EASTERN CAPE
Department of Education

OR TAMBO INLAND DISTRICT

GRADE 12

MATHEMATICS CONTROLLED TEST

TERM 1 2021

Date: 16 April 2021

Marks: 50

Time: 1 hour

This question paper consists of 5 pages including cover page

INSTRUCTIONS
1. This question paper consists of **THREE** questions. Answer all the questions.
2. Clearly show all calculations you have used in determining your answers.
3. Write neatly and legibly.
4. Give all your answers to **TWO** decimal places, except stated otherwise.
5. Diagrams are NOT necessarily drawn to scale.

QUESTION 1

1.1 The following geometric sequence is given: $10\,;5\,;2{,}5\,;1{,}25\,;...$

1.1.1 Calculate the value of the 5th term, T_5 , of this sequence (1)

1.1.2 Determine the n^{th} term, T_n, in terms of n. (2)

1.1.3 Explain why the infinite series $10+5+2{,}5+1{,}25+...$ exists (2)

1.2 The sum of the first p terms of a sequence of numbers is given by:

$$S_p = p(p+1)(p+2)$$

Calculate the value of T_{20} (3)

1.3 Evaluate: $\sum_{k=1}^{\infty}\left(\frac{1}{3}\right)^k - \sum_{k=2}^{40} 5$ (5)

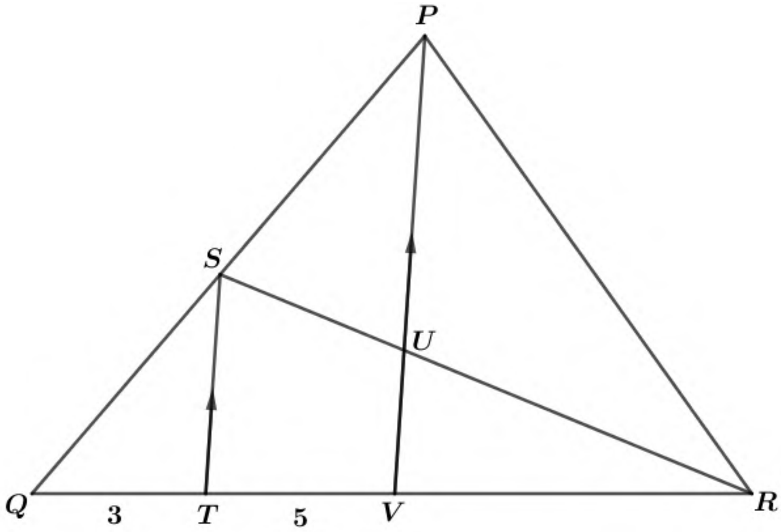
1.4 Atlehang generates a sequence, which he claims that the sequence can be arithmetic and also be geometric if and only if the first term in the sequence 1.

Is he correct? Motivate your answer by showing all your calculations (5)

(18)

QUESTION 2

2.1 In the diagram below, ΔPQR is given such that QT = 3 units, TV = 5 units, TR = 8 units and ST || PV



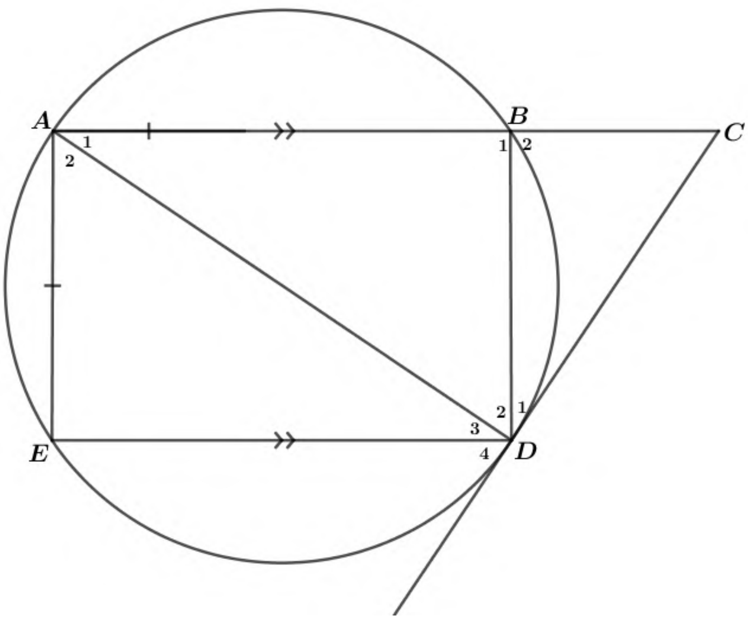
Determine the following with reasons:

2.1.1 $\frac{QS}{PS}$ (2)

2.1.2 $\frac{\text{Area } \Delta PQR}{\text{Area of quadrilateral } PSTR}$ (5)

2.2 In the diagram, a circle with a tangent CD is drawn. A, B, D, and E are points on the circumference of the circle. AE = AB and AB || ED.

