## Worksheet 8: Functions - Polynomials (Factor and Remainder Theorem)

## **Grade 12 Mathematics**

1. Factorise the following third degree polynomials:

a) 
$$3x^3 - 28x^2 + 52x + 48$$

$$2x^3 - 17x^2 + 41x - 30$$

c) 
$$30x^3 + 53x^2 - 4x - 15$$

d) 
$$x^3 + 10x^2 + 8x - 64$$

e) 
$$x^3 + 15x^2 + 75x + 125$$

f) 
$$x^3 - 7x^2 - 14x + 48$$

g) 
$$2x^3 - 39x^2 + 157x + 330$$

h) 
$$15x^3 + 53x^2 - 58x - 120$$

i) 
$$x^3 + 14x^2 + 41x - 56$$

j) 
$$x^3 + 3x^2 - 88x + 240$$

2. Solve for x in of the following equations:

a) 
$$x^3 - 2x^2 - x + 2 = 0$$

b) 
$$x(x^2 - 67) = 126$$

c) 
$$3(x^3 + 8) = 7x(x + 10)$$

d) 
$$x(3x^2 + 8x - 48) = 128$$

e) 
$$10(x^3 + 10) = 3x(11x + 35)$$

f) 
$$x^3 - 3x^2 - 126x + 648 = 0$$

g) 
$$x(x^2 - 41) = -4(x^2 + 9)$$

h) 
$$2x^3 = 3x^2 + 98x + 48$$

i) 
$$5x^3 - 31x^2 = 4(17x + 8)$$

j) 
$$3x^3 + 7x^2 - 22x - 8 = 0$$

3. Use the factor and remainder theorem to prove that each of the factors given is a factor of the expression given.

a) Factor: 
$$x - 1$$

Expression: 
$$f(x) = x^{3} + 2x^{2} - x - 2$$

b) Factor: 
$$x - 4$$

Expression: 
$$f(x) = x^3 - 9x^2 + 26x - 24$$

c) Factor: 
$$x + 6$$

Expression: 
$$f(x) = x^3 + 13x^2 + 54x + 72$$

d) Factor: 
$$2x + 1$$

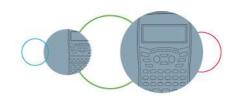
Expression: 
$$f(x) = 2x^3 + 11x^2 - 23x - 14$$

e) Factor: 
$$5x - 1$$

Expression: 
$$f(x) = 5x^3 - 26x^2 - 65x + 14$$

- 4. Use the factor and remainder theorem to find the value of p in the equations below:
  - a) When  $f(x) = px^2 + 25x + 4$  is divided by x 2 the remainder is 78.
  - b) When  $f(x) = x^2 7x p$  is divided by x + 1 the remainder is -52.
  - c) When  $f(x) = 5x^3 + px^2 47x 10$  is divided by x + 4 the remainder is 114.
  - d) When  $f(x) = x^2 + px 21$  is divided by x 5 the remainder is -16.
  - e) When  $f(x) = x^3 + 11x^2 + px 216$  is divided by x 7 it gives a remainder of 624.





5. Determine the remainder if each of these equations are divided by the factor given

a) 
$$f(x) = x^3 - 5x^2 + 2x + 8$$
 divided by  $x + 10$ 

b) 
$$f(x) = x^2 + 2x - 15$$
 divided by  $x - 12$ 

c) 
$$f(x) = x^2 - 14x + 40$$
 divided by  $x + 5$ 

d) 
$$f(x) = 4x^3 - 12x^2 - 67x - 30$$
 divided by  $x + 4$ 

e) 
$$f(x) = x^3 + 2x^2 - 111x + 108$$
 divided by  $5x - 1$ 

6. Which of the 3 divisors given for each equation is a perfect factor of that equation?

a) A 
$$\rightarrow x + 2$$

B 
$$\rightarrow x - 2$$

C 
$$\rightarrow 5x - 1$$

$$g(x) = 6x^2 + 11x - 2$$

b) A 
$$\rightarrow 2x + 1$$

B 
$$\rightarrow x - 4$$

$$C \rightarrow x-2$$

$$g(x) = x^3 - 7x^2 - 6x + 72$$

c) A 
$$\rightarrow 2x - 1$$

$$\mathsf{B} \qquad \to x+3$$

C 
$$\rightarrow x - 1$$

$$g(x) = x^3 - 3x^2 - x + 3$$



