Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

KEREITI YA 12

LOETSE 2023

MMETSE P2

MATSHWAO: **150**

NAKO: **Dihora tse 3**

Pampiri ena e na le maqephe a 13, ho kenyelletsa le leqephe le 1 la tlhahisoleseding mmoho le buka ya dikarabo e nang le maqephe a 25.

DITAELO LE TLHAHISOLESEDING

Bala ditaelo tse latelang ka hloko pele o ka araba dipotso.

1. Pampiri ena e na le dipotso tse 10.
2. Araba dipotso KAOFELA ho BUKA E KGETHEHILENG YA HO ARABELA.
3. Bontsha ka ho hlakileng KAOFELA dikhaltjhuleishene, didayakeramo, dikerafo, jwalo jwalo, tseo o di sebedisitseng ho fumana dikarabo.
4. Dikarabo feela di KE KE tsa abelwa matshwao a felletseng.
5. O ka sebedisa khaltekhuleitha ya saentifikhi e dumelletsweng(e sa porokeramuwang le e se nang dikerafo) ntle le ha ho boletswe.
6. Haeba ho hlokeha, atametsa dikarabo ho didesimale tse PEDI, ntle le ha ho boletswe.
7. Didayakeramo HA DI latele ditekanyetso tse nepahetseng.
8. Pampiri ya tlhahisoleseding e nang le difomula e kenyelleditswe qetellong ya pampiri ena.
9. Ngola ka makgethe le ka mongolo o hlakileng.

POTSO YA 1

1.1 Sehlopha sa hockey sa sekolo se rekotile lenane la diphushapho le phethilweng ke sebapadi ka seng ka motsotso. Dinomoro tsa dipapadi tse supileng ke tsena:

29 27 24 31 22 19 30

1.1.1 Khalekhuleitha:

(a) Mini (2)

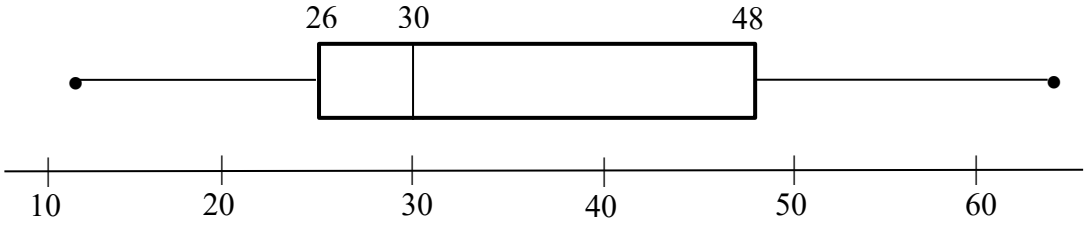
(b) Standade divieshene (1)

1.1.2 Ke dibapadi tse kae tse neng di le ho divieshene e le nngwe ya mini? (3)

1.1.3 Dibapadi tse supileng tsa sehlopha sa rugby sa sekolo le tsona di rekotile lenane la diphushapho tseo ba di phethileng ka motsotso. Dinomoro tsa bona di fane ka mini ya 26 le standade divieshene ya 3,2.

Sebedisa distandade divieshene le dimini ho bapisa lenane la diphushapho tsa dibapadi tsa dihlopha tsa rugby le hockey. (2)

1.2 Lenane la dintlha tse korilweng ke sehlopha sa rugby ho papadi ka nngwe dipapading tse 10 le bontshitswe ho box and whisker dayakeramo e ka tlase. Dintlha tsa dipapadi tse 10 di ne di sa tshwane.



