Province of the
EASTERN CAPE
EDUCATION

OR TAMBO INLAND DISTRICT

GRADE 12

MATHEMATICS INVESTIGATION

DATE: 28/02/2023

MARKS: 70

This question paper consists of 12 pages including the cover sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

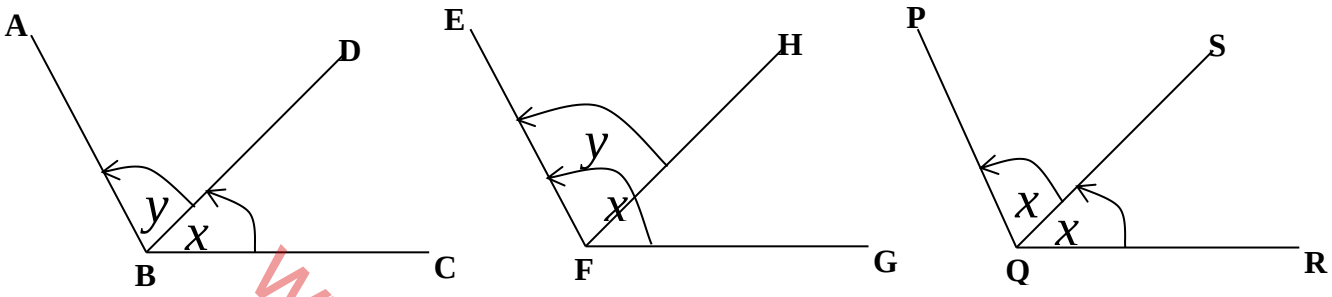
1. This task paper consists of 4 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper
4. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining your answers.
5. Answers only will not necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.

10. Write neatly and legibly.

INVESTIGATING COMPOUND ANGLES AND THEREFORE, DOUBLE ANGLES.

QUESTION 1

1.1. In the following diagrams, $A\hat{B}D = y$, $D\hat{B}C = x$, $E\hat{F}H = y$, $E\hat{F}G = x$ and $P\hat{Q}S = S\hat{Q}R = x$



Write each of the following in terms of x and/ or y

1.1.1 $A\hat{B}C$ (1)

1.1.2 $H\hat{F}G$ (1)

1.1.3 $P\hat{Q}R$ (1)

1.2 In the table below, different methods are used by learner X and learner Y to answer given questions. Study the table and answer the questions that follow:

1.2.1 Redraw the table in your answer book and use a calculator to complete it. (4)

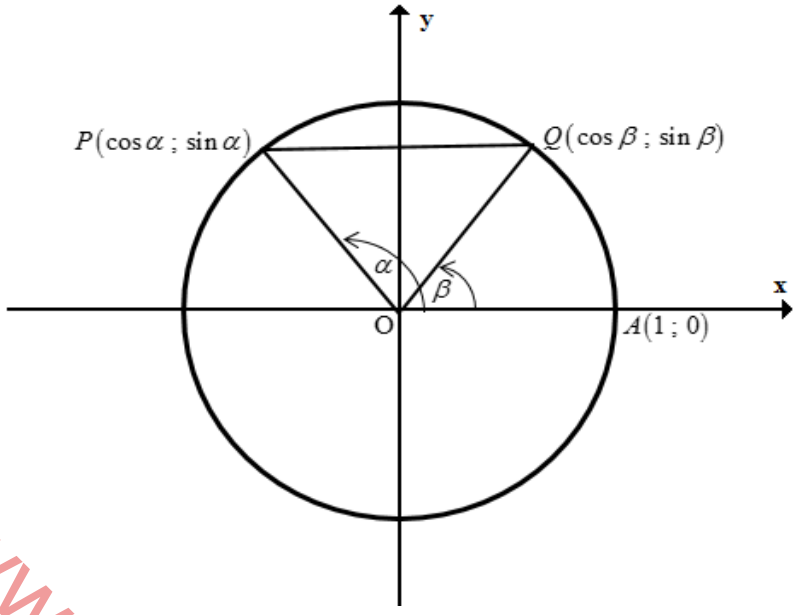
Learner X			Learner Y		
Question	**Working**	**Answer**	**Question**	**Working**	**Answer**
cos (45°–15°)	cos 30°	$\frac{1}{2}$	cos (45°–15°)	cos45°–cos15°	$\frac{-\sqrt{6}+\sqrt{2}}{4}$
cos (150°–30°)	cos 120°		cos (150°–30°)	cos150°–cos30°	
sin (150°+60°)			sin (150°+60°)		
cos (90°–150°)			cos (90°–150°)		
sin (60°–300°)			sin (60°–300°)		

