Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIORSERTIFIKAAT*

GRADE/*GRAAD 12*

JUNE/*JUNIE* 2023

TECHNICAL MATHEMATICS P2/*TEGNIESE WISKUNDE V2* MARKING GUIDELINE/*NASIENRIGLYN*

MARKS/*PUNTE*: **150**

This marking guideline consists of 17 pages./
Hierdie nasienriglyn bestaan uit 17 bladsye.

NOTE:

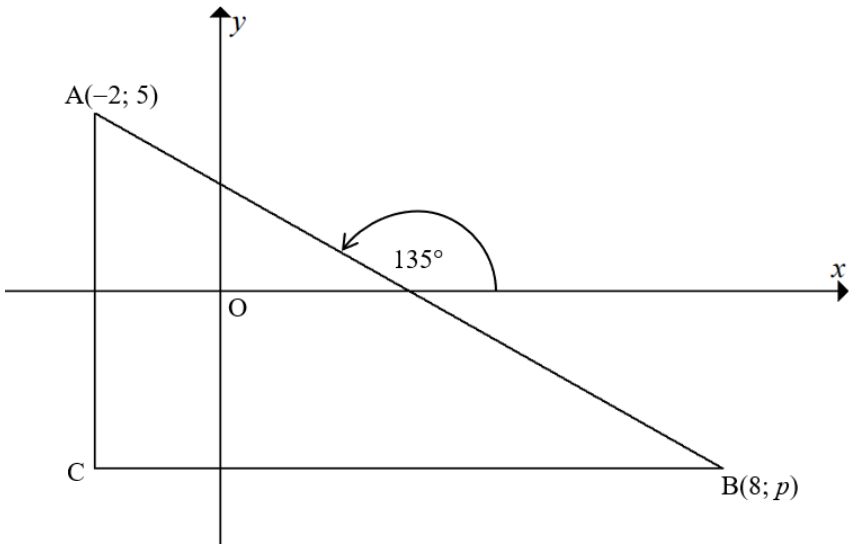
- Continuous accuracy (CA) applies only where indicated in this marking guideline.
- Assuming values/answers in order to solve a problem is unacceptable.

LET WEL:

- *Volgehoue akkuraatheid (CA) is slegs van toepassing soos aangedui in hierdie nasienriglyn.*
- *Aanvaarding van waardes/antwoorde om ’n probleem op te los, is onaanvaarbaar.*

MARKING CODES / *NASIENKODES*	
M	Method/*Metode*
A	Accuracy/*Akkuraatheid*
AO	Answer only/*Slegs antwoord*
CA	Consistent accuracy/*Deurlopende akkuraatheid*
F	Formula/*Formule*
I	Identity/*Identiteit*
R	Rounding/*Afronding*
S	Simplification/*Vereenvoudiging*
ST	Statement/*Bewering*
RE	Reason/*Rede*
ST RE	Statement and correct reason/*Bewering en korrekte rede*
SF	Substitution correctly in correct formula/*Korrekte vervanging in die korrekte formule*
NPU	No penalty for omitting units/*Geen penalisering vir eenhede uitgelaat*

