



**NATIONAL
SENIOR CERTIFICATE/
NASIONALE
SENIOR SERTIFIKAAT**

GRADE/GRAAD 11

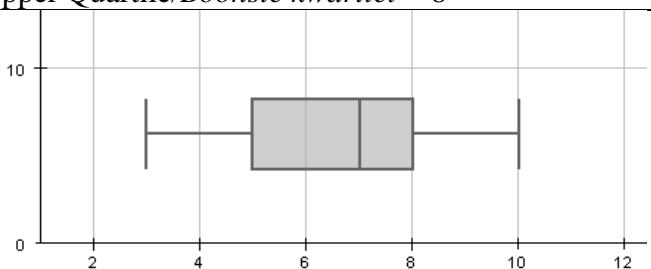
NOVEMBER 2021

**MATHEMATICS P2/WISKUNDE V2
MARKING GUIDELINE/NASIENRIGLYN**

MARKS/PUNTE: 150

This marking guideline consists of 16 pages./
Hierdie nasienriglyn bestaan uit 16 bladsye.

QUESTION 1/VRAAG 1

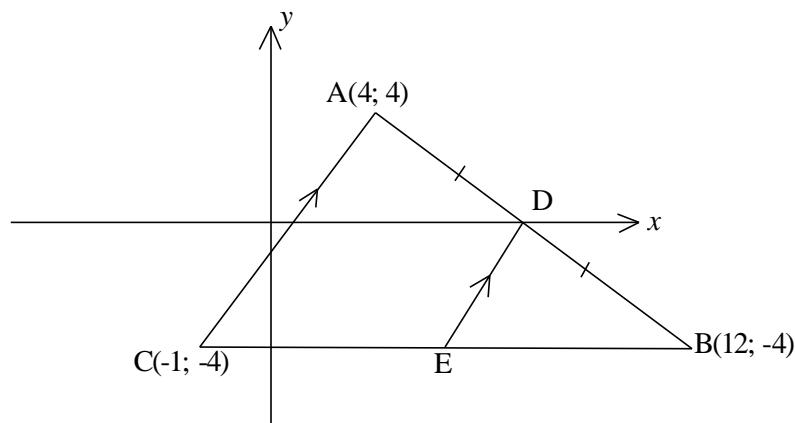
1.1	$\bar{x} = 6,4$	✓ answer / antwoord	(1)
1.2	Min/Min = 3; Max./Maks. = 10; Lower Quartile/Onderste kwartiel = 5; Median/Mediaan = 7; Upper Quartile/Boonste kwartiel = 8	✓ for both min. and max. <i>vir beide min. en maks.</i> ✓ for the rest / vir die res	(2)
1.3		✓ for the box / <i>vir die mond</i> ✓ whiskers / <i>snorre</i>	(2)
1.4	Skewed to the left, mean is smaller than the median OR longer rectangle on the left. <i>Skeef na links, Gemiddelde is kleiner as mediaan OF langer reghoek op linkerkant.</i>	✓ answer / antwoord ✓ reason / rede	(2)
1.5	Standard deviation/Standaardafwyking = 2,01	✓✓ answer / antwoord	(2)
1.6	Slept well, if bigger than 8,61. / <i>Het goed geslaap, indien groter as 8,61.</i> Answer 1 learner./ Antwoord 1 leerder	✓ 8,61 ✓ answer / antwoord	(2)

[11]

