higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE (VOCATIONAL)

MATHEMATICS
(First Paper)
NQF LEVEL 3

NOVEMBER EXAMINATION

(10501053)

31 October 2014 (X-Paper)
09:00–12:00

This question paper consists of 6 pages, 1 formula sheet and 3 diagram sheets.

TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Write your EXAMINATION NUMBER in the space provided on the DIAGRAM SHEETS (attached) and put them inside your ANSWER BOOK.
5. Write neatly and legibly.

QUESTION 1

1.1 Determine $f'(x)$ from the first principles if $f(x) = -2 - 3x$ (4)

1.2 Determine the following:

$$\lim_{x \to -2} \frac{6x^2 + 10x - 4}{2x + 4}$$ (3)

1.3 Use differentiation rules to determine $\frac{dy}{dx}$ of the following. (Leave your answer with a POSITIVE exponent and in SURD form where applicable.)

1.3.1 $y = \frac{x^3 - x^2 + 8x}{-2x}$ (3)

1.3.2 $y = -3x(2x-1)^2$ (3)

1.3.3 $y = \frac{x^3}{3} + \frac{1}{3x^3}$ (3)

1.4 The dimensions of a cylinder are such that $r + h = 12$.

Determine the value of r if the volume of the cylinder is at its maximum.

Hint: $V = \pi r^2 h$ (4)
[20]

QUESTION 2

2.1 Without the use a calculator and without conversion to polar form, simplify the given complex numbers to the standard form of $a + bi$:

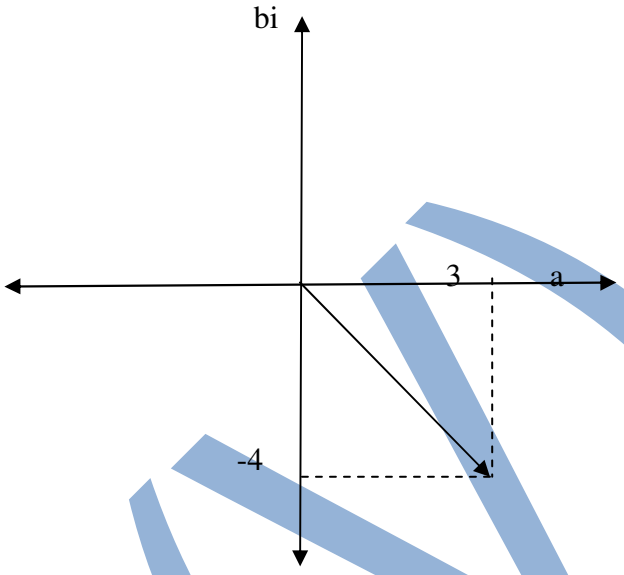
2.1.1 $j(j+2)^2 - (2-j)^2$ (4)

2.1.2 $(1-i)(1+i)(2-3i)$ (3)

2.1.3 $i(2 + 3i^7) + (-2 - i)$ (4)

2.1.4 $(-i)(4 + 2i + i^2) + 3i + 2$ (4)

2.2 Given:



2.2.1 Express the above diagram in the form $a+bi$ (2)

2.2.2 Write down the conjugate of z. (1)

2.2.3 Calculate the modulus (r) and argument (θ) of $\bar{z}$. (3)

2.2.4 Express $\bar{z}$ in polar form. (1)

2.3 Simplify the following complex numbers and leave the answer in standard form, $a+bi$:

2.3.1 $$\frac{4-3j+2+6j-1}{(3-j)(3-j)}$$ (5)

2.3.2 $6(\cos 30°+i\sin 30°)\times 3(\cos 10^0+i\sin 10^0)$ (3)

[30]

QUESTION 3

3.1 Given:

$$f(x)=\frac{2}{x-4}+1 \text{ and } g(x)=\left(\frac{1}{4}\right)^x+1$$

3.1.1 Write down the asymptotes of $f(x)$ (2)

3.1.2 Calculate the x and y intercepts of $f(x)$ (3)

3.1.3 Draw graphs of $f(x)$ and $g(x)$ on the same system of axis on DIAGRAM SHEET 1 (attached). Clearly indicate the intercept and asymptotes. (6)

3.2 Given $y=-x^2+2x+3$

3.2.1 Draw a neat graph of $y=-x^2+2x+3$ on DIAGRAM SHEET 2 (attached) and show all calculated values.

Hint: *x-intercepts, y-intercepts* and turning point (7)

3.2.2 Write down the range of $y=-x^2+2x+3$ (1)

3.3 Lesedi is a small company that makes two types of cards, type A and type B. With the available labour and material, the following are the constraints inequalities:

$x+y\leq 200$
$x\geq 40$
$y\geq 10$
$x\leq 150$
$y\leq 120$

3.3.1 Sketch the graph with the given constraints on DIAGRAM SHEET 3 (attached). (5)

3.3.2 Determine the feasible region. (1)

3.3.3 Find the values of x and y which will maximize and minimize the objective equation: P = 5x + 10y (4)

3.4 Solve for x:

$$(x-1)(x-2)\leq 6$$ (4)

