



**NATIONAL  
SENIOR CERTIFICATE**

**GRADE 11**

**NOVEMBER 2022**

**TECHNICAL MATHEMATICS P2**

**MARKS: 150**

**TIME: 3 hours**

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This question paper consists of 16 pages, including a 2-page information sheet.

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**INSTRUCTIONS AND INFORMATION**

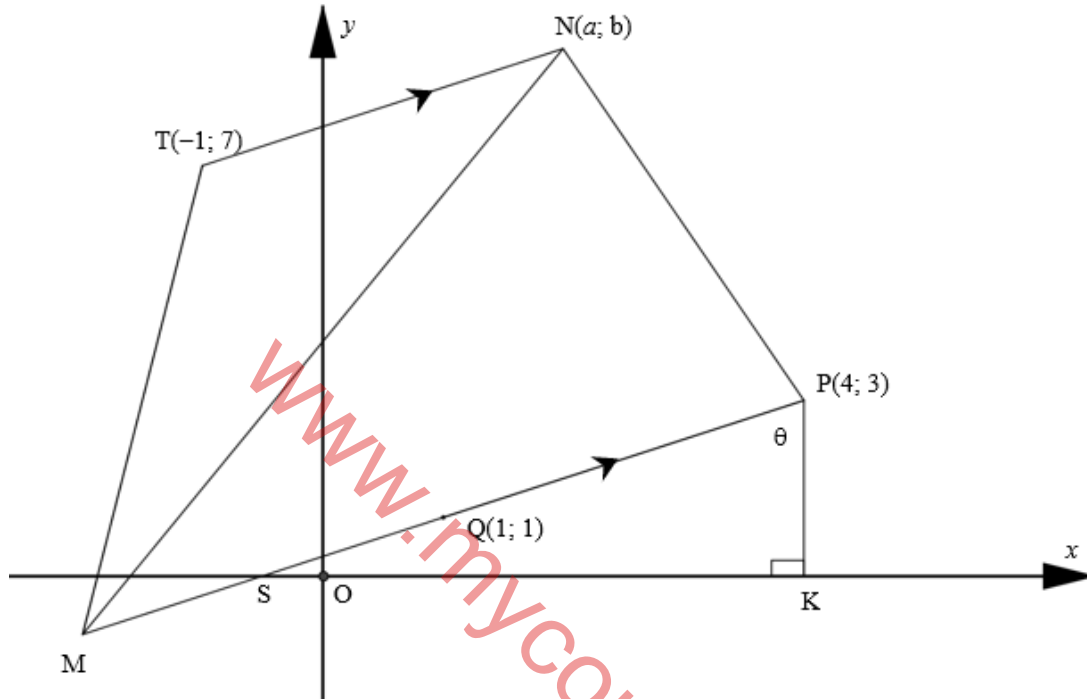
Read the following instructions carefully before answering the questions.

1. This question paper consists of TEN questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used to determine your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical) unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

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**QUESTION 1**

In the diagram below M, T(-1;7), N(a;b) and P(4;3) are vertices of a trapezium MTNP having  $TN \parallel MP$ . Q(1;1) is the midpoint of MP. PK is a vertical line with  $\widehat{SPK} = \theta$ . The equation of NP is  $y = -3x + 15$ .



- 1.1 Write down the coordinates of K. (2)
- 1.2 Determine the coordinates of M. (6)
- 1.3 Determine the gradient of PM. (3)
- 1.4 Calculate the size of  $\theta$ . (5)
- 1.5 Hence, or otherwise, determine the length of PS. (3)
- 1.6 Determine the coordinates of N. (7)

**[26]**

**QUESTION 2**

2.1 Determine the value of the following:

2.1.1  $51,5 \cos 18^\circ \cdot \sin 58^\circ$  (1)

2.1.2  $\frac{1,28 \cot 32,3^\circ \cdot \tan 81,5^\circ}{\sec 16,1^\circ \cdot \operatorname{cosec} 41,8^\circ}$  (2)

2.2 Consider  $5 \cos \theta = 3$  and  $0^\circ < \theta < 90^\circ$ .

Determine the value of the following, WITHOUT the use of a calculator, but with the aid of a diagram:

2.2.1  $\sin \theta \cdot \sec \theta$  (5)