QUESTION 2/VRAAG 2

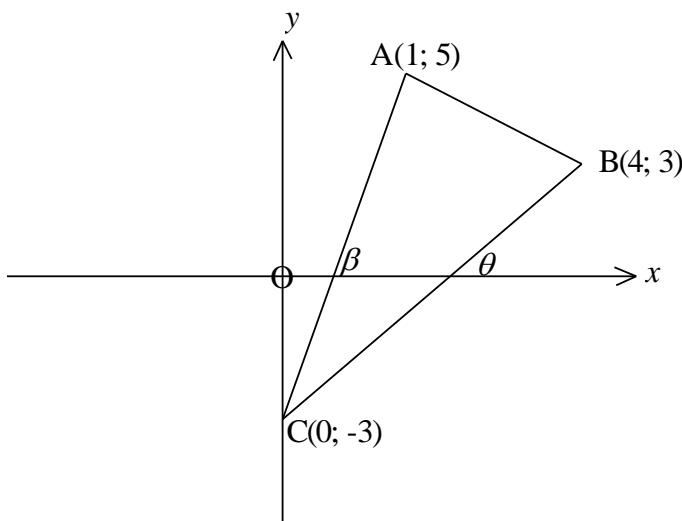
2.1	<table border="1"> <thead> <tr> <th>Age <i>Ouderdom</i></th><th>Frequency <i>Frekvensie</i></th><th>Cumulative Frequency <i>Kumulatiewe Frekvensie</i></th></tr> </thead> <tbody> <tr><td>$25 < A \leq 30$</td><td>2</td><td>2</td></tr> <tr><td>$30 < A \leq 35$</td><td>8</td><td>10</td></tr> <tr><td>$35 < A \leq 40$</td><td>4</td><td>14</td></tr> <tr><td>$40 < A \leq 45$</td><td>5</td><td>19</td></tr> <tr><td>$45 < A \leq 50$</td><td>11</td><td>30</td></tr> <tr><td>$50 < A \leq 55$</td><td>19</td><td>49</td></tr> <tr><td>$55 < A \leq 60$</td><td>20</td><td>69</td></tr> <tr><td>$60 < A \leq 65$</td><td>6</td><td>75</td></tr> </tbody> </table>	Age <i>Ouderdom</i>	Frequency <i>Frekvensie</i>	Cumulative Frequency <i>Kumulatiewe Frekvensie</i>	$25 < A \leq 30$	2	2	$30 < A \leq 35$	8	10	$35 < A \leq 40$	4	14	$40 < A \leq 45$	5	19	$45 < A \leq 50$	11	30	$50 < A \leq 55$	19	49	$55 < A \leq 60$	20	69	$60 < A \leq 65$	6	75	<ul style="list-style-type: none"> ✓ for first 4 <i>vir eerste 4</i> ✓ last 4 <i>laaste 4</i> 	(2)
Age <i>Ouderdom</i>	Frequency <i>Frekvensie</i>	Cumulative Frequency <i>Kumulatiewe Frekvensie</i>																												
$25 < A \leq 30$	2	2																												
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$55 < A \leq 60$	20	69																												
$60 < A \leq 65$	6	75																												
2.2	<p style="text-align: center;">Ages of Teachers <i>Ouderdomme van Onderwysers</i></p>	<ul style="list-style-type: none"> ✓ start point <i>beginpunt</i> ✓ end point <i>eindpunt</i> ✓ shape <i>vorm</i> 	(4)																											
2.3	Median / <i>Mediaan</i> = 52 (Accept/Aanvaar 51 – 53)	✓✓ answer / <i>antwoord</i>	(2)																											
2.4	Percentage / <i>Persentasie</i> = $\frac{75 - 57}{75} = \frac{18}{75} = 24\%$	<ul style="list-style-type: none"> ✓ calculation / <i>berekening</i> ✓ answer / <i>antwoord</i> 	(2)																											
			[10]																											

QUESTION 3/VRAAG 3



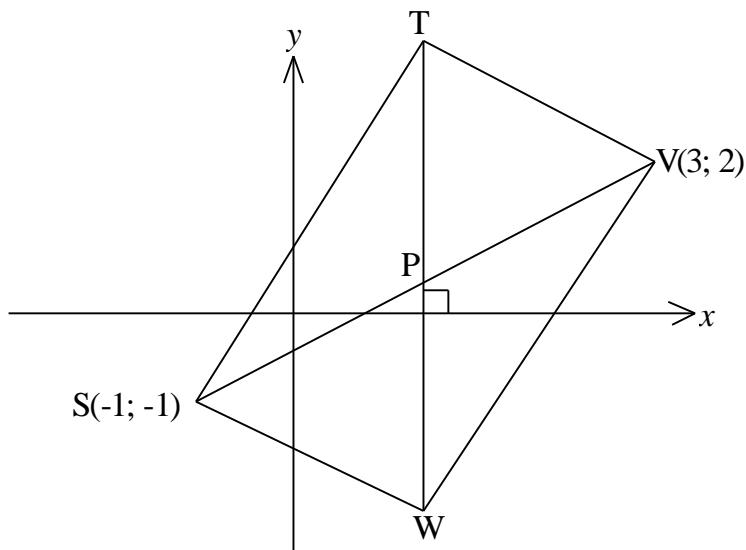
3.1	$AB = \sqrt{(12 - 4)^2 + (-4 - 4)^2}$ $AB = 11,31$ units / eenhede	✓ substitution / vervanging ✓ answer / antwoord	(2)
3.2	$D\left(\frac{4+12}{2}; \frac{4-4}{2}\right)$ $= D(8; 0)$	✓ substitution / vervanging ✓ answer / antwoord	(2)
3.3	$m = \frac{4 - (-4)}{4 - (-1)} = \frac{8}{5}$ $y = mx + c$ $0 = \frac{8}{5} \times 8 + c$ $c = -\frac{64}{5}$ $\therefore y = \frac{8}{5}x - \frac{64}{5}$	$m = \frac{4 - (-4)}{4 - (-1)} = \frac{8}{5}$ $y - y_1 = m(x - x_1)$ $y - 0 = \frac{8}{5}(x - 8)$ $y = \frac{8}{5}x - \frac{64}{5}$	✓ gradient / gradiënt ✓ substitution / vervanging ✓ y-intercept / y-afsnit ✓ answer / antwoord
3.4	$y = \frac{8}{5}x - \frac{64}{5}$ $-4 = \frac{8}{5}x - \frac{64}{5}$ $\therefore x = \frac{11}{5}$ $E\left(\frac{11}{2}; -4\right)$ <p style="text-align: center;">OR / OF</p> $E\left(\frac{11}{2}; -4\right)$ (Midpoint Theorem) / (Middelpunt-Stelling)	✓ substitution -4 vervanging -4 ✓ value of x waarde van x ✓ coordinates of E koördinate van E OR / OF ✓ x – value / waarde ✓ y – value / waarde ✓ reason / rede	(3) [11]

