



**NATIONAL
SENIOR CERTIFICATE/
NASIONALE
SENIOR SERTIFIKAAT**

GRADE/GRAAD 12

JUNE/JUNIE 2022

**MATHEMATICS P2/WISKUNDE V2
MARKING GUIDELINE/NASIENRIGLYN**

MARKS/PUNTE: 150

This marking guideline consists of 15 pages.
Hierdie nasienriglyn bestaan uit 15 bladsye.

QUESTION/VRAAG 1

1.1	B	✓ answer/antwoord	(1)
1.2	B	✓ answer/antwoord	(1)
1.3	75%	✓ answer/antwoord	(1)
1.4	Nothing. It remains the same. No change in standard deviation. <i>Niks. Dit bly dieselfde. Geen verandering in standaardafwyking.</i>	✓ reason/rede	(1)
1.5	Semi – IQR: $IKV = \frac{75 - 30}{2}$ Semi – IQR: $IKV = 22,5$	✓ answer/antwoord	(1)
			[5]

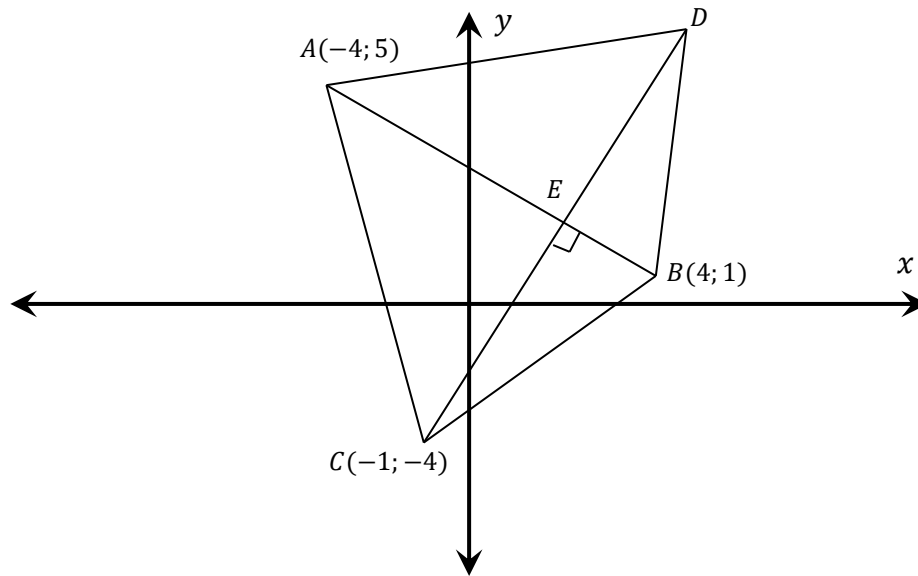
QUESTION/VRAAG 2

2.1	<table border="1"> <thead> <tr> <th>Time taken <i>Tyd geneem</i></th> <th>No. of pupils <i>Aantal leerlinge</i></th> <th>Cumulative frequency <i>Kummulatiewe frekwensie</i></th> </tr> </thead> <tbody> <tr> <td>$60 \leq t \leq 90$</td> <td>3</td> <td>3</td> </tr> <tr> <td>$90 \leq t \leq 120$</td> <td>6</td> <td>9</td> </tr> <tr> <td>$120 \leq t \leq 150$</td> <td>7</td> <td>16</td> </tr> <tr> <td>$150 \leq t \leq 180$</td> <td>8</td> <td>24</td> </tr> <tr> <td>$180 \leq t \leq 210$</td> <td>6</td> <td>30</td> </tr> </tbody> </table>	Time taken <i>Tyd geneem</i>	No. of pupils <i>Aantal leerlinge</i>	Cumulative frequency <i>Kummulatiewe frekwensie</i>	$60 \leq t \leq 90$	3	3	$90 \leq t \leq 120$	6	9	$120 \leq t \leq 150$	7	16	$150 \leq t \leq 180$	8	24	$180 \leq t \leq 210$	6	30	✓ for values <i>vir waardes</i>	(1)