$\hat{A}_1 = x$



2.2.1 Give, with reasons, three more angles equal to x. (3)

2.2.2 Prove that $\Delta DEA ||| \Delta DBC$ (2)

2.2.3 Prove that:

(a) $DC = \dfrac{BC \times AD}{DB}$ (3)

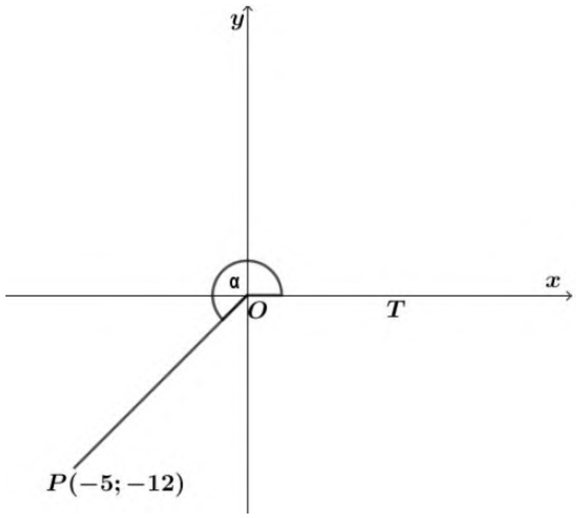
(b) $EA = \dfrac{DC \times DE}{AD}$ (3)

2.2.4 Hence, prove that $AE^2 = BC.ED$ (2)

(20)

QUESTION 3

In the diagram below, reflex $T\hat{O}P = \alpha$ and P has coordinates $(-5;-12)$.



3.1 Determine the value of each of the following trigonometric ratios WITHOUT using a calculator:

3.1.1 $\cos\alpha$ (1)

3.1.2 $\tan(180^\circ - \alpha)$ (2)

3.2 Simplify the following expression into a single trigonometric ratio:

$$\frac{\sin(x-180^\circ).\cos(x-90^\circ)}{\cos(-x-360^\circ).\sin(90^\circ+x)}$$ (6)

3.3 Given: $\cos(A-B) = cosAcosB + sinAsinB$

Use the identity for $\cos(A-B)$ to derive an identity for $\sin(A+B)$ (3)

(12)

TOTAL [50]

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $T_n = a + (n-1)d$ $S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

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MATHEMATICS CONTROLLED MARKING GUIDELINE

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This MARKING GUIDELINE consists of 5 pages including cover page

NOTE:

- If a candidate answers a question TWICE, mark the FIRST attempt ONLY.
- Consistent accuracy applies in ALL aspects of the marking guideline.
- If a candidate crossed out an attempt of a question and did not redo the question, mark the crossed-out attempt.
- The mark for substitution is awarded for substitution into the correct formula.

QUESTION 1		
1.1.1	$T_5 = 0,625$ OR $\frac{5}{8}$	✓ answer (1)
1.1.2	$T_n = 10\left(\frac{1}{2}\right)^{n-1}$ OR $20\left(\frac{1}{2}\right)^{n}$	✓✓ answer (2)
1.1.3	$-1 < r < 1$ $\therefore -1 < \frac{1}{2} < 1$	✓✓answer (2)
1.2	$S_{20} = 20(20+1)(20+2) = 9240$ $S_{19} = 19(19+1)(19+2) = 7980$ $T_{20} = S_{20} - S_{19}$ $\therefore T_{20} = 1260$	✓S_{20} ✓S_{19} ✓T_{20} (3)
1.3	$\sum_{k=1}^{\infty}\left(\frac{1}{3}\right)^k = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$ $S_\infty = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{1}{2}$ $\sum_{k=2}^{40} 5 = 5 + 5 + 5 + \dots + 5$ $n = 40 - 2 + 1 = 39$ $S_{39} = 5(39) = 195$ or $S_{39} = \frac{39}{2}(5+5) = 195$ $S_\infty - S_{39}$ $= \frac{1}{2} - 195$ $= \frac{-389}{2}$ OR -194,5	✓ substitution ✓S_∞ ✓S_{39} ✓$S_\infty - S_{39}$ ✓answer (5)

