higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

T1010**(E)**(J31)T

NATIONAL CERTIFICATE

MATHEMATICS N3

(16030143)

31 July 2019 (X-Paper)
09:00–12:00

This question paper consists of 8 pages and a formula sheet of 3 pages.

DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N3
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer ALL the questions.
2. Read ALL the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but do NOT separate subsections.
5. Show ALL calculations and intermediate steps.
6. Do ALL graph work in the ANSWER BOOK. NO graph paper needed.
7. Accurately approximate ALL final answers to THREE decimal places.
8. Diagrams are NOT drawn to scale.
9. Write neatly and legibly.

QUESTION 1

1.1 Prove that $x - 1$ is a factor of $f(x)$ and then factorise completely if:

$f(x) = 6x^3 - 11x^2 + x + 4$ (5)

1.2 Factorise the following algebraic expression as far as possible in prime factors:

$(a+b)^2 - 2(a+b) + 1$ (2)

1.3 Simplify each of the following:

1.3.1 $$\frac{\frac{1}{\sqrt{a}} + \frac{3}{\sqrt{a}}}{\frac{7}{\sqrt{a}} - \frac{3}{\sqrt{a}}}$$ (2)

1.3.2 $$\frac{-ax - bx + by + ay}{x - y} \div \frac{2a^2 - 3ab - 5b^2}{8a^2 - 50b^2}$$ (7)

[16]

QUESTION 2

2.1 If $\log 2 = a$ write $\log 50$ in terms of a. (2)

2.2 Solve for x in each of the following:

2.2.1 $$\frac{2^{2x-1} - 2^{2x}}{4^x} = \sqrt{2^{-3x}} - 1$$ (6)

2.2.2 $$\log_2 x - 3\log_x 2 = 2$$ (7)

[15]

QUESTION 3

3.1 If $(x-5)(y+3)=0$, determine x if:

3.1.1 $y=-2$ (1)

3.1.2 $y=-3$ (2)

3.2 Make V the subject of the formula:

$$\frac{P}{Q}(R^2-V^2)=D$$ (3)

3.3 In a class test one student obtained 6 marks less than the class average. When his marks are doubled it becomes 40 marks more than the class average.

Determine the class average and the student's marks. (4)

3.4 Solve for a by completing the square:

$$-2ab-1=-a^2$$ (4)

[14]

QUESTION 4

Consider FIGURE 1 where $A(-5;-2)$, $B(-1;-6)$ and $C(5;4)$ are the vertices of ΔABC.