Time taken <i>Tyd geneem</i>	No. of pupils <i>Aantal leerlinge</i>	Cumulative frequency <i>Kummulatiewe frekwensie</i>																			
$60 \leq t \leq 90$	3	3																			
$90 \leq t \leq 120$	6	9																			
$120 \leq t \leq 150$	7	16																			
$150 \leq t \leq 180$	8	24																			
$180 \leq t \leq 210$	6	30																			
2.2	<p style="text-align: center;">Time taken to complete course.</p>	✓ anchor point/ <i>ankerpunt</i> (60;0) ✓ (120; 9) ✓ (150; 16) ✓ (210; 30)	(4)																		
2.3	2.3.1 See diagram above / <i>Sien diagram hierbo</i>	✓ A	(1)																		
	2.3.2 See diagram above / <i>Sien diagram hierbo</i>	✓ B	(1)																		
	2.3.3 See diagram above / <i>Sien diagram hierbo</i>	✓ C	(1)																		
			[8]																		

QUESTION/VRAAG 3

3.1	Median score / <i>Mediaan telling</i> = $2x$	✓ answer/ <i>antwoord</i>	(1)
3.2	$\text{Mean/Gemiddelde} = \frac{\sum x}{n}$ $= \frac{4(x + 3) + 3(2x) + 2(x - 1) + 2(6)}{11}$ $= \frac{12x + 22}{11}$	✓ substitution/ <i>vervanging</i> ✓ simplification/ <i>vereenvoudiging</i> ✓ answer/ <i>antwoord</i>	(3)
3.3	Use of a calculator where the four values are as follows: <i>Gebruik van 'n sakrekenaar waar die vier waardes soos volg is:</i> 8 ; 10 ; 4 and/en 6 $sd(\sigma) = \sqrt{5}$	✓ four values/ <i>vier waardes</i> ✓ answer/ <i>antwoord</i>	(2)
			[6]

