Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

IBANGA 12

SEPTEMBA 2023

IMATHEMATIKA P2

AMANQAKU: **150**

IXESHA: **3 iiyure**

Eliphepha lemibuzo linamaphepha ali13, lidibene nephepha eli1 leenkukacha, nencwadi yokuphendulela enamaphepha angama25.

IMIYALELO NEENKCUKACHA

Funda imiyalelo elandelayo ngocoselelo phambi kokuphendula imibuzo.

1. Eliphepha lemibuzo linemibuzo eli10.

2. Phendula YONKE imibuzo kwiNCWADI EKHETHEKILEYO YOKUPHENDULELA enikiweyo.

3. Bonisa ngokucacileyo ZONKE iikhaltyhuleyishini, iidayagram, iigrafu, njl ozisebenzisileyo ukubonisa iimpendulo.

4. Iimpendulo kuphela AZINYANZELEKANGA ukunikwa amanqaku apheleleyo.

5. Ungayisebenzisa ikhaltyhuleyitha esayentifikhi (engaprogranywanga nengenagrafikhi), ngaphandle kokuba uxelelwe ngeny'indlela.

6. Ukuba kunyanzelekile, sondeza iimpendulo kwiindawo EZIMBINI zedesimali, ngaphandle kokuba uxelelwe ngeny'indlela.

7. Iidayagram AZIZOTYWANGA ngokwesikeyile.

8. Iphepha leenkcukacha elineefomyula lifakiwe ekugqibeleni kwephepha lemibuzo.

9. Bhala ngokucocekileyo nangokucacileyo.

UMBUZO 1

1.1 Iqela lesikolo lehockey lirekhode inani leepush-up umdlali ngamnye azigqibe ngomzuzu. Amanani wabadlali abasixhenxe ngala:

29 27 24 31 22 19 30

1.1.1 Khaltyhuleyitha i:

(a) Min (2)

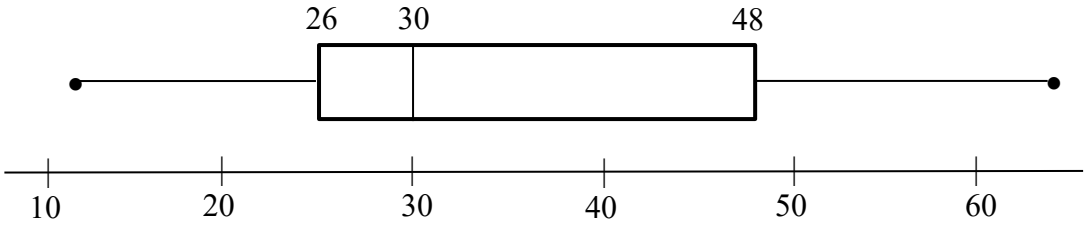
(b) Standadi diviyeyishini (1)

1.1.2 Bangaphi abadlali ababekwi diviyeyishini enye yemin? (3)

1.1.3 Abadlali abasixhenxe kwiqela lesikolo lombhoxo lirekhode inani leepush-up elizigqibe ngomzuzu. Amanani wabo anike imin yama26 nestandadi diveyishini ka3,2.

Sebenzisa iistandadi diveyishini neemin ukuthelekisa inani leepush-up zabadlali kumaqela ombhoxo nehockey. (2)

1.2 Amanqaku eqela lombhoxo kumdlalo ngamnye kweli10 ibonakaliswe kwibhokisi nakwi whisker dayagram engezantsi. Amanqaku emidlalo eli10 ebengafani.