QUESTION 4/VRAAG 4



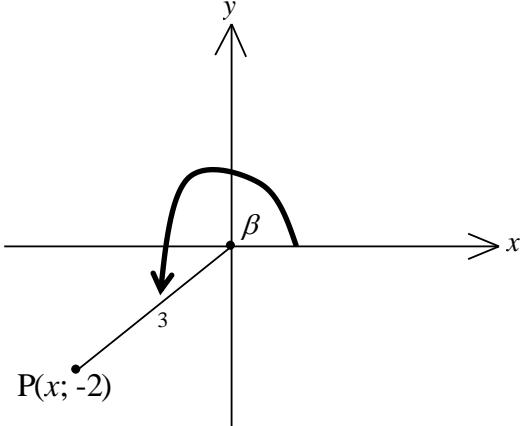
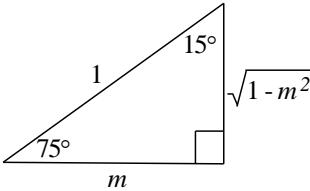
4.1	D(-3; -1)	✓ x-coordinate / koördinaat ✓ y-coordinate / koördinaat	(2)	
4.2	$\text{FC} = \sqrt{(8-0)^2 + (p+3)^2} = 12$ $64 + p^2 + 6p + 9 = 144$ $p^2 + 6p - 71 = 0$ <p>Using the quadratic formula:</p> $p = 5,94 \text{ or } p = -11,94$ $p = 6 \text{ or } p = -12$	✓ substitution into the formula and equating to 12 <i>vervanging in die formule en stel gelyk aan 12</i> ✓ squaring both sides <i>kwadreer albei kante</i> ✓ standard form/standaardvorm ✓ $p = 6$ ✓ $p = -12$	(5)	
4.3	$m_{BC} = \frac{3 - (-3)}{4 - 0} = \frac{3}{2}$ $\therefore \tan \theta = \frac{3}{2}$ $\therefore \theta = 56,31^\circ$	$\tan \beta = 8$ $\therefore \beta = 82,87^\circ$ $\therefore \widehat{ACB} = 82,87^\circ - 56,3^\circ$ $\therefore \widehat{ACB} = 26,56^\circ$	✓ gradient of BC ✓ $\tan \theta = \frac{3}{2}$ ✓ for θ ✓ for β ✓ for \widehat{ACB}	(5)
			[12]	

QUESTION 5/VRAAG 5



$P\left(\frac{-1+3}{2}; \frac{-1+2}{2}\right)$ $= P\left(1; \frac{1}{2}\right)$ $SV = \sqrt{(3+1)^2 + (2+1)^2}$ $SV = 5 \text{ units / eenhede}$ $TW = 5 \text{ units / eenhede (diagonals of a rectangle)}$ $\quad \quad \quad (\text{hoeklyne van 'n reghoek})$ $T(1; 3)$ $W(1; -2)$	✓ coordinates of P <i>koördinate van P</i> ✓ substitution / <i>vervanging</i> ✓ for/vir SV ✓ for/vir TW ✓ coordinates of T <i>koördinate van T</i> ✓ coordinates of W <i>koördinate van W</i>	(6)
		[6]

QUESTION 6/VRAAG 6

6.1	$\sin \beta = -\frac{2}{3}$  $\begin{aligned}x^2 &= (3)^2 - (-2)^2 \\x &= -\sqrt{5} \\\therefore 1 + \tan^2 \beta &= 1 + \left(\frac{-2}{-\sqrt{5}}\right)^2 \\&= \frac{9}{5}\end{aligned}$	✓ for solving for sin oplos vir sin ✓ sketch in the correct quadrant skets in die korrekte kwadrant ✓ value of x/ waarde van x ✓ substitution / vervanging ✓ answer / antwoord	(5)
6.2			
6.2.1	$\begin{aligned}\cos^2 105^\circ &= [\cos(180^\circ - 75^\circ)]^2 \\&= (-\cos 75^\circ)^2 \\&= m^2\end{aligned}$	✓ for/vir - cos 75° ✓ for/vir m^2	(2)
6.2.2	$\begin{aligned}\sin 15^\circ &= \cos 75^\circ \\&= m\end{aligned}$	✓ for/vir $\cos 75^\circ$ ✓ for/vir m	(2)
6.2.3	$\tan 15^\circ = \frac{m}{\sqrt{1-m^2}}$ OR / OF $\tan 15^\circ = \frac{\sin 15^\circ}{\cos 15^\circ} = \frac{\cos 75^\circ}{\sin 75^\circ} = \frac{m}{\sqrt{1-m^2}}$	✓✓ for correct answer only vir korrekte antwoord (accuracy / akkuraatheid)	(2)

