $= 16\ \text{mm}^2$, $h = 12$ mm

3. Calculate the volume of cubes with the following edge lengths:
   (a) 7 cm
   (b) 12 mm

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Answers

4. (a) $5 \times 5 \times 12 = 300\ mm^3$
   (b) (800 cm = 8 m)
   $11 \times 11 \times 8 = 968\ m^3$
5. $V = l \times b \times h$
   $375 = 8 \times 15 \times h$
   $375 = 120 \times h$
   So the height is $375 \div 120 = 3{,}125$ m

10.3 Converting between cubic units

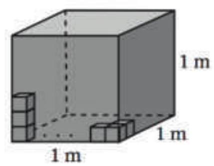
CUBIC UNITS TO MEASURE VOLUME

Teaching guidelines

We use cubic units in these formulae, such as 1 mm^3, 1 cm^3 or 1 m^3. Teach learners to say "cubic millimetre", "cubic centimetre" and "cubic metre".

Converting between units is very important. Help the learners to understand the units by showing them physical objects of different size. Try to have a big box of 1 m × 1 m × 1 m in the class, as well as a die (usually 1 cm × 1 cm × 1 cm). An object of 1 mm × 1 mm × 1 mm is as small as a grain of sugar or sand. Learners should see and experience these relationships – it is not enough for them to see the 2D representations of the objects in the Learner Book.

A cubic metre is a cube with sides 1 m. This means the sides are each 100 cm, therefore the cubic metre consists of 100 × 100 × 100 = 1 000 000 cm^3. In other words, there are 100 layers, each consisting of 10 000 cubes with volume 1 cm^3.



The unit is written in the exponential form as it indicates that three measurements are multiplied, for example, length in centimetres and breadth in centimetres and height in centimetres gives us volume in cm^3. The same argument applies for any unit, m^3, km^3, mm^3, etc.

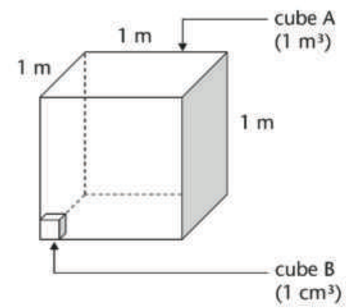
The two units that learners use most to measure volume is the cubic centimetre (cm^3) or the cubic metre (m^3).

4. Calculate the volume of the following square-based prisms:
   (a) side of the base = 5 mm, h = 12 mm (b) side of the base = 11 m, h = 800 cm
5. The volume of a prism is 375 m^3. What is the height of the prism if its length is 8 m and its breadth is 15 m?

10.3 Converting between cubic units

CUBIC UNITS TO MEASURE VOLUME

This drawing shows a cube (A) with an edge length of 1 m. Also shown is a small cube (B) with an edge length of 1 cm.



How many small cubes can fit inside the large cube?

- 100 small cubes can fit along the length of the base of cube A (because there are 100 cm in 1 m).
- 100 small cubes can fit along the breadth of the base of cube A.
- 100 small cubes can fit along the height of cube A.

Total number of 1 cm^3 cubes in 1 m^3 = 100 × 100 × 100
= 1 000 000
∴ 1 m^3 = 1 000 000 cm^3

Work out how many mm^3 are equal to 1 cm^3. Copy and complete:

1 cm^3 = 1 cm × 1 cm × 1 cm
= 10 mm × 10 mm × 10 mm
= 1 000 mm^3

Cubic units:

1 m^3 = 1 000 000 cm^3
(multiply by 1 000 000 to change m^3 to cm^3)

1 cm^3 = 0,000001 m^3
(divide by 1 000 000 to change cm^3 to m^3)

1 cm^3 = 1 000 mm^3
(multiply by 1 000 to change cm^3 to mm^3)

1 mm^3 = 0,001 cm^3
(divide by 1 000 to change mm^3 to cm^3)

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WORKING WITH CUBIC UNITS

Teaching guidelines

We use well-known cubic units in these formulae, such as mm^3, cm^3 or m^3.

We want learners to have a sense of which unit to use in problem-solving situations. For example, it would be inappropriate to use cm^3 to measure the amount of cement used to build a house or the amount of sand on a truck.

Misconceptions

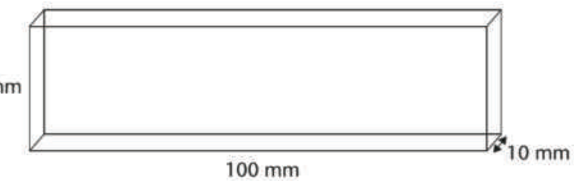
A common misconception learners have is to think that if the measurements of an object are changed by a given scale factor, the volume will change by the same scale factor. For example, a learner may think that if all the measurements are doubled, the volume will double as well. It will in fact change by a factor of 8.

Answers

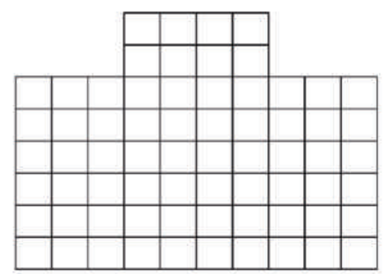
1. (a) cm^3 (b) cm^3
   (c) cm^3 (d) m^3
   (e) m^3 (f) cm^3
   (g) m^3 (h) cm^3
2. (a) 1 cm^3 (b) 3 cm^3
   (c) 2,5 cm^3 (d) 4,45 cm^3
   (e) 7,824 cm^3 (d) 0,05 cm^3
3. (a) 1 m^3 (b) 4 m^3
   (c) 1,5 m^3 (d) 2,35 m^3
   (e) 0,5 m^3 (f) 0,35 m^3
4. (a) 2 cm^3 (b) 4,12 cm^3
   (c) 1 500 000 cm^3 (d) 34 000 000 cm^3
   (e) 50 cm^3 (f) 2 230 000 cm^3
5. $7 \times 4 \times 1 = 28\ m^3$
6. $5\ cm \times 10\ cm \times 1\ cm = 50\ cm^3$
   or $50\ mm \times 100\ mm \times 10\ mm = 50\ 000\ mm^3 = 50\ cm^3$
7. (a) $(6 \times 10) + (2 \times 4) = 60 + 8 = 68\ cm^2$
   $68\ cm^2 \times 8\ cm = 544\ cm^3$
   (b) See LB page 154 alongside.

WORKING WITH CUBIC UNITS

1. Which unit, the cubic centimetre (cm^3) or the cubic metre (m^3), would be used to measure the volume of each of the following?
   (a) a bar of soap (b) a book
   (c) a wooden rafter for a roof (d) sand on a truck
   (e) a rectangular concrete wall (f) a die
   (g) water in a swimming pool (h) medicine in a syringe
2. Write the following volumes in cm^3:
   (a) 1 000 mm^3 (b) 3 000 mm^3
   (c) 2 500 mm^3 (d) 4 450 mm^3
   (e) 7 824 mm^3 (f) 50 mm^3
3. Write the following volumes in m^3:
   (a) 1 000 000 cm^3 (b) 4 000 000 cm^3
   (c) 1 500 000 cm^3 (d) 2 350 000 cm^3
   (e) 500 000 cm^3 (f) 350 000 cm^3
4. Write the following volumes in cm^3:
   (a) 2 000 mm^3 (b) 4 120 mm^3
   (c) 1,5 m^3 (d) 34 m^3
   (e) 50 000 mm^3 (f) 2,23 m^3
5. A rectangular hole has been dug for a children's swimming pool. It is 7 m long, 4 m wide and 1 m deep. What is the volume of earth that has been dug out?
6. Calculate the volume of wood in the plank shown below. Answer in cm^3.



7. The drawing shows the base (viewed from below) of a stack built with 1 cm^3 cubes. The stack is 80 mm high everywhere.
   (a) What is the volume of the stack?
   (b) Copy and complete the following:
   Volume of stack = area of base × height



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Answers

8. (a) $20 \times 15 \times 10 = 3\,000\ cm^3$
   (b) $13 \times 10 \times 0{,}5 = 65\ cm^3$ or $130 \times 100 \times 5 = 65\,000\ mm^3$
   (c) $12 \times 5{,}5 \times 3 = 198\ m^3$
   (d) $1{,}2 \times 2{,}25 \times 4 = 10{,}8\ m^3$
   (e) $300 \times 15 = 4\,500\ cm^3$
   (f) $12 \times 2{,}25 = 27\ m^3$

10.4 Volume and capacity

EQUIVALENT UNITS FOR VOLUME AND CAPACITY

- Capacity is the amount a container can hold.
- Volume is the space taken up by something.

Teaching guidelines

When you deal with the volume and capacity of a 3D object, holder or container in class, ask clear questions. For instance, when working with liquid in a container you can ask:

- What is the volume of the container?
- What is the capacity of the container?
- What is the volume of the liquid in the container?

If the container is full, then the volume of the liquid and the capacity of the container are the same. If the container is not full, the volume of the liquid in the container will differ from the capacity of the container. Remind learners that the capacity of the container is the same whether the container is full or empty.

If a 2 ℓ bottle is half-full, it contains a volume of 1 ℓ but still has a capacity of 2 ℓ, because it can hold 2 ℓ. Volume focuses on what is inside the bottle, but capacity on how much the bottle can hold.

Misconceptions

Learners think of volume as a measurement for solids and capacity as a liquid measurement.

Learners often think that the tallest container has the greatest capacity and do not take width into consideration. They believe that the amount of liquid has changed when a set amount has been poured from one container to another of a different size and that there is more liquid in the one that has the highest level.

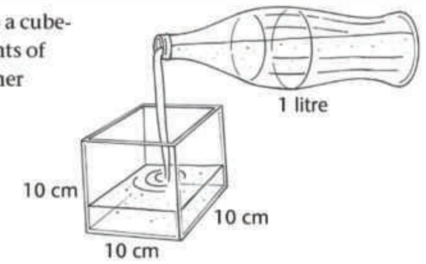
8. Calculate the volume of each of the following rectangular prisms:
   (a) length = 20 cm; breadth = 15 cm; height = 10 cm
   (b) length = 130 mm; breadth = 10 cm; height = 5 mm
   (c) length = 1 200 cm; breadth = 5,5 m; height = 3 m
   (d) length = 1,2 m; breadth = 2,25 m; height = 4 m
   (e) area of base = 300 cm^2; height = 150 mm
   (f) area of base = 12 m^2; height = 2,25 m

10.4 Volume and capacity

The space inside a container is called the internal volume, or **capacity**, of the container. Capacity is often measured in units of millilitres (ml), litres (ℓ) and kilolitres (kl). However, it can also be measured in cubic units.

EQUIVALENT UNITS FOR VOLUME AND CAPACITY

If the contents of a 1 ℓ bottle are poured into a cube-shaped container with internal measurements of 10 cm × 10 cm × 10 cm, it will fill the container exactly. Thus:



(10 cm × 10 cm × 10 cm) = 1 ℓ
or 1 000 cm^3 = 1 ℓ

Since 1 ℓ = 1 000 ml
1 000 cm^3 = 1 000 ml [1 ℓ = 1 000 cm^3]
∴ 1 cm^3 = 1 ml [divide both sides by 1 000]

Since 1 kl = 1 000 ℓ
= 1 000 × (1 000 cm^3) [1 ℓ = 1 000 cm^3]
= 1 000 000 cm^3
= 1 m^3 [1 000 000 cm^3 = 1 m^3]

This means that an object with a volume of 1 cm^3 will take up the same amount of space as 1 ml of water. Or an object with a volume of 1 m^3 will take up the space of 1 kl of water.

Notes on the conversions

The standard unit of capacity is the litre (ℓ).

1 ℓ fills a space 10 cm long, 10 cm wide and 10 cm deep.

To measure capacities smaller than one litre, the litre is divided into 1 000 parts called millilitres, which is written as ml.

1 ml fills a space 1 cm long, 1 cm wide and 1 cm deep. This means that a ml will fill a cube of 1 cm^3.

An ordinary teaspoon has a capacity of about 5 ml.

VOLUME AND CAPACITY CALCULATIONS

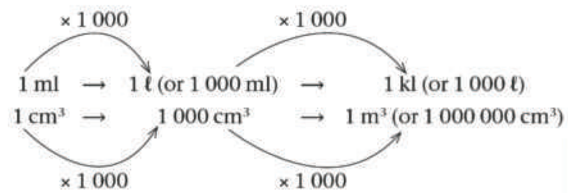
Teaching guidelines

Learners could write their own conversion diagram on a sheet of paper and keep it in front of them when they do the conversions. This may help to reinforce the conversion factors and help them to understand the relationships between different units.

Answers

1. (a) 2 000 ml (b) 250 ml
   (c) 1 000 ml (d) 4 000 ml
   (e) 2 500 ml (f) 6 850 ml
   (g) 500 ml (h) 0,5 ml
2. (a) 2 kl (b) 2,5 kl
   (c) 5 kl (d) 6 500 kl
   (e) 3 kl (f) 1,423 kl
   (g) 0,02 kl (h) 0,0025 kl
3. (a) 250 ml (b) 250 cm^3

The following diagram shows the conversions in another way:



Conversion is the changing of something into something else. In this case, it refers to changes between equivalent units of measurement.

From the diagram on the previous page, you can see that:

- 1 ℓ = 1 000 ml; 1 ml = 0,001 ℓ
- 1 kl = 1 000 ℓ; 1 ℓ = 0,001 kl
- 1 ml = 1 cm^3
- 1 ℓ = 1 000 cm^3
- 1 kl = 1 000 000 cm^3 or 1 m^3

Remember these conversions:
1 ml = 1 cm^3
1 kl = 1 m^3

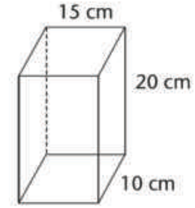
VOLUME AND CAPACITY CALCULATIONS

1. Write the following volumes in ml:
   (a) 2 000 cm^3 (b) 250 cm^3
   (c) 1 ℓ (d) 4 ℓ
   (e) 2,5 ℓ (f) 6,85 ℓ
   (g) 0,5 ℓ (h) 0,5 cm^3
2. Write the following volumes in kl:
   (a) 2 000 ℓ (b) 2 500 ℓ
   (c) 5 m^3 (d) 6 500 m^3
   (e) 3 000 000 cm^3 (f) 1 423 000 cm^3
   (g) 20 ℓ (h) 2,5 ℓ
3. A glass can hold up to 250 ml of water. What is the capacity of the glass:
   (a) in ml? (b) in cm^3?

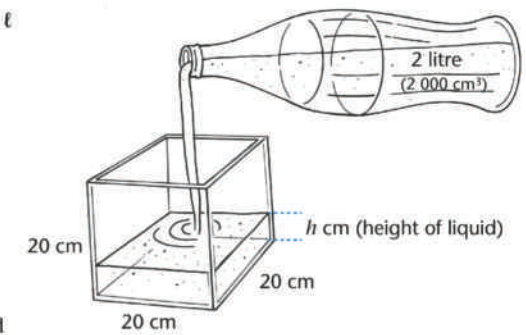
Answers

4. $15 \times 10 \times 20 = 3\,000 \text{ cm}^3 = 3\,000 \text{ ml}$
5. (a) $2\,000 \text{ cm}^3$ or $2\ \ell$
   (b) (At least) $2\,000 \text{ cm}^3$ or $2\ \ell$
   (c) $2\,000 \text{ cm}^3$ or $2\ \ell$
   (d) $20 \times 20 \times 20 = 8\,000 \text{ cm}^3$ or $8\ \ell$
   (e) $2\,000 \div (20 \times 20) = 5 \text{ cm}$

4. A vase is shaped like a rectangular prism. Its inside measurements are 15 cm × 10 cm × 20 cm. What is the capacity of the vase (in ml)?



5. A liquid is poured from a full 2 ℓ bottle into a glass tank with inside measurements of 20 cm by 20 cm by 20 cm.
   (a) What is the volume of the liquid when it is in the bottle?
   (b) What is the capacity of the bottle?
   (c) What is the volume of the liquid after it is poured into the tank?
   (d) What is the capacity of the tank?
   (e) How high does the liquid go in the tank?



In question 5 above, you should have found the following:

$$\begin{aligned} \text{Volume of liquid in tank} &= \text{Volume of liquid in bottle} \\ 20 \times 20 \times h \text{ (liquid's height in tank)} &= 2\,000 \text{ cm}^3 \\ h &= \frac{2000}{(20 \times 20)} \\ &= 5 \text{ cm} \end{aligned}$$

Note: The capacity of the tank is $20 \text{ cm} \times 20 \text{ cm} \times 20 \text{ cm} = 8\,000 \text{ cm}^3$ ($8\ \ell$).
The volume of liquid in the bottle is $2\,000 \text{ cm}^3$ ($2\ \ell$).

WORKSHEET

Teaching guidelines

The worksheet can be used as formative assessment or as a test.

Answers

1. See LB page 158 alongside.
2. (a) $2 \times (8 \times 4 + 8 \times 3 + 4 \times 3)$
   $= 2 \times (32 + 24 + 12) = 136\ m^2$
   (b) $8 \times 4 \times 3 = 96\ m^3$
3. (a) $27 \times (20 \times 20 \times 20) = 216\ 000\ mm^3$ (or $216\ cm^3$)
   (b) $\sqrt[3]{216} = 6$ cm
   (c) $6(6^2) = 6 \times 36 = 216\ cm^2$
4. (a) 250 mm = 25 cm; 120 mm = 12 cm; 100 mm = 10 cm
   $25 \times 12 \times 10 = 3\ 000\ cm^3$
   (b) $3\ 000\ cm^3 = 3\ 000$ ml
   (c) $3\ 000\ cm^3 = 3\ 000\ ml = 3\ \ell$
5. See LB page 158 alongside.
6. $11\ 250 \div (15 \times 15) = 11\ 250 \div 225 = 50$ cm

WORKSHEET

1. Do the following unit conversions:
   (a) $2\ 348\ cm^2$ = 0,2348 m^2
   (b) $5,104\ m^2$ = 51 040 cm^2
   (c) $1\ m^3$ = 1 kl
   (d) $250\ cm^3$ = 250 ml = 0,25 ℓ
   (e) 0,5 kl = 500 ℓ = 500 000 ml
   (f) 6,850 ℓ = 6 850 ml = 6 850 cm^3
2. A rectangular prism measures 8 m × 4 m × 3 m. Calculate:
   (a) its surface area
   (b) its volume
3. A boy has 27 cubes, with edges of 20 mm. He uses these cubes to build one big cube.
   (a) What is the volume of the cube if he uses all 27 small cubes?
   (b) What is the edge length of the big cube?
   (c) What is the surface area of the big cube?
4. A glass tank has the following inside measurements: length = 250 mm, breadth = 120 mm and height = 100 mm. Calculate the capacity of the tank:
   (a) in cubic centimetres
   (b) in millilitres
   (c) in litres
5. Calculate the capacity of each of the following rectangular containers. The inside measurements have been given. Copy and complete the table.

	Length	Breadth	Height	Capacity
(a)	15 mm	8 mm	5 mm	0,6 cm^3
(b)	2 m	50 cm	30 cm	300 ℓ
(c)	3 m	2 m	1,5 m	9 kl

6. A water tank has a square base with internal edge lengths of 150 mm. What is the height of the tank when the maximum capacity of the tank is $11\ 250\ cm^3$?

SURFACE AREA AND VOLUME OF 3D OBJECTS

Term 3

Numeric and geometric patterns

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
11.1 Number patterns in sequences	Completing sequences using recursion; exploring relationships between dependent and independent variables; using the position of a term as the independent variable	Pages 159 to 162
11.2 Geometric patterns	Distinguishing constant quantities and variable quantities; patterns with matches; alphabetic patterns; square and cube patterns; generating patterns	Pages 162 to 168

CAPS time allocation	6 hours
CAPS content specification	Pages 58 to 61

Mathematical background

A list of numbers that follow each other in order is called a **sequence**. For example:

1; 3; 5; 7; ... are the first four terms of an infinite sequence. The three dots indicate that the terms go on forever.

- If the number of terms in a sequence does not end, it is called an **infinite sequence**. This means there is no last term. If there is a last term in a sequence, it is called a **finite sequence**. This means that the number of terms in the sequence comes to an end.
- In a **number sequence** each term has its own position which we indicate with the symbol n. This position, n, is the input value in the relationship between the position of a term and the output value, which shows the value of that term.
- A sequence usually has a **rule**, which is a formula to find the value of each term. For example: the sequence 1; 3; 5; 7; 9; ... starts at 1 and increases by two every time. This description of the rule makes it difficult to find a term further on in the sequence, for example the twentieth or hundredth and second term. To find a usable version of the rule we us n to indicate any term number. A rule helps to find the nth term. The formula must have n in it where n represents any term.
- In Chapter 8 we discussed ways of finding the rule that describes a relationship. We use the same method to determine the rule for a sequence. The difference between the terms is two each time. The multiplicative operator is 2. Multiply the number that indicates the position of the term by 2 ($2n$) and add or subtract the correct amount to get the value of the term, for example the first term is given by 2×1 which is 2. We must subtract 1 to get the value of the first term. The rule is $2n - 1$. Test for $n = 2$: $2(2) - 1 = 3$; $n = 3$: $2(3) - 1 = 5$; each gives the correct answer.
- The rules for some sequences are formed in different ways, for example 1; 3; 6; 10; 15; 21; ... each term is formed by adding one more to a term than the difference between that term and the previous term: term 2: $1 + \boxed{2} = 3$; term 3: $3 + \boxed{3} = 6$; term 4: $6 + \boxed{4}$; etc. Other examples are the sequence of squares (1; 4; 9; 16; ...) and cubes (1; 8; 27; 64; ...).
- Patterns formed by numbers are called **numeric patterns**, and patterns formed by geometric shapes are called **geometric patterns**.

11.1 Number patterns in sequences

WHAT COMES NEXT?

Teaching guidelines

A number sequence is a list of numbers that grow according to a certain rule. The elements of a sequence are called **terms** and are in order, which means they follow on from each other.

To make a number sequence we need the following:

- a number to start with
- a rule to make more terms: rules like +4 (add 4 to each term), ×2 (multiply each term by 2) or ÷2 (divide each term by 2).

Sometimes terms do not differ by a constant value or there is not a constant ratio between terms, but the difference between terms increases by a fixed amount, for example the third sequence on LB page 159: 4; 8; 14; 22; 32; ... where the difference between the terms is 4; 6; 8; ... etc.

Work through the first four sequences on the top of LB page 159 with learners and guide them to write the next three terms in the sequences by asking questions like: "How can we get each term from the previous term?" "Can we add a number to each term or multiply each term by a number?"

Discuss vocabulary such as **consecutive** and **recursive** (see LB pages 159–160).

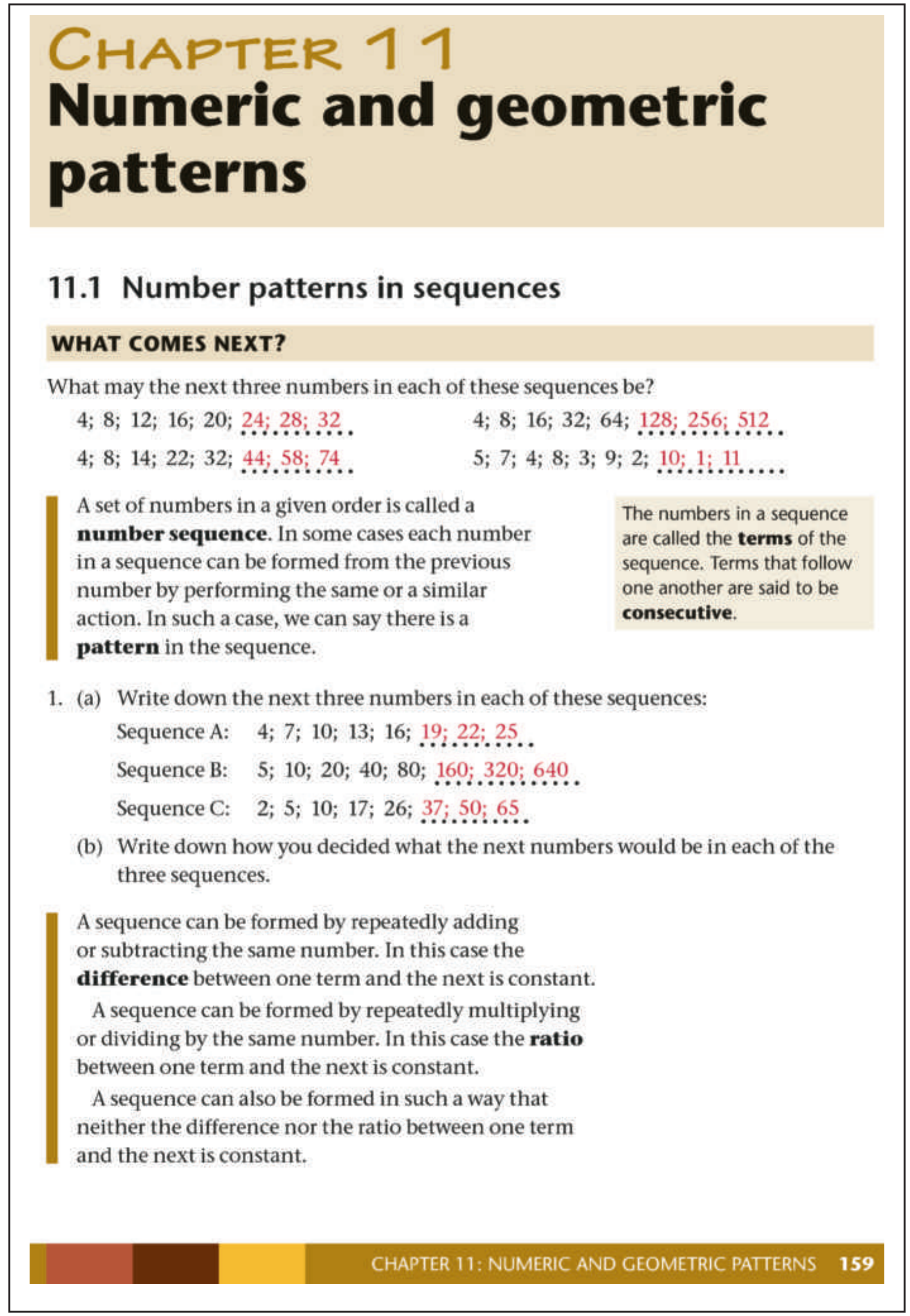
Notes on the questions

The introductory sequences are continued as follows:

- first sequence: add 4 to each term to get the next term
- second sequence: multiply each term by 2 to get the next term
- third sequence: add 4 to get the second term, then add 6; then 8 ... each time add 2 more
- start with 5 and add 2 to get the next term, then subtract 3, add 4, subtract 5, add 6, subtract 7, add 8, etc.

Answers

1. (a) See LB page 159 alongside.
   (b) Let learners discuss in small groups how they arrived at the answers.
   Sequence A: There is a common difference of 3 between terms.
   Sequence B: There is a common ratio of 2 between terms.
   Sequence C: The difference between consecutive terms is 2 more each time. (See the discussion on LB page 160.)



CHAPTER 11

Numeric and geometric patterns

11.1 Number patterns in sequences

WHAT COMES NEXT?

What may the next three numbers in each of these sequences be?

4; 8; 12; 16; 20; 24; 28; 32   4; 8; 16; 32; 64; 128; 256; 512

4; 8; 14; 22; 32; 44; 58; 74   5; 7; 4; 8; 3; 9; 2; 10; 1; 11

A set of numbers in a given order is called a **number sequence**. In some cases each number in a sequence can be formed from the previous number by performing the same or a similar action. In such a case, we can say there is a **pattern** in the sequence.

The numbers in a sequence are called the **terms** of the sequence. Terms that follow one another are said to be **consecutive**.

1. (a) Write down the next three numbers in each of these sequences:
   Sequence A: 4; 7; 10; 13; 16; 19; 22; 25
   Sequence B: 5; 10; 20; 40; 80; 160; 320; 640
   Sequence C: 2; 5; 10; 17; 26; 37; 50; 65
   (b) Write down how you decided what the next numbers would be in each of the three sequences.

A sequence can be formed by repeatedly adding or subtracting the same number. In this case the **difference** between one term and the next is constant.

A sequence can be formed by repeatedly multiplying or dividing by the same number. In this case the **ratio** between one term and the next is constant.

A sequence can also be formed in such a way that neither the difference nor the ratio between one term and the next is constant.

Answers

2. (a) 80; 75; 70; 65; 60. Each term is 5 less than the previous term.
   (b) 1,1; 1,3; 1,5; 1,7; 1,9. Each term is 0,2 more than the previous term.
   (c) 486; 1 458; 4 374; 13 122; 39 366. Each term is three times the previous term.
   (d) 21; 28; 36; 45; 55. The difference between the first two terms is 2. Thereafter, the difference is one more than the previous difference in each case. OR:
   Add 2 to the first term, then add one more than previously to get the next term in each case.
   (e) 64; 75; 86; 97; 108. Each term is 11 more than the previous term.
   (f) 8,8; 8,5; 8,2; 7,9; 7,6. Each term is 0,3 less than the previous term.
   (g) 1,8; 0,18; 0,018; 0,0018; 0,00018. Each term is a tenth of the previous term.
   (h) $\frac{1}{3}$; $\frac{2}{3}$; $1\frac{1}{3}$; $2\frac{2}{3}$; $5\frac{1}{3}$. Each term is two times the previous term.
   (i) 25; 36; 49; 64; 81. The differences between consecutive terms are the odd numbers starting at 3.
   (j) 1; 0,2; 0,04; 0,008; 0,0016. Each term is a fifth of the previous term.

RELATIONSHIPS BETWEEN DEPENDENT AND INDEPENDENT VARIABLES

Teaching guidelines

Point out that there is a relationship between the number of hours and the cost of the parking. The cost depends on the number of hours parked. These two quantities are the variables in the relationship.

Introduce the words **independent variable** and **dependent variable**. Work through the explanation on LB page 161 with learners.

Notes on the questions

Question 1 requires learners to interpret words to make a mathematical formula. It is very important in the development of learners' future ability to use mathematics to model a situation.

The rules can be given in two ways:

- working with the output. Describe where the start was and what was done to each term to get the next term: Start with R5 and add R2 each time to get the next term. This is the recursive process.
- working with the input and making a rule: Input × 2 and add 3; for example, to find term 8 (input is 8): 8 × 2 + 3 = 19.

In sequence A of question 1 there is a **constant difference** between consecutive terms, as shown below.

Sequence A:	4	7	10	13	16
Difference:	+ 3	+ 3	+ 3	+ 3	

In sequence B of question 1 there is a **constant ratio** between consecutive terms, as shown below.

Sequence B:	5	10	20	40	80
Ratio:	× 2	× 2	× 2	× 2	

In sequence C of question 1 there is neither a constant difference nor a constant ratio between consecutive terms. There is, however, a pattern in the differences between the terms, which makes it possible to extend the sequence. Consecutive odd numbers, starting with 3, are added to form the next term.

Sequence C:	2	5	10	17	26
Difference:	+ 3	+ 5	+ 7	+ 9	

2. Write down the next five terms in each of the sequences below. In each case, describe the relationship between consecutive terms.
   (a) 100; 95; 90; 85; .....
   (b) 0,3; 0,5; 0,7; 0,9; .....
   (c) 6; 18; 54; 162; .....
   (d) 1; 3; 6; 10; 15; .....
   (e) 20; 31; 42; 53; .....
   (f) 10; 9,7; 9,4; 9,1; .....
   (g) 18 000; 1 800; 180; 18; .....
   (h) $\frac{1}{48}$; $\frac{1}{24}$; $\frac{1}{12}$; $\frac{1}{6}$; .....
   (i) 1; 4; 9; 16; .....
   (j) 625; 125; 25; 5; .....

In all of the above cases it was possible to extend the sequence by repeatedly adding or subtracting a number to get the next term, or by repeatedly multiplying or dividing by a number to get the next term, or by adding different numbers according to some pattern to get the next term.

The word "recur" means "to happen again". The extension of a number sequence by repeatedly performing the same or similar action is called **recursion**. The rule that describes the relationship between consecutive terms is called a **recursive rule**.

RELATIONSHIPS BETWEEN DEPENDENT AND INDEPENDENT VARIABLES

1. (a) Mr Twala pays a fee to park his car in a parking lot every day. He has to pay R3 to enter the parking lot and then a further R2 for every hour that he leaves his car there. Copy and complete the table on the following page to show how much his parking costs him per day for various numbers of hours.

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Answers

1. (a) See the completed table on LB page 161 alongside.
   (b) Learners could discuss in small groups or pairs how they completed the table.
   (c) One way: add 2 to each answer to get the next answer (recursive).
   Another way: multiply the number of hours by 2 and add 3 to the answer.
   (d) See the completed flow diagram on LB page 161 alongside.

Teaching guidelines

- Let learners see that there is a relationship between the number (position) of the term and the corresponding term in a number sequence. The number of the term (or the position) can be assigned a variable, for example, n.
- The rule is formulated using the assigned variable and the specific description of how the dependent variable is calculated. For example, multiply by 4 and add 11.
- Discuss the strong points of using the formula and not the recursive method. Using the formula allows you to calculate any term, but if the recursive method is used, you have to add on (or subtract) the required number of times until you reach the term. For example, if you add 4 to each term to get the next and you want to know the value of term 50, you have to start at term 1 and add 4 repeatedly 49 times, whereas the rule allows you to replace the value of n with 50 and calculate term 50 immediately:

$$4 \times 50 + 11 = 211$$

Misconceptions

Learners confuse the dependent and independent variables.

Notes on the questions

2. (a) See the completed table on LB page 161 alongside.
   (b) Starting at 43 and adding 4 repeatedly 42 times is the most likely method, though some learners may do $43 + 42 \times 4 = 43 + 168$. Or even, if they recognise the relationship, $50 \times 4 + 11$ (this would be very smart).
   (c) Learners should write $11 + n \times 4 = ...$ for $n = 3; 4; 5; 6; 10$ and 50 and complete the calculation on the right-hand side.

Number of hours	1	2	3	4	5	6	7	8	9
Cost of parking in R	5	7	9	11	13	15	17	19	21

(b) How did you complete this table? Describe your method.
(c) Is there another way that you could complete the table? Describe it.
(d) Thembi multiplied the number of hours by 2 and then added 3 to calculate the cost for any specific number of hours. Copy and complete the flow diagram to show Thembi's rule.

Number of hours → × 2 → + 3 → Cost in rands

The R3 that is added is a **constant** in this situation. The number of hours and the cost are **variables**.

The rule *multiply by 2 and then add 3* describes the relationship between the two variables in this situation. The number of hours is the **independent variable**. The cost of Mr Twala's parking is the **dependent variable** because the amount he has to pay *depends on* the number of hours that he parks.

This rule describes how you can calculate the value of the *dependent* variable if the corresponding value of the *independent* variable is known. It differs from a recursive rule, which describes how you can calculate the value of the *dependent* variable that follows on a given value of the *dependent* variable.

In the case of a number sequence, the **position** (number) of the term can be taken as the independent variable, as shown for the sequence 15; 19; 23; 27; 31; . . . in this table:

Term number	1	2	3	4	5	6	7	8	50
Term	15	19	23	27	31	35	39	43	211

2. (a) Copy and complete the above table.
   (b) How did you calculate term number 50?
   (c) Lungile reasoned like this:
   I added 4 each time to complete the table. I counted backwards to see what comes before term 1. I got 11 and then I knew I had to add one 4 to 11 to get the first term.

Lungile remembered that multiplication is done before addition, unless otherwise indicated by brackets.

Complete the pattern below to show Lungile's thinking:

Term 1: $11 + 1 \times 4 = 11 + 4 = 15$ Term 2: $11 + 2 \times 4 = 11 + 8 = 19$

Term 3: $11 + 3 \times 4 = 23$ Term 4: $11 + 4 \times 4 = 27$

Term 5: $11 + 5 \times 4 = 31$ Term 6: $11 + 6 \times 4 = 35$

Term 10: $11 + 10 \times 4 = 51$ Term 50: $11 + 50 \times 4 = 211$

Answers

2. (d) Add 49 times 4 to 15. OR: Multiply 50 by 4 and add the answer to 11.
   (e) Term 4: $15 + 3 \times 4 = 15 + 12 = 27$; Term 5: $15 + 4 \times 4 = 15 + 16 = 31$;
   Term 6: $15 + 5 \times 4 = 15 + 20 = 35$; Term 10: $15 + 9 \times 4 = 15 + 36 = 51$;
   Term 50: $15 + 49 \times 4 = 15 + 196 = 211$
   (f) Subtract 1 from the term number, multiply the result by 4, then add 15.
3. (a) Multiply the term number by 4 and add to 11.
   (b) Multiply one less than the term number by 4 and add to 15.
   (c) Term $= (n - 1) \times 4 + 15 = 4 \times (n - 1) + 15$

11.2 Geometric patterns

CONSTANT QUANTITIES AND VARIABLE QUANTITIES

Teaching guidelines

The sequence is formed by the yellow tiles that increase from drawing to drawing, while the number of red and blue tiles remains constant.

The yellow tiles do not grow by a fixed number of tiles, but the number added to get the next tile increases by 7 from the first to the second; then by 9 from the second to the third tile; by 11 from the third to the fourth tile and so on.

Misconceptions

Because the pattern does not grow linearly (common difference between terms), learners might think this pattern grows by a common ratio.

Notes on the questions

The words **geometric patterns** in this section refer to the drawings and do not necessarily mean the patterns are geometric sequences (where there is a common ratio between terms).

Resources required

Grid paper

Answers

1. See the completed drawing of tile no. 5 on LB page 162 alongside.

(d) Describe in your own words how term number 50 can be calculated.
(e) Tilly reasoned like this: *The constant difference between the terms is 4. I must add four 49 times to the first term to get the fiftieth term. So, $15 + 49 \times 4 = 15 + 196 = 211$.*
Complete the pattern below to demonstrate Tilly's thinking:

Term 1: 15	Term 2: $15 + 1 \times 4 = 15 + 4 = 19$
Term 3: $15 + 2 \times 4 = 15 + 8 = 23$	Term 4: .....
Term 5:	Term 6: .....
Term 10:	Term 50: .....

(f) Write the rule to calculate term number 50 in your own words.

In the example in question 2, the term number is the independent variable and the term itself is the dependent variable. So, if we know the rule that links the dependent variable and the independent variable, we can use it to determine any term for which we know the term number.

3. Write a rule to calculate the term for any term number in the sequence 15; 19; 23; 27; 31; . . . by using:
   (a) Lungile's thinking (b) Tilly's thinking

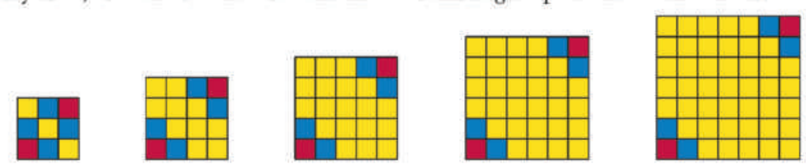
We can use n as a symbol for "any term number".
The rule to calculate the term for any term number when using Lungile's thinking will then be:
Term $= n \times 4 + 11$

(c) Write down the rule to calculate the term for any term number in terms of n by using Tilly's thinking.

11.2 Geometric patterns

CONSTANT QUANTITIES AND VARIABLE QUANTITIES

Small yellow, blue and red tiles are combined to form larger square tiles as shown below:



Tile no. 1 *Tile no. 2* *Tile no. 3* *Tile no. 4* *Tile no. 5*

1. Draw tile no. 5 on grid paper. (Shade the blue and red tiles in different ways. You don't have to use colours.)

Answers

2. See the completed table on LB page 163 alongside.
3. 2
4. The number grows. Start with 3 and add 7; then add two more each time.
   OR: (The tile number plus 2) squared minus 6.
5. The number of red tiles and the number of blue tiles are constants, while the number of yellow tiles and the tile number are variables.
6. No, other patterns could also start with the same first tile.

PATTERNS WITH MATCHES

Teaching guidelines

These patterns with matches lead to arithmetic number sequences.

Learners should be able to recognise the variable quantities and the values that remain constant in each rule.

Let learners explain the way each of the patterns in this section was made. The learners may give different rules. You could write all the possible rules on the board and let learners evaluate these rules to find the one that is most useful for them.

For example, a recursive rule leads to a lot of work if one has to find, for example, the hundredth term. Let learners choose the best rule for this purpose.

Misconceptions

The learners can see the common difference, but cannot write the formula correctly.

Answers

1. (a) After the initial triangle, each further triangle uses two more matches.
   (b) See the completed table on LB page 163 alongside.

2. Copy and complete the table.

	Tile no. 1	Tile no. 2	Tile no. 3	Tile no. 4	Tile no. 5	Tile no. 10
Number of yellow tiles	3	10	19	30	43	138
Number of red tiles	2	2	2	2	2	2
Number of blue tiles	4	4	4	4	4	4

3. How many red tiles are there in each bigger tile?
4. How many yellow tiles are there in each bigger tile?
5. Some of the quantities in this situation are variables and some are constants. Which are variables and which are constants?
6. Was it possible to predict the pattern on tile no. 2 by looking only at tile no. 1?

The number of red tiles is constant and the number of blue tiles is constant. It is clear that the design is such that there is always a red tile in the top right corner, and also in the bottom left corner, and that the red tiles are always "bordered" by two blue tiles each. So the number of red and blue tiles is **constant** in this situation.

The number of yellow tiles in the arrangements varies. The number of yellow tiles is a **variable** in this situation.

PATTERNS WITH MATCHES

1. A pattern with matches is shown below.

Figure 1

Figure 2

Figure 3

(a) Explain how the pattern is formed.
(b) Copy and complete the table.

Figure number	1	2	3	4	5	6	7	8
Number of matches	3	5	7	9	11	13	15	17

Answers

1. (c) Learners could say: add 2 to get the next term (recursive rule), or they could use a formula: multiply by 2 and add 1.
   (d) 19
   (e) 35. Multiply the figure number by 2, then add 1. OR: Keep on adding 2. OR: Add 16 times 2 to 3.
   (f) Multiply the figure number by 2, then add 1.
   (g) Term 4: $1 + 4 \times 2 = 9$ Term 5: $1 + 5 \times 2 = 11$
   Term 10: $1 + 10 \times 2 = 21$ Term 17: $1 + 17 \times 2 = 35$
   (h) The 1 in the number expression and the 2 that the term number is multiplied, by stay the same. The term (figure) number and the number of matches vary.
   (i) See the completed flow diagram on LB page 164 alongside.
   (j) You add two matches each time, so that is the number that you multiply the figure number by in your rule.
2. (a) After the initial square, each further square uses three more matches.
   (b) See the completed table on LB page 164 alongside.
   (c) Add 3 to the number of matches in the preceding figure (or term).
   (d) 28 (Add 3 to the value of term 8).

(c) What rule did you use to complete the table?
(d) How many matches are needed to form Figure number 9?
(e) How many matches are needed to form Figure number 17? Explain.
(f) If you used the recursive rule to complete the table, it would have taken a long time to answer question (e) because you had to add the same number many times. Try to find an easier way to answer question (e). Describe your method.
(g) Copy and complete the pattern below.
Hint: It may help to think of Figure number 1 or term 1 like this: There is one match at the beginning and two more are added every time. It helps to "see" the two matches that are added each time.

Term 1: $1 + 1 \times 2 = 3$ Term 2: $1 + 2 \times 2 = 5$
Term 3: $1 + 3 \times 2 = 7$ Term 4: .....
Term 5: ..... Term 10: .....
Term 17: .....

(h) What stays the same in the pattern in (g) and what varies?
(i) Copy the flow diagram below and use it to write down the rule that you can use to calculate the number of matches needed for any figure in the pattern.

Figure number → × 2 → + 1 → *Number of matches*

(j) Can you link the number of matches added each time to the number that you multiply by in the flow diagram? Explain.

2. Another pattern with matches is shown below.

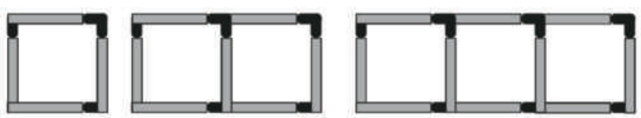


Figure 1 *Figure 2* *Figure 3*

(a) Explain how the pattern is formed.
(b) Copy and complete the table.

Figure number	1	2	3	4	5	6	7	8
Number of matches	4	7	10	13	16	19	22	25

(c) What rule did you use to complete the table?
(d) How many matches are needed for Figure 9 (or term 9)?

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Answers

2. (e) 61 matches are needed. Learners could use the recursive method, or find the formula and use it.
   (f) Multiply the term number by 3 and add 1.
   (g) Term 3: $1 + 3 \times 3 = 10$ Term 4: $1 + 4 \times 3 = 13$
   Term 5: $1 + 5 \times 3 = 16$ Term 10: $1 + 10 \times 3 = 31$
   Term 17: $1 + 17 \times 3 = 52$
   (h) The 1 in each number expression and the 3 that the term is multiplied by, stay the same. The figure number (or term) and the number of matches vary.
   (i) See the completed flow diagram on LB page 165 alongside.
3. In both cases, 1 is the added constant and the figure number is multiplied by a constant (changed from 2 to 3) to determine the number of matches.

MORE GEOMETRIC PATTERNS

Teaching guidelines

Learners should see that five dots are added to figure 1 to get figure 2 and then five dots again to get figure 3.

Let learners explain the way the pattern was made. The learners may give different rules. You could write all the possible rules on the board and let learners evaluate these rules to find the one that is most useful.

Answers

1. 27
2. See figure 5 on LB page 165 alongside.
3. See the completed table on LB page 165 alongside.
4. See the completed flow diagram on LB page 165 alongside.

(e) How many matches are needed for Figure 20 (or term 20)?
(f) What rule did you use to calculate the number of matches in question (e)?
(g) Copy and complete the pattern:
Term 1: $1 + 1 \times 3 = 4$ Term 2: $1 + 2 \times 3 = 7$
Term 3: $1 + 3 \times 3 =$ ..... Term 4: .....
Term 5: ..... Term 10: .....
Term 17: .....
(h) What stays the same in the pattern in (g) and what varies?
(i) Copy the flow diagram below and use it to write down the rule that you can use to calculate the number of matches needed for any figure in the pattern.

Figure number → × 3 → + 1 → *Number of matches*

3. Compare the way in which the number of matches increases in question 1 to the way in which it increases in question 2. What is the same and what is different?

MORE GEOMETRIC PATTERNS

Consider the figures below formed with red dots.

Figure 1 *Figure 2* *Figure 3* *Figure 4* *Figure 5*

1. How many dots are used to form Figure 5?
2. Draw Figure 5.
3. Copy and complete the table.

Figure number	1	2	3	4	5	6	7	8
Number of dots	7	12	17	22	27	32	37	42

4. Copy and complete the flow diagram.

Figure number → × 5 → + 2 → Number of dots

CHAPTER 11: NUMERIC AND GEOMETRIC PATTERNS 165

Answers

5. Learners could give the recursive rule: Add 5 to the preceding term; or they could give the rule using the figure number as the input variable.
6. Figure number + (figure number × 2 + 1) × 2 [based on the H-shapes]

   = Figure number × 5 + 2
7. Independent variable: figure number

   Dependent variable: number of dots

SQUARES AND CUBES

Teaching guidelines

Learners start using a symbol (n) for the independent variable to describe the rule.

Show learners that this is a powerful way of writing all the rules described in question 1(c) and the rule to find the number of squares in any figure.

Answers

1. (a) See the completed table on LB page 166 alongside.

   (b) Add the next odd number, starting from 3.

   (c) Figure 1: $1 \times 1 + 1 = 1 + 1 = 2$

   Figure 2: $2 \times 2 + 1 = 4 + 1 = 5$

   Figure 3: $3 \times 3 + 1 = 9 + 1 = 10$

   Figure 4: $4 \times 4 + 1 = 16 + 1 = 17$

   Figure 5: $5 \times 5 + 1 = 25 + 1 = 26$

   Figure 6: $6 \times 6 + 1 = 36 + 1 = 37$

   Figure 7: $7 \times 7 + 1 = 49 + 1 = 50$

   Figure 8: $8 \times 8 + 1 = 64 + 1 = 65$

   Figure 50: $50 \times 50 + 1 = 2\,500 + 1 = 2\,501$

   (d) Square the figure number, then add 1.

   (e) Number of squares = $n^2 + 1$

   (f) The letters H just used five more red dots each time, while in the next sequence, the number of extra small squares needed was at first just three, but increased each time you added the next odd number to the preceding term [+ 3; + 5; + 7; + 9; ...]. The letters H had a constant difference, while the difference between consecutive terms in the squares of this activity is not constant.

5. What rule did you use to complete the table? Describe your rule.
6. Can you think of another rule to complete the table? Describe your rule.
7. Name the dependent variable and the independent variable in this situation.

SQUARES AND CUBES

1. Squares are arranged to form figures as shown below, according to a rule.

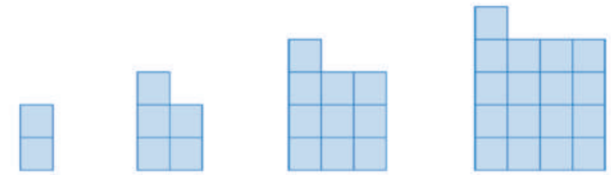


Figure 1 *Figure 2* *Figure 3* *Figure 4*

(a) Copy and complete the table. Then determine the differences between consecutive terms.

Figure number	1	2	3	4	5	6	7	8
Number of squares	2	5	10	17	26	37	50	65

+ 3 + 5 + 7 + 9 + 11 + 13 + 15

(b) Describe the recursive rule that you can use to extend the pattern in words.

(c) Nombuso played around with the differences between consecutive terms. She noticed that the pattern (+ 3; + 5; + 7; ...) was similar to the one that you get when you calculate the differences between square numbers. This made her think that she should investigate square numbers to help her find a rule that could link the figure number and the number of squares.

Complete the following pattern along the lines of Nombuso's thinking:

Figure 1: $1 \times 1 + 1 = 1 + 1 = 2$ Figure 2: $2 \times 2 + 1 = 4 + 1 = 5$

Figure 3: ..... Figure 4: .....

Figure 5: ..... Figure 6: .....

Figure 7: ..... Figure 8: .....

Figure 50: .....

(d) Write a rule to calculate the number of squares for any figure number.

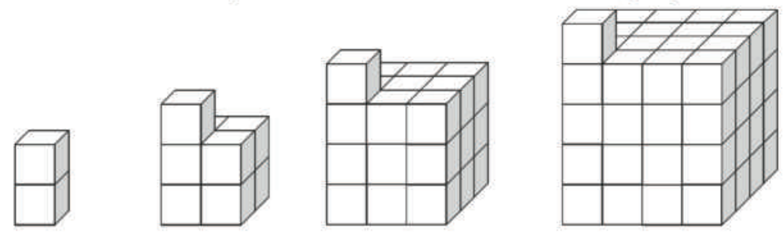
(e) Write your rule in (d) in terms of n where n is the symbol for any figure number.

(f) Compare the sequence in this activity to the sequence in the previous activity where dots were arranged to form the letter H. Describe the way in which the dependent variable (the output number) changed in each of the sequences.

Answers

2. (a) See the completed table and differences between terms on LB page 167 alongside.
   (b) Learners' own answer, for example: I noticed a pattern in the "differences" and followed that pattern. OR: I cubed the stack number and added one.
   (c) Stack 1: $1 \times 1 \times 1 + 1 = 1 + 1 = 2$
   Stack 2: $2 \times 2 \times 2 + 1 = 8 + 1 = 9$
   Stack 3: $3 \times 3 \times 3 + 1 = 27 + 1 = 28$
   Stack 4: $4 \times 4 \times 4 + 1 = 64 + 1 = 65$
   Stack 5: $5 \times 5 \times 5 + 1 = 216 + 1 = 216$
   Stack 6: $6 \times 6 \times 6 + 1 = 216 + 1 = 217$
   Stack 7: $7 \times 7 \times 7 + 1 = 343 + 1 = 344$
   Stack 8: $8 \times 8 \times 8 + 1 = 512 + 1 = 513$
   Stack 9: $9 \times 9 \times 9 + 1 = 729 + 1 = 730$
   Stack 10: $10 \times 10 \times 10 + 1 = 1\,000 + 1 = 1\,001$
   (d) $50^3 + 1 = 50 \times 50 \times 50 + 1 = 125\,000 + 1 = 125\,001$
   (e) Cube the stack number and add 1.
   (f) Number of cubes = $n^3 + 1$
3. (a) The difference became constant, 6, in the third round.
   (b) The differences were consecutive odd numbers, starting from 3. Thus, the second round of differences produces a constant, 2.

2. Identical cubes are arranged to form stacks of cubes in the following way:



Stack 1 Stack 2 Stack 3 Stack 4

(a) Copy and complete the table and the arrows. Then find the differences between consecutive terms. Do it a second and a third time. Write the differences below the arrows.

Stack number	1	2	3	4	5	6	7	8
Number of cubes	2	9	28	65	126	217	344	513

7 19 37 61 91 127 169

12 18 24 30 36 42

6 6 6 6 6

(b) Describe the way in which you completed the table.
(c) David looked carefully at the structure of the stacks and did the following to link the stack number with the number of cubes in a stack. Complete the pattern.

Stack 1: $1 \times 1 \times 1 + 1 = 1 + 1 = 2$ Stack 2: $2 \times 2 \times 2 + 1 = 8 + 1 = 9$

Stack 3: $3 \times 3 \times 3 + 1 = 27 + 1 = 28$ Stack 4: $4 \times 4 \times 4 + 1 = 64 + 1 = 65$

Stack 5: ..... Stack 6: .....

Stack 7: ..... Stack 8: .....

Stack 9: ..... Stack 10: .....

(d) How many cubes will there be in stack 50?
(e) Write the rule that you used to calculate the number of cubes in stack 50 in words.
(f) Write your rule in (e) in terms of n where n is the symbol for any stack number.

3. In questions 1(a) and 2(a) you calculated the differences between the consecutive terms.
   (a) What did you find when you kept on finding the differences, as suggested in question 2(a)?
   (b) Go back to question 1(a). What do you find when you keep on finding the differences between consecutive terms, like you did in question 2(a)?

CHAPTER 11: NUMERIC AND GEOMETRIC PATTERNS 167

MY OWN PATTERNS

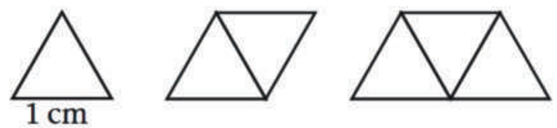
Resources required

Grid paper or a copy of LB page 168 alongside.

Teaching guidelines

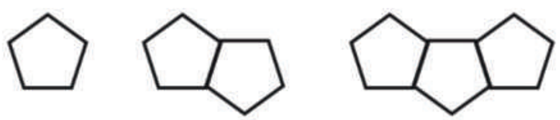
Learners make their own patterns with matches, or squares, etc. Below are a few suggestions that you could draw on the board.

1. The perimeter of equilateral triangles.

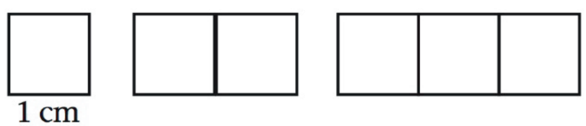


2. The number of matches to make a shape.

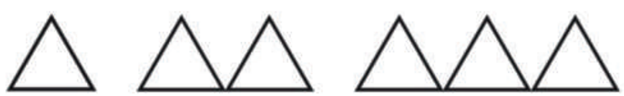
3. The perimeter of a shape (side length of a pentagon is 1 cm).



4. Perimeter of each set of squares (side length is 1 cm).



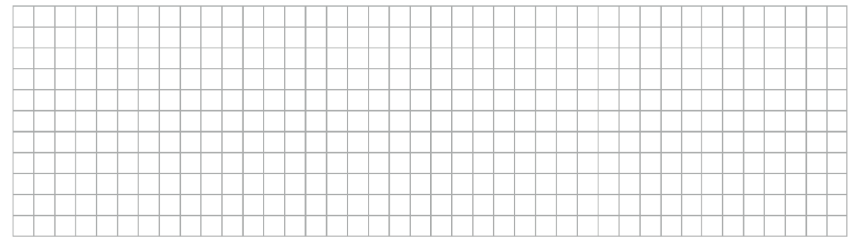
5. Perimeter of each figure (side length of equilateral triangle is 1 cm).



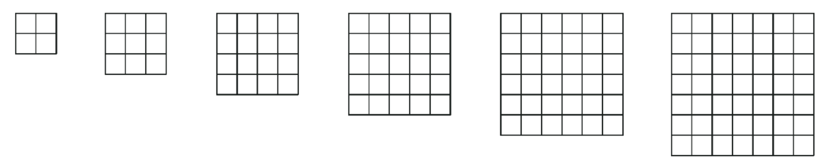
MY OWN PATTERNS

Copy the grid, the tables and the tile template to create and describe your own geometric patterns.

Pattern A



Pattern B



Grade 7 Term 3 Chapter 12 Functions and relationships 2

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
12.1 From counting to calculation	Finding formulae to describe relationships	Pages 169 to 170
12.2 What to calculate and how	Choosing symbols to represent the variables in a situation and writing formulae that represent situations mathematically; generating values of the dependent variable using values of the independent variable; comparing descriptions for similar situations using the formulae to calculate values for the same value of the independent variable	Pages 170 to 172
12.3 Input and output numbers	Using formulae to generate input and output numbers in tables; finding formulae to describe patterns	Pages 172 to 173

CAPS time allocation	3 hours
CAPS content specification	Page 62

Mathematical background

Using letters as placeholders for numbers is a powerful tool in describing situations mathematically by one formula or equation.

An example of such a description of a situation is the simplified description of an electricity account. There is a fixed value (the constant, let's say R60) which is the availability cost and then there is a fixed cost per unit, let's say R1,85. The size of the account depends on the number of units used. Therefore, the two variables are the number of units (x) and the final amount of the account (y in rand). Each household's account will differ, because x, the number of units used, will be different. The formula that gives the amount owed will be $y = 1{,}85x + 60$. The power of this formula lies in the fact that thousands of accounts can be generated with this one formula.

In a formula like this x and y are the variables, with x the independent variable and y the dependent. The cost depends on the number of units used.

Formulae can be used to substitute input values and generate tables of values. It is important that learners do this correctly as it is a skill that they will need to do future work, for example when working with graphs.

Patterns (geometrical and numerical) can be described by formulae. Sometimes it is helpful to make a table of input and output values and to find a rule from there.

Resources required

Grid paper

12.1 From counting to calculating

Teaching guidelines

In this section learners are guided to find formulae with which to calculate the dependent values in a relationship.

By using different formulae, some of which are different versions of the same formula, learners substitute values and learn intuitively about simplifying algebraic expressions, for example that $2 \times (2 \times x + 1)$ gives the same values as $4 \times x + 2$.

When learners answer question 3, point out that it does not matter which symbols we use for the variables, as long as the rule is correct.