1.2.1 Ithini ipesenti yemidlalo apho iqela lifumane ngaphezu kwama30 amanqaku? (1)

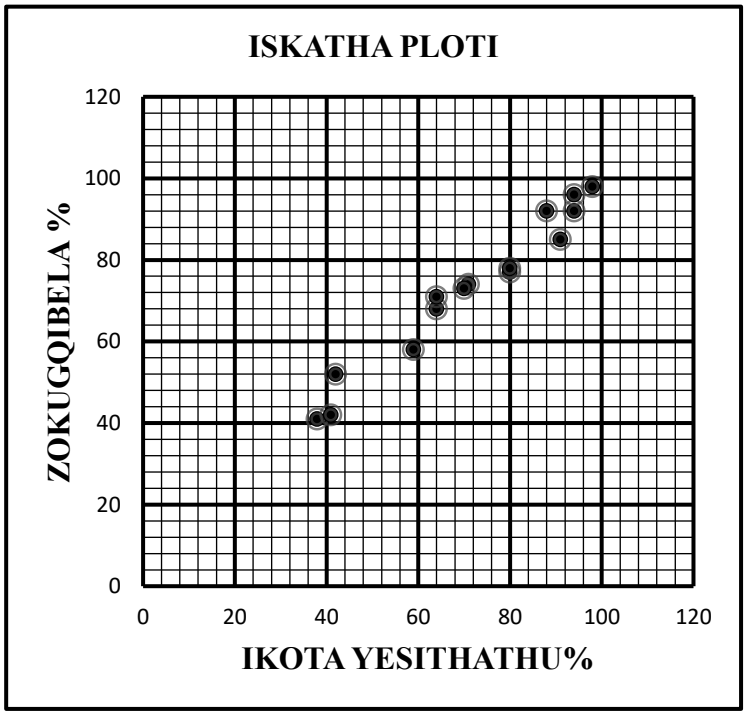
1.2.2 Yeyiphi kwimin okanye imidiyeni enokuba nkulu? Nika isizathu sempendulo yakho. (2)

[11]

UMBUZO 2

Itheyibhile ibonisa iipesenteji ezifunyenwe kwisampuli yabaviwa abali15 kwikota yesithathu nakwiimviwo zokugqibela zika2022. Itheyibhile neskatha ploti ezingezantsi zibonisa lamanqaku.

Yesithathu	71	80	59	38	41	98	80	88	91	94	64	94	70	42	64
Zokugqibela	74	77	58	41	42	98	78	92	85	92	68	96	73	52	71



2.1 Fumana i-ikhweyizhini yeleast squares regression layini yedatha, sondeza iimpendulo zakho kwiindawo ezi3 zedesimali. (3)

2.2 Bhala ivelyu yekhorileyishini khoefishiyenti, *r,* phakathi kwe3rd kota neepesenti zemviwo zokugqibela. (1)

2.3 Umviwa ufumene ama48% kwikota yesithathu.

2.3.1 Sebenzisa i-ikhweyzhini yeleast squares regression layini ukuqikelela ipesenteji yakhe yokugqibela. Sondeza impendulo yakho kwiHowuli namba ekufutshane. (2)

2.3.2 Nika isizathu sokuba uqikelelo luthembeke. (1)

2.4 Ileast squares regression layini isetyenziswa ukuqikelela ukuba ipesenteji yokugqibela yomviwa ofumene ama50% kwikota yesithathu ngama80%.

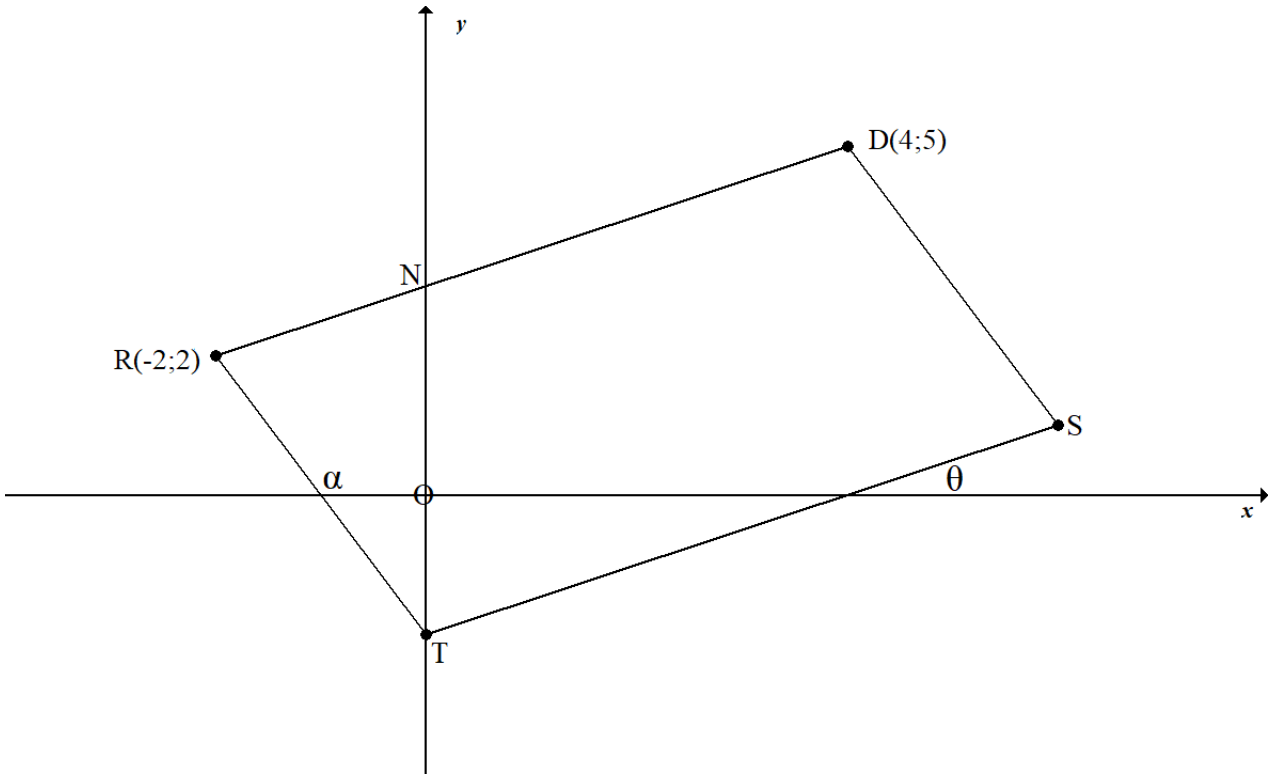
2.4.1 Kutheni olu qikelelo lungenakuthembakala? (1)