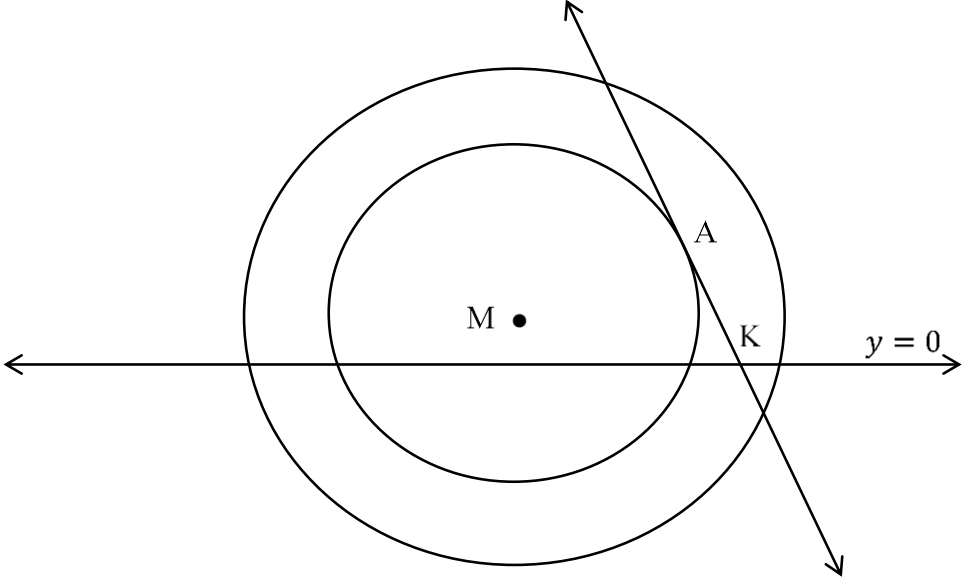
QUESTION/VRAAG 4



4.1	$m_{AB} = \frac{5 - 1}{-4 - 4}$ $= \frac{4}{-8}$ $m_{AB} = -\frac{1}{2}$ $\therefore m_{CD} = 2$	<ul style="list-style-type: none"> ✓ subst. into gradient formula/ verv. in gradiënt formule ✓ $m_{AB} = -\frac{1}{2}$ 	(2)
4.2	$\therefore m_{CD} = 2$ $\overline{CD} : y = 2x + c$ $-4 = 2(-1) + c$ $-2 = c$ $\overline{CD} : y = 2x - 2$	<ul style="list-style-type: none"> ✓ $m_{CD} = 2$ ✓ sub of point/verv. van punt (-1; -4) ✓ $-2 = c$ ✓ equation of CD / vergelyking van CD 	(4)
4.3	$\overline{AB} : y = -\frac{1}{2}x + c$ $1 = -\frac{1}{2}(4) + c$ $3 = c$ $y = -\frac{1}{2}x + 3$ $-\frac{1}{2}x + 3 = 2x - 2$ $-x + 6 = 4x - 4$ $5x = 10$ $x = 2$ $y = 2(2) - 2$ $y = 2$ $E(2; 2)$	<ul style="list-style-type: none"> ✓ substitution of point (4; 1) vervanging van punt (4; 1) ✓ equation of \overline{AB} vergelyking van \overline{AB} ✓ equating of AB and CD gelykstel van AB en CD ✓ x-value/waarde ✓ substitution of/vervanging van x-value/waarde ✓ y-value/waarde 	(6)

4.4	$D(x; y)$ $\frac{x-1}{2} = 2$ $x - 1 = 4$ $x = 5$ $D(5; 8)$	$\frac{y-4}{2} = 2$ $y - 4 = 4$ $y = 8$	$\checkmark x = 5$ $\checkmark y = 8$	(2)
4.5	$m_{AC} = \frac{5+4}{-4+1}$ $m_{AC} = -3$ Line parallel to AC has same gradient. <i>Lyn ewewydig aan AC het dieselfde gradient.</i> $y = -3x + c$ $8 = -3(5) + c$ $c = 23$ $y = -3x + 23$		\checkmark substitution into gradient formula/ <i>vervanging in gradiënt formule</i> $\checkmark m_{AC} = -3$ $\checkmark c = 23$ \checkmark equation of line/ <i>vergelyking van lyn</i>	(4)
4.6	x intercept of CD : x afsnit van CD $2x - 2 = 0$ $x = 1$ $m_{BC} = 1$ Equation of Altitude/ <i>Vergelyking van hoogtelyn</i> $y = -x + c$ $5 = -(-4) + c$ $c = 1$ $\therefore y = -x + 1$ x intercept of Altitude / x -afsnit van hoogtelyn $x = 1$ x intercept of CD = x intercept of altitude x -afsnit van CD = x -afsnit van hoogtelyn		$\checkmark x = 1$ $\checkmark m_{BC} = 1$ \checkmark gradient of altitude -1 <i>gradiënt van hoogtelyn -1</i> \checkmark substitution of point <i>vervanging van punt</i> \checkmark equation of altitude <i>vergelyking van hoogtelyn</i> $\checkmark x = 1$	(6)
				[24]