1.2.1 Ke persente efe ya dipapadi moo sehlopha se korileng dintlha tse ka hodimo ho tse 30? (1)

1.2.2 Ke efe ha e le mini le mediene e ka bang kgolo? Tshehetsa karabo ya hao. (2)

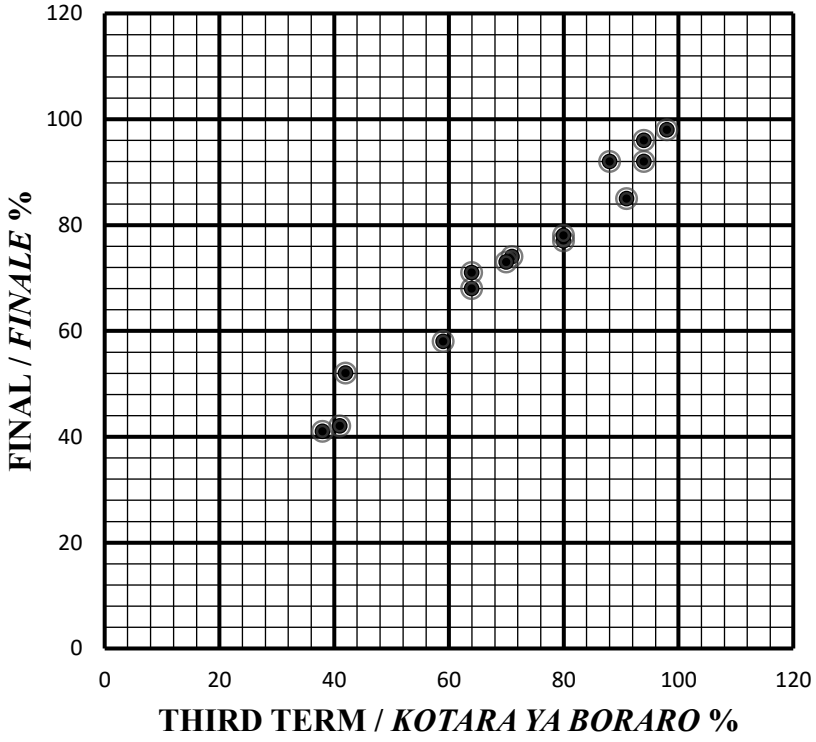
[11]

POTSO YA 2

Theibole e bontsha dipersente tse fumanweng ke sampole ya bahlahlobuwa ba 15 ho kotara ya borara le ditlhahlobo tsa mafelo a selemo sa 2022. Theibole le scatter plot tse ka tlase di bontsha matshwao ana.

Third	71	80	59	38	41	98	80	88	91	94	64	94	70	42	64
Final	74	77	58	41	42	98	78	92	85	92	68	96	73	52	71

SCATTER PLOT / *SKATHA PLOTO*



2.1 Fumana ikhweijene ya least squares regression line ya datha, atametsa dikarabo tsa hao ho didesemale tse 3. (3)

2.2 Ngola velyu ya khoreleishene khoefishiente, *r,* pakeng tsa dipersente tsa 3rd kotara le tlhahlobo ya mafelo a selemo. (1)

2.3 Mohlahlobuwa o fumane 48% ho kotara ya boraro.

2.3.1 Sebedisa ikhweijene ya least squares regression line ho nahanela dipersente tsa diphetho tsa hae. Atametsa karabo ya hao ho whole namba e haufi. (2)