2.2.2  $\frac{\tan \theta}{\cot \theta}$  (3)

2.2.3 Determine the size of  $\theta$ , with the use of a calculator. (3)

2.3 Solve for  $\theta \in [0^\circ; 360^\circ]$ , rounded off to ONE decimal digit.

$3 \sin \theta = -1,026$  (5)  
[19]

**QUESTION 3**

3.1 Complete the following identities:

3.1.1  $1 - \sin^2 x = \dots$  (1)

3.1.2  $\sec^2 x - \tan^2 x = \dots$  (1)

3.2 Simplify:  $\frac{\sin(\pi - \theta) \cdot \tan \theta \cdot \sin 270^\circ}{\cos(2\pi - \theta) \cdot \tan(\pi - \theta)}$  (5)

3.3 Prove the identity:  $\sin x + \cot x \cdot \cos x = \operatorname{cosec} x$  (4)  
[11]

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**QUESTION 4**

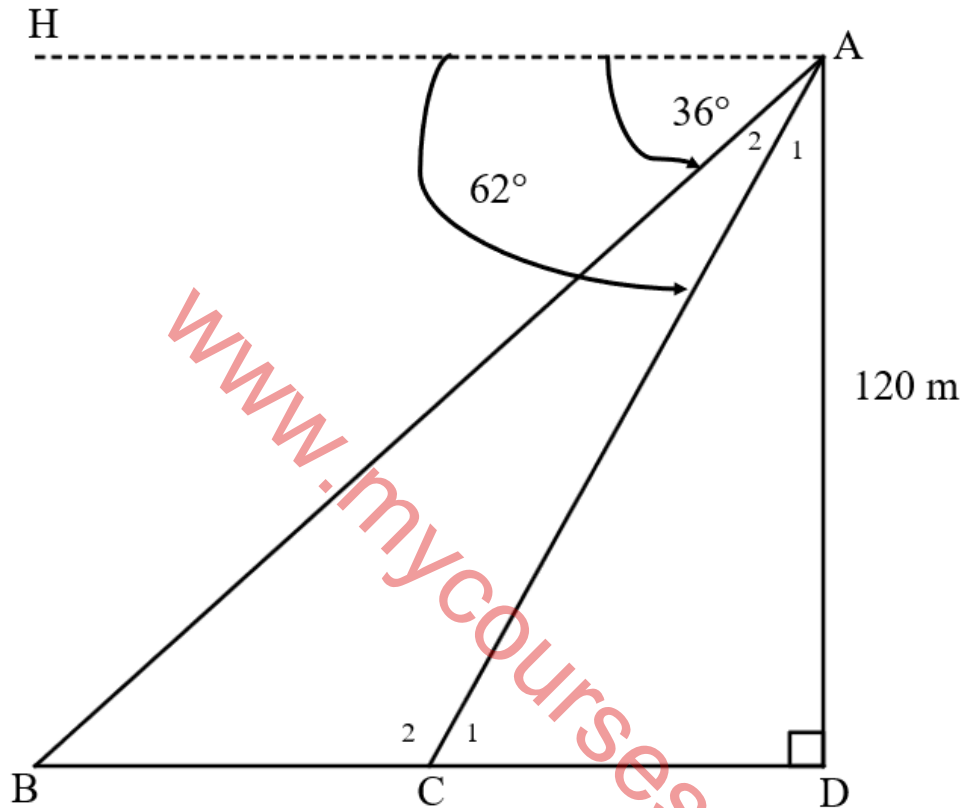
Given  $f(x) = \sin x + 2$  and  $g(x) = 3\cos x$  for  $x \in [0^\circ; 360^\circ]$ .

- 4.1 Use the set of axes provided in the SPECIAL ANSWER BOOK to draw sketch graphs of the curves of  $f$  and  $g$  for  $x \in [0^\circ; 360^\circ]$ . Clearly show ALL intercepts with the axes, coordinates of all turning points and end points of both curves. (6)
- 4.2 Use the graphs drawn in QUESTION 4.1, or otherwise, to determine the following:
- 4.2.1 The amplitude of  $g$  (1)
- 4.2.2 The value(s) of  $x \in [0^\circ; 360^\circ]$  for which  $g(x) \leq 0$ . (2)
- [9]**

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**QUESTION 5**

From the top of a cliff, 120 m above sea level, a person at point **A** notices two ships in the distance (at points **B** and **C** respectively). The angles of depression in the direction of the two ships are  $36^\circ$  and  $62^\circ$  respectively. The diagram below represents the above scenario.