2.4.2 Ingaba ukudibanisa upoyinti (20;10) kwidatha seti ubuyinikwe ekuqaleni kuyayinyusa okanye iyayehlisa igradiyenti yeleast squares regression layini? (1)

[9]

UMBUZO 3

Kwidayagram engezantsi, uD(4; 5), uR(-2; 2), uT noS benza ikhwadrilatherali. uRD unqumla i *y*-ekhsisi kuN aze uT abeyipoyinti kwi y-ekhsisi. I-inklineyishini kaRT noTS ngu α noθ ngokulandelelana. uRD∥TS ize i-ikhweyizhini kaTS ibe ngu $y = \frac{1}{2}x - 2$.



3.1 Bhala iikho-odineyithi zikaT. (1)

3.2 Khaltyhuleyitha i:

3.2.1 gradiyenti kaRT (2)

3.2.2 sayizi ka$R\hat{T}S$ (5)

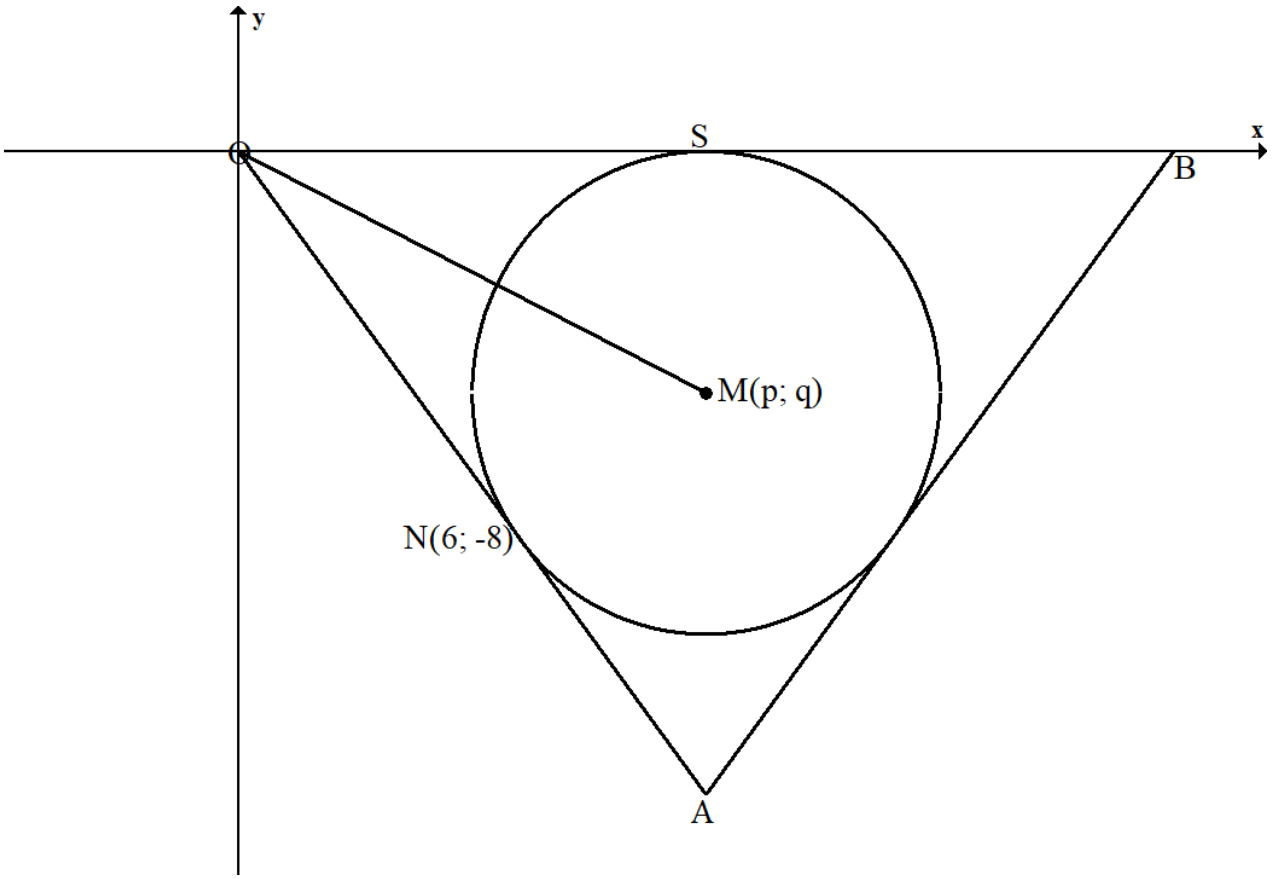
3.3 Fumana i-ikhweyizhini kaRD ngokwefom: $y = mx + c$. (3)

