LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

WATERBERG

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS TERM TEST 1

07 MARCH 2022

MARKS : 100

TIME : 2 HOURS

This question paper consists of 9 pages, including information sheet and 2 diagram sheets.

Please turn over

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 7 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. You may use ONLY approved scientific non-programmable and non-graphical calculator.
6. If necessary, round answers off to TWO decimal places, unless stated otherwise.
7. An information sheet, with formulae, is included at the end of the question paper and TWO diagram sheets.
8. Number the answers correctly according to the numbering system used in this question paper.
9. Start each QUESTION on a new page.
10. Write legibly and present your work neatly.

QUESTION 1

1.1. Consider: 9; 19; 33; 51; ...

1.1.1. Write the next TWO terms of the pattern. (2)

1.1.2. Determine the n^{th} term of the sequence. (4)

1.1.3. Prove that all the terms of the quadratic sequence are odd. (3)

1.2. Given:

$$\sum_{k=1}^{\infty} 4(0,2)^{k-1}$$

1.2.1. Write down the first THREE terms of the series. (1)

1.2.2. Calculate sum to infinity of the series. (3)

1.2.3. Hence calculate the smallest number of the terms of the series whose sum will differ by less than 0,0001 from the sum to infinity of the series. (5)

1.3. Evaluate the sum of : $3 + 11 + 3 + 15 + 3 + 19 + \cdots + 107$. (5)

[23]

QUESTION 2

2.1. Simplify : $\frac{\cos(-\theta).\tan(180°-\theta).\cos(90°-\theta)}{\sin(180°-\theta).\sin(540°+\theta)}$ (7)

2.2. Given that $\cos(A-B)=\cos A\cos B+\sin A\sin B$,

Prove that: $\sin(A+B)=\sin A\cos B+\cos A\sin B$ (3)

2.3. If $5\cos A+3=0$ and $A\epsilon[0°;180°]$, determine WITHOUT the use of calculator:

2.3.1. $\sin A$ (3)

2.3.2. $\sin 2A$ (3)

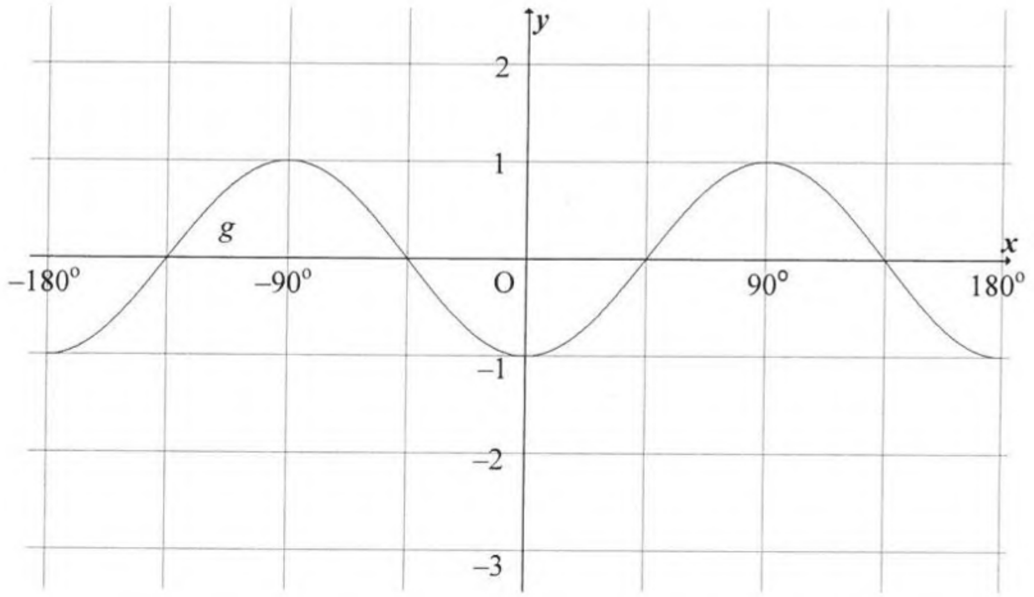
2.4. Prove that $\tan x=\frac{1-\cos 2x-\sin x}{\sin 2x-\cos x}$ (5)

[21]