QUESTION/VRAAG 5

		
5.1	$x^2 + y^2 = 4y - 2x + 44$ $x^2 + 2x + 1 + y^2 - 4y + 4 = 44 + 1 + 4$ $(x + 1)^2 + (y - 2)^2 = 49$ $\therefore M(-1; 2)$	<ul style="list-style-type: none"> ✓✓ completing the square <i>voltooiing van die vierkant</i> ✓ factorizing / <i>faktorisering</i> ✓ $M(-1; 2)$
5.2	$m_{MA} = \frac{y - 2}{x + 1}$ $\frac{y - 2}{x + 1} = 1$ $y - 2 = x + 1$ $y = x + 3$ $x + 3 = -x + 5$ $2x = 2$ $x = 1$ $\therefore y = 4$ $A(1; 4)$	<ul style="list-style-type: none"> ✓ gradient of MA/<i>gradiënt van MA</i> ✓ equating it to 1/<i>stel dit gelyk aan 1</i> ✓ making y or x the subject. <i>maak y of x die onderwerp</i> ✓ equating the two linear functions. <i>gelykstel van twee lineêre funksies</i> ✓ coordinates of $A(1; 4)$ <i>koördinate van $A(1; 4)$</i>
5.3	$(x + 1)^2 + (y - 2)^2 = r^2$ $(1 + 1)^2 + (4 - 2)^2 = r^2$ $8 = r^2$ $(x + 1)^2 + (y - 2)^2 = 8$	<ul style="list-style-type: none"> ✓ substitution of A/<i>vervanging van A</i> ✓ $8 = r^2$ ✓ equation of the circle/<i>vergelyking van die sirkel</i>

5.4	$K(5; 0)$	✓ $K(5; 0)$	(1)
5.5	$AK = \sqrt{32}$ Area of/van $\Delta AMK = \frac{1}{2} AK \times AM$ $Area\ of/van\ \Delta\ AMK = \frac{1}{2} \sqrt{32} \times \sqrt{8}$ $Area\ of/van\ \Delta\ AMK = 8\ units^2/eenhede^2$	✓ $AK = \sqrt{32}$ ✓ $AM = \sqrt{8}$ ✓ $8\ units^2/eenhede^2$	(3)
			[16]

QUESTION/VRAAG 6

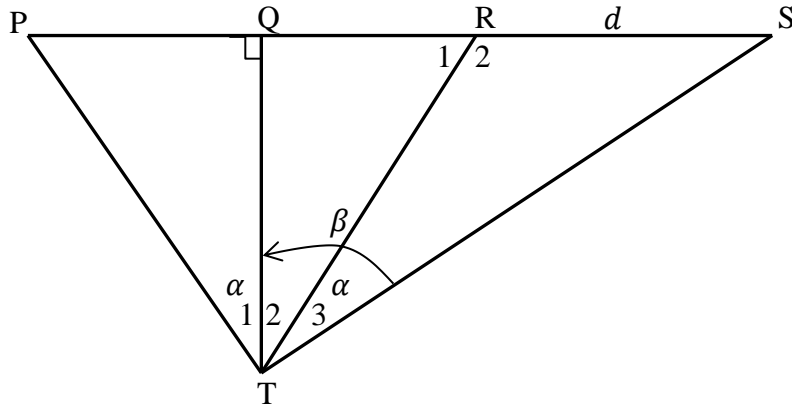
6.1			
6.1.1	$\sin 26^\circ = \frac{\sqrt{p^2 - 1}}{p}$	✓ sketch in Quadrant 1 skets in kwadrant 1 ✓ $\sqrt{p^2 - 1}$ ✓ answer/antwoord	(3)
6.1.2	$\begin{aligned} \cos 52^\circ &= \cos 2(26^\circ) \\ &= 2 \cos^2 26^\circ - 1 \\ &= 2 \left(\frac{1}{p}\right)^2 - 1 \\ &= \frac{2}{p^2} - 1 \end{aligned}$	✓ $\cos 2(26^\circ)$ ✓ $2 \cos^2 26^\circ - 1$ ✓ answer/antwoord	(3)
6.1.3	$\begin{aligned} &\tan^2 64^\circ \times (p + 1) \\ &= \left(\frac{1}{\sqrt{p^2 - 1}}\right)^2 \times (p + 1) \\ &= \frac{1}{p^2 - 1} \times (p + 1) \\ &= \frac{1}{(p - 1)(p + 1)} \times (p + 1) \\ &= \frac{1}{p - 1} \end{aligned}$	✓ $\left(\frac{1}{\sqrt{p^2 - 1}}\right)^2$ ✓ $\frac{1}{p^2 - 1}$ ✓ $(p - 1)(p + 1)$ ✓ answer/antwoord	(4)

