basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

TECHNICAL MATHEMATICS

GUIDELINES FOR PRACTICAL ASSESSMENT TASKS

GRADE 12

2021

These guidelines consist of 37 pages.

TABLE OF CONTENTS

1. INTRODUCTION

The 18 Curriculum and Assessment Policy Statement subjects which contain a practical component all include a practical assessment task (PAT). These subjects are:

- AGRICULTURE: Agricultural Management Practices, Agricultural Technology
- ARTS: Dance Studies, Design, Dramatic Arts, Music, Visual Arts
- SCIENCES: Computer Applications Technology, Information Technology, Technical Sciences, Technical Mathematics
- SERVICES: Consumer Studies, Hospitality Studies, Tourism
- TECHNOLOGY: Civil Technology, Electrical Technology, Mechanical Technology, Engineering Graphics and Design

A practical assessment task (PAT) mark is a compulsory component of the final promotion mark for all candidates offering subjects that have a practical component and counts 25% (100 marks) of the examination mark at the end of the year. The practical assessment task for Technical Mathematics Grade 12 consists of three tasks (one task per term) which should be completed by end of Term 3. The tasks are COMPULSORY for ALL candidates offering Technical Mathematics in Grade 12.

The PAT is implemented during the first three terms of the school year. The PAT allow learners to be assessed regularly during the school year and it also allows for the assessment of skills acquired and it applies the science of Mathematics to the technical field where the emphasis is on application. It is therefore important that schools ensure that all learners complete the practical assessment tasks within the stipulated period to ensure that learners are promoted at the end of the school year. The planning and execution of the PAT differs from subject to subject.

The tasks should be administered under supervised conditions. Moderation may be done onsite.

2. TEACHER GUIDELINES

2.1 How to administer the PATs

- The following documents must be available for all formal tasks:
  - Task instructions explaining the procedures to be followed
  - The worksheets which include questions to be answered under examination conditions
  - The teacher guidelines with task instructions, worksheets and marking guidelines (The teacher guidelines MUST NOT be released to the learners.)
  - Teachers should compile marking guidelines (memoranda) for the actual results of the task (teachers should do the tasks themselves FIRST)
- The tasks must be done individually.
- Each learner must record his/her OWN INDIVIDUAL data and observations.
- Each learner must be provided with his/her OWN worksheet and answer the questions INDIVIDUALLY under examination conditions.
- Only once all the learners are ready to do the task and they are seated and ready to answer questions may teachers hand out a worksheet to each learner. Examination conditions have to be applied.
- If it is not possible to do the task and complete the worksheet on the same day, the teacher must collect the learners' tasks. These tasks must be kept at school.

2.2 Moderation of the PATs

For moderation the following documents are required in the teacher's file:

- Index indicating all tasks with raw and weighted marks
- All task instructions
- Marking guidelines for all tasks, with ticks and totals
- Composite working mark sheet for all learners showing raw and weighted marks
- Evidence of internal moderation

For moderation the following documents are required in the learner's file:

- Index stating all tasks with raw and weighted marks
- Answer sheets for all tasks

3. LEARNER GUIDELINES

3.1 This PAT for Grade 12 consists of THREE tasks.

3.2 The PAT contributes 25% towards your final promotion mark for Grade 12.

3.3 All the work in the PAT must be your own work. Group work will NOT be allowed.

3.4 Show ALL calculations clearly and include units. Round off answers to TWO decimal places. Use correct SI units where necessary.

4. EVIDENCE OF MODERATION

Learner's name:	
School:	

MARK ALLOCATION

TASK	MAXIMUM MARK	WEIGHTING	LEARNER'S MARK (TEACHER)	MODERATED MARK (SCHOOL)	MODERATED MARK (DISTRICT)	MODERATED MARK (PROVINCE)
1	40	40				
2	30	30				
3	30	30				
TOTAL	100	100				
NAME						
SIGNATURES						
DATE						

DECLARATION OF AUTHENTICITY

I hereby declare that the tasks submitted for assessment is my own original work and have not been submitted for assessment or moderation previously.

_____ _____

SIGNATURE OF LEARNER DATE

As far as I know, the above declaration by the candidate is true and I accept that the work offered is his/her own.

_____ _____

SIGNATURE OF TEACHER DATE

SCHOOL STAMP

5. CONCLUSION

On completion of the practical assessment task learners should be able to demonstrate their understanding of the industry, enhance their knowledge, skills, values and reasoning abilities as well as establish connections to life outside the classroom and address real-world challenges. The PAT furthermore develops learners' life skills and provides opportunities for learners to engage in their own learning.

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REPUBLIC OF SOUTH AFRICA

TECHNICAL MATHEMATICS

PRACTICAL ASSESSMENT TASK 1

GRADE 12

2021

SURNAME AND NAME	
SCHOOL	

TERM: **1**

MARKS: **40**

This task consists of 12 pages.

TECHNICAL MATHEMATICS TASK 1

TOPIC: COMPLEX NUMBERS

AIM: To apply and develop mathematical skills, reasoning and demonstrate an understanding of complex numbers in real-life technical problems

A complex number is any number that can be written in the form $a + bi$ where a and b are real numbers, where a is a real part, b is an imaginary part and i is an imaginary unit.