QUESTION 3

3.1. Determine the general solution of : $4\sin x + 2\cos 2x = 2$ (6)

3.2. The graph of $g(x) = -2\cos 2x$ for $x\epsilon[-180°; 180°]$ is drawn below.



3.2.1. Draw the graph of $f(x) = 2\sin x - 1$ for $x\epsilon[-180°; 180°]$ on the same set of axes with $g(x)$ (3)

3.2.2. For which values of x is $g(x)$ strictly decreasing in the interval $x\epsilon[-180°; 0°]$ (2)

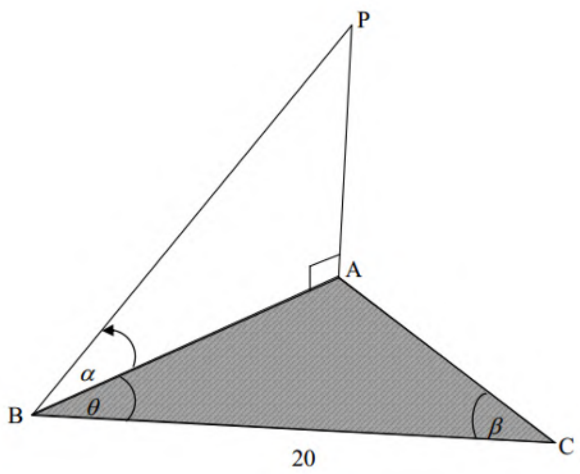
3.2.3. For which value(s) of x is $f(x + 30°) - g(x + 30°) = 0$ for $x\epsilon[-180°; 180°]$ (2)

[13]

QUESTION 4

In the diagram below, A, B and C are in the same horizontal plane. P is a point vertically above A. The angle of elevation from B to P is $\propto$.

$A\hat{C}B = \beta, A\hat{B}C = \theta$ and $BC = 20$ units.



4.1. Write AP in terms of AB and $\propto$. (2)

4.2. Prove that $AP = \frac{20 \sin \beta \tan \alpha}{\sin(\theta + \beta)}$ (3)

4.3. Given that $AB = AC$, determine AP in terms of $\propto$ and β in its simplest form. (3)

[8]

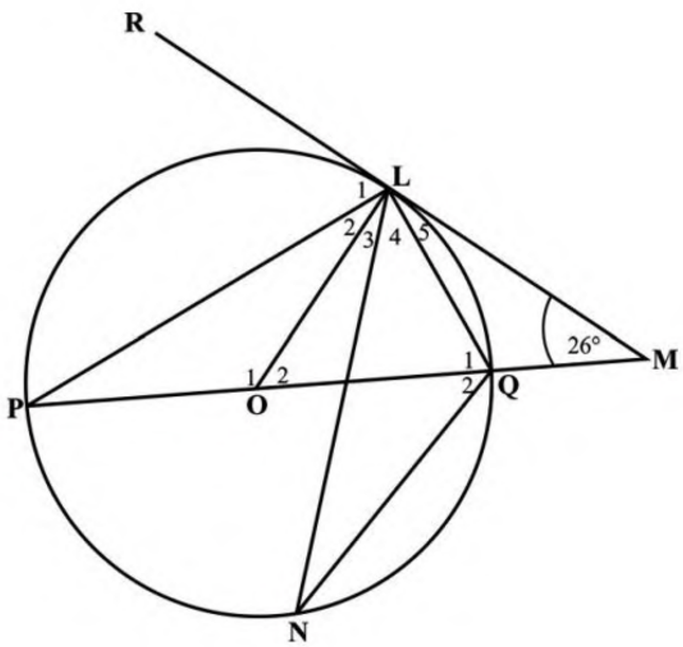
QUESTION 5

5.1. Complete the statement below by filling in the missing word(s) so that the statement is CORRECT:

An angle subtended by an arc at centre of a circle is … (1)

5.2. In the diagram, O is the centre of the circle and L is a point on the circumference.

RLM is a tangent at L.