2.3.2 Fana ka lebaka hobaneng ho nahanela hona ho ka nkuwa jwalo ka ho tshepahetseng. (1)

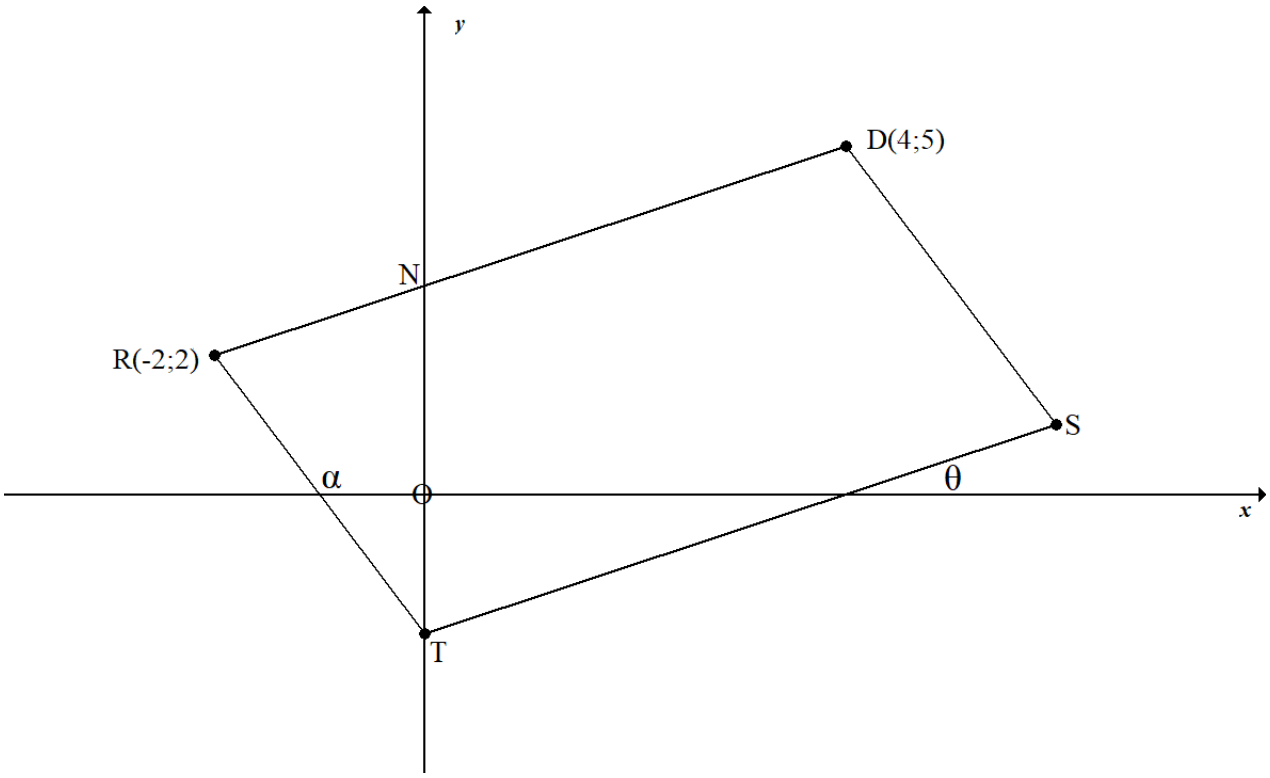
2.4 Least squares regression line e sebedisitswe ho nahanela hore persente ya mohlahlobuwa ya fumaneng 50% ho kotara ya boraro ke 80%.

2.4.1 Hobaneng ho nahanela hona e ka ba ho sa tshepahalang? (1)

2.4.2 Ekaba ho eketsa (20;10) ho datha sete ya motheo ho nyolla kapa ho theola gradiente ya least squares regression line? (1)
[9]

POTSO YA 3

Ho dayakeramo e ka tlase, D(4; 5), R(-2; 2), T le S ba etsa khwadrilatherale. RD o feta y-axis ho N mme T ke poente ho y-axis. Diinclination tsa RT le TS ke α le θ ka ho latellana. RD∥TS mme ikhweishene ya TS ke $y = \frac{1}{2}x - 2$.