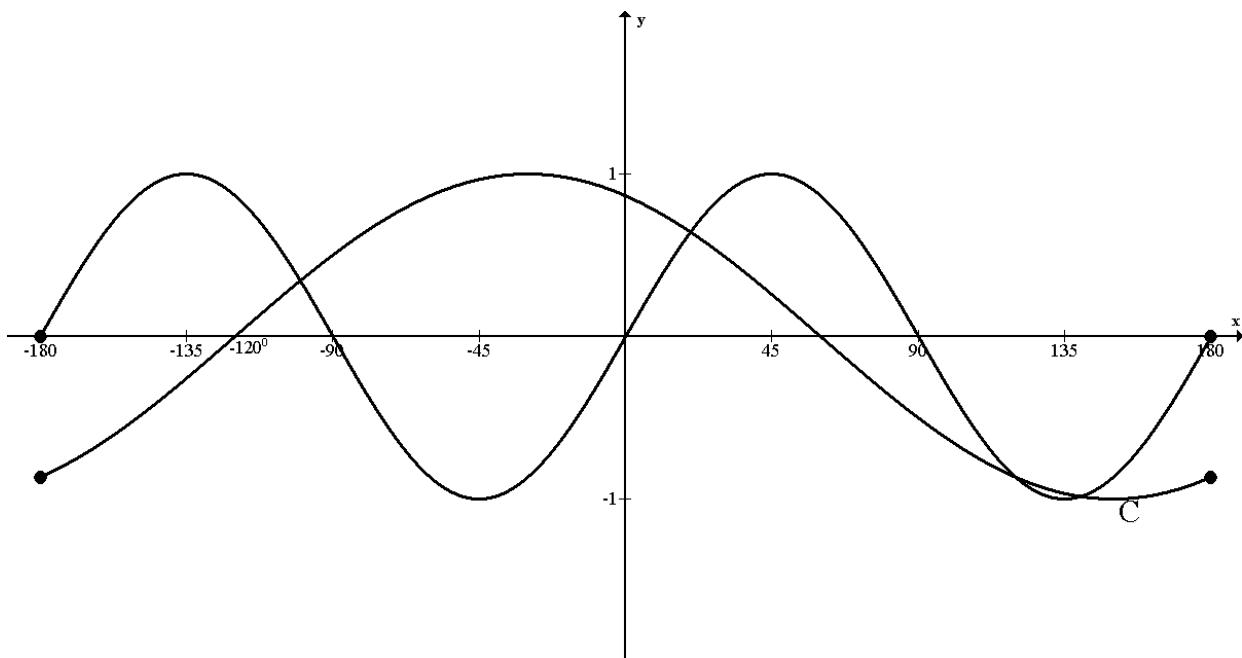
6.3.1	$\begin{aligned} & \frac{\cos(180^\circ - k) \cdot \sin(k - 90^\circ) - 1}{\tan^2(540^\circ + k) \cdot \sin(90^\circ + k) \cdot \cos(-k)} \\ &= \frac{-\cos k \cdot -\cos k - 1}{\tan^2 k \cdot \cos k \cdot \cos k} \\ &= \frac{\cos^2 k - 1}{\frac{\sin^2 k}{\cos^2 k} \cdot \cos^2 k} \\ &= \frac{-\sin^2 k}{\sin^2 k} \\ &= -1 \end{aligned}$	✓ for/vir $-\cos k$ ✓ for/vir $-\cos k$ ✓ for/vir $\tan^2 k$ ✓ for/vir $\cos k$ ✓ for/vir $\cos k$ ✓ for changing $\tan^2 k$ <i>vir verandering van $\tan^2 k$</i> ✓ for answer / vir antwoord	(7)
6.3.2	$\begin{aligned} \tan^2 k \cdot \cos^2 k &= 0 \\ \tan^2 k = 0 \text{ or/of } \cos^2 k &= 0 \\ \tan k = 0 \text{ or/of } \cos k &= 0 \\ \therefore k = 0^\circ \text{ or/of } k = 90^\circ \text{ or/of } k &= 180^\circ \\ \text{or/of } k &= 270^\circ \text{ or/of } k = 360^\circ \end{aligned}$	✓ $\tan^2 k \cdot \cos^2 k = 0$ $\tan^2 k = 0$ or/of $\cos^2 k = 0$ ✓ $\tan k = 0$ or/of $\cos k = 0$ ✓ $k = 0^\circ ; k = 90^\circ$ ✓ $k = 180^\circ ; k = 270^\circ$ ✓ $k = 360^\circ$	(6)
6.4	$\begin{aligned} \frac{1+\sin\theta}{1-\sin\theta} - \frac{1-\sin\theta}{1+\sin\theta} &= \frac{4\tan\theta}{\cos\theta} \\ \text{LHS/LK} &= \frac{1+\sin\theta}{1-\sin\theta} - \frac{1-\sin\theta}{1+\sin\theta} \\ &= \frac{1+2\sin\theta+\sin^2\theta-1+2\sin\theta-\sin^2\theta}{(1-\sin\theta)(1+\sin\theta)} \\ &= \frac{4\sin\theta}{1-\sin^2\theta} \\ &= \frac{4\sin\theta}{\cos^2\theta} \\ &= \frac{4\tan\theta}{\cos\theta} = \text{RHS/RK} \end{aligned}$	✓ for/vir LCD/KGD ✓ for/vir $1+2\sin\theta+\sin^2\theta$ ✓ for/vir $-1+2\sin\theta-\sin^2\theta$ ✓ for/vir $\frac{4\sin\theta}{1-\sin^2\theta}$ ✓ for/vir $\cos^2\theta$	(5)