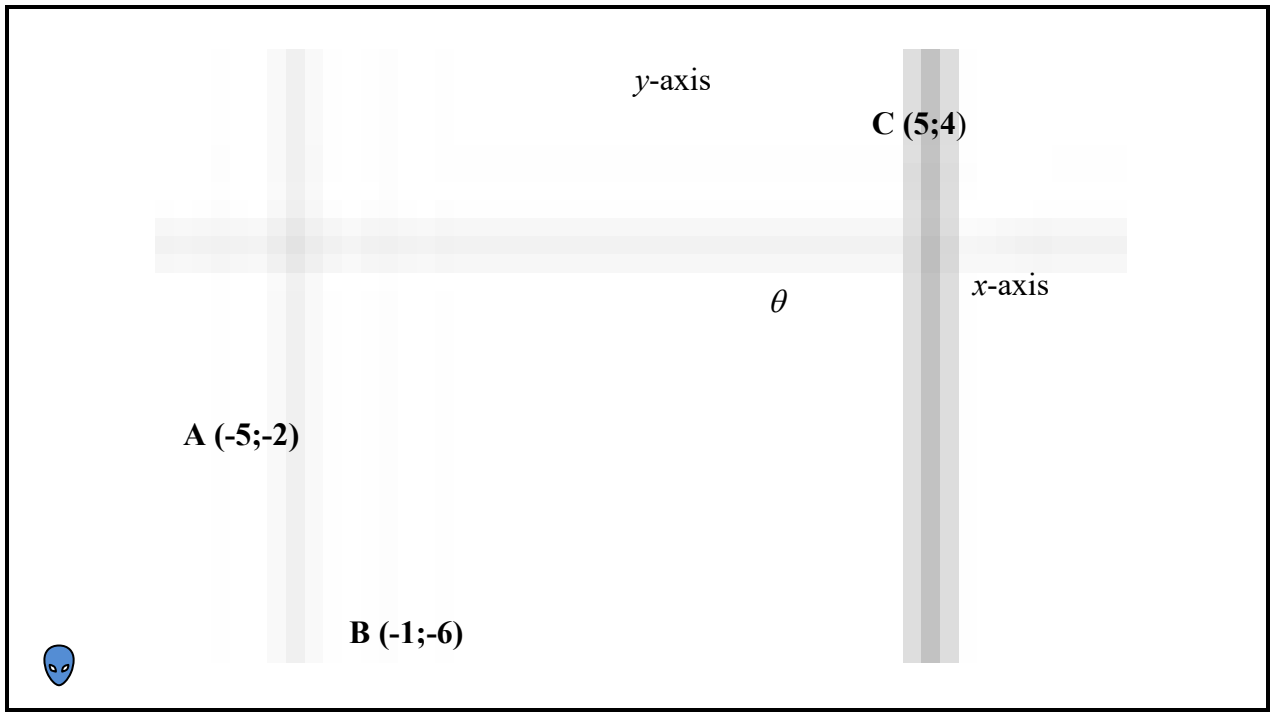

FIGURE 1

Determine:

4.1 The gradient of line AB (2)

4.2 The length of line AB in the simplest surd form (2)

4.3 The coordinates of the midpoint of line segment BC (2)

4.4 The equation of the line segment in the form $y = mx + c$ parallel to BC and passing through the point A (5)

4.5 The angle of inclination θ which is between BC and the x-axis (2)

4.6 The equation of the line segment in the form $y = mx + c$ perpendicular to BC and passing through the midpoint of BC (3)
[16]

QUESTION 5

5.1 Given: function f defined by $y = x - 1$ and function g defined by $x = 2y^2$.

5.1.1 Determine algebraically the coordinates of the points of intersection of functions f and g. (4)

5.1.2 Draw the graphs of the given functions to show the solution obtained in QUESTION 5.1.1 as well as the applicable turning point and intercepts with the axes. (7)

5.2 Determine $\frac{dy}{dx}$ of the following function by using the rules of differentiation. Leave the answers with positive indices and in surd form where applicable.

$$y = 3\sqrt[3]{x^2} + \frac{5}{\sqrt{x}}$$

(5)

5.3 The diagram in FIGURE 2 shows the graph of $f(x) = -x^3 + 6x^2 - 9x$.
T has x - coordinator 2. Find the gradient of the tangent to the curve at T.

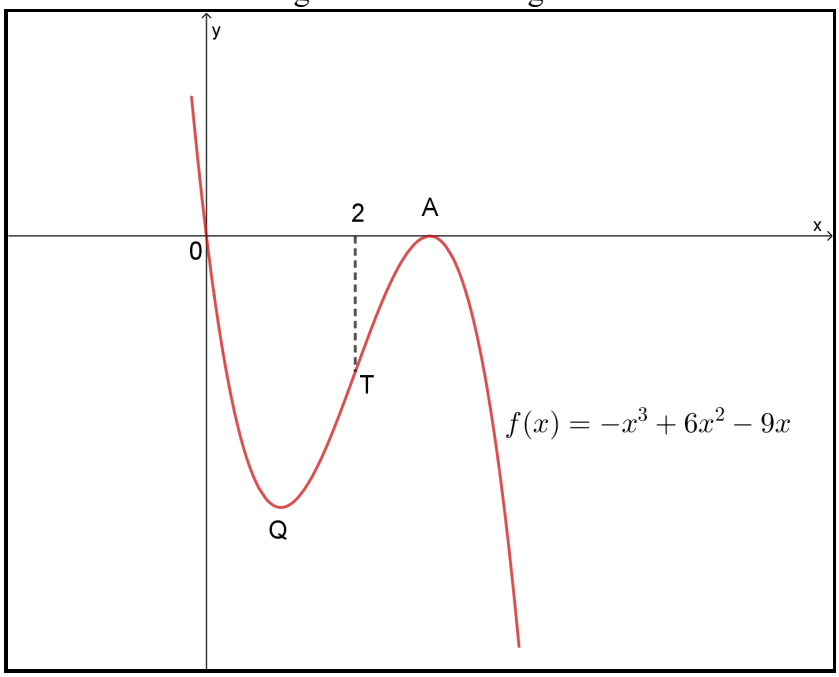

FIGURE 2

(3)
[19]

QUESTION 6

6.1 Calculate the value(s) of x that will satisfy the equation for $0^\circ \le x \le 360^\circ$ if $\tan(x - 30^\circ) = \frac{1}{\sqrt{3}}$.

(3)

6.2 Prove the following trigonometry identity:

$$\cos(360^\circ - A) = 1 - \frac{\sin^2(180^\circ + A)}{1 - \cos(180^\circ + A)}$$

(6)

6.3 In FIGURE 3, a land surveyor wants to determine the height of a tower AB when he finds that from a certain position C, which is in the same horizontal plane as B, an angle of elevation to the top of a tower is 56°. From another position D, 20 m away from C, which is between B and C, an angle of elevation to the top of the tower is 74°.