Determine with reasons the sizes of:

5.2.1. $\hat{O}_2$ (2)

5.2.2. $\hat{L}_2$ (3)

5.2.3. $\hat{L}_5$ (2)

5.2.4. $\hat{Q}_1$ (2)

5.2.5. $\hat{N}$ (1)

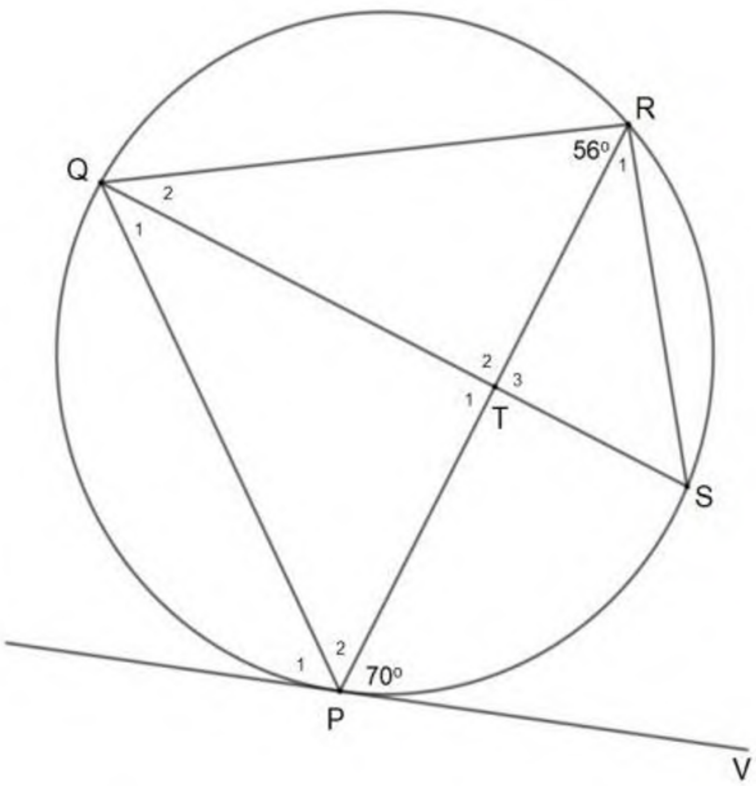
[11]

QUESTION 6

In the diagram below, P, Q, R and S are points on the circle. QS and PR intersect at point T.

The line from V is a tangent at P.

$Q\hat{R}P = 56°$ and $R\hat{P}V = 70°$



6.1. Find the size of $R\hat{S}T$ (5)

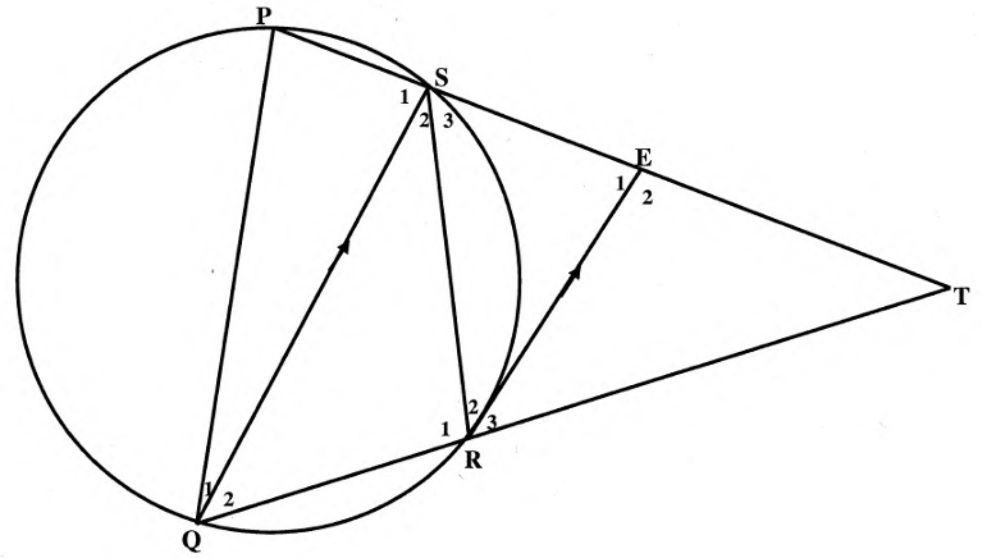
6.2. If $\hat{Q}_1 = 37°$, then explain why QS is not a diameter of the circle. (4)

6.3. Is QP parallel to RS? Justify your answer appropriately (2)

[11]

QUESTION 7

In the diagram, PQRS is a cyclic quadrilateral. PS and QR are produced to meet at T. RE is a tangent to the circle at R, with E on PT and RE ∥ QS.