3.1 Ngola dikhoodineithe tsa T. (1)

3.2 Khaltekhuleitha:

3.2.1 Gradiente ya RT (2)

3.2.2 Saeze ya $R\hat{T}S$ (5)

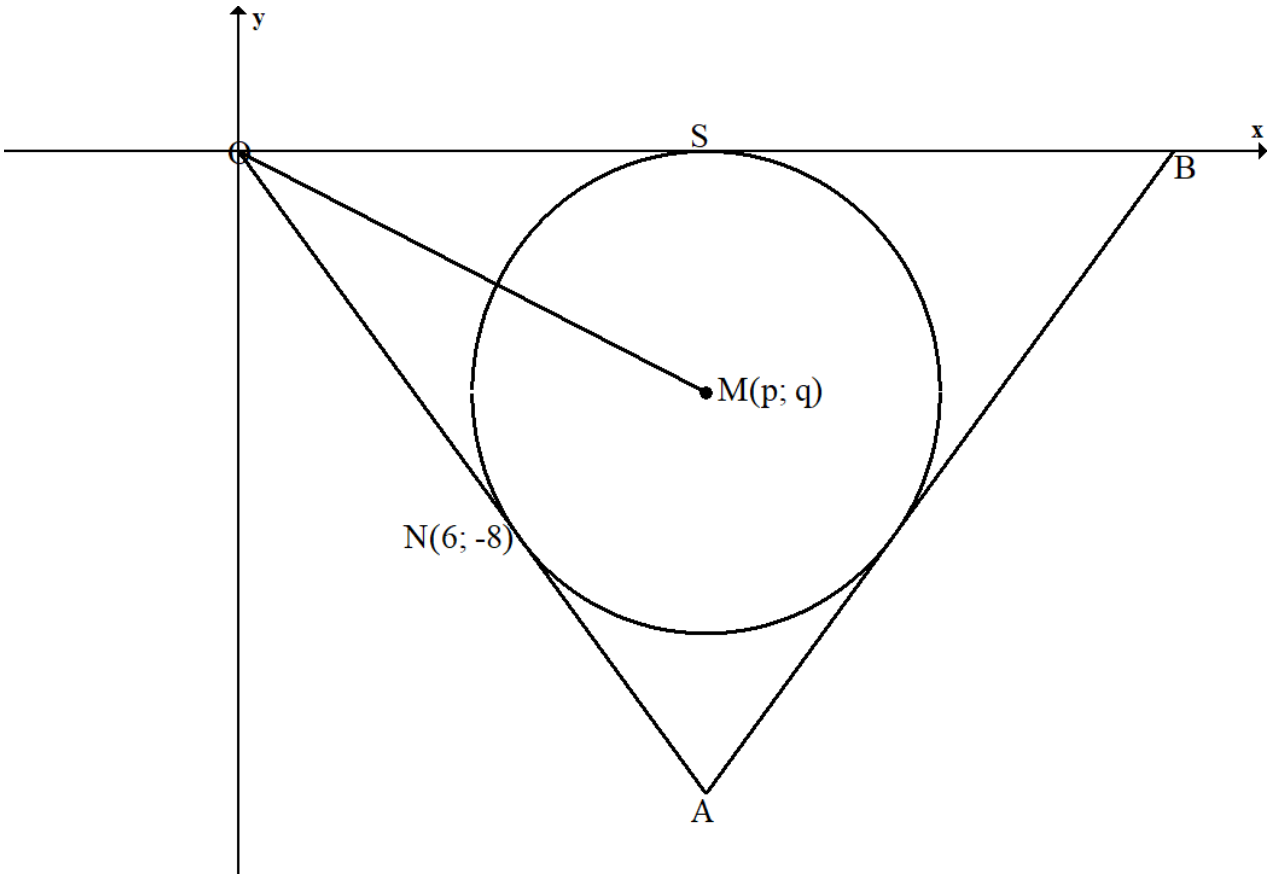
3.3 Fumana ikhweijene ya RD ka mokgwa ona $y = mx + c$. (3)

3.4 Haeba RT∥DS, khaltekhuleitha dikhoodineithe tsa M, midpoente ya RS. (3)

3.5 Khalekhuleitha eriya ya ΔRTN. (4)
[18]

POTSO YA 4

Ho dayakeramo e ka tlase, e na le mokgubu wa sekele $M(p;q)$, e thetsa x-axis ho S mme mola OA ke thanjente ya sekele ho $N(6;-8)$.



4.1 Khalekhuleitha:

4.1.1 Bolelele ba ON (2)

4.1.2 Velyu ya p (2)

4.1.3 Gradiente ya NM (3)

4.1.4 Velyu ya q (2)

4.2 Fumana ikhweishene ya sekele ka mokgwa: $(x-a)^2+(y-b)^2=r^2$. (3)

4.3 $x=k$ ke thanjente ya sekele. Ngola velyu(di) ya /tsa k. (2)

4.4 Mola $y=-\frac{4}{3}x+t$ o feta sekele ho dintlha tse pedi tse sa tshwaneng. Fumana divelyu tsa t. (6)

4.5 O nehilwe sekele e nngwe e leng $(x-10)^2+(y-6)^2=25$.

Disekele tse pedi di tla thetsana, fetana kapa di ke ke tsa thetsana/fetana? Tshehetsa karabo ya hao. (2)

[22]

POTSO YA 5

5.1 Haeba $\sin 54° = p$, bontsha tse latelang ka mokgwa wa *p*, **ntle le ho sebedisa khalekhuleitha.**

5.1.1 $\sin 594°$ (2)

5.1.2 $\cos 36°$ (2)

5.1.3 $\cos 18°$ (4)

5.2 Simplifaya **ntle le ho sebedisa khalekhuleitha**.

$$\frac{\cos 140° - \sin(90 - \theta)}{\sin 410° + \cos(-\theta)}$$ (6)

