KWAZULU-NATAL PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

GRADE 12 MATHEMATIC INVESTIGATION 2023

FUNCTIONS AND INVERSES

.

"The aim of this task is to investigate the relationship between functions and their inverses."

DATE	**09 FEBRUARY 2023**
TOTAL	**60 MARKS**
TIME	**2 HOURS**

INSTRUCTIONS

1. ***Answer all the questions for each activity.***
2. ***Answers are to be done on this question paper.***
3. ***Write neatly and legibly.***
4. ***When drawing linear graphs, use a ruler. In the case of curves ensure that smooth curves are drawn.***
5. ***Calculators may be used. However, it is an individual task and NO GROUP WORK is allowed.***

This investigation consists of NINE pages and FIVE activities.

Pre-requisite Knowledge for this task:

- Grade 9,10 & 11 knowledge on functions
- Changing the subject of the formula from Grade 10.

Additional stationery requirements for this task:

- Calculator, Ruler, Pencil.

NAME OF LEARNER: ___________________________________

NAME OF SCHOOL: _____________________________________

______
60

1 - RECAP OF WORK DONE IN EARLIER GRADES

In earlier grades you learnt about Mathematical relations. We will now determine how some of these relations may be considered as functions while others are **not** functions.

These are the conditions that make a relation a **FUNCTION**:

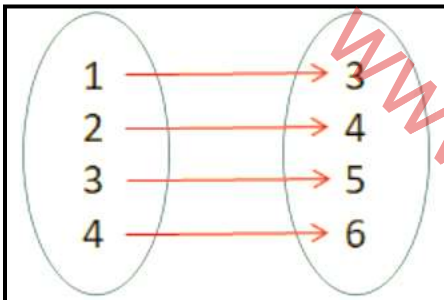
1. **When each element of the domain is associated with only one element of the range.**
2. **When two or more elements of the domain are associated with the same element of the range.**

This is the condition that makes a relation **NOT a FUNCTION**:

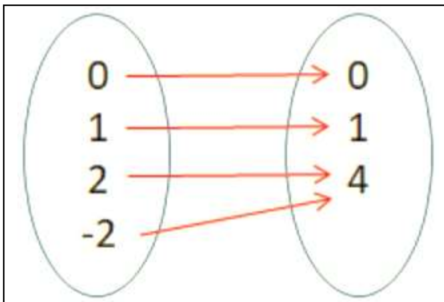
1. **A relation is not a function if one element of the domain is associated with more than one element of the range.**

Consider the examples below using sets that illustrate the conditions above:

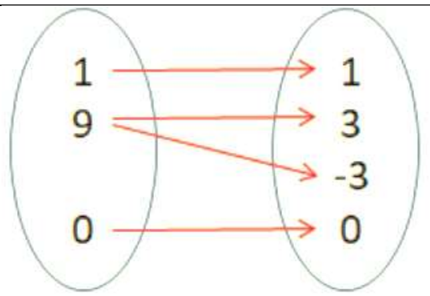
Example 1

 Domain (x) Range (y)	In the diagram alongside each element of the domain is associated with only one element of the range. This means that each x-value associates with only one y-value.	In this scenario, the relation is said to be a **ONE-TO-ONE FUNCTION**.

Example 2

 Domain (x) Range (y)	In the diagram alongside each element of the domain is associated with only one element of the range. However, the x-values 2 and - 2 are associated with the same element of the range (namely 4).	In this scenario, the relation is said to be a **MANY-TO-ONE FUNCTION**. Note that each x-value still associates with only **one** y-value.

Example 3

 Domain (x) Range (y)	In the diagram alongside an element in the domain is associated with more than one element of the range. The x-value 9 is associated with the y - values 3 and - 3.	In this scenario, the relation is **ONE-TO-MANY** and is **NOT A FUNCTION**. Note that the same x-value is associated with **more** than one y-value.

