GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

JOHANNESBURG WEST DISTRICT

TERM 1
CONTROLLED TEST
01 MARCH 2023

GRADE 12

MATHEMATICS

MARKS: 50 **DURATION: 1 HOUR**

This question paper consists of 6 pages including the formula sheet.

INSTRUCTIONS AND INFORMATION

1. This question paper consists of 6 questions.
2. Answer ALL the questions in your answer book.
3. Use the appropriate and correct numbering system as it is used on this paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
7. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of this paper.
10. It is in your own interest to write legibly and to present your work neatly.

QUESTION 1

1.1 Solve for x.

1.1.1 $x(2x-3)=0$ (2)

1.1.2 $0=3x^2-5x-11$ (3)

1.1.3 $x(3x+4)-2(3x+4)\leq 0$ (3)

1.2 The roots of a quadratic equation, in terms of p, are given as:

$$x=\frac{4\pm\sqrt{8-p^3}}{p}$$

Determine the value(s) of p for the roots to be real. (3)

[11]

QUESTION 2

The first four (4) terms of a quadratic pattern are: $11\ ;\ 20\ ;\ 33\ ;\ 50\ ;\ ...$

2.1 Determine the general term of this pattern in the form $T_n=an^2+bn+c$. (4)

2.2 Prove that the sum of the first n first-differences of this quadratic pattern can be given by $S_n=2n^2+7n$. (2)

[6]

QUESTION 3

A convergent geometric series is given by: $\frac{5(x+1)}{3}+\frac{5(x+1)^2}{9}+\frac{5(x+1)^3}{27}+\ ...$

3.1 Calculate the values of x . (3)

3.2 If $x=1$, calculate the sum to infinity, S_∞ . (2)

[5]

QUESTION 4

$(1-x)$; $(x+2)$ and $(2x-5)$ are the first three (3) terms of an arithmetic sequence.

4.1 Determine the value of x . (3)

4.2 If the first three (3) terms of this pattern are: 9 ; -6 ; -21 ; ... , calculate the numerical value of the sum of the first 100 terms, S_{100} . (2)

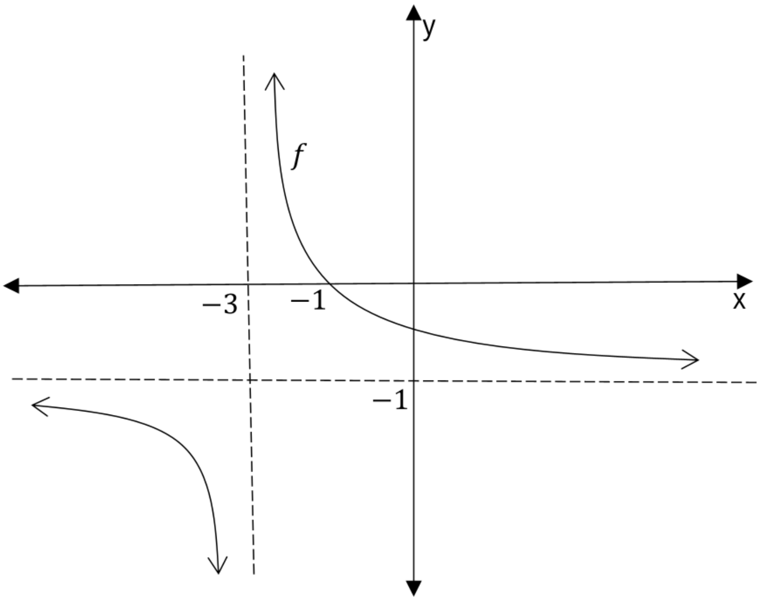
4.3 Hence, or otherwise, calculate m if :

$$\sum_{n=0}^{m-1}(24-15n)=S_{100}+73\,320$$ (4)

[9]

QUESTION 5

The graph of $f(x)=\frac{2}{x+p}+q$ is sketched below:



5.1 Write down the values of p and q (2)

5.2 The straight line $g(x)=-x+k$ is one of the axes of symmetry of the graph of f . Determine the value of k. (2)

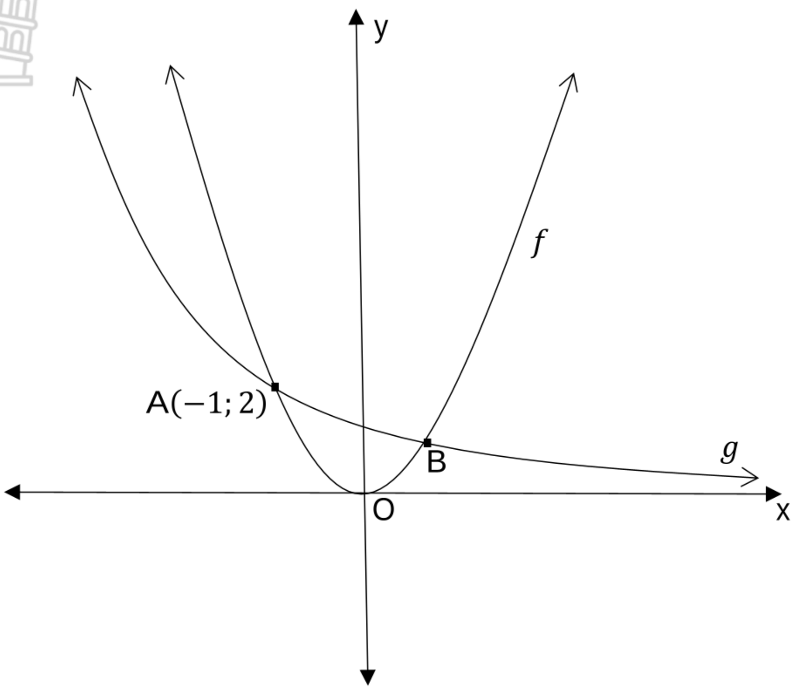
5.3 If $h(x)=-2[g(x)]$, determine the equation of the inverse of h , h^{-1} , in the form $h^{-1}(x)=mx+c$. (3)

5.4 Draw a neat sketch of the graphs of h and h^{-1} on the same set of axes. Clearly show all intercepts with axes, point of intersection and the axis of symmetry. (4)

[11]

QUESTION 6

The graphs of $f(x) = ax^2$ and $g(x) = b^x$ are sketched on the same set of axes. Points $A(-1; 2)$ and B are points of intersection of f and g. The graph of f has the turning point at the origin:



6.1 Calculate the values of a and b. (2)

6.2 The inverse of f is NOT a function. Write down at least one condition which can be used to restrict the domain of f such that its inverse will be a function. (1)

6.3 For which value(s) of x, where $x \in (-\infty; 0]$, will $g(x) \leq f(x)$? (2)

6.4 If $h(x) = g(x+3)$, write down the coordinates of …

6.4.1 A', the new coordinates of A on the graph of h. (1)

6.4.2 A'', the new coordinates of A on the graph of h^{-1}, the inverse of h. (2)

[8]

TOTAL = 50 MARKS

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}\ ; r \neq 1$ $\quad S_\infty = \frac{a}{1-r}\ ;\ -1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

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MARKING MEMORANDUM
CONTROLLED TEST 1
01 MARCH 2023

GRADE 12

MATHEMATICS

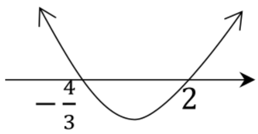
MARKS: 50

This Marking Guidelines consists of 6 pages including this cover page.

IMPORTANT NOTES AND INFORMATION

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Accept any other Mathematically valid attempt which yields a correct answer and credit full marks.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming answers/values in order to solve a problem is NOT acceptable.

QUESTION 1

Q#	Suggested Solutions	Descriptors
1.1.1	$x(2x-3)=0$ $x=0$ or $x=\frac{3}{2}$	✓ $x=0$ ✓ $x=\frac{3}{2}$ (2)
1.1.2	$0=3x^2-5x-11$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-(-5)\pm\sqrt{(-5)^2-4(3)(-11)}}{2(3)}$ $x=-1{,}25$ or $x=2{,}92$	✓ correct substitution ✓ $x=-1{,}25$ ✓ $x=2{,}92$ (3)
1.1.3	$x(3x+4)-2(3x+4)\leq 0$ $(x-2)(3x+4)\leq 0$ **Critical values**: $x=2$; $x=-\frac{4}{3}$ $-\frac{4}{3}\leq x\leq 2$ 	✓ factors/ method ✓ critical values ✓ solution (3)
1.2	For real roots: $8-p^3\geq 0$; $p\neq 0$ $-p^3\geq -8$ $p^3\leq 8$ $p\leq 2$	✓ $8-p^3\geq 0$ ✓ $p\neq 0$ ✓ $p\leq 2$ (3)
	Subtotal	**[11]**