6.5	$6\sin^2 \theta + \cos \theta = 4$ $6(1 - \cos^2 \theta) + \cos \theta = 4$ $6 - 6\cos^2 \theta + \cos \theta - 4 = 0$ $-6\cos^2 \theta + \cos \theta + 2 = 0$ $6\cos^2 \theta - \cos \theta - 2 = 0$ $(3\cos \theta - 2)(2\cos \theta + 1) = 0$ $\cos \theta = \frac{2}{3} \quad \text{or/of} \quad \cos \theta = -\frac{1}{2}$ $\theta = 48,19^\circ + 360^\circ \cdot k \quad \text{or/of} \quad \theta = 311,81^\circ + 360^\circ \cdot k$ <p style="text-align: center;">OR/OF</p> $\theta = 120^\circ + 360^\circ \cdot k \quad \text{or/of} \quad \theta = 240^\circ + 360^\circ \cdot k$ <p>where/waar $k \in \mathbb{Z}$</p>	<ul style="list-style-type: none"> ✓ for <i>vir</i> $1 - \cos^2 \theta$ ✓ for the two general solutions/ <i>vir die twee algemene oplossings</i> ✓ for standard form/ <i>vir standaardvorm</i> ✓ for factors / <i>vir faktore</i> ✓ for the other two general solutions/ <i>vir die ander twee algemene oplossings</i> ✓ for answers of $\cos \theta = \frac{2}{3}$ <i>vir antwoorde van</i> $\cos \theta = \frac{2}{3}$ ✓ for answers of $\cos \theta = -\frac{1}{2}$ <i>vir antwoorde van</i> $\cos \theta = -\frac{1}{2}$ 	(7)
6.6	$p = \tan A + \sin A \quad \text{and/en} \quad q = \tan A - \sin A$ $pq = (\tan A + \sin A)(\tan A - \sin A)$ $pq = \tan^2 A - \sin^2 A$ $pq = \frac{\sin^2 A}{\cos^2 A} - \sin^2 A$ $pq = \frac{\sin^2 A - \sin^2 A \cdot \cos^2 A}{\cos^2 A}$ $pq = \frac{\sin^2 A(1 - \cos^2 A)}{\cos^2 A}$ $pq = \tan^2 A \cdot \sin^2 A$	<ul style="list-style-type: none"> ✓ for substitution <i>vir vervanging</i> ✓ for multiplication <i>vir vermenigvuldiging</i> ✓ for changing $\tan^2 A$ <i>vir verandering van</i> $\tan^2 A$ ✓ for the numerator <i>vir die teller</i> ✓ for the common factor <i>vir die gemene faktor</i> 	(5)
			[41]

QUESTION 7/VRAAG 7

7.1	Exterior angle of a triangle / Buitehoek van 'n driehoek	✓ answer / antwoord	(1)
7.2	$\frac{PS}{\sin 29^\circ} = \frac{10}{\sin 11^\circ}$ $PS = \frac{10 \cdot \sin 29^\circ}{\sin 11^\circ}$ $PS = 25,41 \text{ m}$	✓ for sub. in sine rule <i>vir verv. in sinus-reël</i> ✓✓ for the answer <i>vir die antwoord</i>	(3)
7.3	$PQ^2 = (30)^2 + (25,41)^2 - 2 \times 30 \times 25,41 \times \cos 40^\circ$ $PQ = 19,44 \text{ m}$ $\frac{\sin Q_1}{25,41} = \frac{\sin 40^\circ}{19,44}$ $\sin Q_1 = \frac{25,41 \times \sin 40^\circ}{19,44}$ $\therefore \hat{Q}_1 = 57,16^\circ$ $\therefore \theta = 32,84^\circ$	✓ for sub. in the cosine rule <i>verv. in die cosinus-reël</i> ✓ for the answer PQ <i>vir die antwoord PQ</i> ✓ for using the sine rule <i>vir gebruik van die sinus-reël</i> ✓ for/vir \hat{Q}_1 ✓ for/vir θ	(5)
		[9]	