Prove that:

7.1. $QR = RS$ (4)

7.2. ΔRST ||| Δ PQT (4)

7.3. $\frac{PQ}{PT} = \frac{SE}{ET}$ (5)

[13]

TOTAL: 100

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $T_n = a + (n-1)d$ $S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$ $y = mx + c$

$y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$ $(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$

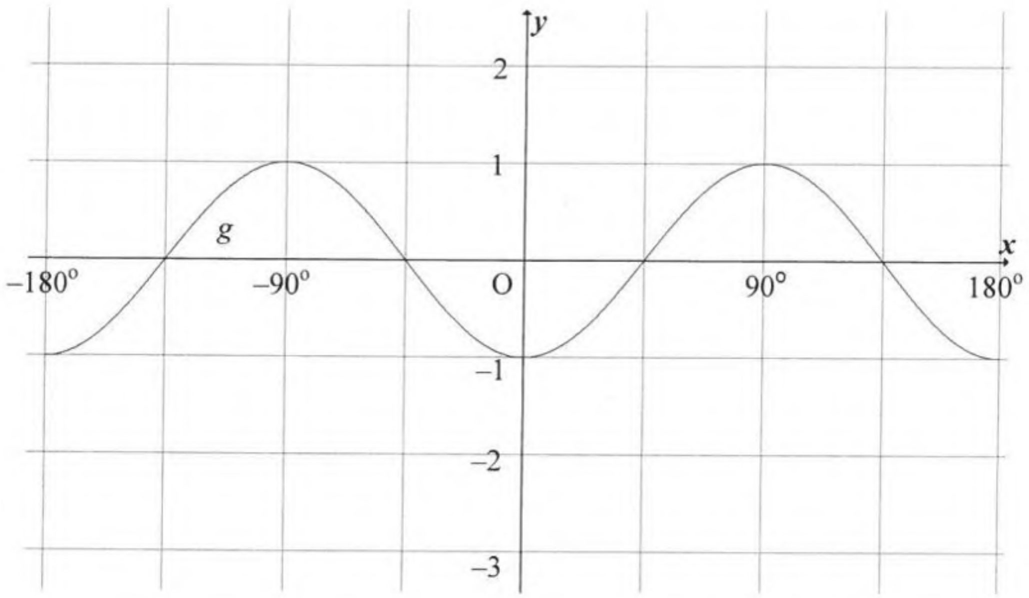
$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$