1.2.2 Are the answers obtained by learner X the same as that of learner Y for the same question? (1)

1.2.3 Whose method do you think is mathematically correct? (1)

1.2.4 Give a reason why you think there is a difference in the answers. (2)

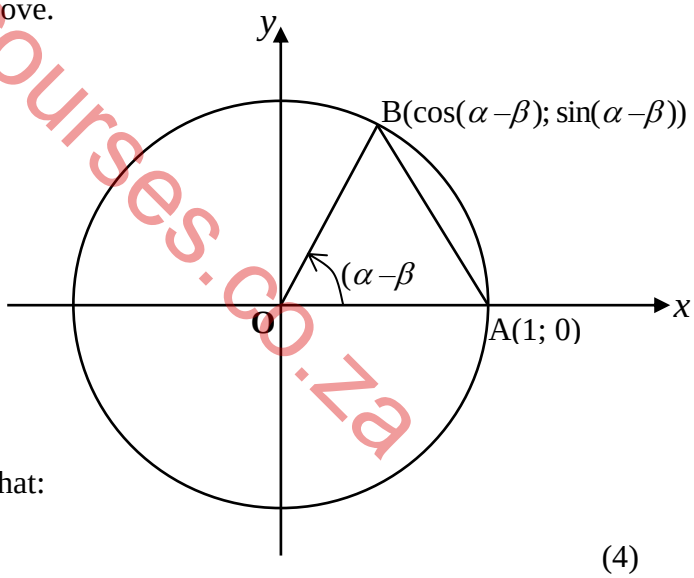
1.3 Consider the sketched unit circle. By the definition of trigonometric functions, the points P and Q on the terminal sides of angles α and β are labelled as shown in figure below.



Use the distance formula to show that: $PQ^2 = 2 - 2(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$ (4)

1.4 Consider ΔPOQ in the sketched above.

Now place angle $(\alpha - \beta)$ in standard position and label the point B on the unit circle corresponding to the terminal side of angle $(\alpha - \beta)$, as shown in the figure alongside.



Use the distance formula to show that:

$AB^2 = 2 - 2\cos(\alpha - \beta)$ (4)

1.5 Use 1.2 and 1.3 above to prove that $\Delta POQ \equiv \Delta BOA$ (4)

1.6 Hence, compare sides PQ and AB and write a conclusion about $\cos(\alpha - \beta)$. (4)

1.7 Use $\cos(\alpha - \beta)$ to derive a formula for $\sin(\alpha - \beta)$. (Hint: use co-function) (3)

1.8 By applying the compound angles and without using a calculator.
Determine the value of $\sin 105°$ (3)

[33]

QUESTION 2

Using your CALCULATOR, go to TABLE mode.

2.1 Insert the function $f(x)=2\cos^2 x-1$

- Start: $-180°$; End: $180°$
- Step $45°$

2.1.1 Complete the following table: (1)

x	$-180°$	$-135°$	$-90°$	$-45°$	$0°$	$45°$	$90°$	$135°$	$180°$
$2\cos^2 x-1$									

2.1.2 Use the table to sketch the graph of $f(x)=2\cos^2 x-1$, $x\in[-180° ; 180°]$. (3)

2.2 Using your calculator insert the following function $g(x)=1-2\sin^2 x$

2.2.1 Complete the following table: (1)

x	$-180°$	$-135°$	$-90°$	$-45°$	$0°$	$45°$	$90°$	$135°$	$180°$
$1-2\sin^2 x$									

2.2.2 Use the table to sketch the graph of $g(x)=1-2\sin^2 x$; $x\in[-180° ; 180°]$. (3)

2.3 Using your calculator insert the following function $h(x)=\cos^2 x-\sin^2 x$

2.3.1 Complete the following table: (1)

x	$-180°$	$-135°$	$-90°$	$-45°$	$0°$	$45°$	$90°$	$135°$	$180°$
$\cos^2 x-\sin^2 x$									