QUESTION/*VRAAG* 1

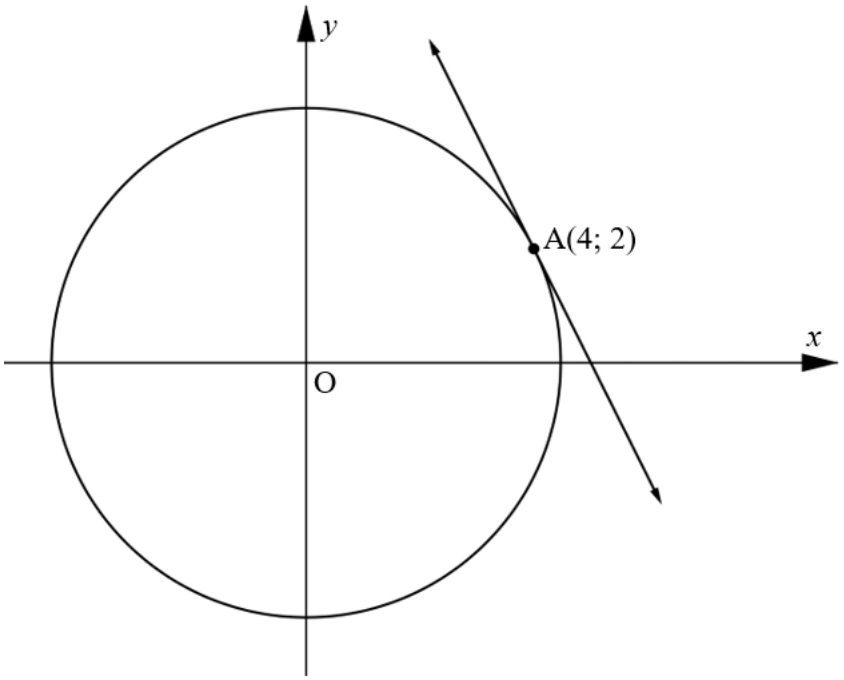


1.1	$m_{AB} = \tan 135° = -1$ **AO: Full marks / *Volpunte***	✓ **M** ✓ **S** **A** (2)
1.2	$m_{AB} = \frac{p-5}{8+2}$ $\therefore \frac{p-5}{8+2} = -1$ $\therefore p-5 = -10$ $\therefore p = -5$	✓ **M** ✓ **S** **A** ✓ **S** **A** (3)
1.3	$M_{AB} = \left(\frac{-2+8}{2}; \frac{5-5}{2}\right) = (3;0)$	✓ *x*-value **A** ✓ *y*-waarde **A** (2)
1.4	$y = -5$	✓ **A** (1)
1.5	$C(-2;-5)$	✓ *x*-value **A** ✓*y*-waarde **A** (2)

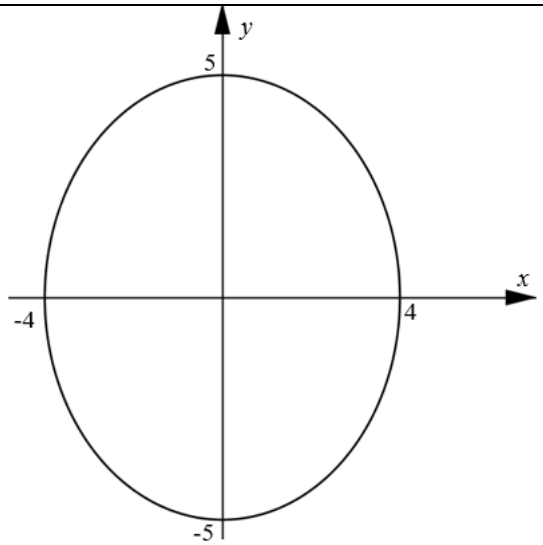
1.6	$m_{CM} = \frac{-5-0}{-2-3} = 1$ $m_{AB} \times m_{CM} = -1 \times 1 = -1$ $\therefore$ CM $\perp$ AB	✓ grad **CA** ✓ product/ *produk* **A** ✓ conclusion/ *gevolgtrekking* (3)
1.7	grad of line $= m_{CM} = 1$ $y - y_1 = m(x - x_1)$ **OR/*OF*** $y = mx + c$ $y - 5 = 1(x - (-2))$ $5 = 1(-2) + c$ $\therefore y = x + 7$	✓ grad. **CA** ✓ **SF** **A** ✓ equation/*vergelyking* **CA** (3)
		[16]