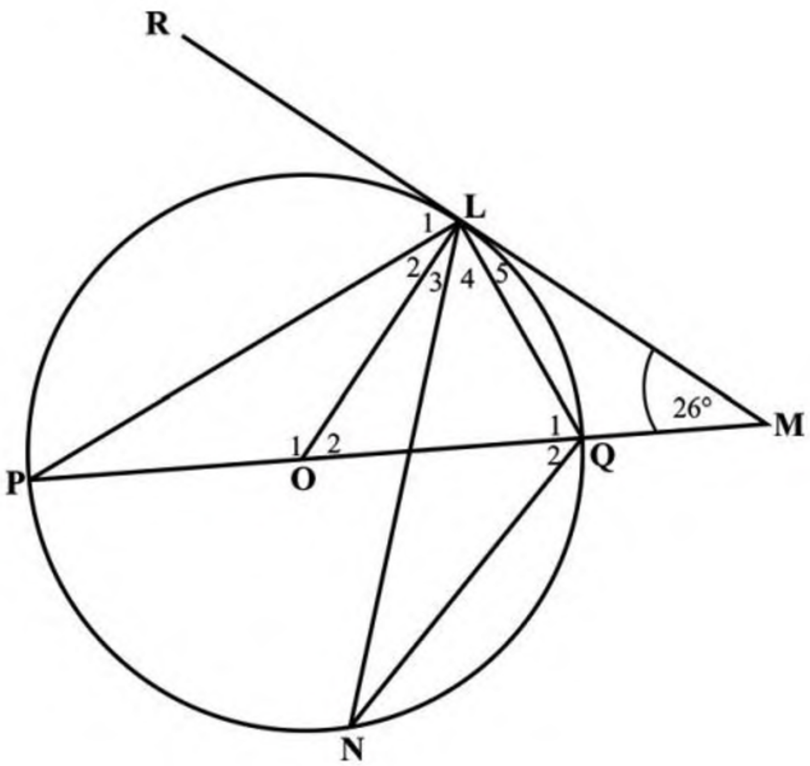
NAME AND SURNAME: ____

DIAGRAM SHEET 1

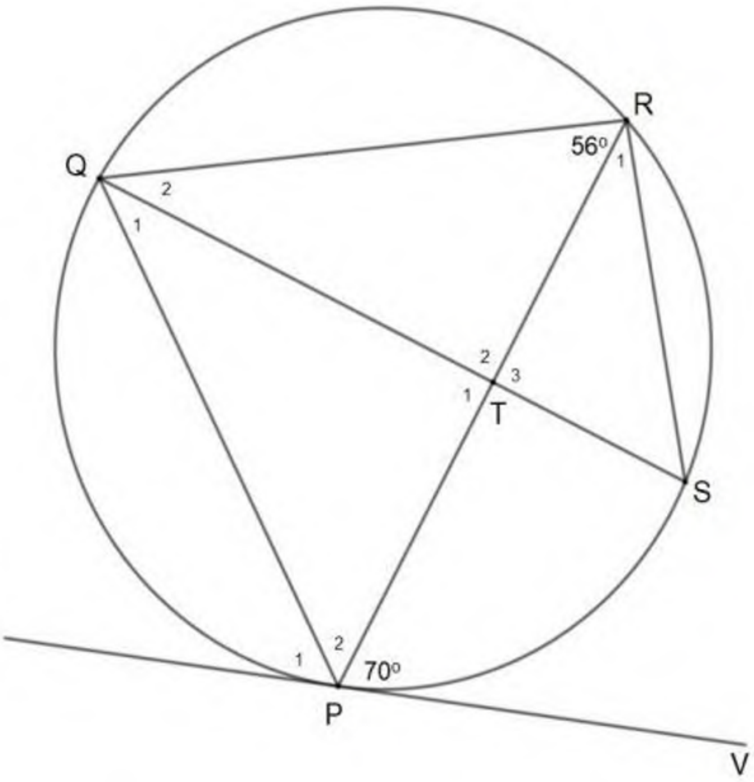
QUESTION 3.2



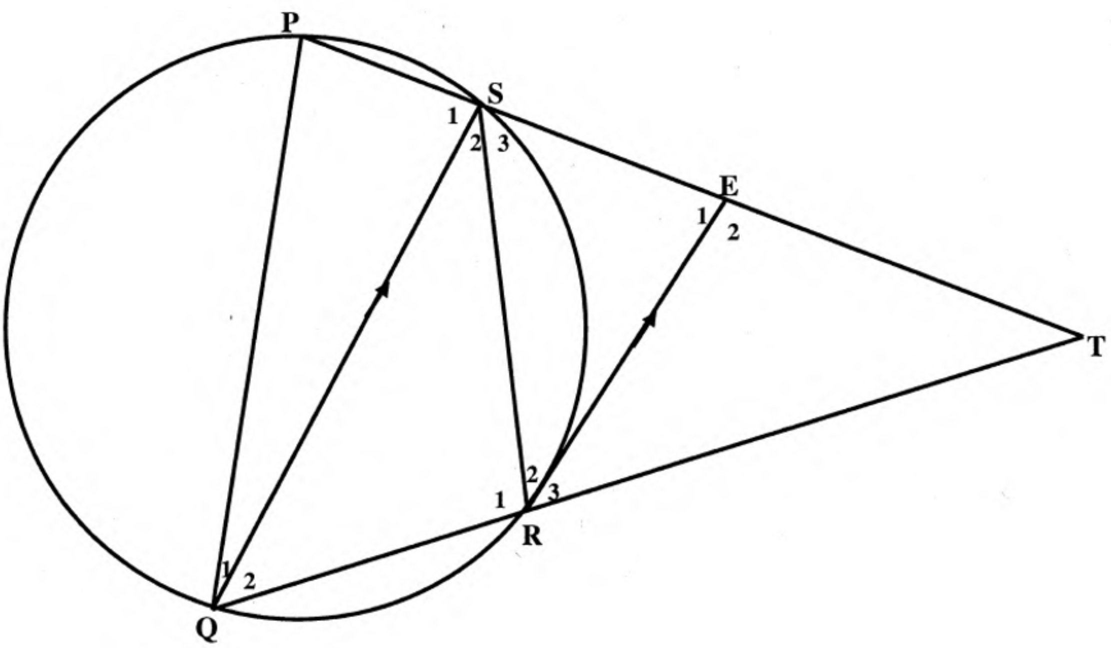
QUESTION 5



QUESTION 6



QUESTION 7



Question number	TOPIC	Sub quest.	Knowing			Performing Routine Procedures			Performing Complex Procedures			Solving problems		
			Easy	Med	Diff	Easy	Med	Diff	Easy	Med	Diff	Easy	Med	Diff
1	SEQUENCE AND SERIRIES	1.1.1	2											
		1.1.2			4									
		1.1.3							3					
		1.2.1	1											
		1.2.2					3							
		1.2.3											5	
		1.3						5						
2	TRIGONOMETRY	2.1		7										
		2.2				3								
		2.3.1					3							
		2.3.2							3					
		2.4								5				
3		3.1									6			
		3.2.1			3									
		3.2.2										2		
		3.2.3											2	
4		4.1		2										
		4.2							3					
		4.3				3								
5	EUCLIDEAN GEOMETRY	5.1	1											
		5.2.1					2							
		5.2.2								3				
		5.2.3				2								
		5.2.4				2								
		5.2.5	1											
6		6.1					5							
		6.2										4		
		6.3				2								