5.3 Fumana, **ntle le ho sebedisa khalekhuleitha**, velyu ya trigonometrikhi ekspreshene e latelang

$$\cos(x + 65°).\cos(x + 20°) - \sin(x + 245°).\sin(x + 20°)$$ (4)

5.4 Fumana general solution ya: $\cos^2 x - \sin^2 x = \frac{1}{2}$ (4)

5.5 O filwe ayedentithi:

$$\frac{\sin 2\theta . \tan \theta}{\cos 2\theta + 1} = \tan^2\theta$$

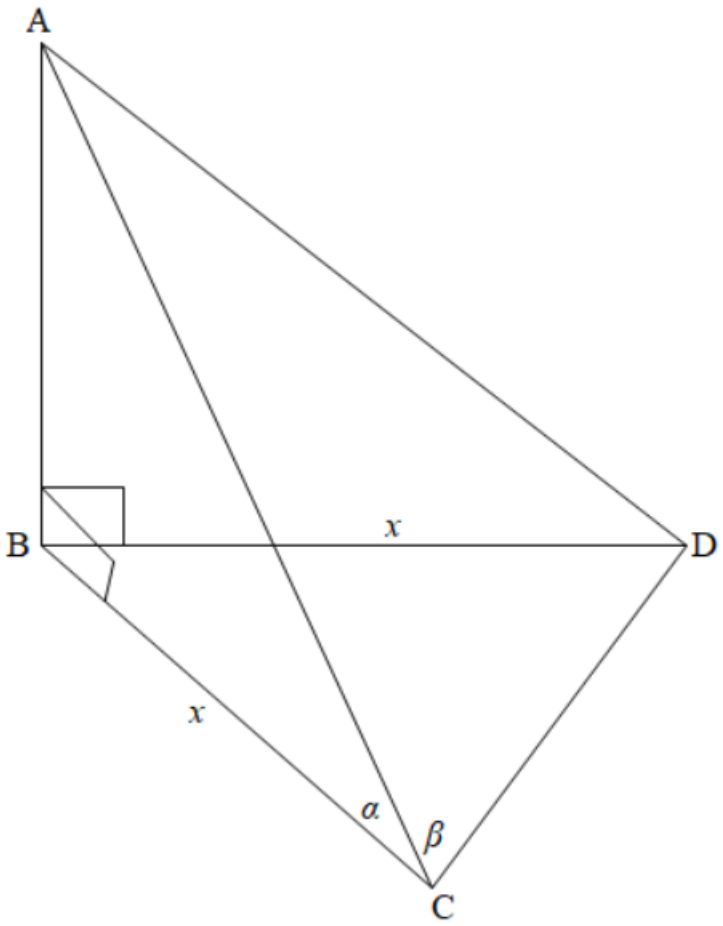
5.5.1 Pruva ayedentithi (4)

5.5.2 Fumana divelyu tsa θ moo ayedentithi e le undefined haeba $0° \leq \theta \leq 180°$. (4)

[30]

POTSO YA 6

Ho setshwantsho se latelang , B, C le D ke dipoente ho horizontale pleine e le nngwe. AB ke vethikhale thawa e nang le enkele ya eleveishene ho tloha ho C ho ya ho A e lekana le α le $A\hat{C}D = \beta$. $BD = BC = x$.