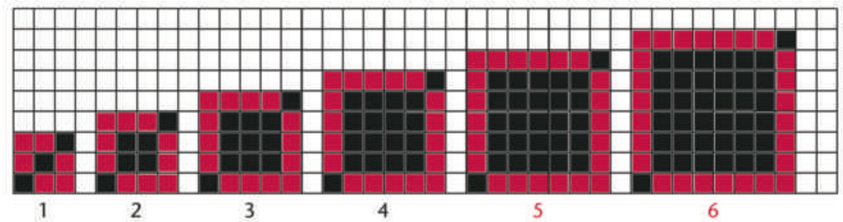
Answers

1. (a) See the table on LB page 169 alongside.
   (b) See the table on LB page 169 alongside.
   (c) Learners draw the arrangements. Answers for arrangements 5 and 6 are shown on LB page 169 alongside. Arrangement 7 follows the pattern and covers 9 by 9 squares.
   (d) 82, 86 and 90 red squares, and 402, 443 and 486 black squares. It is fine if learners do not manage to get this right now. Advise them to continue to the next question after some time (at most 20 minutes).
2. (a) $y = 2 \times (2 \times x + 1)$ and $y = 4 \times x + 2$
   (b) x is the number of the arrangement; y is the number of red squares.
   (c) 82, 86 and 90 red squares.

Chapter 12
Functions and relationships 2

12.1 From counting to calculating

1. (a) How many red squares and how many black squares are there in each of the arrangements 1, 2, 3 and 4 below? Copy and complete the table.



Arrangement number	1	2	3	4	5	6	7
Number of red squares	6	10	14	18	22	26	30
Number of black squares	3	6	11	18	27	38	51

   (b) Imagine that arrangements 5, 6 and 7 are made according to the same pattern. How many red and how many black squares do you think there will be in each of these arrangements? Write your answers in the table you copied.
   (c) Complete arrangements 5 and 6 on grid paper, if you have not done so already.
   (d) Try to figure out how many red and how many black squares there will be in arrangements 20, 21 and 22.

2. It will be useful to have formulae to calculate the numbers of red and black squares in different arrangements like the one above.
   (a) Which of the formulae below can be used to calculate the numbers of red squares in the above arrangements? There is more than one formula that works.
   $y = 2 \times x + 4$ $y = 2 \times (2 \times x + 1)$ $y = x^2 + 2$ $y = 4 \times x + 2$
   (b) Natasha decided to use the formula $y = 4 \times x + 2$ to calculate the number of red squares in an arrangement. What do the symbols x and y mean in this case?
   (c) Use the formula $y = 4 \times x + 2$ to calculate the numbers of red squares in arrangements 20, 21 and 22.

Answers

2. (d) Learners could work in pairs and help correct each other's mistakes if there are any.
   (e) See the completed table on LB page 170 alongside.
3. (a) $z = x^2 + 2$ and $p = n^2 + 2$
   (b) See the completed table on LB page 170 alongside.
4. (a) Both formulae can be used; they give the same answers.
   (b) $y = 4 \times x - 6$ and $4 \times (x - 2) + 2$
   (c) See the completed table on LB page 170 alongside.

12.2 What to calculate and how

REPRESENTING SITUATIONS MATHEMATICALLY

Teaching guidelines

The situations described in this section are "word" sums that learners have to find formulae for to represent mathematically and then use the formulae to generate values.

Learners should ask the questions: What do I have to calculate? What information do I have that I can use. This means they have to find the dependent and independent variables in the situation. For example, the cost of using the buses depends on distance travelled and not the other way around.

Misconceptions

Notes on the questions

In the discussion of the problems learners are guided through the process of describing the situations mathematically. Guide learners through the discussion about the different formulae and help them to test the formulae to find out which describes the situation and which of them do not (and why not).

Answers

1. (a) 60
   (b) 120
   (c) 180
   (d) I multiplied the number of hours by the number of minutes (60) in one hour.

(d) If your answers differ from the answers you gave in question 1(d), you have made mistakes somewhere. Find your mistakes and correct them.

(e) Copy and complete the table.

x	1	2	3	4	5	6	7	8	9
$2 \times (2 \times x + 1)$	6	10	14	18	22	26	30	34	38
$2 \times x + 4$	6	8	10	12	14	16	18	20	22
$4 \times x + 2$	6	10	14	18	22	26	30	34	38

3. (a) Which of the formulae below can be used to calculate the numbers of black squares in the arrangements in question 1?
   $z = (x + 2)^2$ $\quad z = x^2 + 2$ $\quad p = n^2 + 2$
   (b) Copy and complete the table.

x	1	2	3	4	5	6	7	8	9
$x^2 + 2$	3	6	11	18	27	38	51	66	83
$(x + 2)^2$	9	16	25	36	49	64	81	100	121

4. Hilary uses x to represent the *number of squares in each side* of the arrangements.
   (a) Which of these formulae can Hilary use to calculate the numbers of black squares in the arrangements in question 1?
   $y = x^2 - 4 \times x + 6$ $\quad y = (x - 2)^2 + 2$
   (b) Which of these formulae can Hilary use to calculate the numbers of red squares?
   $y = 3 \times x - 3$ $\quad y = 4 \times x - 6$ $\quad 4 \times (x - 2) + 2$
   (c) Copy and complete this table to check your answers.

x	3	4	5	6	7	8	9
$x^2 - 4 \times x + 6$	3	6	11	18	27	38	51
$(x - 2)^2 + 2$	3	6	11	18	27	38	51
$3 \times x - 3$	6	9	12	15	18	21	24
$4 \times x - 6$	6	10	14	18	22	26	30
$4 \times (x - 2) + 2$	6	10	14	18	22	26	30

12.2 What to calculate and how

REPRESENTING SITUATIONS MATHEMATICALLY

1. (a) How many minutes are there in an hour?
   (b) How many minutes are there in two hours?
   (c) How many minutes are there in three hours?
   (d) Explain how you determined the answers for questions 1(b) and (c).

Answers

1. (e) Number of minutes = 60 × number of hours.
   (f) See the completed table on LB page 171 alongside.
2. (a) Formula A
   (b) Formula C
   (c) Formula F
   (d) See the completed table on LB page 171 alongside.

The formula $m = 60 \times h$ can be used to calculate the number of minutes when the number of hours is known. The symbol h represents the number of hours and m the number of minutes.

(e) Express the formula $m = 60 \times h$ in words.
(f) Copy and complete the table.

Number of hours	1	2	3	15	24
Number of minutes	60	120	180	900	1 440
How to calculate	60 × 1	60 × 2	60 × 3	60 × 15	60 × 24

2. Three bus companies placed the following advertisements in a newspaper:

(a) Which of the formulae below can be used to calculate the fare for a journey with Hamba Kahle Tours?
(b) Which of the formulae can be used to calculate the fare for a journey with Saamgaan Tours?

Saamgaan Tours
We criss-cross every province and stop in every town and dorpie. Pay only R450 per trip plus 60c per km.

Hamba Kahle Tours
Long distance travel is our business: R500 per trip plus 50c per km!

Comfort Tours
Experience what it means to travel in style. Only R480 per trip plus 55c per km.

Some formulae to calculate fares:

A. Fare = 0,50 × distance + 500
B. Fare = 50 × distance + 500
C. Fare = 0,60 × distance + 450
D. Fare = 60 × distance + 450
E. Fare = 55 × distance + 480
F. Fare = 0,55 × distance + 480

(c) Which of the above formulae can be used to calculate the fare for a journey with Comfort Tours?

We write 50c as R0,50 or 0,50 when we do calculations.

(d) Copy and complete the table by making use of the formulae below. You may use a calculator for this question.
Fare for Hamba Kahle Tours = 0,50 × distance + 500
Fare for Saamgaan Tours = 0,60 × distance + 450
Fare for Comfort Tours = 0,55 × distance + 480

Distance in km	150	200	250	300
Hamba Kahle Tours	R575	R600	R625	R650
Saamgaan Tours	R540	R570	R600	R630
Comfort Tours	R562,50	R590	R617,50	R645

Answers

2. (e) Saamgaan Tours. For all four distances above it offers the cheapest fare.
   (f) See the completed flow diagram on LB page 172 alongside.
   (g) y is the fare; x is the distance (for all three of Wandile's formulae)
   (h) Hamba Kahle : $y = 0{,}5 \times x + 500 = \text{R}1\ 000$
   Saamgaan Tours: $y = 0{,}6 \times x + 450 = \text{R}1\ 050$
   Comfort Tours: $y = 0{,}55 \times x + 480 = \text{R}1\ 030$
   Hamba Kahle Tours is the cheapest.

12.3 Input and output numbers

FROM FORMULAE TO TABLES

Teaching guidelines

Learners must be able to substitute values and do the calculations correctly to find values of the dependent variable.

In these problems learners have to check which formula fits which table. They do this by substituting the values of x into the formula to see if they get the given values of y.

They use given formulae to generate values by substitution. They must keep the order of operations in mind and remember to multiply before they add or subtract.

Notes on the questions

Learners must be able to describe a function using different representations: from tables to formulae; from formulae to tables and later also from formulae or tables to graphs. They should also be able to describe a function in words.

Answers

1. See LB page 172 alongside.

(e) Which bus company is the cheapest? Explain.
(f) Copy and complete this flow diagram for the bus company that you named in question (e):

distance → $\times 0{,}6$ → $+ 450$ → fare

(g) Wandile wrote the formulae for calculating the fares for the different bus companies using the letter symbols x and y. Say what each letter symbol stands for in each of the following:
(i) $y = 0{,}50 \times x + 500$
(ii) $y = 0{,}60 \times x + 450$
(iii) $y = 0{,}55 \times x + 480$
(h) Which of the three bus companies would be the cheapest to use for a journey of 1 000 km?

12.3 Input and output numbers

FROM FORMULAE TO TABLES

1. For each of the tables (a) to (f) below, determine which of the following formulae could have been used to complete it:

A. $y = 5 \times x + 3$ B. $y = 3 \times x$ C. $y = 3 \times x + 2$
D. $y = 4 \times x$ E. $y = 3 \times x + 1$ F. $y = 2 \times x$
G. $y = 3 \times x + 10$ H. $y = 2 \times x - 1$ I. $y = 5 \times x$

(a)

x	1	2	3	4	5
y	13	16	19	22	25

Formula used: G

(b)

x	1	2	3	4	5
y	8	13	18	23	28

Formula used: A

(c)

x	1	2	3	4	5
y	4	8	12	16	20

Formula used: D

(d)

x	1	2	3	4	5
y	5	8	11	14	17

Formula used: C

(e)

x	1	2	3	4	5
y	5	10	15	20	25

Formula used: I

(f)

x	1	2	3	4	5
y	1	3	5	7	9

Formula used: H

We can complete a table of values if we are given a formula. For example, for the formula $y = 7 \times x - 3$ we can complete the table below, as shown:

For $x = 1$, $y = 7 \times 1 - 3 = 7 - 3 = 4$
For $x = 2$, $y = 7 \times 2 - 3 = 14 - 3 = 11$
For $x = 3$, $y = 7 \times 3 - 3 = 21 - 3 = 18$

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Answers

2. See the completed tables on LB page 173 alongside.

FROM PATTERNS TO FORMULAE

Teaching guidelines

If learners struggle to find a formula from the geometric representation of the pattern, let them write the sequence of numbers formed by the pattern, or make a table to describe the pattern. They can find the formula by working from the table.

Misconceptions

Learners get confused with the number (position) of the pattern and the side length of the square. First work with the pattern number and then with the side length. Make sure that learners understand the difference.

Notes on the questions

The sequence for the black squares is 6; 11; 18; 27; ... The number in a term increases by two more than the difference between the previous two terms, starting with an increase of 5 between the first and the second term. The increases are 5; 7; 9; 11; 13; ...

The red squares increase by two between terms: 3; 5; 7; 9; ...

Answers

1. (a) 27 + 11 = 38 black squares in arrangement 5, and 38 + 13 = 51 black squares in arrangement 6.
   (b) z is the number of red squares, x is the length of a side, y is the number of black squares.
   (c) z is the number of red squares, n is the number of the arrangement (its position from the left in the above picture), y is the number of black squares.
2. Possible answers, with letter symbols having the same meanings as in question 1:
   $z = 2 \times n + 2$ or $z = 2 \times x - 2$ and $y = (n + 1)^2 + 1$ or $y = x^2 \times (2 \times x - 2)$

x	1	2	3	4	5	6	7
y	4	11	18	25	32	39	46

2. Copy and complete the tables using the given formulae.
   (a) $y = 6 \times x - 5\frac{1}{2}$

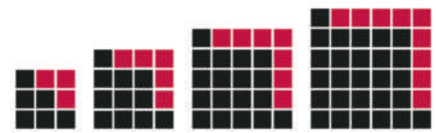
x	1	2	3	4	5	6	7
y	$\frac{1}{2}$	$6\frac{1}{2}$	$12\frac{1}{2}$	$18\frac{1}{2}$	$24\frac{1}{2}$	$30\frac{1}{2}$	$36\frac{1}{2}$

   (b) $y = 30 \times x + 1$

x	0,1	0,2	0,3	0,4	0,5	0,6	0,7
y	4	7	10	13	16	19	22

FROM PATTERNS TO FORMULAE

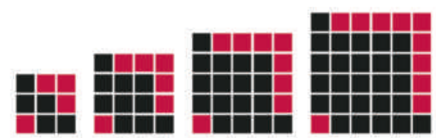
1. Some arrangements with black and red squares and some formulae are given below.



Formula A: $z = 2 \times n + 1$ Formula B: $z = 2 \times x - 3$
Formula C: $y = (n + 1)^2 + 2$ Formula D: $y = x^2 - (2 \times x - 3)$

   (a) How many black squares will there be in the next two similar arrangements?
   (b) Susan uses formulae B and D to calculate the numbers of red and black squares. What do the letter symbols z, x and y mean in Susan's work?
   (c) Zain uses formulae A and C to calculate the numbers of red and black squares. What do the letter symbols z, n and y mean in Zain's work?

2. Write formulae that can be used to calculate the numbers of red and black squares in arrangements like those below. Use letter symbols of your own choice and state clearly what each of your symbols represents.



CHAPTER 12: FUNCTIONS AND RELATIONSHIPS 2 173

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
13.1 Describing and doing computations	Looking at different ways of describing a computation; describing patterns using algebraic expressions	Pages 174 to 177
13.2 Relationships represented in formulae	Understanding variables and constants in formulae and expressions	Pages 177 to 178

CAPS time allocation	3 hours
CAPS content specification	Page 63

Mathematical background

An **algebraic expression** is a symbolic description of a set of calculations that can be performed on different values of a variable. For example, the expression $3x + 5$ means "multiply the value of x by three and add five to the answer".

To find the value of an expression for particular values of the variable or variables, the normal rules of arithmetic apply – i.e. do the calculations in brackets first, then do multiplication and division in the order they appear and lastly, do the additions and subtractions in the order that they appear.

If a relationship is given as a table or in words, it can usually be given in an expression as well, for example: the table that describes a relationship is:

Input	1	2	3	4	5	6
Output	7	11	15	19	23	27

And the words are: three more than a number multiplied by 4. It can be written algebraically as $3 + 4 \times x$ or $4 \times x + 3$.

An **equation** is a statement about an unknown number (a constant, not a variable). An equation states what result (answer) is obtained if certain calculations are performed on an unknown constant. For example, the equation $2x + 3 = 15$ states that "when a certain unknown number is multiplied by two and three is added to the result, the answer is fifteen". To "solve an equation" means to find out what the unknown number is.

Substitution is used to find out when expressions are equal in value. Algebraic expressions that have the same numerical value for the same values of x, but look different, are called **equivalent expressions**.

13.1 Describing and doing computations

DIFFERENT WAYS OF DESCRIBING A COMPUTATION

Teaching guidelines

The diagrams in question 1 represent the even numbers. Work from the pattern to the table and the flow diagram and then to the expression; following the explanation in the Learner Book, and represent the expression as another way of describing the patterns. Let learners see that the rule $2 \times n$ generates even numbers for:

$n = 1, 2, 3, 4, ...$

It is critically important that learners are made to realise that the letter symbols in algebraic expressions are placeholders for numbers.

Notes on the questions

Note that we are still using the × sign in algebraic expressions. In terms of computational procedures and flow diagrams, it is natural for learners to write $2 \times x$ for some time. The idea of dropping the × sign by convention will be introduced later. Neither form of notation should be prescribed. Different learners will adopt the shorter notation at different times. Eventually, everybody in the class will naturally write $2x$ and not $2 \times x$.

Answers

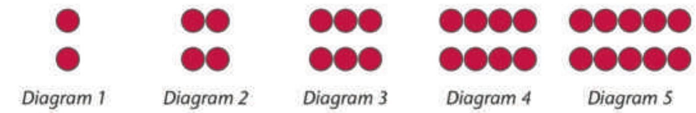
1. (a) See the completed table on LB page 174 alongside.
   (b) The number of rows.
   (c) The number of circles per diagram and the number of circles per row.
   (d) See the completed flow diagram on LB page 174 alongside.
   (e) There will be 22 circles because diagram 11 will have 2 rows of 11 circles per row.
   (f) It represents the number of rows.
   (g) The letter symbol n represents the number of circles in a row.

CHAPTER 13
Algebraic expressions 1

13.1 Describing and doing computations

DIFFERENT WAYS OF DESCRIBING A COMPUTATION

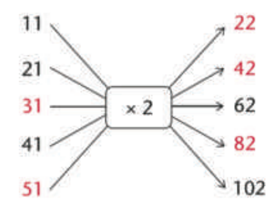
1. The diagrams below represent arrangements of small circles. In every arrangement there are two rows of circles.



(a) The table below relates to the diagrams. Copy and complete it.

Diagram number	1	2	3	4	5
Number of circles per row	1	2	3	4	5
Number of rows	2	2	2	2	2
How to calculate the total number of circles per diagram (**rule**)	2×1	2×2	2×3	2×4	2×5

In every diagram, we can identify:
- the number of rows
- the number of circles per row
- the total number of circles per arrangement.



(b) What remains the same in the diagrams above?
(c) What changes in the diagrams? In other words, what are the variable quantities in the situation?
(d) Copy and complete the flow diagram on the right.
(e) How many circles will diagram 11 have if the pattern is extended? Explain.
(f) What does the number 2 in the rule $2 \times n$ represent?
(g) What does the letter symbol n represent in the rule $2 \times n$?

The rule $2 \times n$ can be used to determine the total number of circles in a diagram. The number 2 in the rule $2 \times n$ remains the same all the time. We say it is a **constant**. The letter symbol n represents the number of circles per row and that is a **variable**, because it changes.

Teaching guidelines

The rule for the odd numbers can be derived from the rule for the even numbers. If 2 is the first even number, generated by 2 × n, the first odd number is one less, therefore 2 × 1 – 1, and so on.

In each expression in this section point out which numbers are constants and which are variables.

Let learners know that any letter symbol can be chosen to represent a variable, but that those most often used are x, y, z and n.

Remind learners what we mean by input and output numbers in a flow diagram or a table. When the rule to generate output numbers is written as an expression, for example $5 \times x + 4$, x represents the input and $5 \times x + 4$ represents the output values.

Answers

2. (a) $2 \times 10 - 1 = 20 - 1 = 19$
   (b) $2 \times 30 - 1 = 60 - 1 = 59$
   (c) $2 \times 100 - 1 = 200 - 1 = 199$
   (d) $2 \times n - 1$
3. The letter symbol n represents the position of the odd number in the sequence.
4. (a) See the completed flow diagram on LB page 175 alongside.
   (b) C. Add 4 to the input number and then multiply by 5.
5. (a) $10 \times 5 + 4 = 54$
   (b) $10 + 45 = 55$
   (c) $(10 + 4) \times 5 = 14 \times 5 = 70$

Consider the sequence 1; 3; 5; 7; 9; ...

The first odd number can be written as 2 × **1** – 1.

The second odd number can be written as 2 × **2** – 1.

The third odd number can be written as 2 × **3** – 1.

> The numbers 2 and –1 remain the same all the time; we call them **constants**. The numbers in blue change according to the position of the odd number in the sequence. We call them **variables**.

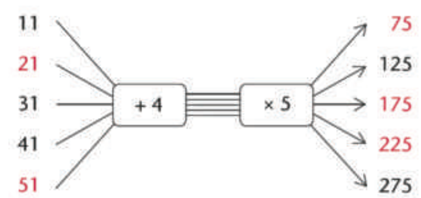
2. (a) What is the tenth odd number?
   (b) What is the thirtieth odd number?
   (c) What is the hundredth odd number?
   (d) What is the nth odd number?

3. The rule $2 \times n - 1$ can be used to determine any odd number in the sequence 1; 3; 5; 7; 9; ...
   What does the letter symbol n represent in the rule $2 \times n - 1$?

> The rule **2 × *n*** can be used to calculate the total number of circles in a diagram if the number of circles per row is known.
> The rule **2 × *n* – 1** can be used to determine any odd number in the sequence of odd numbers if its position is known.

In the questions above we have used the letter symbol n to represent:
- a changing number in the rule $2 \times n$ (n represents the number of circles in a row)
- the position of the odd number in a sequence in the rule $2 \times n - 1$.

4. (a) Copy and complete the flow diagram.



> We call the numbers on the left in the flow diagram the **input numbers**.
> The numbers on the right in the flow diagram, and whose values depend on the input numbers, are called the **output numbers**.

   (b) Which of the following instructions did you follow to calculate the output values of —[+ 4]—[× 5]→ in question 4(a)?
   Write out and place a tick mark (✓) next to the correct answer.
   A. Multiply the input number by 5 and then add 4.
   B. Add 45 to the input number.
   C. Add 4 to the input number and then multiply by 5. ✓

5. Use 10 as the input number and calculate the output number for each of the word formulae in question 4(b).

Teaching guidelines

Talk about the fact that x in an expression like $5 \times x + 4$ is a variable, but if the expression equals a value, as in $5 \times x + 4 = 29$, then x is an unknown number that makes the number sentence true, namely 5 in this case.

Learners should be able to make a clear distinction between a letter symbol as a variable and as an unknown.

Answers

6. (a) $\frac{1}{2} \times x + 2$
   (b) $x \times 6 - 2$ or $6 \times x - 2$
   (c) $(x + 3) \times 10$
   (d) $(x - 4) \times 7$
7. (a) Yes, Paul is correct.
   (b) Cardo's expression means you first add 5 and 2 to get 7 and then multiply the answer by 3. By convention, the operation in brackets is done before any other operation.
   (c) See LB page 176 alongside.
8. See LB page 176 alongside.

We may write $(x + 4) \times 5$ as an abbreviation for *add 4 to the input number, then multiply by 5*.
$(x + 4) \times 5$ can be called a computational instruction or an **algebraic expression**.

In the expression $(x + 4) \times 5$, the letter symbol x can be replaced by many different input numbers. The symbol x represents a **variable quantity** or a **variable**. If, however, the expression $(x + 4) \times 5$ is equal to 35, as in the number sentence $(x + 4) \times 5 = 35$, the symbol x represents only one value, and that is 3.

In the expression $(x + 4) \times 5$, the numbers 4 and 5 are **constants**. In the number sentence $(x + 4) \times 5 = 35$, x is an **unknown value**.

6. Write the abbreviations for the following computational instructions by using x for "the input number":
   (a) Halve the input number and plus 2.
   (b) Multiply the input number by 6 and subtract 2.
   (c) Multiply the sum of the input number and 3 by 10.
   (d) Subtract 4 from the input number and multiply the answer by 7.
7. Cardo's teacher writes on the board: "Add 2 and then multiply the answer by 3." The class must use 5 as an input number and apply the computational instruction.
   (a) Cardo uses 5 as the input number and writes: $(5 + 2) \times 3$.
   Paul says $(5 + 2) \times 3$ is 7×3 which is 21. Is Paul right?
   (b) Explain your answer in (a).
   (c) Represent this flow diagram as an algebraic expression:

   x → [+ 2] → [× 3] → $(x + 2) \times 3$

8. Express each computational instruction as a flow diagram and then write the abbreviation (algebraic expression) with x as input number:
   (a) Multiply by 4 and then subtract 8.

   x → [× 4] → [− 8] → $x \times 4 - 8$

   (b) Subtract 8 and then multiply by 4.

   x → [− 8] → [× 4] → $(x - 8) \times 4$ or $4 \times (x - 8)$

   (c) Add 15 and then divide by 5.

   x → [+ 15] → [÷ 5] → $(x + 15) \div 5$ or $\frac{x + 15}{5}$

   (d) Divide by 5 and then add 15.

   x → [÷ 5] → [+ 15] → $(x \div 5) + 15$ or $\frac{x}{5} + 15$

Answers

9. (a) Multiply by 4 and then add 7.
   (b) Add 7 and then multiply the answer by 4.
   (c) Multiply by 9 and then subtract 5.
   (d) Subtract 5 and then multiply the answer by 9.
10. See the first completed table on LB page 177 alongside.

13.2 Relationships represented in formulae

MAKING SENSE OF VARIABLES AND CONSTANTS IN FORMULAE

Teaching guidelines

Let learners test the two formulae:

$P = 2 \times l + 2 \times b$ and

$P = 2 \times (l + b)$

to show that they give the same result for the same values of *l* and *b*.

Show learners how to change the formula:

$P = 2 \times l + 2 \times b$ to

$P = 2 \times (l + b)$

by inserting the brackets.

Answers

1. (a) See the completed table on LB page 177 alongside.
   (b) $P = 2 \times (l + b)$ Testing for rectangle 1: $2 \times (24 + 1) = 2 \times 25 = 50$
   (c) There are two lengths (longer sides) and two breadths (shorter sides) in every rectangle.
   (d) It is called a constant.
   (e) The letter symbols *P*, *l* and *b*. All (perimeter, length and breadth) have changing values.
   (f) The area of all of the rectangles is 24, so the area stays constant.

9. Describe each computational instruction in words:

(a) → ×4 → +7 → (b) → +7 → ×4 →

(c) → ×9 → −5 → (d) → −5 → ×9 →

10. Two algebraic expressions are given in the table. Copy the table and use the given input values (*x* values) to determine the corresponding output values.

x	1	2	3	4	5	6
$6 \times x + 8$	14	20	26	32	38	44
$2 \times x \times (3 + 4)$	14	28	42	56	70	84

13.2 Relationships represented in formulae

MAKING SENSE OF VARIABLES AND CONSTANTS IN FORMULAE

1. (a) Chris uses the formula $P = 2 \times l + 2 \times b$ to calculate the perimeters of rectangles of differing lengths and breadths as indicated in the table. He also calculates the area of each rectangle using the formula $A = l \times b$. Copy and complete the table.

Rectangle	1	2	3	4
Length (*l*)	24	6	8	12
Breadth (*b*)	1	4	3	2
Perimeter $P = 2 \times l + 2 \times b$	50	20	22	28
Area $A = l \times b$	24	24	24	24

(b) Rita calculates the perimeter of a rectangle in a different way. She adds the value of the length of the rectangle to the value of the breadth of the rectangle and then multiplies the answer by 2. Write down the formula that Rita uses to calculate the perimeter of each rectangle. Test whether or not Rita's formula produces the same results as Chris's.

Questions 1(c) to (e) refer to the formula $P = 2 \times (l + b)$.

(c) What does the number 2 represent in the formula?
(d) What is the number 2 called?
(e) Which letter symbols represent variables in the formula $P = 2 \times (l + b)$? Explain.
(f) What can you say about the area of all of these rectangles?

Teaching guidelines

Remind learners that constants in a formula or expression do change their value. The variable quantities do change their value and are usually indicated by letter symbols.

Answers

2. Substitute Sindi's age then for x:

   $F = 43 + 37$

   $\therefore F = 80$

   Sindi's father was 80 years old when he died.

3. (a) See the completed table on LB page 178 alongside.

   (b) No, because by the eighth week he will have saved only $45 + 5 \times 8 = 45 + 40 =$ R85.

   (c) See the completed table on LB page 178 alongside.

4. See the completed table on LB page 178 alongside.

2. Sindi calculates her father's age by using the formula $F = x + 37$, where x is Sindi's age. Her father passed away when Sindi was 43 years old. How old was he then?

3. Jacob wants to buy the cheapest cell phone in the market. He has already saved R45 and decides to save R5 per week until he has enough money to buy the phone. The formula $y = 45 + 5 \times w$ gives the amount of money (in rands) that Jacob has saved to buy the cell phone after w weeks.

   (a) Copy and complete the table. The first row has been done as an example.

Number of weeks (w)	How to calculate $45 + 5 \times w$	Amount saved (y)
0	$45 + 5 \times 0 = 45 + 0$	45
1	$45 + 5 \times 1 = 45 + 5$	50
2	$45 + 5 \times 2 = 45 + 10$	55
4	$45 + 5 \times 4 = 45 + 20$	65
5	$45 + 5 \times 5 = 45 + 25$	70

   (b) The cell phone that Jacob wants to buy costs R90. Will Jacob have saved enough money to be able to buy the cell phone by the eighth week? Explain.

   (c) Copy and complete the table.

	Formula: $y = 45 + 5 \times w$	Explanation
Which are constants in the formula?	45 and 5	These numbers remain the same all the time.
Which letter symbols represent variable quantities in the formula?	y and w	The number of weeks that he has been saving money and the total of his savings after each week change all the time.

4. Copy the table in each of the formulae in the table, identify the symbols that represent variables and constants and fill them in.

		Symbols for variable(s)	Constant(s)
(a)	$y = 5 \times x + 7$	y and x	5 and 7
(b)	$y = 100 + x$	y and x	100
(c)	$y = x \div 5$	y and x	5
(d)	$y = 5 \times x$	y and x	5
(e)	$y = 0{,}7 \times x + 2{,}3$	y and x	0,7 and 2,3

Grade 7 Term 3 Chapter 14 Algebraic equations 1

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
14.1 Solving by inspection	Use number puzzles to introduce mathematical sentences (equations) where an unknown value, represented by a letter symbol, has to be found; solve equations by inspection	Pages 179 to 180
14.2 Solving by the trial and improvement method	The solving of equations using trial and improvement; then solve problems either by inspection or by trial and improvement	Pages 180 to 181
14.3 Describing problem situations with equations	Descriptions of situations in words have to be translated to mathematical equations that can be solved; equations that describe real-life situations are interpreted as to the meaning of the variables and constants used	Pages 181 to 182

CAPS time allocation	3 hours
CAPS content specification	Page 64

Mathematical background

In an equation, the equal sign expresses equality; expressions written on either side of the equal sign have the same value: $15 = 18 - x$ or $3 + x = 9$.

Solving an equation means working out the value of the unknown, for example *x,* that would make the equation true. There are different methods of solving an equation, depending how complex the equation is. We usually solve equations using **inverse operations**. In order to build the algebraic reasoning to do that, ask: "Which value(s) makes this equation true?"

Solving equations **by inspection** means we can answer this question immediately. For example, the solution to "what plus 7 is 12?" can be written as $x + 7 = 12$ and $x = 5$ because we can see at a glance that $5 + 7 = 12$.

Questions like $24 - x = 14$ are often done wrong when inverse operations are used, but if the question is asked "24 minus what is 14?", the answer by inspection is quite simple.

The numbers in these problems (solving by inspection) should be easy to work with as the computations here take second place to the insight into the algebraic structure of solving equations.

Solving equations using **trial and improvement** involves substituting values for the unknown until the value that makes the equation true is found.

14.1 Solving by inspection

NUMBER PUZZLES

Notes on the questions

A few word problems are used to introduce learners to equations and the need to solve an equation.

Teaching guidelines

You could start by writing a sentence like this on the board:

- A certain number plus 3 equals 13.

We can change the sentence into mathematical language as follows:

- A certain number + 3 = 13.

This mathematical statement is called an equation.

The phrase (words) "a certain number" is a placeholder for the number that will make the sentence true. It can be replaced by any letter symbol, for example x. The sentence then becomes $x + 3 = 13$.

Repeat the explanation by writing sentences like these on the board:

- A certain number multiplied by 6 equals 30 i.e. $x \times 6 = 30$.
- A certain number is multiplied by 5 and 2 is added to the result to give 17 i.e. $x \times 5 + 2 = 17$.

Learners choose variables to write the sentences.

Discuss the fact that only one value of x makes the left-hand side equal in value to the right-hand side. In that case, we say the number sentence is true. That value is called the solution to the equation.

Misconceptions

For many learners, from their experience with number sentences in the lower grades, the equal sign had the meaning "to do something", for example: 13 + 8 = ? Especially if they were never given examples like 21 – ? = 8. In order to have insight into solving equations, learners have to understand the meaning of the equal sign as relating two sides, "making them equal".

Answers

1. 10 2. 6

3. (a) No, because $3 \times 3 + 4 = 9 + 4 = 13$
   (b) No, because $3 \times 4 + 4 = 12 + 4 = 16$
   (c) Yes, because $3 \times 5 + 4 = 15 + 4 = 19$
   (d) No, because $3 \times 6 + 4 = 18 + 4 = 22$

CHAPTER 14
Algebraic equations 1

14.1 Solving by inspection

NUMBER PUZZLES

Solve these number puzzles.

1. I am thinking of a certain number. If I add 3 to that number, the answer is 13. What is the number?
2. I am thinking of a certain number. If I multiply that number by 5, the answer is 30. What is the number?
3. I am thinking of a certain number. If I multiply that number by 3 and then add 4 to the result, the answer is 19.
   (a) Is the number 3? Give a reason for your answer.
   (b) Is the number 4? Give a reason for your answer.
   (c) Is the number 5? Give a reason for your answer.
   (d) Is the number 6? Give a reason for your answer.

Number puzzles like those above can be shortened by using letter symbols as placeholders for unknown numbers. In the case of question 1 we can write the following number sentence: $\boldsymbol{x + 3 = 13}$.

In the case of a number sentence such as $x + 3 = 13$ we cannot say whether it is true or false until we have determined the value of the unknown. The value of the unknown that makes the number sentence (an **equation**) true is called the **solution** of the number sentence.

For the number sentence $x + 3 = 13$, the solution is $x = 10$ because it makes the number sentence true.

A mathematical statement such as $x + 3 = 13$ that could be true or false depending on the value of x, is called an **open number sentence** or an **equation**.

To make a number sentence **true** means to find its **solution**.

THE SOLUTION IS THERE TO SEE

The solution to the number sentence $x + 4 = 20$ can be seen at once. The value of x is 16 simply because $16 + 4 = 20$. In this case, we say we solve the number sentence **by inspection**.

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THE SOLUTION IS THERE TO SEE

Notes on the questions

If learners are only taught what steps to follow when solving an equation, it could lead to rote learning. Solving equations by inspection strengthens learners' understanding of what it means to solve an equation.

Teaching guidelines

Learners ask the question: "What must x be so that the left-hand side will have the same value as the right-hand side?"

Answers

1. (a) $x = 16$ (b) $x = 13$
   (c) $x = 2$ (d) $x = 32$
   (e) $x = 8$ (f) $x = 5$
2. (a) $x = 12$ (b) $x = 9$
   (c) $x = 44$ (d) $x = 44$

14.2 Solving by the trial and improvement method

Teaching guidelines

Let learners work through the example given on LB page 180 alongside. Discuss the way in which the first value for x, the unknown, was chosen.

Misconceptions

Learners are sometimes confused and call anything with variables equations when they are not. For example, you may see something like this referred to as an equation: $2x + 12$. Since it does not have an equal sign, it does not have two sides that are compared, therefore it is not an equation. This is called an expression.

Some learners report the value of the expression as the answer instead of the value of x; for example, a learner might say 43 is the answer instead of $x = 9$ is the answer to the example on LB page 180.

Answers

1. to 5. Let learners work in pairs or alone. They choose the values to start with and make a table as shown on LB page 180 alongside for each sum in their books.

The solutions are written next to each question on LB page 180 alongside and LB page 181 on the next page.

Solve these number sentences (equations) by inspection.

1. (a) $x - 8 = 8$ (b) $x + 7 = 20$
   (c) $\frac{16}{x} = 8$ (d) $\frac{x}{16} = 2$
   (e) $5 \times x = 40$ (f) $8 \times x = 40$
2. (a) $84 \div x = 7$ (b) $36 \div x = 4$
   (c) $x + 56 = 100$ (d) $100 - x = 56$

14.2 Solving by the trial and improvement method

Sometimes you cannot see the solution of a number sentence (an equation) at once. Look at the following number puzzle or equation, for example:

I am thinking of a number. 6 × *the number* – 11 = 43. What is the number?

In this case, you will have to try many different possible solutions until you identify the correct one. Here we can use a method known as **trial and improvement** to determine the solution. It is shown in the table below.

Possible solution	Test	Conclusion
Try 5	$6 \times 5 - 11 = 30 - 11 = 19$	5 is too small
Try 10	$6 \times 10 - 11 = 60 - 11 = 49$	10 is too big
Try 8	$6 \times 8 - 11 = 48 - 11 = 37$	8 is too small
Try 9	$6 \times 9 - 11 = 54 - 11 = 43$	9 is the solution

Copy the tables below. Solve the following equations by means of the trial and improvement method. In each case, the solution is a number between 1 and 20.

1. $2 \times x + 13 = 37$ The solution is $x = 12$

Possible solution	Test	Conclusion

2. $14 \times x - 21 = 77$ The solution is $x = 7$

Possible solution	Test	Conclusion

3. $7 \times x + 8 = 71$ The solution is $x = 9$

Possible solution	Test	Conclusion

SOLVING BY INSPECTION OR TRIAL AND IMPROVEMENT

Teaching guidelines

Learners could work on their own and then discuss their methods and answers in pairs or small groups.

Misconceptions

Learners find different values of x on either side of the equation. Stress the fact that in a specific equation, all the instances of the unknown with the same variable will have the same value.

Answers

1. (a) $x = 5$ (b) $k = 5$ (c) $q = 6$ (d) $t = 11$
2. (a) $y = 2$ (b) $p = 6$ (c) $z = 6$
   (d) $x = 4$ (e) $m = 7$ (f) $x = 4$

14.3 Describing problem situations with equations

WRITE NUMBER SENTENCES TO DESCRIBE PROBLEM SITUATIONS

Teaching guidelines

Discuss with learners what steps to take when they solve word problems. Steps like the following:

- Read the problem and think what it is you have to find.
- Write a variable to represent the quantity you have to find.

In the problems the first two steps have already been done.

- Write down what the variable represents.
- Write an equation using the quantities given in the problem.
- Solve the equation.
- Check your solution.

Answers

1. $30 - x = 19$ or $19 + x = 30$ or $30 - 19 = x$
2. $70 - m = 23$ or $m + 23 = 70$ or $70 - 23 = m$
3. $90 \times x = 1\ 260$ or $x = 1\ 260 \div 90$
4. $5 \times c = 240$ or $c = 240 \div 5$
5. $30 \times x + 100 = 400$ or $x = (400 - 100) \div 30$
6. $6 \times x = 54$ or $x = 54 \div 6$ or $54 \div x = 6$

4. $4 \times x + 7 = 31$ The solution is $x = 6$

Possible solution	Test	Conclusion

5. $10 \times x + 11 = 141$ The solution is $x = 13$

Possible solution	Test	Conclusion

SOLVING BY INSPECTION OR TRIAL AND IMPROVEMENT

Solve the following equations by inspection or by the trial and improvement method:

1. (a) $x + 5 = 2 \times x$ (b) $k \times 5 = 20 + k$
   (c) $2 \times q = 18 - q$ (d) $3 \times t = t + 22$
2. (a) $y + 6 = 4 \times y$ (b) $5 \times p = 18 + 2 \times p$
   (c) $4 \times z = 18 + z$ (d) $x \times 5 = 20$
   (e) $42 \div m = 35 - 29$ (f) $3 \times x - 2 = x + 6$

14.3 Describing problem situations with equations

WRITE NUMBER SENTENCES TO DESCRIBE PROBLEM SITUATIONS

Write an equation using a letter symbol as a placeholder for the unknown number to describe the problem in each of the situations below.

1. There are 30 learners in a class. x learners are absent and 19 are present.
2. There are 70 passengers on a bus. At a bus stop m passengers get off. There are now 23 passengers on the bus.
3. A boy buys a bicycle for R1 260 on lay-by. How many payments of R90 each must he make to pay for the bicycle? Let x be the number of payments to be made.
4. Five people share a total cost of R240 equally amongst themselves. Let c be the cost per person.
5. A school charges R100 a day for the use of its training facilities for athletes plus R30 per athlete per day for food and use of equipment. A team of athletes paid R400 for a day's practice. Let x be the number of athletes attending the training.
6. Bennie has R54 with which to buy chocolates for his friends. Each chocolate costs R6. How many chocolates can he buy for that amount? Let x be the number of chocolates that Bennie can buy.

7. $A = 2{,}5 \times 2$
8. $2 \times x + 6 = 38$
9. $x = 3 \times 12 + 7$ or $x = 7 + 3 \times 12$

MAKING SENSE OF EQUATIONS

The emphasis should be to analyse and interpret number sentences that describe a given situation.

Notes on the questions

Not only should learners be able to interpret words to make equations, but they should also be able to interpret equations that describe a situation, knowing what the variable represents and what the constants represent.

Teaching guidelines

These equations describe real-life situations. Discuss that the symbols are not critical, but what they represent is important. Any other symbols could have been used to represent the unknown values in the equations.

Answers

1. (a) 80 is the total amount charged for the trip.
   t is the number of kilometres travelled.
   10 is the cost per kilometre.
   30 is the standard charge.
   (b) Because each kilometre travelled costs R10.
2. (a) The cost of a child's ticket.
   (b) Because the cost of an adult's ticket is four times that of a child's ticket.
   (c) $4 \times x = 240$; $4 \times 60 = 240 \div 4$ so $x = 60$
   (d) R60
3. (a) The number of cartons.
   (b) $c = 6$
   (c) The total number of eggs in c cartons.

7. Write an equation to calculate the area of a rectangle with length 2,5 cm and breadth 2 cm. Let A represent the area of the rectangle.
8. There are 38 girls in Grade 7. This is 6 more than double the number of boys.
9. Janine is 12 years old. Her father's age is 7 years plus three times Janine's age.

MAKING SENSE OF EQUATIONS

1. Rajbansi Taxi Service charges R10 per kilometre travelled and a standard charge of R30 per trip. Consider the equation below about a taxi trip:
   $10 \times t + 30 = 80$
   (a) Explain what each number and letter symbol stands for in the equation.
   (b) Why is t multiplied by 10 in the equation?
2. The cost of an adult's ticket for a music concert is four times the cost of a child's ticket. An adult's ticket costs R240. The equation below represents this problem:
   $4 \times x = 240$
   (a) What does x represent?
   (b) Why is x multiplied by 4?
   (c) Solve the equation by inspection.
   (d) How much does a child's ticket cost?
3. There are 12 eggs in a carton. Consider the equation below:
   $12 \times c = 72$
   (a) What does the letter symbol c represent in the equation?
   (b) What value of c makes the equation true?
   (c) What does the number 72 represent?

Grade 7 Term 3 Chapter 15 Graphs

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
15.1 A graph can tell a story	Describing a situation that involves a relationship mathematically using visual means	Page 183
15.2 Investigating rate of change in situations	Representing situations in tables, graphs and verbal descriptions; understanding rate of change	Pages 184 to 186
15.3 Interpreting graphs	Understanding how to read information from a graph; relating the way the output numbers change to the rate of change; relating the rate of change to the steepness of the graph and to the shape and direction of the graph	Pages 187 to 192
15.4 Drawing graphs	Interpreting information and representing it in a graph	Pages 192 to 193

CAPS time allocation	6 hours
CAPS content specification	Page 65

Mathematical background

A graph shows information about a situation and can be used to reveal trends.

Graphs are used by businesses, governments, organisations, and all kinds of people in many different contexts to represent information and study trends. We therefore need to be able to interpret graphs so that we can make sense of the information offered to us.

In this unit, the focus is on being able to discover the trend that a graph shows and understand the "story" it tells rather than the basics, such as using formulae to generate coordinates, plotting points, etc. We also focus on identifying the variables in the "story" that a graph represents and how these variables relate to each other at different points, and how that relationship changes if they change.

The direction of the graph is linked to the increase or decrease of the dependent values. The increase or decrease is determined by the rate of change of the output values in the relationship.

Working with the rate of change in this context prepares learners to have a better understanding in later, more formal work, of what a gradient means.

Interpreting graphs involves making up a story that contains the facts shown by the graph.

Drawing graphs involves another way of thinking. The story is known and has to be represented in a graph while making sure that all the elements of the story are represented and then checking that the graph actually tells the story that was intended from the start.

15.1 A graph can tell a story

Teaching guidelines

We can represent a relationship between two variables graphically, for example the time of day and the feeling of hunger. Although, strictly speaking, we cannot really describe feeling hungry in terms of numbers (in other words, we cannot quantify it), time and feeling hungry can be seen as variables as the feelings of hunger depend on the time of the day.

We label the axes to represent the variables. The independent variable is always shown on the horizontal axis.

The purpose of this discussion is to show how to interpret a graph and how to use a graph to describe a situation or to tell a story.

Answers

1. Possible answers:

   At the start of the day Jena was fairly hungry. She ate something (probably at breakfast) and her feelings of hunger decreased sharply. After a while she started feeling hungry again. She felt hungrier than at the start of the day. She had something to eat (probably at break time) after which her feelings of hunger decreased rapidly. From feeling very little hunger after break time, her hunger grew so that at lunch time she was very hungry. Once she had eaten, her feelings of hunger dropped to not feeling hungry. Throughout the afternoon her hunger grew so that at supper time she was feeling moderately hungry, ate something and her hunger decreased a little.

   OR:

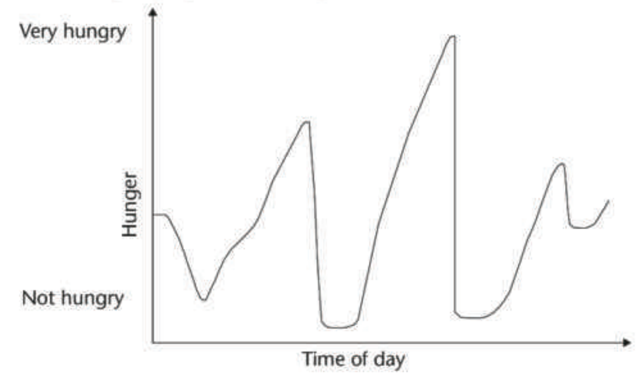
   She had breakfast, lunch, dinner and a snack before bed time (hunger dropped). Her hunger increased after each meal, reaching a maximum before dinner.

2. Learners draw their own graph.

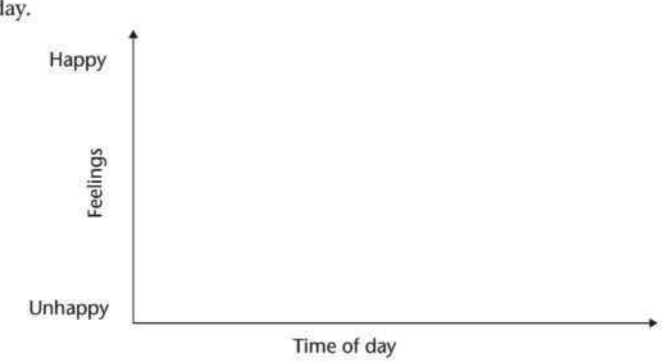
Chapter 15
Graphs

15.1 A graph can tell a story

1. Jena drew this graph to show how her feelings of hunger changed during the day. Describe in a short paragraph how her day went as far as need for food is concerned.



2. Think about a specific day and things that happened to you on that day. Use the example below to draw your own graph to show how your feelings changed during that day.



CHAPTER 15: GRAPHS 183

15.2 Investigating rate of change situations

COMPARE SITUATIONS AND REPRESENT THEM IN A DIFFERENT WAY

Teaching guidelines

It is not the intention that learners should find the rules for these relationships; but there may be learners to whom it is a challenge to find the rule.

The first situation shows a relationship where the output numbers steadily increase each week. Learners could have predicted this as the output numbers grow by a fixed amount from week to week. Learners may see that the rule in this case is: multiply the number of weeks by 4, or if the number of weeks is represented by x, the rule is $4x$.

In the second situation, the output is determined by dividing the fixed number of chocolates by the input numbers that increase. This leads to decreasing output numbers that give the shape of the graph. As the number of friends increase, the number of chocolates per friend decreases. The rule in this case is: divide 24 by the number of friends. If the number of friends is represented by x, the rule is $24 \div x$.

Notes on the questions

The fact that learners represent the output numbers using bars should make it easier for them to relate the shape of the graph to the relationship: linear increase in the first case and non-linear decrease in the second case.

Answers

1. (a) The completed table is shown on LB page 184 alongside.
   (b) The completed table is shown on LB page 184 alongside.
   (c) The completed graphs are shown next to each other on LB page 184 alongside.

15.2 Investigating rate of change in situations

COMPARE SITUATIONS AND REPRESENT THEM IN A DIFFERENT WAY

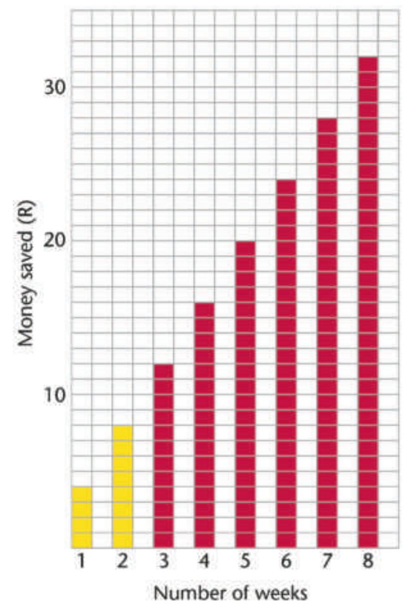
1. Consider the situations in (a) and (b) below and copy and complete the tables to represent the relationships.
   (a) Sally is saving money to buy a CD that she really wants. She saves R4 per week.

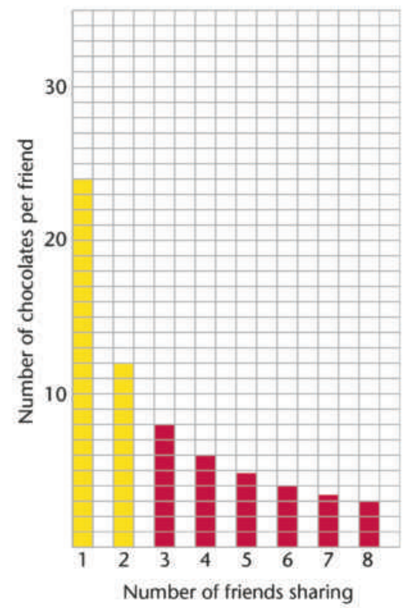
Number of weeks	1	2	3	4	5	6	7	8
Money saved in rand	4	8	12	16	20	24	28	32

   (b) Nathi has a box of 24 chocolates. He is thinking about sharing the chocolates equally between different numbers of friends and is working out how many chocolates each friend would get.

Number of friends	1	2	3	4	5	6	7	8
Chocolates per friend	24	12	8	6	4,8	4	3,4	3

   (c) Copy the grids below, and draw bar graphs to represent the relationships in the situations described in (a) and (b). The length of each bar should represent an output number.





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Teaching guidelines

The situation in 2(a) shows a relationship where the output numbers double with each new bag added. This relationship leads to increasing output numbers; each successive number is double the previous number. We would expect the bars to increase in length rapidly. (For the teacher: this is an exponential pattern with rule 2^{n-1} if the number of bags is given by n.)

In the second situation, the output numbers (area of the square) are determined by squaring the side length of the square. The output numbers that give the shape of the graph are therefore the squares of the input numbers. The rule in this case is: if the side length is represented by x, the rule to get the output numbers is x^2.

Answers

2. (a) The completed table is shown on LB page 185 alongside.
   (b) The completed table is shown on LB page 185 alongside.
   (c) The completed graphs are shown next to each other on LB page 185 alongside.

2. Consider the situations in (a) and (b) below and copy and complete the tables to represent the relationships.

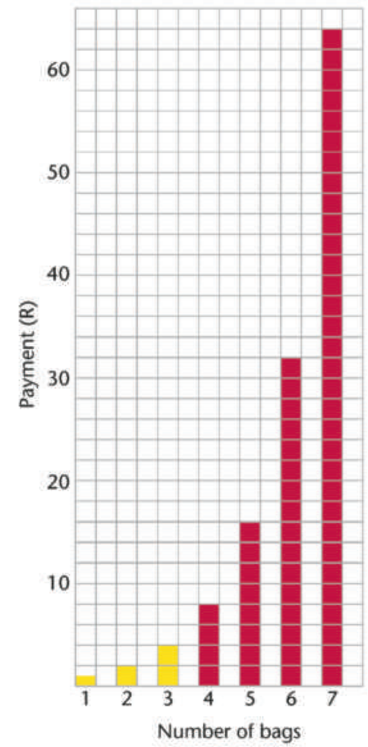
(a) Vusi collects acorns for a pig farmer who pays him per bag. Vusi thinks: "I wish Mr Bengu would agree to pay me R1 for one bag of acorns, R2 for two bags, R4 for three bags, and then keep doubling the money when another bag is added."

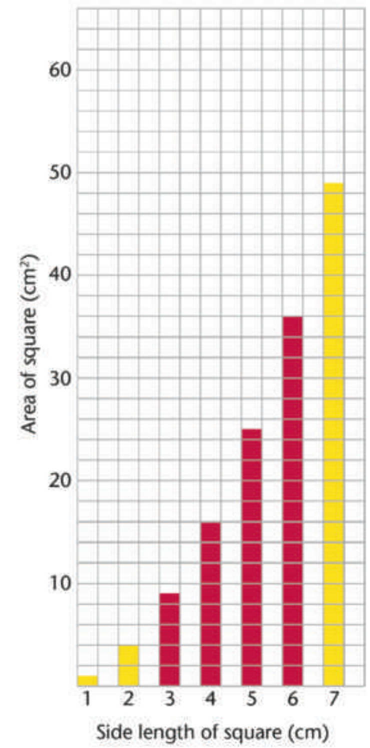
Number of bags	1	2	3	4	5	6	7
Payment (R)	1	2	4	8	16	32	64

(b) Judy works out the areas of squares with different side lengths.

Side length of square (cm)	1	2	3	4	5	6	7
Area of square (cm^2)	1	4	9	16	25	36	49

(c) Copy the grids below, and draw bar graphs to represent the relationships in the situations described in (a) and (b). The length of each bar should represent an output number.





Teaching guidelines

Let learners work in pairs or small groups and discuss the changes in the output numbers using their tables and graphs. See the discussion above for comments on questions 3 and 4.

Guide them to find descriptions for the shapes of the bar graphs, working through the answers with them.

Work through the concept of the rate of change with learners, using the situations in questions 1 and 2 to explain the different possibilities that it can have. Work through the discussion about rate of change on LB page 186 alongside.

Answers

3. 1(a) They increase steadily/by the same amount every month (constant rate).
   1(b) There is a rapid decrease and then the decrease becomes slower and slower.
   2(a) They increase slowly in the beginning and then faster and faster.
   2(b) They increase slowly in the beginning and then faster and faster.
4. 1(a) is a straight line, increasing.
   1(b) is a curve that is steep at first and then becomes flatter and flatter.
   2(a) is a curve that increases quite steeply in the beginning and then becomes even steeper.
   2(b) is a curve that slopes upwards slowly and then the slope becomes steeper.
5. See the answers on LB page 186 alongside, as well as below.
   (a) Sally's savings grow by **a constant amount** every week.
   (b) The number of chocolates per friend **decreases rapidly and then the decrease becomes slower**.
   (c) The amount grows slowly at first and then **increases faster and faster**.
   (d) The areas of the squares **increase faster and faster**.
6. If the rate of change is constant, the tops of the bars in the graph form a straight line.
   If the rate of change is changing, the shape of the graph is curved. If it increases, the graph slants upwards and if it decreases, the graph slants downwards.

3. The input numbers for the different relationships in questions 1 and 2 are the same, but the output numbers differ. Describe how the output numbers change in each of the four situations.
4. Describe briefly how the shape of the bar graphs differ.
5. Turn back to the tables of values that you made for the four relationships in questions 1 and 2. Find out how the output values changed by calculating the differences between consecutive output values, and describe the differences for each case as suggested:
   (a) Sally's savings: 4 8 12 16 20 24 28 32
   Differences: 4 4 4 4 4 4 4
   Sally's savings grow by ....a constant amount.... every week.
   (b) Chocolates per friend: 24 12 8 6 4,8 4 3,4 3
   Differences: 12 4 2 1,2 0,8 0,6 0,4
   The number of chocolates per friend .....
   (c) Vusi's payment per bag: 1 2 4 8 16 32 64
   Differences: 1 2 4 8 16 32
   The amount grows slowly at first and then .....
   (d) Areas of Judy's squares: 1 4 9 16 25 36 49
   Differences: 3 5 7 9 11 13
   The area of the squares .....

The amount of money that Sally saves per week stays **constant**. The relationship therefore has a **constant rate of change**.

With every additional friend, the number of chocolates per friend changes. The number of chocolates changes (decreases) **rapidly at first and then it changes slower**. The **rate of change is not constant**.

The amount that Vusi would like to earn per bag of acorns grows faster and faster with each bag that he collects. The **rate of change increases**.

The area of a square also **increases faster and faster** for every centimetre added to the side length.

The **rate of change** means how fast or slow change happens per unit of time.

The **shape** of a bar graph shows the **rate of change** of the relationship. If the rate of change is constant, the shape is a **straight line**. If it is changing, the shape is a **curve**.

6. Refer to the bar graphs in questions 1 and 2 and link the shape of the graphs to the rate of change of the relationship.

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15.3 Interpreting graphs

READING GRAPHS – TO ANALYSE AND INTERPRET GLOBAL GRAPHS

Teaching guidelines

Extracting information from graphs is an important skill. Let learners work in pairs or small groups and discuss each situation. The first graph gives them an opportunity to read the information on the two axes, interpret the shape of the graph and make deductions.

The second situation requires learners to interpret the information and choose the correct graphical representation. Discuss the different trends of growth of plants A, B and C, and how to relate these trends to the information learners have about the plants, so that they can match the plant name with a trend.