Determine AB which is the height of the tower.

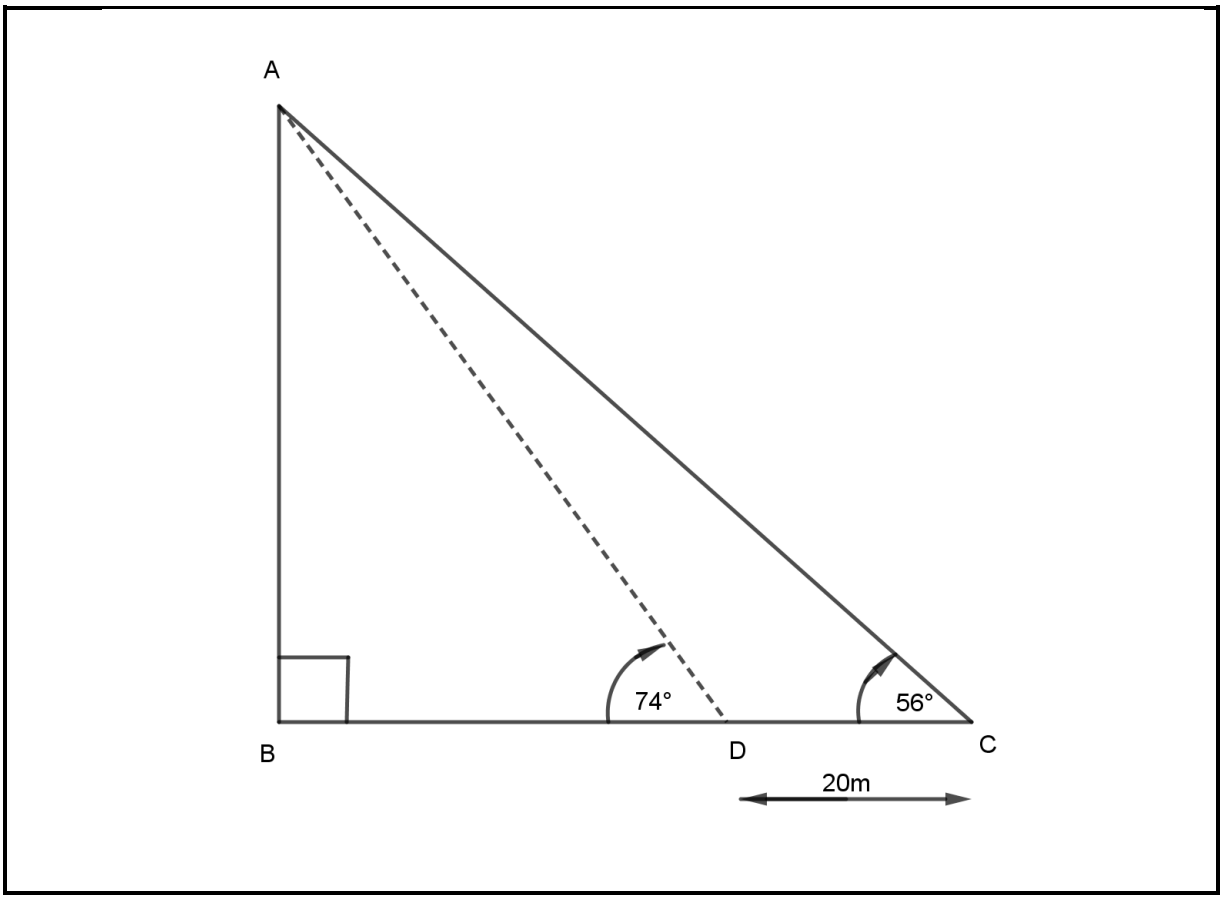

FIGURE 3

(7)