QUESTION/*VRAAG* 2

2.1

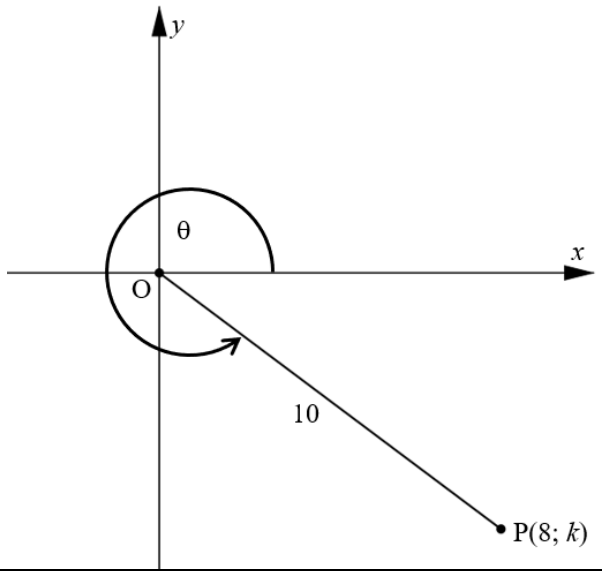


2.1.1	$r=\sqrt{20}=2\sqrt{5}$	✓ **A** (1)
2.1.2	$xx_1+yy_1=r^2$ $\therefore x(4)+y(2)=20$ $\therefore 2y=-4x+20$ $\therefore y=-2x+10$ **OR/*OF*** $\therefore m_{radius}=\frac{2}{4}=\frac{1}{2}$ $\therefore m_{\text{tangent/raaklyn}}=-2$ $y-y_1=m(x-x_1)$ **OR/*OF*** $y=mx+c$ $\therefore y-2=-2(x-4)$ $2=-2(4)+c$ $\therefore y=-2x+10$	✓ **F** ✓ **SF** **A** ✓ **S** ✓ equation / *vergl* **CA** **OR/*OF*** ✓ grad. radius **A** ✓ grad. tan / *raaklyn* **CA** ✓ **SF** **A** ✓ equation / *vergl* **CA** (4)
2.1.3	$(-4;-2)$	✓ *x*-value **A** ✓ *y*-waarde **A** (2)

2.2		✓ elliptical shape / elliptiese *vorm* **A** ✓ *x*-intercepts/*afsnitte* **A** ✓ *y*-intercepts/*afsnitte* **A** (3)
		[10]

QUESTION/*VRAAG* 3

3.1

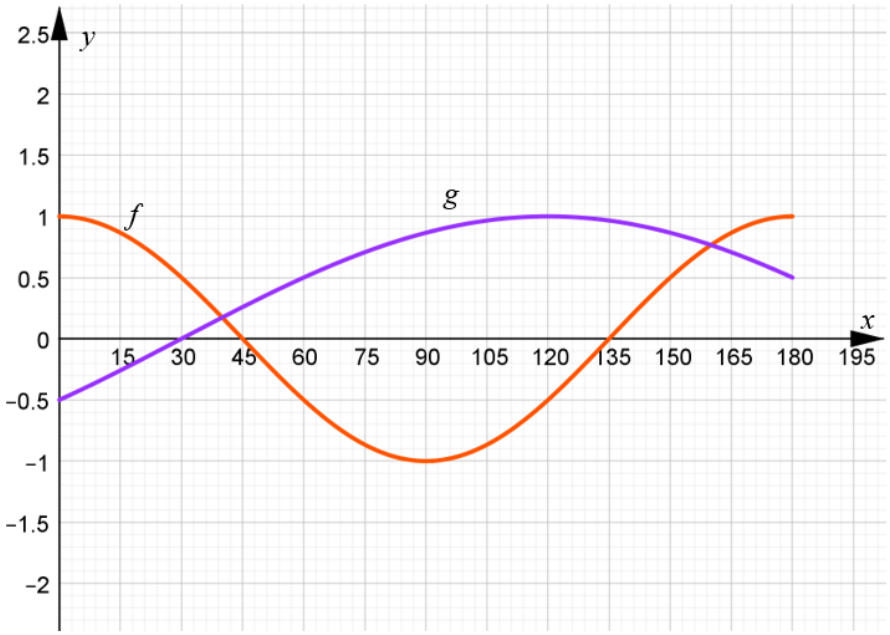


3.1.1	$\cos\theta = \frac{8}{10} = \frac{4}{5}$	✓ **A** (1)
3.1.2	$k^2 + 8^2 = 10^2$ *Pythagoras* $\therefore k^2 = 36$ $\therefore k = -6$ 4*th* quadrant /*4de kwadrant*	✓ **M** ✓ **S** ✓ value of / *waarde van k* (3)
3.1.3	$\frac{\tan q}{\text{cosec} q} = \frac{-6/8}{10/-6}$ $= \frac{9}{20}$	✓ tan ratio / *verh.* **A** ✓ cosec ratio / *verh.* **A** ✓ **S** **CA** (3)
3.2	$3\cos x - 1 = -1,5$ $3\cos x = -0,5$ $\therefore \cos x = -0,1666...$ Ref / *Verw* $\angle = 80,41°$ $\therefore x = 180° - 80,41°$ **OR/OF** $x = 180° + 80,41°$ $\therefore x = 99,59°$ **OR/*OF*** $x = 260,41°$	✓ **S** **A** ✓ Ref / *Verw* $\angle$ **CA** ✓ Quadrants / *Kwadrante* **A** ✓ values of *x* / *waardes van x* **CA** (4)
		[11]