INSTRUCTIONS AND INFORMATION

1. This PAT Task 1 worksheet consists of FIVE questions.

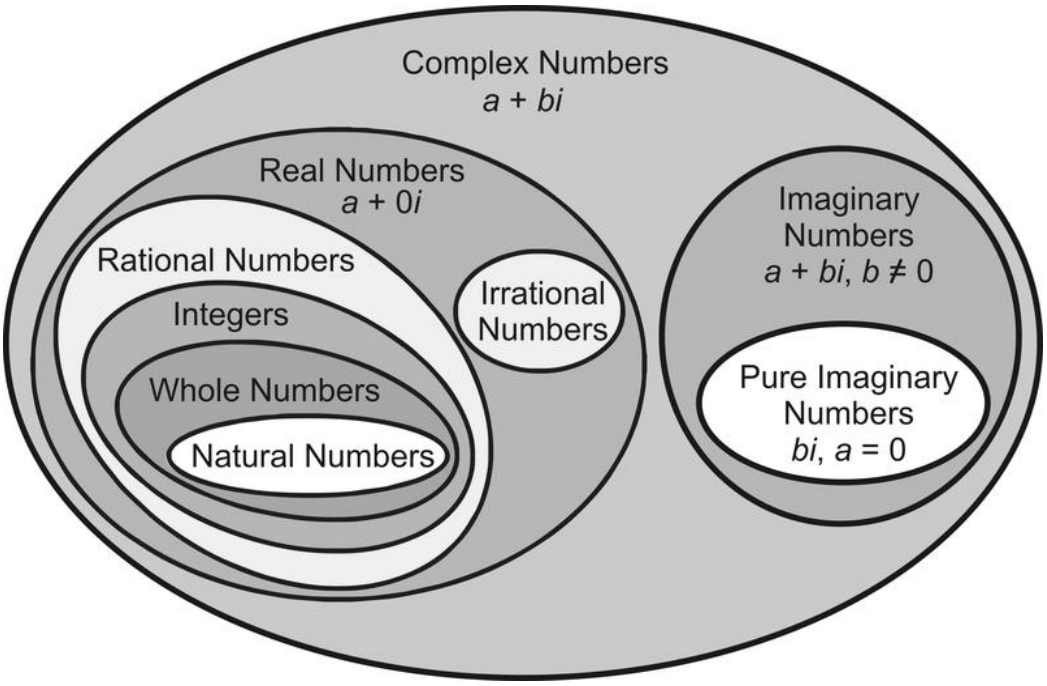
2. Answer ALL the questions.

3. Resources required are calculators, grids (provided) and mathematical sets.

4. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

QUESTION 1

CLASSIFICATION OF COMPLEX NUMBERS AND INTRODUCTION TO COMPLEX CONJUGATES



Classify each complex number by placing a value in the appropriate column(s).

	Solution	**Marks**

No.	**Complex Number**	**Imaginary Part**	**Real Part**	**Complex Conjugate**
1.	$5 + 2i$			
2.	$-8 + \frac{1}{2}i$			
3.	$-3i$			

[3]

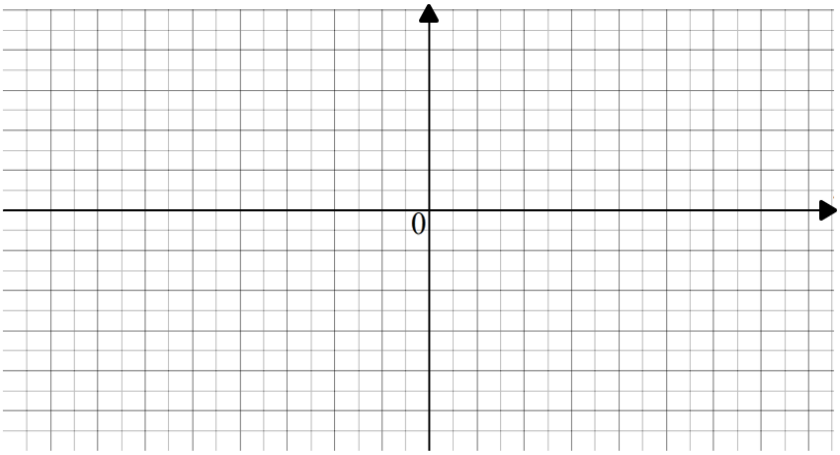
QUESTION 2

BASIC OPERATIONS AND GRAPHICAL REPRESENTATION OF COMPLEX NUMBERS

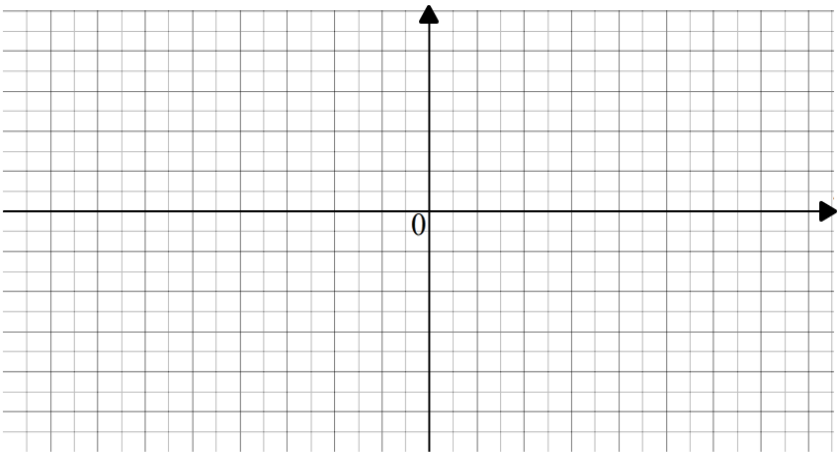
Complex numbers can be represented on a complex plane, the Argand diagram with the horizontal axis as the real part and the vertical axis as the imaginary part.
A Complex number $z = a + bi$ in rectangular form has a, representing the distance along the real axis and b, the distance along the imaginary axis.

