## GRADE 11

## TECHNICAL MATHEMATICS P1

MARKS: 150
TIME: 3 hours

This question paper consists of 12 pages, including a 1-page answer sheet and a 2-page information sheet.

## INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of SEVEN questions.
2. Answer ALL the questions.
3. An ANSWER SHEET is attached for QUESTION 4.4. Write your name in the spaces provided and then hand in with your ANSWER BOOK.
4. Number the answers correctly according to the numbering system used in this question paper.
5. Clearly show ALL calculations, diagrams, graphs, et cetera, that you have used in determining your answers.
6. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.

If necessary, ALL answers should be rounded off to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. Write neatly and legibly.


## QUESTION 1

1.1 Simplify the following completely WITHOUT using a calculator:

$$
\begin{equation*}
\text { 1.1.1 } \quad(3-\sqrt{x})(3+\sqrt{x}) \tag{2}
\end{equation*}
$$

1.1.2 $\left(\frac{3}{3^{3 x}}+\frac{2}{3^{3 x}}\right) \div \frac{10}{27^{x}}$
1.1.3 $\frac{\sqrt{2}(\sqrt{12}+\sqrt{75})}{\sqrt{6}}$
1.1.4 $\log _{5}\left(\frac{1}{5}\right)+\log _{5} 30-\log _{5} 6$
1.2 Prove that:

$$
\frac{2(\log 1-\log 3-\log 2)}{\log 36}=-1
$$

1.3 Given binary numbers: $100111_{2}$ and $11_{2}$
1.3.1 Use the long division method to calculate $100111_{2} \div 11_{2}$ in binary form.
1.3.2 Convert your answer in QUESTION 1.3. 1 to a decimal form without using a calculator.
1.4 A tender to build 280000000 houses within a period of 5 years has been awarded to a construction company. Write the number of houses in Scientific notation.

## QUESTION 2

2.1 Solve for $x \in$ R, WITHOUT using a calculator:

$$
\text { 2.1.1 } \frac{1}{(x)^{\frac{5}{3}}}=32
$$

2.1.2 $\sqrt{x+5}-x=3$
2.1.3 $\log _{x} 3=-1$
2.1.4 $\log _{a}(x-8)-\log _{a} 24=-\log _{a}(x+2)$
2.2 Show that:

2.3 The formula below is used to calculate the Body Mass Index (BMI) of a human being:
$\operatorname{Height}(\mathrm{m})=\sqrt{\frac{\text { Weight }(\mathrm{kg})}{\text { BMI }}}$
BMI $=$ Body Mass Index (kg.m ${ }^{-2}$ )
Weight measured in kilograms ( kg )
Height measured in metres (m)
2.3.1 Make Weight the subject of the formula.
2.3.2 Determine the Weight of a person whose BMI is $29,9 \mathrm{~kg} \cdot \mathrm{~m}^{-2}$ and height 1960 mm .

## QUESTION 3

3.1 Solve for $x$ :

$$
\begin{equation*}
\text { 3.1.1 } x(x-2)-15=0 \tag{4}
\end{equation*}
$$

3.1.2 $2 x-\frac{7}{x}=-3$ (correct to TWO decimal places)
3.1.3 $x^{2}-x-12 \leq 0$ (represent the solution set on a number line)
3.2 Solve for $x$ and $y$ simultaneously in the following equations:

$$
\begin{equation*}
y-x=-2 \quad \text { and } \quad x^{2}-x-10=y \tag{6}
\end{equation*}
$$

3.3 The diagram below shows a rectangular fish pond and its rectangular geometric model alongside it. The length of the pond is $x$ metres, width is $y$ metres and height is $h$ metres.

3.3.1 Show that $x=9-y$, if the perimeter of the pond is 18 m .
3.3.2 Write down the formula for the area of the pond in terms of $y$.
3.3.3 Hence, determine the numerical values of the length and the width that will yield a perimeter that is equal to the area of the pond, $x>y$.
3.4 Determine, without solving the equation, the nature of roots of $f(x)=x^{2}+x+1$.
3.5 Determine for which value(s) of $c$ the equation $g(x)=x^{2}+x+c$ will have equal roots.

## QUESTION 4

Given function $f$ defined by $f(x)=2(x-3)^{2}-8$
4.1 Write down the coordinates of the turning point of $f$.
4.2 Determine the $y$ intercept of $f$.
4.3 Determine the $x$ intercept of $f$.
4.4 Sketch the graph of $f$ on the ANSWER SHEET provided at the end of the question paper. Clearly indicate the intercepts with the axes and the coordinates of the turning point.
4.5 Write down the range of $f$.
4.6 Determine the coordinates of the turning point of function $h$, that results from shifting $f, 2$ units vertically upwards.

## QUESTION 5

The graphs drawn below represent functions defined by $g(x)=\frac{a}{x-p}+q$ and $h(x)=m x+c$. The graphs, $g$ and $h$ intersect at point $(0 ; 5)$ and $\left(\frac{5}{3} ; 0\right)$ respectively. $x=1$ and $y=3$ are the asymptotes of $g$.

5.1 The numerical value of $m$
5.2 The numerical value of $c$
5.3 The numerical value of $q$
5.4 The numerical value of $p$
5.5 The numerical value of $a$
5.6 The domain of $g$
5.7 The values of $x$ for which $g(x) \leq h(x)$.
5.8 The equation of the function $f$, a reflection of $g$ about the $x$-axis

## QUESTION 6

The graphs below represent functions defined by $f(x)=\sqrt{r^{2}-x^{2}}$ and $g(x)=2^{x}-4$.
AB is an asymptote of $g$.
C is the $y$-intercept of $g$.
D is the common $x$-intercept for both the graph of $f$ and $g$.
F and E are the $x$ - and $y$-intercepts of $f$, respectively.