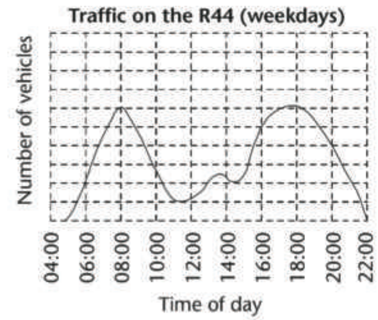
Answers

1. (a) There are two peak periods when traffic is heavy: early morning and late afternoon/evening. The number of vehicles increases slightly over the lunch hour.
   (b) The graph has a maximum at about 08:00 and at about 18:00. There is a small maximum during the lunch hour.
2. (a) It means it grows by the same amount per unit of time, for example per week.
   (b) Plant A: 0,3; 0,1; 0,8; 0,1; 0,1; 1,7.
   The amount by which the plant grows varies from week to week.
   Plant B: 1,9; 2,0; 1,9; 2,1; 2,1; 1,9.
   The plant grows by about 2 cm each week.
   Plant C: 0,5; 0,6; 0,7; 0,9; 1,1; 1,9.
   The plant grows slowly at first, then faster and faster.
   (c) Plant B = Glamiolus. The salesman told Mr Thatcher that Glamiolus grows at a constant rate.
   Plant C = Bouncy Bess. The salesman said Bouncy Bess grows slowly at first, then faster and faster.
   Therefore, Plant A = Samara.
   (d) A: rate of change is constant, i.e. Glamiolus
   B: rate of change varies, i.e. Samara
   C: rate of change increases (becomes faster and faster), i.e. Bouncy Bess

15.3 Interpreting graphs

READING GRAPHS

1. Look carefully at the graph.
   (a) What does this graph tell you?
   (b) Explain your answer in (a).



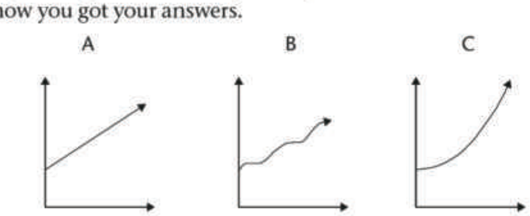
2. Mr Thatcher bought three plants in containers. The salesman at the nursery told him that one of the plants, Glamiolus, grows at a constant rate. The second plant, Bouncy Bess, grows slowly at first but then grows faster and faster. The salesman was not sure about the rate at which the third plant, Samara, grows.
   (a) What does "grows at a constant rate" mean?

   Mr Thatcher measured the three plants every week and recorded the heights in a table, given below.

   (b) Calculate the differences in height from week to week, to find the rate at which each plant grows per week.

Week	Height of A (cm)	Height of B (cm)	Height of C (cm)
1	6	8,3	10,1
2	6,3	10,2	10,6
3	6,4	12,2	11,2
4	7,2	14,1	11,9
5	7,3	16,2	12,8
6	7,4	18,3	13,9
7	9,1	20,2	15,8

   (c) Identify the plants. Which plant is plant A, which plant is plant B and which plant is plant C? Explain how you got your answers.
   (d) The three graphs on the right show the growth of the three plants. Which graph belongs to which plant? Explain.



CHANGE AND RATE OF CHANGE

Teaching guidelines

As learners work through the questions, make sure that they discover and understand the facts about the rate of change of a relationship.

The rate of change of the output values in a relationship influences the appearance of the graph:

- the higher the rate of change, the faster the output numbers change and the steeper the graph
- if the rate of change is constant, the graph is a line (linear)
- if the rate of change is not constant, the graph is non-linear and is a curve
- if the rate of change is increasing, constantly or not, the graph slopes upward from left to right
- if the rate of change is decreasing, constantly or not, the graph slopes downward from left to right.

Show learners an example of graphs where the rate of change is 0, in other words, the relationship is constant and the graph is a horizontal straight line.

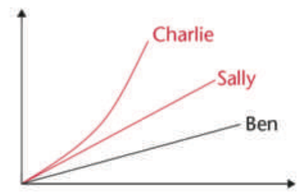
Answers

1. (a) The graphs have been drawn on LB page 188 alongside.
   (b) Sally's graph is steeper than Ben's graph. Charlie's savings will grow faster than Ben's and Sally's savings.
2. (a) See the completed table on LB page 188 alongside.
   (b) See the completed table on LB page 188 alongside.

CHANGE AND RATE OF CHANGE

In section 5.2 you compared the way in which relationships changed.

1. Consider the following situations:
   Ben saves R5 per week. Sally saves R7 per week.
   Charlie saves R5 in the first week, R6 in the second week and R7 in the third week. Every week he increases the amount that he saves by R1.
   (a) The graph shows Ben's savings. Copy the graph and draw Sally's savings and Charlie's savings. Do it on the same graph.
   (b) Describe and explain the shape of the graphs showing Sally, Ben and Charlie's savings.



The rate of change in a relationship influences the **steepness** of the graph. The higher the rate of change (i.e. the faster the output numbers change), the steeper the graph.

2. Examine the following relationships. Copy and complete the tables and calculate the differences between the output numbers indicated with arrows.
   (a) Christine wants to buy a book for her favourite teacher. The book costs R240. This is a lot of money for Christine to spend. She realises that if she asks her friend Beatrice to share the cost, she will only have to spend R120. She could even ask more classmates to join in and share the cost. Christine investigates the situation and calculates what amount everyone must pay if they share the cost equally.

Number of learners sharing the cost	1	2	3	4	6	8	10	12
Amount each learner will pay	240	120	80	60	40	30	24	20

120 40 20

   (b) Investigate the relationship between the length of a side of a square and the perimeter of the square.

Length of a side of the square	1	2	3	4	6	7	8	9
Perimeter of the square	4	8	12	16	24	28	32	36

4 4 4 4 4 4

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Teaching guidelines

Discuss with learners how to relate the change in the output numbers to the rate of change and from there to choose a graph that fits the situation.

Learners should be clear about the difference between linear increasing or linear decreasing as constant rates of change, and graphs that do not show a constant rate of change, but that are decreasing faster and faster or increasing faster and faster.

Answers

2. (c) See the completed table on LB page 189 alongside.
   (d) See the completed table on LB page 189 alongside.
3. (a) B
   (b) C
   (c) D
   (d) A

(c) Investigate the relationship between the length of a side of a square and the area of the square.

Length of a side of the square	1	2	3	4	6	7	8	9
Area of the square	1	4	9	16	36	49	64	81

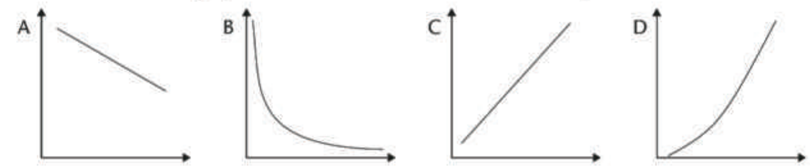
3 5 7 13 15 17

(d) A tall candle was lit and its length was measured and recorded every hour while it was burning.

Number of hours the candle is burning	1	2	3	4	6	7	8	9
Length of candle in centimetres	33	31	29	27	23	21	19	17

2 2 2 2 2 2

3. Match each of the graphs below to one of the situations in question 2.



Write the letter of the graph next to the description of the situation:

(a) Buying a book for the teacher
(b) Length of a side of a square and the perimeter of the square
(c) Length of a side of a square and the area of the square
(d) Length of the candle and the number of hours it is burning

When we investigate the growth (or change) in a relationship, we look at the way the output numbers change.

The change can be:

- an increase or a decrease
- a constant increase, for example the perimeter of a square as the side length increases
- a constant decrease, for example the length of a burning candle
- an increase that is not constant but happens faster and faster, for example the area of a square as the side length increases
- a decrease that is not constant but happens faster and faster, for example the amount of money each friend has to pay as more and more friends share the cost.

Teaching guidelines

Make sure that learners understand that when the output values increase, graphs slope upward, and when output values decrease, the graphs slope downward.

You could make various drawings on the board of linear and non-linear graphs, that increase and decrease, as well as graphs that show no increase or decrease (horizontal lines) and let learners classify them.

Answers

4. (a) Graph A shows a linear decrease. Graph C shows a linear increase.
   (b) Graph B shows a non-linear decrease. Graph D shows a non-linear increase.
5. They left home and their speed increased to 60 km/h. They travelled at a constant speed and then slowed down (perhaps at a corner). They picked up speed again up to 60 km/h and continued driving at that speed for a while. They then slowed down (perhaps at another corner); then the speed increased again. They drove at a constant speed of 60 km/h for a while until they slowed down and the car came to a standstill at the school.
6. See the completed questions on LB page 190 alongside.

Misconceptions

Learners wrongly see the graph as a picture of the physical event instead of recognising that it represents the data. For example, in question 5, learners can confuse a sloped line showing how the speed changes with the actual movement, thinking the car moved uphill, then level, downhill, then uphill again, etc. This sort of thinking shows that they do not understand the connection between the graph and the information it represents.

In the case of an increase, the graph slopes like this: 

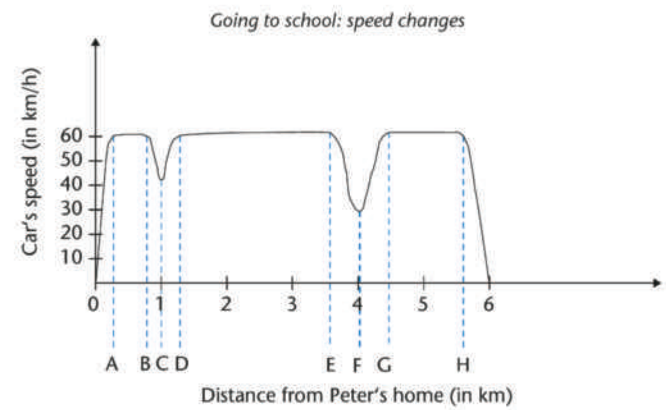
In the case of a decrease, the graph slopes like this: 

When the increase or decrease is constant, the graph is a straight line and it is called a **linear graph**.

If the increase or decrease is not constant, the graph is curved and is called a **non-linear graph**.

If there is no change in the output variable, the graph is a straight horizontal line.

4. Consider the graphs in question 3 on the previous page.
   (a) Which graphs indicate a linear increase or decrease?
   (b) Which graphs indicate a decrease or increase which is not constant?
5. Peter's father drives him to school in the mornings. Below is a graph of their journey to school. Describe the story that the graph tells. What do you know about the route that they are taking?



6. Consider the graph in question 5 above. Identify and indicate for the different parts of the graph listed below whether they are increasing, decreasing or constant.

(a) 0 to A increasing	(b) A to B constant	(c) B to C decreasing
(d) C to D increasing	(e) D to E constant	(f) E to F decreasing
(g) F to G increasing	(h) G to H constant	(i) H to 6 decreasing

EXPLORING MORE GRAPHS

Teaching guidelines

Explain that the independent variable is always shown on the horizontal axis. Discuss why time is the independent variable; the amount of water that runs into the bath has no influence on the time passed. In question 2, the distance from home also has no influence on the time passed.

Misconceptions

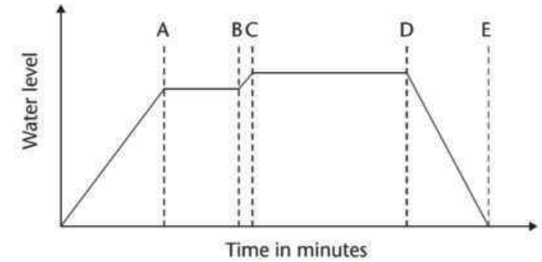
Learners wrongly see the graph as a picture of the data instead of recognising that it represents the data. For example, in question 2, learners can think the graph shows the actual movement, thinking the movement was uphill, then level and then downhill again, etc. This shows that they do not understand the connection between the graph and the information it represents.

Answers

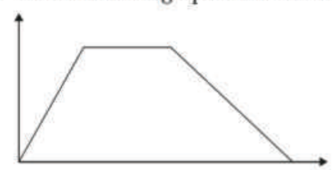
1. Water flows into the bathtub at a constant rate up to A. At point A, the tap is closed and remains closed. At point B, Janet gets into the bathtub, so the water level rises to point C. The water level stays the same up to point D. Janet then pulls the plug and the water runs out of the bathtub. At point E, the bathtub is empty (and Janet climbs out of the bath).
2. (a) The set of labels in B could fit the graph.
   (b) A person walks (or travels) at a constant speed away from home. The person stays at one place for a period of time and then starts walking back home – again at a constant speed, but slower than in the first part (the slope is less steep).
3. (a) A started at a good speed but slowed down as she neared the finish line. B was the slowest at the start, but picked up speed to pass A close to the finish line.
   C had a good start but fell and was at the same spot for a while, then ran the rest of the race.

EXPLORING MORE GRAPHS

1. Janet takes a bath. The graph below shows the height of the water level in the bathtub as time passes. The water runs into the bath at a constant rate. Study the graph and describe what happens.



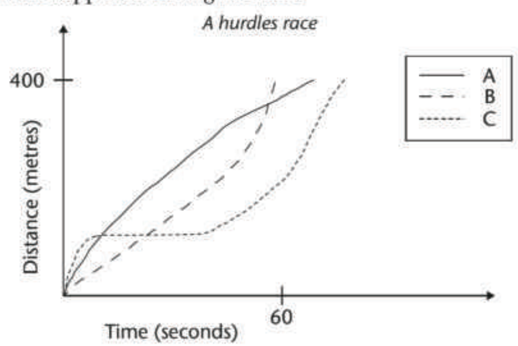
2. The axes of the graph below are not labelled.



The **vertical axis** is the one that goes from bottom to top. The **horizontal axis** is the one that goes from left to right. (**Axes** is the plural of **axis**.)

(a) Which of the following sets of labels could fit the graph?
   A: **vertical axis:** time passed; **horizontal axis:** distance from home
   B: **vertical axis:** distance from home; **horizontal axis:** time passed
   C: **vertical axis:** rainfall; **horizontal axis:** temperature
(b) Describe the story told by the graph, with the axes that you chose.

3. The graph that follows shows the distance that three athletes, A, B and C, covered in a hurdles race in a certain time.
   (a) Describe what happened during the race.



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Answers

3. (b) 400 m
   (c) B
   (d) Not necessarily. B won, but was the slowest over the first part of the race. C had a very good start and was ahead when she fell. She might have won the race if she had not fallen.
4. All the parts of the graphs in questions 1 and 2 are linear and all of the parts of the graphs in question 3 are non-linear.

15.4 Drawing graphs

Teaching guidelines

You could ask learners how high they think the water level would be in each of the containers after the same amount of time has elapsed (for example, five minutes). Then ask them what they think the water level would be in each container after a further ten minutes. Questions like these might help learners who struggle to visualise the shapes of the graphs.

Misconceptions

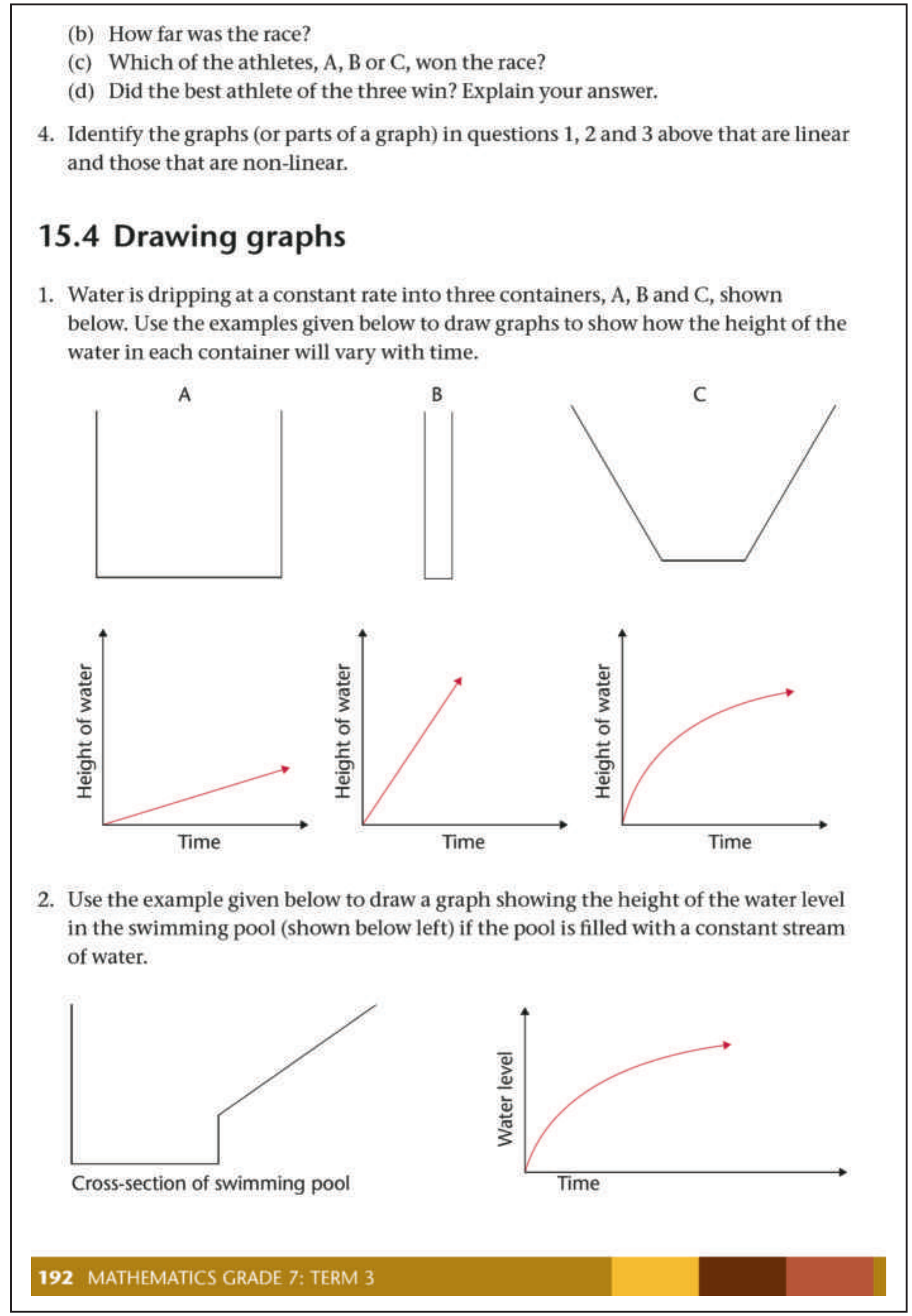
Learners may want to draw horizontal lines to represent the height of the water in the containers.

Answers

1. The graphs have been drawn on LB page 192 alongside.
2. See graph drawn on LB page 192 alongside.
   Note: The water level will increase at a constant rate for the first part, when the walls of the pool are going "straight upwards". The first part of the graph will therefore initially be a straight line.

   One wall of the pool then changes shape (not vertical any more) and the water level will increase (rise) slower and slower because the pool becomes wider and wider.

   Compare this to question 1, example C.



(b) How far was the race?
(c) Which of the athletes, A, B or C, won the race?
(d) Did the best athlete of the three win? Explain your answer.

4. Identify the graphs (or parts of a graph) in questions 1, 2 and 3 above that are linear and those that are non-linear.

15.4 Drawing graphs

1. Water is dripping at a constant rate into three containers, A, B and C, shown below. Use the examples given below to draw graphs to show how the height of the water in each container will vary with time.

2. Use the example given below to draw a graph showing the height of the water level in the swimming pool (shown below left) if the pool is filled with a constant stream of water.

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Teaching guidelines

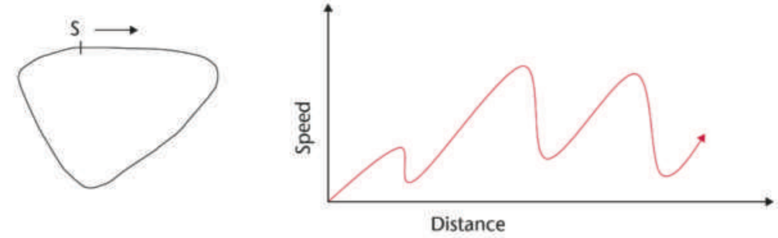
Let learners work in pairs and discuss the speed of the car in question 3. The car should go slower when going around the bends. Draw an answer on the board and ask questions about the speed at the start and the general shape of the graph, for example: Why is the first peak in the speed not so high and why is there a dip in the speed?

Let them work on question 5 on their own and share their answers with a partner to check correctness.

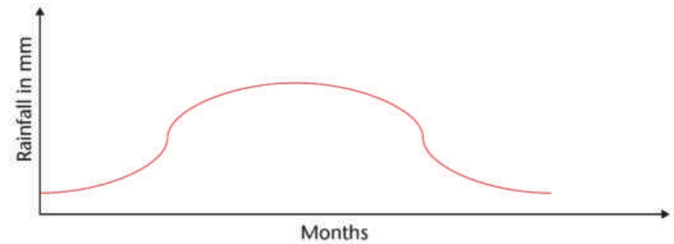
Answers

3. The graph is shown on LB page 193 alongside.
4. The graph is shown on LB page 193 alongside.
5. The graph is shown on LB page 193 alongside.

3. Use the example given below to draw a graph of the speed of a racing car as it travels once around the track shown below. S is the starting point.



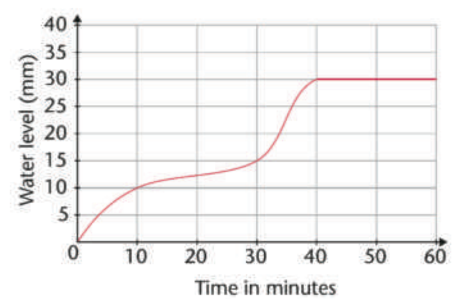
4. The Western Cape gets rain during the winter months, but in summer it is usually dry. Using the example below, draw a global graph of the average rainfall in the Western Cape during one year.



5. Use the example given below to draw a graph of the following story:

During a rainstorm, Lydia put a measuring cup outside to measure the rainfall. After 10 minutes of hard rain the water level was 10 mm. It started to rain softer, and after 20 more minutes the water level was 15 mm.

When Lydia went back 10 minutes later, the level was 30 mm. An hour after the storm started, the water level was still 30 mm.



CHAPTER 15: GRAPHS 193

Grade 7 Term 3 Chapter 16 Transformation geometry

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
16.1 Lines of symmetry	Introduce lines of symmetry and congruent figures	Pages 194 to 196
16.2 Original figures and their images	Introduce images of figures, translation, reflection and rotation	Page 197
16.3 Translating figures	Investigate properties of translation; practise translating figures	Pages 197 to 200
16.4 Reflecting figures	Investigate properties of reflection; practise reflecting figures	Pages 200 to 204
16.5 Rotating figures	Investigate properties of rotation; practise rotating figures	Pages 205 to 208
16.6 Enlarging and reducing figures	Investigate properties of enlargements and reductions; practise resizing figures	Pages 209 to 212

CAPS time allocation	5 hours
CAPS content specification	Pages 78 to 79

Mathematical background

Transformation geometry deals with the operations that may be used on a figure to affect its position, size or shape, or any combination thereof.

- The **figure** (also referred to as the **object**) is the original shape before transformation is applied.
- The **image** is the shape which appears after transformation has been applied to the figure.

Transformations that affect the **position** of a figure are translations, reflections and rotations:

- During a **translation**, every point in the figure is shifted in the **same direction** and over the **same distance**.
- During a **reflection**, every point in the figure is flipped **perpendicularly** over a **line of symmetry** (mirror line) so that the point and its image are the **same distance** from the line of symmetry.
- During a **rotation**, every point in the figure is turned **clockwise** or **anti-clockwise** through the same angle about a fixed point, the **centre of rotation**, so that the point and its image are the **same distance** from the centre of rotation.

Transformations that affect the **size** of a figure are enlargements and reductions:

- During an **enlargement**, every side of a figure is multiplied by a positive number bigger that 1 to produce an image larger than the figure.
- During a **reduction**, every side of a figure is multiplied by a positive number smaller than 1 (but not 0) to produce an image smaller than the figure.

Transformations that affect the **shape** of a figure are shears and stretches:

- During a **shear**, all points along a fixed line remain fixed while all other points are shifted parallel to the fixed line (turning a square into a rhombus).
- During a **stretch**, all points along a fixed line remain fixed while all other points are stretched away from the fixed line (turning a square into a rectangle).

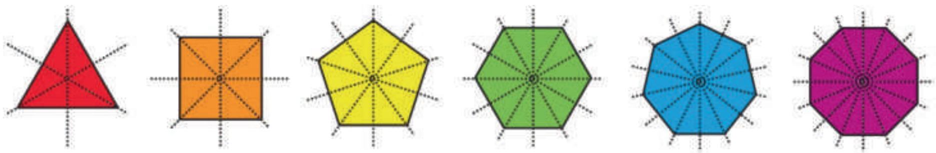
16.1 Lines of symmetry

WHAT IS THE LINE OF SYMMETRY?

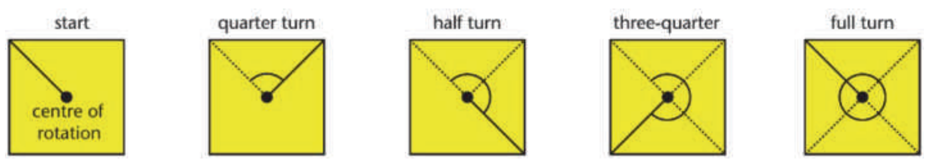
Background information

The **symmetry of a figure** describes how parts of that figure can, under certain rules of movement, fit exactly on to other parts of the same figure. Figures can have three **types of symmetry**:

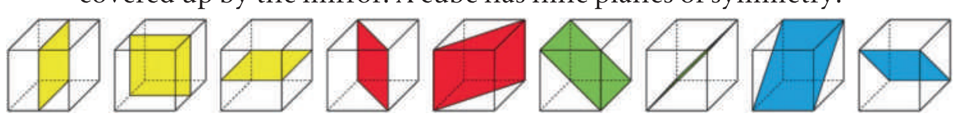
- A figure has **line symmetry** if it can be folded along a line running through the figure so that one half of the figure fits exactly on the other half. The fold line is called the **line of symmetry** or **axis of symmetry**. Figures such as the following have more than one line of symmetry.



- A figure has **rotational symmetry** if it can be turned around a point and fitted on to itself somewhere other than in its original position. The point about which the figure has rotational symmetry is called the **centre of symmetry**. During one revolution, a square fits four times on to itself.



- A solid has **plane symmetry** if a flat mirror can be placed so that the reflection in the mirror looks exactly like the part of the solid which is covered up by the mirror. A cube has nine planes of symmetry.



Teaching guidelines

Use paper folding to find the lines of symmetry of regular and irregular figures.

Answers

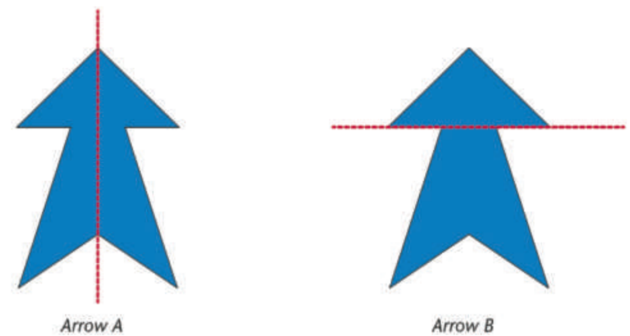
In the arrow diagrams, A has a line of symmetry.

CHAPTER 16
Transformation geometry

16.1 Lines of symmetry

WHAT IS THE LINE OF SYMMETRY?

In the diagrams below, the red dotted lines divide the arrows into two parts. In which diagram does the red dotted line divide the arrow into two parts that are exactly the same?



If you were to cut out arrow A and fold it along the red dotted line, the two parts would fit perfectly on top of one another (all edges would match). The fold line is called a **line of symmetry** or an **axis of symmetry**.

A line or axis of symmetry is a line that divides a figure into two parts that have an equal number of sides, and all the corresponding sides and angles are equal. The two parts on either side of the line of symmetry are mirror images of each other. We also say the parts are **congruent**.

A geometric figure can have no line of symmetry, one line of symmetry, or more than one line of symmetry.

Congruent figures are figures that are the same size and shape. All the sides and angles of the figures match.

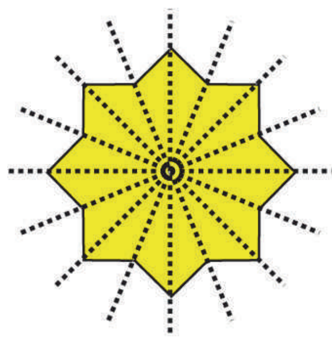
Teaching guidelines (continued)
Discuss the concept of congruent figures.

Misconceptions
Some learners may think that a line of symmetry of a single figure can be positioned alongside the figure. This is NEVER possible.

IDENTIFYING LINES OF SYMMETRY

Teaching guidelines
Remind learners that any line of symmetry has to run through the middle of the figure in order to divide it into two congruent halves.

Note on question 1(d)
If the figure is changed to the one below, it will have eight lines of symmetry.

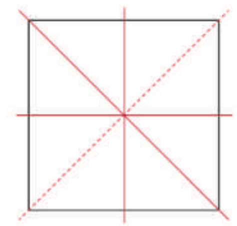


Answers
1. See LB page 195 alongside.

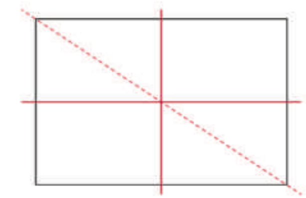
IDENTIFYING LINES OF SYMMETRY

1. (a) Copy each of the figures shown below. Make a tick next to each figure in which the red line is a line of symmetry.
   (b) In the figures where the red line is not a line of symmetry, draw in a line of symmetry if this is possible. If there is more than one line of symmetry, draw it in too. If a figure doesn't have any lines of symmetry, write this above the figure.

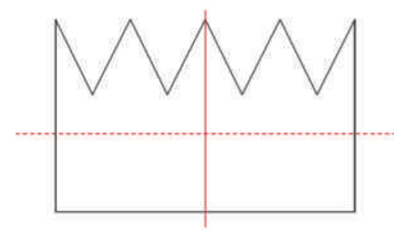
A Four possibilities ✓



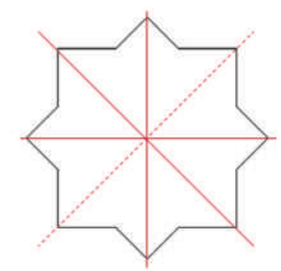
B Two possibilities



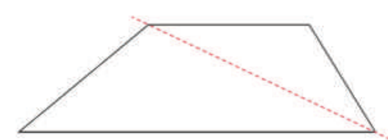
C One line of symmetry



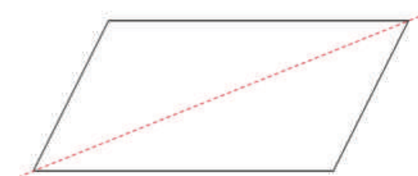
D Four possibilities ✓



E No line of symmetry



F No line of symmetry



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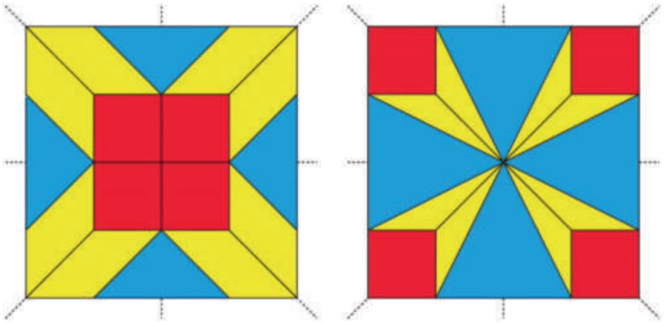
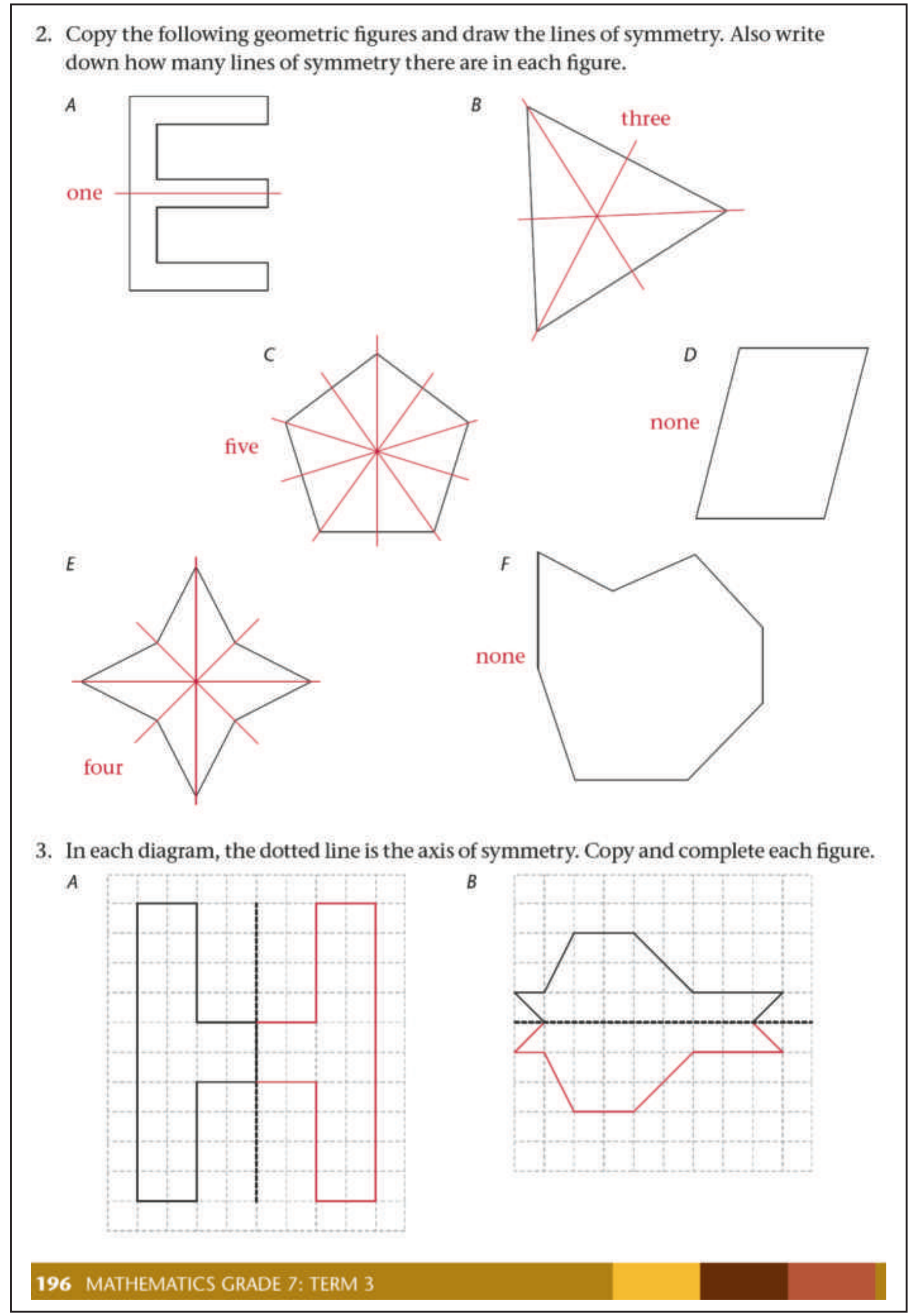
Answers

2. See LB page 196 alongside.
3. See LB page 196 alongside.

Note

Learners can use congruent figures to design interesting patterns with lines of symmetry.





2. Copy the following geometric figures and draw the lines of symmetry. Also write down how many lines of symmetry there are in each figure.

3. In each diagram, the dotted line is the axis of symmetry. Copy and complete each figure.

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16.2 Original figures and their images

Figures can, without changing their shape or size, be moved around in three different ways:

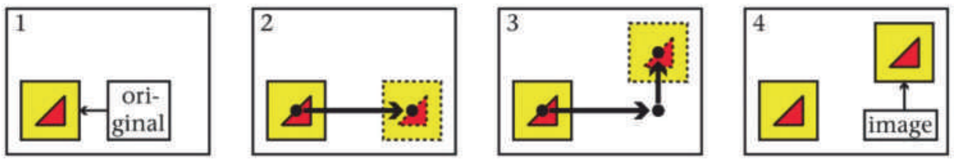
- **Translation** is often referred to as "sliding".
- **Reflection** is often referred to as "flipping" (turning over).
- **Rotation** is often referred to as "turning" (swinging).

16.3 Translating figures

INVESTIGATING THE PROPERTIES OF TRANSLATION

Background information

- During a **translation** the figure is shifted to another position without changing the direction in which it is facing.



- A translation has the following **properties**:
  - All the points on the figure move in the **same direction**.
  - All the points on the figure move by the **same distance**.
  - A figure and its image are **congruent figures** because they have the same shape and size.
- The image of a figure ABC is denoted by using the same letters but we add a prime symbol after each letter: A′B′C′.

Teaching guidelines

Discuss the concept of movement in the same direction over the same distance.

Explain the use of the prime symbol (′) to label the image of a figure.

Answers

1. (a) Refer to the drawing on LB page 198 on following page.

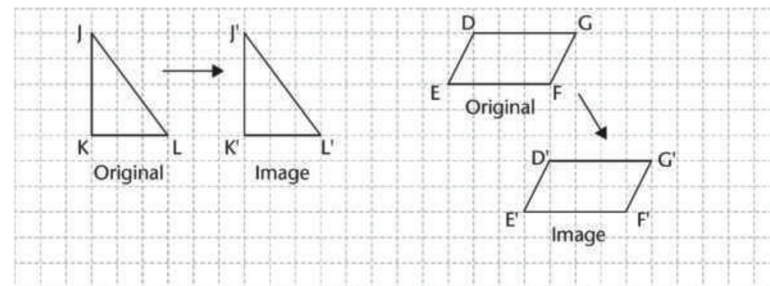
16.2 Original figures and their images

Figures can be moved around in different ways – they can be shifted, swung around and turned over. When the movement is done, the figure in its new position is called the **image** of the original figure.

Figures can be moved in three ways: through **translation**, **reflection** and **rotation**. These transformations are often referred to as "sliding" (shifting), "flipping" (turning over) and "turning" (swinging) respectively.

16.3 Translating figures

Here are two original figures and their images after the figures were **translated**:



When we name the image, we use the same letters for the points that correspond to those of the original figure, but we add the prime symbol (') after each letter. The image of ΔJKL is ΔJ'K'L'. The image of parallelogram DEFG is parallelogram D'E'F'G'.

INVESTIGATING THE PROPERTIES OF TRANSLATION

In a **translation**, all the points on the figure move in the same direction by the same distance. For example, look at ΔJKL above. All of its points have moved six units to the right. Also look at parallelogram DEFG above. All of its points have moved three units to the right and five units down.

1. Look at ΔABC on the following page.
   (a) Copy the triangle on the following page. Translate each of the points A, B and C five units to the right and two units down. Then join the translated points to form the image ΔA'B'C'.

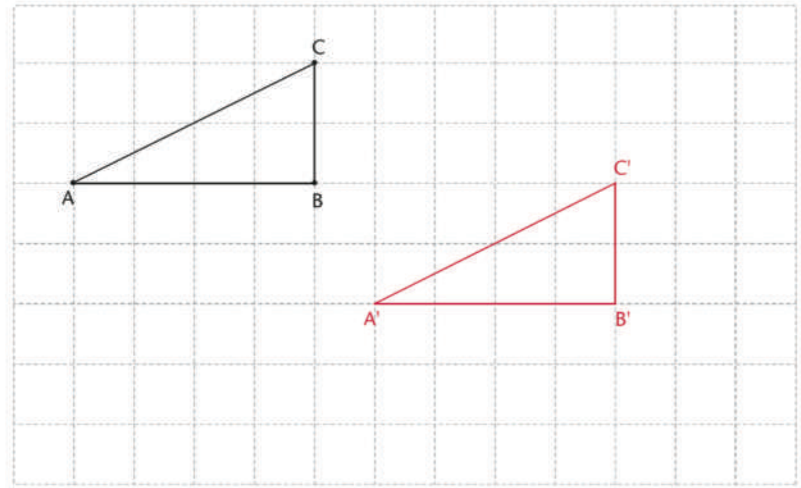
Answers

1. (b) Yes
   (c) Yes
2. (a) Refer to the drawing on LB page 198 alongside.
   (b) Refer to drawing on LB page 198 alongside.
   (c) Yes
   (d) Yes

An important note

Translations can be used to draw simple three-dimensional objects. Follow these simple steps:

1. Draw any figure. This figure will eventually form the front face of the object.
2. Translate the figure four steps to the right and two steps up. Join the corresponding vertices of the figure and its image.



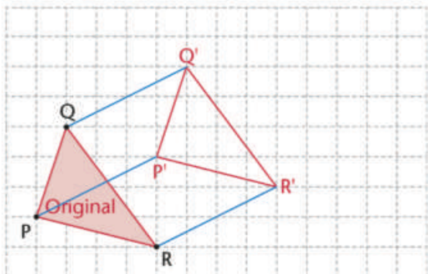
Look at the completed translation.

(b) Are the side lengths of the original triangle and those of its image the same?

(c) Is the area of the original triangle the same as the area of its image?

2. Look at ΔPQR below.

(a) On grid paper, copy the triangle below. Translate each of the points P, Q and R four units to the right and two units up. Then join the translated points to form the image ΔP'Q'R'.



(b) Join point P and its image, point Q and its image, and point R and its image.

(c) Are the line segments that join the original points to their image points equal in length?

(d) Are the line segments that join the original points to their image points parallel?

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Teaching guidelines (continued)

Make sure that learners understand the three important properties of translations listed on the right.

PRACTISE TRANSLATING FIGURES

Background information

- The first two properties of translations are used to produce the required image. The distance and direction of the translation is explained by describing the **horizontal movement** (right or left) in a chosen measuring unit, followed by the **vertical movement** (up or down) measured in the same unit.
- The third property of translations can be used to evaluate the result.

Teaching guidelines

- Apply the first two properties of translations (same distance; same direction) to draw the image of a figure. Use the information provided.
- Apply the third property of translations (congruent figures) to check the result.
- Describe a translation by providing the distance and direction of the horizontal movement, followed by the distance and direction of the vertical movement.

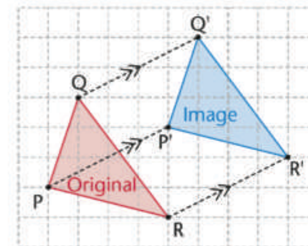
Answers

1. See LB page 199 alongside.
2. See LB page 199 alongside.

Properties of translation

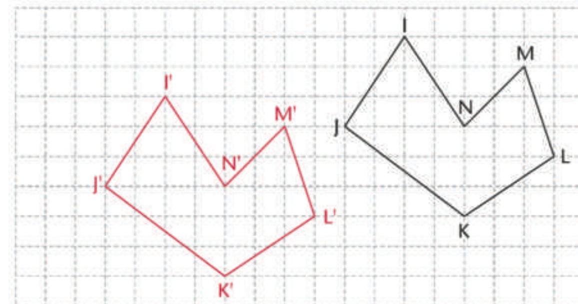
Use the diagram on the right to check if the following is true:

- The line segments that connect the vertices of the original figure to those of the image are all equal in length:
  PP' = RR' = QQ'
- The line segments that connect the vertices of the original figure to those of the image are all parallel to one another:
  PP' || RR' || QQ'
- When a figure is translated, its shape and size do not change. The original and its image are therefore congruent.

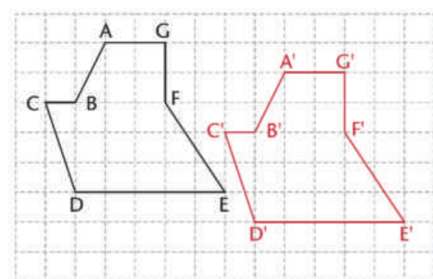


PRACTISE TRANSLATING FIGURES

1. On grid paper, copy the figure. Translate the figure eight units to the left and two units down.



2. On grid paper, copy the figure. Translate the figure six units to the right and one unit down.



Answers

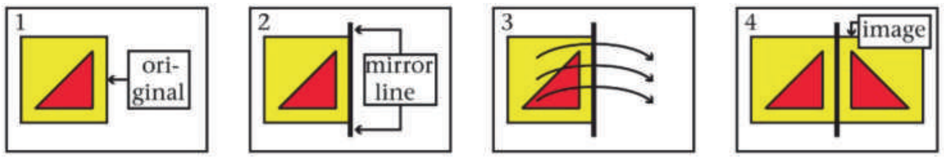
3. (a) Seven units to the right
   (b) Five units down
   (c) Seven units to the right and three units up

16.4 Reflecting figures

INVESTIGATING THE PROPERTIES OF REFLECTION

Background information

- During a **reflection**, the figure is flipped over a **line of reflection** (mirror line) to its new position, like a page of a book.



- A reflection has the following **properties**:
  - The figure and its image lie on **opposite sides** of the line of reflection.
  - An original point and its image lie at **equal distances** from the line of reflection.
  - The line that connects an original point to its image is always **perpendicular** to the line of reflection.
- A figure and its image are **congruent figures** because they have the same shape and size.

Teaching guidelines

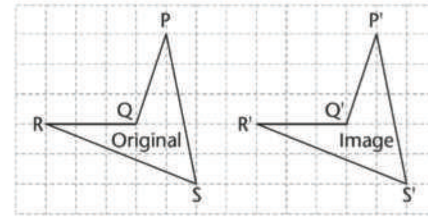
Discuss the concept of being on opposite sides of and at equal distances from a line of reflection. Recall the use of the prime symbol (′) to label the image of a figure.

Misconceptions

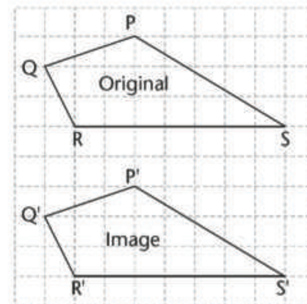
“The mirror line must be drawn on top of a side of the figure.” This statement is NOT true. On the contrary, it can be drawn across the figure, through any of the vertices (corners) of the figure or anywhere alongside the figure.

3. Describe the translation in each of the following diagrams:

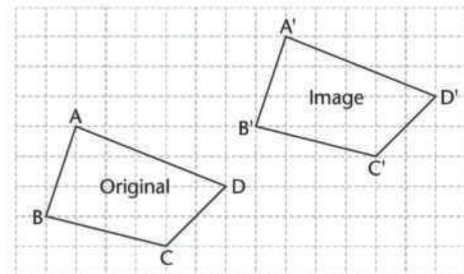
(a)



(b)



(c)



16.4 Reflecting figures

When a figure is **reflected**, it is flipped or turned over. The image that is produced is the mirror image of the original figure. The **line of reflection** is like a mirror in which the original figure is reflected.

The image is produced on the opposite side of the line of reflection. Each point on the original figure and its corresponding point on the image are the same distance away from the line of reflection.

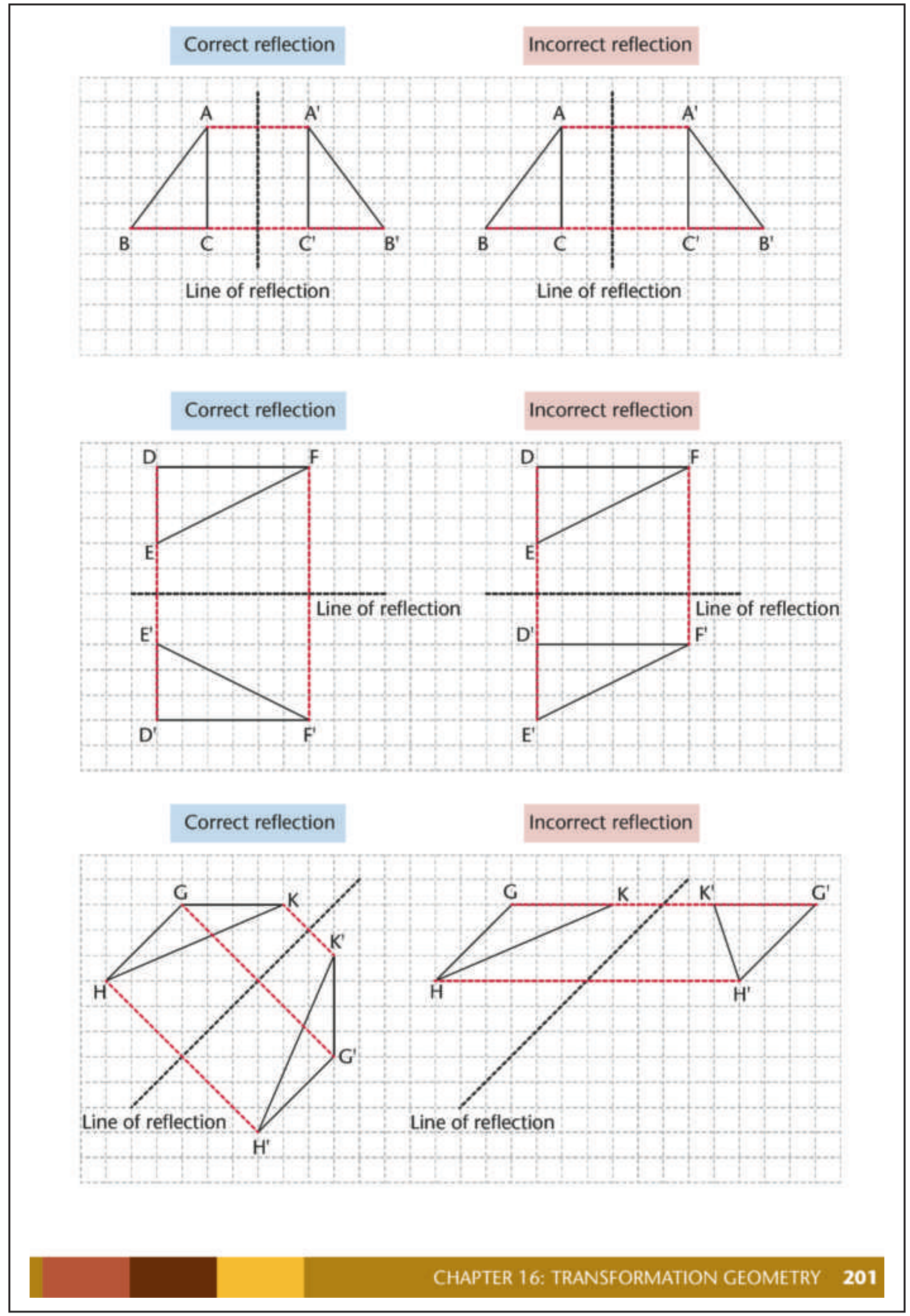
INVESTIGATING THE PROPERTIES OF REFLECTION

The diagrams on the next page show examples of figures that have been correctly and incorrectly reflected in the lines of reflection.

Teaching guidelines (continued)

Refer to the figures on the right.

- The **second reflection** is incorrect because point A and its image A′ are not at equal distances from the line of reflection. The same applies to point B and B′ as well as point C and C′.
- The **fourth reflection** is incorrect because point D and its image D′ are not at equal distances from the line of reflection. The same applies to points E and E′ as well as points F and F′.
- The **sixth reflection** is incorrect because the line that connects points G and G′ is not perpendicular to the line of reflection. The same applies to the lines that connect points H and H′ as well as points K and K′.



Answers

1. See LB page 202 alongside.
2. (a) Yes
   (b) Yes
3. (a) Refer to the relevant drawings on LB page 201 on the previous page.
   (b) Yes
4. (a) Refer to the relevant drawings on LB page 201 on the previous page.
   (b) Not always; in the third reflection it is not.

Teaching guidelines (continued)

Make sure that learners understand the four important properties of reflections listed on the right and at the top of the next page.

1. Copy the table and write down the distance from each of the following points to the line of reflection.

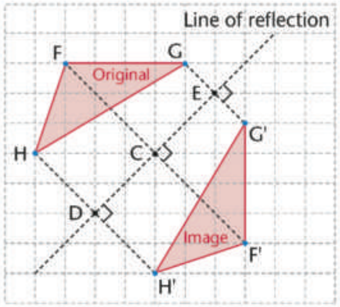
Original figure	Correct reflection	Incorrect reflection
A: two units	A': 2 units	A': 3 units
B: 5 units	B': 5 units	B': 6 units
C: 2 units	C': 2 units	C': 3 units
D: 5 units	D': 5 units	D': 2 units
E: 2 units	E': 2 units	E': 5 units
F: 5 units	F': 5 units	F': 2 units
G: 6 units	G': 6 units	G': 6 units
H: 6 units	H': 6 units	H': 6 units
K: 2 units	K': 2 units	K': 2 units

2. Look at each set of *correct* reflections.
   (a) Are the side lengths of the image the same as those of the original figure?
   (b) Are the size and shape of the image the same as the size and shape of the original figure?
3. (a) Copy each diagram that shows the *correct* reflection, and draw a dotted line to join each point on the original figure to its corresponding reflected point (A to A', B to B', C to C' and so on).
   (b) Is the line that joins the original point to its correct reflection perpendicular to the line of reflection?
4. (a) Copy each diagram that shows the *incorrect* reflection, and draw a dotted line to join each point on the original figure to its corresponding reflected point.
   (b) Is the line that joins the original point to its incorrect reflection perpendicular to the line of reflection?

Properties of reflection

The diagram on the right shows ΔFHG and its reflection ΔF'H'G'. Notice the following properties of reflection:

- The image of ΔFHG lies on the opposite side of the line of reflection.
- The distance from the original point to the line of reflection is the same as the distance from the reflected point to the line of reflection: GE = G'E; FC = F'C and HD = H'D



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PRACTISE REFLECTING FIGURES

Teaching guidelines

To reflect a figure in a line of reflection, follow these steps:

- Reflect the points (vertices) of the figure perpendicularly across the line of reflection.
- Check that a point and its image are at equal distances from the line of reflection.
- Join the reflected points.
- Check whether the figure and its image are still congruent.

Answers

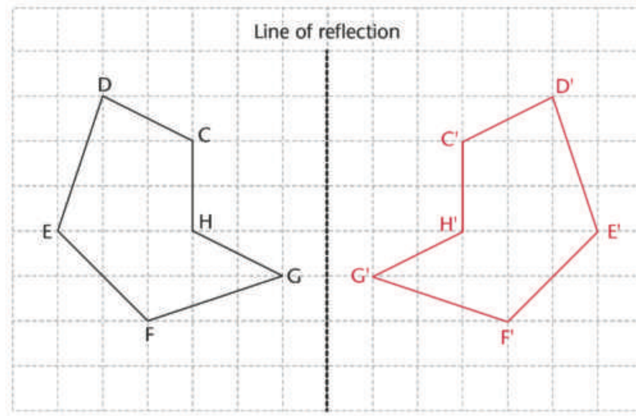
1. (a) Refer to the image on LB page 203 alongside.
   (b) Refer to the image on LB page 203 alongside.

- The line that connects an original point to its image is always perpendicular (⊥) to the line of reflection: HH' ⊥ line of reflection; FF' ⊥ line of reflection, and GG' ⊥ line of reflection.
- When a figure is reflected, its shape and size do not change. The original and its image are therefore congruent.

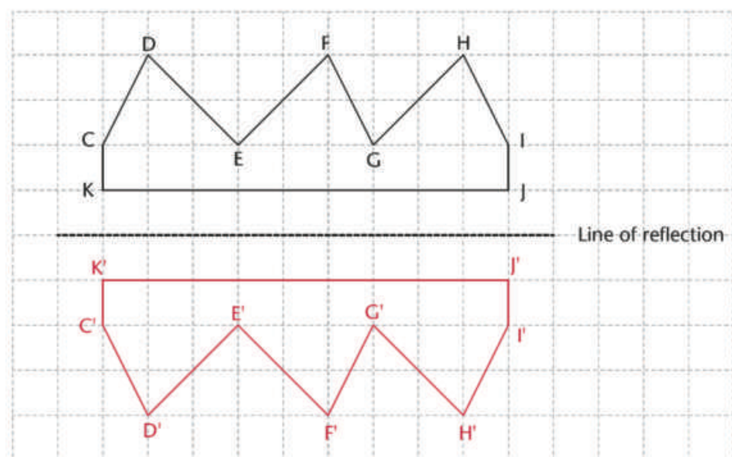
PRACTISE REFLECTING FIGURES

1. Copy the figures below and reflect the figures in the given line of reflection. (*Hint*: First reflect the points; then join the reflected points.)

(a)



(b)



Note on question 2

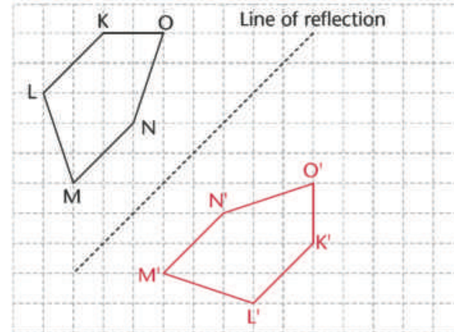
Remember the following:

- The line of reflection should lie between a figure and its image.
- Each point and its image must be at equal distances from the line of reflection.
- The line that connects any point and its image must be perpendicular to the line of reflection.

Answers

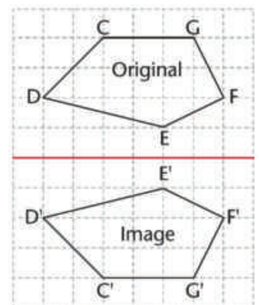
1. (c) Refer to the image on LB page 204 alongside.
2. (a) Refer to the line of reflection on LB page 204 alongside.
   (b) Refer to the line of reflection on LB page 204 alongside.
   (c) Refer to the line of reflection on LB page 204 alongside.

(c)

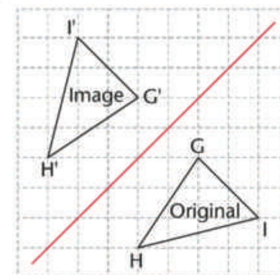


2. Copy the following and draw the line of reflection.

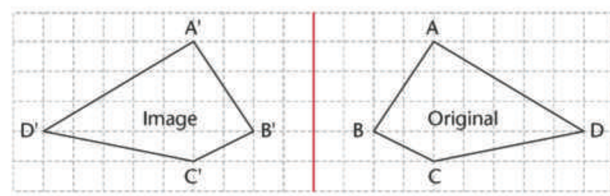
(a)



(b)



(c)

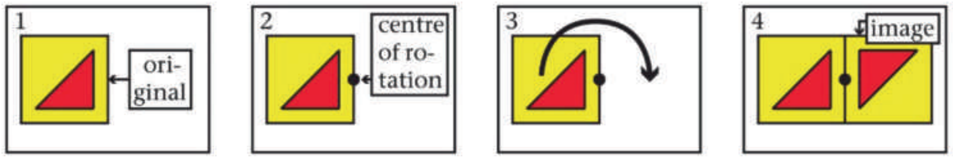


16.5 Rotating figures

INVESTIGATING THE PROPERTIES OF ROTATION

Background information

- During a **rotation** the figure is swung around a fixed point into its new position.



- A rotation has the following **properties**:
  - An original point and its image lie at **equal distances** from the point of rotation.
  - The **angle** formed by the connecting lines between any original point, the centre of rotation and the original point's image is called the angle of rotation.
  - A figure and its image are **congruent figures** because they have the same shape and size.

Teaching guidelines

Discuss different ways of performing a rotation.

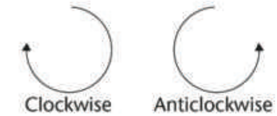
Recall the use of the prime symbol (′) to label the image of a figure.

Misconceptions

"The point of rotation must be part of the figure." This statement is NOT true. On the contrary, it can be positioned inside the figure, on an edge (side) or a vertex (corner) of the figure, or anywhere outside the figure.

