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Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T1010**(E)**(J23)T

NATIONAL CERTIFICATE

MATHEMATICS N3

(16030143)

23 July 2018 (X-Paper)
09:00–12:00

This question paper consists of 6 pages and a formula sheet of 2 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N3
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but do NOT separate subsections.
5. Show ALL calculations and intermediary steps.
6. ALL graph work must be done in the ANSWER BOOK. NO graph paper is needed.
7. ALL final answers must be accurately approximated to THREE decimal places, rounded off.
8. Diagrams are NOT drawn to scale.
9. Write neatly and legibly.

QUESTION 1

1.1 Simplify the following algebraic fraction:

$$\frac{x^2-4}{3x^2+4x-4} \div \frac{x^2-4x+4}{2x^2-11x+14} \times \frac{3x-2}{6x-21}$$ (5)

1.2 When $f(x)$ is divided by $2x-3$ the remainder is 4.

Determine the value of p if $f(x)=2x^3-px^2+6x-5$. (4)

1.3 Factorise the following expression completely:
$ax+9by-3ay-3bx$ (3)

1.4 Simplify the following expression:

$$\frac{x+9x^{\frac{1}{2}}}{x+7x^{\frac{1}{2}}-18} \div \left(1-\frac{2}{x^{\frac{1}{2}}}\right)^{-1}$$

(5)
[17]

QUESTION 2

Solve for *x:*

2.1 $2^0+2^{1-2x}=9.2^{-1-x}$ (6)

2.2 $\log x+\dfrac{1}{\log_x 10}=2$ (4)

2.3 $2(\log x)^2+\log x^3-2=0$ (6)
[16]

QUESTION 3

3.1 Make h the subject of the formula:

$$M=\frac{K(H^2+h^2)}{B}$$ (4)

3.2 Solve for x:

$$\frac{x-2}{x-1}-\frac{5}{x^2-4x+3}-\frac{2}{3-x}=1$$ (7)

3.3 The perimeter of a rectangular triangle is 40 m. Determine the length of the hypotenuse and the area of the rectangular triangle if the length of one of the other two sides of the triangle is 8m. (6)

[17]

QUESTION 4

Consider FIGURE 1. O is the centre of the circle and chord AB cuts the circle at the points A (0; 10) and B (*x*; y). The midpoint of AB is P (4; 8). Line AC makes an angle of θ with the *x*-axis.