- 5.1 Write down the size of  $\hat{B}$ . (2)
- 5.2 Write down the size of  $\hat{C}_1$ . (1)
- 5.3 Determine the length of AC. (3)
- 5.4 Hence, determine the distance between the two ships (BC). (5)
- 5.5 Determine the area of  $\Delta ABC$ . (3)
- [14]**

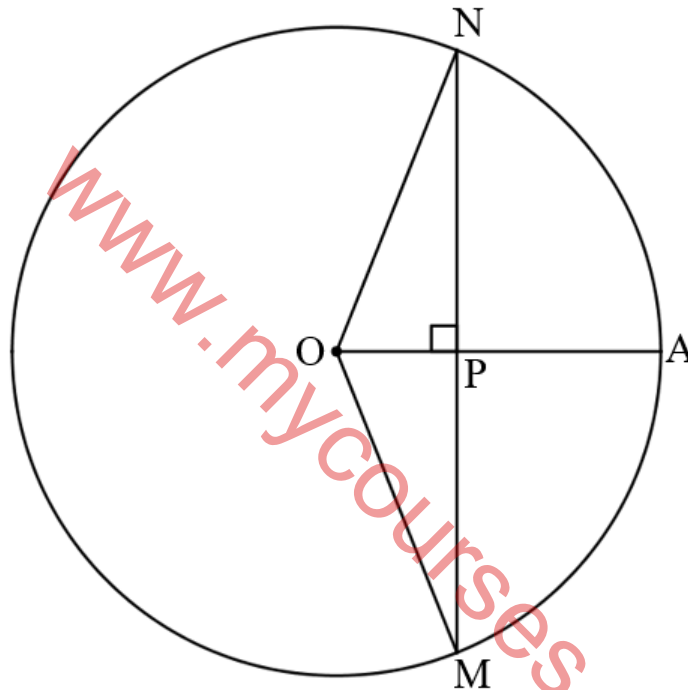
## QUESTION 6

6.1 Complete the following statement:

*“The line drawn from the centre of a circle perpendicular to the chord ... the chord.”*

(1)

6.2 The diagram below shows a circle with centre O.  $OPA \perp MPN$ ;  $MN = 48$  units and  $OP = 7$  units.



Determine, stating reasons, the length of PA.

(7)

[8]

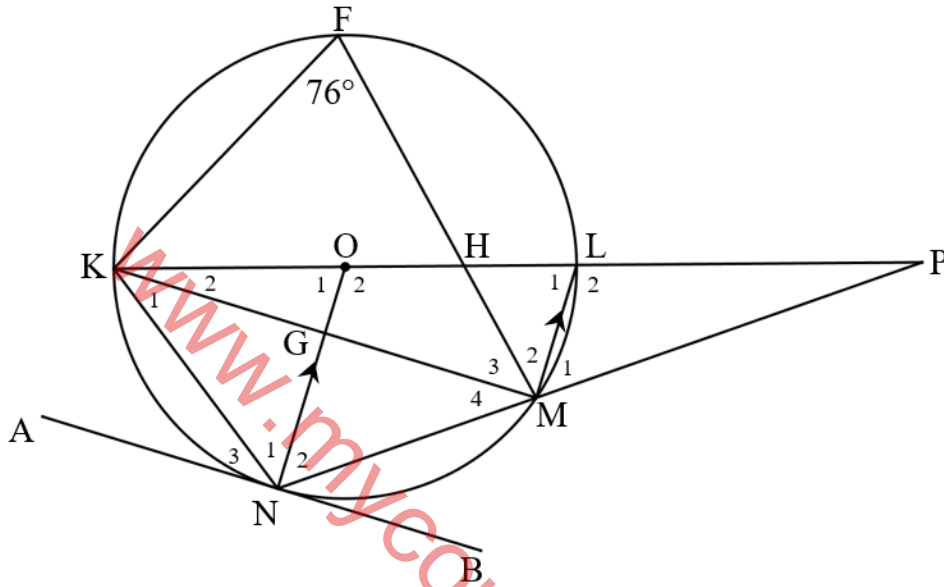


**QUESTION 7**

7.1 Complete the following statement:

“The angle subtended by the diameter at the circumference of the circle is ...” (1)

7.2 O is the centre of a circle, diameter KL and line NM are produced to meet at P.  $ON \parallel LM$ . ANB is a tangent to the circle at N and  $\hat{F} = 76^\circ$ .