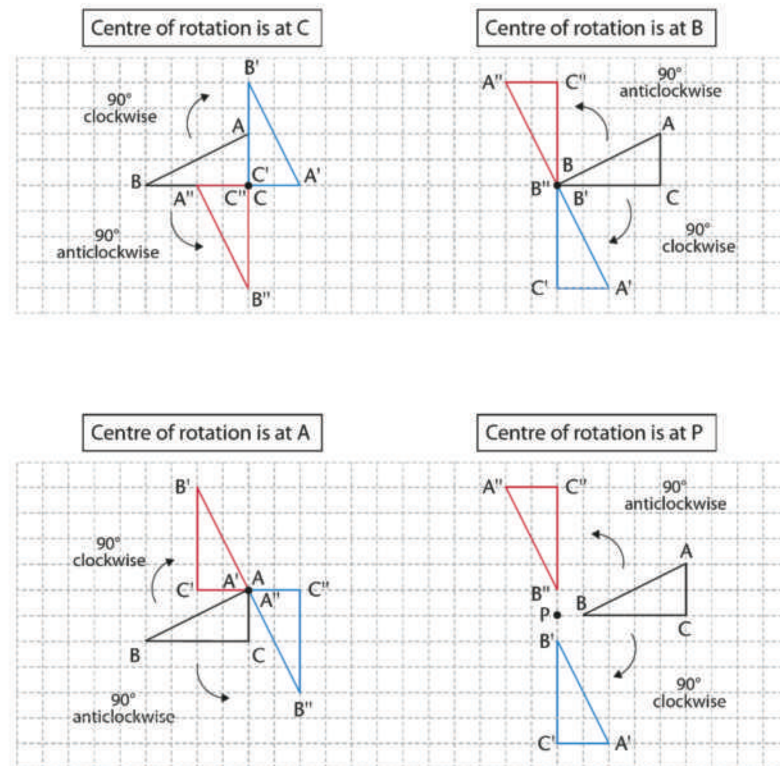
16.5 Rotating figures

When a figure is **rotated** it is turned in a clockwise direction or in an anticlockwise direction around a particular point. This point is called the **centre of rotation** and could be inside the figure or outside of the figure.



The following diagrams show ΔABC rotated 90° clockwise and 90° anticlockwise about different centres of rotation.

In this case, **about** means "around".



Note on question 1

Rotate through 90° anti-clockwise from AS to AS′.

Note on question 2

Rotate through 90° anti-clockwise from AP to AP′.

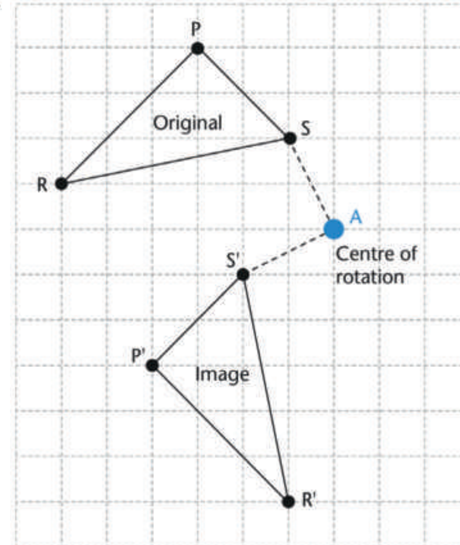
Answers

1. (a) 20 mm
   (b) 20 mm
   (c) They are the same.
   (d) It is 90°.
2. (a) 45 mm
   (b) 45 mm
   (c) They are the same.
   (d) It is 90°.

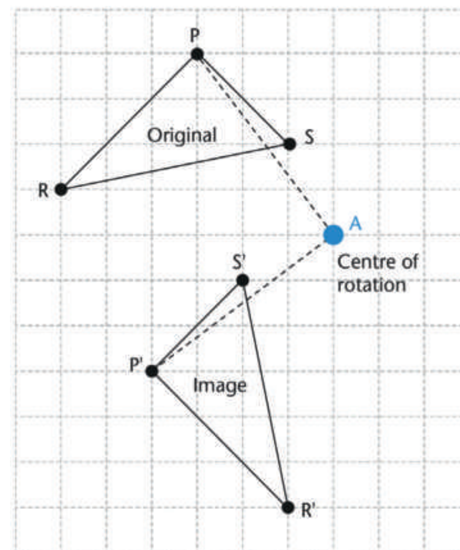
INVESTIGATING THE PROPERTIES OF ROTATION

In the following diagrams, the centre of rotation is point A. ΔPRS has been rotated anticlockwise through 90° about point A.

1. Lines have been drawn to join A to point S, and A to point S'.
   (a) Measure the distance from A to S.
   (b) Measure the distance from A to S'.
   (c) What do you notice about the distances in (a) and (b) above?
   (d) Measure the size of the angle SAS'. What do you notice?



2. Lines have been drawn to join A to P, and A to P'.
   (a) Measure the distance from A to P.
   (b) Measure the distance from A to P'.
   (c) What do you notice about the distances in (a) and (b) above?
   (d) Measure the size of the angle PAP'. What do you notice?



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Note on question 3

Rotate through 90° anti-clockwise from AR to AR′.

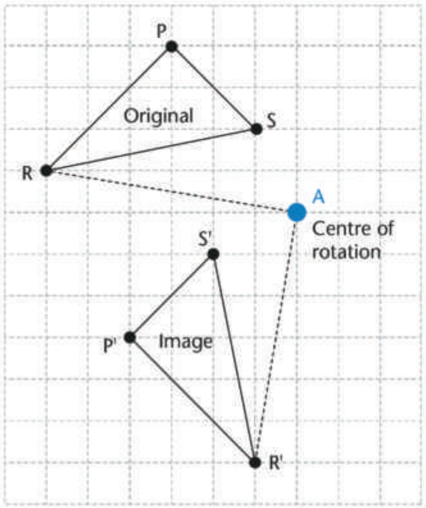
Answers

3. (a) 50 mm
   (b) 50 mm
   (c) They are the same.
   (d) It is 90°.
4. All the corresponding sides are equal, so the triangles are congruent.

Teaching guidelines (continued)

Make sure that learners understand the three important properties of rotation listed on the right.

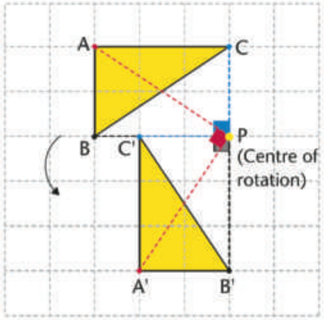
3. Lines have been drawn to join A to R, and A to R'.
   (a) Measure the distance from A to R.
   (b) Measure the distance from A to R'.
   (c) What do you notice about the distances in (a) and (b) above?
   (d) Measure the size of the angle RAR'. What do you notice?



4. In any of the diagrams in questions 1 to 3 above, measure the sides of the original triangle and the corresponding sides of the image. What do you notice?

Properties of rotation

- The distance from the centre of rotation to any point on the original is equal to the distance from the centre of rotation to the corresponding point on the image. In the diagram on the right: PA = PA', PB = PB' and PC = PC'.
- The angle formed by the connecting lines between any point on the original figure, the centre of rotation and the corresponding point on the image is equal to the angle of rotation. For example, if the image is rotated through 90°, this angle will be equal to 90°. If the image is rotated through 45°, the angle will be 45°.
- When a figure is rotated, its shape and size do not change.



PRACTISE ROTATING FIGURES

Teaching guidelines

To rotate a figure 90° clockwise about a centre of rotation, follow these steps:

- Rotate each point (vertex) of the figure 90° clockwise about the centre of rotation.
- Check that each point and its image are at equal distances from the centre of rotation.
- Check that the angle formed by a point, the centre of rotation and the point's image is 90° clockwise.
- Join the reflected points.
- Check whether the figure and its image are still congruent.

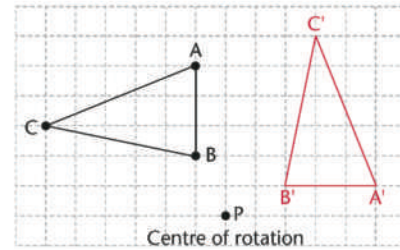
Answers

1. Refer to the image on LB page 208 alongside.
2. Refer to the image on LB page 208 alongside.
3. Refer to the image on LB page 208 alongside.

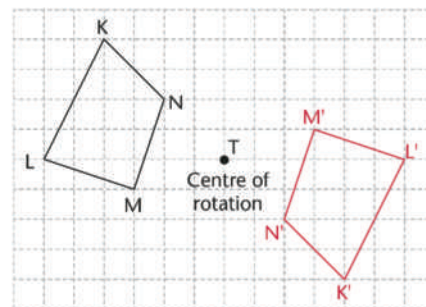
PRACTISE ROTATING FIGURES

1. Rotate triangle ΔABC 90° clockwise about point P as follows:
   (a) Plot the image of each vertex on grid paper. Remember:
   - The image point must be the same distance from P as the original point.
   - The angle that is formed between the line connecting an original point to point P and the line connecting its image point to point P must be the same as the angle of rotation. In this case, it must be 90°.

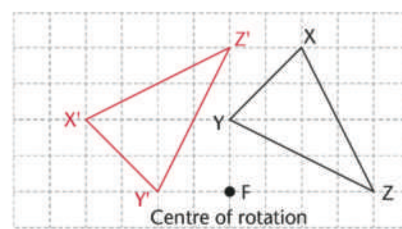
   (b) Join the image points to create ΔA'B'C'.



2. On grid paper, rotate KLMN 180° about point T.



3. On grid paper, rotate ΔXYZ 90° anticlockwise about point F.



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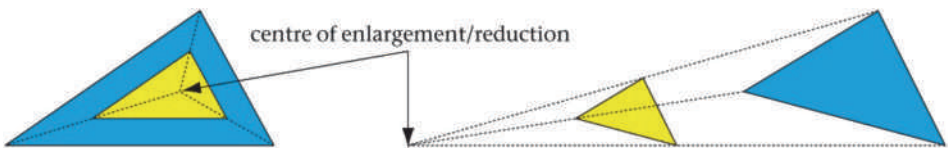
16.6 Enlarging and reducing figures

INVESTIGATE THE PROPERTIES OF ENLARGEMENTS AND REDUCTIONS

Background information

- An **enlargement** is the image of a figure which was made bigger by multiplying the lengths of all its sides by a number.
- A **reduction** is the image of a figure which was made smaller by multiplying the lengths of all its sides by a number smaller than 1, but larger than 0.
- The number by which the sides of a figure are multiplied is called the **scale factor**.
- When the corresponding points of a figure and its image are joined by straight lines, then all those lines will cross at a common point called the **centre of enlargement** or **reduction**.



- The blue triangle is an enlargement of the yellow triangle.
- The yellow triangle is a reduction of the blue triangle.

Teaching guidelines

Discuss the meaning of enlargement and reduction.

Discuss the meaning of scale factor.

Answers

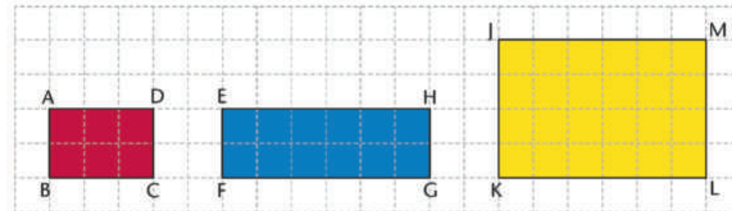
1. (a) Two times
   They are the same.
   (b) Two times
   Two times

16.6 Enlarging and reducing figures

Enlarging a figure means that we make it bigger in a specific way. **Reducing** a figure means that we make it smaller in a specific way. Enlarging or reducing figures is also called **resizing**.

INVESTIGATE THE PROPERTIES OF ENLARGEMENTS AND REDUCTIONS

1. Look at the following rectangles and answer the questions below.



(a) Rectangle EFGH:

How many times is FG longer than BC?

How many times is EF longer than AB?

(b) Rectangle JKLM:

How many times is KL longer than BC?

How many times is JK longer than AB?

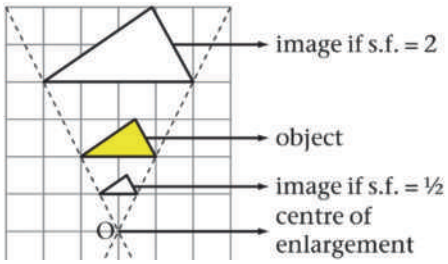
When the lengths of **all the sides** of a figure are **multiplied by the same number** to produce a second figure, the second figure is an **enlargement** or **reduction** of the first figure.

The number by which the sides are multiplied to produce an enlargement or reduction is called the **scale factor**. The scale factor in question 1(b) above is 2. We say that figure ABCD has been enlarged (or resized) by a scale factor of 2 to produce figure JKLM.

Figure EFGH is not an enlargement of figure ABCD because not *all* its sides have been increased by the *same* scale factor.

Background information (continued)

- The **size of the scale factor** determines whether the image will be larger or smaller than the figure.
  - If the scale factor is a **positive number bigger than 1**, the image will be larger than the figure (enlargement).
  - If the scale factor is a **positive number smaller than 1** (but not 0), the image will be smaller than the figure (reduction).



Teaching guidelines (continued)

Discuss the facts about the scale factor listed at the top of LB page 210.

Discuss the difference between congruent and similar figures.

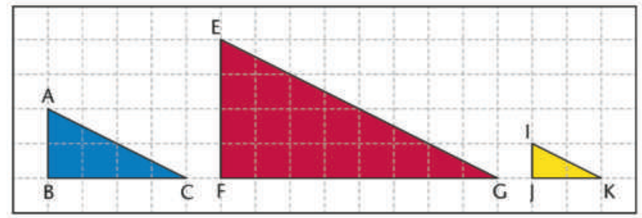
Answers

2. (a) 2 2 2 2 2 2
   (b) Yes. The lengths of all corresponding sides have been multiplied by 2.
   (c) Yes. The lengths of all corresponding sides have been multiplied by 0,5.

The scale factor

- When the scale factor is 1, the image is the same size as the original.
- When the scale factor is <1, the image is a reduction. For example, if the scale factor is $\frac{1}{2}$ or 0,5, each side of the image is half the length of its corresponding side in the original figure.
- When the scale factor is >1, the image is an enlargement. For example, if the scale factor is 2, each side of the image is double the length of its corresponding side in the original figure.

2. Look at the following triangles and answer the questions that follow.



(a) How many times is:

- FG longer than BC?
- EF longer than AB?
- EG longer than AC?
- JK shorter than BC?
- IJ shorter than AB?
- IK shorter than AC?

(b) Is ΔEFG an enlargement of ΔABC? Explain your answer.

(c) Is ΔIJK a reduction of ΔABC? Explain your answer.

Similar figures

When figures are enlarged or reduced, the enlarged or reduced image is **similar** to the original figure. ΔABC, ΔEFG and ΔIJK above are all similar. We also say that the lengths of their corresponding sides are **in proportion**.

If two or more figures are **similar**:

- their corresponding angles are equal, and
- their corresponding sides are longer or shorter by the same scale factor.

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PRACTISE RESIZING FIGURES

Teaching guidelines

To draw an enlargement with a scale factor of 2, draw a similar figure of which the sides are twice as long as the sides of the original figure.

To draw a reduction with a scale factor of 0,5, draw a similar figure of which the sides are half as long as the sides of the original figure.

Answers

1. (a) larger
   (b) smaller
   (c) larger
   (d) smaller
2. Refer to the enlargement on LB page 211 alongside.
3. Refer to the reduction on LB page 211 alongside.
4. Refer to the reduction on LB page 211 alongside.

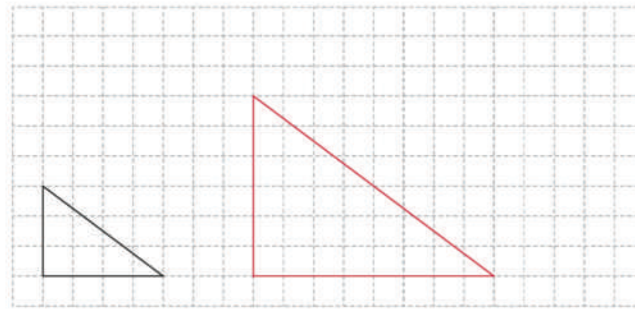
PRACTISE RESIZING FIGURES

1. State whether the following scale factors will produce a larger or smaller image:

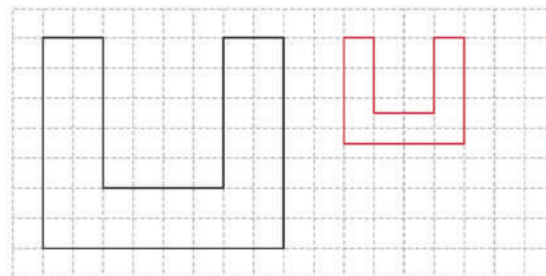
   (a) 5 (b) 0,25

   (c) 1,2 (d) $\frac{3}{8}$

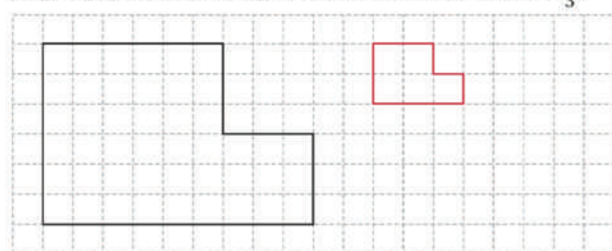
2. On grid paper, enlarge the triangle below with a scale factor of 2.



3. On grid paper, resize the following figure. Use a scale factor of 0,5.



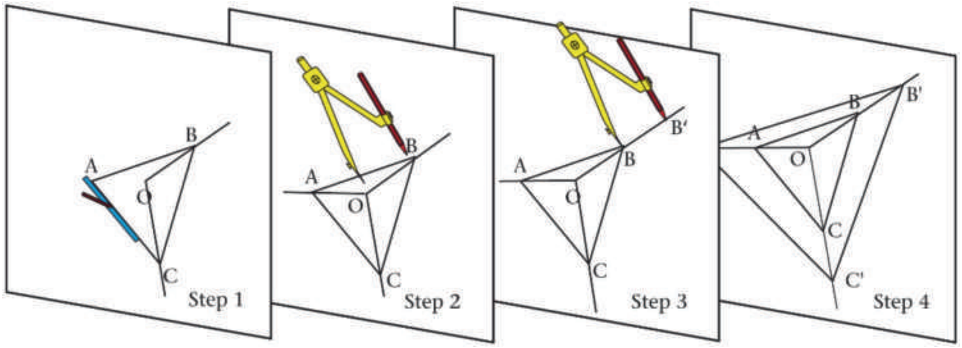
4. On grid paper, resize the figure below. Use a scale factor of $\frac{1}{3}$.



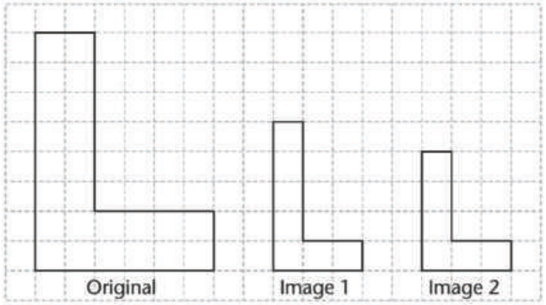
Answers

5. (a) Image 2
   (b) Scale factor of 0,5
6. Image 1: Scale factor 1,5
   Image 2: Scale factor 2

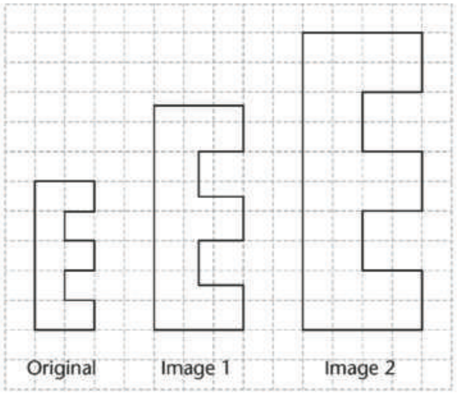
Note



5. (a) Which image below is similar to the original?
   (b) State the scale factor by which it has been resized.



6. What scale factors were used to produce image 1 and image 2 from the original?



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Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
17.1 Classifying 3D objects	Defining a polyhedron; using properties to identify and describe 3D objects	Pages 213 to 215
17.2 Prisms and pyramids	The difference between prisms and pyramids; using properties to identify prisms and pyramids	Pages 215 to 217
17.3 Describing, sorting and comparing 3D objects	Using properties of 3D objects to classify them	Pages 218 to 219
17.4 Nets of 3D objects	Defining a net of a 3D object; evaluating possible nets of a cube; identifying nets of a cube; working with nets of other 3D objects	Pages 220 to 224
17.5 Using nets to construct cubes and prisms	How to draw the net of a prism; drawing nets and using them to construct 3D models	Pages 225 to 226

CAPS time allocation	9 hours
CAPS content specification	Page 66

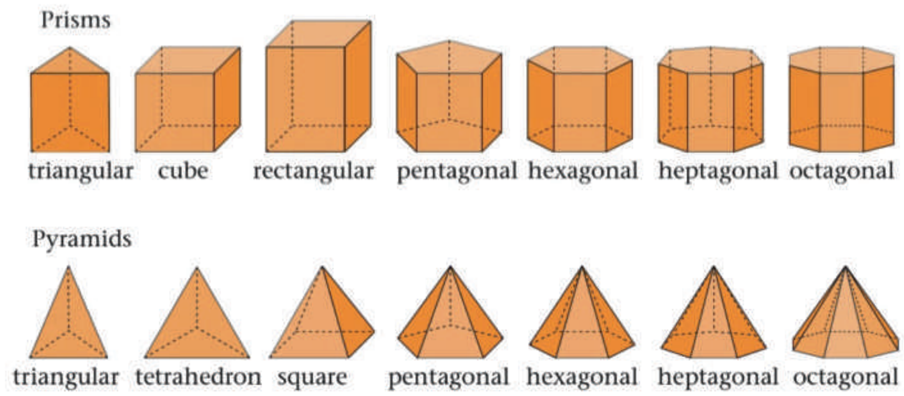
Mathematical background

A three-dimensional object (**3D object**) has length, breadth and height and takes up space. 3D objects can have curved surfaces only (spheres) or curved and flat surfaces (like cylinders and cones) or only flat surfaces. A **cylinder** is shaped like a pipe and has one curved surface (on the side) and two flat surfaces or faces (at each end). The 3D objects that have flat surfaces only, like a cube or a pyramid, are called **polyhedra** (singular: polyhedron).

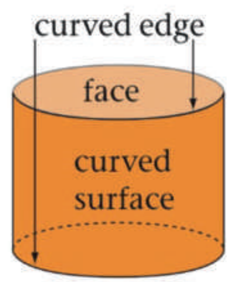
Prisms have identical faces at the top and bottom, and all remaining faces are either rectangles or squares. The base of a prism (the face on which it rests) can be a triangle, quadrilateral, pentagon, hexagon, heptagon or octagon or any polygon. The name of a prism or a pyramid is related to its base, for example a prism with an octagon as a base is called an octagonal prism and has eight rectangles or squares (depending on its height) as side faces because its base has eight sides. A rectangular prism has six faces of which at least four are rectangles. A cube is a prism with all six of its faces being identical squares.

Pyramids rest on a particular 2D shape with straight sides only. All remaining faces of pyramids are isosceles triangles which meet at one point (called the apex).

The net of a 3D object is a flat 2D arrangement of all the faces that make up the 3D object when joined to each other at one side.



17.1 Classifying 3D objects



Try to have 3D objects when you explain what curved surfaces and flat surfaces are. Objects like a ball, a tin (for example a tin of baked beans), a milk carton or any other rectangular box.

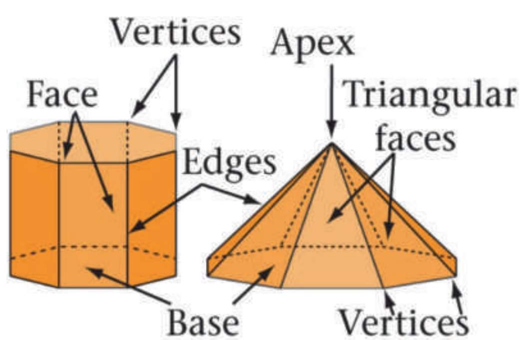
- A flat surface is called a **face**.
- An **edge** of a 3D object is where the sides of two of its surfaces come together.
- A **straight edge** is formed when both surfaces are faces (flat).
- A **curved edge** is formed when at least one of the two surfaces is curved.

WHAT IS A POLYHEDRON?

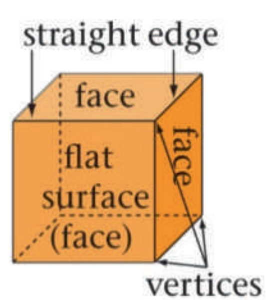
Teaching guidelines

If possible, have an example of a prism and a pyramid in the class or have pictures available or draw them on the board. Point out that each of these objects is a polyhedron. Let learners point out objects around them that are polyhedra.

Point to the faces, edges and vertices on a polyhedron and let learners name the different properties.

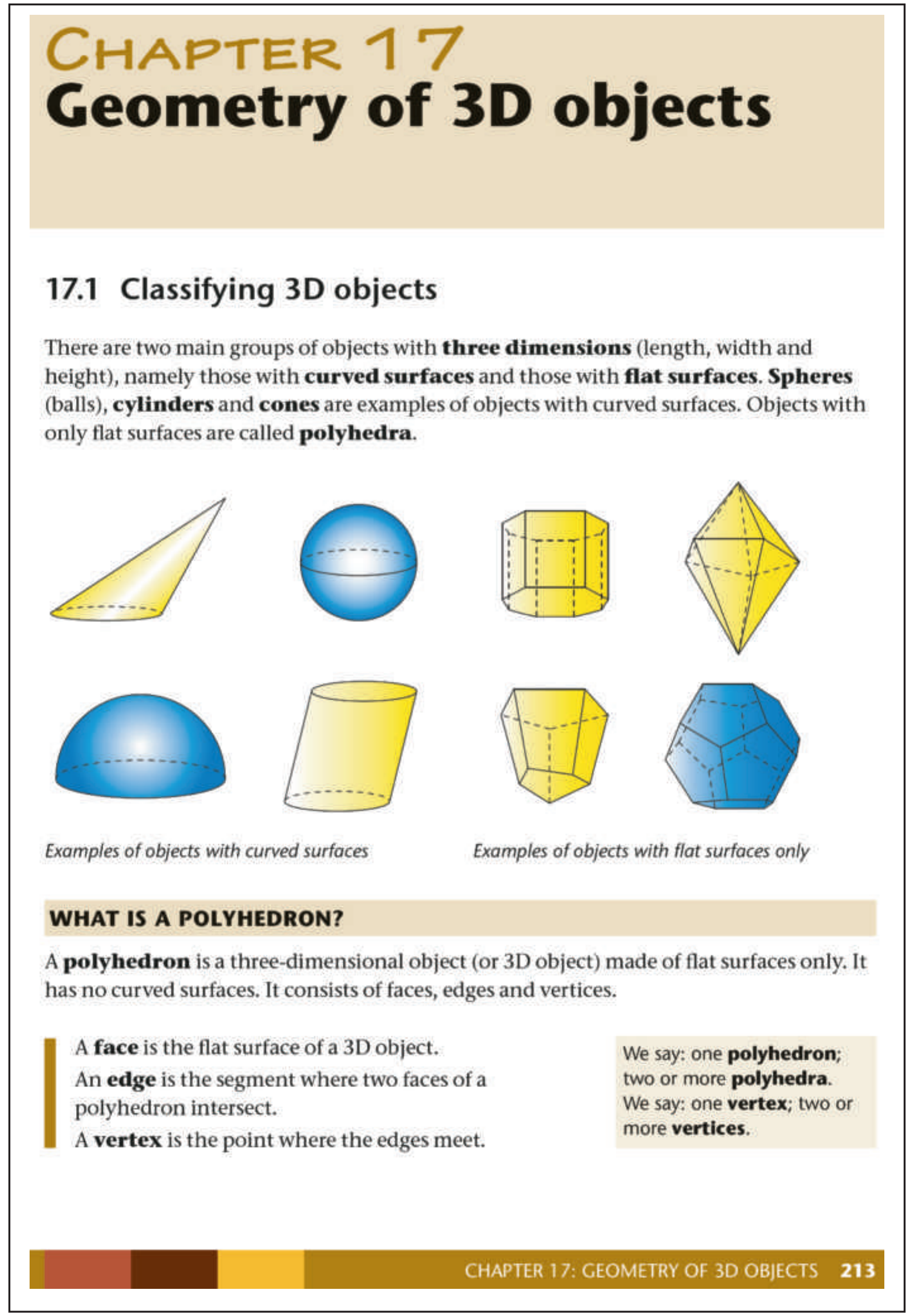


Let them draw a polyhedron of their choice and identify and label the vertices, edges and faces on their drawing using arrows.



Misconceptions

Learners do not make a distinction between polyhedra and objects with curved surfaces.



CHAPTER 17 Geometry of 3D objects

17.1 Classifying 3D objects

There are two main groups of objects with **three dimensions** (length, width and height), namely those with **curved surfaces** and those with **flat surfaces**. **Spheres** (balls), **cylinders** and **cones** are examples of objects with curved surfaces. Objects with only flat surfaces are called **polyhedra**.

Examples of objects with curved surfaces

Examples of objects with flat surfaces only

WHAT IS A POLYHEDRON?

A **polyhedron** is a three-dimensional object (or 3D object) made of flat surfaces only. It has no curved surfaces. It consists of faces, edges and vertices.

A **face** is the flat surface of a 3D object.
An **edge** is the segment where two faces of a polyhedron intersect.
A **vertex** is the point where the edges meet.

We say: one **polyhedron**; two or more **polyhedra**.
We say: one **vertex**; two or more **vertices**.

CHAPTER 17: GEOMETRY OF 3D OBJECTS 213

IDENTIFYING AND DESCRIBING 3D OBJECTS

Teaching guidelines

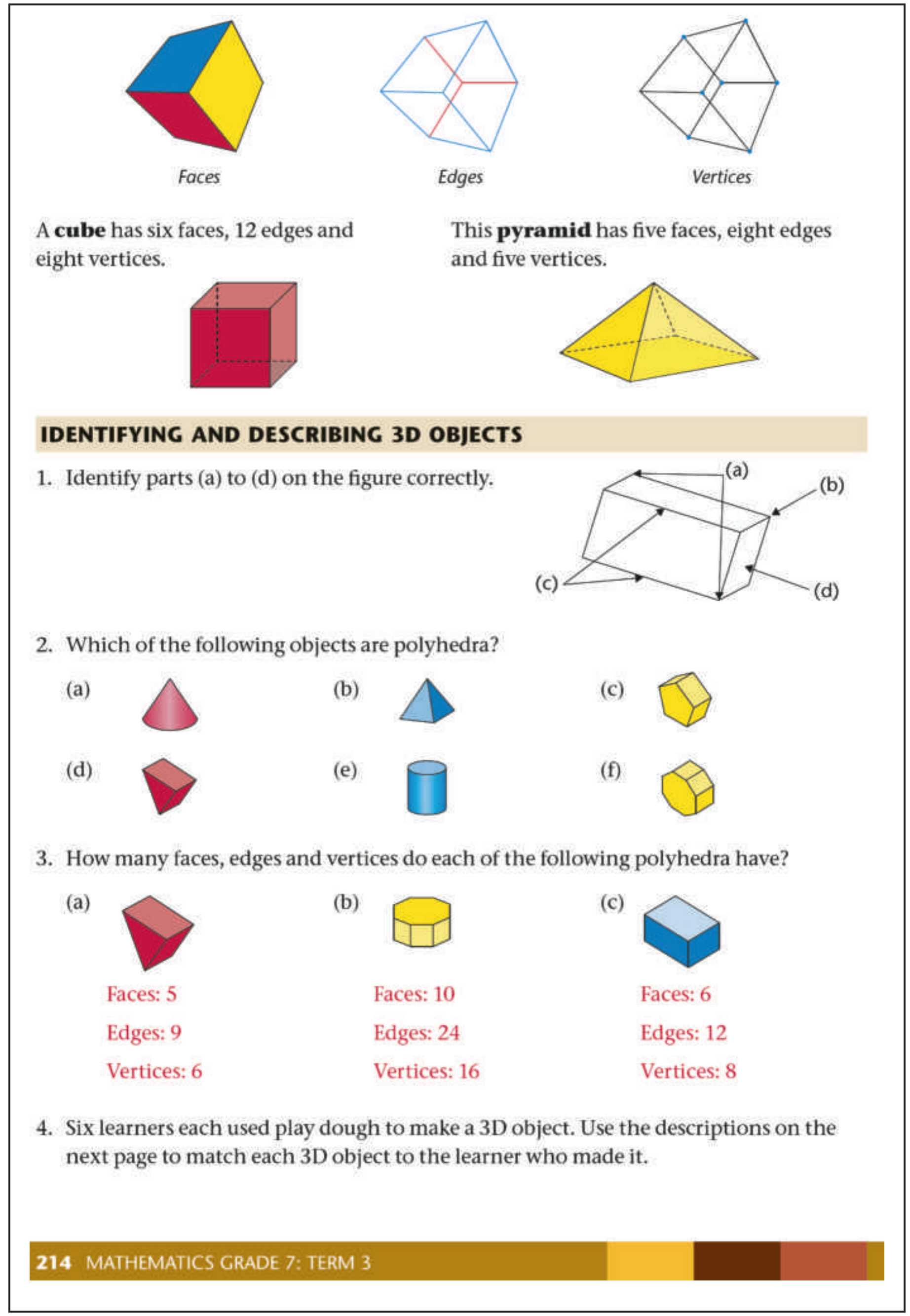
Point out that polyhedra can be distinguished from objects that are not polyhedra by the fact that they have only flat surfaces (faces). Objects with curved surfaces are not polyhedra.

Notes on the questions

The objects in question 2 can be used to increase learners' ability to recognise and identify the faces, vertices and edges of objects.

Answers

1. (a) vertices
   (b) vertex
   (c) edges
   (d) face
2. (a) not a polyhedron
   (b) polyhedron
   (c) polyhedron
   (d) polyhedron
   (e) not a polyhedron
   (f) polyhedron
3. See answers on LB page 214 alongside.
4. (a) Debbie (b) Maggie (c) Tumi
   (d) Xola (e) Mpuka (f) Brad



Faces Edges Vertices

A **cube** has six faces, 12 edges and eight vertices.

This **pyramid** has five faces, eight edges and five vertices.

IDENTIFYING AND DESCRIBING 3D OBJECTS

1. Identify parts (a) to (d) on the figure correctly.

(a) (b) (c) (d)

2. Which of the following objects are polyhedra?

(a) (b) (c) (d) (e) (f)

3. How many faces, edges and vertices do each of the following polyhedra have?

(a) Faces: 5 Edges: 9 Vertices: 6

(b) Faces: 10 Edges: 24 Vertices: 16

(c) Faces: 6 Edges: 12 Vertices: 8

4. Six learners each used play dough to make a 3D object. Use the descriptions on the next page to match each 3D object to the learner who made it.

214 MATHEMATICS GRADE 7: TERM 3

17.2 Prisms and pyramids

DIFFERENCE BETWEEN PRISMS AND PYRAMIDS

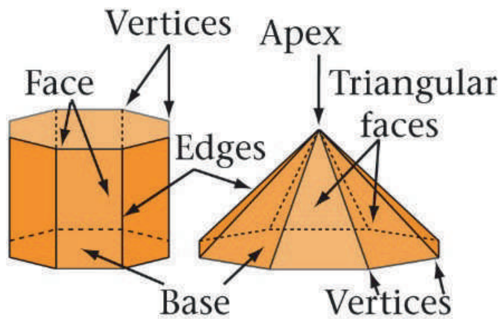
Background

The **apex** of a pyramid is the vertex at which all the triangular faces meet.

An **edge** of a 3D object is where the sides of two of its surfaces come together.

A **straight edge** is formed when both surfaces are faces (flat).

Describe, sort and compare prisms and pyramids using their edges.



Prisms and pyramids have only flat surfaces. The face on which a prism or pyramid rests is called its **base**.

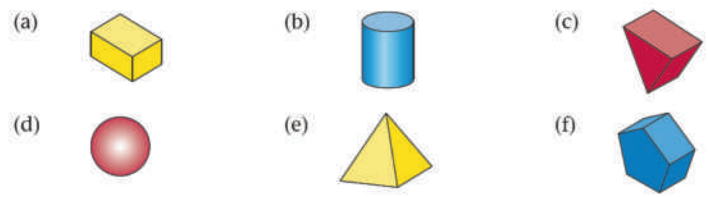
Teaching guidelines

Explain the properties of prisms.

Mention that the lateral faces of **right prisms** are perpendicular to their bases.

Oblique prisms have their lateral faces at an angle to their bases.

Discuss the fact that **right pyramids** have only one base and that the lateral faces are isosceles triangles that meet in a vertex, called the apex. Show learners a drawing of an oblique pyramid. The lateral faces of such a pyramid are not isosceles triangles.



- Tumi's object has six vertices and five faces.
- Debbie's object has eight vertices and 12 edges.
- Brad made an object that has seven faces and ten vertices.
- Xola made an object with no vertices.
- Mpuka's object has eight edges and five faces.
- Maggie made an object with two circles and no vertices.

17.2 Prisms and pyramids

DIFFERENCE BETWEEN PRISMS AND PYRAMIDS

Prisms and **pyramids** are two special groups of polyhedra.

Prisms

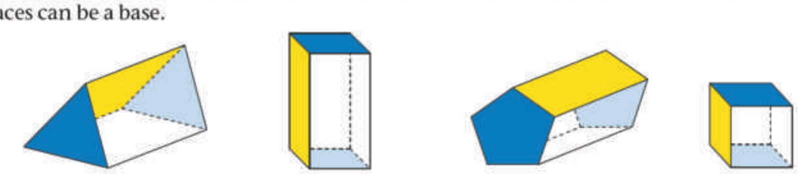
A **prism** is a polyhedron with two faces that are congruent and parallel polygons. These faces are called **bases** and they are connected by **lateral faces** that are parallelograms.

Congruent means exactly the same shape and size.
Lateral faces are faces that aren't bases.

In the case of **right prisms** the bases are connected by rectangles which are perpendicular to the base and the top. This means the lateral faces of a right prism make a 90° angle with the bases.

A prism is named according to the shape of its base. So a prism whose base is a triangle is called a triangular prism; a prism whose base is a rectangle or square is called a rectangular prism; and a prism with a pentagonal base is called a pentagonal prism.

Any pair of faces in a prism that are congruent and parallel can be the bases of that prism. A cube is a special type of prism. It has six congruent faces; therefore any of its faces can be a base.



Triangular prism *Rectangular prism* *Pentagonal prism* *Cube*

CHAPTER 17: GEOMETRY OF 3D OBJECTS 215

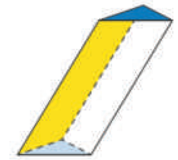
Teaching guidelines

Explain the difference between prisms and pyramids.

Discuss the fact that a pyramid has only one base, whereas a prism has two congruent faces parallel to each other and joined by perpendicular lateral faces. These faces are rectangular.

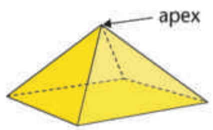
A pyramid has only one base and the lateral faces are isosceles triangles that meet in a vertex, called the apex. Show learners a drawing of an oblique pyramid. The lateral faces of such a pyramid are not isosceles triangles.

These are two **oblique prisms** – the one on the left is a triangular prism and the one on the right is an oblique pentagonal prism.



Pyramids

A **pyramid** has only one base. The lateral faces of a pyramid are always triangles. These triangles meet at the same vertex at the top. This vertex is called the **apex** of the pyramid. In a **right pyramid**, the lateral faces are isosceles triangles.



There are many types of pyramids. The type of pyramid is determined by the shape of its base. For example, a triangular-based pyramid has a triangle as its base, a square-based pyramid has a square as its base and a hexagonal-based pyramid has a hexagon as its base.

A triangular-based pyramid is also called a **triangular pyramid**; a square-based pyramid is also called a **square pyramid**; a hexagonal-based pyramid is also called a **hexagonal pyramid**, etc.

Square-based pyramid
It has five faces:
one square,
four triangles

Hexagonal-based pyramid
It has seven faces:
one hexagon,
six triangles

If a pyramid is not a right pyramid, it is called an **oblique pyramid**, like the two shown below. The lateral faces of an oblique pyramid are not necessarily isosceles triangles.

IDENTIFYING PRISMS AND PYRAMIDS

Teaching guidelines

Let learners work in pairs and take turns to explain to each other what type of object the drawing shows and why the base is where it is indicated to be.

Misconceptions

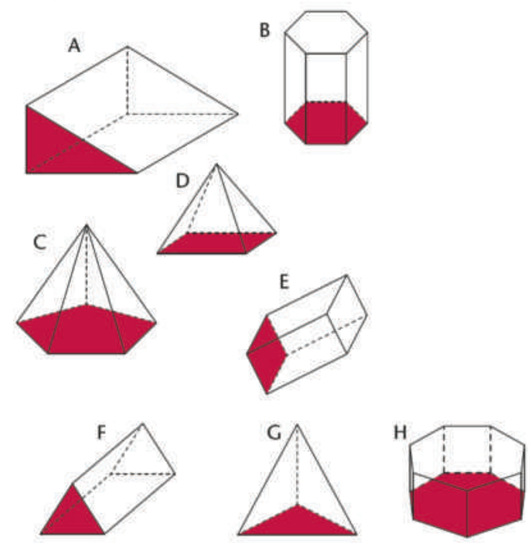
Learners think that the face that is drawn to be horizontal is always the base, for example in F, the base is triangular.

Answers

1. See LB page 217 alongside.
   Note: In E and G any of the faces can be the base.
2. See LB page 217 alongside.

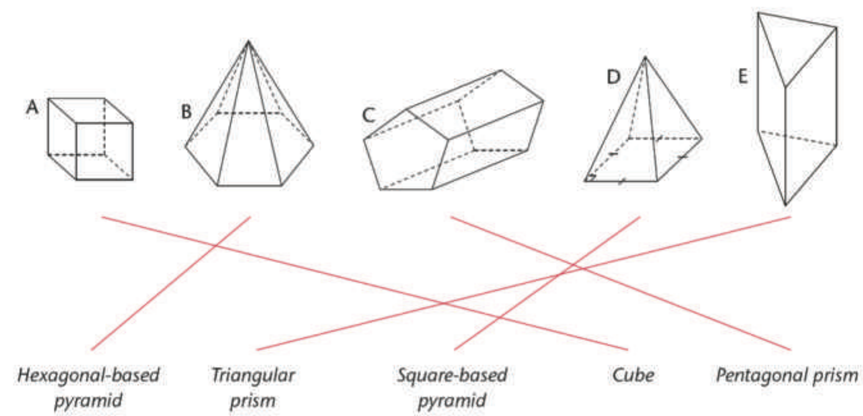
IDENTIFYING PRISMS AND PYRAMIDS

1. Copy the following figures and shade the base of each figure. Write down whether it is a prism or a pyramid. In some cases there is more than one possibility for the base.



A: Prism
B: Prism
C: Pyramid
D: Pyramid
E: Prism
F: Prism
G: Pyramid
H: Prism

2. Join each 3D object with its correct name. Write the letter and the 3D object's name.



CHAPTER 17: GEOMETRY OF 3D OBJECTS 217

17.3 Describing, sorting and comparing 3D objects

PRACTISE DESCRIBING AND CLASSIFYING 3D OBJECTS

Teaching guidelines

Explain how to use the following properties to classify the objects:

- First look for types of surfaces and edges; straight edges and faces as opposed to curved edges and curved surfaces.
- Secondly, separate prisms from pyramids.
- Thirdly, sort the prisms into cubes, rectangular prisms, triangular prisms, etc.

Answers

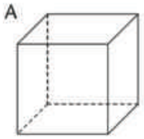
1. See LB page 218 alongside.

17.3 Describing, sorting and comparing 3D objects

PRACTISE DESCRIBING AND CLASSIFYING 3D OBJECTS

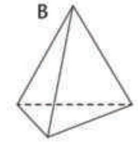
1. For each of the following 3D objects, name the object and describe the number of faces it has and the shapes of these faces.

A



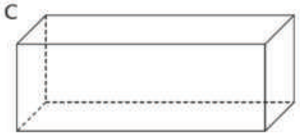
cube
(regular hexahedron)
6 square faces

B



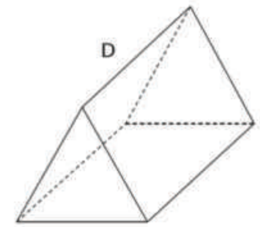
triangular pyramid
(tetrahedron)
4 triangular faces

C



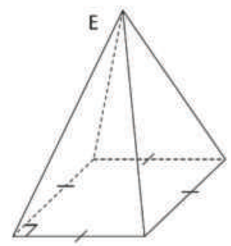
rectangular prism
(hexahedron)
6 rectangular faces

D



triangular prism
2 triangular faces
3 rectangular faces

E



square-based pyramid
1 square face
4 triangular faces

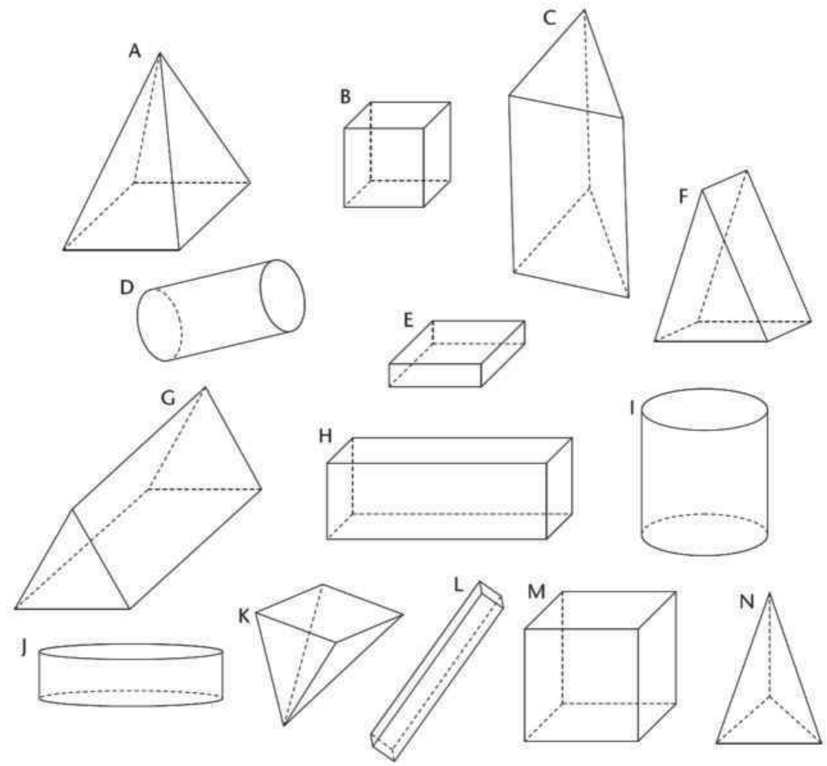
F

cylinder
2 circular faces
1 curved surface (rectangular)

Answers

2. (a) Prisms: B, C, E, F, G, H, L, M
   Pyramids: A, K, N
   Cylinders: D, I, J
   (b) Cubes: B, M
   Rectangular prisms: E, H, L
   Triangular prisms: C, F, G

2. (a) Sort the 3D objects below into the three groups, namely prisms, pyramids and cylinders. Write down the correct letter under each group.
   (b) Further divide the prisms into three groups (cubes, rectangular prisms and triangular prisms). Write the letter under each group.



CHAPTER 17: GEOMETRY OF 3D OBJECTS 219

17.4 Nets of 3D objects

WHAT IS A NET?

Teaching guidelines

The net of a 3D object is a flat, two-dimensional arrangement of all the faces that make up the 3D object, joined to each other at one side.

Show learners how to cut open a box and lay it flat to see the net.

Let learners bring boxes to class to cut open and draw the nets. They should measure the dimensions and draw it more or less to scale.

Learners can draw the four nets A to D, cut them out and try to fold them to form a cube.

Misconception

Learners do not realise that only certain arrangements of the faces of an object will be a net.

Note on the question

Learners can try to find as many different nets of a cube as they can.

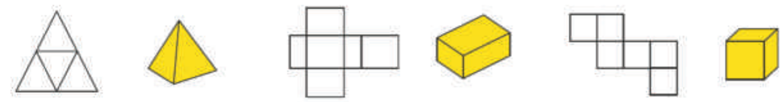
Answer

Net A

17.4 Nets of 3D objects

WHAT IS A NET?

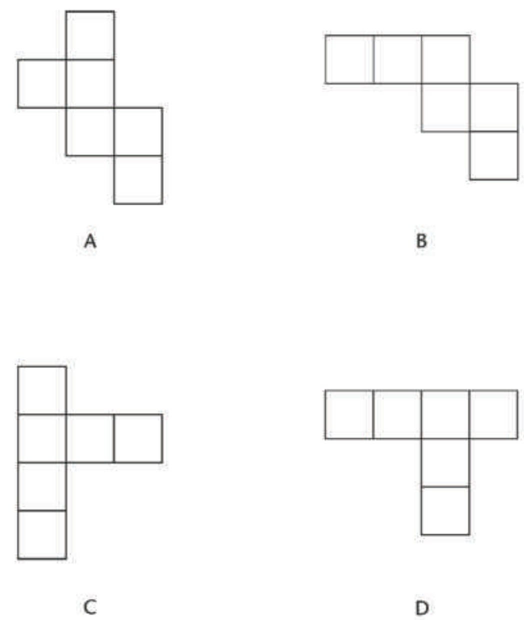
In mathematics, a **net** is a flat pattern that can be folded to form a 3D object. Different 3D objects have different nets. Sometimes the same 3D object can have different nets. Here are examples of different 3D objects and their nets.



Triangular pyramid *Rectangular prism* *Cube*

In this section, you are going to focus on the net of a cube. In order for a net to form a cube, it must consist of six equal squares. But not all net patterns that consist of six squares will fold into a cube.

Only one of the nets below will fold into a cube. Write down which one it is.



A CUBE OR NOT A CUBE

Teaching guidelines

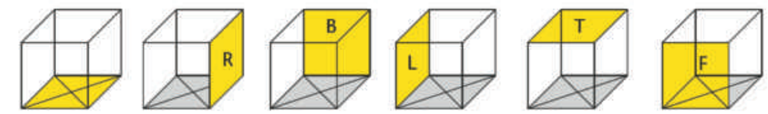
Work through the steps to fold a cube from this net. You could let the learners draw the net on a sheet of paper first, cut it out and fold it with you.

Let learners choose another face as the bottom face and try to fold the cube from there.

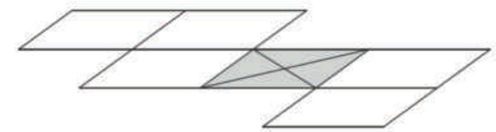
A CUBE OR NOT A CUBE

In order to decide whether or not a net will fold into a cube, you have to imagine what will happen when you fold the net. Read through the following steps.

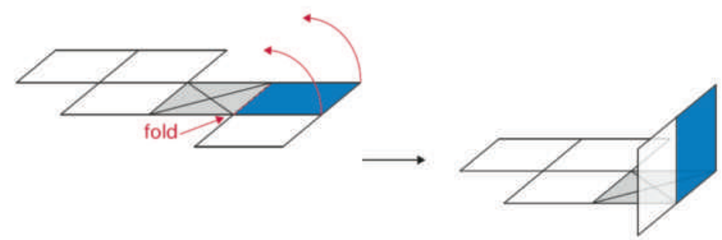
1. We can label the faces of a cube: bottom face (X), right face (R), back face (B), left face (L), top face (T) and front face (F). We will use these terms in the rest of the steps.



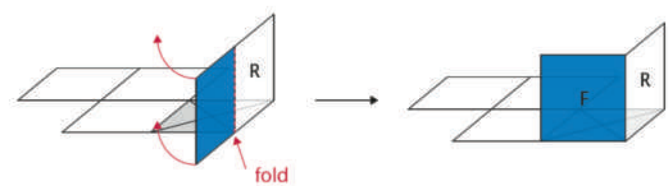
2. Start by choosing one square of the net as the bottom face (marked with an X).



3. Look at the square to the right of the X. (It is coloured blue.) If you fold the net on the red line, the blue square will be the right face of the cube.



4. Look at the next square to the left of the right face. If you fold this square on the red line, it will become the front face of the cube.



CHAPTER 17: GEOMETRY OF 3D OBJECTS 221

IDENTIFYING THE NETS OF A CUBE

Teaching guidelines

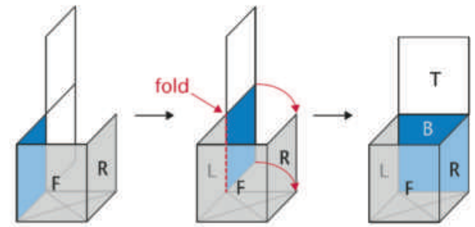
Let learners first imagine the folds and decide whether they will be able to fold the cube or not.

You could let them draw the nets, cut them out and try to fold the cubes.

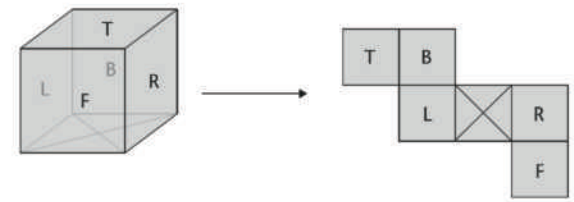
Answers

1. (a) Forms a cube
   (b) Forms a cube
   (c) Forms a cube
   (d) Does not form a cube

5. Look at the square to the left of the front face. It will fold to become the left face of the cube.
6. The square to the left of the left face of the cube will become the back face of the cube.
7. The last square will form the top.



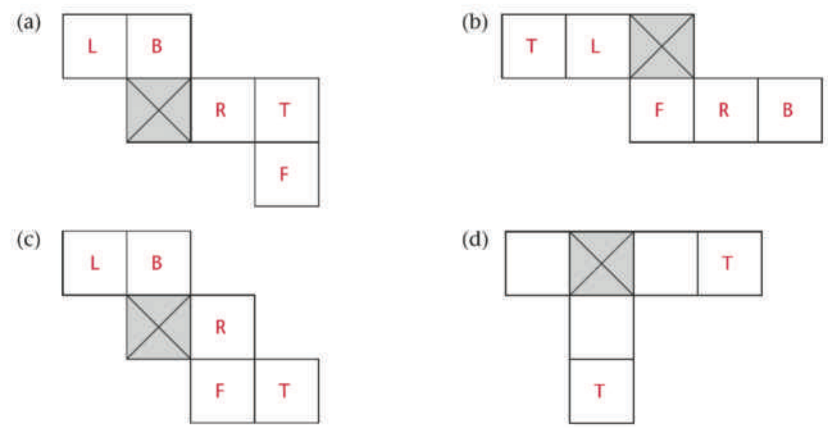
8. Therefore you can label the squares on the net as follows:



9. Since each square on the net corresponds with a face of the cube, this net can be folded into a cube.

IDENTIFYING THE NETS OF A CUBE

1. For each of the following nets, determine whether it will fold into a cube or not by copying and labelling the squares to match the faces of a cube.



NETS OF OTHER 3D OBJECTS

Teaching guidelines

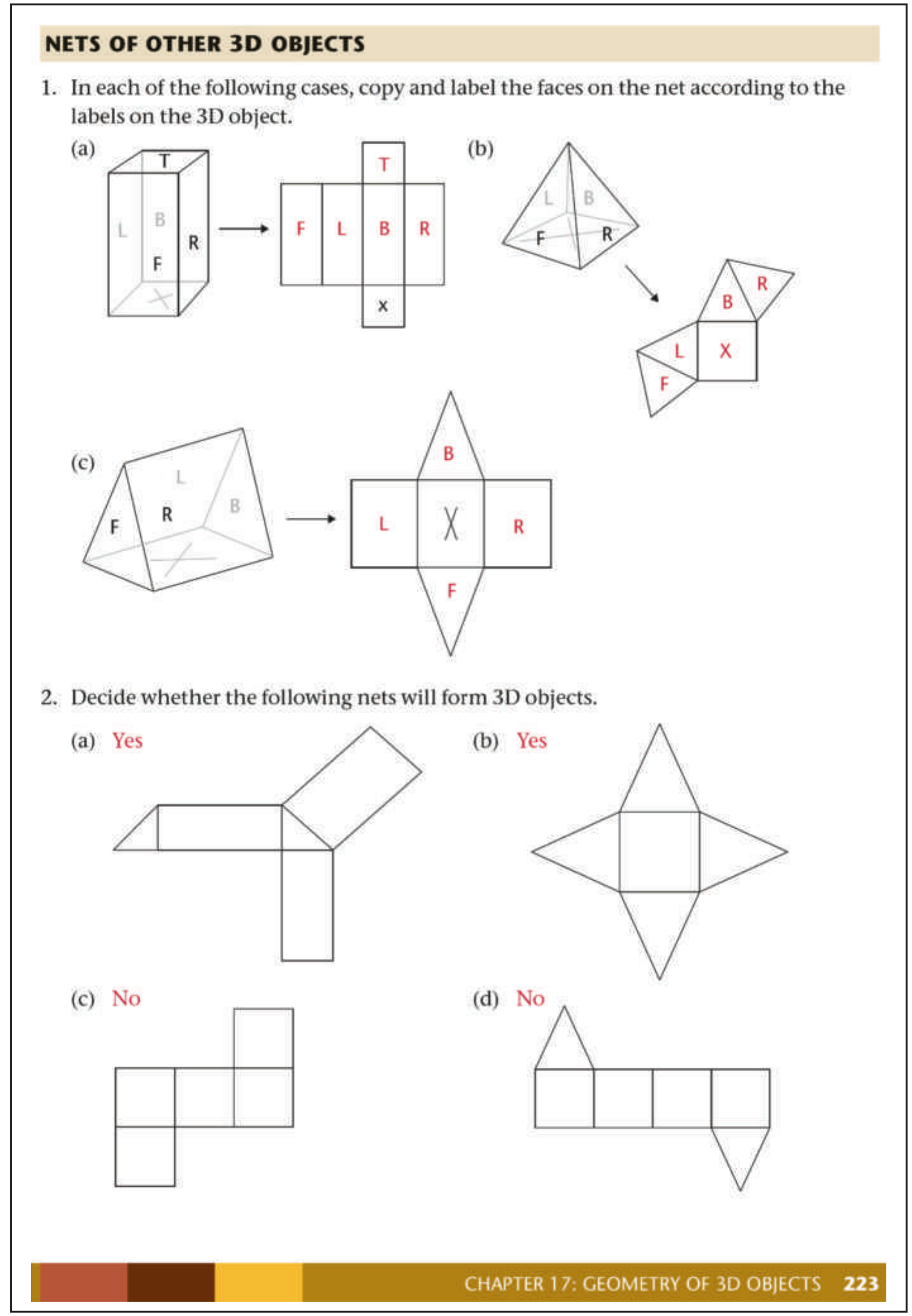
Learners can cut open a box to see how the shapes in the net lie.