6.2	$\frac{\sin(-\beta) + \sin(360^\circ - \beta)}{\sin(180^\circ - \beta) + \sin 180^\circ}$ $= \frac{-\sin \beta + (-\sin \beta)}{\sin \beta + 0}$ $= \frac{-2 \sin \beta}{\sin \beta}$ $= -2$	<ul style="list-style-type: none"> ✓ $-\sin \beta$ ✓ $-\sin \beta$ ✓ $\sin \beta$ ✓ simplification <i>vereenvoudiging</i> ✓ answer/antwoord 	(5)
6.3	$2p \tan\left(\frac{\theta}{2}\right) = \sin(2\theta)$ $2p \tan\left(\frac{82^\circ}{2}\right) = \sin(2 \times 82^\circ)$ $p = \frac{\sin 162^\circ}{2 \tan 41^\circ}$ $p = 0,16$	<ul style="list-style-type: none"> ✓ substitution/<i>vervanging</i> ✓ simplification/<i>vereenvoudiging</i> ✓ answer/antwoord 	(3)
6.4	$4 \sin \theta \cdot \cos^3 \theta - 4 \cos \theta \cdot \sin^3 \theta$ $= \sin 4\theta$ $LHS/LK = 4 \sin \theta \cdot \cos^3 \theta$ $- 4 \cos \theta \cdot \sin^3 \theta$ $= 4 \sin \theta \cdot \cos \theta (\cos^2 \theta - \sin^2 \theta)$ $= 2 \times 2 \sin \theta \cos \theta (\cos 2\theta)$ $= 2 \cdot \sin 2\theta \cdot \cos 2\theta$ $= \sin 4\theta$ $= RHS/RK$	<ul style="list-style-type: none"> ✓ common factor/<i>gemene faktor</i> ✓ $2 \times 2 \sin \theta \cos \theta$ ✓ $(\cos 2\theta)$ ✓ $\sin 2\theta$. ✓ $2 \cdot \sin 2\theta \cdot \cos 2\theta$ ✓ answer/antwoord 	(6)
			[24]

QUESTION/VRAAG 7

<p>7.1</p>	$\cos 3x = \sin(x - 30^\circ)$ $\cos 3x = \cos[90^\circ - (x - 30^\circ)]$ $\cos 3x = \cos[120^\circ - x]$ $3x = 120 - x \text{ (ref angle)}$ $3x = 120^\circ - x + k.360^\circ \text{ OR/OFF } 3x = 360^\circ - [120^\circ - x]k.360^\circ$ $4x = 120^\circ + k.360^\circ \qquad 2x = 240^\circ + k.360^\circ$ $x = 30^\circ + k.90^\circ \qquad x = 120^\circ + k.180^\circ$ $x = 30^\circ ; -60^\circ ; 120^\circ$	<ul style="list-style-type: none"> ✓ Co- ratio Ko-verhoud. ✓ ref angle verwys. \sphericalangle ✓ quadrant 1 kwadrant 1 ✓ quadrant 4 kwadrant 4 ✓ 30° ✓ -60° ✓ 120° 	<p>(7)</p>
<p>7.2</p>	 <ul style="list-style-type: none"> ✓ Shape of f. ✓ x intercepts ✓ start and end points Vorm van f x afsnitte begin en eindpunte ✓ Shape of g. ✓ x intercepts ✓ start and end points Vorm van g x afsnitte begin en eindpunte 		<p>(6)</p>
<p>7.3</p>	$-30^\circ < x < 30^\circ$ $\text{OR/OFF } 30^\circ < x < 90^\circ$ $\text{OR/OFF } 150^\circ < x < 180^\circ$	<ul style="list-style-type: none"> ✓✓ $-30^\circ < x < 30^\circ$ ✓ $30^\circ < x < 90^\circ$ ✓ $150^\circ < x < 180^\circ$ 	<p>(4)</p>
			<p>[17]</p>