2.3.2 Use the table to sketch the graph of where $h(x)=\cos^2 x-\sin^2 x$; $x\in[-180° ; 180°]$. (3)

2.4 Using your calculator insert the following function $j(x)=\cos 2x$

2.4.1 Complete the following table: (1)

x	$-180°$	$-135°$	$-90°$	$-45°$	$0°$	$45°$	$90°$	$135°$	$180°$
$\cos 2x$									

2.4.2 Use the table to sketch the graph of $j(x)=\cos 2x$, $x\in[-180° ; 180°]$. (3)

2.5 If the graphs *f*, *g* and *h* were drawn on the set of axes as *j* what will your observation be? (1)

2.6 Make a comparison of the four expressions. (3)

[20]

QUESTION 3

Using your CALCULATOR, go to TABLE mode.

3.1 Using your calculator insert the following function $u(x) = 2\sin x\cos x$

3.1.1 Complete the following table: (1)

x	$-180°$	$-135°$	$-90°$	$-45°$	$0°$	$45°$	$90°$	$135°$	$180°$
$2\sin x\cos x$									

3.1.2 Use the table to sketch the graph of $u(x) = 2\sin x\cos x$, $x \in [-180° ; 180°]$. (3)

3.2 Using your calculator insert the following function $v(x) = \sin 2x$

3.2.1 Complete the following table: (1)

x	$-180°$	$-135°$	$-90°$	$-45°$	$0°$	$45°$	$90°$	$135°$	$180°$
$\sin 2x$									

3.2.2 Use the table to sketch the graph of $v(x) = \sin 2x$; $x \in [-180° ; 180°]$. (3)

3.3 If the graph of u was drawn on the same set of axes as v, what will you observe? (1)

3.4 Make a comparison of the two expressions. (1)

[10]