Let learners investigate if there are other nets that can be made for the objects shown in question 1.

Let learners discuss how to change the nets in question 2 that do not form 3D objects, so that they will each form an object. For example, in 2(c) another square should be added to the row of three squares in the middle; in 2(d) a square should be removed and the triangles moved to be opposite each other.

Answers

1. See the answers on LB page 223 alongside.
2. (a) Yes
   (b) Yes
   (c) No
   (d) No


NETS OF OTHER 3D OBJECTS

1. In each of the following cases, copy and label the faces on the net according to the labels on the 3D object.

(a)

(b)

(c)

2. Decide whether the following nets will form 3D objects.

(a) Yes

(b) Yes

(c) No

(d) No

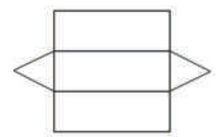
CHAPTER 17: GEOMETRY OF 3D OBJECTS 223

Answers

3. See LB page 224 alongside.
4. (a) A: Hexagon
   B: Rectangle
   C: Hexagon
   D: Rectangle
   E: Rectangle
   F: Rectangle
   G: Rectangle
   H: Rectangle
   (b) six
   (c) two; hexagons
   (d) A prism
   (e) It has two congruent and parallel bases. Pyramids don't use rectangles for the sides as they converge to a point.
   (f) Hexagonal prism
5. (a) Triangles and an octagon
   (b) eight triangles
   (c) one octagon
   (d) A pyramid
   (e) There is only one base and the lateral faces are triangles.
   (f) Octagonal-based pyramid

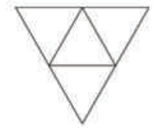
3. Use the nets to identify each of the following 3D objects.

(a)



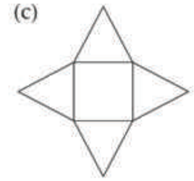
Triangular prism

(b)



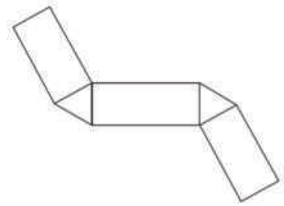
(Tetrahedron)
Triangular-based pyramid

(c)



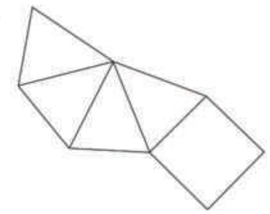
Square-based pyramid

(d)



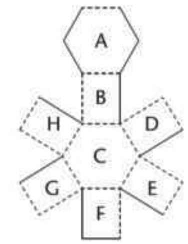
Triangular prism

(e)

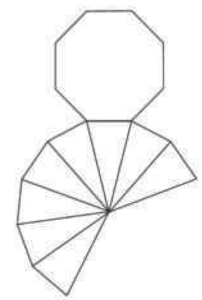


Square-based pyramid

4. (a) Identify the shapes that the net on the right consists of.
   (b) How many rectangular faces does the net have?
   (c) How many other shapes does the net have? What are they?
   (d) Will this form a pyramid or a prism?
   (e) How do you know?
   (f) Name the 3D object that the net will form.



5. Answer the following questions about this net.
   (a) Which shapes does this net consist of?
   (b) How many triangular faces does the net have?
   (c) How many other shapes does the net have?
   (d) Will this form a pyramid or a prism?
   (e) How do you know?
   (f) Name the 3D object that the net will form.



224 MATHEMATICS GRADE 7: TERM 3

17.5 Using nets to construct cubes and prisms

HOW TO DRAW A NET OF A PRISM

Teaching guidelines

Explain to learners that a prism will have the same number of lateral faces as the number of edges of its base.

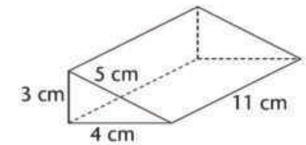
Work through the process of drawing the triangular prism with the learners.

When they draw the net, let learners make a rough drawing first and then they can decide if their net will work or not.

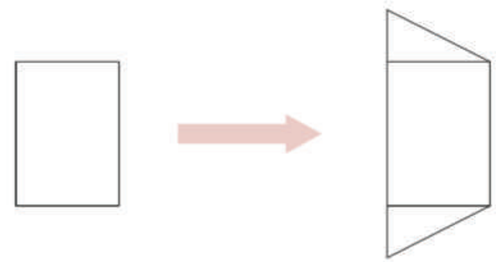
17.5 Using nets to construct cubes and prisms

HOW TO DRAW A NET OF A PRISM

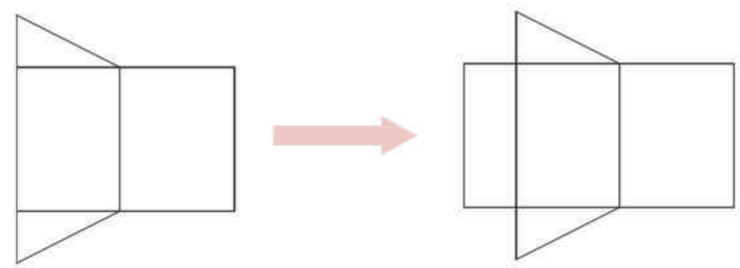
Look at the triangular prism alongside. Its faces are two right-angled triangles and three rectangles. (Since the base has three edges, there will be three rectangles.)



1. Draw a rectangle, which will be the bottom of the prism. Then add a right-angled triangle at the top and a mirror image of the triangle at the bottom of the rectangle.



2. Draw the other two rectangles next to the centre one. The rectangle on the right must fit onto the side opposite the right angle in the triangle when it is folded, so that rectangle will be the biggest. The rectangle on the left is the smallest of the three rectangles.



CHAPTER 17: GEOMETRY OF 3D OBJECTS 225

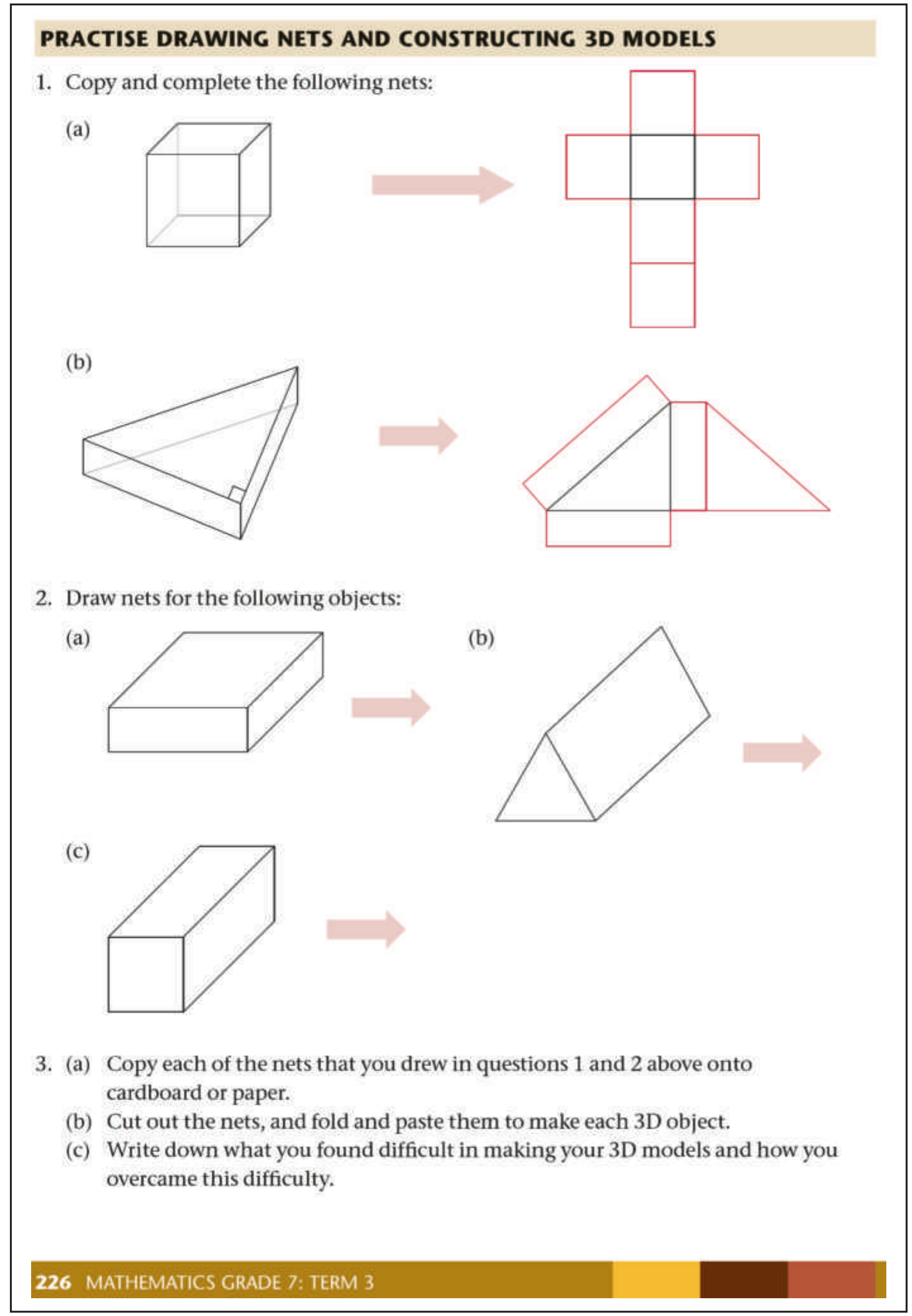
PRACTISE DRAWING NETS AND CONSTRUCTING 3D MODELS

Teaching guidelines

Learners should be able to draw the nets for the objects, but if they still have difficulty, let them work with a partner that can help.

Answers

1. See answers LB page 226 alongside.
2. The nets of (a) and (c) look like the net in question 1(a) on LB page 223. The net of (b) looks like the net in question 1(c) on LB page 223.
3. (c) Example of a response: I found it difficult to make the sides exactly the right size and I overcame this by measuring accurately. I found it difficult to close the object, and I overcame this by adding a small, overlapping flap just for the tape.



PRACTISE DRAWING NETS AND CONSTRUCTING 3D MODELS

1. Copy and complete the following nets:

(a)

(b)

2. Draw nets for the following objects:

(a) (b)

(c)

3. (a) Copy each of the nets that you drew in questions 1 and 2 above onto cardboard or paper.
(b) Cut out the nets, and fold and paste them to make each 3D object.
(c) Write down what you found difficult in making your 3D models and how you overcame this difficulty.

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Term 4

Grade 7 Term 4 Chapter 18 Integers

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
18.1 The need for numbers called integers	Investigate the concept of integers; focus on the use of negative integers in real-life situations	Pages 227 to 231
18.2 Finding numbers that make statements true	Solve simple open number sentences involving integers	Pages 232 to 234
18.3 Adding and subtracting integers	Discover and use the addition and subtraction rules for integers	Pages 234 to 237

CAPS time allocation	5 hours
CAPS content specification	Pages 78 to 79

Mathematical background

- In Chapter 1 we worked with the set of **natural numbers** $N = \{1; 2; 3; 4; 5; ...\}$, which has the following properties:
  - It is **closed under addition** because the sum of natural numbers is always a natural number.
  - It is **closed under multiplication** because the product of natural numbers is always a natural number.
  - It contains the **identity element for multiplication**: Any natural number multiplied by 1 stays the same.
- By adding 0 to the set of natural numbers we created the set of **whole numbers** $N_0 = \{0; 1; 2; 3; 4; 5; ...\}$, which has the following additional property:
  - It contains the **identity element for addition**: Any whole number added to 0 stays the same.
- We extend the set of whole numbers as follows: For every natural number we add another number called the **additive inverse**.
  For example, –24 is the additive inverse of 24. When you add 24 and its additive inverse, the answer is 24 + (–24) = 0. By doing so, we create the set of **integers** $Z = \{...; -5; -4; -3; -2; -1; 0; 1; 2; 3; 4; 5; ...\}$, which has the following additional properties:
  - It is **closed under subtraction** because the difference between integers is always an integer.
  - It enables us to **describe real-life situations** by using a single number, for example:
    - A temperature of 7 °C below freezing point can be described as –7 °C.
    - A bank balance of R200 in the red can be described as –R200.
    - If a golfer is three shots under par for her round, we can describe her score as –3.
    - If a diver is 4 m below sea level, his depth can be described as –4 m.
- This chapter focuses on **integers** and how to add and subtract them.

18.1 The need for numbers called integers

SAYING HOW COLD IT IS

Background information

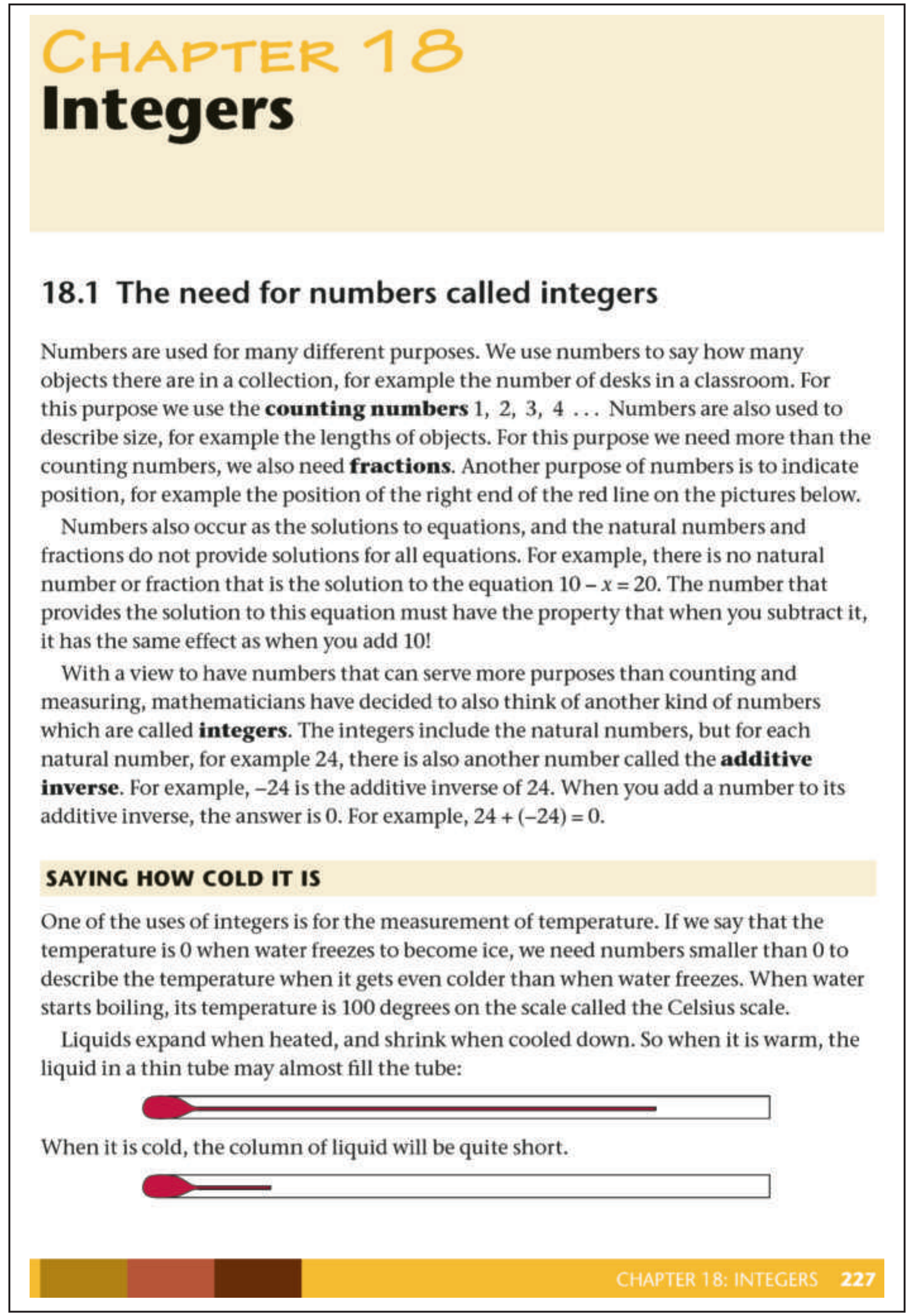
- For every natural number there is an **additive inverse**:
  - The additive inverse for 15 is –15 (read as "negative fifteen").
  - The additive inverse for –10 is 10.
- The sum of any number and its additive inverse is 0:
  - 15 + (–15) = 0: Read as "fifteen plus negative fifteen equals nought".
  - –10 + 10 = 0: Read as "negative ten plus ten equals nought".
- The set of natural numbers and their additive inverses, together with 0, form the set of **integers** Z = {...; –5; –4; –3; –2; –1; 0; 1; 2; 3; 4; 5; ...}
- The set of integers is **closed under subtraction**:
  - 15 – 10 = 5, which is an integer
  - 10 – 15 = –5, which is an integer.
- Integers enable us to **describe real-life situations** by using a single number.
- The most common real-life situation where integers are used is for the **measurement of temperature** with a thermometer.
  - Water boils at 100 °C at sea level.
  - Water freezes at 0 °C.
  - Temperatures to the nearest degree above 0 °C are described by using positive integers.
  - Temperatures to the nearest degree below 0 °C are described by using negative integers.

Teaching guidelines

Discuss the concepts of natural numbers, fractions, integers and additive inverses as reflected in the paragraph on LB page 227.

Learners should know what a thermometer is and how it works.

Use a thermometer to explain how negative integers can describe real-life situations by means of a single number.



CHAPTER 18
Integers

18.1 The need for numbers called integers

Numbers are used for many different purposes. We use numbers to say how many objects there are in a collection, for example the number of desks in a classroom. For this purpose we use the **counting numbers** 1, 2, 3, 4 . . . Numbers are also used to describe size, for example the lengths of objects. For this purpose we need more than the counting numbers, we also need **fractions**. Another purpose of numbers is to indicate position, for example the position of the right end of the red line on the pictures below.

Numbers also occur as the solutions to equations, and the natural numbers and fractions do not provide solutions for all equations. For example, there is no natural number or fraction that is the solution to the equation 10 – x = 20. The number that provides the solution to this equation must have the property that when you subtract it, it has the same effect as when you add 10!

With a view to have numbers that can serve more purposes than counting and measuring, mathematicians have decided to also think of another kind of numbers which are called **integers**. The integers include the natural numbers, but for each natural number, for example 24, there is also another number called the **additive inverse**. For example, –24 is the additive inverse of 24. When you add a number to its additive inverse, the answer is 0. For example, 24 + (–24) = 0.

SAYING HOW COLD IT IS

One of the uses of integers is for the measurement of temperature. If we say that the temperature is 0 when water freezes to become ice, we need numbers smaller than 0 to describe the temperature when it gets even colder than when water freezes. When water starts boiling, its temperature is 100 degrees on the scale called the Celsius scale.

Liquids expand when heated, and shrink when cooled down. So when it is warm, the liquid in a thin tube may almost fill the tube:

When it is cold, the column of liquid will be quite short.

CHAPTER 18: INTEGERS 227

Teaching guidelines (continued)

Learners recall that water boils at 100 °C at sea level and freezes at 0 °C.

Point out that:

- negative integers lie to the left of 0 on a horizontal number line
- negative integers lie below 0 on a vertical number line.

Misconceptions

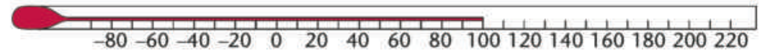
Reading –15 as "minus fifteen" instead of "negative fifteen" could confuse learners when they have to simplify problems like 15 + (–15).

Answers

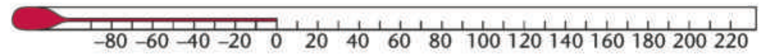
1. (a) 80 °C
   (b) 190 °C
   (c) –20 °C
   (d) –60 °C
   (e) –30 °C
   (f) 10 °C
2. (a) 50 °C
   (b) 50 °C
   (c) 10 °C

This property of liquid is used to measure temperature, and an instrument like the one shown on the previous page is called a **thermometer**.

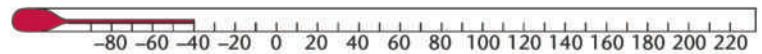
This is what a thermometer will show when it is put in water that is boiling. It shows a temperature of 100 degrees Celsius, which is written as 100 °C.



On the diagram below, you can see what a thermometer will show if it is in water that is starting to freeze. It shows a temperature of 0 °C.

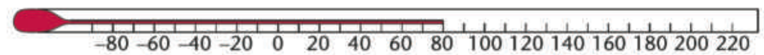


On the next diagram, you can see what a thermometer will show when the temperature is –40 °C, which is colder than any winter night you may have experienced.

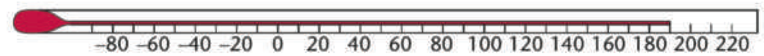


1. Write down the temperature that is shown on each of the thermometers below.

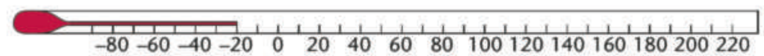
(a)



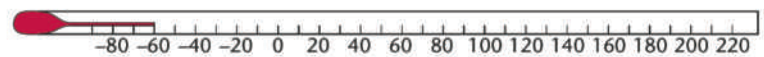
(b)



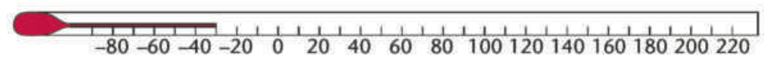
(c)



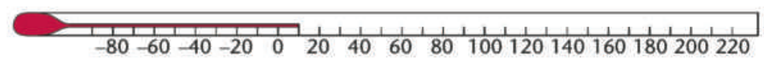
(d)



(e)



(f)



2. (a) The temperature of water in a pot is 20 °C. It is heated so that it gets 30 °C warmer. What is the temperature of the water now?
   (b) The temperature of water in a bottle is 80 °C. During the night it cools down to 30 °C. By how much has it cooled down?
   (c) In the middle of a very cold winter night the temperature outside is –20 °C. At nine o'clock in the morning it has become 30 degrees warmer. What is the temperature at nine o'clock?

Answers

3. (a) –2 °C
   (b) –12 °C
   (c) 2 °C
   (d) –14 °C
4. See LB page 229 alongside.

SAYING HOW MUCH MONEY IT IS

Background information

Another real-life situation where integers are frequently used is in the **financial world**.

- A **surplus** in a bank account, to the nearest rand, is described by using positive integers.
- A **shortage** in a bank account, to the nearest rand, is described by using negative integers.

Teaching guidelines

Discuss how integers feature in the financial world.

Answers

1. R360
2. R160 + R180 = R340
   R340 + R90 = R430
   The total cost is R430.
3. R360 – R160 = R200
   He will have R200 left.
4. R360 – R160 = R200
   R200 – R180 = R20
   He will have R20 left.
5. R360 – R160 = R200
   R200 – R180 = R20
   R20 – R90 = –R70
   He won't have any money left. He is R70 short.

3. (a) The temperature is 8 °C. What will the temperature be if it gets 10 degrees colder?
   (b) The temperature is 8 °C. What will the temperature be if it gets 20 degrees colder?
   (c) The temperature is –8 °C. What will the temperature be if it gets 10 degrees warmer?
   (d) The temperature is –24 °C. What will the temperature be if it gets 10 degrees warmer?
4. Some numbers are shown on the number lines below. Copy the number lines and fill in the missing numbers.

–9 –8 –7 –6 –5 –4 –3 –2 –1 0 1 2 3 4 5 6 7 8 9

–14 –13 –12 –11 –10 –9 –8 –7 –6 –5 –4 –3 –2 –1 0 1 2 3 4

–45 –40 –35 –30 –25 –20 –15 –10 –5 0 5 10 15 20 25 30 35 40 45

SAYING HOW MUCH MONEY IT IS

Simon is in Grade 5. He saved money in a tin. When he turned 10, his grandmother gave him R100. He also opened his savings tin on his tenth birthday and there was R260 in the tin. Simon was very happy. He said to himself: "I am very rich!"

Simon decides to buy some things that he has always wanted. This is what he decides to buy:

- a soccer ball at R160
- a pair of sunglasses at R180
- a book about animals at R90

1. How much money did Simon have in total on the day that he thought he was rich?
2. What is the total cost of the three items he wants to buy?
3. Simon decides to first buy the soccer ball only. How much money will he have after paying for the soccer ball?
4. How much money will Simon have if he buys the soccer ball and the sunglasses?
5. How much money will Simon have if he buys the soccer ball, the sunglasses and the book about animals?

Simon did these calculations while he was thinking about buying the various items:

R360 – R160 = R200
R200 – R180 = R20
R20 – R90 = (–) R70 ?

Background information (continued)

- When a number is subtracted from a larger number, the answer is positive.
- When a number is subtracted from a smaller number, the answer is negative.

Teaching guidelines (continued)

Ask the following questions:

- When will the answer of a subtraction be positive?
- When will the answer of a subtraction be negative?

Answers

6. (a) R210 + R180 = R390
   R390 – R160 = R230
   (b) R150 + R130 = R280
   R280 – R460 = –R180
7. See the table on LB page 230 alongside.
8. (a) –4 (b) –14 (c) –24
   (d) –44 (e) –184 (f) –995
9. R200 + R40 + R50 = R290
   She has R290, provided that her brother will pay her back the R50.

6. Fatima owns a small shop. One afternoon when she closed the shop, she had R120 cash, clients owed her R90, and she owed her suppliers R310. In Fatima's view her financial position was as follows: R120 + R90 – R310 = –R100.
   (a) On another day, Fatima ended the business day with R210 cash, clients owed her R180 and she owed her suppliers R160. What was her financial position?
   (b) On another day, Fatima ended the business day with R150 cash, clients owed her R130 and she owed her suppliers R460. What was her financial position?

About 500 years ago, some mathematicians proposed that a "negative number" may be used to describe the result in a situation like the above, where a number is subtracted from a number smaller than itself.

For example, we may say 10 – 20 = (–10)

Mathematicians are people who do mathematics for a living. Mathematics is their profession, like health care is the profession of nurses and medical doctors.

This proposal was soon accepted by other mathematicians, and it is now used all over the world.

7. Copy the table below. Continue the lists of numbers to complete the table.

(a)	(b)	(c)	(d)	(e)	(f)	(g)
10	100	3	–3	–20	150	0
9	90	6	–6	–18	125	–5
8	80	9	–9	–16	100	–10
7	70	12	–12	–14	75	–15
6	60	15	–15	–12	50	–20
5	50	18	–18	–10	25	–25
4	40	21	–21	–8	0	–30
3	30	24	–24	–6	–25	–35
2	20	27	–27	–4	–50	–40
1	10	30	–30	–2	–75	–45
0	0	33	–33	0	–100	–50
–1	–10	36	–36	2	–125	–55
–2	–20	39	–39	4	–150	–60
–3	–30	42	–42	6	–175	–65

8. Calculate each of the following:
   (a) 16 – 20 (b) 16 – 30 (c) 16 – 40
   (d) 16 – 60 (e) 16 – 200 (f) 5 – 1 000
9. Jeminah has R200 in a savings account and R40 in her purse. Her brother owes her R50. How rich is she? In other words, how much money does she have?

Answers

10. R290 – R60 – R150 = R80
    Actually, she only has R80.
11. R80 – R250 = –R170
    She is R170 short.

ORDERING AND COMPARING INTEGERS

Background information

Number lines can be used to discuss the following **features of integers**:

- All **positive** integers lie to the **right of 0**.
- All **negative** integers lie to the **left of 0**.
- The further an integer lies to the **right of 0**, the **larger its value**.
- The further an integer lies to the **left of 0**, the **smaller its value**.
- The larger of **two positive integers** lies to the right of the smaller one because the larger one lies **further away from 0** than the smaller one.
    - 5 lies further from 0 than 3, therefore 5 > 3
    - 3 lies closer to 0 than 5, therefore 3 < 5
- The larger of **two negative integers** lies to the right of the smaller one because the larger one lies **closer to 0** than the smaller one.
    - –3 lies closer to 0 than –5, therefore –3 > –5
    - –5 lies further from 0 than –3, therefore –5 < –3

Teaching guidelines

Discuss how to compare a pair of positive or negative integers by using their distances from 0 on a number line.

Answers

1. –9 °C; –6 °C; –4 °C; –1 °C; 0 °C; 4 °C; 7 °C; 12 °C
2. See LB page 231 alongside.
3. See LB page 231 alongside.
4. See LB page 231 alongside.

10. Oops! Jeminah forgot that she borrowed R60 from her mother, and that she still has to pay R150 for a dress she bought last month. So how rich (or poor) is she really? In other words, how much money does she actually have?

11. In fact, Jeminah's financial situation is even worse. She has received an outstanding bill from her doctor, for R250. So how much money does she really have?

ORDERING AND COMPARING INTEGERS

1. On a certain day the following minimum temperatures were provided by the weather bureau:

Bethlehem	–4 °C	Bloemfontein	–6 °C
Cape Town	7 °C	Dordrecht	–9 °C
Durban	12 °C	Johannesburg	0 °C
Pretoria	4 °C	Queenstown	–1 °C

Arrange the temperatures from the coldest to the warmest.

2. Copy the number line and place the following numbers on the number line as accurately as you can: 50; –2; –23; 5; –36

3. Copy the number lines. In each case, write the numbers in the boxes provided:

(a) 125 000; –178 000; –100 900; 180 500

(b) –1 055 500; –1 010 100; –1 100 100; –1 032 800; –1 077 500

4. Write > or < to indicate which number is the smaller of the two.

(a) 978 543 > 978 534 (b) –1 043 724 < –1 034 724

(c) –864 026 > –864 169 (d) –103 232 > –104 326

(e) –710 742 < 710 741 (f) –904 700 > –904 704

18.2 Finding numbers that make statements true

Background information

- A **number sentence** is a statement about numbers, for example,
  $8 \times 2 = 16$.
- A **number sentence** is a sentence with the verb "=", "is equal to" or "is equivalent to".
- A **calculation plan** (expression) tells how to execute a certain sequence of calculations, for example,
  $98 - 20 + 12 \times 2$ or $98 - (20 + 12) \times 2$.
- A **true number sentence** is a statement of equivalence with equivalent calculation plans on both sides, for example,
  $3 \times 12 + 5 \times 12 = 8 \times 12$.

Teaching guidelines

Discuss the concept of a number sentence as explained above.

Learners should realise that answers to problems are influenced by the set of numbers from which the answer can be chosen.

Note on question 2

All these sentences are open number sentences because they contain an unknown number.

Note on question 4

Answers are influenced by the set of numbers from which the answer can be chosen.

Note on question 5

All these questions are calculation plans (expressions) because there is no verb.

Answers

1. See LB page 232 alongside.
2. See LB page 232 alongside.
3. (a) No (b) $2\frac{1}{2}$
4. (a) No (b) Learners' own answer.
5. See LB page 232 alongside.

18.2 Finding numbers that make statements true

The numbers 1, 2, 3, 4 and so on that we use for counting are called the **natural numbers**. Natural numbers are **whole numbers** – they do not contain fraction parts.

1. Is there a natural number that can be put in the brackets below to make the statement true?
   $12 + (5) = 17$
2. In each case below, copy the statement and insert a natural number in the space between the brackets that will make the statement true.
   (a) $15 + (6) = 21$
   (b) $15 - (5) = 10$
   (c) $(24) + 10 = 34$
   (d) $(44) - 10 = 34$
   (e) $3 \times (6) = 18$

Here is a different way to ask the same questions:
(a) What is x if $15 + x = 21$?
(b) What is x if $15 - x = 10$?
(c) What is x if $x + 10 = 34$?
(d) What is x if $x - 10 = 34$?
(e) What is x if $3 \times x = 18$?

3. (a) Can you think of a natural number that will make this statement true?
   $2 \times (....) = 5$
   (b) Can you think of any other number that will make the statement true?
4. (a) Can you think of a natural number that will make this statement true?
   $8 + (....) = 5$
   (b) Can you think of any other number that will make the statement true?

We normally think of adding as making something bigger. Question 4(a) requires us to change our mind about this. We have to consider the possibility that adding a number may make something smaller.

We are looking for a number that will make the following statement true:

$8 + (....) = 5$

Consider this plan:
Let us agree that we will call this number *negative 3* and write it as (–3).
If we agree to this, we can say $8 + (-3) = 5$.

This may seem a bit strange to you. You do not have to agree now. But even if you do not agree, let us explore how this plan may work for other numbers. What answers will a person who agrees to the plan give to the following question?

5. Calculate each of the following:
   (a) $10 + (-3) = 7$ (b) $12 + (-3) = 9$
   (c) $12 + (-5) = 7$ (d) $10 + (-9) = 1$
   (e) $8 + (-8) = 0$ (f) $1 + (-1) = 0$

What may each of the following be equal to?
$5 + (-8) = -3$
$(-5) + (-8) = -13$

Background information (continued)

- The **sum** of an integer and its additive inverse is always equal to 0.
- The number **0** is regarded as an integer.
- The **additive inverse** can be used to add integers:
  - $20 + (-15) = \boxed{5 + 15} + (-15) = 5 + \boxed{15 + (-15)} = 5 + 0 = 5$
  - $-18 + 12 = \boxed{(-6) + (-12)} + 12 = (-6) + \boxed{(-12) + 12} = (-6) + 0 = -6$

Teaching guidelines (continued)

Investigate how the idea of additive inverses can be used to explain addition of integers.

Answers

6.	(a) –24	(b) 24	(c) 103	(d) –2 348
7.	(a) 13 + 30 + (–30) = 13 + 0 = 13			
	(b) 70 + 80 + (–80) = 70 + 0 = 70			
8.	(a) 10	(b) 10	(c) 3	(d) 3
9.	(a) 30	(b) 30	(c) 12	(d) 12
	(e) 12	(f) 12	(g) 12	(h) 12
	(i) 8	(j) 8	(k) 8	(l) 8
	(m) 8	(n) 8		
10.	(a) 5	(b) 5	(c) 12	(d) 12
	(e) 20	(f) 10	(g) 0	(h) –10
	(i) 2	(j) 4	(k) –6	

You might agree that:

5 + (–5) = 0 10 + (–10) = 0 and 20 + (–20) = 0

We may say that for each "positive" number there is a **corresponding** or **opposite** negative number. Two positive and negative numbers that correspond, for example 3 and (–3), are called **additive inverses**. They wipe each other out when you add them.

When you add any number to its additive inverse, the answer is 0. For example, 120 + (–120) = 0
So, the **set of integers** consists of all the natural numbers and their additive inverses and zero.

The number zero is regarded as an integer.

6. Write the additive inverse of each of the following numbers:
   (a) 24 (b) –24
   (c) –103 (d) 2 348

The idea of additive inverses may be used to explain why 8 + (–5) is equal to 3:

$$8 + (-5) = 3 + \boxed{5 + (-5)} = 3 + 0 = 3$$

7. Use the idea of additive inverses to explain why each of these statements is true:
   (a) 43 + (–30) = 13 (b) 150 + (–80) = 70

8. Calculate each of the following:
   (a) 10 + 4 + (–4) (b) 10 + (–4) + 4
   (c) 3 + 8 + (–8) (d) 3 + (–8) + 8

Natural numbers can be arranged in any order to add and subtract them. It would make things easy if we agree that this should also be the case for negative numbers.

9. Calculate each of the following:
   (a) 18 + 12 (b) 12 + 18
   (c) 2 + 4 + 6 (d) 6 + 4 + 2 (e) 2 + 6 + 4
   (f) 4 + 2 + 6 (g) 4 + 6 + 2 (h) 6 + 2 + 4
   (i) 6 + (–2) + 4 (j) 4 + 6 + (–2) (k) 4 + (–2) + 6
   (l) (–2) + 4 + 6 (m) 6 + 4 + (–2) (n) (–2) + 6 + 4

10. Calculate each of the following:
   (a) (–5) + 10 (b) 10 + (–5)
   (c) (–8) + 20 (d) 20 – 8
   (e) 30 + (–10) (f) 30 + (–20)
   (g) 30 + (–30) (h) 30 + (–40)
   (i) 10 + (–5) + (–3) (j) (–5) + 7 + (–3) + 5
   (k) (–5) + 2 + (–7) + 4

Background information (continued)

- A **closed number sentence** is one in which all numbers are known, for example, 8 + 2 = 10.
- An **open number sentence** is one in which some numbers are not known at first, for example, 8 – (a number) = 10 or 8 – x = 10.

Teaching guidelines

Discuss the difference between closed and open number sentences.

Answers

11. (a) 20 + 30 = 50 (b) 50 + (–30) = 20
(c) 20 + (–10) = 10 (d) 75 + (–25) = 50
(e) (–25) + (–25) = (–50)

12. (a) 43 + (–43) + (–7) = 0 + (–7) = –7
(b) 60 + (–60) + (–25) = 0 + (–25) = –25

STATEMENTS THAT ARE TRUE FOR MANY DIFFERENT NUMBERS

Background information

- Some statements are true for a **limited number of pairs of numbers**, for example, the statement
(natural number) + (natural number) = 10 has five different solutions:
1 + 9 = 10; 2 + 8 = 10; 3 + 7 = 10; 4 + 6 = 10; 5 + 5 = 10
- Some statements are true for an **infinite number of pairs of numbers**, for example, the statement (integer) + (integer) = 10 has infinite different solutions: all those above as well as solutions like
11 + (–1) = 10; 12 + (–2) = 10; 13 + (–3) = 10; 14 + (–4) = 10;
15 + (–5) = 10; etc.

18.3 Adding and subtracting integers

PROPERTIES OF INTEGERS

Background information

- Refer to question 11 above:
  - In question 11(b): 50 [+ (–30)] = 20, but we know that 50 [– 30] = 20.
  - In question 11(c): 20 [+ (–10)] = 10, but we know that 20 [– 10] = 10.
  - In question 11(d): 75 [+ (–25)] = 50, but we know that 75 [– 25] = 50.

11. In each case find the number that makes the statement true. Give your answer by writing a closed number sentence.
(a) 20 + (an unknown number) = 50
(b) 50 + (an unknown number) = 20
(c) 20 + (an unknown number) = 10
(d) (an unknown number) + (–25) = 50
(e) (an unknown number) + (–25) = (–50)

12. Use the idea of additive inverses to explain why each of the following statements is true:
(a) 43 + (–50) = –7 (b) 60 + (–85) = –25

Statements like these are also called number sentences.
An incomplete number sentence, where some numbers are not known at first, is sometimes called an **open number sentence**, for example: 8 – (a number) = 10.
A **closed number sentence** is where all the numbers are known, for example: 8 + 2 = 10.

STATEMENTS THAT ARE TRUE FOR MANY DIFFERENT NUMBERS

For how many different pairs of numbers can the following statement be true, if only natural (positive) numbers are allowed? five pairs

a number + another number = 10

For how many different pairs of numbers can the statement be true if negative numbers are also allowed? an infinite number of pairs

18.3 Adding and subtracting integers

PROPERTIES OF INTEGERS

1. Calculate:
(a) 80 + (–60) (b) 500 + (–200) + (–200)
2. (a) Do you agree that 20 + (–5) = 15?
(b) What do you think 20 – (–5) should be?
3. (a) Is 100 + (–20) + (–20) = 60, or does it equal something else?
(b) What do you think (–20) + (–20) should be equal to?

We normally think of addition and subtraction as actions that have opposite effects: what the one does is the opposite or **inverse** of what the other does.

4. Copy and complete the following as far as you can:

(a)	(b)	(c)
5 – 9 = –4	5 + 9 = 14	9 – 3 = 6
5 – 8 = –3	5 + 8 = 13	8 – 3 = 5
5 – 7 = –2	5 + 7 = 12	7 – 3 = 4

- All these number sentences prove the **addition rule for integers**: **Adding a negative integer is equivalent to subtracting its additive inverse.**
- Addition and subtraction are **inverse operations** (what one does is the opposite of what the other does). This means the following:
  - If 50 + (–30) = 20 then 50 – (–30) = 80, but 50 + 30 = 80.
  - If 20 + (–10) = 10 then 20 – (–10) = 30, but 20 + 10 = 30.
  - If 75 + (–25) = 50 then 75 – (–25) = 100, but 75 + 25 = 100.
- All these number sentences prove the **subtraction rule for integers**: **Subtracting a negative integer is equivalent to adding its additive inverse.**

Teaching guidelines

In question 4 learners should discover the following rules for integers:

- Adding a negative integer is equivalent to subtracting its additive inverse.
- Subtracting a negative integer is equivalent to adding its additive inverse.

Answers

1. (a) 20 (b) 100
2. (a) Yes (b) 25
3. (a) Yes, it equals 60. (b) (–40)
4. See LB page 234 above and page 235 alongside.
5. (a) 0 (b) 30 (c) 0 (d) (–30)
6. (a) 20 + (–12) = 8 (Read as "twenty plus negative twelve", NOT "twenty plus minus twelve".)

   Because 20 = 12 + 8 and 12 + 8 + (–12) = 12 + (–12) + 8 = 8

   (b) 20 + 8 = 28

   Addition of positive numbers.

   (c) 20 – (–8) = 28 (Read as "twenty minus negative eight", NOT "twenty minus minus eight".)

   To subtract a negative number gives the same result as adding a positive number.

   (d) 20 – 8 = 12

   Because the difference between 12 and 20 is 8.

5 – 6 = –1	5 + 6 = 11	6 – 3 = 3
5 – 5 = 0	5 + 5 = 10	5 – 3 = 2
5 – 4 = 1	5 + 4 = 9	4 – 3 = 1
5 – 3 = 2	5 + 3 = 8	3 – 3 = 0
5 – 2 = 3	5 + 2 = 7	2 – 3 = (–1)
5 – 1 = 4	5 + 1 = 6	1 – 3 = (–2)
5 – 0 = 5	5 + 0 = 5	0 – 3 = (–3)
5 – (–1) = 6	5 + (–1) = 4	(–1) – 3 = (–4)
5 – (–2) = 7	5 + (–2) = 3	(–2) – 3 = (–5)
5 – (–3) = 8	5 + (–3) = 2	(–3) – 3 = (–6)
5 – (–4) = 9	5 + (–4) = 1	(–4) – 3 = (–7)
5 – (–5) = 10	5 + (–5) = 0	(–5) – 3 = (–8)

5. Calculate each of the following:

   (a) 20 – 20 (b) 50 – 20

   (c) (–20) – (–20) (d) (–50) – (–20)

6. In each case, suggest a number that may make the statement true. Also give an argument to support your proposal.

   (a) 20 + (a number) = 8 (b) 20 + (a number) = 28

   (c) 20 – (a number) = 28 (d) 20 – (a number) = 12

SOME HISTORY

The following statement is true if the number is 5:

15 – (a certain number) = 10

A few centuries ago, some mathematicians decided they wanted to have numbers that will also make sentences like the following true:

15 + (a certain number) = 10

But to go from 15 to 10 you have to **subtract 5**.

The number we need to make the sentence *15 + (a certain number) = 10* true must have the following strange property:

If you **add** this number, it should have the **same effect** as to **subtract 5**.

Now the mathematicians of a few centuries ago really wanted to have numbers for which such strange sentences would be true. So they thought:

"Let us just decide, and agree amongst ourselves, that the number we call *negative 5* will have the property that if you add it to another number, the effect will be the same as when you subtract the natural number 5."

This means that the mathematicians agreed that 15 + (–5) is equal to 15 – 5.

Stated differently, instead of adding *negative 5* to a number, you may subtract 5.

SOME HISTORY

This passage confirms that:

- adding a negative number has the same effect as subtracting its additive inverse
- subtracting a negative number has the same effect as adding its additive inverse
- the addition rule for integers above can be investigated by comparing patterns P and Q below
- the subtraction rule for integers above can be investigated by comparing patterns R and S below.

Pattern P	Pattern Q	Pattern R	Pattern S
1 + 3 = ...	1 + 3 = ...	–1 – 3 = ...	–1 + (–3) = ...
1 + 2 = ...	1 + 2 = ...	–1 – 2 = ...	–1 + (–2) = ...
1 + 1 = ...	1 + 1 = ...	–1 – 1 = ...	–1 + (–1) = ...
1 + 0 = ...	1 + 0 = ...	–1 – 0 = ...	–1 + 0 = ...
1 + (–1) = ...	1 – 1 = ...	–1 – (–1) = ...	–1 + 1 = ...
1 + (–2) = ...	1 – 2 = ...	–1 – (–2) = ...	–1 + 2 = ...
1 + (–3) = ...	1 – 3 = ...	–1 – (–3) = ...	–1 + 3 = ...
1 + (–4) = ...	1 – 4 = ...	–1 – (–4) = ...	–1 + 4 = ...
1 + (–5) = ...	1 – 5 = ...	–1 – (–5) = ...	–1 + 5 = ...

Misconceptions

- Read 5 – (–5) as "five minus negative five", NOT "five minus minus five".
- Read 5 + (–5) as "five plus negative five", NOT "five plus minus five".
- Read (–5) – 3 as "negative five minus three", NOT "minus five minus three".

Answers

7. (a) 20 + 10 = 30 (b) 100 + 100 = 200 (c) 20 – 10 = 10
   (d) 0 (e) (–10) (f) 0
   (g) (–30) (h) (–200)
8. See LB page 236 alongside.
9. (a) True: (–10) – (+3) = (–10) + (–3) = –13
   (b) False: (–10) + 5 ≠ (+10) + 5

We may agree that subtracting a negative number has the same effect as adding the additive inverse of the negative number. If we stick to this agreement, the following two calculations should have the same answer:

10 – (–7) and 10 + 7

7. Calculate.
   (a) 20 – (–10) (b) 100 – (–100) (c) 20 + (–10)
   (d) 100 + (–100) (e) (–20) – (–10) (f) (–100) – (–100)
   (g) (–20) + (–10) (h) (–100) + (–100)

8. Copy and complete the following as far as you can:

(a)	(b)	(c)
5 – (–9) = 14	(–5) + 9 = 4	9 – (–3) = 12
5 – (–8) = 13	(–5) + 8 = 3	8 – (–3) = 11
5 – (–7) = 12	(–5) + 7 = 2	7 – (–3) = 10
5 – (–6) = 11	(–5) + 6 = 1	6 – (–3) = 9
5 – (–5) = 10	(–5) + 5 = 0	5 – (–3) = 8
5 – (–4) = 9	(–5) + 4 = –1	4 – (–3) = 7
5 – (–3) = 8	(–5) + 3 = –2	3 – (–3) = 6
5 – (–2) = 7	(–5) + 2 = –3	2 – (–3) = 5
5 – (–1) = 6	(–5) + 1 = –4	1 – (–3) = 4
5 – 0 = 5	(–5) + 0 = –5	0 – (–3) = 3
5 – 1 = 4	(–5) + (–1) = –6	(–1) – (–3) = 2
5 – 2 = 3	(–5) + (–2) = –7	(–2) – (–3) = 1
5 – 3 = 2	(–5) + (–3) = –8	(–3) – (–3) = 0
5 – 4 = 1	(–5) + (–4) = –9	(–4) – (–3) = –1
5 – 5 = 0	(–5) + (–5) = –10	(–5) – (–3) = –2

9. In each case, state whether the statement is true or false and give a numerical example to demonstrate your answer.
   (a) Subtracting a positive number from a negative number has the same effect as adding the additive inverse of the positive number.
   (b) Adding a negative number to a positive number has the same effect as adding the additive inverse of the negative number.

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Answers

9. (c) False: (+15) – (–10) ≠ (+15) – (+10)
   (d) True: (+8) + (–10) = (+8) – (+10) = 8 – 10 = –2
   (e) False: (–10) + (+8) ≠ (–10) + (–8)
   (f) True: (–10) + (+8) = (–10) – (–8) = –2
   (g) False: (–10) – (+8) ≠ (–10) – (–8)
   (h) True: (+10) – (–8) = (+10) + (+8) = 18

PROPERTIES OF OPERATIONS

Background information

- Addition of integers is **commutative**, which means that when integers are added, the numbers can be swopped around.
- Addition of integers is **associative**, which means that the order in which you perform the operations makes no difference.

Teaching guidelines

Learners use the addition and subtraction rules for negative numbers to solve problems.

Answers

1. (a) –8 (b) –8 (c) –2 (d) –2 (e) 4
   (f) 4 (g) 34 (h) 34 (i) –36 (j) –36
2. (a) Yes, it is commutative.
   (b) If you add any two integers, the order of the integers does not affect the answer, for example (–2) + 5 = 3 and 5 + (–2) = 3.
3. (a) 4 (b) –4 (c) –10 (d) 10
   (e) 27 (f) –27 (g) –17 (h) 17
4. (a) No, it is not commutative.
   (b) If you change the order of the numbers, you do not get the same answer, for example 9 – 5 ≠ 5 – 9 and (–40) – (–23) ≠ (–23) – (–40).
5. Yes, it is. For example:
   [(–6) + 4] + (–3) = –5 (–6) + [4 + (–3)] = –5 [(–6) + (–3)] + 4 = –5

(c) Subtracting a negative number from a positive number has the same effect as subtracting the additive inverse of the negative number.
(d) Adding a negative number to a positive number has the same effect as subtracting the additive inverse of the negative number.
(e) Adding a positive number to a negative number has the same effect as adding the additive inverse of the positive number.
(f) Adding a positive number to a negative number has the same effect as subtracting the additive inverse of the positive number.
(g) Subtracting a positive number from a negative number has the same effect as subtracting the additive inverse of the positive number.
(h) Subtracting a negative number from a positive number has the same effect as adding the additive inverse of the negative number.

PROPERTIES OF OPERATIONS

1. Calculate the following:
   (a) (–3) + (–5) (b) (–5) + (–3)
   (c) 5 + (–7) (d) (–7) + 5
   (e) (–13) + 17 (f) 17 + (–13)
   (g) 15 + 19 (h) 19 + 15
   (i) (–21) + (–15) (j) (–15) + (–21)

In Chapter 1 (which was about whole numbers) we said that:
addition is commutative, which means that the numbers can be swopped around.
Or, in symbols: $a + b = b + a$, where a and b are whole numbers.

2. (a) Would you say addition is also commutative when the numbers are integers?
   (b) Explain your answer.
3. Calculate the following:
   (a) 9 – 5 (b) 5 – 9
   (c) (–7) – 3 (d) 3 – (–7)
   (e) 15 – (–12) (f) (–12) – 15
   (g) (–40) – (–23) (h) (–23) – (–40)
4. (a) Do you think subtraction is commutative?
   (b) Explain your answer.

In Chapter 1 we also said that when three or more whole numbers are added, the order in which you perform the calculations makes no difference. We say that: **Addition is associative.**

5. Do you think addition is also associative when we work with integers? Investigate.

Grade 7 Term 4 Chapter 19 Numeric patterns

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
19.1 Investigating and extending numeric patterns	Extending patterns in both directions from given values	Pages 238 to 239
19.2 Making patterns from rules	Using recursive rules to generate terms of patterns	Page 239
19.2 Making patterns from expressions	Using expressions to generate output values for given input values; understanding what decreasing and increasing sequences mean	Pages 239 to 241

CAPS time allocation	3 hours
CAPS content specification	Page 68

Mathematical background

We can think of a **sequence** as a list of numbers (or elements) that are in a certain order. There are many mathematical fields where sequences are useful, for example when we study functions, sequences are useful, and the study of series has sequences as its basis.

The **function concept** is a formal description of the relationships between different quantities. The work we did on number patterns leads directly to the function concept.

Patterns can be extended if a few elements are given and the rule can be deduced, for example "add 5".

Patterns, like those mentioned above can be extended, in both directions.

Patterns can be generated if a rule is given, for example "start at 3 and add 11" or "start at 20 and subtract 3".

Patterns or sequences can be generated if a rule is known in the form of an expression, for example "multiply the input by 4 and add 2".

Patterns can be increasing, which means as the term number increases, the value of the term also increases, or they can be decreasing.

To recognise a **number pattern** is an important problem-solving skill. It helps you to be able to generalise towards a solution. It helps learners to understand the use of symbolic forms to represent and analyse mathematical situations. It also helps learners understand how to use mathematical models and analyse change in real and in abstract contexts.

19.1 Investigating and extending numeric patterns

PATTERNS IN TWO DIRECTIONS

Teaching guidelines

Learners have to find the constant difference between successive terms and use it to continue the sequence forwards and backwards. To go forwards, the constant difference should be added, and to go backwards, it should be subtracted. For example, in sequence A, the constant difference is 2. To get the terms that precede 4, learners have to subtract 2 from each term as it is formed to get the one preceding it.

The rules used in this section are quite simple, given as "add the constant difference".

When learners write their own patterns, let them write their rules down as well. You could let them work in pairs and find the rules to each other's sequences.

Misconceptions

Once learners have the constant difference, they add it to go backwards instead of subtracting.

Note on the questions

In this section the main focus is to recognise the rules by which parts of a sequence were formed.

Answers

1. (a) See the completed table on LB page 238 alongside.
   (b) +2
   (c) +3
2. (a) 120; 140; 160
   (b) 280
3. (a) 80; 60; 40
   (b) –80
4. (a) –2
   (b) The difference is not constant.
5. See the completed table on LB page 238 alongside.

CHAPTER 19
Numeric patterns

19.1 Investigating and extending numeric patterns

PATTERNS IN TWO DIRECTIONS

1. The numbers in each row of the table form a sequence, but not all of the numbers are given.

A	–6	–4	–2	0	2	4	6	8	10	12	14	16	18	20
B	20	18	16	14	12	10	8	6	4	2	0	–2	–4	–6
C	–10	–7	–4	–1	2	5	8	11	14	17	20	23	26	29
D	20	22	23	23	22	20	17	13	8	2	–5	–13	–22	–32

   (a) Copy the table and fill in the missing numbers.
   (b) What is the constant difference in sequence A?
   (c) What is the constant difference in sequence C?

2. The first term of a certain sequence is 100 and the constant difference is 20.
   (a) What is the second term, and the third term, and the fourth term?
   (b) What is the tenth term in this sequence?

A constant-difference sequence is formed by adding the constant difference each time to form the next term.

3. The first term of a certain sequence is 100 and the constant difference is –20.
   (a) What is the second term, and the third term, and the fourth term?
   (b) What is the tenth term in this sequence?

4. (a) What is the constant difference in sequence B in question1?
   (b) What is the constant difference in sequence D in question1?

5. The sixth terms of sequences E, F and G are given in the table. Copy the table and fill in the other terms.

Term number	1	2	3	4	5	6	7	8	9
E with constant difference 10	–20	–10	0	10	20	30	40	50	60
F with constant difference –5	55	50	45	40	35	30	25	20	15
G with constant difference –10	80	70	60	50	40	30	20	10	0

Answers

6. (a) 36 49 64 81 (b) 38 51 66 83
   (c) 5 –1 –8 –16 (d) 35 35 34 32
7. Learners' own answer

19.2 Making patterns from rules

Teaching guidelines

Help learners who struggle with the negative numbers to understand what is required.

The sequences in questions 1 and 2 can be checked for correctness.

Learners can work in pairs to make their own sequences from question 3 to question 8. Let them first write their sequences and then explain to a partner what their rule was. They can check each other's sequences for correctness.

Answers

1.	(a)	30	25	20	15	10	5	0	–5	–10	–15
	(b)	–30	–35	–40	–45	–50	–55	–60	–65	–70	–75
	(c)	–30	–25	–20	–15	–10	–5	0	5	10	15
2.	(a)	–10	–5	0	5	10	15	20	25	30	35
	(b)	–10	–15	–20	–25	–30	–35	–40	–45	–50	–55

3. to 8. Learners' own answers.

19.3 Making patterns from expressions

Teaching guidelines

Remind learners that they use the values of x as input and the output is given by the calculated values of the expression.

Let them check each other's substitution if necessary, i.e. if there are learners who cannot substitute correctly and get the wrong answers.

Misconceptions

Some learners may forget the order of operations and do the calculation incorrectly.

Answers

1. (a) See the completed table on LB page 239 alongside.

6. Investigate each of the patterns below. Find the pattern and write the next four terms in the sequence.
   (a) 1 4 9 16 25 (b) 3 6 11 18 27
   (c) 20 19 17 14 10 (d) 20 25 29 32 34
7. Make some numeric patterns of your own.

19.2 Making patterns from rules

1. (a) Start at 30. Add –5 and write the answer. Add –5 again and write the answer. Continue until you have a number sequence with 10 terms.
   (b) Start at –30. Add –5 and write the answer. Add –5 again and write the answer. Continue until you have a number sequence with 10 terms.
   (c) Start at –30. Add 5 and write the answer. Add 5 again and write the answer. Continue until you have a number sequence with 10 terms.
2. (a) The first term of a sequence is –10 and there is a constant difference of 5 between the terms. Write down the first ten terms of the sequence.
   (b) The first term of a sequence is –10 and there is a constant difference of –5 between the terms. Write down the first ten terms of the sequence.
3. Choose a number to be your first term and another number to be a constant difference. Write the first ten terms of your sequence.
4. Choose a number smaller than –10 to be your first term and another number to be a constant difference. Write the first ten terms of your sequence.
5. Choose a number to be your first term and a negative number to be a constant difference. Write the first ten terms of your sequence.
6. Choose a negative number to be your first term and another negative number to be a constant difference. Write the first ten terms of your sequence.
7. Choose a number to be your tenth term and another number to be a constant difference. Write the first ten terms of your sequence.
8. Choose a negative number to be your tenth term and another negative number to be a constant difference. Write the first ten terms of your sequence.

19.3 Making patterns from expressions

1. (a) Copy and complete the table.

x	0	1	2	3	4	5	6	7	8
$2 \times x - 10$	–10	–8	–6	–4	–2	0	2	4	6

Answers

1. (b) Yes, the constant difference is +2.
   (c) See the completed table on LB page 240 alongside.
   (d) +3
   (e) See the completed table on LB page 240 alongside.
   (f) One may say 3, but since the values decrease by 3, we may describe the difference as –3.
   (g) See the completed table on LB page 240 alongside.
   (h) –2
2. (a) –15 (b) –27 (c) –35
3. (a) 29 (b) 17 (c) 53 (d) 41
4. 5 and 7; or 9 and 11; or any two consecutive odd numbers bigger than 5

Teaching guidelines

Talk about the words we use:

- **consecutive** means one follows directly on the other
- **constant** or **common difference** means if consecutive terms are subtracted, the differences are the same.

Discuss ways that the expressions in question 5 can be matched with the correct sequence. They could use substitution or find the common difference between the terms and match it with an expression where the variable is multiplied by that number. For example, see question 5 on LB pages 240–241, sequence A has a difference of 5 between terms, so test expression (d) to see if it generates the given terms of the sequence.

Explain the meaning of an increasing sequence and a decreasing sequence and connect the idea to the coefficient of the variable (the value by which x is multiplied) – if it is positive, the sequence increases, but if it is negative, the sequence decreases. The rate at which the sequence increases or decreases is determined by the magnitude of the coefficient.

(b) Do the output values of $2 \times x - 10$ in the previous table form a pattern with a constant difference? If they do, what is the constant difference?

(c) Copy and complete the table.

x	0	1	2	3	4	5	6	7	8
$3 \times x - 20$	–20	–17	–14	–11	–8	–5	–2	1	4

(d) What is the constant difference in (c)?