2.1 Determine, with the aid of an Argand diagram:

2.1.1 The sum of $z_1 = -8+3i$ and $z_2 = 3-5i$ (3)

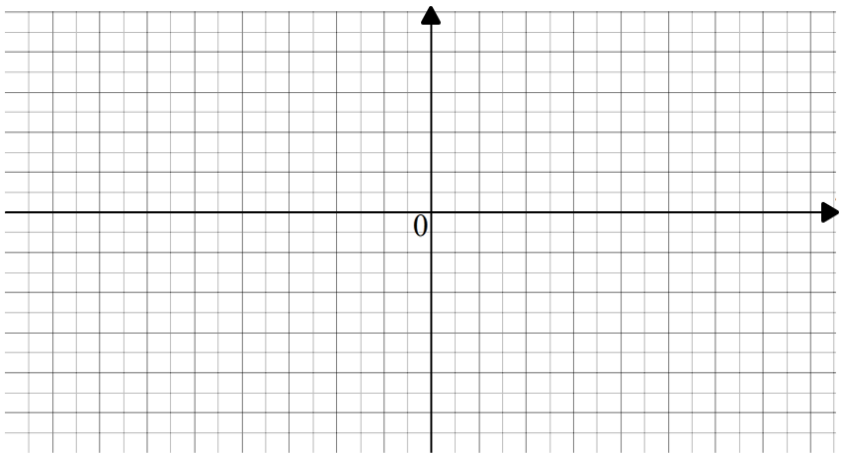
	Solution	Marks
		(3)

2.1.2 The difference between $z_3 = 9-7i$ and $z_4 = 6+i$ (3)

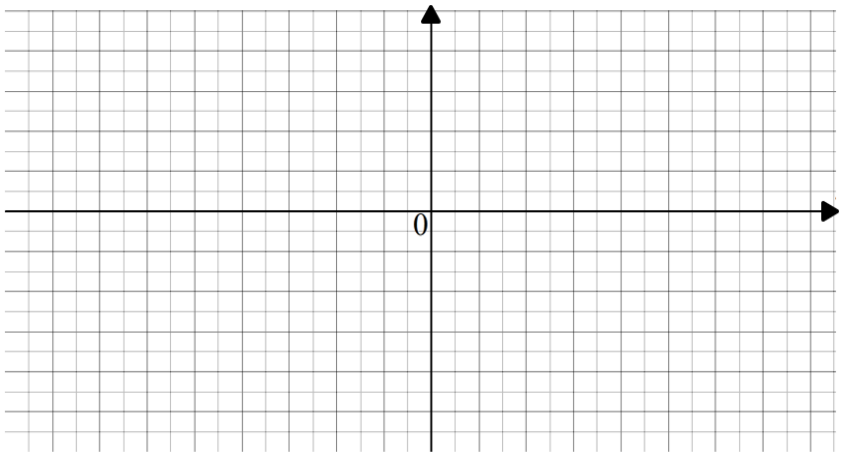
	Solution	Marks
		(3)

2.2 Sketch the given complex number and its complex conjugate on the grid provided:

2.2.1 $z_3 = 9 - 7i$ (3)

	Solution	Marks
		(3)

2.2.2 $z_4 = 6 + i$ (2)

	Solution	Marks
		(2)

2.3 What can you deduce from QUESTIONS 2.2.1 and 2.2.2 about the complex number and its complex conjugate? (1)

	Solution	Marks
		(1)
		[12]

QUESTION 3

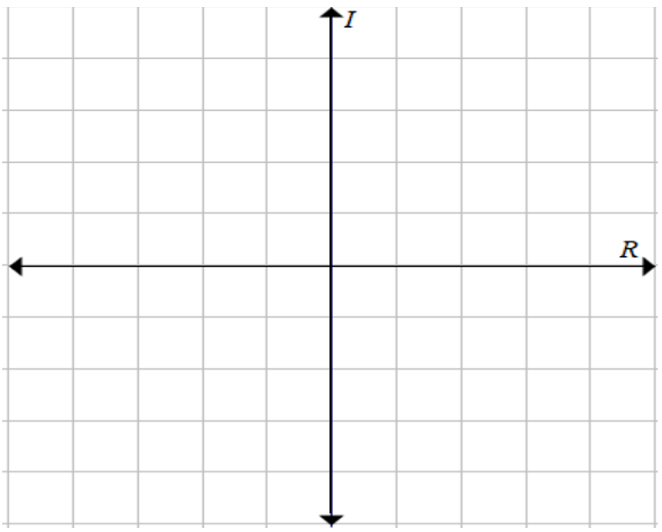
POLAR FORM AND GRAPHICAL REPRESENTATION OF COMPLEX NUMBERS

- A complex number in rectangular form $a + bi$ has polar coordinates $r\,cis\,\theta = r\,(cos\,\theta + i\,sin\,\theta) = r\lfloor\theta$ where:
  - $r = \sqrt{a^2+b^2}$, r is the modulus
  - $tan\,\theta = \frac{b}{a}$, θ is the argument
- In a complex plane, by drawing a vector from the origin to the point representing $z = a + bi$, an angle θ in standard position is formed. The point $a + bi$ is r units from the origin.
- When converting a complex number from a rectangular form to a polar form:
  - The quadrant in which the complex number lies should be identified
  - The required angle is found by using a tangent ratio, the signs of a and b indicate the quadrant in which the angle is found