QUESTION 4

APPLICATION

4.1 Without the use of tables or a calculator prove that $\dfrac{1-\cos 2A}{\sin 2A} = \tan A$ (3)

4.2 Hence calculate, the value of $\tan 15^\circ$ (4)

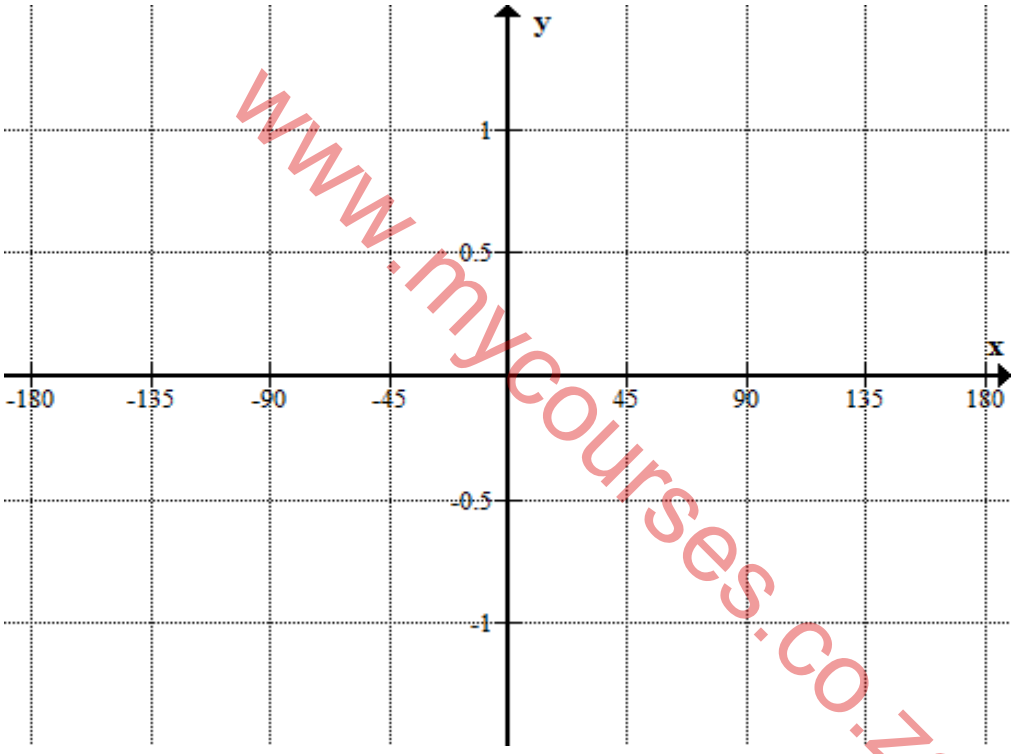
TOTAL: 70

NAME OF LEARNER: …….. CLASS: ……

2.1.1 Complete the following table:

x	–180°	–135°	–90°	–45°	0°	45°	90°	135°	180°
$2\cos^2 x - 1$									

2.1.2

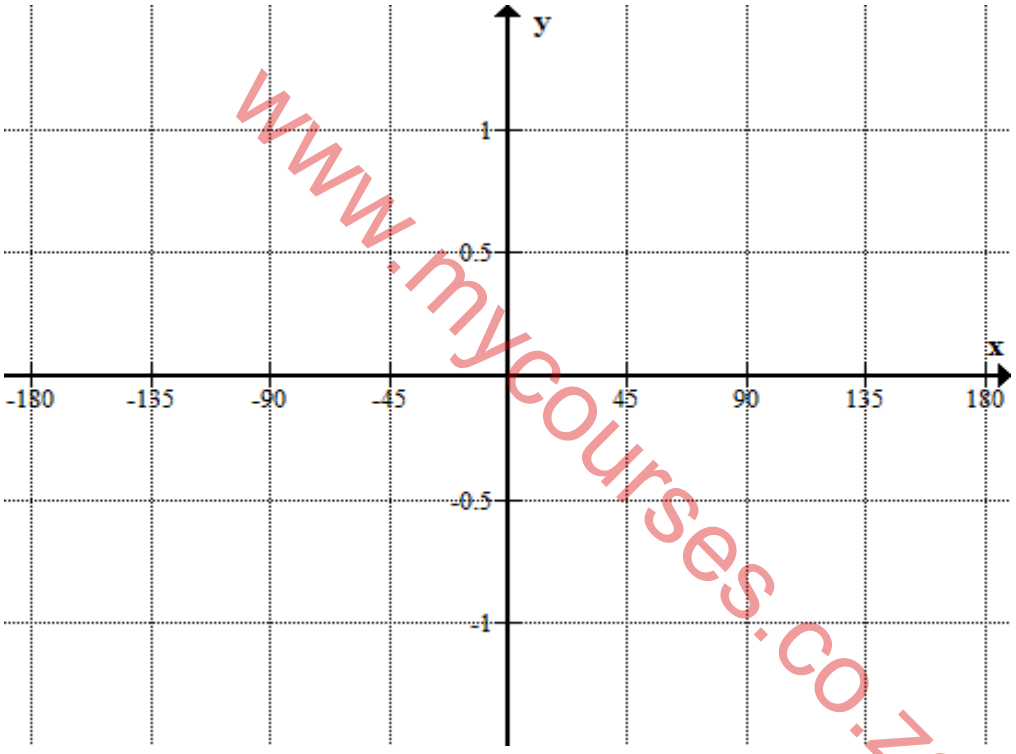