(e) Copy and complete the table.

x	0	1	2	3	4	5	6	7	8
$2 - 3 \times x$	2	–1	–4	–7	–10	–13	–16	–19	–22

(f) What is the constant difference in (e)?

(g) Copy and complete the table.

x	0	1	2	3	4	5	6	7	8
$1 - 2 \times x$	1	–1	–3	–5	–7	–9	–11	–13	–15

(h) What is the constant difference in (g)?

2. Look at the pattern: –15; –19; –23; –27; –31; . . .
   In this pattern, –19 is followed by –23 and –23 is followed by –27.
   (a) What number in the pattern is followed by –19?
   (b) What number in the pattern is followed by –31?
   (c) In the pattern, –19 follows on –15 and –23 follows on –19. What number follows on –31?
3. A certain pattern is formed by a common difference of 6.
   (a) What number follows on 23 in this pattern?
   (b) What number is followed by 23 in this pattern?
   (c) What number follows on 47 in this pattern?
   (d) What number is followed by 47 in this pattern?

Consider the sequence: 10 6 2 –2 –6 . . . .
In this sequence, 2 follows on 6. They are called consecutive terms.

When one number follows another in a sequence they are called **consecutive terms**.

4. Write down any two consecutive terms in the pattern formed by $2 \times x + 3$, when the input numbers are consecutive whole numbers.
5. Each pattern on the following page was formed by using one of the following expressions. Establish which pattern belongs to each expression.

Answers

5. See LB page 241 alongside.
6. (a) A, B, C, E, G and H
   (b) D, F and I
7. (a) 5
   (b) 4
   (c) H increases by the biggest amount, namely 7 from one term to the next.
8. (a) I
   (b) D
9. (a) For example: 50 42 34 26 18
   (b) For example: 50 52 54 56 58
10. (a) $6 \times x - 1$
    (b) One way is to write some terms of the sequence for each of the expressions.
11. increasing decreasing increasing increasing

(a) $2 \times x + 5$	(b) $3 \times x + 2$	(c) $4 \times x + 1$
(d) $5 \times x + 6$	(e) $6 \times x - 5$	(f) $7 \times x - 2$
(g) $1 - 4 \times x$	(h) $5 - 5 \times x$	(i) $-5 - 6 \times x$

Pattern A:	6	11	16	21	26	Expression (d)
Pattern B:	13	17	21	25	29	Expression (c)
Pattern C:	20	23	26	29	32	Expression (b)
Pattern D:	1	–3	–7	–11	–15	Expression (g)
Pattern E:	31	33	35	37	39	Expression (a)
Pattern F:	–20	–25	–30	–35	–40	Expression (h)
Pattern G:	25	31	37	43	49	Expression (e)
Pattern H:	26	33	40	47	54	Expression (f)
Pattern I:	–11	–17	–23	–29	–35	Expression (i)

Sequence I in question 5 is a **decreasing** sequence; the numbers become smaller as the sequence progresses:

–11 –17 –23 –29 –35

Sequence H is an **increasing** sequence; each term is bigger than the previous term:

26 33 40 47 54

6. (a) Which sequences in question 5 are increasing sequences?
   (b) Which sequences in question 5 are decreasing sequences?
7. (a) By how much does sequence A increase from one term to the next?
   (b) By how much does sequence B increase from one term to the next?
   (c) Which of the sequences in question 5 increases by the biggest amount from one term to the next, and by how much does it increase?

Sequence G increases by 6 from term to term, and sequence E increases only by 2. We may say that sequence G **increases faster** than sequence E.

8. (a) Which of the sequences in question 5 decreases fastest?
   (b) Which of the sequences in question 5 decreases slowest?
9. (a) Write five consecutive terms of a sequence which decreases faster than sequence D in question 5.
   (b) Write five consecutive terms of a sequence which increases slower than sequence B in question 5.
10. (a) Each of the expressions below can be used to produce a sequence. Which of the expressions will produce the sequence that increases fastest?
    $3 \times x + 5$ $2 \times x + 10$ $6 \times x - 1$ $20 + 3 \times x$ $4 \times x - 9$
    (b) Think of a way in which you can test your answer, and do it.
11. In each case, state whether the sequence will be decreasing or increasing.
    $10 + 3 \times x$ $10 - 3 \times x$ $10 \times x + 3$ $3 \times x - 10$

Grade 7 Term 4 Chapter 20 Functions and relationships 3

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
20.1 Relationships between variables	Different ways to describe a rule for a relationship; tables to words; tables to flow diagrams; formulae to tables	Pages 242 to 245
20.2 Integers in the rules for relationships	Rules that have integers as constants	Pages 245 to 246

CAPS time allocation	3 hours
CAPS content specification	Page 68

Mathematical background

In algebra, a multiplication sign between a number and a letter symbol, or between two letter symbols, does not need to be written, for example

- $2 \times x$ and $a \times b$ can be written as $2x$ and ab respectively.
- Also, $5(x + 2)$ means $5 \times (x + 2)$.

A relationship between two variables can be described:

- in words
- in tables
- using flow diagrams
- using an expression like $3x - 7$
- using a formula like $y = 3x - 7$.

When we work with rules like $-2x + 5$ care must be taken when substituting negative numbers, for example if $x = -3$: $-2(-3) + 5 = 6 + 5 = 11$.

20.1 Relationships between variables

DIFFERENT WAYS TO REPRESENT THE RULE FOR A RELATIONSHIP

Teaching guidelines

Learners have already worked with the different ways in which a relationship can be represented. Remind them that we work with input numbers to which we apply a rule to get output numbers.

Show learners that a relationship can be represented by:

- a description in words
- a flow diagram
- a table of values
- an expression, such as $x \times 7 + 25$
- a formula, such as $y = 7 \times x + 25$.

Notes on questions

In this section learners substitute input values into the rules to get output values.

Answers

1. (a) $7 \times 10 + 25 = 70 + 25 = 95$
   (b) $7 \times 20 + 25 = 140 + 25 = 165$
   (c) $7 \times 5 + 25 = 35 + 25 = 60$
   (d) $7 \times 15 + 25 = 105 + 25 = 130$
2. (a) 25
   (b) 55
   (c) 70
   (d) 295

CHAPTER 20 Functions and relationships 3

20.1 Relationships between variables

DIFFERENT WAYS TO REPRESENT THE RULE FOR A RELATIONSHIP

A relationship between two variables consists of two sets of numbers as shown in the two rows of the table below. The first row contains the **input numbers** and the second row contains the **output numbers**.

x	1	2	3	4	5	6	7	8	9
y	32	39	46	53	60	67	74	81	88

For the relationship shown in the table, any output number can be calculated by multiplying the input number by 7 and adding 25 to the answer.

The way in which an output number can be calculated is called the **rule** for the relationship. The rule can be described in **words** or with a **formula**, and in some cases with a **flow diagram**.

The input numbers may also be called the values of the **input variable**, and the output numbers may also be called the values of the **output variable**.

The rule "multiply by 7 and add 25" can be represented with this flow diagram:

→ [× 7] → [+ 25] →

The same rule can also be represented with the formula below:

$y = 7 \times x + 25$

1. Calculate the value of $7 \times x + 25$ for each of the following values of x:
   (a) $x = 10$ (b) $x = 20$
   (c) $x = 5$ (d) $x = 15$
2. (a) What is the value of $3 \times x - 5$ if $x = 10$?
   (b) What is the value of $3 \times x - 5$ if $x = 20$?
   (c) What is the value of $3 \times x - 5$ if $x = 25$?
   (d) What is the value of $3 \times x - 5$ if $x = 100$?

Teaching guidelines

We introduce the following two new ideas in this section:

- When we know the output value and need to calculate the corresponding input value, we can write an equation, for example $3 \times x - 5 = 25$ means that the value of x is now an unknown in the equation.
- In an expression like $3 \times x - 5$, the multiplication sign can be left out so that we get $3x - 5$, which is a shorter way of writing expressions.

Misconceptions

Learners misinterpret expressions in flow diagram form like $-5 \longrightarrow \times 3$ and write it as an expression as $x - 5 \times 3$, not inserting brackets. The correct expression would be $(x - 5) \times 3$.

Notes on questions

Learners do not have to solve equations formally, but they may intuitively reverse the flow diagram.

Answers

3. See the completed table on LB page 243 alongside.
4. $3 \times x - 5 = 61$
5. (a) See LB page 243 alongside.
   (b) Flow diagram C
6. (a) $y = 3(x - 5)$
   (b) $y = 5x + 3$ and $y = 3 + 5x$

FORMULAE FOR TABLES

Teaching guidelines

Let learners work in pairs and find the rule and the missing values of the output values. They should find the difference between successive terms, which is 8 in question 1. If x is multiplied by 8, the amount still to be added to get 13 is 5. Therefore, the rule is $y = 8x + 5$. Let learners write a flow diagram for the rule and also give the rule in words.

Answers

1. (a) If $x = 3$, y is 29   If $x = 7$, y is 61   If $x = 8$, y is 69

3. Copy and complete the table for the values of x and $3 \times x - 5$ given in the table.

x	0	1	2	5	15	22	50	200	333
$3 \times x - 5$	–5	–2	1	10	40	61	145	595	994

4. When you worked out the input number that corresponds to the output number 994 in question 3, you solved the **equation** $3 \times x - 5 = 994$.
   Write the equation that you solved when you worked out the input number that corresponds to the output number 61.

5. (a) Express each of the rules below in words.

   A $\rightarrow$ [× 5] $\rightarrow$ [+ 3] $\rightarrow$ Multiply by 5 and then add 3.

   B $\rightarrow$ [– 5] $\rightarrow$ [× 3] $\rightarrow$ Subtract 5 and then multiply by 3.

   C $\rightarrow$ [× 3] $\rightarrow$ [– 5] $\rightarrow$ Multiply by 3 and then subtract 5.

   (b) Which of the above flow diagrams represent the same calculations as the expression $3 \times x - 5$?

Instead of $3 \times x - 5$, we may write $3x - 5$.
$3x$ means $3 \times x$.
The multiplication sign can be left out.
Instead of $3 \times (x - 5)$, we may write $3(x - 5)$.

6. (a) Which of the formulae below provide the same information as flow diagram B in question 5?

   $y = 5x - 3$ $y = 3 + 5x$ $y = 5(x - 3)$
   $y = 3x - 5$ $y = 5x + 3$ $y = 3(x - 5)$

   (b) Which of the above formulae provide the same information as flow diagram A in question 5?

FORMULAE FOR TABLES

1. The table below shows the values of y that correspond to some of the given values of x. In this case, the output numbers form a pattern with a constant difference if the input numbers are the natural numbers.

x	1	2	3	4	5	6	7	8
y	13	21		37	45	53		

   (a) Find the output numbers that correspond to the input numbers 3, 7 and 8.

1. (b) If $x = 20$, y is 165
If $x = 21$, y is 173
If $x = 22$, y is 181
(c) $y = 8x + 5$
2. See the completed tables on LB page 244 alongside.
3. See the completed tables on LB page 244 alongside.
4. See the completed tables at the top of LB page 245 on the following page.

(b) Find the output numbers that correspond to the input numbers 20, 21 and 22.
(c) Which of the formulae below is the rule for the relationship between x and y in the previous table?
$y = 10x + 3$ $y = 8x + 5$ $y = 6x + 7$ $y = 4x + 9$ $y = 2x + 11$

2. Copy and complete the tables below for the formulae in question 1(c).

x	1	2	3	4	5	6	7	8
$10x + 3$	13	23	33	43	53	63	73	83

x	1	2	3	4	5	6	7	8
$8x + 5$	13	21	29	37	45	53	61	69

x	1	2	3	4	5	6	7	8
$6x + 7$	13	19	25	31	37	43	49	55

x	1	2	3	4	5	6	7	8
$4x + 9$	13	17	21	25	29	33	37	41

x	1	2	3	4	5	6	7	8
$2x + 11$	13	15	17	19	21	23	25	27

3. In each table in question 2, the output numbers form a number pattern with a constant difference between consecutive terms. What is the constant difference in the pattern generated by each of the following expressions, when the input numbers are consecutive natural numbers? Copy and complete the table below. Also fill in the values of the expressions for $x = 0$ in the last column.

Expression	Constant difference between output numbers	Value of the expression for $x = 0$
$2x + 11$	2	11
$4x + 9$	4	9
$6x + 7$	6	7
$8x + 5$	8	5
$10x + 3$	10	3

4. What do you think the constant differences between consecutive output numbers, and the values of the expressions for $x = 0$ may be in each of the following cases, when the input numbers are consecutive natural numbers?

20.2 Integers in the rules for relationships

RULES THAT MAY LOOK STRANGE AT FIRST

Teaching guidelines

Explain that we usually write the expressions with the variable term first, for example it is not wrong to write $5 + 12x$, but we normally write it as $12x + 5$.

Care should be taken when there are negative numbers in an expression, for example $3 - 10x$ can be written as $-10x + 3$ and not as $-3 + 10x$.

Misconceptions

Learners do not keep the sign and the value of a negative number together, in other words, rewriting an expression like $50 - 4x$ they write $4x - 50$. Let learners describe the original expression in words so that they can understand the order of the operations.

Answers

1. See answers on LB page 245 alongside.
2. A. Multiply by 3 and then subtract 5
   B. Multiply by 3 and then add (–5)
   C. Subtract 3 and then multiply by 5
   D. Add (–3) and then multiply by 5
   E. Multiply by 3 and then subtract (–5)
   F. Multiply by 3 and then add 5
3. (a) $-(10 \times 5) + 3 = -50 + 3 = -47$
   (b) $-(10 \times 10) + 3 = -100 + 3 = -97$
   (c) $-(10 \times 20) + 3 = -200 + 3 = -197$
   (d) $-(10 \times 1) + 3 = -10 + 3 = -7$
4. A. $y = 3x - 5$ B. $y = 3x + (-5)$
   C. $y = 5(x - 3)$ D. $y = 5(x + (-3))$
   E. $y = 3x - (-5)$ F. $y = 3x + 5$
5. (a) $20 - 5x$ $(-5x) + 20$ $20 + (-5x)$
   (b) $20 + 5x$ $5x + 20$ $20 - (-5x)$ $5(x + 4)$
   (c) $5x - 20$ $(-20) - (-5x)$ $-((-5x) + 20)$

Copy and complete the table.

Expression	Constant difference between output numbers	Value of the expression for $x = 0$
$5x + 7$	5	7
$3x + 10$	3	10
$12x + 5$	12	5
$5x - 5$	5	–5
$(-10x) + 3$	–10	3

20.2 Integers in the rules for relationships

RULES THAT MAY LOOK STRANGE AT FIRST

1. Copy and complete the flow diagrams.

A. 5 → [× 3] → [– 5] → 10   B. 5 → [× 3] → [+ (–5)] → 10

C. 5 → [– 3] → [× 5] → 10   D. 5 → [+ (–3)] → [× 5] → 10

E. 5 → [× 3] → [– (–5)] → 20   F. 5 → [× 3] → [+ 5] → 20

2. Describe each rule in question 1 in words, for example: "*multiply by 6 and then add –3*".

The rule *multiply by 6 and subtract the answer from 100* can be expressed with the formula $y = 100 - 6x$. This formula can also be written as $y = 100 + (-6x)$ or as $y = (-6x) + 100$.

The brackets around the $-6x$ can be left out, so the last formula above can also be written as $y = -6x + 100$.

3. Calculate y if $y = -10x + 3$, for each of the following values of x:
   (a) $x = 5$ (b) $x = 10$
   (c) $x = 20$ (d) $x = 1$
4. Describe each of the rules in question 1 with a formula, for example $y = 5x + 8$.
5. In each case below, predict which of the different expressions will produce the same results. You will test your predictions later, and can then mark your own answers for this question.
   (a) $20 - 5x$ $5x - 20$ $(-5x) + 20$ $20 + (-5x)$
   (b) $20 + 5x$ $5x + 20$ $20x + 5$ $20 - (-5x)$ $5(x + 4)$
   (c) $5x - 20$ $20x - 5$ $(-20) - (-5x)$ $-((-5x) + 20)$

Notes on questions

Rules like $5x + 20$ and $-5x + 20$ may seem different, but learners must be aware that there may be values of x for which the expressions could have the same value, in this case $x = 0$ (both expressions equal 20).

Answers

6. See the completed table on LB page 246 alongside.
7. (a) $x = 0$
   (b) all values of x
   (c) all values of x
   (d) no values of x
   (e) all values of x

6. Copy and complete the table below and then use the results to carefully check your answers to question 5.

x	0	1	5	10	100
$20 - 5x$	20	15	–5	–30	–480
$5x - 20$	–20	–15	5	30	480
$(-5x) + 20$	20	15	–5	–30	–480
$20 + (-5x)$	20	15	–5	–30	–480
$20 + 5x$	20	25	45	70	520
$5x + 20$	20	25	45	70	520
$20x + 5$	5	25	105	205	2 005
$20 - (-5x)$	20	25	45	70	520
$5(x + 4)$	20	25	45	70	520
$5x - 20$	–20	–15	5	30	480
$20x - 5$	–5	15	95	195	1 995
$(-20) - (-5x)$	–20	–15	5	30	480
$-((-5x) + 20)$	–20	–15	5	30	480

7. In each case below, use your results in the above table or other methods to establish for which values of x the two expressions have the same value(s).
   (a) $20 - 5x$ and $20 + 5x$
   (b) $20 - 5x$ and $(-5x) + 20$
   (c) $5x - 20$ and $(-20) - (-5x)$
   (d) $5(x + 4)$ and $5x - 20$
   (e) $20 + 5x$ and $20 - (-5x)$

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
21.1 Interpret rules to calculate values of a variable	Rules to calculate values if variables are given in words and are translated into expressions; expressions are translated into words; rules are compared to find different forms of the same rule; brackets are removed	Pages 247 to 249
21.2 Slightly different kinds of rules	Integers are used as constants in the expressions; additive inverses are used to write rules for relationships as expressions	Pages 249 to 251

CAPS time allocation	3 hours
CAPS content specification	Page 69

Mathematical background

Algebraic expressions are instructions to calculate values.

The **independent variable**, is the input. The expression tells you what calculations should be performed on the input variable to produce an answer, called the output variable. In this way, the expression forms the rule for a relationship.

The **constants** in the expression can be negative numbers and the input values can also be integers.

An understanding of the **additive inverse** of numbers is important when we work with integers in expressions.

Rules can be simplified to make the number of calculations less or easier, for example a rule like:

- $3x + 7 + 2x - 3$ can be written as $5x + 4$
- $3(x + 2)$ can be written as $3x + 6$
- $22 - (-3x)$ can be written as $22 + 3x$, and so on.

21.1 Interpret rules to calculate values of a variable

RULES IN VERBAL AND SYMBOLIC FORM

Teaching guidelines

Learners use the rule given in words to complete the table in question 1. Let learners read through the explanation and discuss in small groups or pairs how to write the rule as an expression.

As they work through the questions remind them to be aware of the expressions that give the same values.

Misconceptions

Learners remove brackets incorrectly so they may think that $3(x + 5)$ and $3x + 5$ give the same values (and are therefore equal).

Notes on the questions

By describing rules in words, learners can become aware of which expressions are equivalent, which means they give the same output values for the same input values. This teaches them about removing brackets correctly; inserting brackets; when to do multiplication first and how to do operations when brackets are involved.

Answers

1. See the completed table on LB page 247 alongside.
2. (a) Multiply the number by 15 and then add 30 to the answer.
   (b) Multiply the number by 15 and then add 30 to the answer.
   (c) Add 30 to the number and multiply the answer by 15.
   (d) Add 2 to the number and multiply the answer by 15.
   (e) Multiply the number by 15 and subtract 30 from the answer.
   (f) Subtract 30 from the number and multiply the answer by 15.
   (g) Subtract 2 from the number and multiply the answer by 15.
3. $3(x + 5)$ means you first add 5 to the number and then multiply the answer by 3, while $3x + 5$ means you first multiply the number by 3 and then add 5 to the answer. The difference is 10.
4. See the completed table on LB page 247 alongside.

CHAPTER 21

Algebraic expressions 2

21.1 Interpret rules to calculate values of a variable

RULES IN VERBAL AND SYMBOLIC FORM

1. Copy the table below. Do this to each of the numbers in the top row of the table, and write your answers in the bottom row: *multiply the input number by 20 and add 50 to the answer.*

x	1	2	3	4	5	6	7	8	9
y	70	90	110	130	150	170	190	210	230

The sentence *multiply the input number by 20 and add 50 to the answer* is the rule that describes how the output number that corresponds to each input number in the above relationship between the variables x and y can be calculated.

The same rule can be described with the algebraic expression $20x + 50$. In this expression, the symbol x represents the input variable (the values of x). The numbers 20 and 50 are constant; they remain the same for all the different values of x.

The rule *add 50 to the input number and multiply the answer by 20* can be described with the expression $20(x + 50)$.

If there are no brackets in an expression, multiplication is done first, even if it appears later in the expression like in $30 + 5x$.

If there are brackets in an algebraic expression, the operations in brackets are to be done first.

2. Describe each of the following rules in words.
   (a) $15x + 30$ (b) $30 + 15x$ (c) $15(x + 30)$
   (d) $15(x + 2)$ (e) $15x - 30$ (f) $15(x - 30)$ (g) $15(x - 2)$
3. What is the difference between $3(x + 5)$ and $3x + 5$?
4. Copy and complete the table.

x	1	2	3	4	5	6	7	8	9
$15x + 30$	45	60	75	90	105	120	135	150	165
$30 + 15x$	45	60	75	90	105	120	135	150	165
$15(x + 30)$	465	480	495	510	525	540	555	570	585
$15(x + 2)$	45	60	75	90	105	120	135	150	165

Answers

5. See the completed table on LB page 248 alongside.
6. (a) A, C and D will produce the same output numbers.
   See the completed table on LB page 248 alongside.
   (b) A: $10x + 20$
   B: $(x + 20) \times 10$ or $10(x + 20)$
   C: $(x + 2) \times 10$ or $10(x + 2)$
   D: $3x + 15 + 7x + 5$
7. (a) A, D, E and F
   (b) A: Multiply the number by 5 and then add 20.
   B: Multiply the number by 4 and then add 19.
   C: Add 20 to the number and then multiply the answer by 5.
   D: Multiply the number by 5 and then add 20.
   E: Add 4 to the number and then multiply the answer by 5.
   F: Multiply the number by 3, add 7, add twice the number and then add 13.
   (c) See the completed table on LB page 248 alongside.
   (d) Learners can work in pairs to check their answers.

5. Copy and complete the table.

x	30	40	50	60	70	80	90
$15x - 30$	420	570	720	870	1 020	1 170	1 320
$15(x - 30)$	0	150	300	450	600	750	900
$15(x - 2)$	420	570	720	870	1 020	1 170	1 320

6. (a) Investigate which of the following rules will produce the same output numbers. You need to check for several different input numbers.
   A: Multiply the input number by 10 and then add 20.
   B: Add 20 to the input number and then multiply by 10.
   C: Add 2 to the input number and then multiply by 10.
   D: Multiply the input number by 3, add 15, add 7 times the input number, and then add 5.

x	1	2	3	4	5	6	7	8
A	30	40	50	60	70	80	90	100
B	210	220	230	240	250	260	270	280
C	30	40	50	60	70	80	90	100
D	30	40	50	60	70	80	90	100

   (b) Describe each of the above rules with an algebraic expression.

7. (a) Which of these rules do you think will produce the same output numbers?
   A: $5x + 20$ B: $4x + 19$ C: $5(x + 20)$
   D: $20 + 5x$ E: $5(x + 4)$ F: $3x + 7 + 2x + 13$
   (b) Express each of the above rules in words.
   (c) Copy and complete this table for the rules given in question (a).

x	0	5	10	15
$5x + 20$	20	45	70	95
$4x + 19$	19	39	59	79
$5(x + 20)$	100	125	150	175
$20 + 5x$	20	45	70	95
$5(x + 4)$	20	45	70	95
$3x + 7 + 2x + 13$	20	45	70	95

   (d) Use your completed table to check your answer in question (a).

Answers

8. (a) A, E and F.

   Learners should be allowed to check their answers themselves when they have done question (c).

   (b) A: Multiply the number by 5 and subtract 20 from the answer.

   B: Multiply the number by 5 and subtract the answer from 20.

   C: Subtract 20 from the number and multiply the answer by 5.

   D: Multiply the number by 3 and subtract 18 from the answer.

   E: Subtract 4 from the number and multiply the answer by 5.

   F: Multiply the number by 9, add 10, then subtract 4 times the number and then subtract 30.

   (c) See the completed table on LB page 249 alongside.

21.2 Slightly different kinds of rules

SUBTRACT POSITIVE AND NEGATIVE QUANTITIES

Teaching guidelines

Use the tables to show learners that expressions like $-100 + 5x$ and $5x - 100$ give the same values, the only difference is the order of the terms in the expression. Use this to reaffirm the idea that we can change the order when adding, but where negative numbers are involved, care should be taken. For example, with $5 + 7x = 7x + 5$, we can exchange the terms, but if we have $5 - 7x$, we should think of it as $5 + (-7x)$, so if we want to change the positions, we have to write $(-7x) + 5$ and we can leave out the bracket and write $-7x + 5$.

Misconceptions

Learners write $10x - 5$ and $5 - 10x$ as if they were the same and give the same values.

Answers

1. See the completed table on LB page 249 alongside.
2. (a) See the completed table on LB page 249 alongside.

   (b) $100 - 5x$ and $5 - 10x$

   (c) The values increase.

8. (a) Which of these rules do you think will produce the same output numbers?

   A: $5x - 20$ B: $20 - 5x$ C: $5(x - 20)$

   D: $3x - 18$ E: $5(x - 4)$ F: $9x + 10 - 4x - 30$

   (b) Express each of the above rules in words.

   (c) Copy and complete this table for the rules given in question (a).

x	20	30	40	50	60	70	80	90
$5x - 20$	80	130	180	230	280	330	380	430
$20 - 5x$	–80	–130	–180	–230	–280	–330	–380	–430
$5(x - 20)$	0	50	100	150	200	250	300	350
$3x - 18$	42	72	102	132	162	192	222	252
$5(x - 4)$	80	130	180	230	280	330	380	430
$9x + 10 - 4x - 30$	80	130	180	230	280	330	380	430

   (d) Use your completed table to check your answer to question (a).

21.2 Slightly different kinds of rules

SUBTRACT POSITIVE AND NEGATIVE QUANTITIES

1. Copy and complete the table.

x	1	10	5	20	25
$10x$	10	100	50	200	250
$50 - 10x$	40	–50	0	–150	–200
$20 - 10x$	10	–80	–30	–180	–230
$0 - 10x$	–10	–100	–50	–200	–250

2. (a) Copy and complete the table.

x	0	5	10	15	20	25	30
$10x - 5$	–5	45	95	145	195	245	295
$5x - 10$	–10	15	40	65	90	115	140
$100 - 5x$	100	75	50	25	0	–25	–50
$-100 + 5x$	–100	–75	–50	–25	0	25	50
$5x - 100$	–100	–75	–50	–25	0	25	50
$5 - 10x$	5	–45	–95	–145	–195	–245	–295

   (b) The values of $10x - 5$ *increase* as the values of x increase from 0 to 30. For which expressions in (a) do the values *decrease* when x is increased?

   (c) Do the values of $-100 + 5x$ *increase* or *decrease* when x is increased from 0 to 30?

Answers

3. (a) They will keep on increasing.
   (b) $100 - 3x$ decreases as x increases, because the bigger x is, the more is subtracted from 100.
4. –20 –30 25 20 –40
5. See the completed table on LB page 250 alongside.
6. See the completed table on LB page 250 alongside.
7. See the completed table on LB page 250 alongside.

3. (a) The values of the expression $5x - 10$ increase when x is increased from 0 to 30. Do you think the values will increase further when x is increased beyond 30, or will they start to decrease at some stage?
   (b) Do you think the values of the expression $100 - 3x$ will increase when x is increased from 0 to 30? Explain why you think they will or will not.

The additive inverse of a number may be indicated by writing a negative sign before the number. For example, the additive inverse of 8 can be written as –8.

When a number is added to the number called its additive inverse, the answer is 0. For example, 45 + (–45) = 0 and (–12) + 12 = 0.

4. Write the additive inverse of each of the following numbers:
   20 30 –25 –20 40
5. Different values for x are given in the first row of the table below. Copy the table. Write the additive inverses of the x values in the second row, and then complete the table.

x	5	10	15	20	25	30
the additive inverse of x	–5	–10	–15	–20	–25	–30
20 + (the additive inverse of x)	15	10	5	0	–5	–10
20 – (the additive inverse of x)	25	30	35	40	45	50
$20 + x$	25	30	35	40	45	50
$20 - x$	15	10	5	0	–5	–10

6. Copy and complete the table.

x	–5	–10	–15	–20	–25	–30
the additive inverse of x	5	10	15	20	25	30
20 + (the additive inverse of x)	25	30	35	40	45	50
20 – (the additive inverse of x)	15	10	5	0	–5	–10
$20 + x$	15	10	5	0	–5	–10
$20 - x$	25	30	35	40	45	50

7. Copy and complete the table.

x	3	2	1	0	–1	–2	–3
$-x$	–3	–2	–1	0	1	2	3
$5 + (-x)$	2	3	4	5	6	7	8
$5 - (-x)$	8	7	6	5	4	3	2
$5 - x$	2	3	4	5	6	7	8
$5 + x$	8	7	6	5	4	3	2

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EXPRESSIONS WITH ADDITIVE INVERSES

Teaching guidelines

Stress the fact that a number and its additive inverse add up to 0.

Show learners that adding the additive inverse of a number is the same as subtracting the number by using the values in the tables alongside.
For example: 5 + (– 5) = 5 – 5.

Misconceptions

Learners ignore the negative sign and for example, work with 100 – (–10*x*) as if it were 100 – 10*x*.

Answers

1. See the completed table on LB page 251 alongside.
2. See the completed table on LB page 251 alongside.
3. See the completed table on LB page 251 alongside.

EXPRESSIONS WITH ADDITIVE INVERSES

1. Copy and complete the table.

x	1	5	10	20	25
5*x*	5	25	50	100	125
the additive inverse of 5*x*	–5	–25	–50	–100	–125
20 + (the additive inverse of 5*x*)	15	–5	–30	–80	–105
20 – (the additive inverse of 5*x*)	25	45	70	120	145
3*x*	3	15	30	60	75
–3*x*	–3	–15	–30	–60	–75
10 + (–3*x*)	7	–5	–20	–50	–65
10 – 3*x*	7	–5	–20	–50	–65
10 – (–3*x*)	13	25	40	70	85

2. Copy and complete the table below.
Note that (–10*x*) indicates the additive inverse of 10*x*.

x	1	2	3	4	–4	–3	–2
10*x* – 1 000	–990	–980	–970	–960	–1 040	–1 030	–1 020
1 000 – (–10*x*)	1 010	1 020	1 030	1 040	960	970	980
1 000 – 10*x*	990	980	970	960	1 040	1 030	1 020
(–10*x*) + 1 000	990	980	970	960	1 040	1 030	1 020
10*x* + 1 000	1 010	1 020	1 030	1 040	960	970	980
10*x* + (–1 000)	–990	–980	–970	–960	–1 040	–1 030	–1 020
(–10*x*) – 1 000	–1 010	–1 020	–1 030	–1 040	–960	–970	–980
1 000 + (–10*x*)	990	980	970	960	1 040	1 030	1 020
1 000 + 10*x*	1 010	1 020	1 030	1 040	960	970	980
10*x* – (+1 000)	–990	–980	–970	–960	–1 040	–1 030	–1 020

Instead of (–10*x*) – 1 000 we may write –10*x* – 1 000, in other words the brackets around the additive inverse may be left out.

Similarly, (–10*x*) + 1 000 may be written as –10*x* + 1 000.

3. Copy and complete the table.

x	1	5	10	20	25	30
–5*x* + 20	15	–5	–30	–80	–105	–130
–5*x* + (–20)	–25	–45	–70	–120	–145	–170

Grade 7 Term 4 Chapter 22 Algebraic equations 2

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
22.1 Describing problem situations	Writing closed number sentences in order to describe situations	Page 252
22.2 Analysing and interpreting equations	Choosing equations to fit situations and interpreting the equations	Page 253
22.3 Solving and completing equations	Using inspection to find solutions to equations; using trial and improvement to find solutions to equations	Pages 253 to 256
22.4 Identifying variables and constants	Using the information in the problem to decide which quantities are variables and which are constants	Page 256
22.5 Numerical values of expressions	Using substitution to find values of expressions and comparing these expressions in various ways	Page 257

CAPS time allocation	4 hours
CAPS content specification	Page 69

Mathematical background

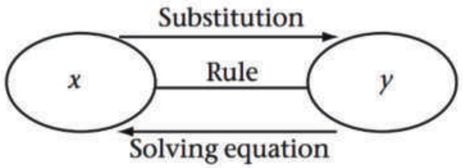
Two mathematical expressions on either side of an equal sign is an **equation**, for example $3x + 5 = 17$ or $5x - 1 = 2x + 2$.

The **equality sign** in an equation tells us that the value of the expression on the left-hand side is equal to the value of the expression on the right-hand side. In other words, an equation tells us that two mathematical expressions have the same value, for example: $12 + x = 30$; $3x - 7 = 14$.

We can interchange the left-hand side and the right-hand side of an equation and it will still be true, for example: $30 = 12 + x$ is the same as $12 + x = 30$ and $14 = 3x - 7$ is the same as $3x - 7 = 14$.

When we **solve an equation**, we find the value (or values) of the unknown (x in the examples) that makes the equation true.

An equation like $y = ax + b$ is an example of a **rule for a function**. The value of y depends on the value of x. Therefore, we can find a set of pairs of values for x and y that will satisfy the equation.



In a sense, most of the work learners will do in algebra will be with rules for functions. When they substitute values into expressions, they will be finding y-values and when they solve equations, they will be finding an x-value that goes with a particular y-value as the diagram alongside illustrates.

In Grade 7 learners will be working with equations to:

- understand that a situation can be described in terms of an open mathematical sentence in order to find a solution to a problem
- learn how to make use of variables and constants in a situation
- solve equations by inspection or trial and improvement, which strengthens the idea that only certain values of the variable make the equation true
- compare the values of different expressions for the same values of the independent variable (to compare y-values for the same x-values).

22.1 Describing problem situations

Teaching guidelines

Learners can work on their own and check their answers with a partner. All the answers are closed sentences.

Misconception

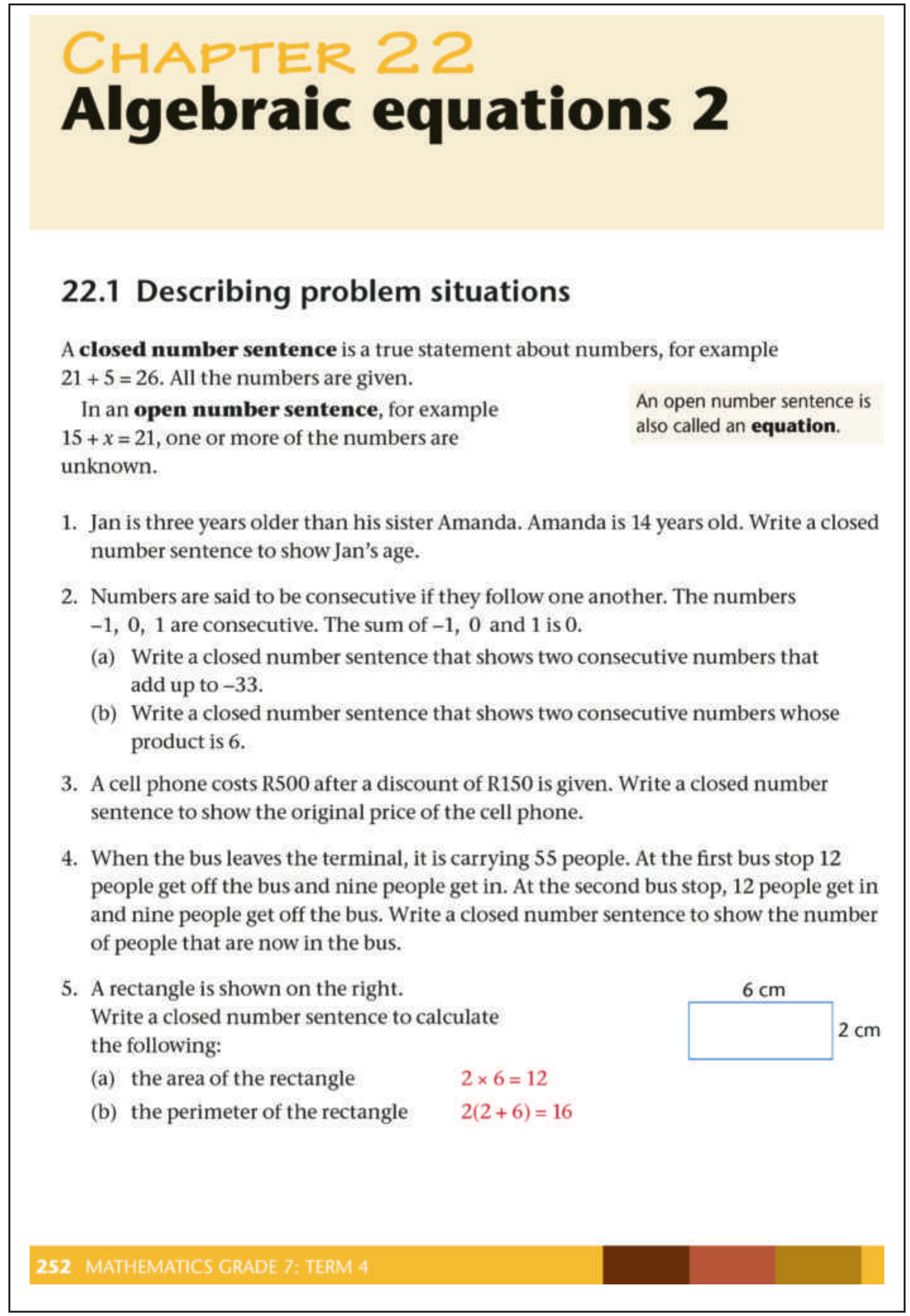
Learners get the two sides of the sentence wrong, or they interpret the given information incorrectly. For example, for question 3 learners might write 500 – 150 if they misunderstand which amount was discounted.

Notes on the questions

Writing the closed sentences in this section gives learners more practice in translating sentences given in words into writing mathematical sentences.

Answers

1. $14 + 3 = 17$
2. (a) $-16 + (-17) = -33$
   (b) $2 \times 3 = 6$
3. $500 + 150 = 650$
4. $55 - 12 + 9 + 12 - 9 = 55$
5. See LB page 252 alongside.



CHAPTER 22

Algebraic equations 2

22.1 Describing problem situations

A **closed number sentence** is a true statement about numbers, for example $21 + 5 = 26$. All the numbers are given.

In an **open number sentence**, for example $15 + x = 21$, one or more of the numbers are unknown.

An open number sentence is also called an **equation**.

1. Jan is three years older than his sister Amanda. Amanda is 14 years old. Write a closed number sentence to show Jan's age.
2. Numbers are said to be consecutive if they follow one another. The numbers -1, 0, 1 are consecutive. The sum of -1, 0 and 1 is 0.
   (a) Write a closed number sentence that shows two consecutive numbers that add up to -33.
   (b) Write a closed number sentence that shows two consecutive numbers whose product is 6.
3. A cell phone costs R500 after a discount of R150 is given. Write a closed number sentence to show the original price of the cell phone.
4. When the bus leaves the terminal, it is carrying 55 people. At the first bus stop 12 people get off the bus and nine people get in. At the second bus stop, 12 people get in and nine people get off the bus. Write a closed number sentence to show the number of people that are now in the bus.
5. A rectangle is shown on the right. Write a closed number sentence to calculate the following:
   (a) the area of the rectangle $2 \times 6 = 12$
   (b) the perimeter of the rectangle $2(2 + 6) = 16$

22.2 Analysing and interpreting equations

Teaching guidelines

Let learners analyse the situation and decide which quantity will be the unknown (variable) represented by a letter symbol. In most of these questions the unknown is given.

The next step is to write an open sentence that describes the situation, using the given numbers and the correct operations, for example in question 2 the number of sweets each learner gets has to be multiplied by 5 (the number of learners) to get the total number of sweets.

When a solution is found, it has to be tested to see if it makes the open sentence true.

Answers

1. (a) C. $x + 20 = 520$ (b) R500
2. (a) B. $5s = 60$
   (b) $5s = 60$ so $s = 60 \div 5 = 12$ Each learner gets 12 sweets.
   (c) s represents the number of sweets each learner should receive.
3. (a) A. $n - 6 = 7$ (b) 13 passengers
4. $3 \times 5 = 15$ or $5 + 5 + 5 = 15$
5. $4 + 3 + 5 = 12$

22.3 Solving and completing equations

SOLVE BY INSPECTION

Teaching guidelines

Remind learners how to add and subtract integers, especially adding two negative numbers (1(b)) and subtracting a negative number from another negative number (1(k)).

Misconception

Learners write negative numbers without the sign.

Answers

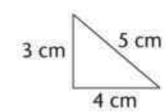
1. (a) $13 + 9 = 22$ (b) $-50 + (-50) = -100$ (c) $7 \times 8 = 56$
   (d) $9 - (-3) = 12$ (e) $-5 + 12 = 7$ (f) $4 \times 7 = 28$
   (g) $6 - 9 = -3$ (h) $9 - 6 = 3$ (i) $5 + (-12) = -7$
   (j) $10 + (-2) = 8$ (k) $(-1) - (-1) = 0$ (l) $0 + (-2) = -2$

22.2 Analysing and interpreting equations

1. The cost of a school uniform in rand is represented by x. An alteration fee of R20 is also charged. Mr Malan paid R520 for both the school uniform and the alterations done on it.
   (a) Which of the following equations describes the above situation?
   A. $20 \times x = 520$ B. $x - 20 = 520$ C. $x + 20 = 520$ D. $20 + 20 = x$
   (b) What is the price of the uniform?
2. Five learners should each receive the same number of sweets. There are 60 sweets in total that they have to share.
   (a) Which equation describes this situation?
   A. $5 + s = 60$ B. $5s = 60$ C. $s - 5 = 60$ D. $\frac{s}{5} = 60$
   (b) How many sweets does each learner get?
   (c) What does the letter s represent in the equation you have chosen?
3. A taxi picks up n passengers at the airport and drives to the nearest hotel. When it leaves the hotel, the number of passengers in the taxi has decreased by six. There are now seven passengers in the taxi.
   (a) Which equation describes this situation?
   A. $n - 6 = 7$ B. $7 - n = 6$ C. $n + 6 = 7$ D. $n - 7 = 6$
   (b) How many passengers were in the taxi when it left the airport?
4. Write a closed number sentence for calculating the perimeter of an equilateral triangle whose sides are 5 cm long.

Remember: An equilateral triangle is a triangle in which all three sides are equal.

5. Write a closed number sentence to calculate the perimeter of the triangle shown on the right.



22.3 Solving and completing equations

SOLVE BY INSPECTION

1. The number sentences given below are not true. Make the number sentences true by changing the numbers in blue.
   (a) $13 + 7 = 22$ (b) $50 + (-50) = -100$ (c) $7 \times 8 = 54$
   (d) $9 - (-3) = 6$ (e) $-5 + 12 = -7$ (f) $4 \times 6 = 28$
   (g) $6 - 9 = 3$ (h) $9 - 6 = -3$ (i) $5 + (-12) = 7$
   (j) $10 + (-2) = 12$ (k) $(-1) - (-1) = -2$ (l) $0 + (-2) = 0$

2. (a) Yes. $-3 + 3 = 0$
   (b) No. $3 - 1 = 2$
   (c) No. $-5 + (-2) + (-2) = -9$
   (d) Yes. $3 - (-1) = 3 + 1 = 4$
3. (a) $x = 2$ (b) $x = -2$ (c) $x = -6$
   (d) $x = -2$ (e) $x = 2$ (f) $x = 6$
   (g) $x = 8$ (h) $x = 8$ (i) $x = \frac{1}{2}$
4. (a) $x = 0$ (b) $x = -8$ (c) $x = -3$
   (d) $x = -5$ (e) $x = 5$ (f) $x = 15$
   (g) $x = -15$ (h) $x = -5$ (i) $x = 15$
   (j) $x = 1$ (k) $x = 0$ (l) $x = 0$
5. (a) $x = 7$ (b) $x = -7$
   (c) $x = -2$ (d) $x = -2$
6. (a) See the completed table on LB page 254 alongside.
   (b) See the completed table on LB page 254 alongside.
   (c) See the completed table on LB page 255 on the following page.

2. Consider the equations given below. Check whether the value given in brackets is the solution. Simply write the other letter and *yes* or *no* with an explanation.
   (a) $x + 3 = 0$ $(x = -3)$
   (b) $3 - x = 4$ $(x = 1)$
   (c) $-5 + x + x = -11$ $(x = -2)$
   (d) $3 - x = 4$ $(x = -1)$

To check whether a given value is the solution or not we have to answer the following question in our minds: **Does the given value make the equation true?** If it does, we say such a value is the **solution**.

3. Find the value of the unknown that makes the equation true in each case:
   (a) $x + 6 = 8$ (b) $x + 6 = 4$ (c) $x + 6 = 0$
   (d) $6 - x = 8$ (e) $6 - x = 4$ (f) $6 - x = 0$
   (g) $\frac{x}{4} = 2$ (h) $x = 4 \times 2$ (i) $\frac{x}{2} = \frac{1}{4}$
4. Three possible solutions are given in brackets below each equation, but only one is correct. Find the correct solution in each case.
   (a) $x + 27 = 27$ {−27; 0; 1} (b) $12 = 4 - x$ {8; 16; −8} (c) $x + 3 = 0$ {−3; 0; 3}
   (d) $5 - x = 10$ {−5; 0; 5} (e) $5 + x = 10$ {−5; 0; 5} (f) $-5 + x = 10$ {−5; −15; 15}
   (g) $-5 - x = 10$ {−5; −15; 15} (h) $-5 - x = 0$ {−5; −15; 15} (i) $5 - x = -10$ {−5; −15; 15}
   (j) $x = \frac{10}{10}$ {0; 1; 100} (k) $10x = 0$ {0; 1; $\frac{1}{10}$} (l) $\frac{x}{10} = 0$ {0; 1; 10}
5. What value for x would make each equation below true?
   (a) Let $x = ...$ then $x + 3 = 10$ (b) Let $x = ...$ then $x + 3 = -4$
   (c) $x + x + x = -6$ is true for $x = ...$ (d) $x + x + x + x = -8$ is true for $x = ...$
6. Copy the tables below. In each case, fill in the table until you can see for what value of x the equation given above the table is true. You may add more x values of your own choice. To save time and work, you may skip columns that you think will not help you to find the solution.
   (a) $37 - 4x = 5$

x	1	10	5	6	7	8			
$37 - 4x$	33	−3	17	13	9	5			

   (b) $50 - 7x = 22$

x	1	10	5	6	4				
$50 - 7x$	43	−20	15	8	22				

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SOLVE BY TRIAL AND IMPROVEMENT

Teaching guidelines

Learners should understand that when we solve an equation we are working out the value of an unknown quantity. That value will make the left-hand side of the equation equal to the right-hand side. In other words, we want to know, "what must the unknown value be so that the equation is true?" For example, "what should x be in $42x + 8 = 302$?" One of the ways we can use to find the answer is by trial and improvement.

- Try $x = 10$: it is too large and gives 428 when substituted into $42x + 8$ and not 302.
- Try $x = 5$ next: it is too small and gives a number less than 302 when substituted.
- Now we know the answer should be between 5 and 10, so we try $x = 6$: $42 \times 6 + 8 = 260$, x is still too small; try $x = 7$: $42 \times 7 + 8 = 302$, so $x = 7$.

Misconception

Learners become confused with what they are calculating and forget to give the value of the unknown that satisfies the equation as the answer.

Answers

1. See LB page 255 alongside.
2. See LB page 255 alongside.
3. See LB page 255 alongside.

(c) $100 - 3x = 49$

x	10	20	25	15	16	17			
$100 - 3x$	70	40	25	55	52	49			

SOLVE BY TRIAL AND IMPROVEMENT

We can think of an equation as a question asking for a value that we can assign to the **unknown** to make the equation true.

Consider the equation $82 + m = 23$. We need to assign values to m until we find a value that makes the equation true, as shown in the table below.

	Equation	True/False
Let $m = -50$	$82 + (-50) = 82 - 50 = 32$	False
Let $m = -30$	$82 + (-30) = 82 - 30 = 52$	False
Let $m = -60$	$82 + (-60) = 82 - 60 = 22$	False
Let $m = -59$	$82 + (-59) = 82 - 59 = 23$	True

So $m = -59$ because $82 + (-59) = 82 - 59 = 23$

1. Determine the value of t that makes the equation $28 - t = 82$ true by making use of the trial and improvement method. Copy and complete the table.

	Equation	True/False

Choice of numbers depends on the learner. Solution: $t = -54$

2. Consider the equation $w + 32 = -68$. Use the trial and improvement method to find the solution of the equation. Copy and complete the table.

	Equation	True/False

Choice of numbers depends on the learner. Solution: $w = -100$

3. The equation $200 - 5t = 110$ is given. What value of t makes the equation true? Copy the table below and use it to determine the solution.

	Equation	True/False

Choice of numbers depends on the learner. Solution: $t = 18$

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Answers

4. See LB page 256 alongside.
5. See LB page 256 alongside.

22.4 Identifying variables and constants

Teaching guidelines

Explain to the learners that when we write an equation to describe a situation we need to be clear on what the variables are and what the constants are.

Misconceptions

Learners cannot recognise the variable, for example the number of pockets of cement, and confuse it with the mass. Make sure that they can isolate the quantities that can vary in a description of the situation.

Answers

1. (a) y is a variable. The combined mass varies or changes according to the number of pockets of cement.
   (b) 90 is a constant. The mass of one pocket of cement is 90 kg.
   (c) x is a variable. The number of pockets of cement to be transported may vary.
   (d) 2 680 is a constant. The mass of the empty truck is 2 680 kg.
2. (a) y is a variable. The length of the spring depends on the number of mass pieces hooked.
   (b) 8 is a constant. It remains the same all the time.
   (c) x is the variable. It is the number of mass pieces hooked to the bottom and it changes.
   (d) 40 is a constant.

4. What value of p makes the equation $18p = 90$ true? Copy and complete the table.

	Equation	True/False

Choice of numbers depends on the learner. Solution: $p = 5$

5. What value of x makes the equation $88 - 6x = 46$ true? Copy and complete the table.

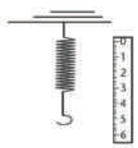
	Equation	True/False

Choice of numbers depends on the learner. Solution: $x = 7$

22.4 Identifying variables and constants

1. The mass of an empty truck is 2 680 kg. The truck is used to transport cement. Each pocket of cement has a mass of 90 kg. The combined mass of the truck and the cement can be calculated by means of the formula: $y = 90 \times x + 2\,680$.
   Use the terms **variable** or **constant** to describe the meaning of each symbol used in the formula. Explain your answer.
   (a) y (b) 90 (c) x (d) 2 680

2. A steel spring is suspended from a stand. Mass pieces of equal mass are hooked onto the bottom end of the spring. The length of the spring is measured with one mass piece hooked, two mass pieces hooked, three mass pieces hooked and so on. The results are shown in the table below.



Number of mass pieces	1	2	3	4	5	7	10
Length of spring in cm	48	56	64	72	80	96	120

The formula $y = 8x + 40$ is used to predict the length of the spring for the various number of mass pieces hooked.
Use the terms **variable** or **constant** to describe each symbol used in the formula. Explain your answer.
(a) y (b) 8 (c) x (d) 40

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22.5 Numerical values of expressions

SUBSTITUTING NUMBERS INTO EXPRESSIONS

Teaching guidelines

Substituting the values creates the sequences from which learners can make deductions about finding an unknown, for example 100 – 9x = –800 means –9x has to be –900, so x has to be 100.

Substituting values also reinforces the use of x as a variable.

Furthermore, the concept of two different expressions having the same value will be important in later work when learners solve two equations simultaneously.

Comparing two expressions for the same values of the variable (x) could help learners to think about the way the values of the expressions increase or decrease, and at which x values the one is less than the other, equal to and greater than the other.

Looking at expressions such as 3x – 3; 3x – 2 and 3x – 1, shows learners that the difference between the first and the second is always 1 and between the first and the last is always 2. Let them try to give a reason why this is so (the constants differ by those values).

Answers

1. (a) See the completed table on LB page 257 alongside.
   (b) The sequence generated by 100 – 9x decreases fastest, and the sequence generated by 100 – 3x decreases slowest.
2. (a) See the completed table on LB page 257 alongside.
   (b) x = 4
   (c) For x values 5, 6 and 7 (in fact for all x values bigger than 7, but it would be unfair to expect learners to realise this now).
   (d) Yes (because 3x – 1 increases by +3 while 2x + 3 increases by +2).
   (e) The sequence generated by 3x – 3.

22.5 Numerical values of expressions

SUBSTITUTING NUMBERS INTO EXPRESSIONS

1. (a) Copy the table below. Calculate the values of each expression for the given values of x, and write your answers in the table.

x	0	2	5	10	20	50	100
100 – 9x	100	82	55	10	–80	–350	–800
100 – 8x	100	84	60	20	–60	–300	–700
100 – 7x	100	86	65	30	–40	–250	–600
100 – 6x	100	88	70	40	–20	–200	–500
100 – 5x	100	90	75	50	0	–150	–400
100 – 4x	100	92	80	60	20	–100	–300
100 – 3x	100	94	85	70	40	–50	–200

   (b) Which sequence in the above table decreases fastest, and which sequence decreases slowest?

2. (a) Copy and complete the table.

x	1	2	3	4	5	6	7
2x + 3	5	7	9	11	13	15	17
3x – 3	0	3	6	9	12	15	18
3x – 2	1	4	7	10	13	16	19
3x – 1	2	5	8	11	14	17	20

   (b) For which value of x is 2x + 3 equal to 3x – 1?
   (c) For which values of x is 2x + 3 smaller than 3x – 1?
   (d) Do you think 2x + 3 is smaller than 3x – 1 for all values of x greater than 4? You may try a few numbers to help you think about this.
   (e) Which sequence increases fastest, the sequence generated by 2x + 3 or the sequence generated by 3x – 3?

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
23.1 Collecting data	Distinguish between populations and samples; construct a questionnaire	Pages 258 to 262
23.2 Organising data	Classify data; organise data using tallies in frequency tables, dot plots and stem-and-leaf displays; group data into intervals	Pages 262 to 270
23.3 Summarising data	Find the mode, median, mean and range of a set of numerical data	Pages 270 to 273

CAPS time allocation	5 hours
CAPS content specification	Pages 78 to 79

Mathematical background

Data handling is the part of Mathematics that deals with numbers and facts that we collect about the world around us. Data can be many different things, for example, people's opinions on politics or the success rates of treating people with a certain kind of medicine. We use data to help us make decisions and solve problems about the world around us.

The **data handling cycle** consists of the following phases:

- **Pose a question**: Identify a real-life problem and pose (formulate) a question that requires the collection of data.
- **Collect data**: Identify the data source (the population), which is the whole group you are asking the question about. If the population is too large to handle, select a smaller group (the sample) to represent the population. Find the most suitable method to collect the data, for example through observation (by watching something closely) or by using a questionnaire (a list of questions) or research material. Decide whether to use a data collection sheet (during observation) or a questionnaire (during interviews) to collect the data.
- **Classify and organise data**: Identify whether the data is categorical (words) or numerical (numbers) and whether the numerical data is discrete (fixed numbers) or continuous (measurements). Sort the data into categories or into ungrouped or grouped intervals. Organise the data using tallies in frequency tables as well as dot plots or stem-and-leaf displays.
- **Summarise data**: Find the mode, median and mean, which are measures of central tendency (balance), as well as the range, which is a measure of spread (width).
- **Represent data**: Draw a graph of the data, for example a bar graph, histogram or pie chart.
- **Interpret and analyse data**: Ask questions about the data and identify and describe trends or patterns in the data in order to draw conclusions about the data.
- **Report on the data**: Explain what the data tells about the problem or question and predict how the data can be used to solve problems about the world around us.

23.1 Collecting data

Background information

The **data handling cycle** is started by identifying a real-life problem and posing (formulating) a question that requires the collection of data. Before data is collected, questions should be asked about the following:

- The **question**: What was asked?
- The **population**: Who/What is the whole group the question is about?
- The **sample**: Must a smaller group be chosen to represent the population?
- The **data collection method**: Which method is best to obtain the data?
    - **Observation** (by watching something closely)
    - **Interview** (by talking to someone face-to-face)
- The **data collection instrument**: How will the data be recorded?
    - On a data collection sheet (during observation)
    - On a questionnaire (during an interview or survey).

POPULATIONS AND SAMPLES: FROM WHOM TO COLLECT DATA

Teaching guidelines

Discuss the following:

- the bullet points in the Learner Book on page 258 are to be considered when data is collected
- the difference between the population and a sample of an investigation
- Thandeka's method of choosing a sample.

CHAPTER 23

Collect, organise and summarise data

23.1 Collecting data

Think of something that you really want to know about your own community or about children your age in other schools. For example, "How many Grade 7 learners in South Africa have access to a computer?" What would *you* find interesting to know about?

When you start the cycle of data handling, you start with at least one question. But there can of course be many more questions.

Once you have a research idea in mind, you can start planning how you will collect the data. When you collect data, you need to consider:

- what question you are asking
- where you will find the data to answer the question (for example, from people such as your peers, family or the wider community; or from published sources such as newspapers, books or magazines)
- how you will collect the data (for example, by using questionnaires or conducting interviews)
- who you will collect the data from (the entire population or a sample).

POPULATIONS AND SAMPLES: FROM WHOM TO COLLECT DATA

In data handling, **population** refers to the whole group you are asking the question about.
Sample refers to a small number of the group that you think will represent the whole group.

Here is an example: Thandeka wants to know about the home languages of all Grade 7s across the whole of South Africa. All Grade 7s in all of South Africa would be the population of that data. But it is not possible to reach every single Grade 7 learner in South Africa, so Thandeka could choose a sample of Grade 7 learners. For example, she could choose to collect data from her own Grade 7 class and from two other Grade 7 classes from two other schools.

But if Thandeka chose her own Grade 7 class and only two other classes from other schools, her sample would not really give information about learners across the whole

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Background information (continued)

- **How to choose a representative sample** from the learners in a school:
  - **Simple random sampling**: Choose the sample by drawing names from a bag that contains the names of all the learners in the school.
  - **Systematic random sampling**: Choose a number from 1 to 20 at random, say 14. Then choose every fourteenth name on an alphabetical list of all the learners in the school.
  - **Stratified sampling**: Determine the ratio of boys to girls in the school. Now choose a random sample from the whole school in that ratio.
  - **Cluster sampling**: Divide the learners in the school into grades and each grade into classes. Choose one class at random from each grade.
- **How NOT to choose a sample** from the learners in a school:
  - **Convenience sampling**: Choose only your friends to be part of the sample.
  - **Self-selection sampling**: Ask who in school would like to be part of the sample.
  - **Quota sampling**: Choose only boys in Grade 12 to be part of the sample.