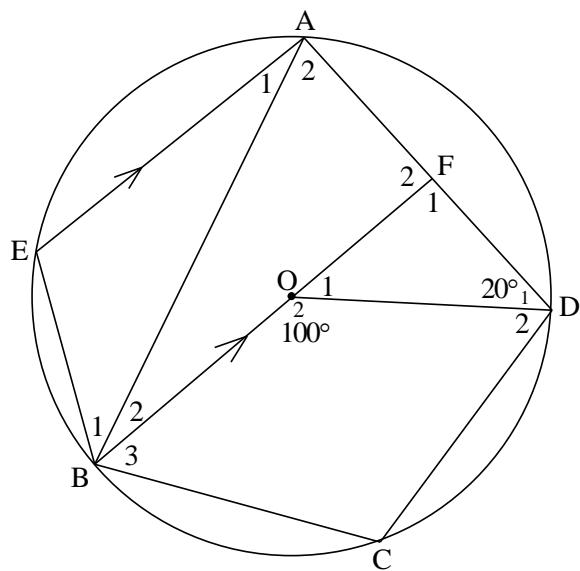
QUESTION 8 / VRAAG 8



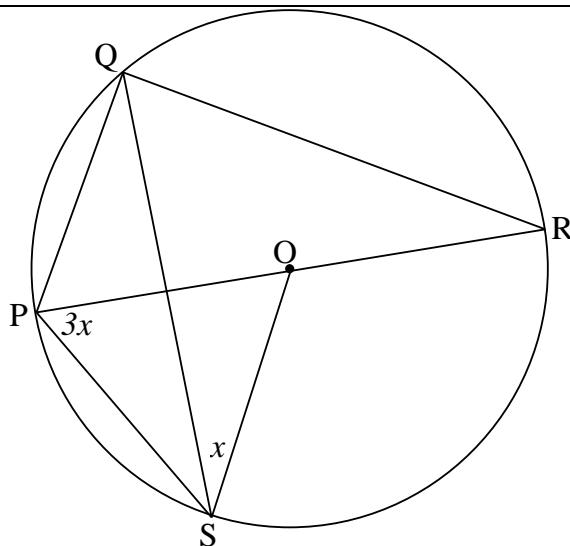
8.1	$a = 2 \quad b = -30^\circ$	✓ for/vir a ✓ for/vir b	(2)
8.2	$C(150^\circ; -1)$	✓ for/vir 150° ✓ for/vir -1	(2)
8.3	$-120^\circ \leq x \leq -90^\circ$	✓ for/vir -120° ✓ for/vir -90° ✓ for correct notation <i>vir korrekte notasie</i>	(3)
8.4	$f(x) = \sin 2(x - 30^\circ)$	✓ for correct answer <i>vir korrekte antwoord</i>	(1)

[8]

QUESTION 9/VRAAG 9

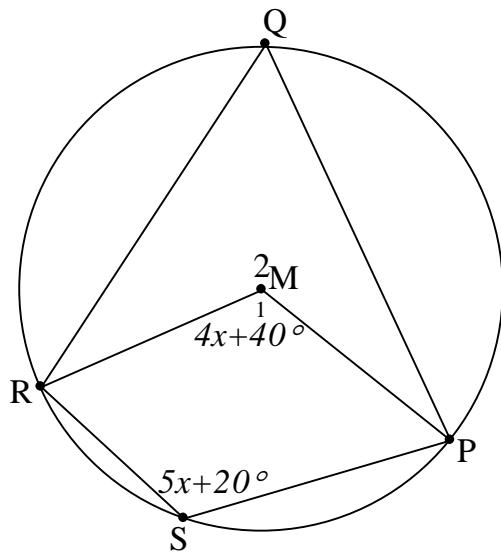


9.1.1	< at centre = 2 x angle at the circumference <i>Middelpunts ∠ = 2 × Omtreks ∠</i>	✓ for the answer <i>vir die antwoord</i>	(1)
9.1.2	<'s on a straight line <i>∠'e op 'n reguitlyn</i>	✓ for the answer <i>vir die antwoord</i>	(1)
9.1.3	Exterior angle of a triangle / <'s of a triangle <i>Buitehoek van 'n driehoek / ∠e van 'n driehoek</i>	✓ for the answer <i>vir die antwoord</i>	(1)
9.1.4	$\hat{A} = \hat{F}_1$; corresponding angles = ; EA BOF <i>ooreenkomsige ∠e = ; EA BOF</i>	✓ for the answer <i>vir die antwoord</i>	(1)
9.1.5	alternate angles = ; EA BOF <i>verwisselende hoeke = ; EA BOF</i>	✓ for the answer <i>vir die antwoord</i>	(1)