QUESTION/VRAAG 8

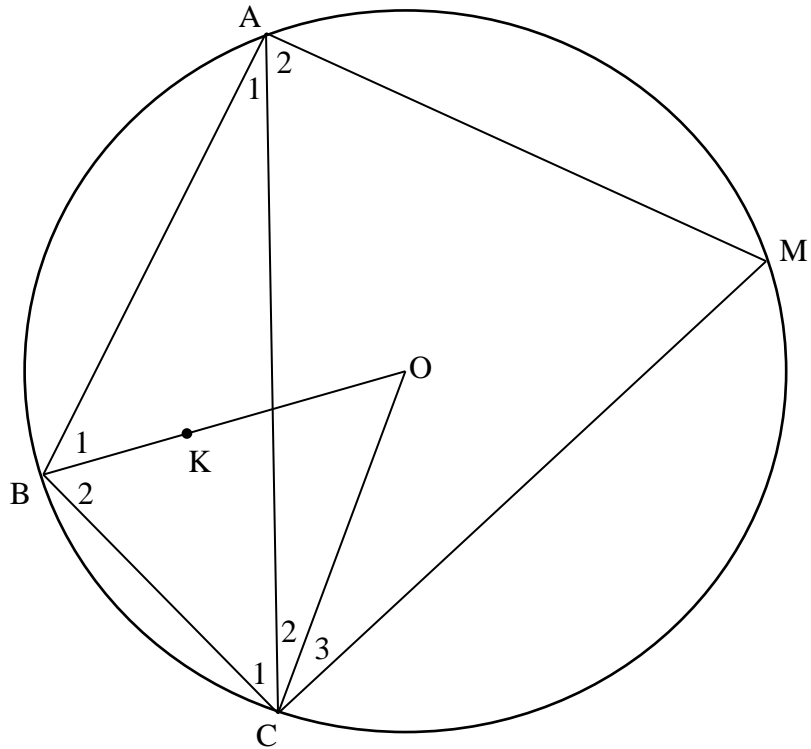


8.1	$\widehat{QTR} = \beta - \alpha$	✓ answer/antwoord	(1)
8.2	$\widehat{S} = 90^\circ - \beta$	✓ answer/antwoord	(1)
8.3	$\widehat{S} = 90^\circ - \alpha$	✓ answer/antwoord	(1)
8.4	In ΔRST $\frac{d}{\sin \alpha} = \frac{RT}{\sin(90^\circ - \beta)}$ $RT = \frac{d \cos \beta}{\sin \alpha}$	✓ use of sine rule <i>gebruik van sinusreël</i> ✓ use of co-function <i>gebruik van ko-funksie</i> ✓ answer/antwoord	(3)
8.5	$\frac{PR}{\sin \beta} = \frac{RT}{\sin(90^\circ - \alpha)}$ $PR = \frac{RT \sin \beta}{\cos \alpha}$ $PR = \frac{d \cos \beta \sin \beta}{\sin \alpha \cos \alpha}$	✓ use of sine rule <i>gebruik van sinusreël</i> ✓ sub of RT <i>vervanging van RT</i> ✓ answer/antwoord	(3)
			[9]

QUESTION/VRAAG 9

9.1	centre / middelpuntshoek	✓ answer/antwoord	(1)
9.2	$\widehat{BOC} = 2x$ [angle at centre = 2 × angle at circum] [Middelpuntshoek = 2 × Omtrekshoek] $\widehat{BCO} = \frac{x}{3}$ [angles opp = sides ; OB = OC] [hoeke teenoor = sye ; OB = OC] $\therefore \frac{x}{3} + \frac{x}{3} + 2x = 180^\circ$ [sum of angles of Δ] [som van hoeke van Δ] $8x = 540^\circ$ $x = 67,5^\circ$	✓ statement / stelling (S) ✓ reason/rede (R) ✓ statement / stelling (S) ✓ reason/rede (R) ✓ S/R ✓ answer/antwoord	(6)