Teaching guidelines (continued)

Discuss the following:

- the points to remember when choosing a sample, on LB page 259
- Ganief's method of choosing a sample.

THINKING ABOUT POPULATIONS AND SAMPLES

Teaching guidelines

The sample is a smaller group than the population.

Answers

1. See LB page 259 alongside.

of South Africa, because the learners in all three of the schools could be from the same language group.

So, how can you try to make sure that a sample gives information about the whole population? In other words, how can you make sure that your sample is **representative** of the population?

1. *Choose a big enough sample.* Generally, the bigger the sample is the more likely it is to represent the characteristics of the population.
2. *Ensure that you do not take a sample from only one of the groups within the population.* For example, if you want to find out if people like watching soccer, you cannot survey people at a Chiefs versus Pirates match. The majority of these people will almost certainly be there because they love watching soccer!

Example

Ganief wishes to find out if learners at his school like the style and colour of their school uniform and surveys ten learners in Grade 7. There are 2 000 learners at the school.

Give two reasons to explain why the sample chosen is not likely to be representative of the population.

Answer

1. The sample is too small.
2. He is only getting the views of Grade 7s, not of the learners in any of the other grades (who might have very different views).

THINKING ABOUT POPULATIONS AND SAMPLES

1. Here are some research questions. Copy the statements below and use a **P** to show which statement describes the population and an **S** to show which statement describes a sample of the population.

   (a) What percentage of plants in the vegetable patch is affected by disease?

   P All the plants in the vegetable patch

   S Every fourth or fifth plant in the vegetable patch

   (b) How often do teenagers recycle plastic?

   P Every teenager in South Africa

   S About 40 teenagers in the community

   (c) How many hours of sleep do 10-year-olds in my community get per night?

   P All 10-year-olds in the community

   S About ten 10-year-olds in the community

Answers

2. (a) All the learners in every grade in my school.
   (b) Possible answer: Ten learners from each grade in my school.
3. (a) 2 500 ÷ 26 000 × 100% = 9,6%
   (b) To make sure that they were not collecting data from only one group.
   (c) Learners' own answer.
4. No, because her sample is only girls (who don't often play rugby) and because she is sampling from only one school (and learners in other schools in her town may have different views). Also, there are 13-year-olds in other grades too, which she needs to include in her sampling method.

CONSTRUCTING QUESTIONNAIRES: HOW TO COLLECT DATA

Background information

- A **questionnaire** is a sheet with questions used to collect data from people.
- The person who answers these questions is called the **respondent**.
- When a questionnaire is constructed, different **types of questions** can be asked:
  - Questions with "yes" or "no" responses (answers).
  - Multiple choice questions with a selection of answers.
  - Questions that ask for a rating like "never", "sometimes" or "always".
  - Open questions where the respondents may enter their own view or information.
- If you use the wrong method or instrument for collecting data, the data may be **flawed**. This could lead to unreliable conclusions and predictions.

Teaching guidelines

Discuss:

- what questionnaires and respondents are
- the types of questions listed on LB page 260.

2. You want to know the most popular colour of the learners in your school.
   (a) Write down the population of your data collection.
   (b) Write down what sample you would use.
3. Census@School took place in 2001 and 2009. These were surveys that Statistics South Africa did to show learners how information about people is collected and analysed. The Census@School wanted to know personal, community and household information about learners from Grades 3 to 12. This is how they chose their sample:
   - *A sample of 2 500 schools was selected from the Department of Basic Education's database of approximately 26 000 registered schools.*
   - *The schools were divided into groups depending on their province, school type (primary: Grades 3 to 7 only; intermediate: Grades 5 to 9 only; secondary: Grades 8 to 12 only; combined: Grades 3 to 12), and education district.*
   - *A sample of schools was selected from each of these groups.*
   - *Approximately 790 000 learners participated in the Census@School 2009.*

   This information was included in their final report.

   (a) What percentage was the sample of all the schools in the country?
   (b) Why do you think they separated the schools into groups first?
   (c) Do you think the information that they obtained from this survey would be interesting to you? Explain.
4. Unathi goes to River View Girls' Primary School. She wants to find out whether 13-year-olds in her town prefer rugby or netball. She surveys ten learners from each of the three Grade 7 classes at her school. Is the sample chosen likely to be representative of the population (13-year-olds in her town)? Explain your answer.

CONSTRUCTING QUESTIONNAIRES: HOW TO COLLECT DATA

A **questionnaire** is a sheet with questions used to collect data from people. Each **respondent** in the sample completes a questionnaire. The questions on the sheet can be structured differently, for example:

- The questions may require "yes" or "no" answers.
- A selection of answers (multiple-choice answers) may be provided for respondents to choose from.
- The respondents may enter their own views or information on the questionnaire.

A **respondent** is a person who fills in a questionnaire or from whom you collect data.

The type of responses you need (for example a simple "yes" or "no" or more detailed information) depends on the data you intend to collect.

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Background information (continued)

The type of questions that you choose depends on the data you want to collect. The following should be kept in mind when constructing a questionnaire:

- Ask clear and short questions.
- Start with easy questions.
- Avoid leading questions as they tell you what the answer should be.
- Avoid biased questions as they favour someone or something.
- Avoid offensive questions as they are too personal and may upset respondents.
- Give clear instructions.
- Make the questionnaire as short as possible.

Teaching guidelines (continued)

Work through the five examples on LB page 261. Point out the following:

- Each question is worded to be as clear as possible.
- Each question allows the data to be collected easily.

MAKING QUESTIONNAIRES

Background information

Multiple choice questions should be constructed with the following in mind:

- The options should **not overlap**.
- There should be **no gaps** between the options.
- The options should cover **all possible answers**.

Teaching guidelines

Discuss the requirements for options of multiple questions listed above.

Answers

1. (a) The question does not make it clear whether this is per week, per month, per term or per year. The rand symbols are missing. The options overlap (e.g. if you receive R10 you could tick two different boxes). There is no option that someone who gets more than R40 a month could tick.

   (b)

How much pocket money do you get each month?	
☐ less than R10	☐ R10–R19,99
☐ R20–R29,99	☐ R30 or more

Look at the examples below. Notice how each question is worded to be as clear as possible and to allow the data to be collected easily. (The questions in examples 4 and 5 were used by Census@School in their 2009 questionnaire.)

Example 1

Do you help with chores at home?

☐ Yes ☐ No

Example 2

Which of these chores do you help with?

☐ cleaning dishes ☐ washing clothes
☐ sweeping/vacuuming ☐ making beds

Example 3

How old are you?

☐ 5–8 years ☐ 9–11 years
☐ 12–15 years ☐ 16–19 years

Note in example 3 how all the ages from 5 to 19 are covered, but without any overlaps.

Example 4

Tick the box if you have:

1 ☐ Running water inside your home
2 ☐ Electricity inside your home
3 ☐ A radio at home
4 ☐ A TV at home
5 ☐ A telephone at home
6 ☐ A cell phone
7 ☐ Access to a computer
8 ☐ Access to the internet
9 ☐ Access to a library

Example 5

6 How tall are you without your shoes on? Answer to the nearest centimetre.
☐☐☐ centimetres

7 What is the length of your right foot, without a shoe? Answer to the nearest centimetre.
☐☐☐ centimetres

8 What is your arm span? (Open arms wide, measure the distance across your back from the tip of your right hand middle finger to the tip of your left hand middle finger.) Answer to the nearest centimetre.
☐☐☐ centimetres

MAKING QUESTIONNAIRES

1. (a) Refue wants to find out how much pocket money learners in her class receive each month. She draws up the following multiple-choice question:

How much pocket money do you get?			
☐ 0–10	☐ 10–20	☐ 20–30	☐ 30–40

Explain why this question is not clear. Give at least three reasons.

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Answers

2. (a) All the learners at the school.
   (b) *Possible answer*: ten learners in each grade from Grade 4 to Grade 7.
3. *Sample answer of a question with yes/no responses:*

Do you play the following sport?	Yes	No
soccer		
hockey		
netball		
rugby		
cricket		

Sample answer of a question with multiple-choice responses:

Which of the following sports do you play?	
☐ soccer	☐ hockey
☐ netball	☐ rugby

4. Learners collect data from their population or a sample that they chose. They keep the data for the next chapter.

23.2 Organising data

Background information

The **next phase in the data handing cycle** is to classify, sort and organise the data.

- **Classify** the data as either categorical (words) or numerical (numbers), of which the latter can be discrete (fixed numbers) or continuous (measurements).
- **Sort** any categorical data into categories and numerical data into ungrouped or grouped intervals.
- **Organise** the data using tallies in frequency tables as well as dot plots or stem-and-leaf displays.

(b) Draw up the multiple-choice question so that it will allow Refue to collect the data that she needs.

2. You want to find out which sports learners at your school play.
   (a) Describe the population of your data.
   (b) Describe the sample you will use.
3. Make a question with yes/no or multiple-choice responses to help you collect the data you need.
4. Collect your data from your population or the sample you chose. Keep your data for the next chapter.

23.2 Organising data

To organise data that we have collected, we can use tally marks and tables, dot plots, and stem-and-leaf displays. We can also group the data when there are many data values. The ways that we organise the data depends on the type of data we collected.

DIFFERENT TYPES OF DATA

Look at the five examples of questions for questionnaires on page 261.

1. Which of the examples will give you data that looks like this?

Yes	1 235 learners
No	1 265 learners

2. Which of the examples might give you data that looks like this?
   132 cm; 141 cm; 160 cm; 132 cm; 154 cm; 145 cm; 147 cm; 129 cm; 121 cm; 143 cm; 135 cm; 154 cm; 156 cm; 133 cm; 156 cm; 123 cm; 137 cm etc.
3. What could the data for example 4 look like? Copy and fill in this table to give a possible example for 30 learners. Use numbers that you have made up.

	Number of learners
1. Running water inside your home	24
2. Electricity inside your home	26
3. A radio at home	28
4. A TV at home	25
5. A telephone at home	12
6. A cell phone	14
7. Access to a computer	16
8. Access to the internet	17
9. Access to a library	13

DIFFERENT TYPES OF DATA

Background information

- Data in the form of words is called **categorical data**, for example, the colour of people's eyes or the type of car they own.
- Data in the form of fixed numbers is called **discrete data**, for example, the number of siblings per family or the possible scores when a die is thrown.
- Data in the form of numbers obtained through measurement is called **continuous data**, for example the weight and height of respondents.
- **An important note**: Although age and shoe size are measurements, both are treated as discrete data because, in real life, they are expressed in fixed numbers.

Teaching guidelines

Discuss the difference between categorical and numerical data.

Answers

1. Example 1
2. Question 6 in Example 5
3. See the completed table on LB page 262 on the previous page.
4. Example 3
5. See LB page 263 alongside.

ORGANISING CATEGORICAL DATA

Background information

Data can be **organised** in ordinary tables, frequency (tally) tables, dot plots and stem-and-leaf displays.

- **Ordinary tables** show **raw data**, which is data as it is collected, before any sorting is done (refer to the table on LB page 263).
- **Frequency (tally) tables** show **sorted data**, after it has been sorted into categories or intervals (refer to the table in question 4 on LB page 265). Tally marks (|) are used to record data items in clusters of five (~~||||~~), which makes it easy to count the number of tally marks in a particular category.

4. Which of the examples might give you a data set that looks like this?

5–8 years	15
9–11 years	45
12–15 years	32
16–19 years	28

The type of data in questions 1 and 3 is called **categorical data**. This is often described by words. The categories don't have to be given in order.

The type of data in questions 2 and 4 is called **numerical data**. Numerical data can be whole numbers only, or it can include fractions.

For both of these kinds of data, your results give you a list of responses. You will soon learn how to organise these responses.

5. Classify the following data sets as categorical or numerical.
   (a) the number of pages in books — numerical
   (b) the length of learners' arm spans — numerical
   (c) learners' favourite soccer teams — categorical
   (d) the time it takes 13-year-olds to run 1,5 km — numerical
   (e) the cost of different types of cell phones — numerical
   (f) colours of new cars manufactured — categorical

ORGANISING CATEGORICAL DATA

Thandeka asked the following question: "Which of South Africa's official languages are the home languages of the learners in my class?"

Thandeka drew up a table with each learner's name. She then asked each learner what his or her home language was, and wrote it down as follows:

Name	Language	Name	Language	Name	Language
Nonkhanyiso	isiXhosa	Marike	Afrikaans	Herbert	Sepedi
Anna	Afrikaans	Jennifer	Sepedi	Thabo	isiXhosa
Mpho	Ndebele	Nomonde	isiXhosa	Nomi	isiXhosa
Nontobeko	isiZulu	Thandeka	Sepedi	Manare	Sepedi
Jonathan	English	Siza	isiZulu	Unathi	Sesotho
Sibongile	isiZulu	Prince	Sesotho	Gabriel	Ndebele
Dumisani	isiZulu	Duma	isiZulu	Marlene	Afrikaans
Matshediso	Sesotho	Thandile	Sepedi	Simon	Sesotho

Teaching guidelines

Categorical data can be organised in an ordinary table by recording the data as it is collected, before any sorting is done.

Answers

1. I need to know how many times each language appears in the list. This will tell me how many learners speak each language.
2. No, there is no particular order as the data is categorical.

Background information (continued)

A **dot plot** uses dots to show the number of data values in each category of the data set (refer to question 3(a) on LB page 264 alongside).

- The different **categories** of data are shown below a horizontal line. To avoid bias (prejudice and unfairness), the categories are usually listed in alphabetical order.
- The number of data items in each category is shown as **dots** directly above each category.

Teaching guidelines (continued)

Check whether learners understand the features of a dot plot.

Answers

3. (a) See LB page 264 alongside.

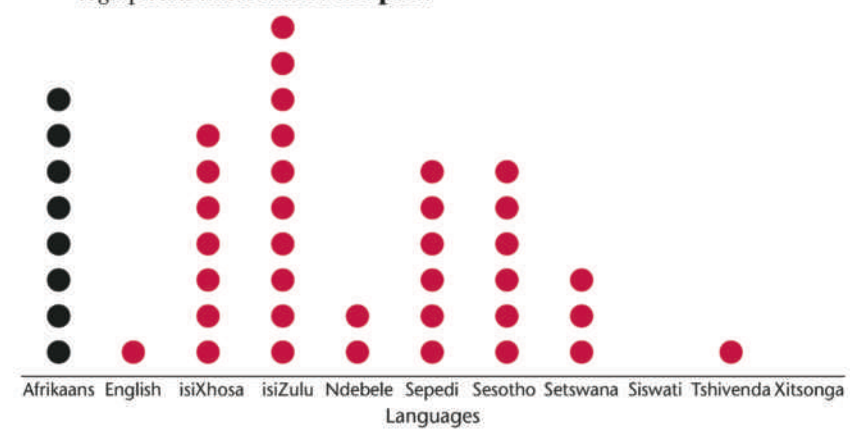
Name	Language	Name	Language	Name	Language
Chokocha	Sepedi	Nicholas	Sesotho	Miriam	Setswana
Khanyisile	isiXhosa	Jabulani	isiZulu	Sibusiso	isiZulu
Ramphamba	Tshivenda	Nomhle	isiXhosa	Mishack	isiZulu
Portia	isiZulu	Frederik	Afrikaans	Peter	Setswana
Erik	Afrikaans	Lola	Afrikaans	Maya	Afrikaans
Jan	Afrikaans	Zinzi	isiXhosa	Thobile	Sesotho
Palesa	isiZulu	Jacob	Setswana		

We don't need the learners' names in the data. This data could be written as a list of the languages, like this:

isiXhosa, Afrikaans, Sepedi, Afrikaans, Sepedi, isiXhosa, Ndebele, isiXhosa, isiXhosa, isiZulu, Sepedi, Sepedi, English, isiZulu, Sesotho, isiZulu, Sesotho, Ndebele, isiZulu, isiZulu, Afrikaans, Sesotho, Sepedi, Sesotho, Sepedi, Sesotho, Setswana, isiXhosa, isiZulu, isiZulu, Tshivenda, isiXhosa, isiZulu, isiZulu, Afrikaans, Setswana, Afrikaans, Afrikaans, Afrikaans, Afrikaans, isiXhosa, Sesotho, isiZulu, Setswana

Now work with this data set to see what story it is telling you. What do you notice about the data?

1. What do you need to find out from this list of languages?
2. Does it matter what order you write the languages in? Why or why not?
3. (a) Copy Thandeka's graph below. Place a dot above each language to show every learner who speaks that language. The languages are in alphabetical order. Try to space out the dots evenly. The dots for Afrikaans have been drawn for you. A graph like this is called a **dot plot**.



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Answers

3. (b) English and Tshivenda; Sepedi and Sesotho

   (c) IsiZulu, Afrikaans, isiXhosa, Sepedi, Sesotho, Setswana, Ndebele, Tshivenda, English, Siswati, Xitsonga

Background information (continued)

A **frequency (tally) table** uses tally marks to show the counts of data values. It consists of three columns (see question 4(a) on LB page 265 alongside):

- Column one shows the different **categories** of data in alphabetical order.
- Column two show the counts of each category in **tallies**.
- Column three shows the **total** number of counts in each category. These numbers are called the **frequency** of each category.

Teaching guidelines (continued)

Check whether learners know how to use tallies to organise data in a frequency (tally) table.

Answers

4. (a) See completed table on LB page 265 alongside.

   (b) 44

   (c) isiZulu

   (d) Siswati and Xitsonga

   (e) The highest number of learners (ten) speak isiZulu. This is followed by Afrikaans, which is spoken by eight learners. One learner speaks English and one Tshivenda. No learners have Xitsonga or Siswati as a home language.

An important note

Empty tally tables and dot plots can be used to record categorical and numerical data as it is collected. This sorts the data at the same time as it is recorded.

(b) Which languages have the same numbers of learners?

(c) List the languages in order from the language spoken by the most learners to the language spoken by the fewest learners.

You can also record results in a **tally table**. To do this, you draw a single line (|) for each item you count. This line is called a **tally mark**.

You group tally marks in groups of five. The fifth tally mark is always drawn horizontally to show that the group of five is complete. Then you start a new group. This makes it easy to quickly count how many tally marks there are in a particular category.

Examples of tally marks:
A count of three = |||
A count of four = ||||
A count of five = ~~||||~~
A count of seven = ~~||||~~ ||

4. (a) Copy and complete the table.

Home language of learners in the Grade 7 class

Language	Number of speakers of each home language	Total
Afrikaans	~~||||~~ |||	8
English			1
isiXhosa	~~||||~~ ||	7
isiZulu	~~||||~~ ~~||||~~	10
Ndebele	||	2
Sepedi	~~||||~~		6
Sesotho	~~||||~~		6
Setswana	|||	3
Siswati		0
Tshivenda			1
Xitsonga		0
Total (whole class)		**44**

(b) How many learners altogether were asked about their home language?

(c) Which home language occurs most often in this class?

(d) Which languages are not spoken as a home language by any of the learners in this class?

(e) Write a short paragraph to describe the home languages in Thandeka's class.

Dot plots and tally tables are used for **numerical data** too. You can write data values on prepared tally tables or dot plots as you record them. This sorts the data at the same time as it is recorded.

INTRODUCING STEM-AND-LEAF DISPLAYS

Background information

A **stem-and-leaf display (plot)** shows numerical data listed in two columns separated by a vertical line (refer to Examples 1 and 2 on LB page 266).

- The **stem column** (on the left of the vertical line) shows the tens and hundreds of all the data values in numerical order, including those which are missing from the sequence.
- The **leaf column** (on the right of the vertical line) shows the units of all the data values in numerical order and in line with their relevant stems, which means that something like 23 | 0, 5 represents the data values 230 and 235.

Teaching guidelines

Learners work through Example 1.

Check whether learners understand the following:

- the features of the stem-and-leaf display
- the key: 1 | 2 means 12.

INTRODUCING STEM-AND-LEAF DISPLAYS

A **stem-and-leaf display** (also called a stem-and-leaf plot) is a way of listing numerical data using two columns divided by a vertical line. Each number is split across the columns.

For example, if the numbers in a set of data consist of digits for tens and units (such as 23, 25 and 34), the column on the right (the leaf column) shows the units digits of the numbers, and the column on the left (the stem column) shows the tens digits of the numbers.

Numerical data is data that consists of numbers.

Example 1

Show the following data set as a stem-and-leaf display:

13, 56, 20, 35, 47, 53, 12, 51, 53, 49, 34, 53

First, we order the values in the data set from smallest to biggest:

12, 13, 20, 34, 35, 47, 49, 51, 53, 53, 53, 56

In this example, the tens digits range from one to five, so we list these in the stem column. Then we fill in the units digits in the leaf column.

The stem-and-leaf display of the above data set looks like this:

Key: 1 | 2 means 12

Stems	Leaves
1	2, 3
2	0
3	4, 5
4	7, 9
5	1, 3, 3, 3, 6

Values with the same stem are written in the same row. Numbers with the same tens value are separated by a space or a comma. The first row shows the numbers 12 and 13, the second row shows 20, and the last row shows 51, 53, 53, 53 and 56.

In this example, the stem column shows the tens digit of each value.

Here the leaf column shows the units digit of each value.

Example 2

The stem-and-leaf display on the following page shows the units digits as the leaves, and both the hundreds and tens digits as the stems:

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Teaching guidelines (continued)

Learners work through Example 2.

Check whether learners understand the following:

- the features of the stem-and-leaf display
- the key: 10 | 2 means 102.

DOT PLOTS AND STEM-AND-LEAF DISPLAYS

Teaching guidelines

Learners work with stem-and-leaf displays and dot plots and answer relevant questions.

Answers

1. (a) 131, 139, 140, 162, 163, 165, 165, 165, 176, 178, 178, 194, 196, 197
   (b) 160s
   (c) 165
   (d) Refer to the stem-and-leaf display in question 1 on LB page 267 alongside.
   (e) No, we cannot because this would mean that there is a value of 150 in the data set, which is not true. (Remind learners not to leave out stems 15 and 18!)
2. (a) 354, 360, 361, 375, 378, 378, 378, 378, 379, 382, 390, 394, 395, 399, 400
   (b)

Key: 35 | 4 means 354

35	4
36	0, 1
37	5, 8, 8, 8, 8, 9
38	2
39	0, 4, 5, 9
40	0

   (c) 378

10	2, 5
11	0, 6
12	1, 4, 4
13	
14	7, 9
15	
16	1, 3, 8

Key: 10 | 2 means 102

The values shown are: 102, 105, 110, 116, 121, 124, 124, 147, 149, 161, 163, 168.

Note that if there is a 0 in the leaf column it means the unit digit is a 0, as in 110 above. When there is nothing written in the leaf column next to a stem, it means that there aren't any numbers with that particular stem. In the case of stem 13 above, for example, it means there are no values between 129 and 140.

When you draw stem-and-leaf displays, it is important that the numbers line up vertically so that you can compare the leaves. Draw lines to help you. (Or use grid paper, if you have some.)

DOT PLOTS AND STEM-AND-LEAF DISPLAYS

1. Look at the following stem-and-leaf display and answer the questions below.

13	1, 9
14	0, 3
15	
16	2, 3, 5, 5, 5, 7
17	6, 8, 8
18	
19	4, 6, 7, 9

Key: 13 | 1 means 131

   (a) Write down the values in the data set shown by the stem-and-leaf display.
   (b) Do most of the values fall in the 160s or 170s?
   (c) Which value occurs the most times?
   (d) Copy the above stem-and-leaf display and add the following values: 143, 167 and 199.
   (e) There are no values in the 150s. Can we add 15 | 0 to the stem-and-leaf display to show that there are no values in the 150s? Explain your answer.

2. (a) Arrange the values in the following data set in order from smallest to largest: 378, 360, 390, 378, 378, 400, 379, 382, 354, 394, 399, 395, 378, 361, 375
   (b) Organise the data set as a stem-and-leaf display. Remember to add the key.
   (c) Which value occurs most often?

Answers

3. (a) Refer to the dot plots in question 3(a) on LB page 268 alongside.
   (b) The whole set of dots for Mercury looks as if it is more to the left than the dots for Jupiter, even though the sets do have some of the same values (they overlap). This means that the sales for Jupiter are generally higher.

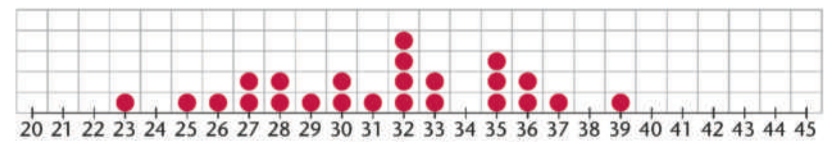
SOMETHING TO THINK ABOUT

Teaching guidelines

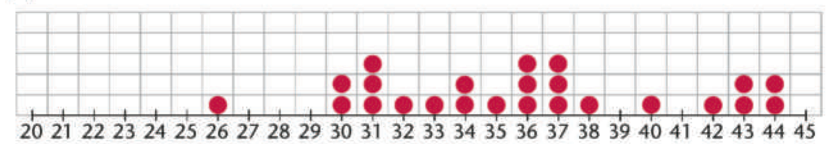
When a stem-and-leaf display is turned through 90° in an anti-clockwise direction, the new graph resembles a dot plot.

3. (a) The data sets below show the sales of two new makes of cars (Jupiter and Mercury) over 24 months. Copy the number lines and draw a dot plot for each set on the number lines.
   Mercury: 23, 27, 30, 27, 32, 31, 32, 32, 35, 33, 28, 39, 32, 29, 35, 36, 33, 25, 35, 37, 26, 28, 36, 30
   Jupiter: 31, 44, 30, 36, 37, 34, 43, 38, 37, 35, 36, 34, 31, 32, 40, 36, 31, 44, 26, 30, 37, 43, 42, 33

Mercury



Jupiter



   (b) If you look at the dots for Mercury and the dots for Jupiter, what can you see about the sales of the two cars? What does this mean?

SOMETHING TO THINK ABOUT

What kind of graph does the stem-and-leaf display look like if you turn it by 90 degrees?

1	2, 5
2	0, 6
3	1, 4, 4, 5, 5, 9
4	
5	2, 7, 8, 9
6	1, 3, 6, 7, 7
7	1, 3, 8

→

		9				
		5			7	
		5		9	7	
		4		8	6	8
5	6	4		7	3	3
2	0	1		2	1	1
1	2	3	4	5	6	7

GROUPING DATA INTO INTERVALS

Background information

- Sets of numerical data are usually grouped when:
  - the set contains a large number of data values
  - the data values are very different in magnitude.
- Data is grouped into intervals to make it easier to handle.
- When data is grouped, the following should be kept in mind:
  - intervals should not overlap
  - there should be no gaps between intervals.

An important note

Once data is grouped, the original data values cannot be found again. The values can only be found in the raw data.

Teaching guidelines

Discuss the concept of class intervals and frequency.

WORKING WITH GROUPED DATA

Teaching guidelines

Remind learners that class intervals should neither overlap, nor have gaps between them.

Answers

1. (a) See the table on LB page 269 alongside.
   (b) Less than 1 km.

GROUPING DATA INTO INTERVALS

When a data set contains many data items, we sometimes group the data items to help us organise the data. For example, the following data set shows the number of milk bottles collected by 24 learners for recycling:

9, 10, 13, 23, 24, 26, 26, 27, 30, 31, 34, 40, 42, 49, 50, 53, 61, 64, 67, 67, 68, 69, 91, 94

We can group the data into categories called **class intervals**, such as 0–9, 10–19, 20–29, and so on. We can then count how many times a value occurs in each interval. The number of times a value occurs in an interval is called its **frequency**.

This table shows the grouped data and the frequency of the values in each interval.

Interval	0–9	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80–89	90–99
Frequency	1	2	5	3	3	2	6	0	0	2

The table shows that one learner collected 0–9 bottles, two learners collected 10–19 bottles, five learners collected 20–29 bottles, and so on. We can clearly see that most learners (six) collected 60–69 bottles.

WORKING WITH GROUPED DATA

1. Anita collected data from a sample of Grade 7 learners about how far they live from the nearest grocery store. Below are the results. The values are in kilometres, correct to one decimal figure.

0,1	0,1	0,2	0,2	0,2	0,2	0,3	0,3	0,3	0,4	0,4	0,5	0,5	0,5	0,6
0,6	0,7	0,7	0,7	0,8	0,8	0,8	0,9	0,9	0,9	1	1	1	1,5	1,5
2	2	2	2	2,5	2,5	3	3	3	3,5	3,5	4	4	4,	4,5
5	5	6	6	7	7	8	8	9	10	10	15	20	23	30

(a) Copy and complete the table alongside to indicate how many of the values appear in each of the given intervals.

(b) How far do most of the learners live from the nearest grocery store?

Interval	Frequency
Less than 1,0 km	25
1,0–5,9 km	22
6,0–9,9 km	7
10 km or further	6

Answers

2. (a)

Key: 13 | 9 means 139 cm

13	5, 8, 8, 8, 9, 9
14	0, 0, 2, 3, 6, 8, 8, 8, 8
15	0, 0, 2, 2, 5, 5, 6, 6, 8, 8, 9
16	0, 0, 0, 0, 0, 0, 0, 0, 2, 3,3, 4, 4, 4, 5, 5, 5, 5, 5, 7, 7
17	1, 8, 9

(b) The interval with the biggest number of heights by far is 160–169 cm. But there are quite a few boys who are between 130 cm and 159 cm tall. Only three boys are taller than 170 cm.

(c) See the table on LB page 270 alongside.

23.3 Summarising data

Background information

The **next phase in the data handling cycle** is to summarise the data.

- We summarise data by finding a few numbers that, together, show us more about the whole data set. Some of these numbers involve all data values. Others involve only a few.
- **Measures of central tendency** tell us more about the balance in a data set.
- The **mode** shows the most common data value in an *ordered data set*. It is equal to the data value with the highest frequency.
  - A data set can have **more than one mode**.
  - If the frequencies of all data values are equal, the data set has **no mode**.
- The **median** cuts an ordered data set in half.
  - If the data set consists of an **odd** number of data values, the median is the **data value in the middle** of the *ordered data set*.
  - If the data set consists of an **even** number of data values, the median is the **value between the two data values in the middle** of the *ordered data set*.
- The **mean** is the **average** of a data set. It is found by adding all the data values and dividing the total by the number of data values.

2. Here are the heights of 50 Grade 7 boys at a school (in centimetres):

165	148	150	160	165	150	156	155	164	162
160	158	138	158	140	146	160	148	152	139
165	148	152	139	165	148	160	163	178	138
142	179	156	160	160	171	140	160	164	135
159	143	167	138	163	164	155	160	167	165

(a) Draw a stem-and-leaf display to show this set of data. Remember to include the key.

(b) Write a short paragraph to describe the data set.

(c) Copy and complete the frequency table below for the grouped data from your stem-and-leaf display in question (a).

Class interval (cm)	Frequency
130–139	6
140–149	9
150–159	11
160–169	21
170–179	3
Total	50

23.3 Summarising data

When you have collected data, you often need to tell someone what you have found out. People want to know what your conclusions are, without looking at all of the data you have collected.

It is often useful to summarise a set of numerical data by using *one* value. For example, which value best summarises or describes the following data set?

0 1 1 5 8 8 9 9 10 10 10 11 11

Statisticians use any of three values that show the most central values in the set, or the value around which the other values tend to cluster. These values are called the **measures of central tendency** or **summary statistics**.

Statisticians are mathematicians who specialise in collecting, organising and analysing data.

- **Measures of spread** tell us how far apart the data values in an *ordered data set* lie.
- The **range** tells us how far apart the smallest and largest data values in an *ordered data set* are. It is equal to the difference between the largest and smallest data values (the width of the ordered data set).

Teaching guidelines

Discuss the following concepts:

- measures of central tendency
- **mode**, **median** and **mean** of a data set.

Point out the following important facts:

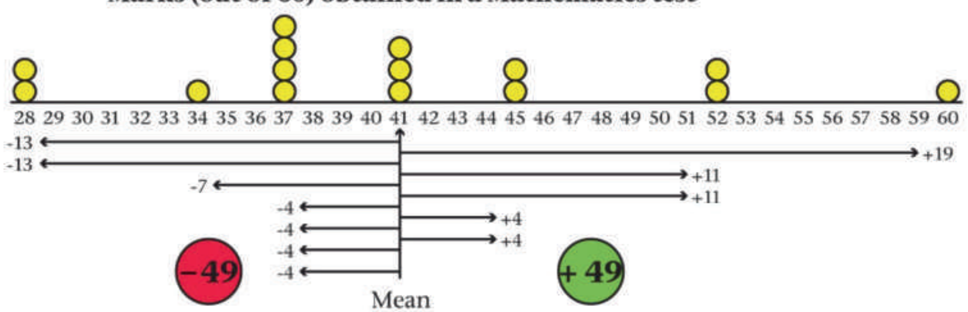
- The mode and median of a data set make no sense if the data values are not arranged in ascending or descending order.
- ALWAYS change the data set into an ordered data set BEFORE you try to find the mode and the median.

UNDERSTANDING THE MEAN

Background information

The diagram below explains why the mean of a data set is seen as a "balancing" measure.

Marks (out of 60) obtained in a Mathematics test



- The number at the end of each arrow shows the difference between the corresponding data value and the mean.
- The sum of differences to the **right** of the mean is +49.
- The sum of differences to the **left** of the mean is –49.
- If the two sums are added, the answer is 0 (the differences from the mean to the right and to the left balance out).

- The **mode** is the value that occurs most frequently in the data set. In the example on the previous page, the mode is 10 because it occurs the most times (three times).
- The **median** is the value exactly in the middle of the data set when the data values are arranged in order from smallest to largest. For the data set on the previous page, the median is 9 because there are six values to the right of the first 9 and six values to the left of it.
- The **mean (average)** is the total (sum) of the values divided by the number of values in the data set. So:

$$\text{Mean} = \frac{\text{Total of values}}{\text{Number of values}} = \frac{93}{13} = 7{,}15$$

A data set can have more than one mode.

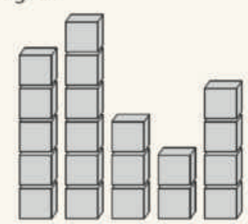
If the data set consists of an even number of items, the median = sum of the two middle values divided by 2.

In the data set on the previous page, either 10 (mode), 9 (median) or 7,15 (mean) could be used to represent the entire data set.

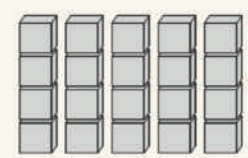
UNDERSTANDING THE MEAN

This activity will help you to understand how the mean represents the whole set of data.

Make piles of blocks of different heights:



Then move blocks from the higher piles to the lower ones to make all the piles equal:



You have just found the mean: Each pile now has four blocks in it. But how do you do this if you only have the numbers 5, 6, 3, 2 and 4 to work with? You add them up and then divide the answer by the total number of values (numbers):

$5 + 6 + 3 + 2 + 4 = 20$ $\qquad$ $20 \div 5 = 4$

What this means is that you are finding a single number that you can use in place of all the different numbers and still get the same total.

Teaching guidelines

- Refer to the stacks of blocks on LB page 271. There are three blocks in a position higher than fourth. These three blocks can be used to fill the gaps where the stacks are lower than four blocks high, therefore the mean height of the five stacks together is four blocks.
- Discuss the concept of **range** of a data set and point out that it makes no sense if the data values are not arranged in ascending or descending order.

DETERMINING THE MODE, MEDIAN, MEAN AND RANGE

Teaching guidelines

Revise the concepts of mode, median, mean and range of a data set.

Answers

1. (a) 5
   (b) 4
   (c) Mean = $\frac{\text{Total of values}}{\text{Number of values}} = \frac{69}{19} = 4$
   (d) $6 - 1 = 5$
2. (a) 23
   (b) 2
   (c) 2
   (d) Mean = $\frac{\text{Total of values}}{\text{Number of values}} = \frac{47}{23} = 2$
   (e) $5 - 0 = 5$
3. (a) 16
   (b) 40
   (c) Median = $\frac{40 + 40}{2} = 40$
   (d) Mean = $\frac{\text{Total of values}}{\text{Number of values}} = \frac{559}{16} = 34{,}9$
   (e) $48 - 15 = 33$
4. (a) 16

It is also useful to know how big the spread of the data is.

The **range** of a data set is the difference between the highest value and the lowest value. For example, for the data set on the previous page, the range is:

$11 - 0 = 11$

The bigger the range, the more the data is spread out. The smaller the range, the more the data is clustered around similar values.

DETERMINING THE MODE, MEDIAN, MEAN AND RANGE

1. The following data set shows the shoe sizes of a sample of learners at a school:
   1, 1, 1, 2, 2, 2, 3, 3, 4, 4, 4, 5, 5, 5, 5, 5, 5, 6, 6
   (a) What is the mode of the data set?
   (b) What is the median of the data set?
   (c) What is the mean? (Round off to the nearest whole number.)
   (d) What is the range of the data set?
2. The following data set shows the number of siblings (that is, brothers and sisters) that the learners in a sample of Grade 7 learners have:
   0, 0, 0, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 4, 4, 5
   (a) How many learners are in the sample?
   (b) What is the mode of the data set?
   (c) What is the median of the data set?
   (d) What is the mean? (Round off to the nearest whole number.)
   (e) What is the range of the data set?
3. The following data set shows the number of hours worked in a week by a sample of parents at School A:
   15, 16, 20, 25, 25, 30, 40, 40, 40, 40, 40, 42, 45, 45, 48, 48
   (a) How many parents are in the sample?
   (b) What is the mode of the data set?
   (c) What is the median of the data set?
   (d) What is the mean? (Round off to one decimal place.)
   (e) What is the range of the data set?

Remember, if the number of items in a data set is even, the median = the sum of the two middle numbers divided by 2.

4. The following data set shows the number of hours worked in a week by a sample of parents at School B:
   25, 30, 35, 35, 35, 40, 40, 40, 40, 40, 42, 45, 45, 45, 48, 50
   (a) How many parents are in the sample?

Answers

4. (b) 40
   (c) Median $= \frac{40 + 40}{2} = 40$
   (d) Mean $= \frac{\text{Total of values}}{\text{Number of values}}$
   $= \frac{635}{16} = 39{,}7$
   (e) $50 - 25 = 25$
5. (a) 10, 13, 23, 30, 31, 34, 40, 42, 44, 49, 50, 53, 61, 64, 67, 67, 68, 69, 91, 94
   (b) 20
   (c) 67
   (d) Median $= \frac{49 + 50}{2} = \frac{99}{2} = 49{,}2$
   (e) Mean $= \frac{\text{Total of values}}{\text{Number of values}}$
   $= \frac{1\,000}{20} = 50$
   (f) $94 - 10 = 84$
6. (a) See LB page 273 alongside.
   (b) five goals and seven goals
   (c) one goal
   (d) 0 and 3
   (e) One goal is the mode.
   (f) Count up to the fifteenth and sixteenth goals (dots) from the left-hand side. The median is 2.
   (g) $61 \div 30 = 2{,}03$ goals ≈ 2 goals

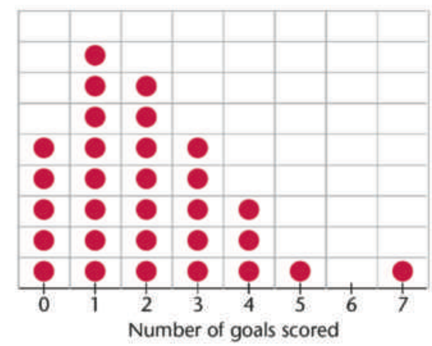
(b) What is the mode of the data set?
(c) What is the median of the data set?
(d) What is the mean? (Round off to one decimal place.)
(e) What is the range of the data set?

5. The following is a list of test scores of learners in a Grade 7 class:
40, 42, 44, 13, 10, 23, 68, 31, 69, 91, 30, 49, 50, 53, 67, 94, 61, 64, 67, 34
(a) Arrange the scores from the lowest to the highest.
(b) How many learners are in the population?
(c) What is the mode of the data set?
(d) What is the median of the data set?
(e) What is the mean?
(f) What is the range of the data set?

6. A hockey player recorded the number of goals she scored in her last 30 matches:

1	1	3	2	0	0	4	2	2	4	3	1	0	1	0
2	1	5	1	3	7	2	2	2	4	3	1	1	0	3

(a) Copy the graph below. Draw a dot plot on the number line to organise these data values.



Now use the dot plot to answer these questions.

(b) Which of the values are quite different to the other values?
(c) Which number of goals has she scored the highest number of times?
(d) Which numbers of goals did she score in the two groups with five matches each?
(e) Use the dot plot to find the mode of the data.
(f) Use the dot plot to find the median.
(g) What is the mean of the goals?

CHAPTER 23: COLLECT, ORGANISE AND SUMMARISE DATA 273

Grade 7 Term 4 Chapter 24 Represent data

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
24.1 Bar graphs and double bar graphs	Draw and interpret bar graphs and double bar graphs	Pages 274 to 277
24.2 Histograms	Draw and interpret histograms	Pages 277 to 283
24.3 Pie charts	Draw and interpret pie charts	Pages 284 to 287

CAPS time allocation	5 hours
CAPS content specification	Pages 78 to 79

Mathematical background

In Chapter 23 we covered the following **phases in the data handling cycle**:

- **Pose a question** about a real-life problem that requires the collection of data.
- **Collect and record data** on data recording sheets during observations and on questionnaires during interviews.
- **Classify, sort and organise data** in categories or intervals on frequency tables, dot plots and stem-and-leaf displays.
- **Summarise data** by finding the mode, median, mean and range of the data set.

In this chapter the focus is on **representing data**, which is the next phase in the data handling cycle. Data is represented by drawing a picture of the tabulated data. This is done for the following reasons:

- A picture makes information **easier to understand**.
- It is easier to **identify patterns** in a picture than in a table. Humans are great at seeing patterns, but they struggle with raw numbers.
- It is easier to **identify trends** in a picture than in a table. These trends can be upward or downward, and they can even be cyclical. Trends are easy to see in a picture, but not easy to see in a table.
- People are more attracted when visual information is presented in an **aesthetically pleasing manner** in the media.

A large variety of statistical displays are used by statisticians and in the media to convey information, for example:

- dot plots, pictographs, single and double bar graphs, pie charts, line and broken-line graphs and scatter plots
- histograms, frequency polygons, ogives (cumulative frequency polygons), regression functions, normal distributions, and so on.

In this chapter we focus on **single bar graphs**, **double bar graphs**, **histograms** and **pie charts**.

24.1 Bar graphs and double bar graphs

DRAWING A BAR GRAPH

Background information

- A **bar graph** uses separate bars to show the frequencies of different categories in a data set.
- Here are some **features of a bar graph**:
  - It represents only one set of data.
  - It represents either categorical or ungrouped numerical data.
  - It never represents grouped numerical data.
  - It uses bars to show the frequencies of the different categories.
  - The heights or lengths of the bars represent the frequencies of the different categories.
- **Bar graphs are suitable** to:
  - identify patterns in the data
  - describe trends shown by the data.
- To **draw a bar graph**, remember the following:
  - All bars must be equally wide.
  - Gaps between bars must be equally wide, but narrower than the bars, unless a bar is missing from the sequence.
  - The first bar should not touch the frequency axis.
- **Vertical bar graphs** are usually used to show change over time at discrete times, for example, absentees per day of the week.
- **Horizontal bar graphs** are usually used to compare or rank items at one point in time, for example, absentees per grade on a specific day.

Teaching guidelines

Discuss the features of a bar graph as reflected by the bar graph on LB page 274.

CHAPTER 24

Represent data

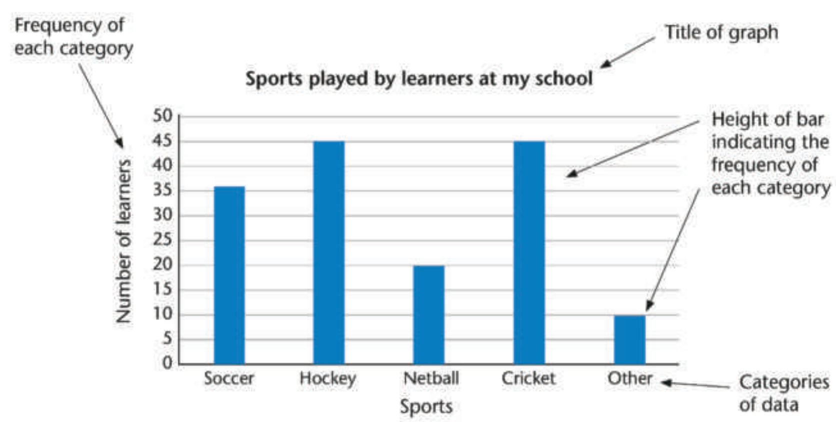
Now that we have collected and organised a set of data, we want to show the results in a useful way.

Remember when you drew dot plots in the previous chapter, you could see which categories or measurements occurred many times and which occurred only a few times. There are a few different graphs that show the important things about the data in such a way that you can see them easily. You need to be able to draw these graphs.

24.1 Bar graphs and double bar graphs

DRAWING A BAR GRAPH

A **bar graph** shows categories (or classes) of data along the horizontal axis, and the frequency of each category along the vertical axis. (Sometimes the axes are swopped around.) Here is an example of a bar graph.



Answers

Learners represent Thandeka's data about languages spoken in her class (Section 23.2 of Chapter 23) in a bar graph like the one on LB page 275.

USING DOUBLE BAR GRAPHS

Background information

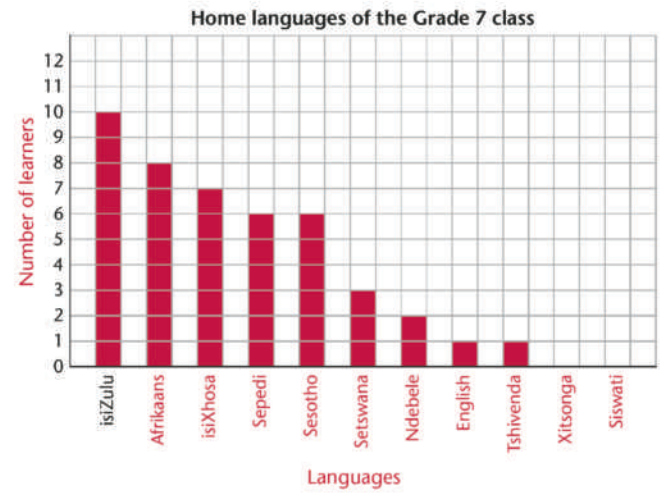
- A **double bar graph** uses separate pairs of bars to show the frequencies of different categories in two related data sets.
- Here are some **features of a double bar graph**:
    - It represents two sets of data with matching categories.
    - It represents either categorical or ungrouped numerical data.
    - It never represents grouped numerical data.
    - It uses pairs of bars to show the frequencies of matching categories.
    - The heights or lengths of the bars represent the frequencies of the matching categories.
- **Double bar graphs are suitable** to:
    - identify similar and different patterns in the two sets of data
    - describe similar and different trends shown by the two sets of data.
- To **draw a double bar graph**, remember the following:
    - All bars must be equally wide.
    - Gaps between pairs of bars must be equally wide, but narrower than the bars, unless a bar is missing from one or both sequences.
    - The first bar should not touch the frequency axis.
    - A key (legend) should explain the colours used to distinguish the two sets of data.

Teaching guidelines

Discuss the features of a double bar graph as reflected by the double bar graph on the right.

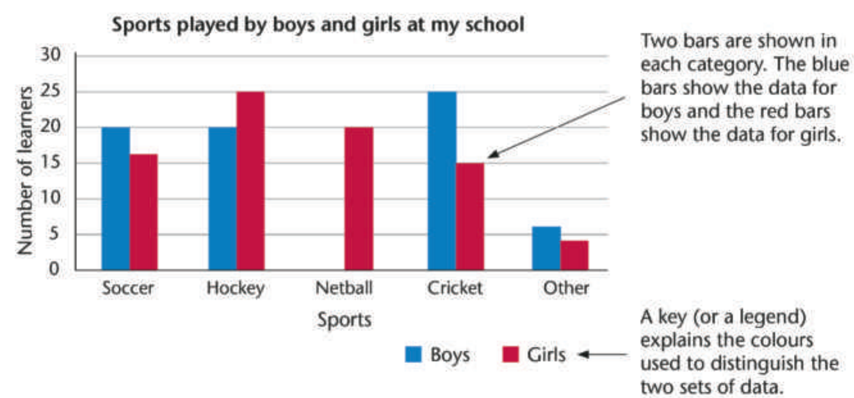
Point out that there is no blue bar for netball because none of the boys play netball.

Go back to section 23.2 of Chapter 23, where you drew a dot plot and made a tally table of Thandeka's data about languages spoken in her class. Copy the grid below and use Thandeka's data to draw a bar graph. Draw the bars to the correct height by looking at the numbers on the vertical axis.



USING DOUBLE BAR GRAPHS

A **double bar graph** shows two sets of data for each category (or class). For example, the double bar graph below shows data collected from girls for each category, and data collected from boys for each category.

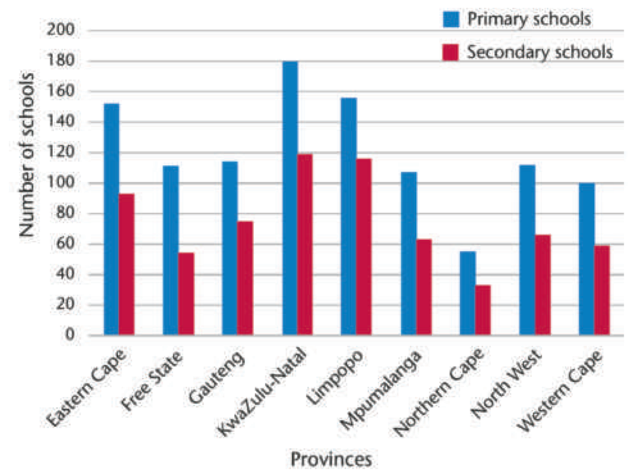


Answers

1. (a) Primary schools
   (b) Northern Cape
   (c) Eastern Cape, KwaZulu-Natal and Limpopo
2. Refer to the double bar graph on the top of LB page 277 on following page.

1. Look at the data below and answer the questions that follow.

Number of schools, by province, participating in a school survey



   (a) Did more primary schools or more secondary schools participate in the survey?
   (b) Which province had fewer than 50 secondary schools participating in the survey?
   (c) Which provinces had more than 150 of its primary schools participating in the survey?

2. Use grid paper, as shown in the example on page 277, to draw a double bar graph to show the following data.

Facilities available at schools in Province A and Province B

Facility	Percentage of schools in Province A	Percentage of schools in Province B
Electricity	73	50
Running water	68	45
Computers	60	20
Internet	30	10

24.2 Histograms

A SITUATION WHERE DATA HAS TO BE ORGANISED

Background information

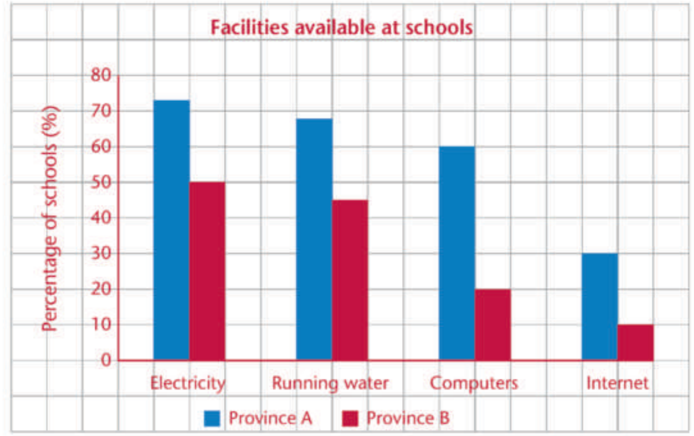
- A **histogram** uses touching bars to show the frequencies of different data intervals in a data set.
- Here are some **features of a histogram**:
  - It represents only one set of data.
  - It represents numerical data which is grouped in class intervals.
  - It uses bars to show the frequencies of the different class intervals.
  - The heights of the bars represent the frequencies of the different class intervals.
- **Histograms are suitable** to:
  - identify patterns in the grouped data
  - describe trends shown by the grouped data.
- To **draw a histogram**, remember the following:
  - All bars must be equally wide if all class intervals are equally wide.
  - There are no gaps between bars unless the frequency of a specific class interval is equal to 0.
  - The first bar should not touch the frequency axis.

Teaching guidelines

Introduce learners to question 1 on LB page 277 alongside and challenge them in groups to find a solution for Mr Makae's problem.

Answers

1. Refer to the teaching guidelines above. The purpose of the question is to engage learners and make them think.



24.2 Histograms

A SITUATION WHERE DATA HAS TO BE ORGANISED

1. Mr Makae wants to buy an orange farm. Three farms are available, each with an orchard of orange trees, and the three farms cost about the same. There are 40 orange trees on each farm. The total mass of oranges (in kilograms) harvested from each tree on each farm over the last three years is given below. Which farm should he buy?

Farm A:

426	628	467	413	862	585	652	600	734	611
741	605	536	643	833	438	613	704	623	719
719	701	501	768	642	444	751	579	695	726
616	619	441	703	902	947	785	952	725	721

Farm B:

822	736	773	674	884	463	644	433	688	487
884	530	448	410	982	638	492	638	725	621
743	661	744	530	560	745	455	943	760	734
888	457	621	969	507	500	542	831	576	801

Farm C:

438	530	743	947	450	777	859	748	473	724
750	852	428	464	725	554	758	997	467	743
722	438	779	690	785	543	752	898	474	483
460	772	544	756	491	576	482	744	701	803

CHAPTER 24: REPRESENT DATA 277

2. Learners put their solutions in words. No specific answer is required.
3. Refer to the tally and frequency tables on LB page 278 alongside.

2. How can the data about the orange trees on the three farms be organised so that the farmer has a clear picture of the difference between the orchards on the three farms? For now, just write down how you think the data may be organised. You will organise the data later when you do the questions that follow.

3. Copy and complete these tally and frequency tables for the data about the masses of oranges harvested on the three orange farms.

Masses of oranges harvested from different trees on Farm A

Mass of oranges harvested from each tree. These are called **class intervals**.	Number of trees that produced masses in the interval	Total
400 kg or more but less than 500 kg	~~||||~~		6
500 kg or more but less than 600 kg	||||	4
600 kg or more but less than 700 kg	~~||||~~ ~~||||~~ ||	12
700 kg or more but less than 800 kg	~~||||~~ ~~||||~~ |||	13
800 kg or more but less than 900 kg	||	2
900 kg or more but less than 1 000 kg	|||	3

Masses of oranges harvested from different trees on Farm B

Class interval	Number of trees that produced masses in the interval	Total
400 kg or more but less than 500 kg	~~||||~~ |||	8
500 kg or more but less than 600 kg	~~||||~~ ||	7
600 kg or more but less than 700 kg	~~||||~~ |||	8
700 kg or more but less than 800 kg	~~||||~~ |||	8
800 kg or more but less than 900 kg	~~||||~~		6
900 kg or more but less than 1 000 kg	|||	3

Masses of oranges harvested from different trees on Farm C

Class interval	Number of trees that produced masses in the interval	Total
400 kg or more but less than 500 kg	~~||||~~ ~~||||~~ ||	12
500 kg or more but less than 600 kg	~~||||~~	5
600 kg or more but less than 700 kg			1
700 kg or more but less than 800 kg	~~||||~~ ~~||||~~ ~~||||~~		16
800 kg or more but less than 900 kg	||||	4
900 kg or more but less than 1 000 kg	||	2

On the next page, you will learn how to draw graphs of the data for the three farms.
The data for Farm A is represented on the following graph.

Background information (continued)

Refer to the histogram on LB page 279 alongside.

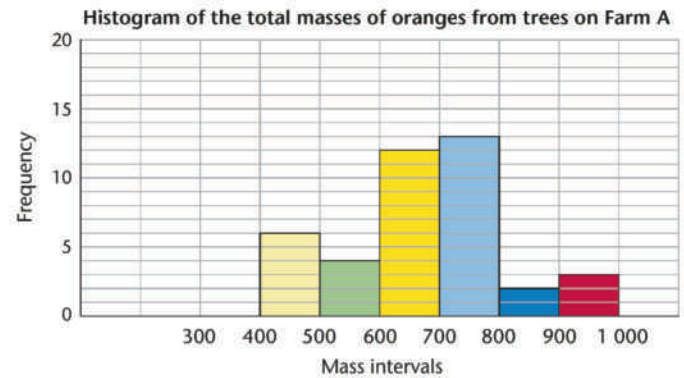
- A **class** of data is **one of the groups** in a collection of grouped data, for example, all data values from 500 up to 599.
- The **class limits** are the **two values at the ends of a class** between which the data values must lie, for example, 500 is the lower class limit of the green bar and 600 is the lower class limit of the yellow bar, which means that the smallest data value in the yellow class can be 500 but the largest data value must be less than 600.
- The **class interval** is the **width of a class** and is measured by the difference between the class limits, for example, the class interval of the green bar is 600 g – 500 g = 100 g.
- The **class boundaries** are the **smallest and largest possible values** that fit inside a given class, for example, equal to 500 g but less than 600 g for the green bar.

Teaching guidelines (continued)

- Use the background information on page 317 of this TG and discuss the features of a histogram as shown by the one on LB page 279.
- Point out that there are no gaps between bars (no gaps between class intervals) and no bars overflow (no class intervals overflow).
- Point out that 500 is the lower boundary of the green bar and 600 is the lower boundary of yellow bar, therefore the green bar contains all masses from 500 g to 599 g.

Answers

4. (a) In the green column.
   This is because all the trees that produced 500 kg or more but less than 600 kg are represented in this column.
   (b) Masses of 900 kg or more, but less than 1 000 kg.
   In this case, 902 kg, 947 kg and 952 kg.
   (c) The class interval 700–800, in other words masses of 700 kg or more but less than 800 kg.
   (d) Four
   (e) The light blue column
5. Refer to the histogram for Farm B alongside and Farm C on the next page.



This type of graph is called a **histogram**.

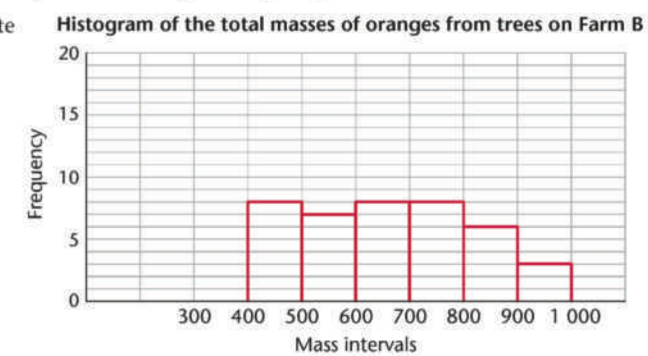
(The columns in a histogram are normally not coloured differently, or even coloured at all. In this histogram the columns are coloured only because some questions are asked about them in question 4 below.)