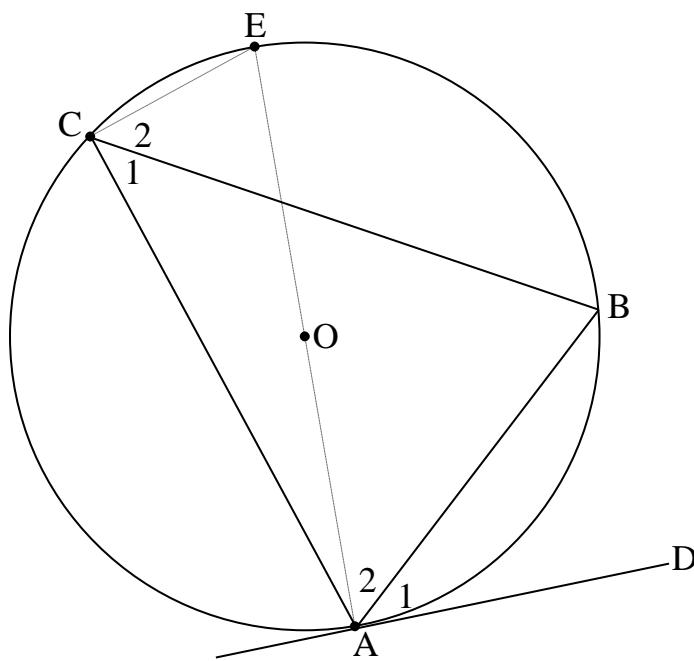
9.2.1	$S\hat{Q}R = 3x$ (<s in the same segment) ($\angle e$ in dieselfde segment)	✓ S ✓ R	(2)
9.2.2	$\widehat{Q} = 90^\circ$ (<s in a semi-circle) ($\angle e$ in 'n halwe-sirkel) $\therefore P\hat{Q}S = 90^\circ - 3x$	✓ S ✓ R ✓ answer / antwoord	(3)
9.2.3	$P\hat{S}O = 3x$ (<s opposite equal sides) ($\angle e$ teenoor gelyke sye) $\therefore P\hat{S}Q = 2x$ ($Q\hat{S}O = x$)	✓ S ✓ R ✓ answer / antwoord	(3)
9.2.4	$P\hat{R}Q = 2x$ (<s in the same segment) ($\angle e$ in dieselfde segment)	✓ S ✓ R	(2)
9.2.5	$Q\hat{P}R = 180^\circ - (2x + 90^\circ)$ (<s of a triangle) ($\angle e$ van 'n driehoek) $= 90^\circ - 2x$	✓ S and/en R ✓ answer / antwoord	(2)
			[17]

QUESTION 10/VRAAG 10



10.1	$\hat{Q} = 2x + 20^\circ$ (< at the centre = $2 \times$ angle at the circumf.) <i>(Middelpunts∠ = 2 × Om trekshoek)</i> $\therefore 2x + 20^\circ + 5x + 20^\circ = 180^\circ$ (opp. <s of a c.q.) <i>(teenoorst. ∠e van 'n k.v.)</i> $7x + 40^\circ = 180^\circ$ $7x = 140^\circ$ $\therefore x = 20^\circ$ $\therefore \hat{Q} = 60^\circ$ OR / OF $\hat{M}_2 = 360^\circ - (4x + 40^\circ)$ (<s around a point) <i>(∠e rondom 'n punt)</i> $= 320^\circ - 4x$ $320^\circ - 4x = 2(5x + 20^\circ)$ (< at the centre) / (<i>Middelpunts ∠</i>) $320^\circ - 4x = 10x + 40^\circ$ $14x = -280^\circ$ $\therefore x = 20^\circ$ $\therefore \hat{Q} = 60^\circ$	✓ for/vir S ✓ for/vir R ✓ for S and R <i>vir S en R</i> ✓ for simplifying <i>vereenvoudiging</i> ✓ the answer <i>die antwoord</i> ✓ for S and R <i>vir S en R</i> ✓ for the answer <i>vir die antwoord</i> ✓ for S and R <i>vir S en R</i> ✓ for simplifying <i>vereenvoudiging</i> ✓ for the answer <i>vir die antwoord</i>	(5)
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10.2



Construct diameter AE and join C to E.