6.1 Hobaneng AC = AD? (1)

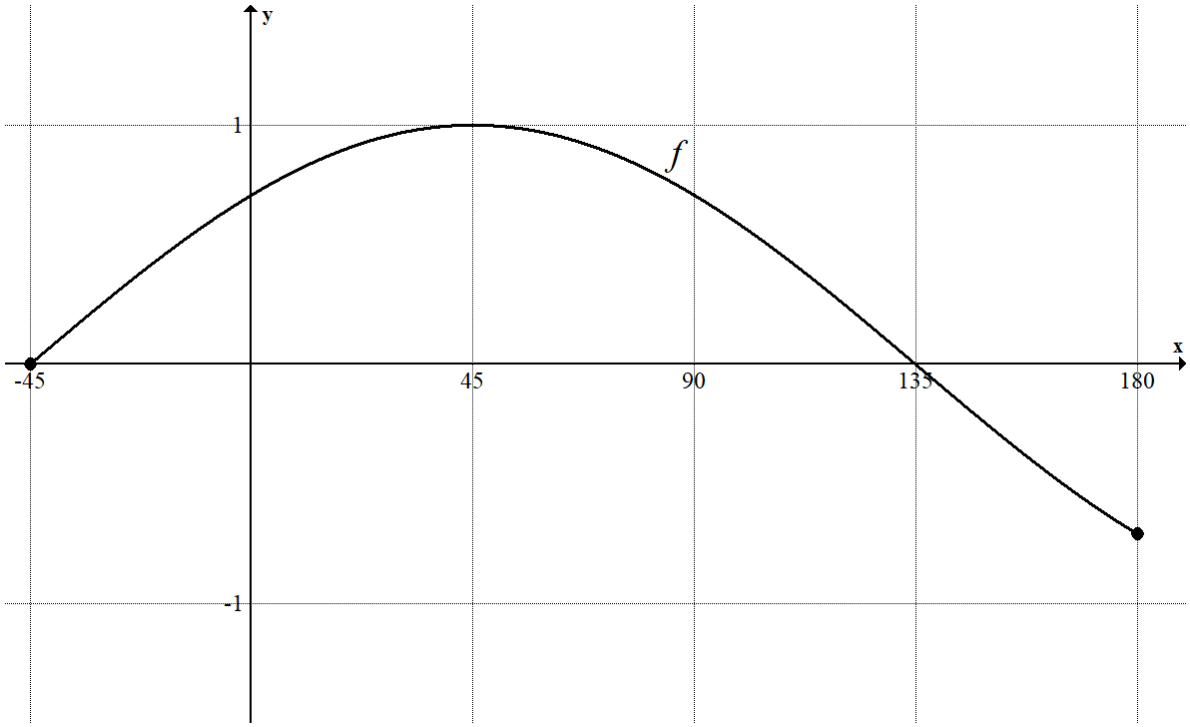
6.2 Ngola AC ka mokgwa wa x le α. (2)

6.3 Bontsha hore $CD = \frac{2x\cos\beta}{\cos\alpha}$ (4)

6.4 O nto, fumana bolelele ba CD haeba $x = 25\ cm, \alpha = 30°$ le $\beta = 65, 62°$. (2)
[9]

POTSO YA 7

Setshwantsho se ka tlase ke kerafo ya $f(x) = \cos(x - 45°)$ moo $-45° \leq x \leq 180°$. Sebedisa kerafo ho araba dipotso tse latelang.



7.1 Ngola reinje ya f , ho inthavale e nehilweng. (2)

7.2 Teroya kerafo ya $h(x) = \sin 2x$, moo $x \in [-45°;180°]$ ho sete ya diekses e le nngwe ho BUKA YA DIKARABO. Bontsha kaofela dikhoodineithe tsa diinthasepthe le diekses moho le ditheneng poente. (3)

7.3 Fana ka pheriode ya *h*. (1)

7.4 Sebedisa kerafo ho fumana divelyu tsa *x* moo f le *h* di nyolohang ka bobedi. (2)

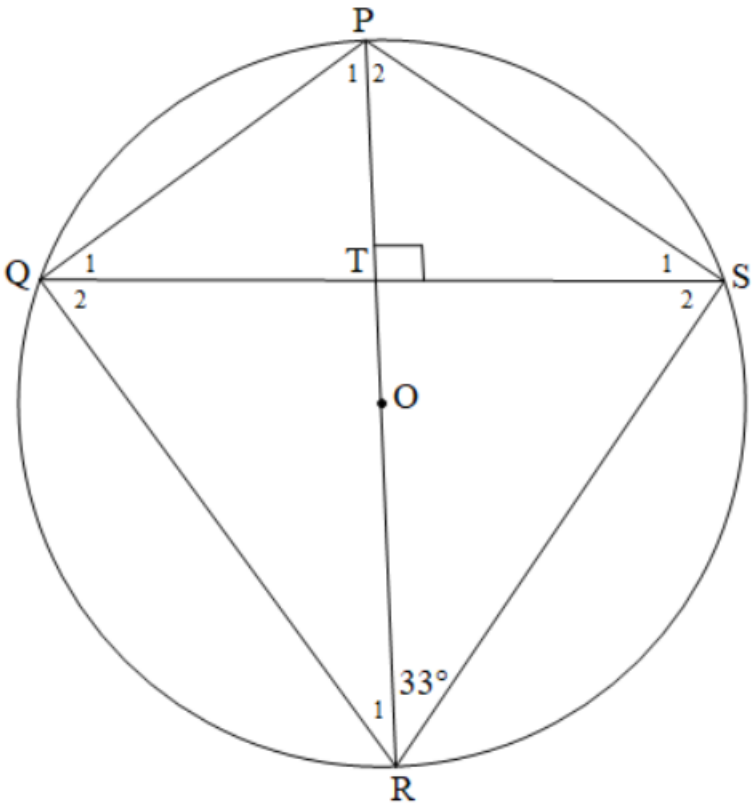
7.5 Fumana divelyu tsa *x* moo $f(x) - h(x) = 1$. (2)