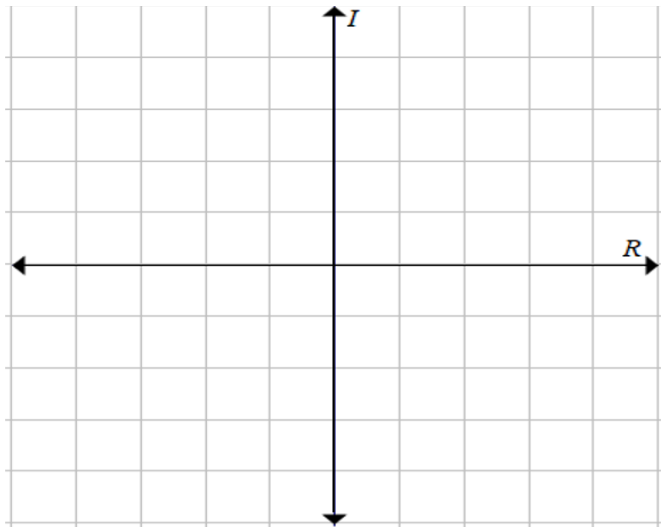
Convert to polar form and represent on an Argand diagram the following rectangular complex numbers:

3.1 $z = 4i$ (4)

	Solution	Marks
		(4)



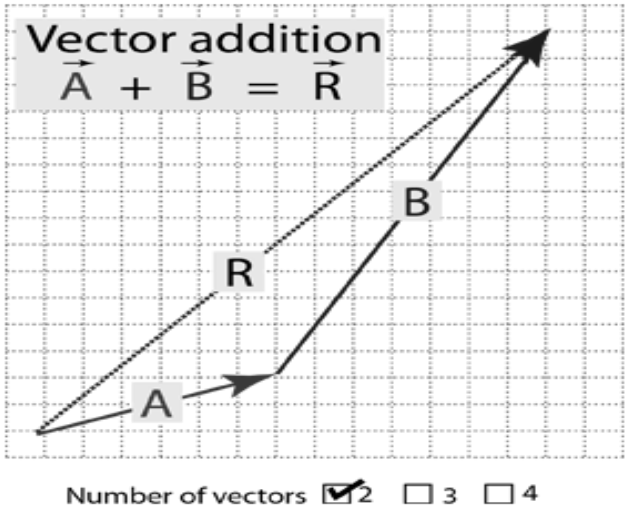
3.2 $z = 2\sqrt{5} + 2i$ (6)

	Solution	Marks
		(6)
		[10]

QUESTION 4

APPLY THE COMPLEX NUMBER TO DETERMINE THE RESULTANT FORCE OF TWO VECTORS BY ADDITION

The picture below shows a force diagram of the addition of two vectors.



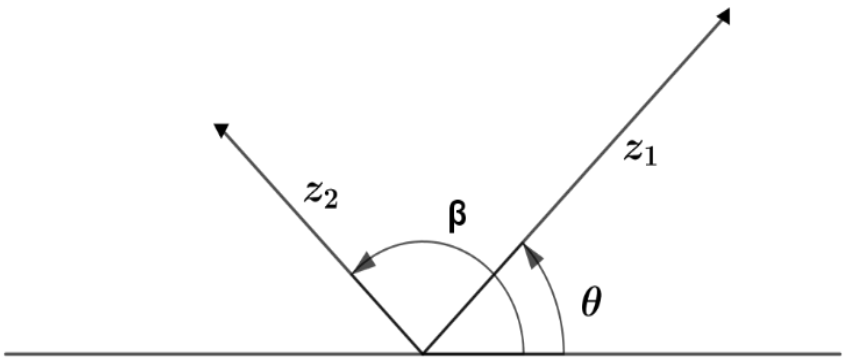
Now use knowledge of complex numbers to add two vectors.

Given in the diagram below:

Force 1: $z_1 = 4\sqrt{2}\,cis\,\theta$

Force 2: $z_2 = \frac{6}{\sqrt{2}}cis\,\beta$

The diagram below shows the two vectors.



Using a protractor:
Step 1: Measure θ in degrees.
Step 2: Measure β in degrees.

4.1 Using a protractor, measure angles θ and β (2)

	Solution	Marks
	$\theta =$ _____ °	
	$\beta =$ _____ °	
		(2)

4.2 Write z_1 and z_2 in polar form. (2)

	Solution	Marks
		(2)

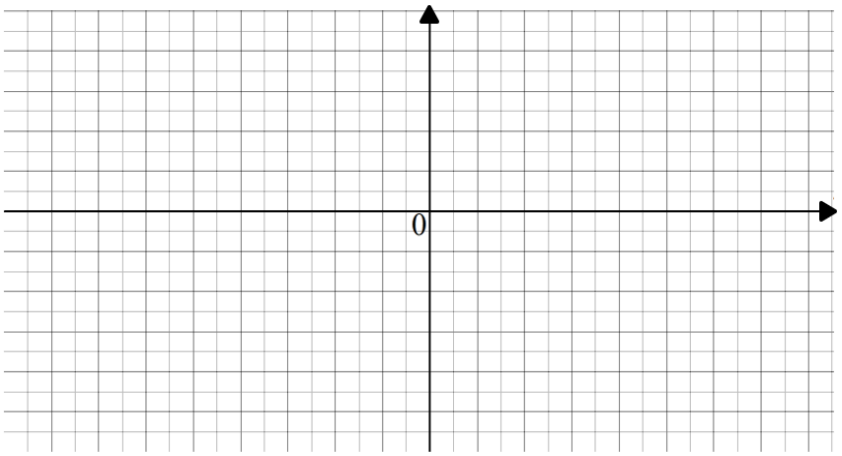
4.3 Determine resultant force z_R if $z_R = z_1 + z_2$. (1)

	Solution	Marks
		(1)