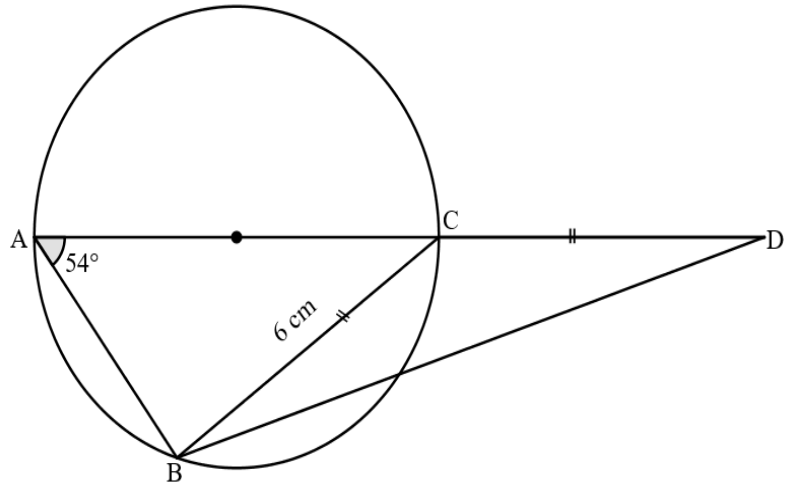
QUESTION/*VRAAG* 4

4.1	$(1+\cos x)(1-\cos x)=1-\cos^2 x$ $=\sin^2 x$	✓ **S** **A** ✓ **I** (2)
4.2	$\dfrac{\cos^2(2\pi-x)\tan^2 x}{\sin(180°+x)\operatorname{cosec}(180°-x)}$ $=\dfrac{\cos^2 x\dfrac{\sin^2 x}{\cos^2 x}}{(-\sin x)(\operatorname{cosec} x)}$ $=\dfrac{\cos^2 x\dfrac{\sin^2 x}{\cos^2 x}}{(-1)}$ $=-\sin^2 x$	✓ $\cos^2 x$ ✓ $\dfrac{\sin^2 x}{\cos^2 x}$ ✓ $-\sin x$ ✓ $\operatorname{cosec} x$ ✓ -1 ✓ $-\sin^2 x$ (6)
4.3	LHS / *LK* $=\cot x+\tan x$ $=\dfrac{\cos x}{\sin x}+\dfrac{\sin x}{\cos x}$ $=\dfrac{\cos^2 x+\sin^2 x}{(\sin x)(\cos x)}$ $=\dfrac{1}{(\sin x)(\cos x)}$ $=\operatorname{cosec} x\sec x=$ RHS / *RK*	✓ $\dfrac{\cos x}{\sin x}$ ✓ $\dfrac{\sin x}{\cos x}$ ✓ **S** ✓ $\cos^2 x+\sin^2 x=1$ (4)
		[12]

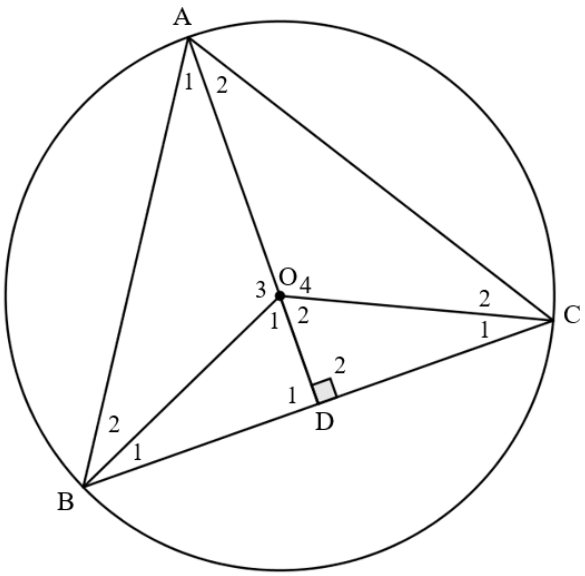
QUESTION/*VRAAG* 5

	$f(x)=\cos 2x$ and $g(x)=\sin(x-30°)$ for $x\in[0°;180°]$	
5.1	$\text{Period}_f=\frac{360°}{2}=180°$	✓ **A** (1)
5.2	$\text{Amplitude}_g=1$	✓ **A** (1)
5.3		*f:* ✓ *y*-intercept at / *y*-afsnit by 1 ✓ *x*-intercepts at 45° and 135° / *x-afsnitte by 45 ° en 135 °* ✓ turning point at / *draaipunt by* (90°; –1) ✓ End point at / *eindpunt by* (180°; 1) *g:* ✓ *y*-intercept at / *y*-afsnit by –0,5 ✓ *x*-intercept at 30° / *x-afsnit by 30* ✓ turning point at / *draaipunt by* (120°; 1) ✓ End point at / *eindpunt by* (180°; 0,5) (8)
5.4.1	$45°\le x\le 135°$	✓ $45°\le x$ **CA** ✓ $x\le 135°$ **CA** (2)
5.4.2	$135°\le x<180°$	✓ $135°\le x$ **CA** ✓ $x<180°$ **CA** (2)
		[14]