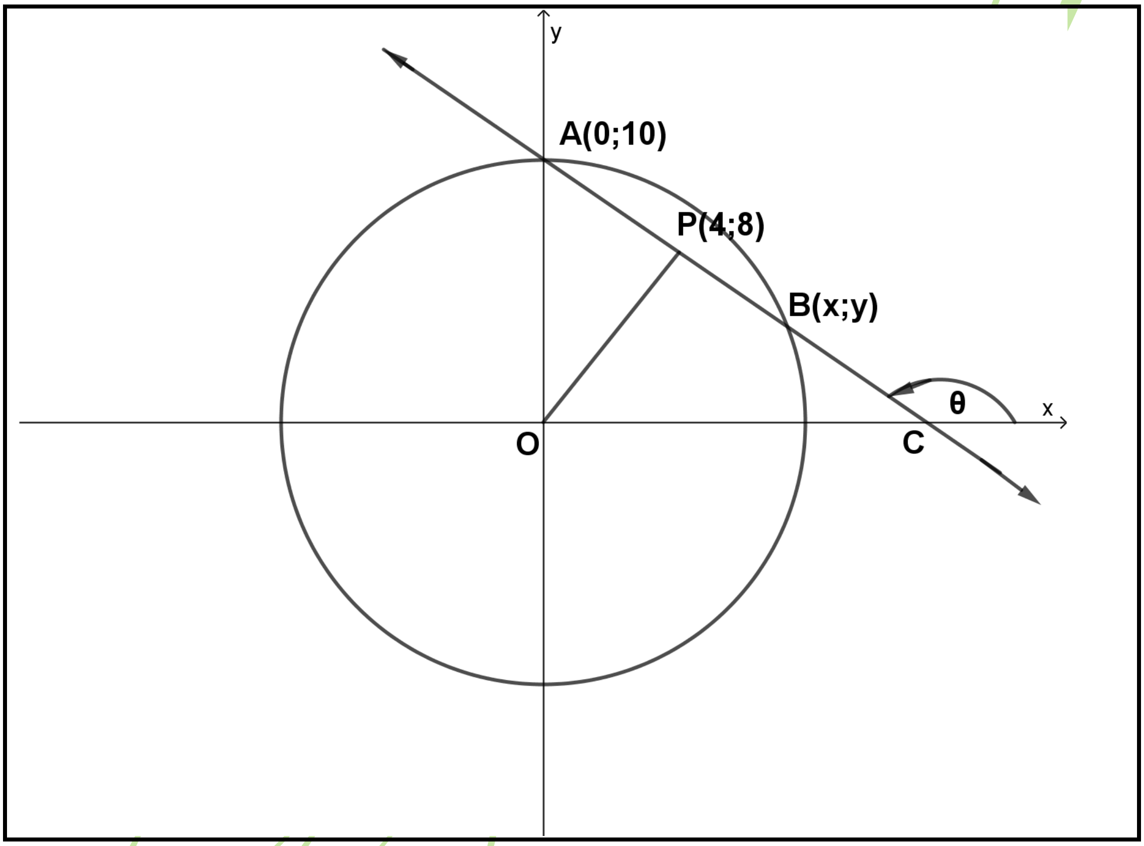


FIGURE 1

4.1 Determine the coordinates of B if P is the midpoint of AB. (3)

4.2 Determine the gradient of chord AB. (2)

4.3 Determine the equation of the straight line through A and B and leave the answer in gradient-intercept form. (2)

4.4 Show that OP is perpendicular to AB. (2)

4.5 Determine the equation of the line parallel to OP and passing through A. (3)

4.6 Calculate the size of θ, the angle of incline between AC and *x*-axis. (3)

[15]

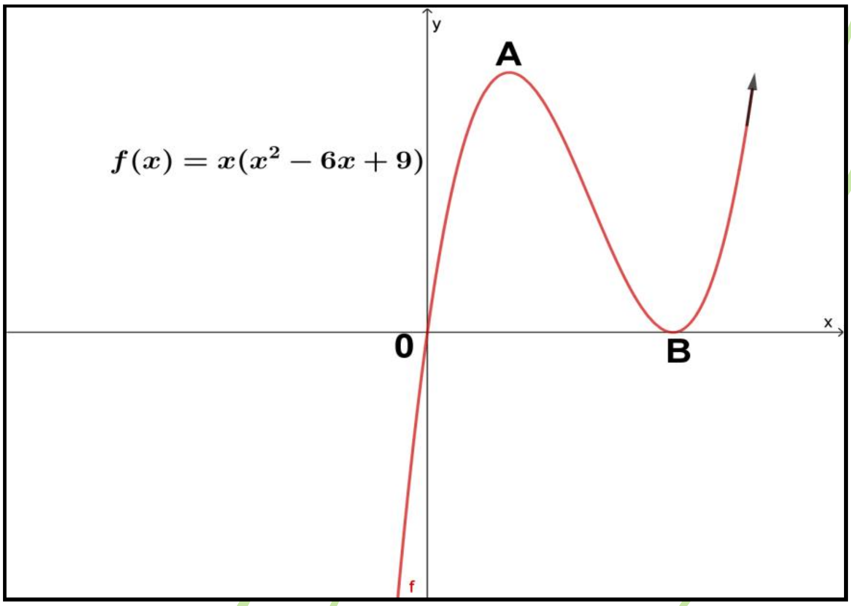
QUESTION 5

5.1 On separate system of axes, sketch the following graphs, indicating the intercepts on the axes.

5.1.1 $x = -\sqrt{16 - y^2}$ (3)

5.1.2 $x = y^2$ (3)

5.2 FIGURE 2 shows the graph of $f(x) = x(x^2 - 6x + 9)$. Find the coordinates of the turning points A and B.



(6)

FIGURE 2

5.3 Determine $\frac{dy}{dx}$ of $f(x) = -4\sqrt{x} + \frac{1}{2x^2}$ by using rules of differentiation. Leave the answer with positive exponents and in surd form. (5)

[17]

QUESTION 6

6.1 Determine the values of α that will satisfy the following equation:
$\operatorname{cosec}\alpha = \cot 20^\circ$ for $\alpha \in [90°; 360°]$ (3)

6.2 Make use of a diagram to determine the value of $\frac{1 - 2\cot^2 A}{\tan A + \operatorname{cosec} B}$ if $A + B = 90^\circ$ and $5\sin B = 3$. (5)

6.3 Proof that $\cos^2(180^\circ + x)\left[2 + \tan^2(360^\circ - x)\right] = 2 - \sin^2 x$ (5)

6.4 FIGURE 3 shows ΔABC with $AB = 50$ m, $BC = 100$ m and $\hat{B} = 45°$. Determine the length of AC.

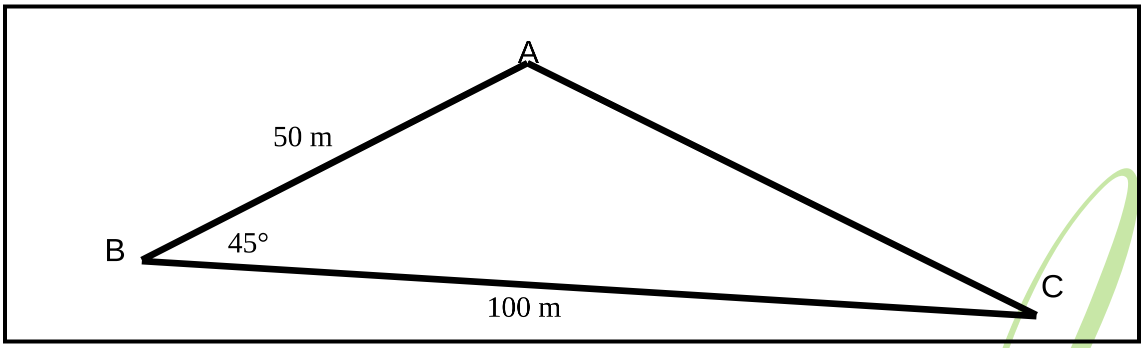


FIGURE 3 (3)

6.5 The sketch in FIGURE 4 represents the graphs of $f(x) = \cos ax$ and $g(x) = \cos bx$ where $0^\circ \leq x \leq 180^\circ$. A, B and C are the points of intersection of f and g. K and T are maximum and minimum points of g and f respectively. Determine the values of a and b.