Label \hat{C}_1 , \hat{C}_2 and \hat{A}_2

$\hat{C}_1 + \hat{C}_2 = 90^\circ$ (< in a semi - circle)

$\hat{A}_1 + \hat{A}_2 = 90^\circ$ (tan \perp rad)

But $\hat{C}_2 = \hat{A}_2$ (<s in the same segment)

$\therefore \hat{A}_1 = \hat{C}_1$

✓ for construction

✓ for S ✓ for R

✓ for S ✓ for R

✓ for S and R

Teken Middellyn AE en verbind C aan E.

Merk \hat{C}_1 , \hat{C}_2 en \hat{A}_2

$\hat{C}_1 + \hat{C}_2 = 90^\circ$ (\angle in 'n halwe - sirkel)

$\hat{A}_1 + \hat{A}_2 = 90^\circ$ (raaklyn \perp radius)

Maar $\hat{C}_2 = \hat{A}_2$ (\angle e in dieselfde segment)

$\therefore \hat{A}_1 = \hat{C}_1$

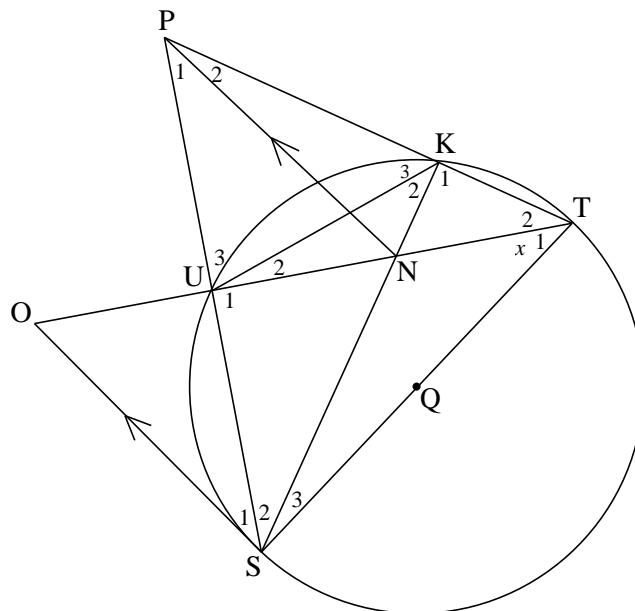
✓ vir konstruksie

✓ vir S ✓ vir R

✓ vir S ✓ vir R

✓ vir S en R

(6)



10.3.1	$\hat{U}_1 = 90^\circ$ ($< s$ in a semi - circle) $(\angle e$ in 'n halwe - sirkel) $\hat{K}_1 = 90^\circ$ ($< s$ in a semi - circle) $(\angle e$ in 'n halwe - sirkel) $\hat{K}_3 + \hat{K}_2 = 90^\circ$ ($< s$ on a straight line) $(\angle e$ op 'n reguitlyn) $\therefore \hat{U}_1 = \hat{K}_2 + \hat{K}_3 = 90^\circ$ $\therefore \text{PUNK is a c.q. (conv. exterior } < \text{ of a c.q)}$ $\text{PUNK is 'n k.v. (omgekeerde Buite}\angle \text{ van k.v)}$	✓ for/vir S and/en R ✓ for/vir S and/en R ✓ for/vir S and/en R ✓ for/vir S ✓ for/vir R	(5)
10.3.2	$\hat{T}_2 = x$ (TO bisects \widehat{SPT}) / (TO halveer \widehat{SPT}) $\hat{T}_1 = \hat{K}_2 = x$ ($< s$ in the same segment) $(\angle e$ in dieselfde segment) $\hat{K}_2 = \hat{P}_1 = x$ ($< s$ in the same segment) $(\angle e$ in dieselfde segment) $\hat{P}_1 = \hat{S}_1 = x$ (alt. $< s$; $PN \parallel OS$) / (Verw. $\angle e$; $PN \parallel OS$) $\therefore \hat{S}_1 = \hat{T}_1 = x$ $\therefore \text{SO is a tangent (conv. tan-chord theorem)}$ SO is 'n raaklyn (omgekeerde raaklyn-koord stelling)	✓ for S /vir S/✓ and/en R ✓ for S /vir S /and/en R ✓ S and/en R ✓ for conclusion vir gevolgtrekking ✓ for R /vir R	(6)
10.3.3	$\hat{T}_1 = \hat{T}_2 = x$ (proven/bewys) $\hat{S}_1 = \hat{T}_2 = x$ (proven/bewys) $\therefore \text{POST is a c.q. (conv. } < s \text{ in the same segment)}$ $\text{POST is 'n k.v. (omgekeerde } \angle e \text{ in dieselfde segment)}$	✓ for/vir S and/en R ✓ for S /vir S and/en R ✓ for R/vir R	(3)

[25]

TOTAL/TOTAAL: 150