3.4 Ukuba uRT∥DS, khaltyhuleyitha iikho-odineyithi zikaM, imidipoyinti kaRS. (3)

3.5 Khaltyhuleyitha ieriya ka ΔRTN. (4)
[18]

UMBUZO 4

Kwidayagram engezantsi, isekile, enombindi oku $M(p;q)$, ikrweca i *x*-ekhsisi kuS ze ulayini OA abe yithanjenti kwisekile ku $N(6;-8)$.



4.1 Khaltyhuleyitha:

4.1.1 Ubude bukaON (2)

4.1.2 Ivelyu ka p (2)

4.1.3 Igradiyenti ka NM (3)

4.1.4 Ivelyu ka q (2)

4.2 Fumana i-ikhweyizhini yesekile ngokwefom: $(x-a)^2+(y-b)^2=r^2$. (3)

4.3 $x=k$ yithanjenti kwisekile. Bhala iivelyu zika k. (2)

4.4 Ulayini $y=-\frac{4}{3}x+t$ unqumla isekile kwipoyinti ezimbini ezahlukileyo. Fumana iivelyu zika t. (6)

4.5 Enye isekile eneikhweyizhini $(x-10)^2+(y-6)^2=25$ inikiwe.

Ingaba isekile ezimbini zizokrwecana, zinqumlane okanye hayi? Nika isizathu sempendulo yakho. (2)

[22]

UMBUZO 5

5.1 Ukuba u $\sin 54° = p$, ekhspresa nganye kwezilandelayo ngokwethem zika *p*, **ungasebenzisi khaltyhuleyitha.**

5.1.1 $\sin 594°$ (2)

5.1.2 $\cos 36°$ (2)

5.1.3 $\cos 18°$ (4)

5.2 Simplifaya okulandelayo **ungasebenzisi khaltyhuleyitha**.

$$\frac{\cos 140° - \sin(90 - \theta)}{\sin 410° + \cos(-\theta)}$$ (6)

5.3 Fumana, **ungasebenzisi khaltyhuleyitha**, ivelyu yaletrigonometrikhi ekhspreshini ilandelayo.

$$\cos(x + 65°).\cos(x + 20°) - \sin(x + 245°).\sin(x + 20°)$$ (4)