QUESTION/*VRAAG* 6

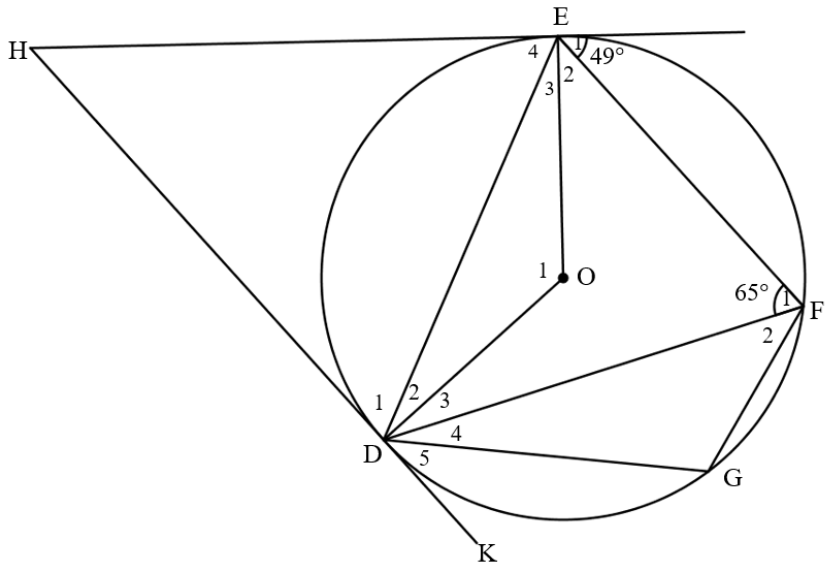
6.1	$p^2 = q^2 + r^2 - 2qr\cos P$ **OR/*OF*** $q^2 = p^2 + r^2 - 2pr\cos Q$ **OR/*OF*** $r^2 = p^2 + q^2 - 2pq\cos R$	✓ **A** (1)
6.2		
6.2.1	$A\hat{B}C = 90°$ (∠ in semi-circle / ∠ *in semi-sirkel*)	✓ **ST A** ✓ **RE** (2)
6.2.2	$B\hat{C}D = 144°$ (ext. ∠ of Δ / *buite ∠ van Δ*)	✓ **ST A** ✓ **RE** (2)
6.2.3	$BD^2 = BC^2 + CD^2 - 2BC\cdot CD\cos BCD$ $= 6^2 + 6^2 - 2\times 6\times 6\cos 144°$ $= 130{,}24922\ldots$ $BD \approx 11{,}41$ cm	✓ **F** ✓ **SF A** ✓ **S CA** ✓ BD value / *waarde* (4)
6.2.4	$\frac{BC}{AC} = \sin 54°$ $AC = \frac{6}{\sin 54°}$ $\approx 7{,}42$ cm	✓ ratio / *verh* **A** ✓ value / *waarde* **CA** (2)
6.2.5	Area of/*van* $\Delta ABC = \frac{1}{2}AC\times BC\sin ACB$ $= \frac{1}{2}\times 7{,}42\times 6\times \sin 36°$ $\approx 13{,}08$ cm^2	✓ **F** ✓ **SF CA** ✓ Area **CA** (3)
		[14]

QUESTION/*VRAAG* 7



7.1	BD = 5,5 cm (line from centre ⊥ to chord / *lyn vanuit midpt ⊥ koord)*	✓ **ST** **A** ✓ **RE** (2)
7.2	$OB^2 = OD^2 + BD^2$ (Pythagoras) $OB^2 = 3^2 + 5,5^2 = 39,25$ $\therefore OB \approx 6,26$ cm	✓ **ST** **CA** ✓ OB length / *lengte* (2)
7.3	In ΔABD and/*en* ΔACD: BD = DC (line from centre ⊥ to chord / *lyn vanuit midpt ⊥ koord)* AD is common / *gemeenskaplik* $\hat{D}_1 = \hat{D}_2 = 90°$ $\therefore \Delta ABD \equiv \Delta ACD \ (S\angle S)$	✓ **ST** **A** ✓ **ST** **A** ✓ **ST** **A** ✓ **RE** (4)
7.4	$\sin B_1 = \frac{OD}{OB} = \frac{3}{6,26}$ $\therefore \hat{B}_1 \approx 28,64°$ **OR/*OF* any alternative trig ratio/ enige *alternatiewe* trig. verhouding**	✓ trig ratio / *verh.* ✓ size of angle / *grootte van hoek* (2)
7.5	$\hat{O}_1 = 61,36°$ (Int ∠ of Δ / *Binne ∠e van Δ*) $\hat{O}_2 = 61,36°$ (Congruency / *Kongruensie*) $\therefore \hat{A} = 61,36°$ (∠ at centre = 2 × ∠ at circumference / *middelpts ∠ = 2 × Omtreks∠)*	✓ **ST** **CA** ✓ **ST** **CA** ✓ **ST** **CA** ✓ **RE** (4)
		[14]