4.4 Express z_R in polar form $r\,cis\,\theta$. (1)

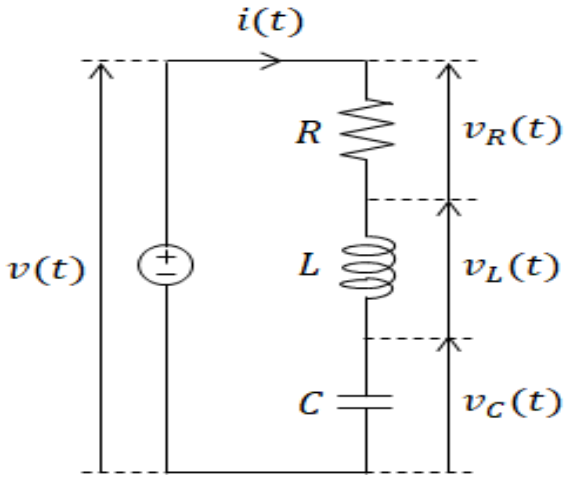
	Solution	Marks
		(1)

4.5 Represent the resultant force in an Argand diagram (3)

	Solution	**Marks**
		(3)
		[9]

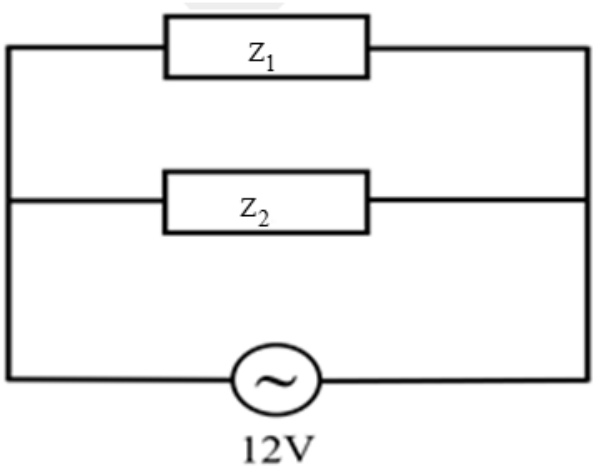
QUESTION 5

APPLICATION OF COMPLEX NUMBERS IN ELECTRONICS (RLC circuits)



The diagram below models an AC circuit with two impedances,
$z_1 = (30 + 10i)\Omega$ and $z_2 = (10 - 30i)\Omega$ connected in parallel.

The total impedance is $z_T = \dfrac{z_1 \times z_2}{z_1 + z_2}$



Determine the total impedance, $z_T = \frac{z_1 \times z_2}{z_1 + z_2}$ (6)

	Solution	**Marks**
		(6)
		[6]

TOTAL: 40

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TECHNICAL MATHEMATICS

PRACTICAL ASSESSMENT TASK 2

GRADE 12

2021

SURNAME AND NAME	
SCHOOL	

TERM: **2**

MARKS: **30**

This task consists of 12 pages.

TECHNICAL MATHEMATICS TASK 2

TOPIC: EUCLIDEAN GEOMETRY

AIM: To verify the midpoint theorem and proportionality theorem and apply the two theorems to solve problems

INSTRUCTIONS AND INFORMATION

1. This PAT Task 2 worksheet consists of SIX questions.

2. Answer ALL the questions.

3. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.

ACTIVITY 1

Objective

- To investigate the relationship between the line joining the mid-points of two sides of a triangle and the third side
- To apply the midpoint theorem to solve problems in a real-life context

Materials required

- Mathematical instruments (ruler and protractor are essential)
- Pen
- Pencil
- Cotton/String
- Scissors
- Tracing paper
- Cellophane tape
- Paper glue

Theory

Midpoint theorem: The line segment joining the midpoints of any two sides of a triangle is parallel to the third side and it is half the length of the third side.

Procedure

Step 1: Draw any ΔABC in the space provided below.

	Solution	
Step 1		

Step 2: Use a ruler to measure the lengths of AB, AC and BC and record them in the space below.

	Solution	
Step 2	AB = _____; AC = _____ and BC = _____	

Step 3: Mark the midpoints D and E of the sides AB and AC respectively. Join D and E.

	Solution	
Step 3	$\therefore$ Length of AD = _____ and AE = _____	

Step 4:
- Place a piece of cotton/string along the length of BC.
- Mark off and cut so that the length of the cotton is the same as BC.
- Fold the piece of cotton/string in half and place that against DE.
- Use cellophane tape to stick the cotton piece along the length of DE.
- Does the folded half fit exactly along the length of DE?
- Use a ruler to measure the length of DE and record the value.

	Solution	
Step 4	• Does the folded half fit exactly along the length of DE? _____ • Use a ruler to measure the length of DE and record the value: DE = _____	

Step 5:
- Use a tracing paper, trace and cut out a copy of ΔADE.
- Clearly label angles $\hat{A}, \hat{D}$ and $\hat{E}$ at the respective vertices.
- Place your traced cut-out of ΔADE so that $\hat{D}$ is placed over $\hat{B}$ of ΔABC.
- Use paper glue to stick the cut-out of ΔADE so that $\hat{D}$ is placed over $\hat{B}$ of ΔABC.
- Does $\hat{D}$ fit exactly over $\hat{B}$?
- Use a protractor to measure the size of $A\hat{B}C$ and $A\hat{D}E$ and record them below.