9.3

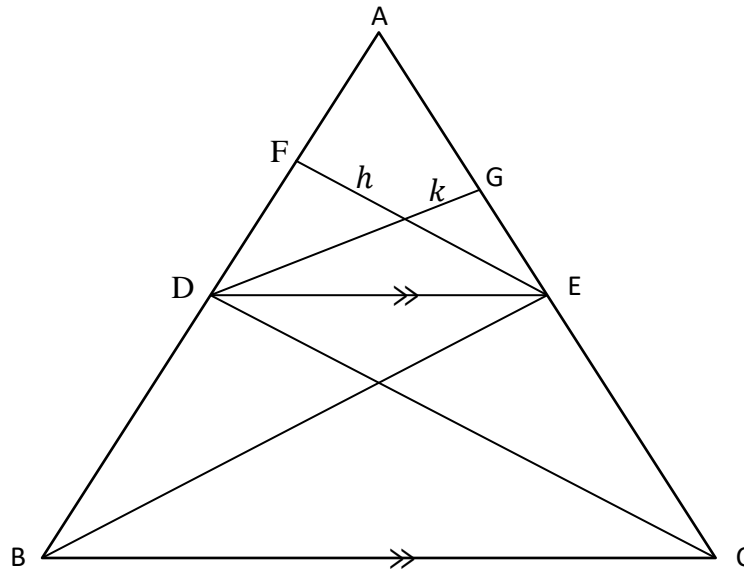


<p>9.3.1</p>	<p>$\hat{A}_1 = 30^\circ$ $\widehat{BOC} = 60^\circ$ [angle at centre = 2 × angle at circum] [middelpuntshoek = 2 × omtrekshoek] $\hat{B}_2 = \hat{C}_1 + \hat{C}_2$ [angles opp = sides ; OB = OC] [hoeke teenoor = sye ; OB = OC] $\therefore 2\hat{B}_2 = 180^\circ - 60^\circ$ [sum of angles of Δ] [som van hoeke van Δ] $\hat{B}_2 = 60^\circ$ $\hat{B}_1 = 60^\circ$ [BO bisects \widehat{ABC}] / [BO halveer \widehat{ABC}]</p>	<p>✓ S/R ✓ S/R ✓ S/R ✓ S ✓ S/R</p>	<p>(5)</p>
<p>9.3.2</p>	<p>$\hat{A}_1 = 30^\circ$ $\hat{B}_1 + \hat{B}_2 + \hat{M} = 180^\circ$ [opp angles of cyclic quad] [teenoorst. hoeke van koordevierhoek] $\hat{M} = 180^\circ - 120^\circ$ $\hat{M} = 60^\circ$ $\therefore \hat{M} = 2 \times \hat{A}_1$</p>	<p>✓ S/R ✓ S ✓ answer/ antwoord</p>	<p>(3)</p>
			<p>[15]</p>

QUESTION/VRAAG 10

<p>10.1</p>	<p>$\widehat{R}_1 = \widehat{R}_2$ [YR bisects $\widehat{P\hat{R}Q}$] / [YR <i>halveer</i> $\widehat{P\hat{R}Q}$] $\widehat{R}_2 + \widehat{R}_3 = \widehat{Y}_2$ [angles opp = sides; RS = YS] [hoeke teenoor = sye ; RS = YS] $\widehat{R}_1 + \widehat{P} = \widehat{Y}_2$ [ext angle of Δ]/[buitehoek van Δ] $\therefore \widehat{R}_3 = \widehat{P}$ \therefore SR = tangent/raaklyn [converse tan – chord theorem] [omgekeerde raaklyn-koord stelling]</p>	<p>✓ S/R ✓ S ✓ R ✓ S ✓ R ✓ S</p>	<p>(6)</p>
<p>10.2</p>	<p>$\widehat{Q}_2 = \widehat{R}_3$ [alt angles = ; SR \parallel QX] [verw. hoeke = ; SR \parallel QX] $\widehat{P} = \widehat{R}_3$ [proven] / [bewys] $\widehat{Q}_2 = \widehat{P}$ \therefore QR = tangent [converse tan – chord theorem] QR = raaklyn [omgekeerde raaklyn-koord stelling]</p>	<p>✓ S/R ✓ S ✓ S</p>	<p>(3)</p>
			<p>[9]</p>