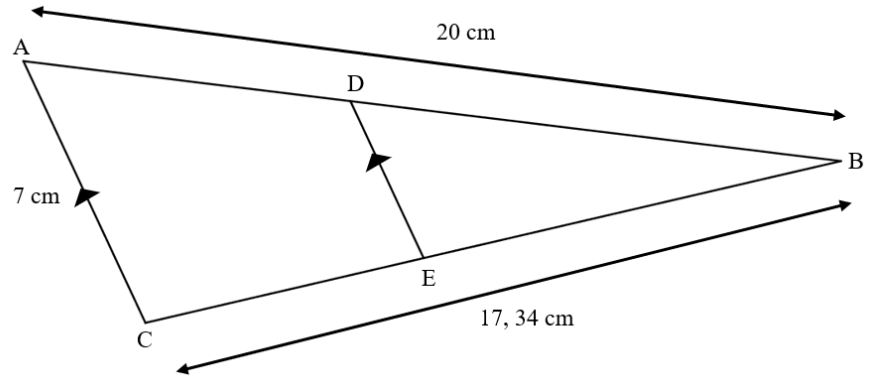
QUESTION/*VRAAG* 8



8.1	Tangents from a common point / *raaklyne vanuit dieselfde punt*	✓ **A** (1)
8.2.1	$\hat{D}_1 = 65°$ (tan – chord th / *raaklyn – koord st*)	✓ **ST** **A** ✓ **RE** (2)
8.2.2	$\hat{D}_2 = 25°$ (Radius ⊥ Tangent / *Raaklyn*)	✓ **ST** **A** ✓ **RE** (2)
8.2.3	$D\hat{E}F = 66°$ (∠ on str line / *∠e op reguit lyn*)	✓ **ST** **A** ✓ **RE** (2)
8.2.4	$\hat{G} = 114°$ (opp. ∠s of cyclic quad / *teenoorst. ∠e van kdvh*)	✓ **ST** **A** ✓ **RE** (2)
8.2.5	$F\hat{D}K = 66°$ (tan – chord th / *raaklyn – koord st*)	✓ **ST** **A** ✓ **RE** (2)

8.3	$\hat{H} = 50°$ (Int ∠s of Δ / *Binne ∠e van Δ*) $\therefore \hat{H} + \hat{F} \neq 180°$ ∴EHDF is not cyclic/*is nie siklies nie* (Opp. ∠s NOT supplementary / *Teenoorst. ∠e is NIE supplementêr NIE)* **OR/*OF*** $\hat{H} = 50°$ (Int ∠s of Δ / *Binne ∠e van Δ*) $\therefore \hat{H} \neq F\hat{D}K$ ∴EHDF is not cyclic/*is nie siklies nie* (Ext ∠ NOT equal to opp. int ∠ *Buite ∠ NIE gelyk aan teenoorst. binne ∠ NIE)*	✓ **ST** **CA** ✓ **RE** ✓ **RE** **OR / *OF*** ✓ **ST** **CA** ✓ **RE** ✓ **RE** (3)
		[14]

