## Worksheet 8: Functions - Polynomials (Factor and Remainder Theorem)

## Grade 12 Mathematics

1. Factorise the following third degree polynomials:
a) $3 x^{3}-28 x^{2}+52 x+48$
b) $2 x^{3}-17 x^{2}+41 x-30$
c) $30 x^{3}+53 x^{2}-4 x-15$
d) $x^{3}+10 x^{2}+8 x-64$
e) $\quad x^{3}+15 x^{2}+75 x+125$
f) $\quad x^{3}-7 x^{2}-14 x+48$
g) $\quad 2 x^{3}-39 x^{2}+157 x+330$
h) $15 x^{3}+53 x^{2}-58 x-120$
i) $x^{3}+14 x^{2}+41 x-56$
j) $\quad x^{3}+3 x^{2}-88 x+240$
2. Solve for $x$ in of the following equations:
a) $x^{3}-2 x^{2}-x+2=0$
b) $\quad x\left(x^{2}-67\right)=126$
c) $3\left(x^{3}+8\right)=7 x(x+10)$
d) $\quad x\left(3 x^{2}+8 x-48\right)=128$
e) $\quad 10\left(x^{3}+10\right)=3 x(11 x+35)$
f) $x^{3}-3 x^{2}-126 x+648=0$
g) $\quad x\left(x^{2}-41\right)=-4\left(x^{2}+9\right)$
h) $\quad 2 x^{3}=3 x^{2}+98 x+48$
i) $\quad 5 x^{3}-31 x^{2}=4(17 x+8)$
j) $3 x^{3}+7 x^{2}-22 x-8=0$
3. Use the factor and remainder theorem to prove that each of the factors given is a factor of the expression given.
a) Factor: $x-1$
Expression: $f(x)=x^{3}+2 x^{2}-x-2$
b) Factor: $x-4$
Expression: $f(x)=x^{3}-9 x^{2}+26 x-24$
c) Factor: $x+6$
Expression: $f(x)=x^{3}+13 x^{2}+54 x+72$
d) Factor: $2 x+1$
Expression: $f(x)=2 x^{3}+11 x^{2}-23 x-14$
e) Factor: $5 x-1$
Expression: $f(x)=5 x^{3}-26 x^{2}-65 x+14$
4. Use the factor and remainder theorem to find the value of p in the equations below:
a) When $f(x)=p x^{2}+25 x+4$ is divided by $x-2$ the remainder is 78 .
b) When $f(x)=x^{2}-7 x-p$ is divided by $x+1$ the remainder is -52 .
c) When $f(x)=5 x^{3}+p x^{2}-47 x-10$ is divided by $x+4$ the remainder is 114 .
d) When $f(x)=x^{2}+p x-21$ is divided by $x-5$ the remainder is -16 .
e) When $f(x)=x^{3}+11 x^{2}+p x-216$ is divided by $x-7$ it gives a remainder of 624 .
5. Determine the remainder if each of these equations are divided by the factor given
a) $\quad f(x)=x^{3}-5 x^{2}+2 x+8$ divided by $x+10$
b) $\quad f(x)=x^{2}+2 x-15$ divided by $x-12$
c) $\quad f(x)=x^{2}-14 x+40$ divided by $x+5$
d) $\quad f(x)=4 x^{3}-12 x^{2}-67 x-30$ divided by $x+4$
e) $\quad f(x)=x^{3}+2 x^{2}-111 x+108$ divided by $5 x-1$
6. Which of the 3 divisors given for each equation is a perfect factor of that equation?
a) $\quad \mathrm{A}$
$\rightarrow x+2$
B $\quad \rightarrow x-2$
C $\quad \rightarrow 5 x-1$
b) $\quad \mathrm{A} \quad \rightarrow 2 x+1$

B $\quad \rightarrow x-4$
C $\quad \rightarrow x-2$
$g(x)=x^{3}-7 x^{2}-6 x+72$
c) $\quad \mathrm{A} \quad \rightarrow 2 x-1$

B $\quad \rightarrow x+3$
C $\quad \rightarrow x-1$
$g(x)=x^{3}-3 x^{2}-x+3$