3.5 Solve for x by completing a square:

$$x^2-10x=24$$ (3)

3.6 Solve for x and y:

$$2x - y = 1$$
$$y - x^2 = -3x + 3$$

(5)

3.7 Simplify the following:

$$\frac{x^2 - 9}{-3x - 9} \times \frac{6x^3 - 2x^2}{6 - 2x}$$

(4)

3.8 Simplify the following:

$$\frac{5}{x+1} + \frac{x-1}{x^2+1}$$

(3)

3.9 Factorize fully:

$$d^2 - 3d + 8d - 24$$

(2)
[50]

TOTAL: 100

FORMULA SHEET

1. $z = r\cos\theta + r\ j\sin\theta$

2. $z = a \pm bj$ or $z = a \pm bi$ *where* $i = j = \sqrt{-1}$

3. $r = \sqrt{a^2 + b^2}$ or $r = \sqrt{z \times \bar{z}}$

4. $\alpha = \tan^{-1}\left(\frac{b}{a}\right)$

5. $r \angle\theta = r\ cis\ \theta$

6. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

7. $y = ax^2 + bx + c$

8. $y = a(x - p)^2 + q$

9. $y = a(x - x_1)(x - x_2)$

10. $y = \frac{a}{(x + p)} + q$

11. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

12. $\frac{d}{dx} x^n = nx^{n-1}$

13. $\frac{d}{dx} k = 0$

14. $Dx[kf(x)] = kDx[f(x)]$

15. $Dx[f(x) \pm g(x)] = Dx[f(x)] \pm Dx[g(x)]$

DIAGRAM SHEET 1

EXAMINATION NUMBER:

QUESTION 3

3.1.3

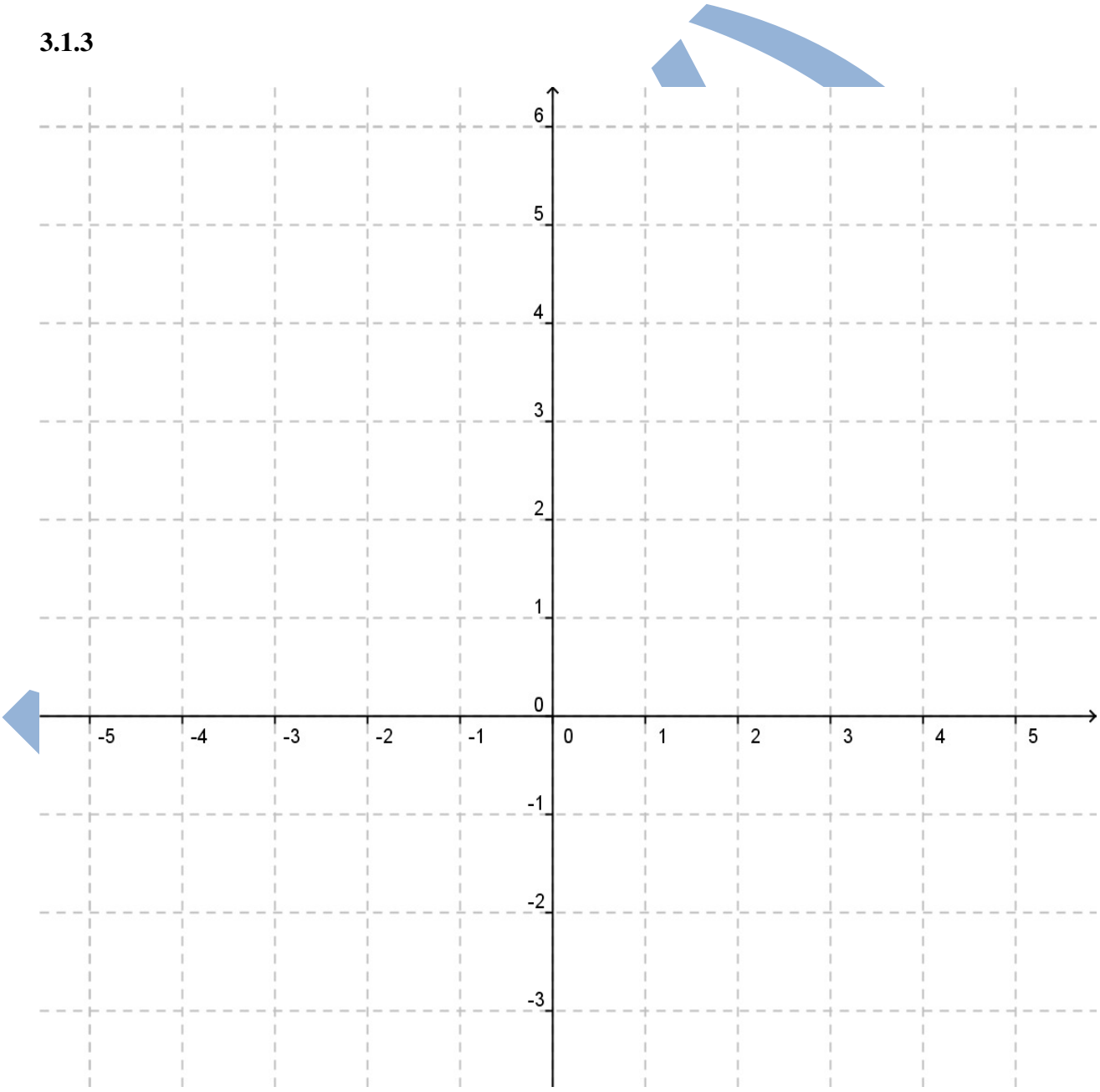


DIAGRAM SHEET 2

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QUESTION 3

3.2.1

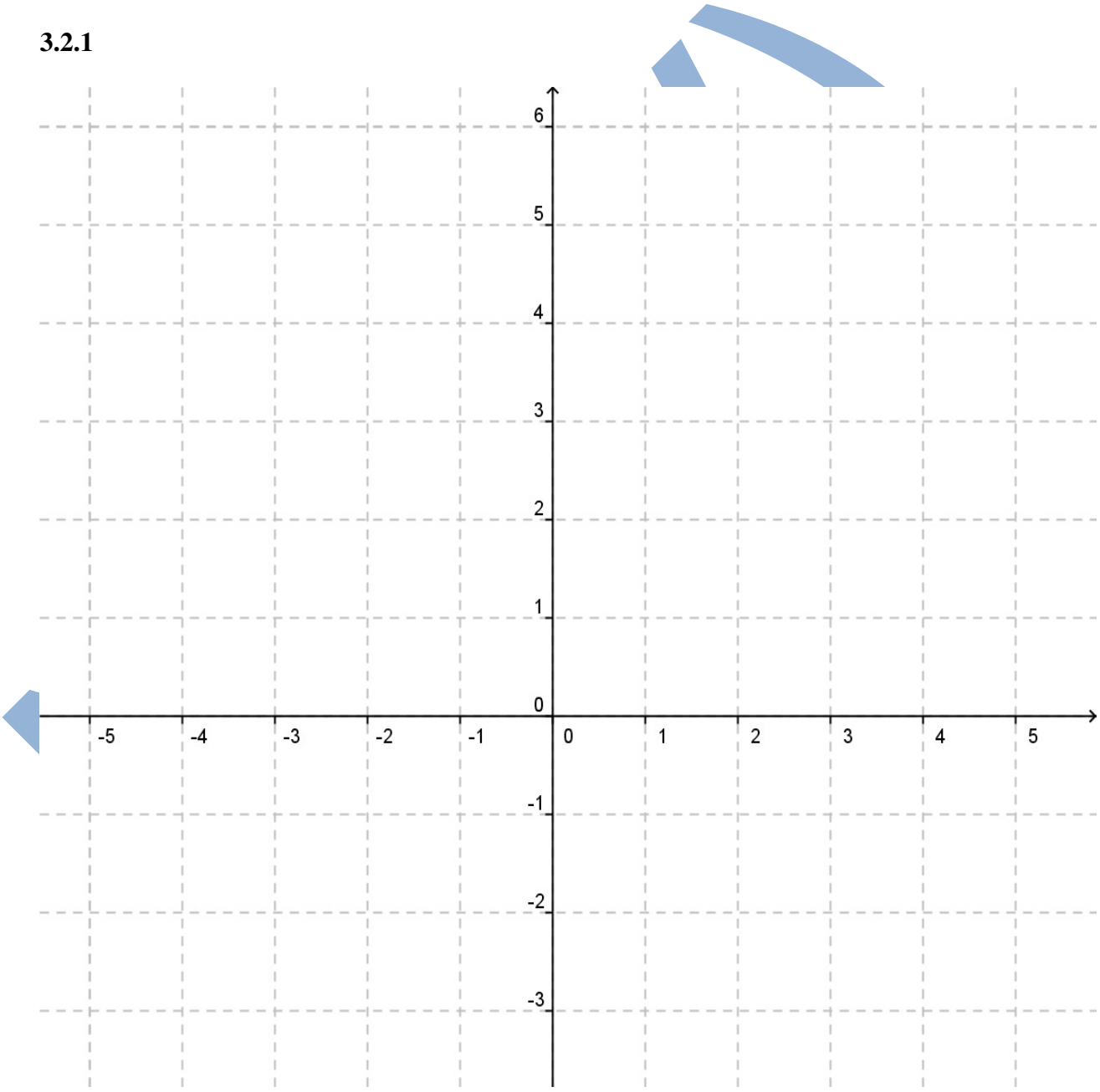

DIAGRAM SHEET 3

EXAMINATION NUMBER:

QUESTION 3

3.3.1