QUESTION/*VRAAG* 9

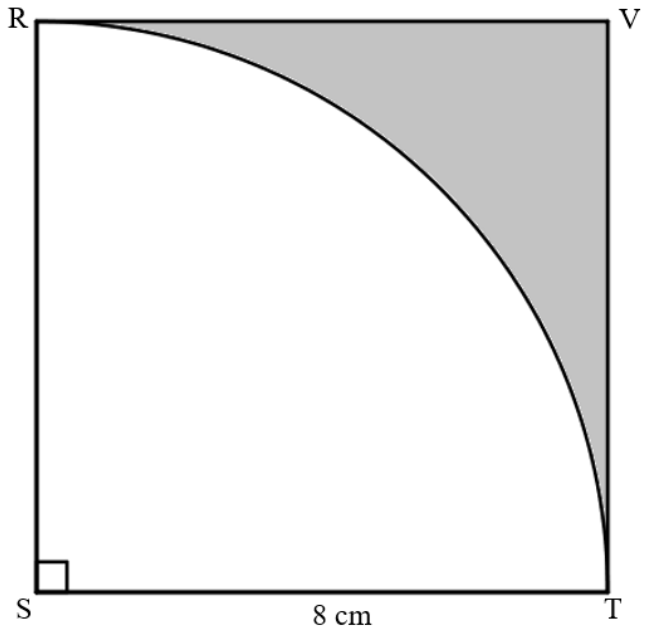


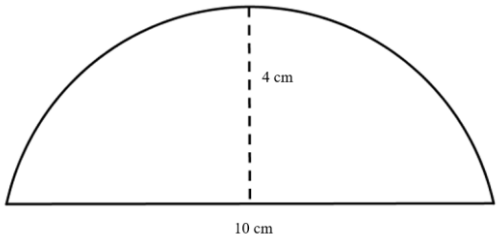
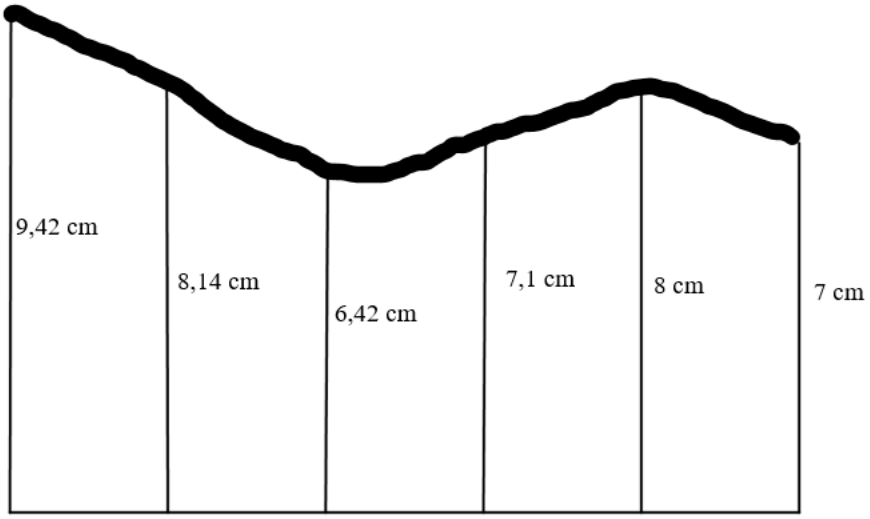
9.1	$2k+3k=20$ for some value k / *vir enige waarde k.* $\therefore 5k=20$ $\therefore k=4$ $\therefore$ AD = 8 cm and/*en* DB = 12 cm **OR/*OF*** $\text{AD}=\frac{2}{5}\times 20=8$ cm $\text{DB}=\frac{3}{5}\times 20=12$ cm	✓ setup equation / *vergelyking opstel* ✓ value of *k* / *waarde van k* ✓AD and/en DB lengths / *lengtes* **OR/*OF*** ✓ fraction $\frac{2}{5}$ ✓ AD length / *lengte* ✓ DB length / *lengte* (3)
9.2	$\frac{\text{BE}}{\text{BC}}=\frac{\text{DB}}{\text{BA}}$ (Prop th, DE || AC / *Ewer st, DE || AC*) $\frac{\text{BE}}{17,34}=\frac{12}{20}$ $\therefore \text{BE}\approx 10,40$ cm	✓ **ST** **A** ✓ **RE** ✓ BE length / *lengte* (3)
9.3	In ΔBDE and/*en* ΔBAC: $\hat{\text{B}}$ is common / *gemeenskaplik* $\hat{\text{D}}=\hat{\text{A}}$ corr. $\angle$s / *ooreenk $\angle$e; DE || AC* $\hat{\text{E}}=\hat{\text{C}}$ corr. $\angle$s / *ooreenk. $\angle$e; DE || AC* **OR/*OF*** Int $\angle$s of Δ / *Binne $\angle$e van Δ* ΔBDE ||| ΔBAC ($\angle,\angle,\angle$)	✓ **ST** **A** ✓ **ST** **RE** **A** ✓ **ST** **RE** **A** Last mark for last statement and reason OR for final reason $\angle\angle\angle$ (3)
9.4	$\frac{\text{DE}}{\text{AC}}=\frac{\text{BA}}{\text{BD}}$ (ΔBDE ||| ΔBAC) $\frac{\text{DE}}{7}=\frac{20}{12}$ $\therefore \text{DE}\approx 11,67$ cm	✓ **ST** **RE** **A** ✓ **ST** **CA** ✓ DE length / *lengte* (3)
		[12]