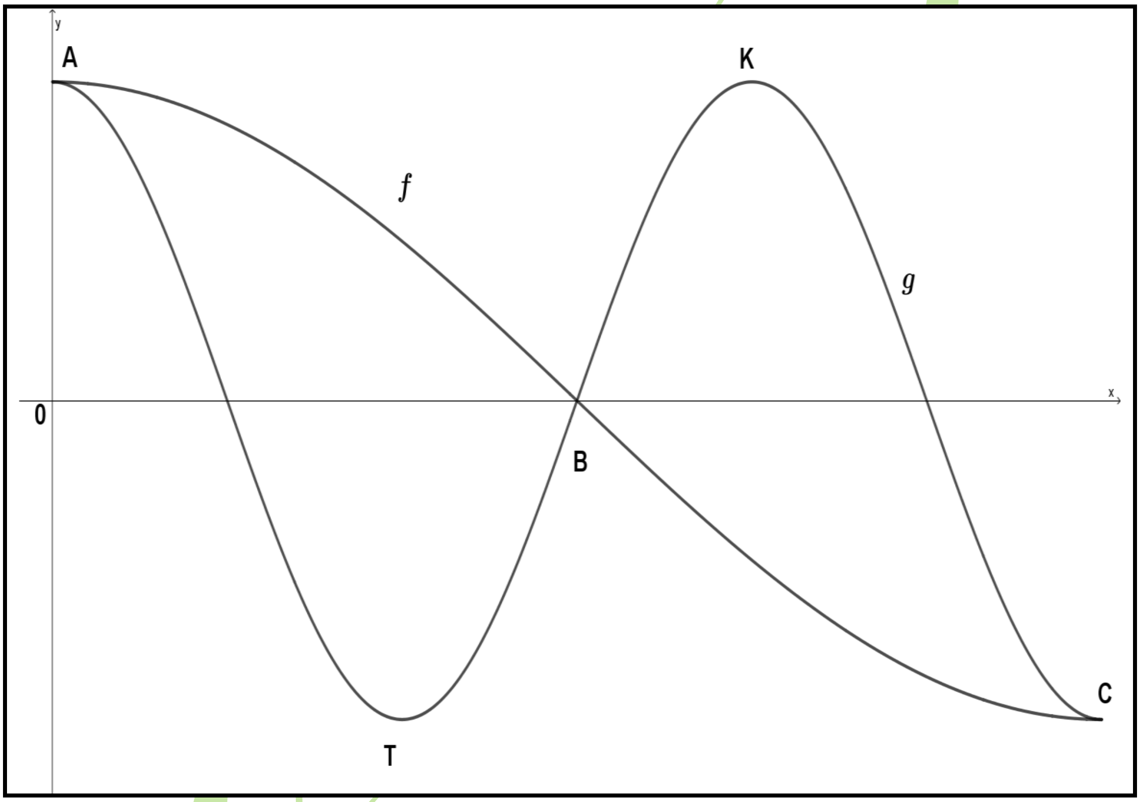


FIGURE 4 (2)

[18]

TOTAL: 100

FORMULA SHEET

Any applicable formula may also be used.

1. Factors

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

2. Logarithms

$\log ab = \log a + \log b$

$\log \frac{a}{b} = \log a - \log b$

$\log_b a = \frac{\log_c a}{\log_c b}$

$\log a^m = m \log a$

$\log_b a = \frac{1}{\log_a b}$

$\log_a a = 1 \therefore \ln e = 1$

$a^{\log_a t} = t \therefore e^{\ln m} = m$

3. Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4. Parabola

$y = ax^2 + bx + c$

$y = \frac{4ac - b^2}{4a}$

$x = \frac{-b}{2a}$

5. Circle

$x^2 + y^2 = r^2$

$D = \frac{x^2}{4h} + h$

$x = \sqrt{4Dh - 4h^2}$

6. Straight line

$y - y_1 = m(x - x_1)$

Perpendicular: $m_1 \cdot m_2 = -1$

Parallel lines: $m_1 = m_2$

Distance: $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint: $P = \left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2} \right)$

Angle of inclination: $\theta = \tan^{-1} m$

7. Differentiation

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

Max/Min

For turning points: $f'(x) = 0$

8. Trigonometry

$$\sin\theta = \frac{y}{r} = \frac{1}{cosec\theta}$$

$$\cos\theta = \frac{x}{r} = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{y}{x} = \frac{1}{\cot\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = cosec^2\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$