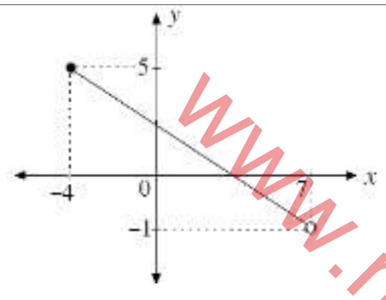
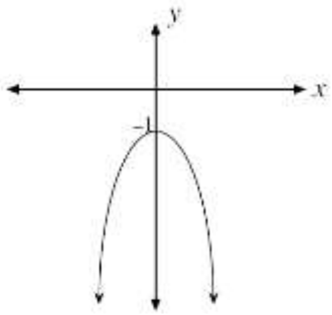
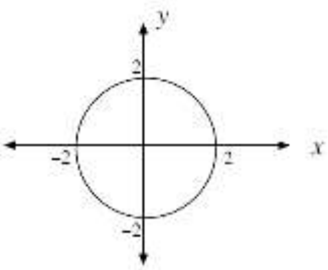
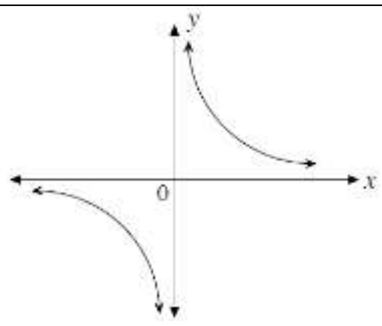
We can use a ruler to perform the **VERTICAL LINE TEST** on a graph to determine whether it is a **function** or not. Hold a **clear plastic ruler parallel to the y – axis, i.e. vertical.** Move it from **left to right** across the graph. The following holds true:

If the ruler only ever ***cuts the graph AT ONE POINT*** as the ruler moves from left to right across the graph…then that graph is a ***function***.

If the ruler ***cuts the graph AT MORE THAN ONE POINT*** …then that graph is **not a *function*.**

ACTIVITY 1 [6 Marks]

Use the information provided above to determine whether the relations below are functions or not and provide a reason for your answer. To assist you the first row is an example.

	Relation	Is the relation a Function or not? Answer **YES or NO.**	Give a reason for your answer from the previous column
Example		***YES***	***A vertical line through the graph cuts it at one point only***
1.1		(1)	(1)
1.2		(1)	(1)
1.3		(1)	(1)

2 - INVESTIGATING FUNCTIONS AND THEIR INVERSES

If a function is called $f(x)$ then its inverse (only if it's also a function, i.e. if it passes the vertical line test) is denoted as $f^{-1}(x)$. The rule (equation) of the inverse of a function is obtained by interchanging/swopping x and y in the rule (equation) of the function.

Consider the example:

Given function is $y=2x+3$, then its inverse is $x=2y+3$. (x and y are interchanged/swopped in the equation).
Often in conventional mathematics one is expected to write ***y*** **as the subject of the formula** when writing equations of functions and inverses. Considering this, then the inverse of the function can be written as:

$x=2y+3$ then $-2y=-x+3$ hence the inverse of the function is $y=\frac{1}{2}x-\frac{3}{2}$.

ACTIVITY 2.1 [11 Marks]

In the table below three examples are provided on how to write down the inverses of functions. Study the examples and fill in the table below by writing down the inverse and the inverse in the form requested for numbers 2.1.1 to 2.1.4.

	Function	**Inverse**	**Inverse in the form ……**
Example 1	$y=2x+11$	$x=2y+11$	$y=....$ $x=2y+11$ $-2y=-x+11$ $y=\frac{1}{2}x-\frac{11}{2}$
Example 2	$f(x)=x^2$	$x=y^2$	$y=....$ $x=y^2$ $y^2=x$ $y=\pm\sqrt{x}$
Example 3	$g(x)=3x+12$	$x=3y+12$	$g^{-1}(x)=.......$ $x=3y+12$ $-3y=-x+12$ $y=\frac{1}{3}x-4$ $g^{-1}(x)=\frac{1}{3}x-4$

	Function	Inverse	Inverse in the form ……
2.1.1	$y = x - 3$	(1)	$y = ...$ (1)
2.1.2	$g(x) = \frac{1}{2}x + 4$	(1)	$g^{-1}(x) = ...$ (2)
2.1.3	$h(x) = 3x^2$	(1)	$y = ...$ (2)
2.1.4	$f(x) = x^2 + 3$	(1)	$y = ...$ (2)