7		7.1									4			
		7.2						4						
		7.3								5				
TOTALS			**5**	**9**	**7**	**12**	**13**	**9**	**9**	**13**	**10**	**6**	**7**	
	100		**21**			**34**			**32**			**13**		

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07 MARCH 2022

MARKING GUIDELINES

MARKS : 100

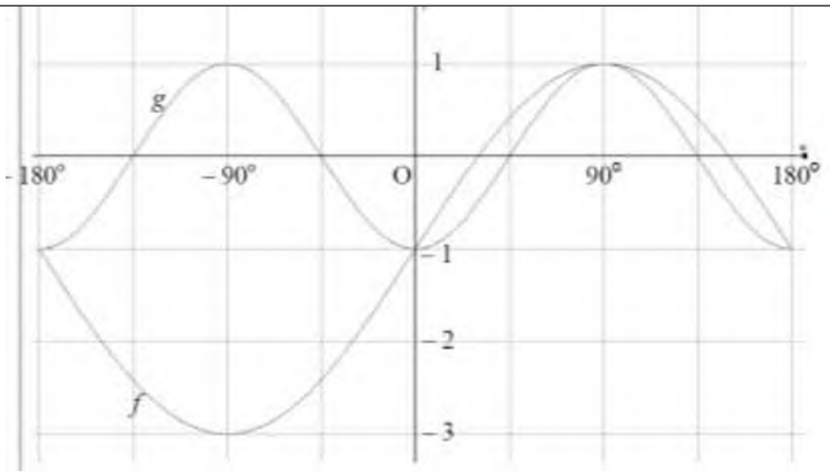
This MEMORANDUM consists of 10 pages.

	QUESTION 1	
1.1.1	73; 99	✓✓ (2)
1.1.2	9; 19; 33; 51; ... 10; 14; 18 4 4 $2a = 4$ $3a + b = 10$ $a + b + c = 9$ $a = 2$ $b = 4$ $c = 3$ $T_n = 2n^2 + 4n + 3$	✓ 2nd diff constant ✓ a ✓ b ✓ c (4)
1.1.3	$T_n = 2n^2 + 4n + 3$ $= 2\left(n^2 + 2n + \frac{3}{2}\right)$ $= 2\left[n^2 + 2n + (1)^2 - (1)^2 + \frac{3}{2}\right]$ $= 2\left[(n+1)^2 + \frac{1}{2}\right]$ $= 2(n+1)^2 + 1$	✓ common factor ✓adding special zero ✓ completed square (3)
1.2.1	$\sum_{k=1}^{\infty} 4(0,2)^{k-1}$ $4; \frac{4}{5}; \frac{4}{25}$	✓ all three (1)
1.2.2	$r = \frac{1}{5}$ $S_\infty = \frac{a}{1-r} = \frac{4}{1-\frac{1}{5}} = 5$	✓ value of r ✓ correct sub into formula ✓ answer (3)