NAME OF LEARNER: …….. CLASS: ……

2.2.1 Complete the following table:

x	–180°	–135°	–90°	–45°	0°	45°	90°	135°	180°
$1-2\sin^2 x$									

2.2.2

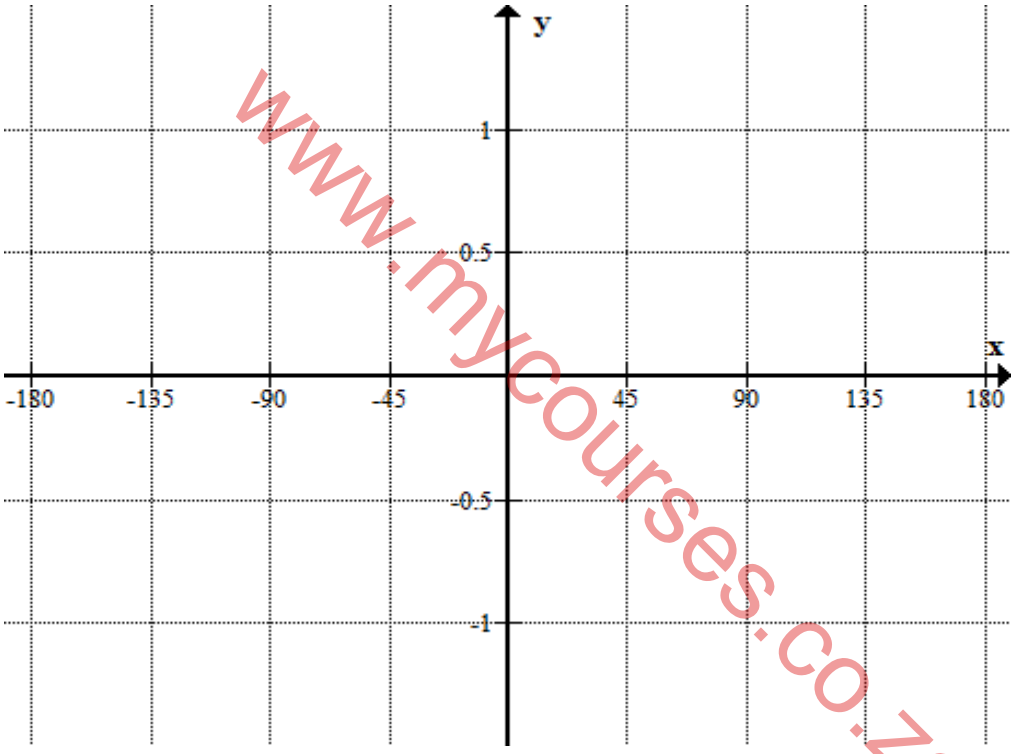


NAME OF LEARNER: …….. CLASS: ……

2.3.1 Complete the following table:

x	–180°	–135°	–90°	–45°	0°	45°	90°	135°	180°
$\cos^2 x - \sin^2 x$									

2.3.2

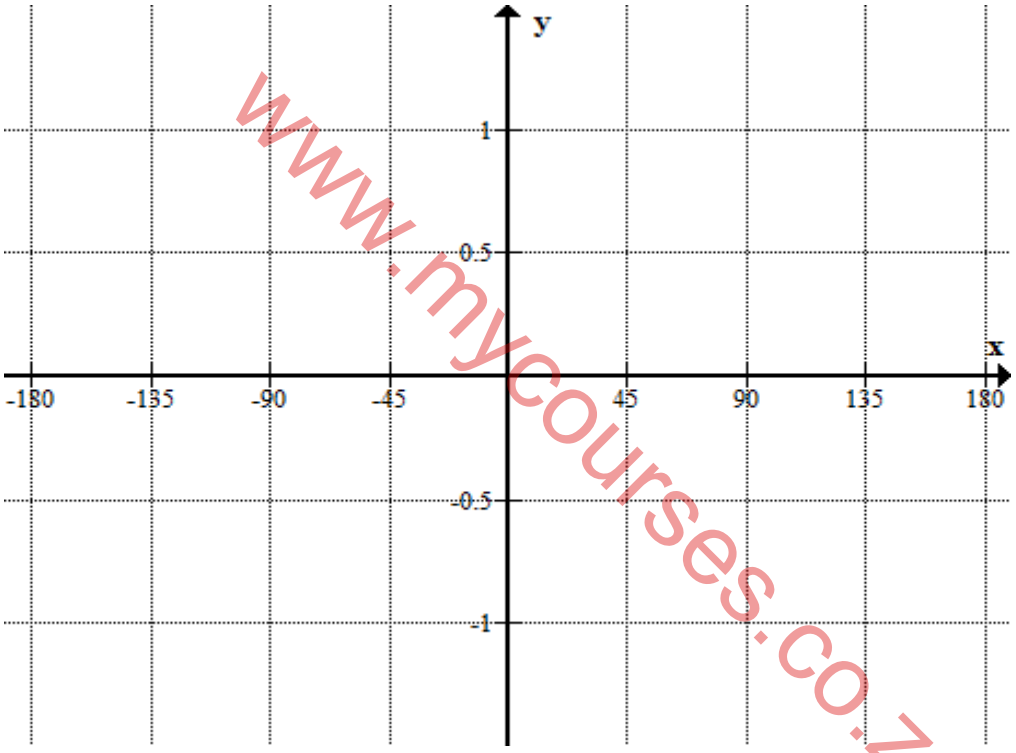


NAME OF LEARNER: …….. CLASS: ……