5.4 Fumana ijenerali solushini ka: $\cos^2 x - \sin^2 x = \frac{1}{2}$ (4)

5.5 Unikwe iayidentithi:

$$\frac{\cos 2\theta + 1}{\sin 2\theta.\tan\theta} = \frac{1}{\tan^2\theta}$$

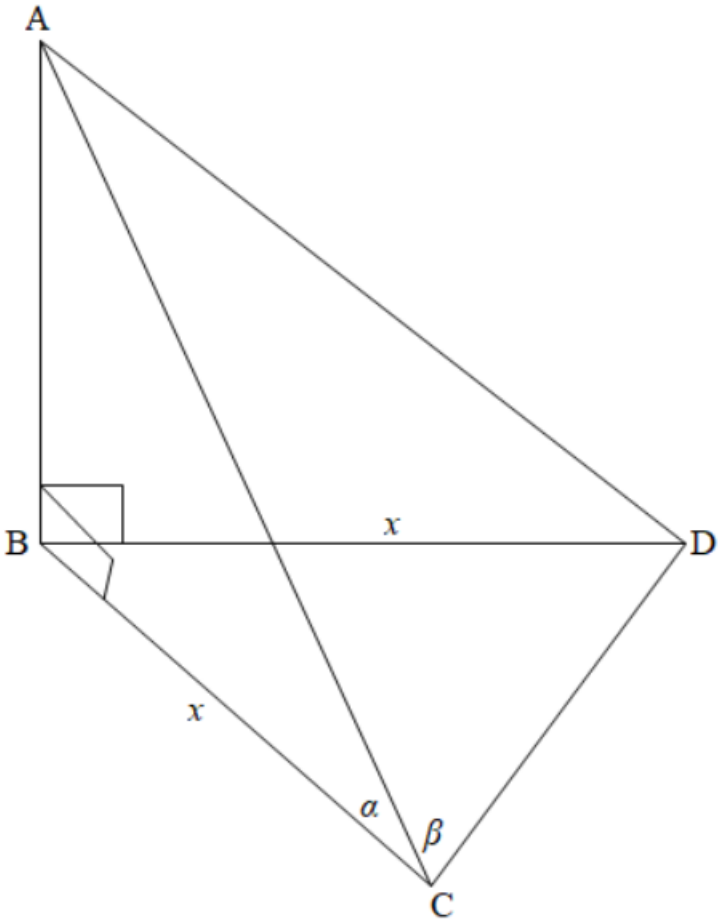
5.5.1 Pruva iayidentithi (4)

5.5.2 Fumana iivelyu zika θ apho iayidentithi iundefined $0° \leq \theta \leq 180°$. (4)

[30]

UMBUZO 6

Kwifiga engezantsi, uB, uC noD ziipoyinti kwihorizontali pleyini enye. uAB yivethikhali thawa ene-engile ye-eleveyishini ukusuka kuC ukuya kuA ilingana no α aze uA$\hat{C}$D $= \beta$. BD $=$ BC $= x$.



6.1 Kutheni uAC = AD? (1)

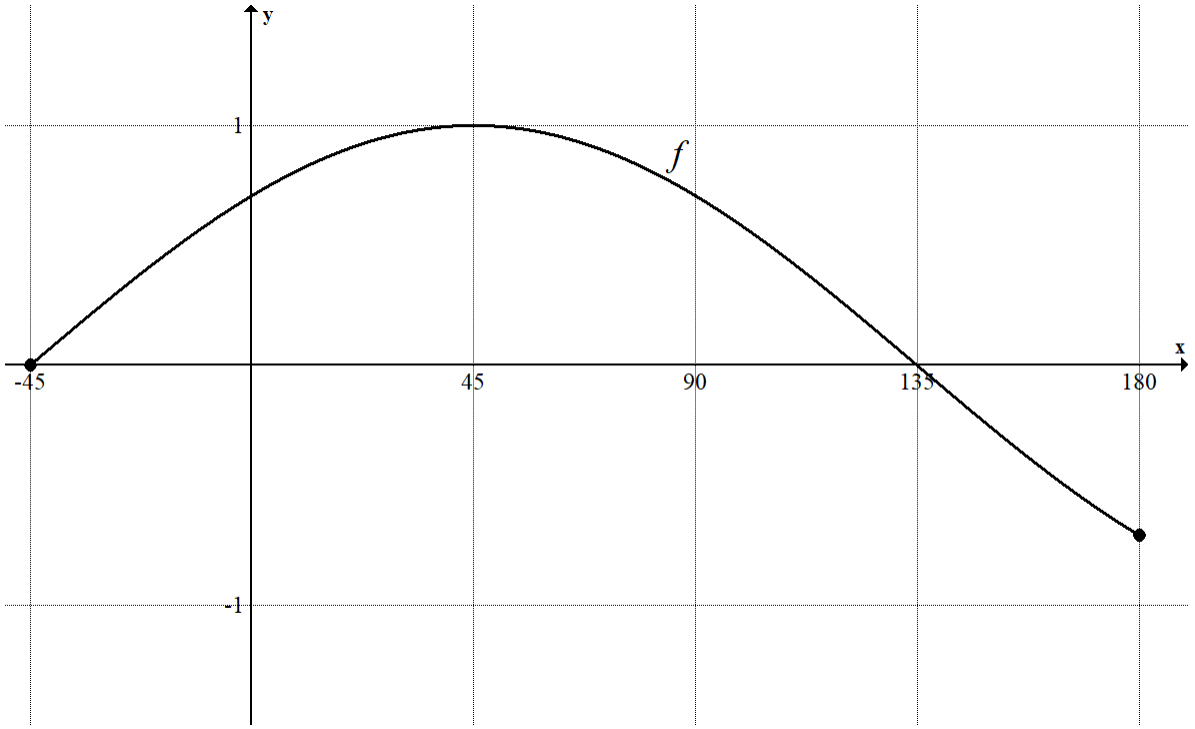
6.2 Bhala uAC ngokweethem zika x no α. (2)