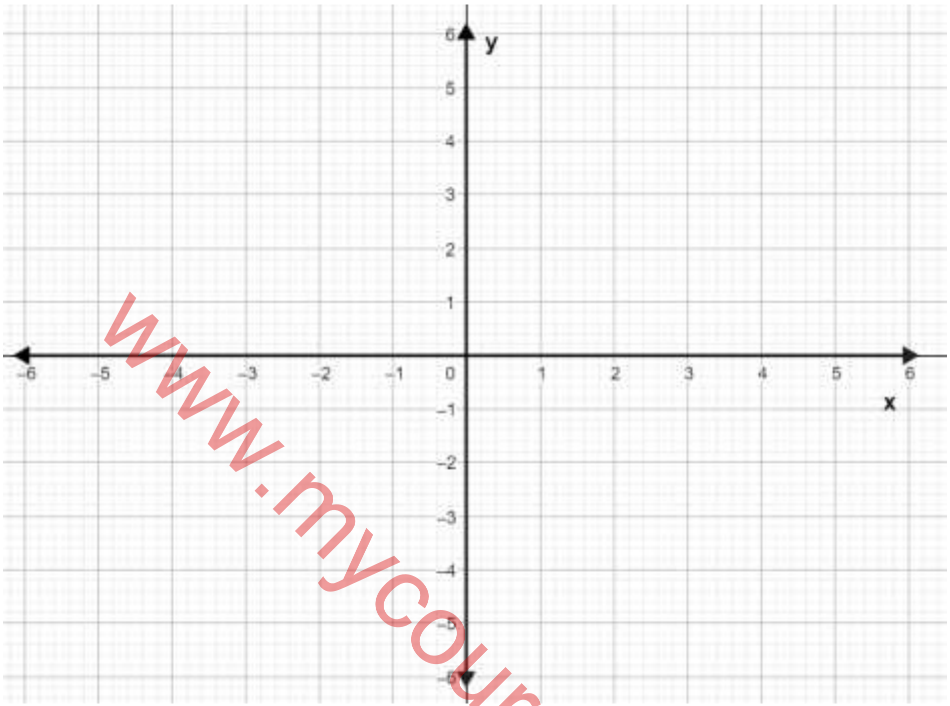
ACTIVITY 2.2 [11 Marks] - Investigating the inverse of a linear function.

Consider the linear function $f(x) = x - 1$

2.2.1 Complete the table below. (1)

x	-4	-3	-2	-1	0	1	2	3	4
$f(x) = x - 1$									

2.2.2 Use the table above to sketch the function $f(x)$ in the grid below. (2)



2.2.3 Draw the line $y = x$ on the same grid in 2.2.2. Indicate it as a dashed line. (1)

2.2.4 Show that the inverse of the function $f(x) = x - 1$ is $f^{-1}(x) = x + 1$ (1)

2.2.5 Complete the table below. (1)

x	-4	-3	-2	-1	0	1	2	3	4
$f^{-1}(x) = x + 1$									

2.2.6 Use the table above to sketch the inverse function $f^{-1}(x)$ on the grid used in 2.2.2 (2)

2.2.7 Write down the domain and range of $f^{-1}(x)$ (2)

2.2.8 Look at your sketches and fill in the missing word (from the options provided below) to make the statement below true: (1)

" The function $f(x)$ is ………………………….about the line $y = x$ to give you the inverse $f^{-1}(x)$."

(Rotated, Translated, Reflected)