QUESTION/*VRAAG* 10

10.1	$108\text{ km/h} = \frac{108\text{ km}}{1\text{ h}} \times \frac{1\,000\text{ m}}{1\text{ km}} \times \frac{1\text{ h}}{3600\text{ s}} = 30\text{ m/s}$	✓ conversion factors / *herleidingsfaktore* ✓ answer / *antwoord* (2)
10.2	$v = \pi Dn$ $30\,\text{m/s} = \pi \times (0,25\,\text{m}) \times n$ $n = \frac{30}{0,25\pi}$ $n \approx 38,20\text{ rev/s}$	✓**F** ✓ conversion / *herleiding* ✓ **SF** **A** ✓ **S** ✓ answer / *antwoord* (5)
10.3	$\omega = 2\pi n$ $= 2\pi \times 38,20$ $\approx 240,02\text{ rad/s}$	✓**F** ✓ **SF** **CA** ✓ answer / *antwoord* (3)
10.4	$s = vt$ **OR/*OF*** $D = S \times T$ $= 30 \times (10\text{ min} \times 60\text{ s})$ $= 18\,000\,\text{m}$ $= 18\,\text{km}$	✓**F** ✓ **SF** **CA** ✓ answer / *antwoord* (3)
10.5	$n = \frac{\text{number of revolutions/}\textit{aantal revolusies}}{\text{time/}\textit{tyd}}$ $38,20 = \frac{20}{t}$ $t \approx 0,52\text{ sec}$	✓ **SF** **CA** ✓ answer / *antwoord* (2)
		[15]

QUESTION/*VRAAG* 11

		
11.1.1	$s = r\theta$ $\text{RT} = 8 \times \frac{\pi}{2}$ $\text{RT} = 4\pi \text{ cm}$ $\approx 12,57 \text{ cm}$	✓**F** ✓ **SF** **A** ✓ RT length / *lengte* (3)
11.1.2	$\text{Area} = \frac{rs}{2}$ $= \frac{8 \times 4\pi}{2}$ $= 16\pi \text{ cm}^2$ $\approx 50,27 \text{ cm}^2$ **OR/*OF*** $\text{Area} = \frac{r^2\theta}{2}$ $= \frac{8^2 \times \frac{\pi}{2}}{2}$ $= 16\pi \text{ cm}^2$ $\approx 50,27 \text{ cm}^2$	✓**F** ✓ **SF** **A** ✓ Area **OR/*OF*** ✓**F** ✓ **SF** **A** ✓ Area (3)
11.1.3	Shaded area/*Gearseerde area* $= \text{Area}_{\text{square/}vierkant} - \text{Area}_{\text{sector/}sektor}$ $= 8 \times 8 - 16\pi$ $\approx 13,73 \text{ cm}^2$	✓M ✓ area of square/ *app van vierkant* ✓ shaded area/ *gearseerde app*

11.2		
	$4h^2 - 4dh + x^2 = 0$ $4(4)^2 - 4d(4) + (10)^2 = 0$ $164 - 16d = 0$ $d = 10,25$ $\therefore r = 5,125 \text{ cm}$	✓**F** ✓**SF** **A** ✓**S** ✓ diameter / *middellyn* ✓ radius (5)
11.3		
	$A_T = a\left(\frac{o_1 + o_n}{2} + o_2 + o_3 + o_4 + ... + o_{n-1}\right)$ $113,61 = a\left(\frac{9,42 + 7}{2} + 8,14 + 6,42 + 7,1 + 8\right)$ $113,61 = a(37,87)$ $\therefore a = 3 \text{ cm}$ **OR/*OF*** $A_T = a(m_1 + m_2 + m_3 + \ldots + m_{n-1})$ $113,61 = a\left(\frac{9,42 + 8,14}{2} + \frac{8,14 + 6,42}{2} + \frac{6,42 + 7,1}{2} + \frac{7,1 + 8}{2} + \frac{8 + 7}{2}\right)$ $113,61 = a(37,87)$ $\therefore a = 3 \text{ cm}$	✓**F** ✓**SF** **A** ✓**S** ✓ value/*waarde* of *a* **OR/*OF*** ✓**F** ✓**SF** **A** ✓**S** ✓ value of *a* (4)
		[18]
	TOTAL/*TOTAAL*:	**150**