The numbers 400 on the left and 500 on the right of the light yellow column indicate that masses of 400 kg or more but less than 500 kg are counted in that interval.

The height of each column represents the number of masses (the frequency) that fall in that interval.

4. (a) A total of 536 kg of oranges was harvested from one of the trees on Farm A over a period of the three years. In which column on the above histogram is this tree represented? Explain your answer.
   (b) Which masses are represented in the red column?
   (c) Which class interval is represented by the light blue column on the above histogram?
   (d) How many masses are represented by the green column?
   (e) Which column represents the highest frequency?

5. Copy and complete the following histograms.



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Teaching guidelines (continued)

- Point out that **class intervals are consecutive** and cannot overlap.
- Explain that the **upper boundary** of each class interval is **not included** in the interval.

INTERPRETING A HISTOGRAM

Background information

To interpret histograms, do the following:

- **extract data** from the histogram
- **identify patterns** shown by the histogram
- **describe trends** shown by the histogram.

Teaching guidelines

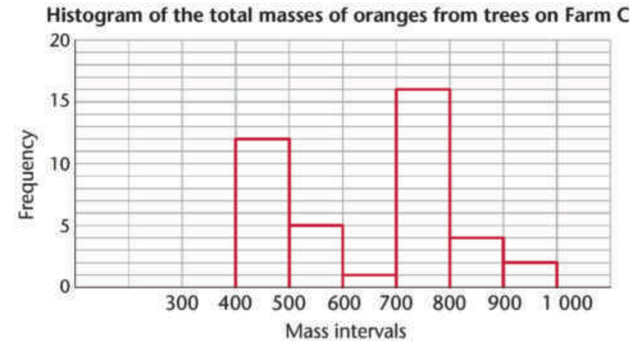
Learners extract data from the histogram on LB page 281 to answer some of the questions below.

Answers

1. Refer to the histogram on LB page 281 on following page.

Age (years)	Frequency
20–30	0
30–40	3
40–50	11
50–60	17
60–70	34
70–80	19

2. 17 members
3. 3 + 11 + 17 + 34 + 19 = 84 members
4. Yes, the order of the columns is important, because the horizontal axis shows a range of age intervals, from smallest to biggest, like a number line.



The different class intervals are **consecutive** and cannot have values that overlap. For example, we can group heights into class intervals of 10 cm, as shown below:

Height (m)	Heights that fall in the class interval	Frequency
1,20–1,30	1,20; 1,25; 1,29	3
1,30–1,40	1,30; 1,31; 1,35; 1,39	4
1,40–1,50	1,40; 1,46; 1,48; 1,48; 1,49	5
1,50–1,60	1,53; 1,53; 1,57; 1,58; 1,59; 1,59	6

We follow the convention that the top value (also called the **upper boundary**) of each class interval is not included in the interval.

So the height of 1,20 m falls into the 1,20–1,30 m interval, but the height 1,30 m falls into the 1,30–1,40 m interval.

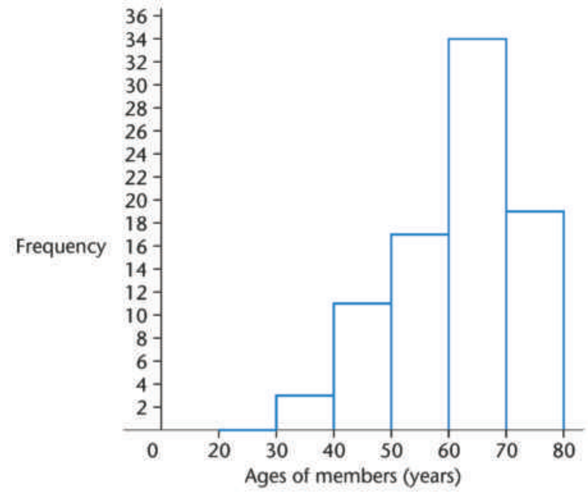
INTERPRETING A HISTOGRAM

Study the histogram (on page 281) showing the numbers of members, in different age groups, of a sports club. Then answer the questions that follow.

1. Complete a frequency table for the information.
2. How many of the members are in their fifties?
3. How many members does the club have?
4. When you drew a bar graph, it did not matter what order the bars were in. Does the order of the columns on the histogram matter? Explain.

Teaching guidelines (continued)

Point out that the individual data values in a histogram cannot be seen because the data was grouped. This is a disadvantage of histograms as well as of grouped data.

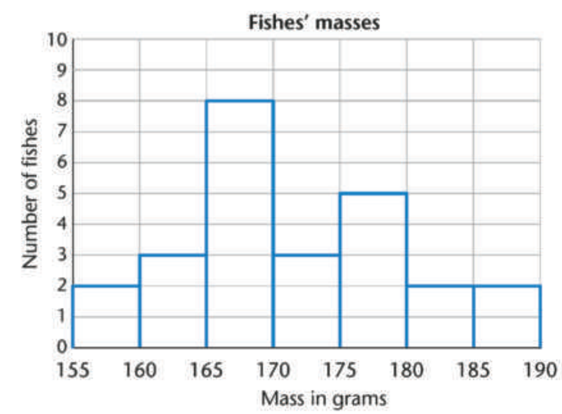


Notice that you cannot see the individual data values in a histogram – they have been "lost". For example, below you can see a stem-and-leaf display and a histogram of the same data set.

Fishes' masses in grams

15	7, 8
16	2, 3, 3
16	5, 5, 6, 7, 8, 8, 9, 9
17	1, 2, 3
17	6, 7, 7, 8, 8
18	0, 3
18	6, 7

Key: 15|7 means 157



A histogram usually has many more data values than a stem-and-leaf display – too many to show in a stem-and-leaf display. It would, for example, be difficult to put the 84 values for the members of the sports club onto a stem-and-leaf display.

CHAPTER 24: REPRESENT DATA 281

DRAWING MORE HISTOGRAMS

Teaching guidelines

Remind learners that:

- bars should be equally wide without any gaps in between because the classes are equally wide with no gaps in between
- the class intervals do not overflow because the upper class boundary of each class is not included in the interval
- the height of the bar that represents a category should match the frequency of that category.

Note on question 1(d)

The first bar touches the frequency axis because the lower boundary of the first class interval is equal to 0.

Answers

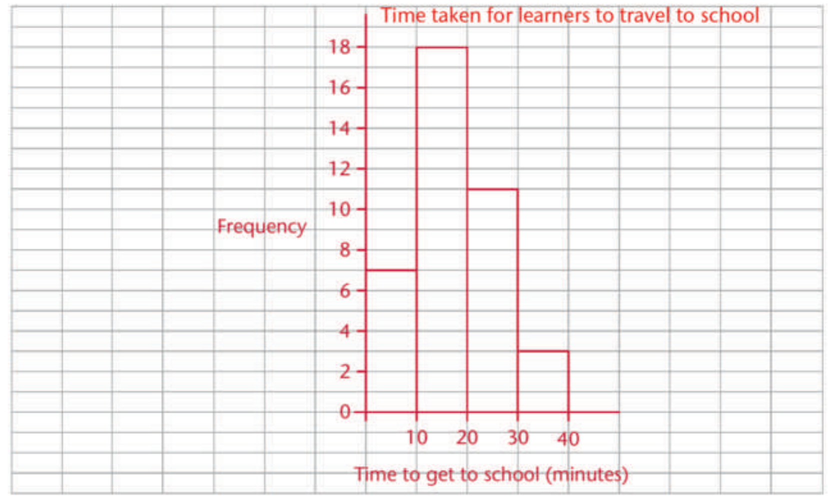
1. (a) 7 + 18 + 11 + 3 = 39
   (b) Look at the maximum frequency (which is 18). Then choose a scale that will cover the greater part of the axis and grid.
   (c) Intervals of 10 because the data is grouped in tens. The columns can be one or two blocks wide.
   (d) See the graph on LB page 282 alongside.
2. (a) 6 + 9 + 11 + 7 + 5 = 38
   (b) Each block represents one or two vendors.
   (c) One block to show R100, or two blocks to show R100.

DRAWING MORE HISTOGRAMS

1. The table shows how long it takes learners from a Grade 7 class at Western Primary to travel to school each day. In question (d) you will use a histogram to represent the data in the table.

Time (minutes)	Frequency
0–10	7
10–20	18
20–30	11
30–40	3

   (a) How many learners were asked about their travelling hours?
   (b) Look at the grid provided in question (d). What do you have to consider in order to help you decide on a scale division for the vertical axis?
   (c) What scale will you use on the horizontal axis? Explain your answer.
   (d) Copy the grid and draw a histogram of the data.



2. The table shows how much money different vendors earn selling their goods every week.
   (a) How many vendors were asked about their earnings?
   (b) Copy the grid on the following page. Decide on a scale for the vertical axis of a histogram and indicate it on the axis.
   (c) Decide on a scale for the horizontal axis and indicate it on the axis.

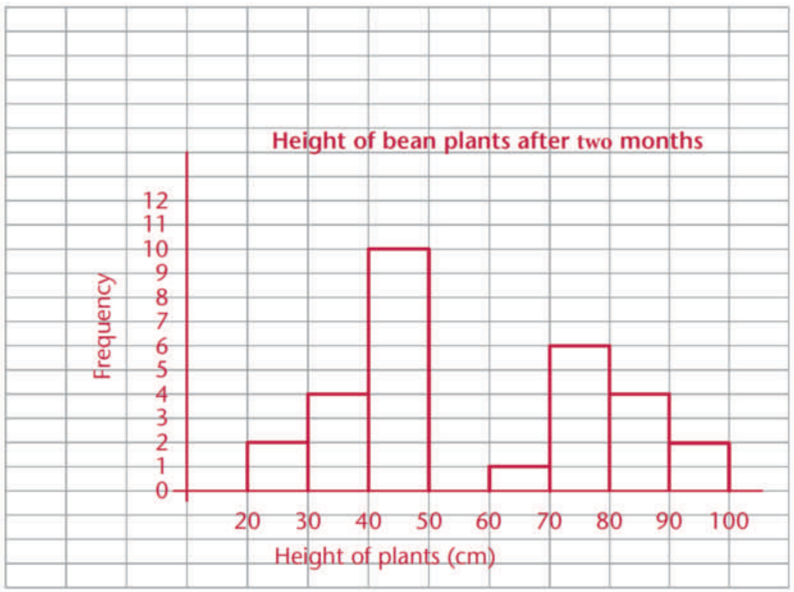
Money (R)	Frequency
0–100	6
100–200	9
200–300	11
300–400	7
400–500	5

Note on question 2(d)

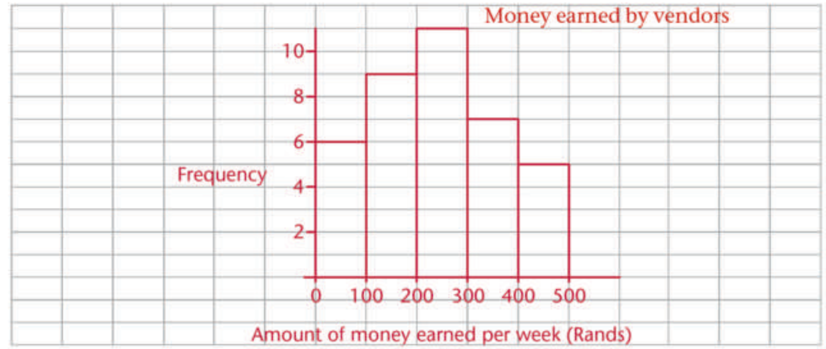
The first bar touches the frequency axis because the lower boundary of the first class interval is equal to 0.

Answers

2. (d) See the graph on LB page 283 alongside.
3. (a) See the table on LB page 283 alongside.
   (b) Refer to the histogram below.



(d) Complete the histogram showing the data.



3. In a Natural Sciences class, learners planted beans and measured the heights of the bean plants after two months. Here is the data they collected (in centimetres):

34 65 72 42 37 29 78 43 79 91 43 45 28 42 79

34 92 87 40 43 43 78 82 47 85 43 32 86 76

(a) Copy and complete this frequency table:

Height of bean plants (cm)	Tally	Frequency
20–30	||	2
30–40	||||	4
40–50	~~||||~~ ~~||||~~	10
50–60		0
60–70	|	1
70–80	~~||||~~ |	6
80–90	||||	4
90–100	||	2
Total	**29**	

(b) Draw a histogram of this data.

24.3 Pie charts

Background information

A **pie chart** uses sectors (slices) of the same circle to compare the frequencies of the different categories within a data set.

- Here are some **features of a pie chart**:
  - It represents only one set of data.
  - It can be used to display any type of data.
  - It compares parts of a whole because it shows how the data set is divided up into different categories and what fraction of the data set each category represents.
  - The sizes of the sectors are proportional to the frequencies of the different categories.
  - It works best if the frequencies of the different categories in the data set are expressed as percentages.
- **Pie charts are suitable** to compare categories within a data set.
- To **draw a pie chart**, express the frequency of each category as a fraction of the total frequency of the data set and convert it to either degrees or a percentage.

Teaching guidelines

Discuss the features of a pie chart as reflected by the pie chart on LB page 284.

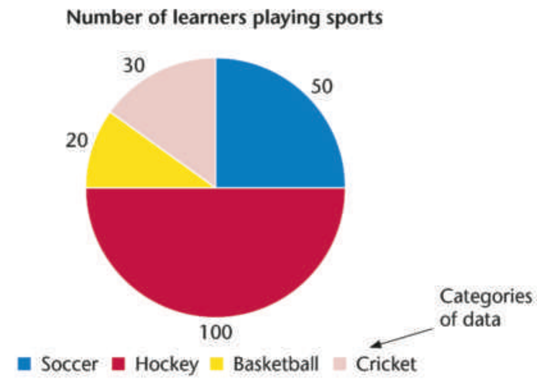
Point out the following:

- The numbers surrounding the pie chart are frequencies. If they were percentages, they would add up to 100.
- The key is only necessary if the different sectors are not labelled separately.

24.3 Pie charts

A **pie chart** consists of a circle divided into slices (**sectors**), where the slices show how the different categories of data make up the whole set of data. Bigger categories of data have bigger slices of the circle.

Look at the example of a pie chart below.



The pie chart shows the following:

- A total of 200 learners were asked about the sports they played:
  $20 + 30 + 50 + 100 = 200$
- The key shows the four categories of data:
  - soccer
  - hockey
  - basketball
  - cricket.
- 100 of the 200 learners play hockey. This is the largest category, and gets the biggest slice (half of the whole).
- 20 of the 200 learners play basketball. This is the smallest category, and gets the smallest slice (one tenth of the whole).

You will learn how to draw accurate pie charts in later grades. In this grade, you will estimate the portions of a pie chart that each category of data requires.

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ESTIMATING SIZES OF SLICES IN A PIE CHART

Background information

If a whole is divided into:

- two equal parts, each part is $\frac{1}{2}$ of the whole
- three equal parts, each part is $\frac{1}{3}$ of the whole
- four equal parts, each part is $\frac{1}{4}$ of the whole, etc.

Teaching guidelines

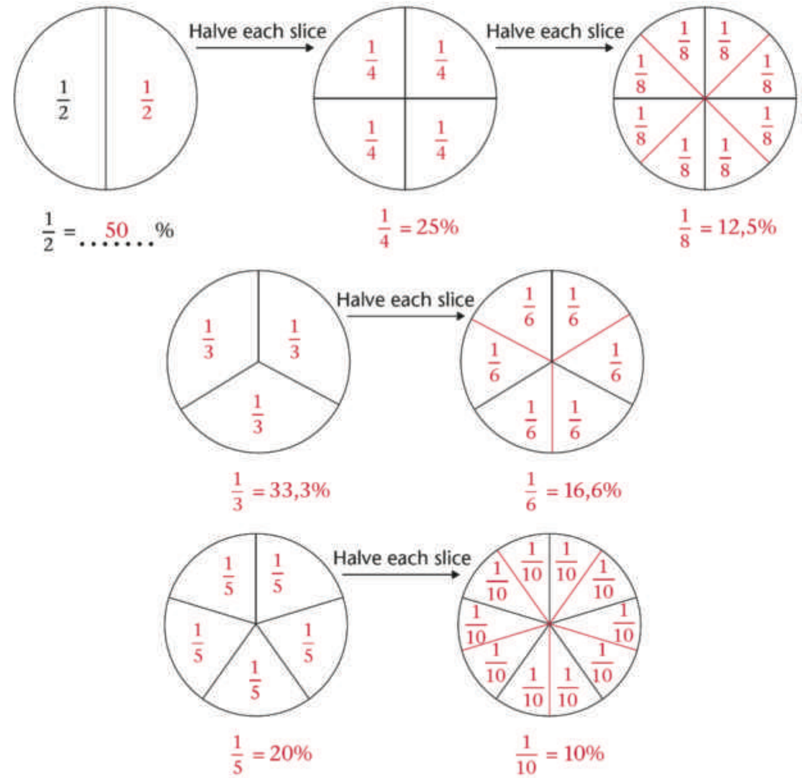
Learners should realise that the given sectors in each pie chart in question 1 are equal in size.

Answers

1. (a) See the pie charts on LB page 285 alongside.
   (b) See beneath the pie charts on LB page 285 alongside.

ESTIMATING SIZES OF SLICES IN A PIE CHART

1. (a) Copy the following diagrams and add the slices indicated. Write down the fraction of a whole that each slice in the following diagrams shows.



   (b) For each diagram in question 1(a), write down what percentage each fraction is equal to.

You can use the diagrams above to estimate the sizes of slices when drawing your own pie charts.

2. Copy the pie charts on the following page. Use the data in each of the following tables to complete the pie charts. You must:
   - label the major sector
   - divide the other sector into the parts that represent the other languages
   - label each sector.

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Teaching guidelines (continued)

Point out that a key is not necessary if the different sectors are labelled separately.

Note on question 2

Each white sector should be divided into smaller sectors in the same ratio as the percentages provided in each table.

Answers

2. (a) See the pie chart on LB page 286 alongside.
   (b) See the pie chart on LB page 286 alongside.
   (c) See the pie chart on LB page 286 alongside.

REPRESENTING DATA AS FRACTIONS AND PERCENTAGES IN PIE CHARTS

Background information

- Before data in a frequency table can be displayed on a pie chart, the following has to be done:
  - Express the frequency of each category as a fraction of the total frequency of the data set.
  - Convert each common fraction to either degrees or a percentage.
- Use the degrees or percentages to divide the circle into sectors.

Teaching guidelines

Revise the conversion of common fractions to percentages.

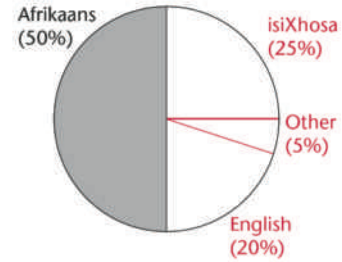
Answers

1. (a) See the fraction column of the table on LB page 287 on the following page.
   (b) See the percentage column of the table on LB page 287 on the following page.

(a)

Province: Western Cape

Major languages	Frequency (in %)
Afrikaans	50%
English	20%
isiXhosa	25%
Other	5%



(b)

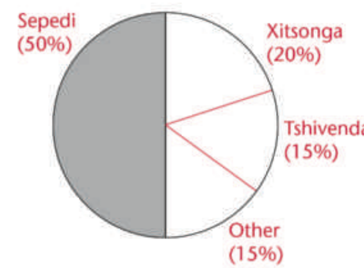
Province: KwaZulu-Natal

Major languages	Frequency (in %)
English	15%
isiZulu	80%
Other	5%

(c)

Province: Limpopo

Major languages	Frequency (in %)
Sepedi	50%
Tshivenda	15%
Xitsonga	20%
Other	15%



REPRESENTING DATA AS FRACTIONS AND PERCENTAGES IN PIE CHARTS

To represent data in a pie chart, you need to know how to convert (change) the frequencies of the different categories into a fraction or percentage of the total.

1. The learners in Class A were asked how many languages they could speak. The table shows the data that was collected. Copy the table.
   (a) Complete the "Fraction" column by determining what fraction of the whole each category is.
   (b) Complete the "Percentage" column by converting the fraction to a percentage.

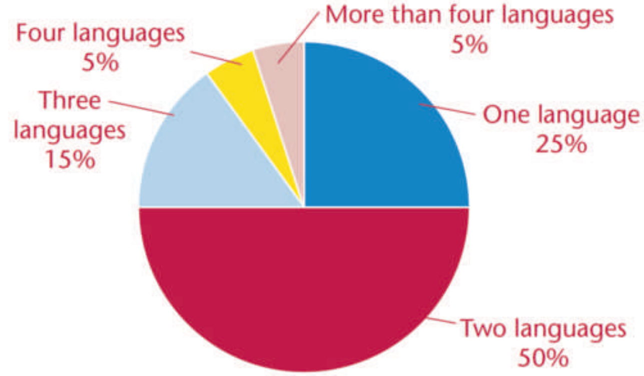
Remember, to convert a common fraction to a percentage you have to multiply by 100%.

286 MATHEMATICS GRADE 7: TERM 4

Answers

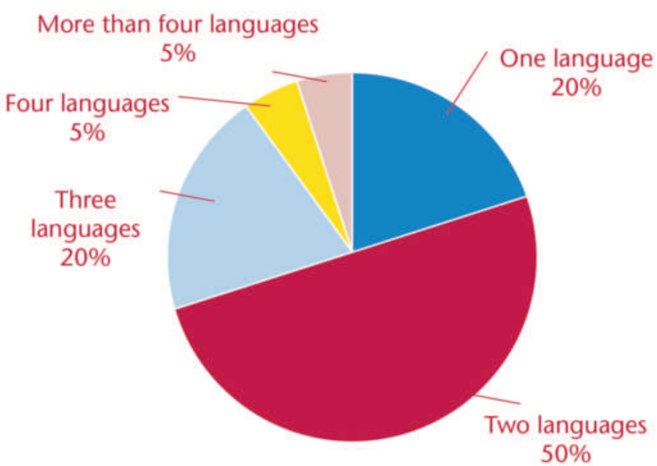
1. (c) Refer to the pie chart below.

Number of languages spoken by learners in Class A



2. (a) Refer to the fraction column of the table on LB page 287 alongside.
   (b) Refer to the percentage column of the table on LB page 287 alongside.
   (c) Refer to the pie chart below.

Number of languages spoken by learners in Class B



Number of languages spoken by learners in Class A

Languages	Frequency	Fraction	Percentage
One language	10	$\frac{10}{40} = \frac{1}{4}$	25%
Two languages	20	$\frac{20}{40} = \frac{1}{2}$	50%
Three languages	6	$\frac{6}{40} = \frac{3}{20}$	15%
Four languages	2	$\frac{2}{40} = \frac{1}{20}$	5%
More than four languages	2	$\frac{2}{40} = \frac{1}{20}$	5%
Total	**40**	$\frac{40}{40}$	**100%**

(c) Draw a pie chart of the data in your completed table. Use a circular object to draw the circle. Then estimate the sizes of the various slices of the pie chart.

2. The learners in Class B were asked how many languages they could speak. The table shows the data that was collected. Copy the table.
   (a) Complete the "Fraction" column by determining what fraction of the whole each category is.
   (b) Complete the "Percentage" column by converting the fraction to a percentage.

Number of languages spoken by learners in Class B

Languages	Frequency	Fraction	Percentage
One language	12	$\frac{12}{60} = \frac{1}{5}$	20%
Two languages	30	$\frac{30}{60} = \frac{1}{2}$	50%
Three languages	12	$\frac{12}{60} = \frac{1}{5}$	20%
Four languages	3	$\frac{3}{60} = \frac{1}{20}$	5%
More than four languages	3	$\frac{3}{60} = \frac{1}{20}$	5%
Total	**60**	$\frac{60}{60}$	**100%**

(c) Draw a pie chart to represent the data in your completed table.

CHAPTER 24: REPRESENT DATA 287

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
25.1 Interpreting and reporting on data	Critically read data; identify patterns in data sets; describe trends shown by data sets; report on a data set	Pages 288 to 290
25.2 Identifying bias and misleading data	Critically analyse data in order to identify bias and misleading data	Pages 290 to 293

CAPS time allocation	5 hours
CAPS content specification	Pages 78 to 79

Mathematical background

In this chapter the focus is on interpretation and analysis of data and reporting on findings.

- To **interpret data** means to extract information directly from the data. This process is often used to determine whether learners understand the given data.
- To **analyse data** means to investigate the given data in order to find patterns and trends shown by the data.
  - **Patterns** can be cyclic (e.g. data may show that learners tend to be absent from school on Mondays).
  - **Trends** are increasing or decreasing patterns (e.g. the percentage of girls per grade increases in higher grades).
- To **report on data** means to summarise the most important findings from the data in a short paragraph.
- **Bias** is a term which refers to how far the information gained from a sample lies from the information hidden by the population. Bias can surface during the sampling process of the data handling cycle.
  - A **biased sampling method** is a method that tends to give non-representative samples. Such samples under-represent or over-represent some characteristics of the population.
  - An **unbiased sampling method** is a method than tends to give representative samples. Such samples give a true reflection of the characteristics of the population.
- **Misleading data** is data which is manipulated in order to favour a specific opinion. This usually occurs when statistical displays in newspapers and financial reports are presented in such a way that the data reflects a positive picture of one party and/or a negative picture of the opposition. It can be done in a variety of ways, for example:
  - Starting the scale on the frequency axis at a point other than 0 (refer to question 1 on LB page 290).
  - Using three-dimensional displays to give a better impression of a specific category (refer to question 2 on LB page 291).
  - Using different scales on the frequency axes of two displays (refer to question 4 on LB page 292).

25.1 Interpreting and reporting on data

CRITICALLY READING AND REPORTING ON DATA

Background information

- We **interpret data** by reading information directly from the data. This proves that we understand the given data.
- We **analyse data** by identifying cyclic patterns and describing increasing and decreasing trends shown by the data.
- We **report on data** by writing a short paragraph to summarise our findings.

Teaching guidelines

Learners should have a thorough understanding of the data that is given before they attempt to answer any questions on the data.

Misconceptions

Learners may misunderstand the given data because their reading abilities are not up to standard. Ask some learners to explain what the given data is about.

Answers

1. (a) 26 000 schools (b) 2 500 schools
   (c) Eastern Cape (d) Northern Cape
   (e)

Province	Number of schools	Percentage of all schools
Eastern Cape	415	16,6%
KwaZulu-Natal	386	15,4%
Limpopo	326	13,0%
North West	275	11,0%
Gauteng	265	10,6%
Mpumalanga	248	9,9%
Free State	238	9,5%
Western Cape	218	8,7%
Northern Cape	129	5,2%

   (f) Refer to the table above.
   (g) The sample of schools surveyed made up almost 10% of the population. Most of the schools (16,6%) surveyed were in the Eastern Cape, followed closely by KwaZulu-Natal (15,4%). The fewest schools were from the Northern Cape (5,2%).

CHAPTER 25
Interpret, analyse and report on data

25.1 Interpreting and reporting on data

CRITICALLY READING AND REPORTING ON DATA

1. Read the following paragraph and answer the questions that follow.

> In 2009, a sample of 2 500 schools from about 26 000 schools across South Africa took part in a survey to provide data about learners and schools. The sample included schools from each province as follows: 415 schools from the Eastern Cape, 238 from the Free State, 265 from Gauteng, 386 from KwaZulu-Natal, 326 from Limpopo, 248 from Mpumalanga, 129 from the Northern Cape, 275 from North West and 218 from the Western Cape.
>
> Adapted from: *Census @ School Results 2009*, Statistics South Africa

(a) What was the population of the survey?
(b) What was the sample of the survey?
(c) Which province were most of the schools from?
(d) Which province were the fewest schools from?
(e) Copy the table below. Complete the first two columns of the table by listing the provinces in order from the province that had the most schools to the province that had the fewest schools participating in the survey.

Province	Number of schools	Percentage of all schools

(f) Complete the last column by working out the percentage of the whole that the schools in each province make up. You may use your calculator for this question. (Round off to one decimal place.)
(g) Write three to five lines as a summary report of the data described in the paragraph above. The summary should give an idea of the highest and lowest data items, as this indicates the range of the data.

2. The graph that follows shows the percentage of male and female learners at schools in Grades 3 to 8 in 2009.

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Note on questions 2(g) and 2(h)

In these questions learners have to analyse the data in order to find an **increasing or decreasing trend**.

Note on question 3(c)

In this question learners have to analyse the data in order to find a **pattern** in the data.

Answers

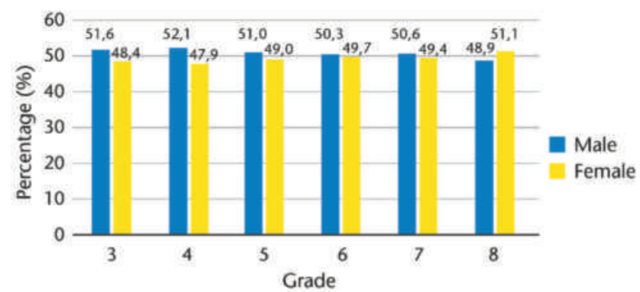
2. (a) Grade 8 (51,1%)
   (b) Grade 4 (47,9%)
   (c) Grade 4 (52,1%)
   (d) Grade 8 (48,9%)
   (e) Girls in Grade 6 = $\frac{(49,7)}{(100)} \times 150\,000 = 74\,550$

   Boys in Grade 6 = $\frac{(50,3)}{(100)} \times 150\,000 = 75\,450$

   (f) See LB page 289 alongside.
   (g) The trend is for the percentage of males to decrease per grade, so we would expect there to be fewer males in Grade 10.
   (h) The trend is for the percentage of females to increase per grade, so we would expect there to be more females in Grade 10.
3. (a) Northern Cape
   (b) Gauteng
   (c) Western Cape, Free State and Limpopo
   (d) $\frac{(30,5)}{(1,4)}$ = about 22 times bigger
   (e) No. A province may have a large land area, but the population in the province does not depend on land area. So we cannot use the pie chart to predict which province has the largest population.
   (f) Northern Cape = $\frac{(30,5)}{(100)} \times 1\,200\,000\text{ km}^2 = 366\,000\text{ km}^2$

   Gauteng = $\frac{(1,4)}{(100)} \times 1\,200\,000\text{ km}^2 = 16\,800\text{ km}^2$

Percentage of male and female learners in Grades 3 to 8



(Source: *Census @ School Results 2009*, Statistics South Africa)

(a) Which grade has the highest percentage of females?
(b) Which grade has the lowest percentage of females?
(c) Which grade has the highest percentage of males?
(d) Which grade has the lowest percentage of males?
(e) If 150 000 Grade 6 learners took part in the survey, how many girls and how many boys were there in Grade 6? You may use your calculator.
(f) Copy and complete the following summary report:

The graph shows that the number of male learners seems to (decrease/~~increase~~) the higher the grade. For example, in Grade 3, 51,6% learners were male compared to 48,9% in Grade 8. The number of female learners seems to (~~decrease~~/increase) the higher the grade. For example, in Grade 3, 48,4% learners were female compared to 51,1% in Grade 8.

(g) Based on the graph, would you expect there to be more or fewer males in Grade 10? Explain your answer.
(h) Based on the graph, would you expect there to be more or fewer females in Grade 10? Explain your answer.

3. The pie chart on the following page shows the land area of each province in 2011.
(a) Which province has the largest land area?
(b) Which province has the smallest land area?
(c) Which three provinces have more or less the same land area?
(d) How much bigger is the Northern Cape than Gauteng? (Use a calculator.)
(e) Are we able to tell from the pie chart which province has the largest population? Explain your answer.
(f) If the total land area of South Africa is 1 200 000 km², how many square kilometres are the largest and the smallest provinces?

Answers

3. (g) The Northern Cape has the largest land area (30,5%) and Gauteng has the smallest land area (1,4%). The second largest province is the Eastern Cape, making up 13,8% of South Africa's land area. The Western Cape, Free State and Limpopo are approximately the same size, each making up about 10% of the land area.

25.2 Identifying bias and misleading data

Background information

- **Bias** refers to how far the information gained from a sample lies from the information hidden by the population.
- **Misleading data** refers to the misuse of graphs or the data itself, to create an impression which favours a specific opinion, for example:
  - The vertical scale is distorted or skips numbers or does not start at 0.
  - The graph is not labelled properly.
  - Some data is left out.
  - The graph is drawn in three dimensions.

Teaching guidelines

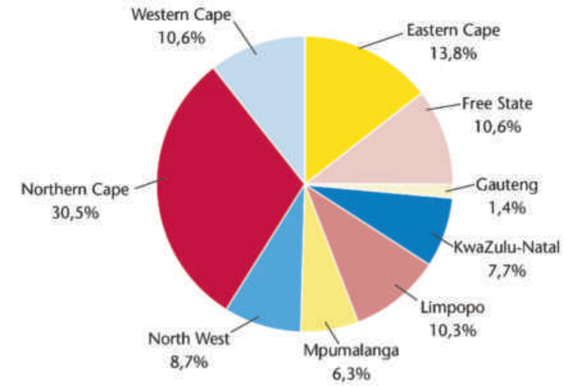
Datasets given in this section reflect a misleading picture. In each question learners should be on the lookout for something that could confuse them.

Discuss the meaning of bias. Discuss how statistical graphs can be misleading.

Note on question 1

The bar graph is misleading because the frequency axis does not start at 0.

(g) Write a short paragraph to summarise the data shown in the pie chart.



(Source: *Census 2011: Census in brief*, Statistics South Africa)

25.2 Identifying bias and misleading data

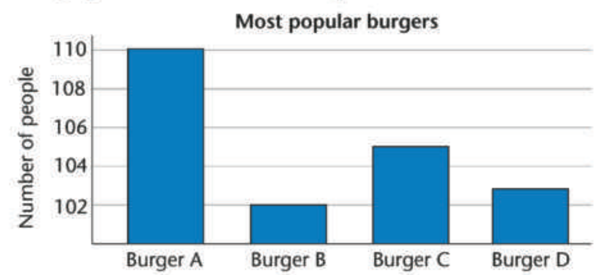
Sometimes the ways in which data is presented could be intentionally or unintentionally **biased** or misleading. As you work through the following activities, think carefully about:

- data that is not necessarily shown by the graph
- when, how and where the data was collected
- which scales are used on the graphs
- which summary statistics (mean, median and mode) are used to summarise the data.

Bias means that a person prefers a certain idea and possibly does not give equal chance to a different idea.

CRITICALLY ANALYSING DATA

1. Look at the bar graph below and answer the questions that follow.



Note on question 2

The pie chart is misleading because it is drawn in three-dimensional perspective.

Note on question 3

The histograms are misleading because different samples are used.

Note on question 4

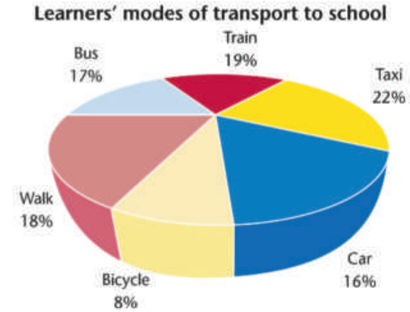
The bar graphs are misleading because different scales are used on the frequency axes.

Answers

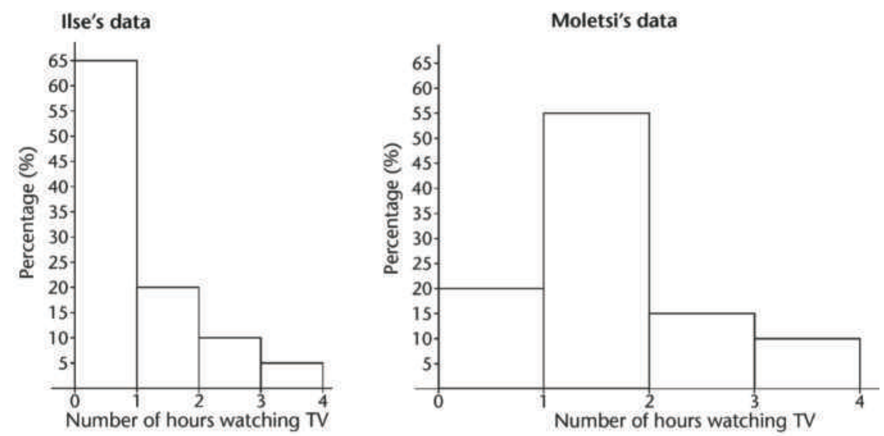
1. (a) A
   (b) The heights of the bars do seem to indicate this; however, the vertical scale does not start at zero, so the full heights of the bars are not shown. Even though it is not incorrect to show the data in this way, it does give the false impression that burger A is much more popular than the other burgers. In reality, 110 people like burger A and 102 people like burger B. There is a difference of only eight people, not five times as many people.
   (c) Learners draw their own bar graphs.
2. (a) The train (19%)
   (b) The bicycle (8%)
   (c) Sample answer: Yes, the 3D perspective drawing gives a false impression. It appears as if most learners come to school either on foot, by car or bicycle, but this is not true. For instance, the bicycle slice is actually the smallest slice (8%) while the taxi slice (22%) is the largest slice.
3. (a) 0 to 1 hour

(a) Which burger is the most popular?
(b) The heights of the bars indicate that burger A is liked by five times as many people as burger B. Is this true? Look at the vertical scale.
(c) Redraw the bar graph, but show the full vertical scale.

2. Look at the pie chart.
   (a) What is the second most common mode of transport that learners use?
   (b) Which mode of transport is the least common one?
   (c) Is the pie chart misleading in any way? Explain.



3. Ilse and Moletsi wanted to find out more about the number of hours people spend watching TV on a particular public holiday. Ilse did her survey on the public holiday from 13:00 to 15:00. She visited a supermarket and asked adult respondents to complete her questionnaire. Moletsi did his survey on the same day from 17:00 to 19:00. He went from door to door in his neighbourhood and asked the children to complete his questionnaire.



   (a) According to Ilse's data, how long did most people spend watching TV on the public holiday?

Answers

3. (b) 1 to 2 hours

   (c) There is a big difference between the two sets of data. Ilse's data shows that most people (65%) spent up to one hour watching TV on the public holiday, and only 5% spent between three and four hours watching TV. Moletsi's data shows that fewer people (20%) spent up to one hour watching TV, and most people (55%) spent between one and two hours watching TV.

   (d) Ilse collected her data earlier in the day than Moletsi did. If she collected the data later in the afternoon, people may have had more time to watch TV on that day, and the data could possibly have shown that people spent more time watching TV on that particular public holiday.

   (e) Ilse collected data from people at a supermarket. These respondents would have had less time at home to watch TV on that day. Moletsi's respondents were all at home at the time of data collection, so they would most likely have had more opportunity to watch more TV.

   (f) Adults and children probably spend different amounts of time watching TV. Ilse's respondents were adults, who would perhaps generally have less time to watch TV than children. Moletsi's respondents were children, who would perhaps generally spend more time watching TV than their parents.

4. (a) The number of CDs sold by Music Note. The graph on the left shows the sales in Year 1 and the graph on the right shows the sales in Year 2.

   (b) 700

   (c) 375

   (d) No, they sold 500 CDs in those months of Year 1 and 300 CDs in those months of Year 2.

   (e) 400 + 200 + 700 + 500 = 1 800

   (f) 250 + 125 + 375 + 300 = 1 050

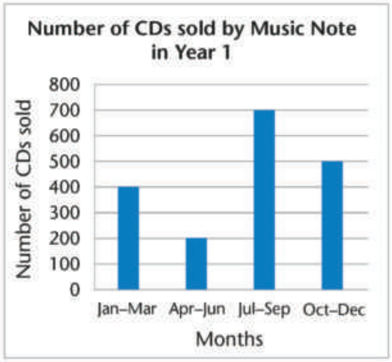
   (g) The scales of the vertical axes on the two graphs are different. For Year 1, the scale increases in units of 100. For Year 2, the scale increases in units of 50, and so each bar is "stretched" higher than it would have been had units of 100 been used, making the number of sales look more than they actually were.

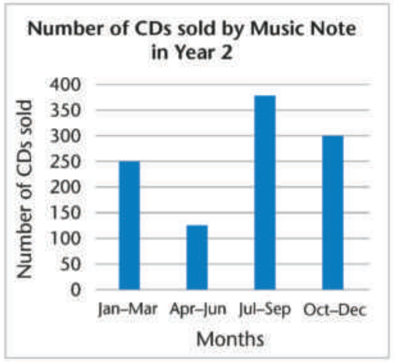
5. (a) See LB page 292 alongside.

   (b) Class A has a wider range of marks, meaning that the spread of marks is wider.

(b) According to Moletsi's data, how long did most people spend watching TV on the public holiday?
(c) Write a paragraph to summarise and compare Ilse's data and Moletsi's data.
(d) How could the time when the data was collected have affected the data?
(e) How could the place where the data was collected have affected the data?
(f) How could the people from whom data was collected have affected the data?

4. Look at the following graphs and answer the questions that follow.





(a) What does each of the graphs show?
(b) How many CDs were sold in July to September of Year 1?
(c) How many CDs were sold in July to September of Year 2?
(d) The heights of the bars indicate that Music Note sold more CDs in October to December of Year 2 than in the same months of Year 1. Is this the case?
(e) How many CDs were sold altogether in Year 1?
(f) How many CDs were sold altogether in Year 2?
(g) Explain why the heights of the bars seem to indicate that Music Note sold more or less the same number of CDs in both years, which is not true.

5. The following table shows the Mathematics marks of Class A and Class B.

Class A	94, 42, 23, 67, 67, 68, 13, 53, 44, 34, 64, 69, 50, 31, 91, 40, 10, 30, 49, 61
Class B	74, 26, 65, 45, 71, 77, 58, 35, 39, 45, 68, 45, 57, 62, 29, 55, 23, 56, 38, 36, 50, 64, 58, 32, 42

(a) Find the range of each set of data.
Class A: 94 – 10 = 84 Class B: 77 – 23 = 54

(b) What can you say about the two classes by looking at the range of marks?

Answers

5. (c) See the completed table on LB page 293 alongside.
   (d) From the means, it looks as though the learners in the two classes are performing at the same level.
   (e) See the completed table on LB page 293 alongside.
   (f) From the medians, it looks as though the learners in the two classes are performing at more or less the same level.
   (g) See the completed table on LB page 293 alongside.
   (h) From the modes, it looks like Class A is performing much better than Class B.
   (i) The mode describes only the most frequent values in the set, and does not take into account all the values. The mean and median take into account all the values, so these probably represent the data better than the mode does.

(c) Copy and complete the table by calculating the mean (average) Mathematics mark for each class. You may use your calculator.

Class	Total marks	Number of marks	Mean
Class A	1 000	20	50
Class B	1 250	25	50

(d) Compare the two sets of data using the means.
(e) Copy and complete the table by finding the median for each class.

Class	Marks from highest to lowest	Middle position	Median
Class A	94, 91, 69, 68, 67, 67, 64, 61, 53, <u>50</u>, <u>49</u>, 44, 42, 40, 34, 31, 30, 23, 13, 10	10 and 11	$\frac{50+49}{2}$ $= 49,5$
Class B	77, 74, 71, 68, 65, 64, 62, 58, 58, 57, 56, 55, 50, 45, 45, 45, 42, 39, 38, 36, 35, 32, 29, 26, 23	13	50

(f) Compare the two sets of data using the medians.
(g) Copy and complete the table by finding the mode for each class.

Class	Highest frequency	Mode
Class A	2	67
Class B	3	45

(h) Compare the two sets of data using the mode.
(i) Which of the following do you think best represents each set of data: mean, median or mode? Explain your answer.

Learner Book Overview		
Sections in this chapter	**Content**	**Pages in Learner Book**
26.1 Possible and actual outcomes, and frequencies	Performing probability experiments in which trials, possible outcomes, actual outcomes and actual frequencies are defined	Pages 294 to 295
26.2 Relative frequencies	Actual frequencies are given as fractions and defined as relative frequencies; relative frequencies are given as fractions, decimal numbers of percentages; the range of relative frequencies, expressed as percentages, is defined	Pages 295 to 296
26.3 More trials and relative frequencies	Many more trials result in the range of relative frequencies becoming smaller	Pages 296 to 297

CAPS time allocation	4,5 hours
CAPS content specification	Page 73

Mathematical background

The **probability** of something happening (an event) is the measure of the likelihood that it will happen.

A **trial** is an action that can have more than one outcome, for example when rolling a die there could be one of six possible scores.

An **experiment** is more than one trial, for example rolling a die ten times.

A **possible outcome** is any of the possible results of a trial, for example scoring 1 or 2 or 3 or 4 or 5 or 6 when rolling a die.

An **actual** outcome is the result of a single trial of an experiment, for example rolling a die and scoring 3.

The frequency or **actual frequency** of an event is the number of times the event happens, for example the number of times 3 was scored when a die was rolled ten times.

The **relative frequency** (experimental probability) of an outcome is given by the ratio $\frac{\text{number of times the outcome happens}}{\text{total number of trials}}$. The relative frequency can be expressed as a fraction, a decimal number or a percentage.

Probability is given by the ratio $\frac{\text{number of favourable outcomes}}{\text{number of possibleoutcomes}}$, for example scoring a prime number (1, 3 or 5) when rolling a die. The probability of scoring a prime number when rolling a die is therefore given by $\frac{3}{6} = \frac{1}{2}$, or 0,5 or 50%.

It takes many trials before the relative frequency of an outcome approaches the probability of the outcome.

26.1 Possible and actual outcomes, and frequencies

WHAT CAN YOU EXPECT?

Teaching guidelines

Explain that a trial is an action that can have more than one outcome. For example, if you take an object from a bag which has three different coloured squares in it, you could get a yellow, red or blue square.

An experiment is more than one trial, for example taking an object from the bag, noting the colour and putting it back and doing this 12 times.

For each of these trials in the experiment there could be three possible outcomes. Let learners name the possible outcomes. What actually happened in a trial was the actual outcome, for example if a blue square was drawn from the bag, it was the actual outcome for that trial.

Misconceptions

Learners confuse the terms "possible outcomes", "actual outcome" and "trial".

Answers

1. (a) No, one cannot say. (The purpose of this question and question (b) is to develop awareness of equally likely events.)
2. (b) While there is some sense in suggesting four occurrences of each colour, an insightful answer would be that it will not necessarily be four occurrences of each.
   (d) No good reason can be cited. (The purpose of the question is to promote formation of the idea of "equally likely" outcomes.)
3. Learners make a similar table in their books and record the outcomes in their table. The number of times for the three colours should add up to 12.

CHAPTER 26

Probability

26.1 Possible and actual outcomes, and frequencies

WHAT CAN YOU EXPECT?

You will soon do an experiment. To do the experiment you need a bag like a plastic shopping bag or a brown paper bag. You also need three objects of the same size and shape, like three buttons, bottle tops or small square pieces of cardboard. The three objects must look different, for example they should have different colours such as yellow, red and blue. If you use cardboard squares, you can write "yellow", "red" and "blue" on them.

1. (a) Put your three objects in your bag. You will later draw one object out of the bag, without looking inside. Can you say whether the object that you will draw will be the yellow one, the blue one or the red one?
   (b) Discuss this with two classmates.
2. (a) Now draw an object out of the bag, write down its colour, and put it back.
   (b) You will soon do this 12 times. Can you say how many times you will draw each of the three colours? If you think you can, write down your prediction.
   (c) Compare your predictions with two classmates.
   (d) Can you think of any reason why you may draw blue more often than red or yellow, when you do the experiment described in (b)?
3. (a) Draw an object out of the bag, write down its colour, and put it back. Do this 12 times and write down the colour of the object each time.
   (b) Write your results in a table like the one below.

Outcome	Yellow	Red	Blue
Number of times obtained			

What you did in question 3 is called a **probability experiment**. Each time you drew an object out of the bag, you performed a **trial**.

Each time you performed a trial, three different things could have happened. These are called the **possible outcomes**.

Answers

4. (a) Drawing a yellow object, drawing a red object, drawing a blue object
   (b) 12 trials
   (c) This will differ from learner to learner. The purpose of the question is to encourage learners to use the idea of "actual outcome" in their own thinking.
   (d) The answer depends on the answer in 3(b).

26.2 Relative frequencies

Teaching guidelines

Let learners use their results from questions 3 and 4 in the previous section and calculate the relative frequency for the outcomes of their experiment.

Remind learners how to calculate percentage – i.e. multiply a common or decimal fraction by 100.

Make sure that when they do the table in question 3 they complete the columns for the relative frequencies using the correct ratio and finding the percentage correctly.

Notes on the questions

When learners have to complete the table in question 3, they may get confused by the terms "actual frequencies" and "relative frequencies".

Answers

1. (a) 20 trials (5 + 7 + 8)
   (b) A quarter of the trials, 5 out of 20, $\frac{5}{20}$
   (c) 7 twentieths of the trials, 7 out of 20, $\frac{7}{20}$
   (d) 8 twentieths, 4 tenths, 2 fifths
2. (a) Possible answers: 0 = 0%; $\frac{1}{12} \cong$ 8,3%; $\frac{2}{12} \cong$ 16,7%; $\frac{3}{12}$ = 25%; $\frac{4}{12} \cong$ 33,3%; $\frac{5}{12} \cong$ 41,7%; $\frac{6}{12}$ = 50%; $\frac{7}{12} \cong$ 58,3%; $\frac{8}{12} \cong$ 66,7%; $\frac{9}{12}$ = 75%; $\frac{10}{12} \cong$ 83,3%; $\frac{11}{12} \cong$ 91,7%; $\frac{12}{12}$ = 1 = 100%. (Note: $\cong$ is being used to communicate equivalence)
   (b) Learners' own answers (i.e. highest percentage minus lowest percentage).
   (c) Learners' own answers.

Each time you performed a trial, one of the possible outcomes actually occurred. This is called the **actual outcome**.

The number of times that a specific outcome occurred during an experiment is called the **actual frequency** of that outcome.

4. (a) What were the possible outcomes in the experiment that you did in question 3?
   (b) How many trials did you perform in the experiment?
   (c) What was the actual outcome in the third trial that you performed?
   (d) What was the actual frequency of drawing a blue object during the 12 trials in the experiment that you did?

26.2 Relative frequencies

Thomas also did the experiment in question 3 on page 294 but he performed more trials and his results were as follows:

Outcome	Yellow	Red	Blue
Number of times obtained	5	7	8

1. (a) How many trials did Thomas perform in total?
   (b) What fraction of the trials produced yellow as an outcome?
   (c) What fraction of the trials produced red as an outcome?
   (d) What fraction of the trials produced blue as an outcome?

The fraction of the trials in an experiment that produce a specific outcome is called the **relative frequency** of that outcome.

Relative frequency of an outcome = $\frac{\text{number of times the outcome occurred}}{\text{total number of trials}}$

A relative frequency can be expressed as a common fraction, as a decimal or as a percentage. The relative frequencies in the results of the experiment Thomas did (question 1) were one quarter for yellow, seven twentieths for red and two fifths for blue. Expressed as percentages, the relative frequencies were 25%, 35% and 40%. The **range** of Thomas's relative frequencies, expressed as percentages, is 15% (40% – 25%).

2. (a) Use your calculator to calculate the relative frequencies that you obtained for the three different outcomes in the experiment you did in question 3 on page 294. Express them both as fractions and percentages.
   (b) Calculate the range of the relative frequencies of the three outcomes for the results of the experiment you did in question 3.
   (c) You will soon repeat the experiment with three possible outcomes and 12 trials that you did. Do you think the results will be the same as the first time you did the experiment?

Answers

3. (c) Teams' answers will differ.

26.3 More trials and relative frequencies

WHAT HAPPENS WHEN YOU CONDUCT MANY TRIALS?

Teaching guidelines

Allow learners enough time to do this activity and to discuss in groups what they learnt.

Notes on the questions

If the range of the relative frequencies becomes smaller when more experiments are done, this means that the relative frequency approaches the probability.

Answers

1. The purpose of the question is not to elicit "good answers", but to instil a sense of expectancy and put questions in learners' minds to induce them to do the experiment reflectively and not as a mere mechanical exercise.
2. Some learners may correctly anticipate that the range will become smaller as more trials are conducted, but it cannot be expected that learners will know this intuitively. For the majority of learners, this extended experiment will be an opportunity to discover that the range diminishes as the number of trials grows.

3. (a) Join with three or four classmates to work as a team, and discuss question 2(c).
   (b) Assign the "names" A, B, C, D and E (if there are five of you) to the team members and copy and complete the table below for the experiment you did in question 3 on page 294. Give the relative frequencies as percentages. Note that to calculate the relative frequencies for the totals as percentages, you have to use your calculators.

	Actual frequencies			Relative frequencies %			Range
	Yellow	Red	Blue	Yellow	Red	Blue	
Experiment 1 by A							
Experiment 1 by B							
Experiment 1 by C							
Experiment 1 by D							
Experiment 1 by E							
Totals for experiment 1							

   (c) Which of the ranges is the smallest?

26.3 More trials and relative frequencies

WHAT HAPPENS WHEN YOU CONDUCT MANY TRIALS?

1. Join up with your teammates of the previous activity. Each of you will soon repeat the experiment you did previously. You will put a yellow object, a red object and a blue object in a bag, draw one object and note the colour. You will do this 12 times. This will be experiment 2.
   (a) Do you expect that the results will, in some ways, be the same as for experiment 1 in the previous section? Do not talk to your teammates yet. Form your own opinion, and also consider *why* you think the results will be different or the same.
   (b) Share your ideas with your teammates.

You will soon repeat the experiment and write the results in the rows for "experiment 2" on the table on the next page. You will repeat it once more and write the results in the rows for "experiment 3". If you have time left, you may repeat it once more as "experiment 4".

2. (a) Look at the table on the next page. Certain rows are for the outcomes that you and your teammates obtain. The shaded rows are for adding different sets of outcomes together. Think about what may happen and predict in what rows the ranges will be smaller than in other rows, and in what row the range will be the smallest of all. Copy the table.

Answers

3. Learners' own answers.
4. Learners' own answers.

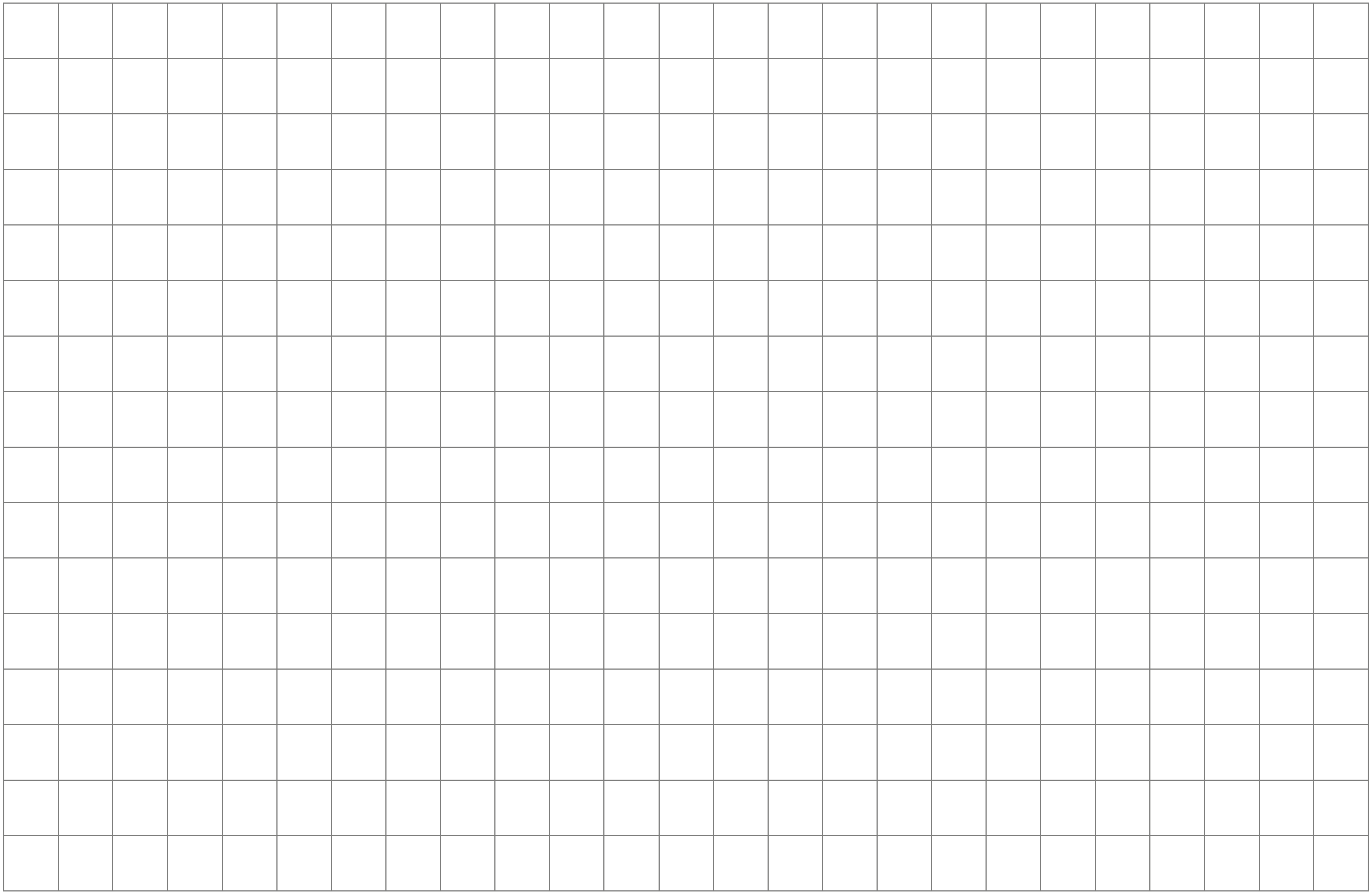
(b) Share your ideas with your teammates.

3. (a) Copy the totals for "experiment 1" into the first row of the table. Do the experiment described in question 1 and enter the results in the rows for "experiment 2". Calculate the relative frequencies and the range.
   (b) Add in the results of your teammates, add up the totals and calculate the relative frequencies and the range of the totals.

4. Repeat question 3, and enter the results in the rows for "experiment 3".

		Actual frequencies			Relative frequencies %			Range
		Yellow	Red	Blue	Yellow	Red	Blue	
1	Totals for experiment 1							
2	Experiment 2 by A							
3	Experiment 2 by B							
4	Experiment 2 by C							
5	Experiment 2 by D							
6	Experiment 2 by E							
7	Totals for experiment 2							
8	Totals for experiments 1 and 2 combined							
9	Experiment 3 by A							
10	Experiment 3 by B							
11	Experiment 3 by C							
12	Experiment 3 by D							
13	Experiment 3 by E							
14	Totals for experiment 3							
15	Totals for experiments 1, 2 and 3 combined							
16	Experiment 4 by A							
17	Experiment 4 by B							
18	Experiment 4 by C							
19	Experiment 4 by D							
20	Experiment 4 by E							
21	Totals for experiment 4							
22	Totals for experiments 1, 2, 3 and 4 combined							

Square grid paper (1 cm × 1 cm)



Graph paper / Square grid paper (0,5 cm × 0,5 cm)

Graph paper

Dotted paper