Use the diagram above to identify angles that are related to the following angles. Find the sizes of these angles, giving reasons for your answers.

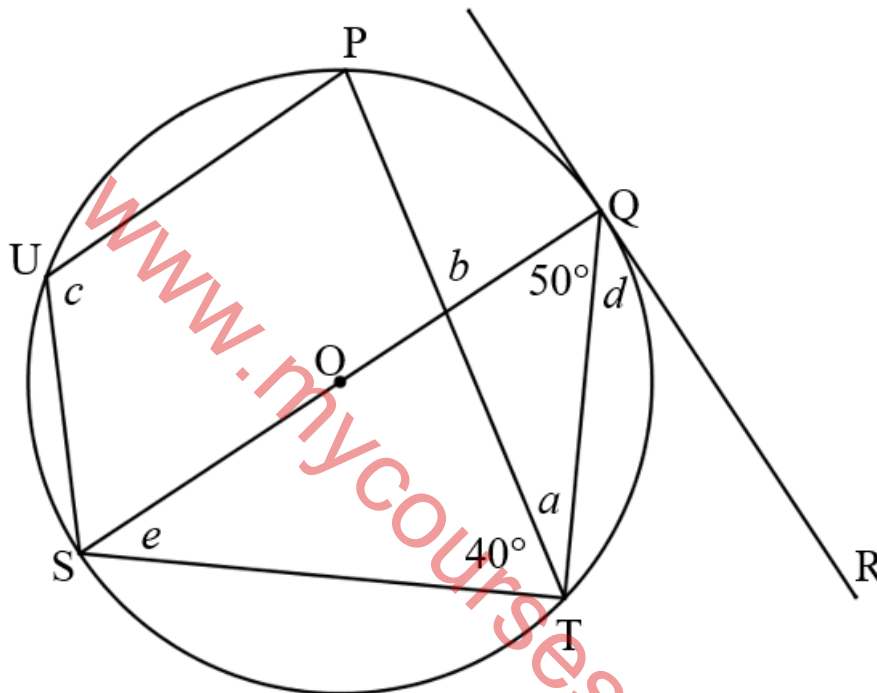
- 7.2.1  $\hat{L}_1$  (2)
  - 7.2.2  $\hat{O}_1$  (2)
  - 7.2.3  $\hat{M}_4$  (2)
  - 7.2.4  $\hat{N}_3$  (2)
  - 7.2.5  $\hat{N}_1 + \hat{N}_2$  (2)
  - 7.2.6  $\hat{N}_1 + \hat{N}_3$  (2)
  - 7.2.7  $\hat{M}_2 + \hat{M}_3$  (2)
  - 7.2.8  $\hat{M}_1$  (2)
- [17]**

**QUESTION 8**

8.1 Complete the following statement:

*“The angle between the tangent to a circle and the chord drawn from the point of contact is ... to the angle in the alternate segment.”* (1)

8.2 Refer to the diagram below. RQ is a tangent to the circle QTSUP with centre O. SOQ and PT are straight lines.  $\widehat{PTS} = 40^\circ$  and  $\widehat{SQT} = 50^\circ$ .



Determine, with reasons, the following:

- 8.2.1  $a$  (2)
- 8.2.2  $b$  (2)
- 8.2.3  $c$  (2)
- 8.2.4  $d$  (2)
- 8.2.5  $e$  (2)

**[11]**

**QUESTION 9**

9.1 If the area of a sector is  $8,5 \text{ cm}^2$  and the radius is  $2,1 \text{ cm}$ .

Calculate the following:

9.1.1 The angle of the sector to the nearest degrees (4)

9.1.2 The arc length of the sector (3)

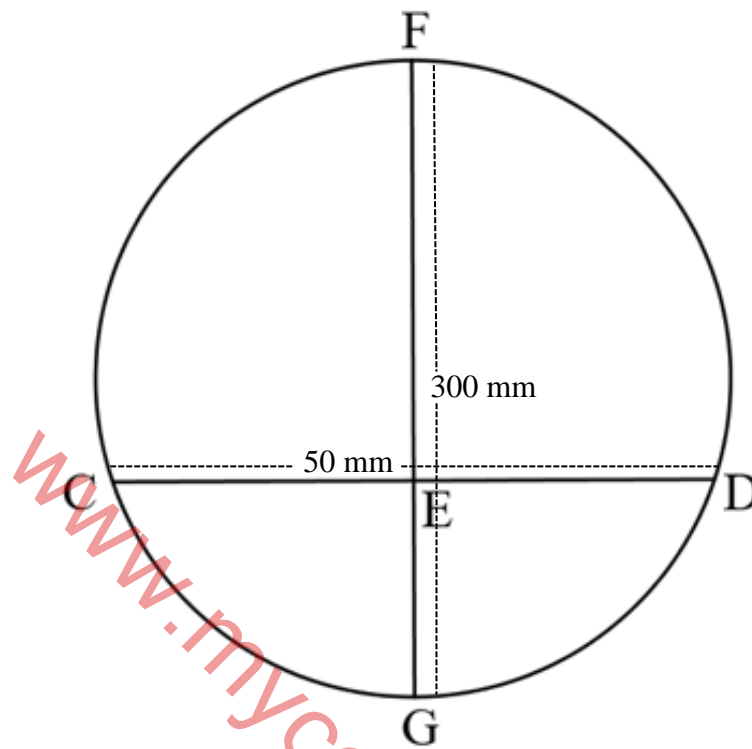
9.2 The diameter of a wheel is  $80 \text{ mm}$  and it turns at 21 revolutions per second.



9.2.1 Calculate the circumferential velocity of the wheel, to the nearest integer. (3)

9.2.2 Calculate the angular velocity of the wheel, to the nearest integer. (3)

- 9.3 In the diagram below,  $FG$  is the diameter of the circle, with length of 300 mm.  $CD$  is a chord of the circle with a length of 50 mm.  $CD$  divides the circle into two segments.

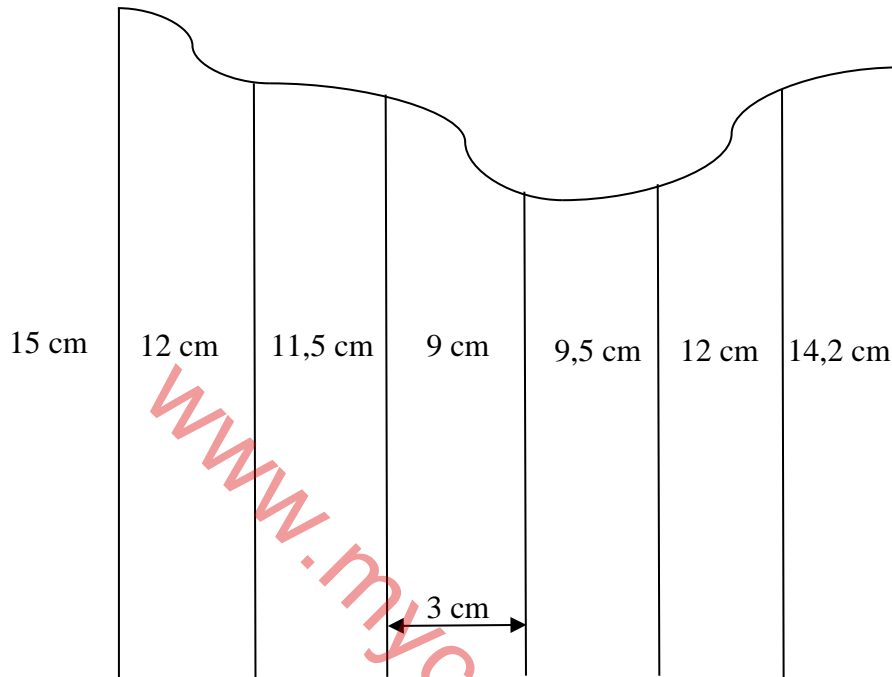


Determine the height of the larger segment in cm.