	Solution	
Step 5	• _____ • $A\hat{B}C$ = _____ ° and $A\hat{D}E$ = _____ °	

Observations and Conclusion

1. Answer the following questions:

1.1 Compare the sizes of $\hat{ABC}$ and $\hat{ADE}$

	Solution	
1.1		

1.2 Fill in the missing word:

The line segment DE is ……. to BC
(corresponding $\angle s$ are =)

	Solution	
1.2		

1.3 Compare the lengths of DE and BC.

	Solution	
1.3		

1.4 Explain the relationship between the lengths of DE and BC,

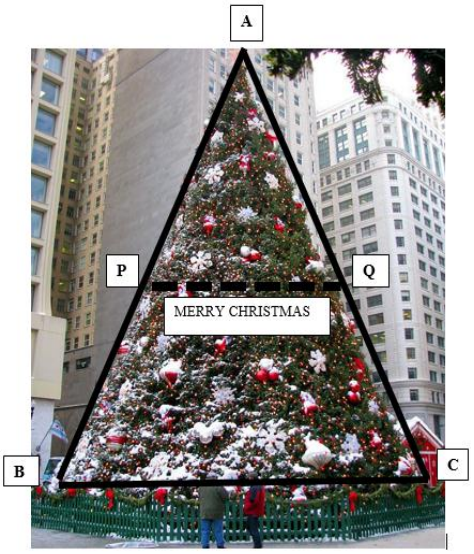
i.e. length of DE = … BC

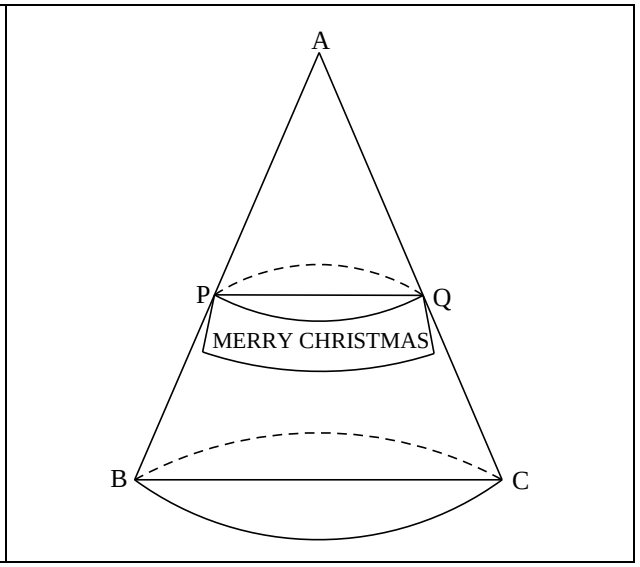
	Solution	
1.4	DE = … BC	

1.5 Explain the relationship that you observe between the line segment joining the midpoints of two sides of a triangle and the third side.

	Solution	
1.5		

2. The picture below shows a large Christmas tree and the diagram alongside models the picture.
Points P and Q are the midpoints of sides AB and AC respectively of ΔABC.
The slogan 'MERRY CHRISTMAS' is attached to a steel cable surrounding the Christmas tree from P to Q.





Use the midpoint theorem to determine the length of the slogan from P to Q, by first determining the length of BC (the base diameter) if the circumference of the base circle of the tree is 40,84 m. Show your calculations in the space below.

Use the following formula: Circumference of circle $= 2\pi r$

	Solution	

ACTIVITY 2

Objective
- To investigate the basic proportionality theorem

Materials required
- Mathematical instruments (ruler and protractor are essential)
- Pen
- Pencil

Theory
Proportionality theorem: The line drawn parallel to one side of a triangle divides the other two sides proportionally.
Equal ratios form proportion.

Procedure

Step 1: Draw ΔKLM in the space provided below.

	Solution	
Step 1		

Step 2: Measure the lengths of KL, KM and LM. Record them below.

	Solution	
Step 2	KL = _____; KM = _____ and LM = _____	

Step 3: Draw line segment NP with N on KL and P on KM such that NP || LM

	Solution	
Step 3	Show in Step 1 above.	

Step 4: Measure the lengths of KN, NL, KP, PM and NP. Record them below.

	Solution	
Step 4	KN = _____ , NL = _____ , KP = _____ , PM = _____ and NP = _____	

Calculations, Observations and Conclusion

1. Answer the following questions:

1.1 Calculate the following:

1.1.1 $\frac{KN}{NL}$ and $\frac{KP}{PM}$

	Solution	
1.1.1	$\frac{KN}{NL} =$ _____ and $\frac{KP}{PM} =$ _____	

1.1.2 $\frac{KN}{KL}$ and $\frac{KP}{KM}$

	Solution	
1.1.2	$\frac{KN}{KL} =$ _____ and $\frac{KP}{KM} =$ _____	

1.1.3 $\frac{NP}{LM}$

	Solution	
1.1.3	$\frac{NP}{LM} =$	

1.2 Compare the values of the ratios calculated above.

	Solution	
1.2		

1.3 Explain the relationship observed between the line segments that are divided by the line drawn parallel to the 3rd side of a triangle.