6.3 Bonisa ukuba u CD $= \frac{2x\cos\beta}{\cos\alpha}$ (4)

6.4 Ngoko, fumana ubude bukaCD ukuba u $x = 25\ cm, \alpha = 30°$ no $\beta = 65{,}62°$. (2)
[9]

UMBUZO 7

Kuzotywe ngezantsi igrafu ka $f(x) = \cos(x - 45°)$ apho u $-45° \leq x \leq 180°$. Sebenzisa igrafu ukuphendula umbuzo olandelayo.



7.1 Bhala ireyinji ka f, kwi-intavali enikiweyo. (2)

7.2 Zoba igrafu ka $h(x) = \sin 2x$, apho u $x \in [-45°;180°]$ kwiseti enye ye-ekhzisi no f KWINCWADI YOKUPHENDULELA. Bonisa iikho-odineyithi zeeintasephthi zonke zine ekhzisi kunye neetheningi poyinti. (3)

7.3 Xela iphiriyodi ka h. (1)

7.4 Sebenzisa igrafu yakho ukufumana iivelyu zika x apho u f no h bekhula bobabini. (2)

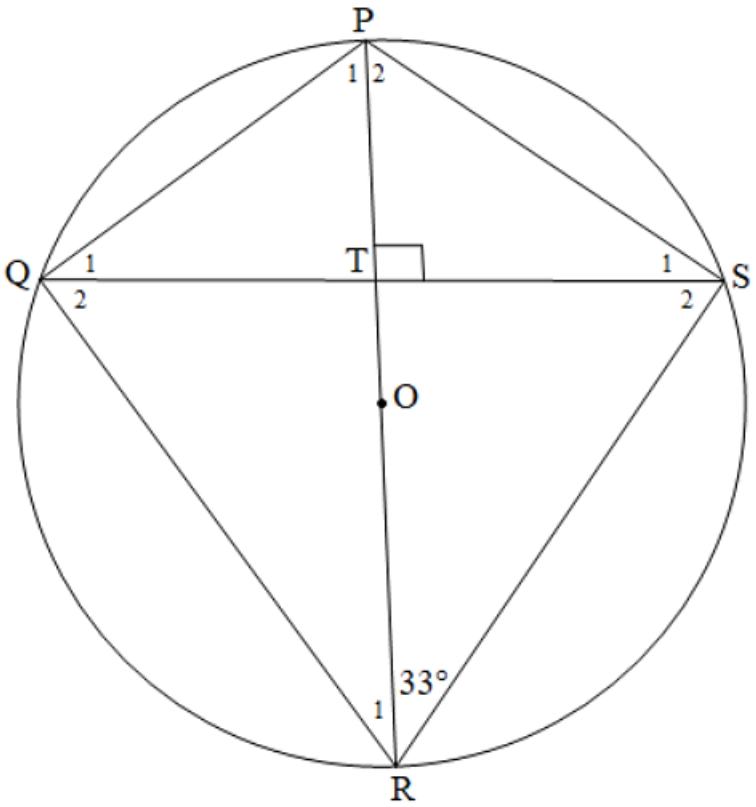
7.5 Fumana iivelyu zika x apho u $f(x) - h(x) = 1$. (2)

7.6 Igrafu ka f itransleyithwe 60° ukuya ekhohlo ukwenza igrafu ka g. Bhala i-ikhweyizhini ka g ngokwefom: $g(x) =$ ____. (1)

[11]

UMBUZO 8

Kwidayagram engezantsi, uPR yidayametha yesekile uPQRS enombindi onguO. uPR udibana nekhodi uQS kuT ze uP$\hat{T}$S = 90°. P$\hat{R}$S = 33°.