QUESTION 2

Q#	Suggested Solutions	Descriptors
2.1	11 ; 20 ; 33 ; 50 1st differences: 9 ; 13 ; 17 2nd differences: 4 ; 4 $\therefore 2a=4$ $3a+b=9$ $a+b+c=11$ $a=2$ $3(2)+b=9$ $2+3+c=11$ $b=3$ $c=6$ $\therefore T_n=2n^2+3n+6$	✓ $a=2$ ✓ $b=3$ ✓ $c=6$ ✓ $T_n=2n^2+3n+6$ (4)
2.2	1st differences: 9 ; 13 ; 17 $a=9$; $d=4$ $S_n=\frac{n}{2}[2a+(n-1)d]$ $=\frac{n}{2}[2(9)+(n-1)4]$ $=\frac{n}{2}[14+4n]$ $=7n+2n^2$	✓ correct substitution ✓ simplification (2)
	Subtotal	**[6]**

QUESTION 3

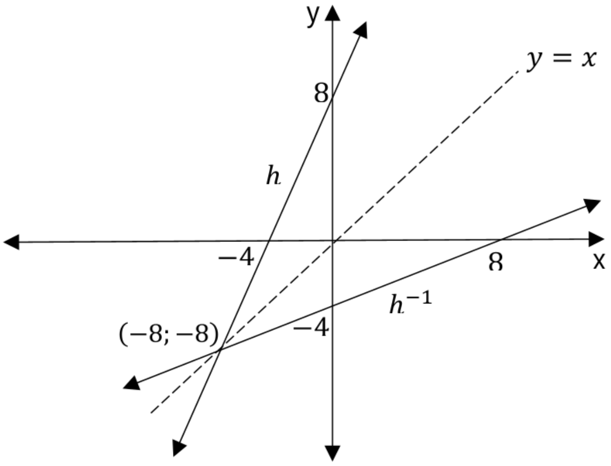
Q#	Suggested Solutions	Descriptors
3.1	$r = \frac{x+1}{3}$ $-1 < \frac{x+1}{3} < 1$ $-3 < x + 1 < 3$ $-4 < x < 2$, but $x \neq -1$	✓ $-1 < \frac{x+1}{3} < 1$ ✓ $-4 < x < 2$ ✓ $x \neq -1$ (3)
3.2	$S_\infty = \frac{a}{1-r}$ $= \frac{\frac{5(1+1)}{3}}{1-\frac{1+1}{3}}$ $= 10$	✓ correct substitution ✓ answer (2)
	Subtotal	**[5]**

QUESTION 4

Q#	Suggested Solutions	Descriptors
4.1	$(2x-5)-(x+2) = (x+2)-(1-x)$ $x - 7 = 2x + 1$ $-8 = x$	✓ method ✓ simplification ✓ $x = -8$ (3)
4.2	$9\ ;\ -6\ ;\ -21$ $a = 9, d = -15, n = 100$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{100} = \frac{100}{2}[2(9) + (100-1)(-15)]$ $= -73\ 350$	✓ correct substitution ✓ answer (2)
4.3	$\sum_{n=0}^{m-1}(24 - 15n) = S_{100} + 73\ 320$ $= -73\ 350 + 73\ 320$ $\frac{m}{2}[2(24) + (m-1)(-15)] = -30$ $m(48 - 15m + 15) = -60$ $63m - 15m^2 = -60$ $0 = 15m^2 - 63m - 60$ $0 = 5m^2 - 21m - 20$ $0 = (5m+4)(m-5)$ $m = 5$ or $m \neq -\frac{4}{5}$	✓ RHS $= -30$ ✓ $\frac{m}{2}[2(24) + (m-1)(-15)]$ ✓ standard form ✓ $m = 5$ and rejection (4)
	Subtotal	**[9]**

QUESTION 5

Q#	Suggested Solutions	Descriptors
5.1	$p = 3$ $q = -1$	✓ $p = 3$ ✓ $q = -1$ (2)
5.2	$g(x) = -x + k$ $-1 = -(-3) + k$ ... at point $(-3; -1)$ $-4 = k$	✓ correct substitution ✓ $-4 = k$ (2)

5.3	$h(x) = -2(-x-4)$ $= 2x+8$ For inverse: $x = 2y+8$ $x-8 = 2y$ $\frac{x-8}{2} = y$ $\therefore y = \frac{x}{2} - 4 \;\Rightarrow h^{-1}(x) = \frac{x}{2} - 4$	✓ $h(x) = 2x+8$ ✓ swapping x and y ✓ $y = \frac{x}{2} - 4$ (3)
5.4		✓ correct graph shapes ✓ x- and y-intercepts interchanging ✓ correct point of intersection ✓ axis of symmetry passes through the correct point of intersection (4)
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QUESTION 6