	Solution	
1.3		

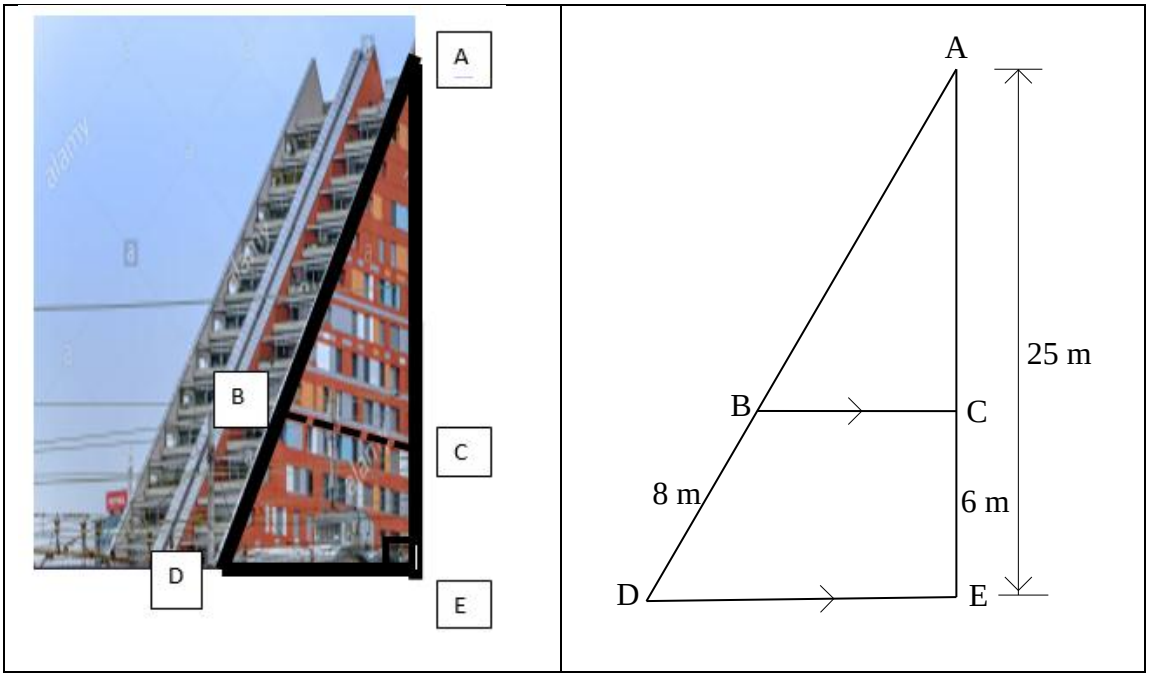
1.4 Write down the relationship between the lengths of NP and LM,
i.e. the length of NP = … LM

	Solution	
1.4	NP = … LM	

1.5 Explain the relationship that you observe between the line segment joining the midpoints of two sides of a triangle and the third side.

	Solution	
1.5		

2. A company has been contracted to place metal spikes on a triangular building along the length of side AD to prevent birds from roosting and messing on the building. The cost of the spikes, including labour, is R165 per meter. The diagram below models the side view of the building, as shown.



If the length of AE = 25 m, CE = 6 m, BD = 8 m and BC // DE, use the proportionality theorem to determine how much will it cost to fit the spikes by first determining the length of AD. Show calculations in the space below.

	Solution	

TOTAL: 30

Name: …… School: ……

The following marking criteria will be used for marking.
(Each criterion will have 4 levels at most.)

No.	Criteria	Marks
1	Ability to carry out instructions	6
2	Accuracy in measurement and calculations	8
3	Observations and conclusion of Activity 1	8
4	Observations and conclusion of Activity 2	8
	TOTAL:	30

RUBRIC FOR MARKING TECHNICAL MATHEMATICS PAT 2021 TASK 2

CRITERIA	1	2	3	4	
1. Ability to carry out instructions	Unable to carry out 75% of the instructions	Able to carry out 50 % of the instructions	Able to follow all the instructions	Able to follow all the instructions	___x 2 = __
2. Accuracy in measurement and calculations	Unable to measure accurately and errors in calculations	Able to do some correct measurements and had some errors in calculations	Able to do correct measurements and correct calculations with few errors of rounding off	Able to do accurate measurements and accurate calculations without errors	___x 2 = __
3. Observations and conclusion of Activity 1	Unable to observe the relationships and conclusion was not related to the theorem	Able to establish one part of the theorem, i.e. that the line segment is parallel to the third side or that the line segment is half the size of the third side	Able to establish both relationships, i.e. that the line segment is parallel to the third side and it is half the size of the third side with some explanation	Able to establish both relationships, i.e. that the line segment is parallel to the third side and it is half the size of the third side with perfect explanation.	___x 2 = __
4. Observations and conclusion of Activity 2	Unable to observe the relationship of the ratios, hence the conclusion was not related to the theorem	Able to establish some relationships of the ratios based on calculations and conclusion made had some errors	Able to identify ratios that are equal and hence form proportion based on calculations. The conclusion was also correct.	Able to clearly identify equal ratios that form proportions based on correct calculations. The conclusion was perfectly explained.	___x 2 = __
TOTAL					**___ 30**

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TECHNICAL MATHEMATICS

PRACTICAL ASSESSMENT TASK 3

GRADE 12

2021

SURNAME AND NAME	
SCHOOL	

TERM: **3**

MARKS: **30**

This task consists of 7 pages.

TECHNICAL MATHEMATICS TASK 3

TOPIC: CIRCLES, ANGLES AND ANGULAR MOVEMENT

AIMS:

- **To apply and develop mathematical skills, reasoning and demonstrate an understanding of radians and degrees**
- **To convert between degrees and radians**
- **To calculate area, arc length and height of the segment**
- **To apply the knowledge gained in circles, angles and angular movement to solve real-life problems**

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.

2. Resources required are a wall chart, ruler, compass, pencil and protractor.

3. Clearly show ALL calculations, diagrams etc. that you have used in determining your answers.

4. Make sure your sketch is neat and constructed according to the instructions.

ACTIVITY 1

Resources required: ruler, protractor, compass and coloured pencils

Step 1: Draw an equilateral triangle PQR with sides of length 6 cm. (1)

	Solution	**Marks**
Step 1		

Step 2: Draw a circle through points A and B such that R is the centre, where A is the midpoint of PR and B the midpoint of QR. (2)

	Solution	**Marks**
Step 2	Must be done on the diagram under Step 1.	

Step 3: Shade the area of the minor sector RAB. (1)

	Solution	**Marks**
Step 3	Must be done on the diagram under Step 1.	

