higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS
(First Paper)
NQF LEVEL 3

NOVEMBER 2012

(10501053)

30 October (X-Paper)
09:00 – 12:00

Scientific calculators may be used.

This question paper consists of 6 pages and a formula sheet.

TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc, which you have used in determining the answers.
5. If necessary, answers should be rounded off to THREE decimal places, unless stated otherwise.
6. Diagrams are NOT drawn to scale.
7. Write neatly and legibly.

QUESTION 1

1.1 Simplify the following and write your answer in the form $a + ib$

1.1.1 $(-3+2j)-(-2+5j)$ (2)

1.1.2 $(-3+j^3)(2-3j^3)$ (4)

1.1.3 $(\sqrt{2}-\sqrt{-16})(\sqrt{2}+\sqrt{-9})$ (3)

1.1.4 $(-1+i)(3+4i)(3i)$ (3)

1.2 Simplify the following expression. Leave your answer in polar form

$$\frac{1.25cis35^0 \times 3.1cis45^0}{2cis60^0}$$ (3)

1.3 Given: $z = -4-7i$

1.3.1 Write down the conjugate of z (1)

1.3.2 Represent z and its conjugate on an Argand diagram (4)

1.3.3 Calculate the modulus (r) of $z = -4-7i$ (2)

1.3.4 Calculate the value of the argument (θ) of $z = -4-7i$ (3)

1.4 Without the use of a calculator and without conversion to polar form, simplify the given complex number to standard form:

$$\frac{4+5j}{-3-2j}$$ (5)

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QUESTION 2

2.1 Solve for x

$2+x-x^2 \leq 0$ (3)

2.2 Solve for a by completing a square: $-2a^2+11a+6=0$ (4)

2.3 Solve for x and y

$y = 3x + 2$

$y = 3x^2 + 6x - 4$ (6)

2.4 Simplify the following:

2.4.1 $\left(1 - \frac{x}{y}\right) \div \left(\frac{x - y}{y}\right)$ (3)

2.4.2 $\frac{2}{(x-y)^2} - \frac{3}{y^2 - x^2}$ (4)

2.5 An entrepreneur sells T-shirts and trousers at the flea market. Suppose x T-shirts and y trousers are sold every week, and that the following constraints are applicable:

$10 \leq x \leq 40$

$x + y \leq 60$

$15 \leq y \leq 45$

$2y + x \geq 60$

2.5.1 Sketch the graph with the given constraints (6)

2.5.2 Determine the feasible region (1)

2.5.3 Determine how many of each can be sold every week so that the maximum profit can be made if $P = 3x + 5y$ (3)

[30]

QUESTION 3

3.1 Given the functions $f(x) = -x^2 - 2x + 3$ and $g(x) = -2.2^{x-1} + 1$:

3.1.1 Draw neat graphs of the above functions on the same set of axes. Clearly indicate the intercepts with the axes. (7)

3.1.2 Write down the axis of symmetry of $f(x)$ (1)

3.1.3 What is the range of $g(x)$ (1)

3.2 Given $f(x)=\frac{6}{x-2}+1$

3.2.1 Determine the asymptote(s) of $f(x)$ (2)

3.2.2 For which value(s) of x is the graph of f(x) undefined? (1)

3.2.3 Draw the graph of $f(x)$ (4)

3.2.4 Determine the value of y at a point where $x=0$ on the graph of $f(x)$ (2)

3.2.5 What is the domain and the range of $f(x)$ (2)

[20]

QUESTION 4

4.1 Find the derivate of $f(x)=2x^2+1$ from first principles (4)

4.2 Determine the following: $\lim_{x\to 3}\frac{\left(3x^2+13x-66\right)}{x-3}$ (3)

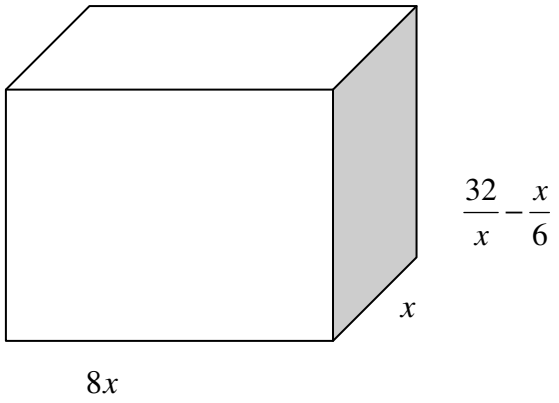
4.3 Determine $\frac{dy}{dx}$ of the following. Leave answers with positive exponents and in surd form where applicable.

4.3.1 $y=\sqrt{x^3}-\frac{1}{3x}+qx^2$ (3)

4.3.2 $y=\frac{3}{x}\left(\frac{4p}{x}-5\sqrt{x}\right)+6p^2$ where p is a constant (4)

4.3.3 $xy=5$ (2)

4.4 The base of a rectangular box has dimensions of $8x$ cm and x cm. The height of the box is given as $\left(\frac{32}{x}-\frac{x}{6}\right)$. If the volume of the box is $V(x)=8x\times x\times\left(\frac{32}{x}-\frac{x}{6}\right)$



Calculate the dimension of the box that would give a maximum volume. (4)
[20]

TOTAL: 100

FORMULAE SHEET

1. $z = r\cos\theta + r\ j\sin\theta$

2. $z = a \pm bj$ or $z = a \pm bi$ *where* $i = j = \sqrt{-1}$

3. $r = \sqrt{a^2 + b^2}$ or $r = \sqrt{z \times \overline{z}}$

4. $\alpha = \tan^{-1}\left(\frac{b}{a}\right)$

5. $r \angle\theta = r\ cis\ \theta$

6. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

7. $y = ax^2 + bx + c$

8. $y = a(x - p)^2 + q$

9. $y = a(x - x_1)(x - x_2)$

10. $y = \frac{a}{(x + p)} + q$

11. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

12. $\frac{d}{dx}x^n = nx^{n-1}$

13. $\frac{d}{dx}k = 0$

14. $Dx[kf(x)] = kDx[f(x)]$

15. $Dx[f(x) \pm g(x)] = Dx[f(x)] \pm Dx[g(x)]$