6.1 Write down the equation of AB , the asymptote of $g$.
6.2 Determine the coordinates of C and D , the intercepts of $g$.
6.3 Determine the defining equation of $f$.
6.4 Write down the value of $x$ for which $f(x)-g(x)=0$.
6.5 Determine the length of EC.
6.6 State a reason, why $f(x)$ is a function.

## QUESTION 7

7.1 A nominal interest rate charged is $6,3 \%$ per annum, compounded quarterly. Determine the effective interest rate per annum.
7.2 The production of chicken eggs grows from 2500 to 8949 , compound production rate over a period of 6 years. Determine the production rate of eggs per annum, compounded annually.
7.3 Mr Faku bought office furniture to the value of R25 000 as shown in the picture below. He paid a deposit of $11 \%$ and took a higher purchase loan for the remaining amount to be paid over a period 4 years, on $16 \%$ interest rate per annum.

7.3.1 Determine the amount paid by Mr Faku as a deposit.
7.3.2 Calculate the total amount paid, in instalments, at the end of 4 years.
7.3.3 Determine the amount he paid monthly over a period of 4 years.

TOTAL:

## ANSWER SHEET

## NAME AND SURNAME:

SCHOOL: $\qquad$

## QUESTION 4.4



INFORMATION SHEET: TECHNICAL MATHEMATICS

$$
\begin{aligned}
& x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \quad x=-\frac{b}{2 a} \quad y=\frac{4 a c-b^{2}}{4 a} \\
& a^{x}=b \Leftrightarrow x=\log _{a} b, \quad a>0, a \neq 1 \text { and } b>0 \\
& \mathrm{~A}=\mathrm{P}(1+n i) \quad \mathrm{A}=\mathrm{P}(1-n i) \quad \mathrm{A}=\mathrm{P}(1+i)^{n} \quad \mathrm{~A}=\mathrm{P}(1-i)^{n} \\
& i_{\text {eff }}=\left(1+\frac{i}{m}\right)^{m}-1 \\
& f^{\prime}(x)=\lim _{h \rightarrow 0} \frac{f(x+h)-f(x)}{h} \\
& \int x^{n} d x=\frac{x^{n+1}}{n+1}+C, n \neq-1 \quad \int k x^{n} d x=k . \frac{x^{n+1}}{n+1}+C, n \neq-1 \\
& \int \frac{1}{x} d x=\ln x+C, x>0 \quad \int \frac{k}{x} d x=k \cdot \ln x+C, x>0 \\
& \int a^{x} d x=\frac{a^{x}}{\ln a}+C, a>0 \quad \int_{k} a^{n x} d x=k \cdot \frac{a^{n x}}{n \ln a}+C, a>0 \\
& d=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}} \\
& \mathrm{M}\left(\frac{x_{2}+x_{1}}{2} ; \frac{y_{2}+y_{1}}{2}\right) \\
& y=m x+c \quad y-y_{1}=m\left(x-x_{1}\right) \\
& m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \quad \tan \theta=m \\
& \frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}=1
\end{aligned}
$$

In $\triangle \mathrm{ABC}: \frac{a}{\sin \mathrm{~A}}=\frac{b}{\sin \mathrm{~B}}=\frac{c}{\sin \mathrm{C}}$
$a^{2}=b^{2}+c^{2}-2 b c \cdot \cos A$
area of $\Delta \mathrm{ABC}=\frac{1}{2} a b . \sin \mathrm{C}$

$$
\begin{array}{ll}
\sin ^{2} \theta+\cos ^{2} \theta=1 & 1+\tan ^{2} \theta=\sec ^{2} \theta \\
1+\cot ^{2} \theta=\operatorname{cosec}^{2} \theta &
\end{array}
$$

$\pi \mathrm{rad}=180^{\circ}$
Angular velocity $=\omega=2 \pi n \quad$ where $n=$ rotation frequency
Angular velocity $=\omega=360^{\circ} n \quad$ where $n=$ rotation frequency
Circumferential velocity $=v=\pi D n \quad$ where $D=$ diameter and $n=$ rotation frequency
Circumferential velocity $=v=2 \pi r n \quad$ where $r=$ radius and $=$ rotation frequency
Arc length $=s=r \theta \quad$ where $r=$ radius and $\theta=$ central angle in radians
Area of a sector $=\frac{r s}{2} \quad$ where $r=$ radius, $s=$ arc length and $\theta=$ central angle in radians
Area of a sector $=\frac{r^{2} \theta}{2} \quad$ where $r=$ radius and $\theta=$ central angle in radians
$4 h^{2}-4 d h+x^{2}=0$
where $h=$ height of segment,$\quad d=$ diameter of circle and $x=$ length of chord
$\mathrm{A}_{\mathrm{T}}=a\left(m_{1}+m_{2}+m_{3}+\ldots+m_{n}\right) \quad$ where $a=$ equal parts, $m_{1}=\frac{o_{1}+o_{2}}{2}$ and $n=$ number of ordinates
$\mathrm{A}_{\mathrm{T}}=a\left(\frac{o_{1}+o_{n}}{2}+o_{2}+o_{3}+\ldots+o_{n-1}\right)$
where $a=$ equal parts, $\mathrm{o}_{i}=i^{\text {th }}$ ordinate and $n=$ number of ordinates