Step 4: Write down the length of side AR and the magnitude of $\hat{R}$. (2)

	Solution	**Marks**
Step 4	AR = _____ $\hat{R}$ = _____	

Step 5: Convert the magnitude of $\hat{R}$ to radians. (2)

	Solution	**Marks**
Step 5		

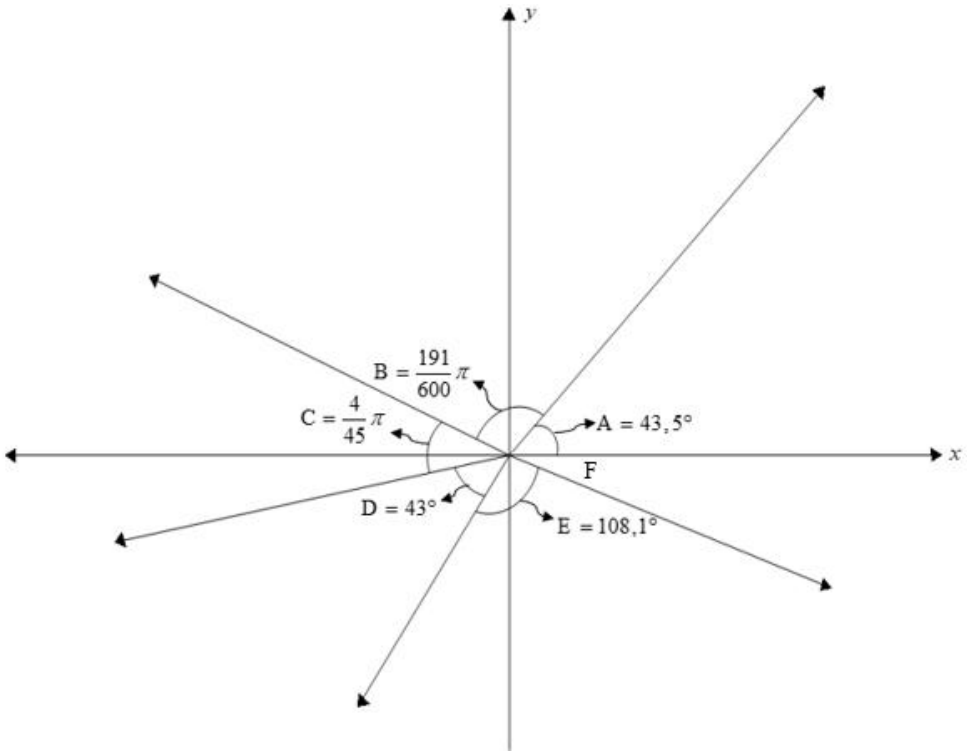
Step 6: Determine the arc length of the minor sector RAB. (3)

	Solution	**Marks**
Step 6		

Step 7: Hence, determine the unshaded area APQB. (5)

	Solution	**Marks**
Step 7		

Step 8: The diagram below shows angles in degrees and radians. (6)



Complete the following table by converting between degrees and radians measures if:

$$180° = \pi \text{ radians}$$

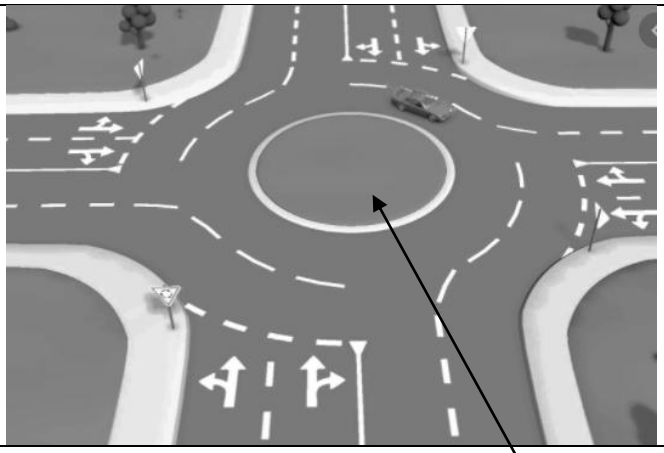
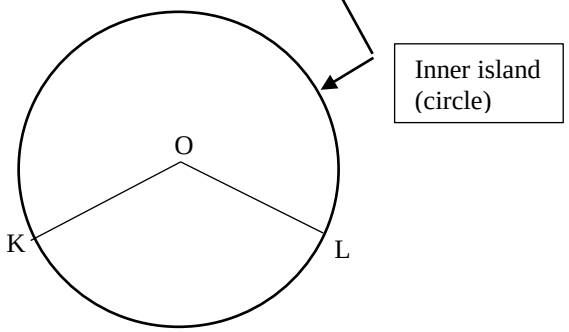
	Solution	Marks
Step 8		

	ANGLES	
	DEGREES	RADIANS
	43,5°	$\frac{29}{240}\pi$
	.....	$\frac{191}{300}\pi$
	.....	$\frac{4}{25}\pi$
	43°	.....
	108,1°	.....
	$x =$.....°	.....
TOTAL	360°	.....

[22]

ACTIVITY 2

The picture below shows a traffic circle at one of the busy intersections in a certain municipality. The municipality wants to pave the inner island of the traffic circle and plans to do so in three consecutive days by paving a third of the island each day. The inner island has a circumference of 18 m. The diagram below the picture models the inner island of the traffic circle.
O is the centre of the circle.
K and L are points on the circle.

PICTURE	
DIAGRAM	

2.1 Calculate:

2.1.1 The length of the radius OK correct to ONE decimal place. (1)

	Solution	Marks
		(1)

2.1.2 The magnitude of the obtuse angle $K\hat{O}L$ (in degrees). (1)

	Solution	Marks
		(1)

2.2 Determine the height of the minor segment of chord KL. (6)
Use $4h^2 - 4dh + x^2 = 0$

	Solution	**Marks**
		(6)
		[8]

TOTAL: 30