8.1 Fumana, unika izizathu, isayizi ka:

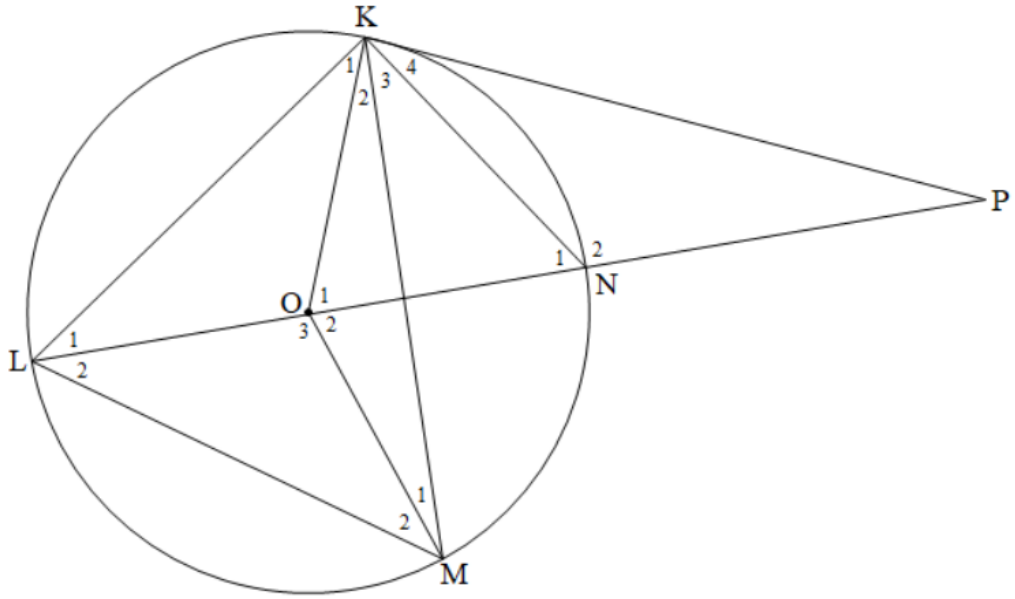
8.1.1 $\hat{P}_1$ (3)

8.1.2 $\hat{Q}_2$ (2)

8.2 Ukuba uQS = 16 cm aze uPR = 20 cm, fumana, unika izizathu, ubude bukaTO. (4)
[9]

UMBUZO 9

Kwidayagram engezantsi, uO ngumbindi wesekile aze uKP abeyithanjenti kwisekile. uLN, yidayametha yesekile, yandisiwe ukuya kudibana noKP ku P. Imigca uOK, uOM, uKM noKN zizotyiwe.



9.1 Bhala iiengile ezimbini ezilingana no $90°$. (2)

9.2 Ukuba u$\hat{K}_4 = x$, bhala iiengile ezilandelayo ngokweethem zika *x*, unika izizathu.

9.2.1 $\hat{L}_1$ (2)

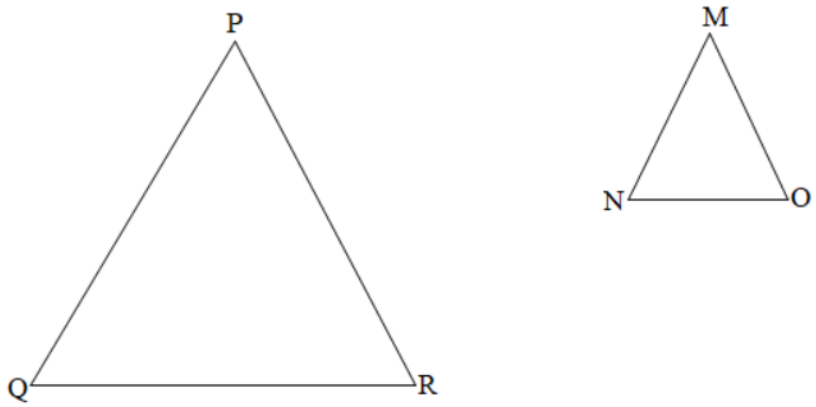
9.2.2 $\hat{K}_1$ (2)