1.2.3	$S_\infty - S_n < 0{,}0001$ $5 - \left[\frac{4\left(1-\left(\frac{1}{5}\right)^n\right)}{1-\frac{1}{5}}\right] < 0{,}0001$ $5 - 5\left(1-\left(\frac{1}{5}\right)^n\right) < 0{,}0001$ $5 - 5 + 5.5^{-n} < 0{,}0001$ $5^{1-n} < 0{,}0001$ $1 - n < \log_5 0{,}0001$ $1 - n < -5{,}7$ $-n < -6{,}7$ $n > 6{,}7$ $n = 7$	✓ inequality ✓substitutions ✓ simplification ✓ use of logarithm ✓answer (5)
1.3	$3 + 11 + 3 + 15 + 3 + 19 + \cdots + 107$ Sub series: $11 + 15 + 19 + \cdots + 107$ $4n + 7 = 107$ $n = 25$ $S_{25} = \frac{25}{2}[11 + 107] = 1475$ Sub series of 3s: $3 + 3 + \cdots + 3$ $S_{25} = 3 \times 25 = 75$ $S_{50} = 1475 + 75 = 1550$	✓ general term for even T ✓equating 107 ✓ n ✓ sum of even sub set ✓ answer **(5)**
		[23]

	QUESTION 2	
2.1	$\dfrac{\cos(-\theta).\tan(180°-\theta).\cos(90°-\theta)}{\sin(180°-\theta).\sin(540°+\theta)}$ $=\dfrac{\cos\theta\,(-\tan\theta)\sin\theta}{\sin\theta\,(-\sin\theta)}$ $=\dfrac{\cos\theta}{\sin\theta}.\dfrac{\sin\theta}{\cos\theta}$ $=1$	✓ $\cos\theta$ ✓ $-\tan\theta$ ✓ $\sin\theta$ ✓ $\sin\theta$ ✓ $-\sin\theta$ ✓ $\tan\theta=\frac{\sin\theta}{\cos\theta}$ ✓ 1 (7)
2.2	$\sin(A+B)=\cos[90°-(A+B)]$ $=\cos[(90°-A)-B]$ $=\cos(90°-A)\cos B+\sin(90°-A)\sin B$ $=\sin A\cos B+\cos A\sin B$	✓ co-fun ✓ $\sin A$ ✓ $\cos A$ (3)
2.3.1	$5\cos A+3=0$ $\cos A=-\frac{3}{5}$ $5^2=(-3)^2+y^2$ $y=\pm 4$ $\therefore y=4$ $\sin A=\frac{4}{5}$	✓ theorem of Pythagoras ✓ value of y ✓ answer (3)
2.3.2	$\sin 2A=2\sin A\cos A$ $=2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right)=-\frac{24}{25}$	✓ expansion ✓ correct sub ✓ answer (3)

2.4	$RHS = \frac{1-\cos 2x - \sin x}{\sin 2x - \cos x}$ $= \frac{1-(1-2\sin^2 x)-\sin x}{2\sin x\cos x - \cos x}$ $= \frac{2\sin^2 x - \sin x}{2\sin x\cos x - \cos x}$ $= \frac{\sin x\,(2\sin x - 1)}{\cos x\,(2\sin x - 1)}$ $= \tan x = RHS$	✓ $1-2\sin^2 x$ ✓ $2\sin x\cos x$ ✓ take $\sin x$ as a common factor ✓ take $\cos\ x$ as a common factor ✓ $\tan x$ (5)
		[21]

	QUESTION 3	
3.1	$4\sin x + 2\cos 2x = 2$ $2\sin x + 1 - 2\sin^2 x = 1$ $2\sin x\,(1 - \sin x) = 0$ $\sin x = 0 \;\; or \;\; \sin x = 1$ $x = 180°k \quad x = 90° + 360°k, k\epsilon\mathbb{Z}$	✓ $1 - 2\sin^2 x$ ✓ factors ✓ $\sin x = 0 \;\; or \;\; \sin x = 1$ ✓ $x = 180°k$ ✓ $x = 90° + 360°k$ ✓ $k\epsilon\mathbb{Z}$ (6)
3.2.1		✓ turning point $(-90°; -3)$ ✓ turning point $(90°; 1)$ ✓ shape (3)
3.2.2	$-90° < x < 0°$	✓ boundaries ✓notation (if boundary correct) (2)
3.2.3	$f(x) = g(x)$ $\therefore -180°; 0°; 90°; 180°$ $f(x + 30°) = g(x + 30°)$ $\therefore -30°; 60°; 150°$	✓ ✓ any ONE correct ✓ OTHER two (2)
		[13]