7.6 Kerafo ya f e transleithuwe 60° ho ya ho lehlakore le letshehadi ho etsa kerafo ya *g*. Ngola ikhweijene ya *g* ka mokgwa o na: $g(x) =$ ____. (1)

[11]

POTSO YA 8

Ho dayakeramo e ka tlase, PR ke dayametha ya sekele PQRS e nang le mokgubu o ho O. PR o inthasektha khode QS ho T hotle $P\hat{T}S = 90°$. $P\hat{R}S = 33°$.



8.1 Fumana, o fana ka mabaka, saeze ya:

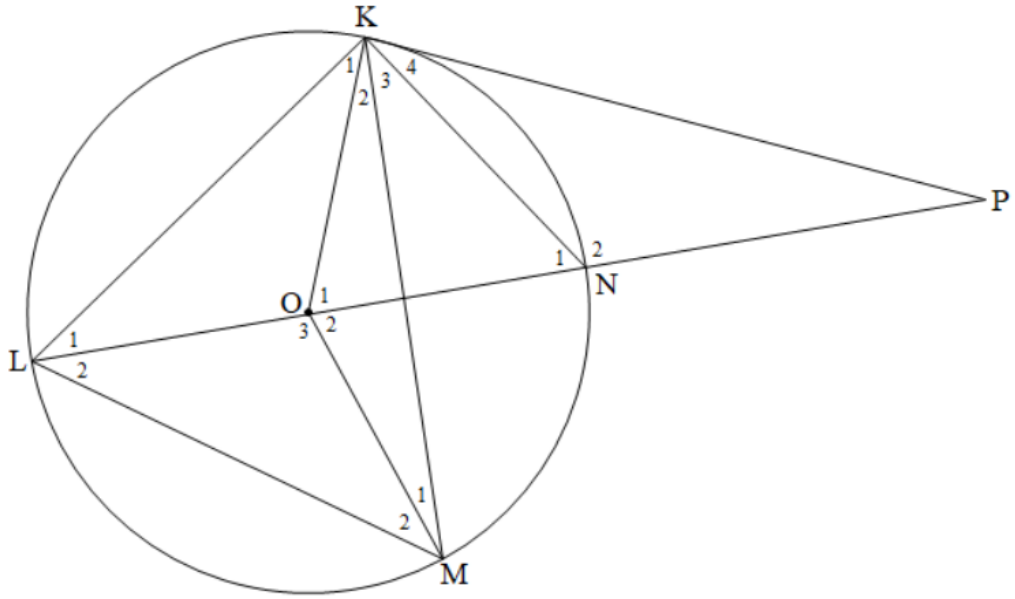
8.1.1 $\hat{P}_1$ (3)

8.1.2 $\hat{Q}_2$ (2)

8.2 Haeba QS = 16 cm mme PR = 20 cm, fumana, o fana ka mabaka, bolelele ba TO. (4)
[9]

POTSO YA 9

Ho dayakeramo e ka tlase, O ke mokgubu wa sekele mme KP ke thanjente ho sekele. LN, e leng dayametha ya sekele, e atisitswe ho kopana le KP ho P. Mela OK, OM, KM le KN e teroilwe.