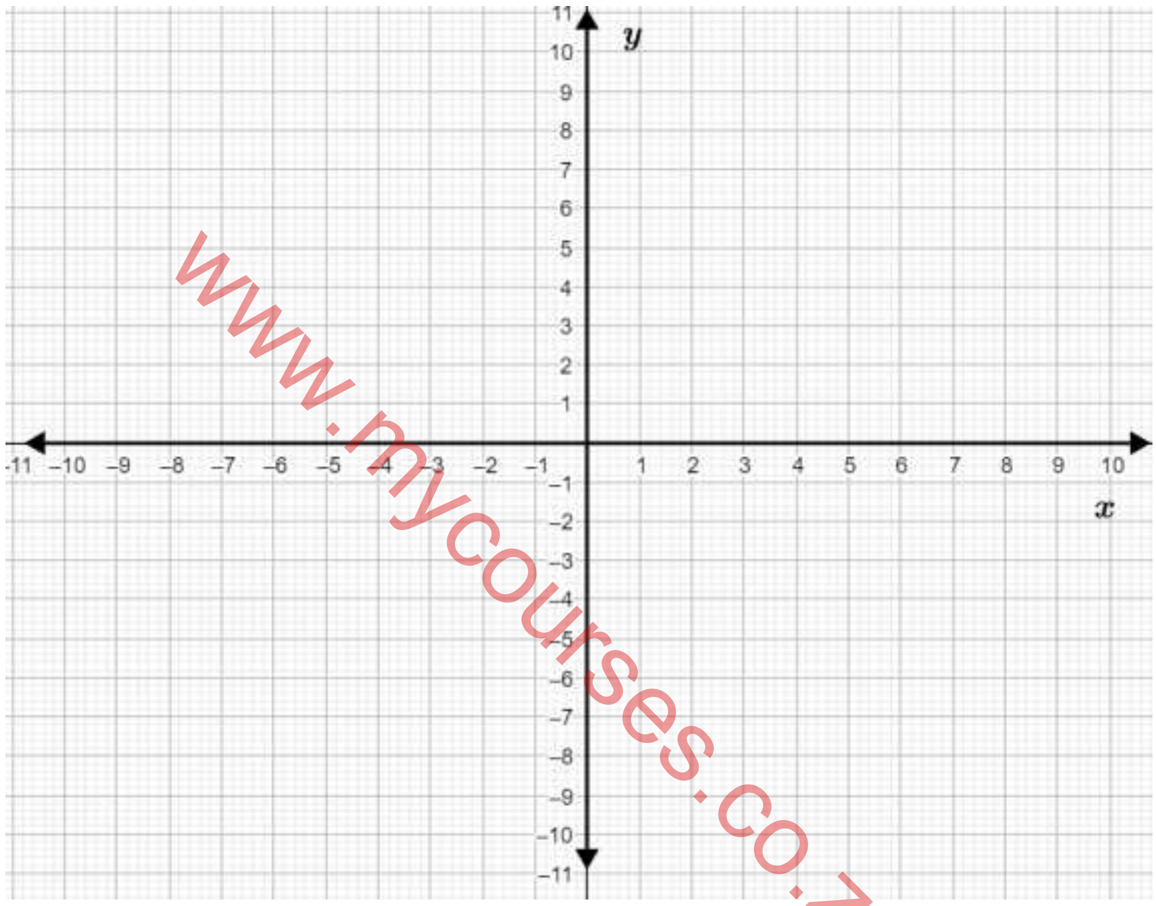
ACTIVITY 2.3 [16 Marks] - Investigating the inverse of a quadratic function.

Consider the quadratic function $g(x) = 2x^2$

2.3.1 Complete the table below. (2)

x	-3	-2	-1	0	1	2	3
$g(x) = 2x^2$							

2.3.2 Use the table above to sketch the function $g(x)$ in the grid below. (2)



2.3.3 Draw the line $y = x$ on the same grid in 2.3.2. Indicate it as a dashed line. (1)

2.3.4 Show that the inverse of the function $g(x) = 2x^2$ is $y = \pm\sqrt{\frac{1}{2}x}$ (2)

2.3.5 Complete the table below. (3)

x	-3	-2	-1	0	1	2	3
$y = \pm\sqrt{\frac{1}{2}x}$							

2.3.6	Use the table from 2.3.5 to sketch the inverse $y=\pm\sqrt{\frac{1}{2}x}$ in the grid used in 2.3.2	(2)
2.3.7	Write down the domain and range of $y=\pm\sqrt{\frac{1}{2}x}$	(2)
2.3.8	Is $y=\pm\sqrt{\frac{1}{2}x}$ a function? (Yes or No) Justify your answer with a reason.	(2)

WRITING THE EQUATION OF THE INVERSE OF AN EXPONENTIAL FUNCTION AS A LOGARITHMIC FUNCTION

Given the exponential function $y=a^x$ then its inverse can be written as $x=a^y$. However, in order to write y as the subject of the formula the concept of **logarithms (logs)** have to be utilised.