Q#	Suggested Solutions		Descriptors
6.1	$f(x) = ax^2$ $2 = a(-1)^2$ $\therefore a = 2$	$g(x) = b^x$ $2 = b^{-1}$ $b = \frac{1}{2}$	✓ $a = 2$ ✓ $b = \frac{1}{2}$ (2)
6.2	$x \leq 0$ or $x \geq 0$		✓ any one of the correct conditions (1)
6.3	$x \leq -1$ OR $x \in (-\infty; -1]$ OR $-\infty < x \leq -1$		✓ correct critical value ✓ correct notation (2)
6.4.1	A' $(-4; 2)$		✓ $(-4; 2)$ (1)
6.4.2	$A"$ $(2; -4)$		✓✓$(2; -4)$ (2)
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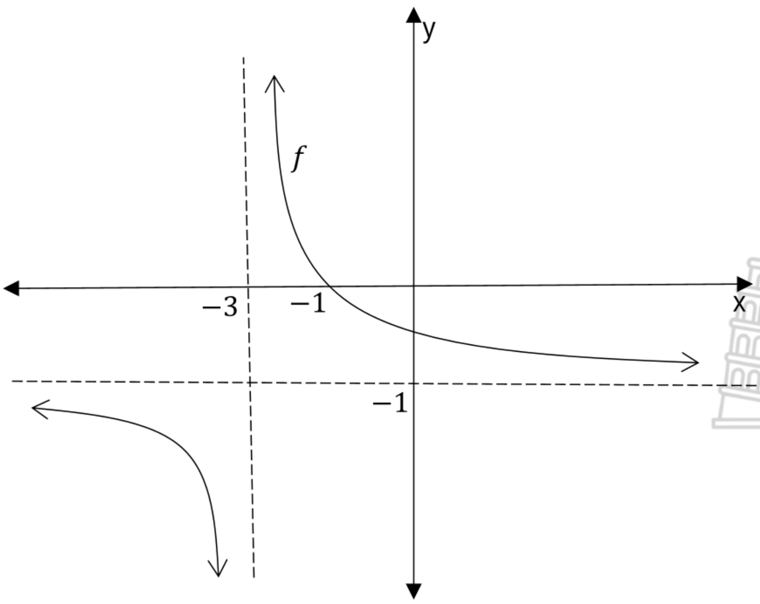

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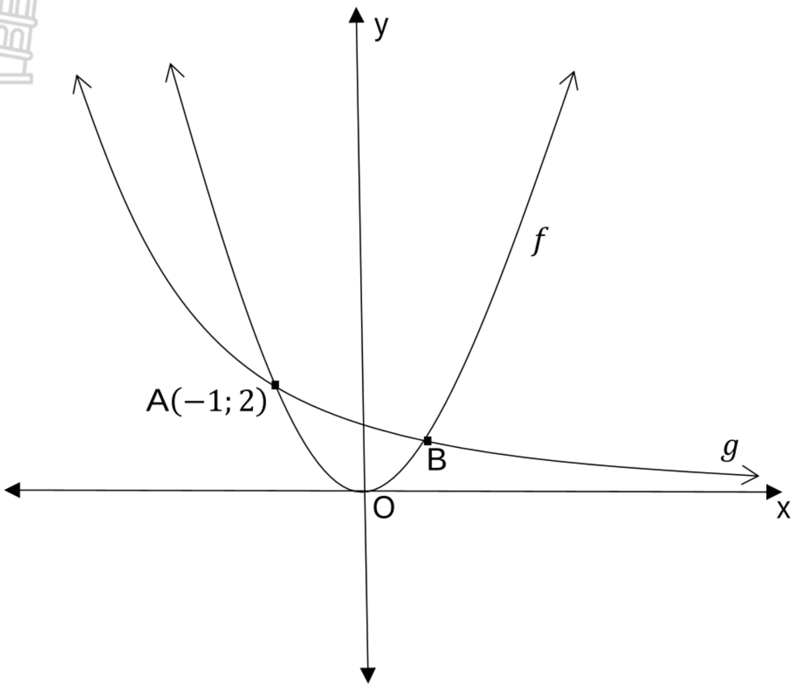

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$\hat{y} = a + bx$ $\quad b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$