	QUESTION 4	
4.1	$\tan\alpha = \frac{AP}{AB}$ $AP = AB\tan\alpha$	✓ trig ratio ✓ answer (2)
4.2	$\frac{AB}{\sin\beta} = \frac{BC}{\sin\left(180° - (\theta+\beta)\right)}$ $AB = \frac{BC\sin\beta}{\sin(\theta+\beta)} = \frac{20sin\ \beta}{\sin(\theta+\beta)}$ $AP = \frac{20sin\ \beta}{\sin(\theta+\beta)}.\tan\alpha$ $= \frac{20sin\ \beta.\tan\alpha}{\sin(\theta+\beta)}$	✓sine rule ✓ AB ✓ substitution of AB on AP (3)
4.3	$AP = \frac{20sin\ \beta.\tan\alpha}{\sin 2\beta}$ $= \frac{20sin\ \beta.\tan\alpha}{2\sin\beta\cos\beta} = \frac{10\tan\alpha}{\cos\beta}$	✓ $\sin 2\beta$ ✓ $2\sin\beta\cos\beta$ ✓ answer (3)
		[8]

	QUESTION 5	
5.1	Twice the angle subtended by the same arc at circumference	✓ (1)
5.2.1	$M\hat{L}O = 90°$ rad ⊥ tan $\hat{O}_2 = 90° - \widehat{M} = 64°$ sum of $< s$ in a Δ	✓ S/R ✓ S/R (2)
5.2.2	$\hat{P} = \frac{\hat{O}_2}{2} = 32°$ < at centre = 2 ×< at circum $\hat{L}_2 = \hat{P} = 32°$ $< s$ opp = sides	✓ S ✓R ✓ S/R (3)
5.2.3	$\hat{L}_5 = \hat{P} = 32°$ tan- chord theorem	✓ S ✓ R (2)
5.2.4	$M\hat{L}P = 90°$ < in a $\frac{1}{2}$ ⊙ $\hat{Q}_1 = 90° - \hat{P} = 58°$ sum of $< s$ in a Δ	✓ S/R ✓ S/R (2)
5.2.5	$\widehat{N} = \hat{P} = 32°$ $< s$ in the same seg	✓ S/R (1)
		[11]

	QUESTION 6	
6.1	$R\hat{P}V = R\hat{Q}P = 70°$ tan-chord theorem $\hat{P}_2 = 180° - R\hat{Q}P - Q\hat{R}P = 54°$ sum of $<s$ in a Δ $R\hat{S}T = \hat{P}_2 = 54°$ $<s$ in the same seg	✓ S ✓ R ✓ S/R ✓ S ✓R (5)
6.2	$\hat{Q}_1 = \hat{R}_1 = 37°$ $<s$ in the same seg $Q\hat{R}S = 93° \neq 90$ ∴ QS is not a diameter, it does not subtend an angle of 90° at circumference	✓ S ✓R ✓ S ✓ R (4)
6.3	$\hat{Q}_1 = 37°$ given $\hat{S} = 54°$ proven $\hat{Q}_1 \neq \hat{S}$ ∴ QP is not parallel to RS, alt $<s$ are $\neq$	✓ S ✓ R (2)
		[11]

	QUESTION 7	
7.1	$\hat{R}_2 = \hat{Q}_2$ tan-chord theorem $\hat{S}_2 = \hat{R}_2$ alt $< s$; QS ∥ RE $\hat{Q}_2 = \hat{S}_2$ both $= \hat{R}_2$ $QR = RS$ sides opp $= < s$	✓ S ✓ R ✓ S/R ✓ R (4)
7.2	In ΔRST and in Δ PQT $\hat{T}$ is common $\hat{S}_3 = P\hat{Q}R$ ext < of a cyclic quad $S\hat{R}T = \hat{P}$ ext < of a cyclic quad / sum of $< s$ in a Δ ∴ ΔRST ||| Δ PQT $<<<$	✓ S ✓ S ✓ R ✓ S (4)
7.3.1	$\frac{RS}{PQ} = \frac{RT}{PT}$ similar Δs $\frac{PQ}{PT} = \frac{RS}{RT}$ $QR = RS$ proven $\frac{PQ}{PT} = \frac{QR}{RT}$ $\frac{QR}{TR} = \frac{SE}{ET}$ a line drawn ∥ line side of a Δ $\frac{PQ}{PT} = \frac{SE}{ET}$ both $= \frac{QR}{RT}$	✓ S ✓R ✓ S ✓S ✓ R (5)
		[13]
	TOTAL	**100**