1.4	$1;1+d;1+2d;...$ $1;r;r^2;...$ $r=1+d.....................(1)$ $r^2=1+2d..................(2)$ sub.(1) in (2) $(1+d)^2=1+2d$ $1+2d+d^2=1+2d$ $d^2=0$ $\therefore d=0$ and $r=1$ $\therefore 1;1;1;...$ Arithmetic sequence $1;1;1;...$ geometric sequence He is correct	✓ eqn (1) and (2) ✓ substitution ✓ comm. diff ✓ comm. ratio ✓conclusion (5)
		(18)
QUESTION 2		
2.1.1	$\frac{QS}{PS}=\frac{3}{5}$ | Line // to one side of a Δ OR prop.theorem; $ST // PV$	✓ S ✓R (2)
2.1.2	lenght: $QS=3a$ and $QP=8a$ $Area\Delta PQR=\frac{1}{2}\times QP\times QR\sin\hat{Q}$ $=\frac{1}{2}\times 8a\times 11\sin\hat{Q}$ $=44a\sin\hat{Q}$ Area of a quad $PSTR$ = Area of ΔPQR − Area ΔSQT $=44x\sin\hat{Q}-\frac{1}{2}\times 3a\times 3\sin\hat{Q}$ $=\frac{79a}{2}\sin\hat{Q}$ $\frac{Area\Delta PQR}{Area of quad PSTR}=\frac{44a\sin\hat{Q}}{\frac{79}{2}a\sin\hat{Q}}=\frac{88}{79}$	✓ area ΔPQR ✓ area ΔSQT ✓area of a Quad PSTR ✓ ratio ✓ answer (5)

2.2.1	$\hat{D}_1 = \hat{A}_1 = x$ tan chord theorem $\hat{D}_3 = \hat{A}_1 = x$ Alt.< 's; AB//ED $\hat{D}_3 = \hat{D}_2 = x$ = chords =< 's OR < 's subtended by = chords	✓ S & R ✓ S & R ✓ S & R (3)
2.2.2	$A\hat{E}D = \hat{B}_2$ | Ext.< of a ʘ quad $\hat{D}_3 = \hat{D}_1$ | proven in 2.2.1 $\hat{A}_1 = \hat{C}_1$ | sum of <'s in a Δ $\therefore \Delta DEA \parallel\!\mid DBC$ | AAA	✓ S &R ✓ S & R (2)
2.2.3 (a)	In ΔDBC and ΔADC $\hat{D}_1 = \hat{A}_2$ tan chord theorem $\hat{C} = \hat{C}$ common < $\hat{B}_2 = A\hat{D}C$ sum of < 's in a Δ $\therefore \Delta DBC \mid\mid\mid \Delta ADC$ AAA $\frac{DB}{AD} = \frac{BC}{DC}$ |||Δ's $\therefore DC = \frac{BC \times AD}{DB}$	✓S & R ✓ΔDBC ||| ΔADC ✓ prop theorem (3)
2.2.3 (b)	$DB = AE$ chords subt. by =<' s But ΔADC |||ΔDBC proved in 2.2.3 (a) and ΔDEA|||ΔDBC proved in 2.2.2 $\therefore \Delta ADC \mid\mid\mid \Delta DEA$ both |||ΔDBC $\frac{AD}{DE} = \frac{DC}{EA}$ |||Δ's $EA = \frac{DC \times DE}{AD}$	✓ S & R ✓ΔADC|||ΔDEA ✓ prop theorem (3)

2.2.4	$EA = \left(\frac{BC \times AD}{DB}\right)\left(\frac{DE}{AD}\right)$ $EA = \frac{BC \times DE}{EA}$ since $DB = EA$ $EA^2 = BC \times DE$	✓ substitution ✓ EA (2)
		(20)
QUESTION 3		
3.1.1	$r = 13$ $\cos\alpha = \frac{-5}{13}$	✓answer (1)
3.1.2	$tan(180° - \alpha)$ $= -\tan\alpha$ $= -\left(\frac{-12}{-5}\right)$ $= -\frac{12}{5}$	✓ $-tan \propto$ ✓answer (2)
3.2	$\frac{\sin(x-180^\circ).\cos(x-90^\circ)}{\cos(-x-360^\circ)\sin(90^\circ+x)}$ $= \frac{-\sin x.\sin x}{\cos x.\cos x}$ $= -\frac{\sin^2 x}{\cos^2 x}$ $= -\tan^2 x$	✓ $-sinx$ ✓ $sinx$ ✓ $cosx$ ✓ $cosx$ ✓ $-\frac{sin^2x}{cos^2x}$ ✓ $-tan^2x$ (6)
3.3	$\sin(A+B) = \cos(90° - (A+B))$ $= \cos((90° - A) - B))$ $= \cos(90° - A)\cos(B) + \sin(90° - A)\sin(B)$ $= sinAcosB + cosAsinB$	✓correct co-function ✓regrouping ✓simplification (3)
		(12)
	TOTAL	**[50]**