9.2.3 $\hat{P}$ (2)

9.3 Joyina uMP, oyithanjenti kwisekile, uze upruve ukuba uKOMP yisayklikhi khwadrilatherali. (3)
[11]

UMBUZO 10

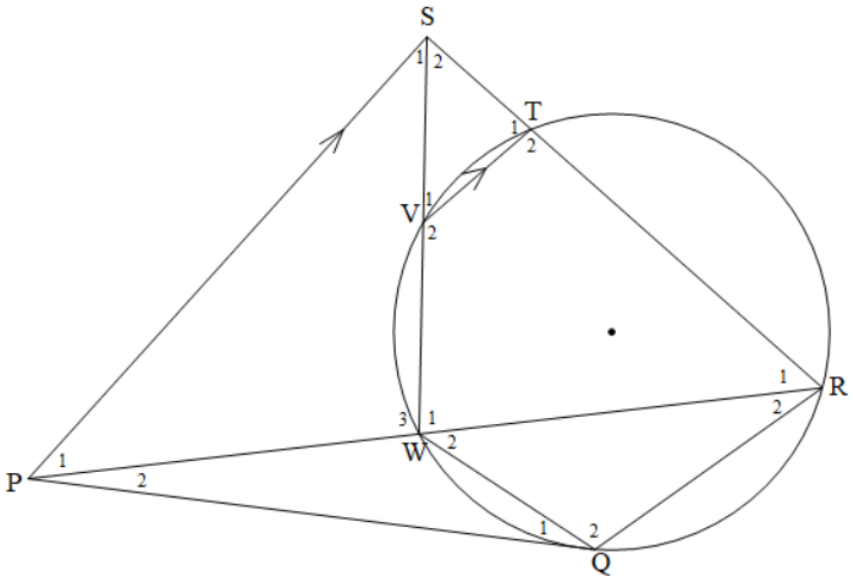
10.1 Kwidayagram engezantsi, ΔPQR noΔMNO zinikeziwe u$\hat{P} = \hat{M}$, $\hat{Q} = \hat{N}$ no$\hat{R} = \hat{O}$.



Sebenzisa idayagram ekwincwadi yakho yokuphendulela ukupruva ithiyorem ethi:

$$\frac{MN}{PQ} = \frac{MO}{PR} \qquad (6)$$

10.2 Kwidayagram engezantsi, uPQ yithanjenti yesekile kuQ. uR yipoyinti kwisekile aze uS alale ngaphandle kwesekile. uPR unqumla isekile kuW ze uRS anqumle isekile kuT. uSW unqumla isekile kuV. VT ∥ PS.



Pruva ukuba:

10.2.1 $\hat{S}_1 = \hat{R}_1$ (3)

10.2.2 $\Delta PWS \,|||\, \Delta PSR$ (3)

10.2.3 $PQ^2 = PW.PR$ (5)

10.2.4 $PQ = PS$ (3)

[20]

EWONKE: 150

IPHEPHA LEENKCUKACHA: IMATHEMATIKA

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1+ni) \qquad A = P(1-ni) \qquad A = P(1-i)^n \qquad A = P(1+i)^n$$

$$\sum_{i=1}^{n} 1 = n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad T_n = a + (n-1)d \qquad \mathrm{S}_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$T_n = ar^{n-1} \qquad S_n = \frac{a\left(r^n - 1\right)}{r-1} \;; \quad r \neq 1 \qquad S_\infty = \frac{a}{1-r} \;; \; -1 < r < 1$$

$$F = \frac{x\left[(1+i)^n - 1\right]}{i} \qquad P = \frac{x[1-(1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c \qquad y - y_1 = m(x - x_1) \qquad m = \frac{y_2 - y_1}{x_2 - x_1} \qquad m = \tan\theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$In\ \Delta ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc.\cos A \qquad area\ \Delta ABC = \frac{1}{2}ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \qquad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \qquad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$\bar{x} = \frac{\sum fx}{n} \qquad \sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \qquad P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$$

$$\hat{y} = a + bx \qquad b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$$