In general, if : $number = base^{exponent}$ then $exponent = \log_{base} number$. From above if $y=a^x$ then the inverse is given by $x=a^y$ making y the subject the equation of the inverse is: $y=\log_a x$. Consider the examples below showing changing from exponential to logarithmic (log) form:

Exponential function	Inverse of the exponential function by interchanging/swopping x and y.	Inverse of the exponential function written in log form.
$y=3^x$	$x=3^y$	$y=\log_3 x$
$g(x)=6^x$	$x=6^y$	$g^{-1}(x)=\log_6 x$
$f(x)=\left(\frac{1}{2}\right)^x$	$x=\left(\frac{1}{2}\right)^y$	$f^{-1}(x)=\log_{\left(\frac{1}{2}\right)} x$

Use this knowledge above and your previous Grades understanding of exponential functions to answer the activity that follows:

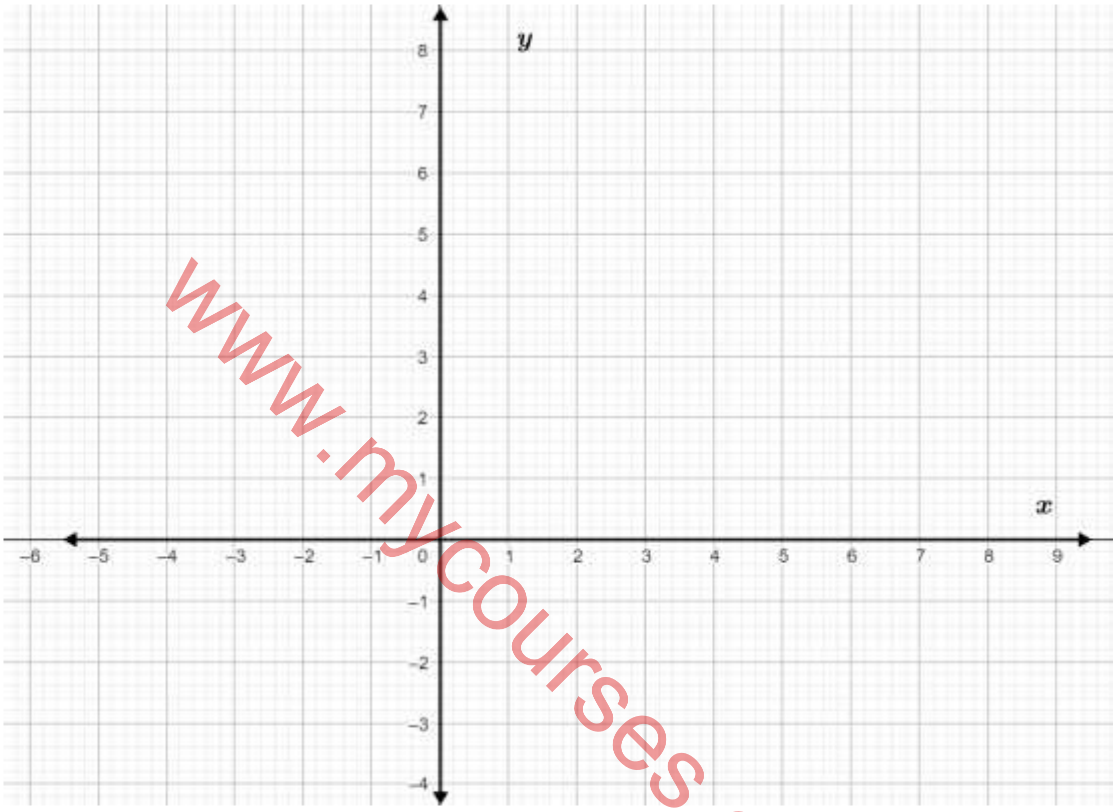
ACTIVITY 2.4 [16 Marks] - Investigating the inverse of an exponential function.

Consider the exponential function $h(x)=2^x$

2.4.1 Complete the table below. (2)

x	-3	-2	-1	0	1	2	3
$h(x)=2^x$							

2.4.2 Use the table above to sketch the function $h(x)$ in the grid below. (2)



2.4.3 Draw the line $y=x$ on the same grid in 2.4.2. Indicate it as a dashed line. (1)

2.4.4 Write down the equation of the inverse of $h(x)$ in the form $h^{-1}(x)=$.......... (2)

2.4.5 Complete the table below. (3)

y	-3	-2	-1	0	1	2	3
$x=2^y$							

2.4.6 Use the table above to sketch the inverse function $x=2^y$ in the grid used in 2.4.2 (2)

2.4.7 Write down the domain and range of $h^{-1}(x)$ (2)

2.4.8 State a conclusion about the x – intercept of a logarithmic function written in the form: $y=\log_a x$. Suggest a reason for your answer. (2)

TOTAL = 60 MARKS