9.1 Ngola dienkele tse pedi tse lekanang le $90°$. (2)

9.2 Haeba $\hat{K}_4 = x$, ngola dienkele tse latelang ka mokgwa wa x, o fana ka mabaka.

9.2.1 $\hat{L}_1$ (2)

9.2.2 $\hat{K}_1$ (2)

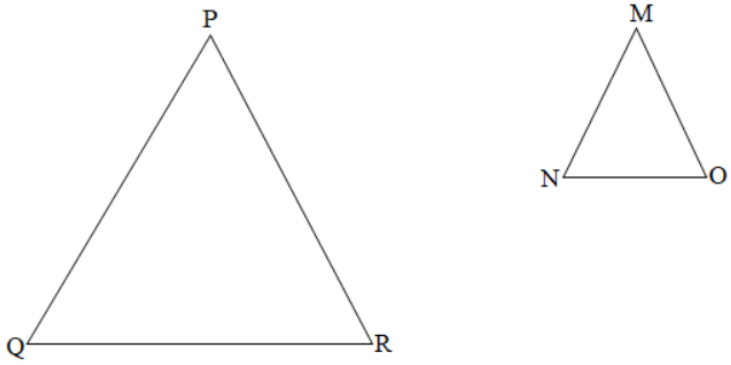
9.2.3 $\hat{P}$ (2)

9.3 Joina MP, e leng thanjente ya sekele, o nto pruva hore KOMP ke saetlekhi khwadrilatherale. (3)

[11]

POTSO YA 10

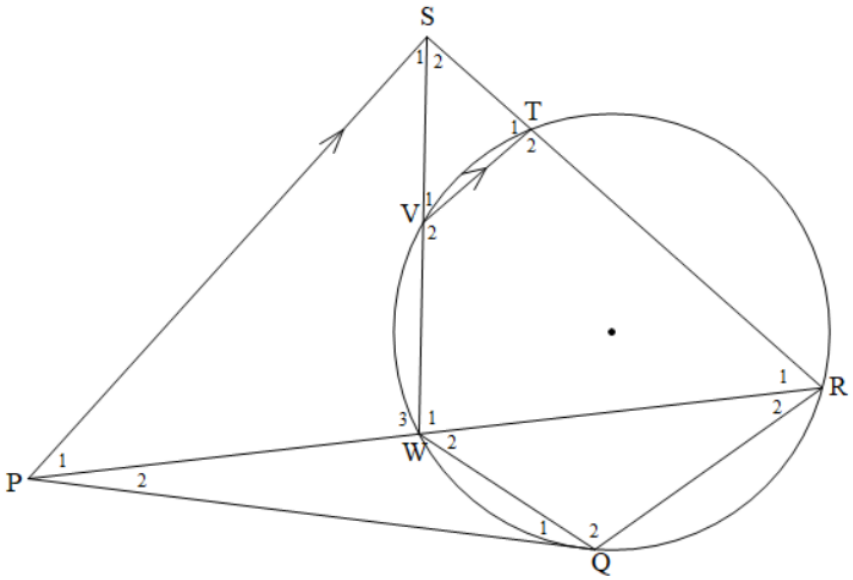
10.1 Ho dayakeramo e ka tlase, re fuwe ΔPQR le ΔMNO moo $\hat{P} = \hat{M}$, $\hat{Q} = \hat{N}$ le $\hat{R} = \hat{O}$.



Sebedisa dayakeramo e ho buka ya hao ya dikarabo ho pruva theyoreme e bolelang hore:

$$\frac{MN}{PQ} = \frac{MO}{PR} \qquad (6)$$

10.2 Ho dayakeramo e ka tlase, PQ ke thanjente ya sekele ho Q. R ke poente e ho sekele mme S o kantle ho sekele. PR o feta sekele ho W mme RS o feta sekele ho T. SW o feta sekele ho V. $VT \parallel PS$.



Pruva hore:

10.2.1 $\hat{S}_1 = \hat{R}_1$ (3)

10.2.2 $\Delta PWS ||| \Delta PSR$ (3)

10.2.3 $PQ^2 = PW.PR$ (5)

10.2.4 $PQ = PS$ (3)

[20]

***KAOFELA*: 150**

LEQEPHE LA TLHAHISOLESEDING: MMETSE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad T_n = a + (n-1)d \qquad \mathrm{S}_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad area\,\Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$