(8)  
[21]

**QUESTION 10**

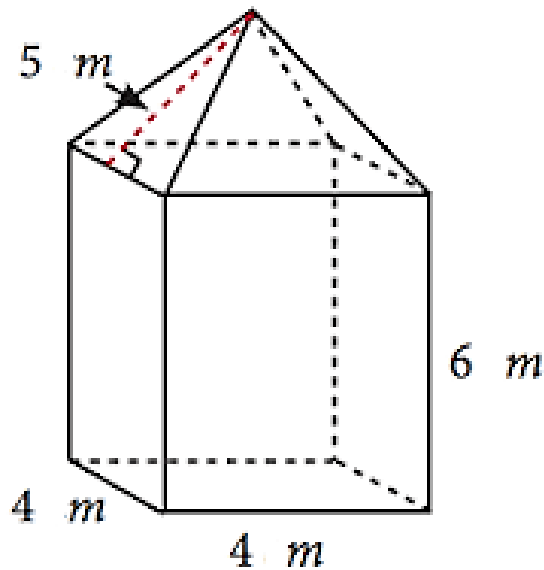
10.1 Consider the irregular figure below.



Determine the area of the figure by using the mid-ordinate rule.

(4)

- 10.2 A person wants to build a shed as depicted in the diagram below. The shed will have a square base with 4 m side.



The following formulae may be used:

Total surface area of a rectangular prism =  $2lw + 2lh + 2wh$

Total surface area of a square pyramid =  $2bs + b^2$

Volume of a rectangular prism =  $lwh$

Volume of a square pyramid =  $\frac{1}{3}(A)(H)$ , where  $A$  is the area of the base and  $H$  is the height of the pyramid

- 10.2.1 Calculate the total surface area of the shed, which includes the roof, that needs to be painted. (4)

- 10.2.2 Calculate the volume of the shed. (6)

[14]

**TOTAL: 150**

## INFORMATION SHEET: TECHNICAL MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a}$$

$$y = \frac{4ac - b^2}{4a}$$

$$a^x = b \Leftrightarrow x = \log_a b, \quad a > 0, a \neq 1 \text{ and } b > 0$$

$$A = P(1+ni)$$

$$A = P(1-ni)$$

$$A = P(1+i)^n$$

$$A = P(1-i)^n$$

$$i_{\text{eff}} = \left(1 + \frac{i}{m}\right)^m - 1$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int kx^n dx = k \cdot \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x} dx = \ln(x) + C, \quad x > 0$$

$$\int \frac{k}{x} dx = k \cdot \ln(x) + C, \quad x > 0$$

$$\int a^x dx = \frac{a^x}{\ln a} + C, \quad a > 0$$

$$\int ka^{nx} dx = k \cdot \frac{a^{nx}}{n \ln a} + C, \quad a > 0$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

In  $\triangle ABC$ :

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\text{Area} = \frac{1}{2} ab \cdot \sin C$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \text{cosec}^2 \theta$$

$$\pi \text{rad} = 180^\circ$$

Angular velocity =  $\omega = 2\pi n$  where  $n$  = rotation frequency

Angular velocity =  $\omega = 360^\circ n$  where  $n$  = rotation frequency

Circumferential velocity =  $v = \pi Dn$  where  $D$  = diameter and  $n$  = rotation frequency

Circumferential velocity =  $v = \omega r$  where  $\omega$  = Angular velocity and  $r$  = radius

Arc length  $s = r\theta$  where  $r$  = radius and  $\theta$  = central angle in radians

Area of a sector =  $\frac{rs}{2}$  where  $r$  = radius and  $s$  = arc length

Area of a sector =  $\frac{r^2\theta}{2}$  where  $r$  = radius and  $\theta$  = central angle in radians

$4h^2 - 4dh + x^2 = 0$  where  $h$  = height of segment,  $d$  = diameter of the circle and  $x$  = length of chord

$A_T = a(m_1 + m_2 + m_3 + \dots + m_{n-1})$  where  $a$  = width of equal parts,  $m_1 = \frac{o_1 + o_2}{2}$   
and  $n$  = number of ordinates

OR

$A_T = a\left(\frac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + \dots + o_{n-1}\right)$  where  $a$  = width of equal parts,  $o_i = i^{\text{th}}$  ordinate and  
 $n$  = number of ordinates