6.4 FIGURE 4 shows the graphs of $f(x) = a\cos x$ and $g(x) = \sin bx + c$ for $0 \le x \le 180^{\circ}$.

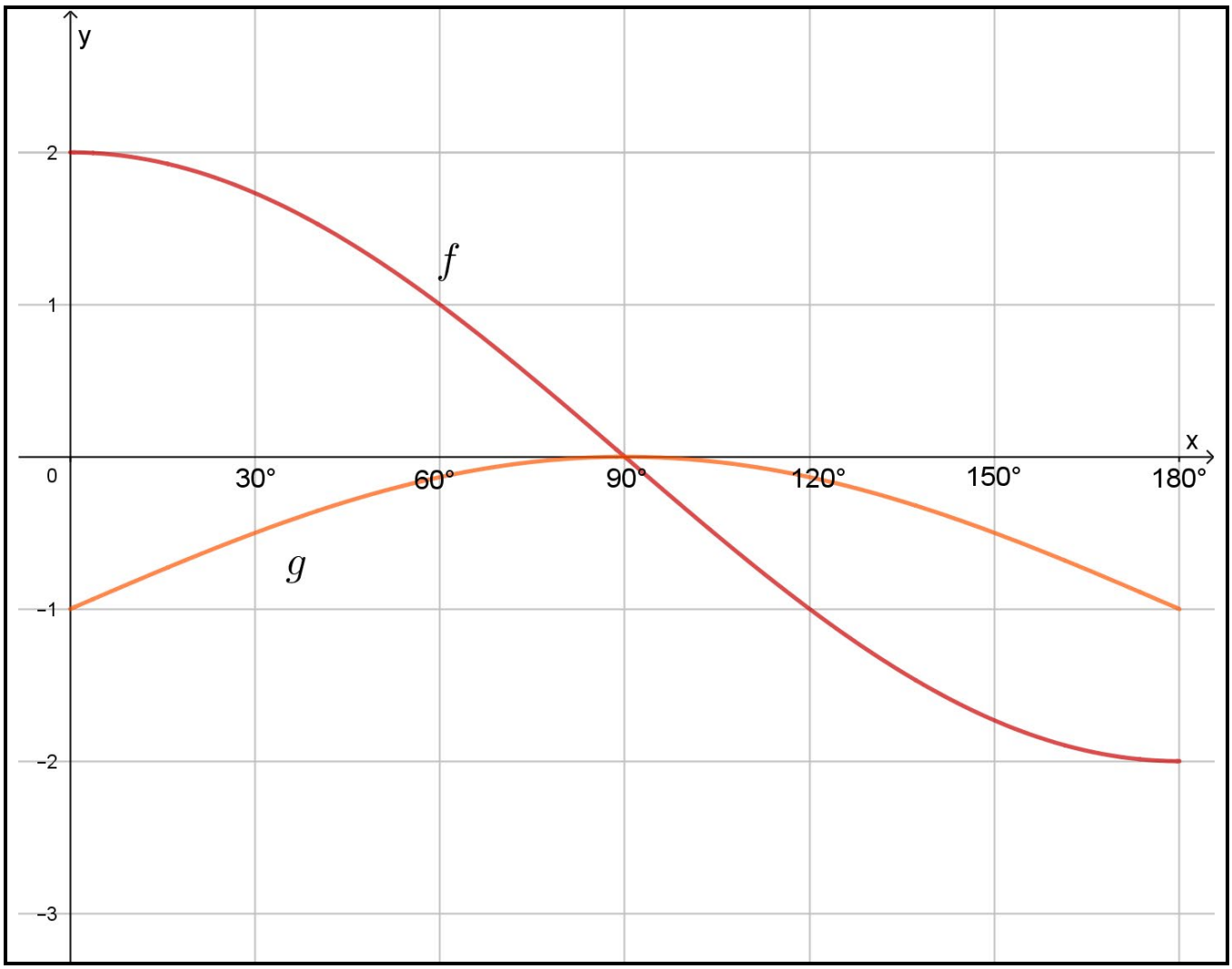

FIGURE 4

6.4.1 Determine the values of a, b and c. (3)

6.4.2 Read from the graphs the values of x where $a\cos x - \sin bx = c$. (1)

[20]

TOTAL: 100

FORMULA SHEET

Any applicable formula may also be used.

1. Factors

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

2. Logarithms

$\log ab = \log a + \log b$

$\log \frac{a}{b} = \log a - \log b$

$\log_b a = \frac{\log_c a}{\log_c b}$

$\log a^m = m \log a$

$\log_b a = \frac{1}{\log_a b}$

$\log_a a = 1 \quad \ln e = 1$

$a^{\log_a t} = t \quad e^{\ln m} = m$

3. Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4. Parabola

$y = ax^2 + bx + c$

$y = \frac{4ac - b^2}{4a}$

$x = \frac{-b}{2a}$

5. Circle

$x^2 + y^2 = r^2$

$D = \frac{x^2}{4h} + h$

$x = \sqrt{4Dh - 4h^2}$

6. Straight line

$y - y_1 = m(x - x_1)$

Perpendicular: $m_1 \cdot m_2 = -1$

Parallel lines: $m_1 = m_2$

Distance: $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint: $P = \left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2} \right)$

Angle of inclination: $\theta = tan^{-1} m$

7. Differentiation

$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$

Max/Min

For turning points: $f'(x) = 0$

8. Trigonometry

$$\sin\theta = \frac{y}{r} = \frac{1}{\operatorname{cosec}\theta}$$

$$\cos\theta = \frac{x}{r} = \frac{1}{\sec\theta}$$

$$\tan\theta = \frac{y}{x} = \frac{1}{\cot\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$