2.4.1 Complete the following table:

x	–180°	–135°	–90°	–45°	0°	45°	90°	135°	180°
$\cos 2x$									

2.4.2

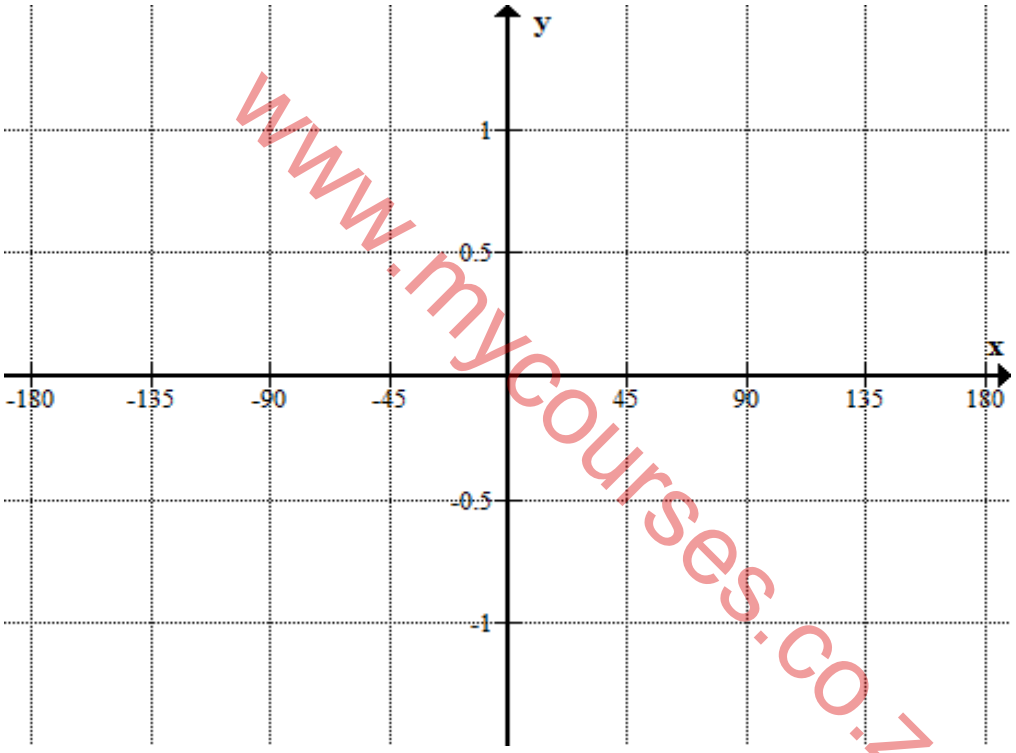


NAME OF LEARNER: …….. CLASS: ……

3.1.1 Complete the following table:

x	–180°	–135°	–90°	–45°	0°	45°	90°	135°	180°
$2\sin x\cos x$									

3.1.2



NAME OF LEARNER: …….. CLASS: ……

3.2.1 Complete the following table:

x	–180°	–135°	–90°	–45°	0°	45°	90°	135°	180°
$\sin 2x$									

3.2.2