QUESTION/VRAAG 11



11.1

Proof / Bewys:
 Construct Perpendicular heights DG (*k*) and EF (*h*) in $\triangle ADE$. Join BE and DC.
*Teken loodregte hoogtes DG (*k*) en EF (*h*) in $\triangle ADE$
 Verbind BE en DC*

$$\frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle BDE} = \frac{\frac{1}{2} AD \cdot h}{\frac{1}{2} BD \cdot h} = \frac{AD}{BD}$$

$$\frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle CED} = \frac{\frac{1}{2} AE \cdot k}{\frac{1}{2} CE \cdot k} = \frac{AE}{CE}$$

But/*Maar* Area $\triangle BDE$ = Area $\triangle CED$ (same base DE, same height)
 (*dieselfde basis DE, dieselfde hoogte*)
 DE \parallel BC

$$\frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle BDE} = \frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle CED}$$

$$\therefore \frac{AD}{BD} = \frac{AE}{CE}$$

✓ constr.
 konstruk.

✓ S/R

✓ S/R

✓ S ✓ R

✓ S

(6)

<p>11.2</p>			
<p>11.2.1</p>	<p>In ΔKOP and/en ΔKLM $\hat{K} = \hat{K}$ [common] / [gemeen] $\hat{O}_1 = \hat{L}$ [corresponding angles = ; $OP \parallel LM$] [ooreenkomstige hoeke = ; $OP \parallel LM$] $\hat{P}_1 = \hat{M}$ [corresponding angles = , $OP \parallel LM$] [ooreenkomstige hoeke = ; $OP \parallel LM$] $\therefore \Delta KOP \parallel \Delta KLM$ [A; A; A]</p>	<p>✓ S/R ✓ S/R ✓ R</p>	<p>(3)</p>
<p>11.2.2</p>	<p>$\frac{KO}{KL} = \frac{OP}{LM}$ [similarity : <i>gelykvormig</i>] $\frac{KO}{KL} = \frac{KY}{KX}$ [line parallel to one side of Δ] [lyn ewewydig aan een sy van Δ] $\therefore \frac{KY}{KX} = \frac{OP}{LM}$</p>	<p>✓ S/R ✓ S/R</p>	<p>(2)</p>
<p>11.2.3</p>	<p>Area of ΔKOP = Area of Quad/<i>Vierhoek</i> OLMP \therefore Area of $\Delta KLM = 2 \times$ Area of ΔKOP $\frac{1}{2} \times LM \times KX = 2 \times \frac{1}{2} \times OP \times KY$ $\frac{1}{2} = \frac{OP \cdot KY}{LM \cdot KX}$ but/maar $\frac{OP}{LM} = \frac{KY}{KX}$ $\frac{OP^2}{LM^2} = \frac{1}{2}$ $\frac{OP}{LM} = \frac{1}{\sqrt{2}}$ $\frac{OP}{LM} = \frac{KO}{KL}$ [$\Delta KOP \parallel \Delta KLM$] $\therefore \frac{KO}{KL} = \frac{1}{\sqrt{2}}$</p>	<p>✓ S ✓ S ✓ S ✓ S ✓ S/R</p>	<p>(6)</p>
			<p>[17]</p>
<p>TOTAL/TOTAAL:</p>			<p>150</p>