higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE

MATHEMATICS N3

(16030143)

18 November 2022 (X-paper)
09:00–12:00

Programmable calculators may not be used.

This question paper consists of 7 pages and a formula sheet of 2 pages.

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DEPARTMENT OF HIGHER EDUCATION AND TRAINING
REPUBLIC OF SOUTH AFRICA

NATIONAL CERTIFICATE
MATHEMATICS N3
TIME: 3 HOURS
MARKS: 100

INSTRUCTIONS AND INFORMATION

1. Answer all the questions.
2. Read all the questions carefully.
3. Number the answers according to the numbering system used in this question paper.
4. Questions may be answered in any order, but keep sub-sections together.
5. Show all calculations and intermediate steps.
6. All graph work must be done in the ANSWER BOOK.
7. All final answers must be accurately rounded off to THREE decimal places.
8. Diagrams are NOT drawn to scale.
9. Use only a black or a blue pen.
10. Write neatly and legibly.

QUESTION 1

1.1 Factorise the following polynomial fully and simplify the answer completely:

1.1.1. $-x^3-4y^2+x^2y+4xy$ (3)

1.1.2. $b^2-4bd+4d^2-(2b+d)^2$ (4)

1.2 Solve for x by completing the square:

$x^2-4x+2=0$ (4)

1.3 Use the factor and remainder theorem to factorise $h(x)=3x^3-9x^2+12$ completely. (6)

1.4 Simplify the following algebraic expression:

$$\frac{x+3}{x^2-7x+12}-\frac{x-3}{x^2-9}$$ (5)

[22]

QUESTION 2

2.1. Solve for x in the following equation:

$$\frac{e^x}{e^{-x^2+6}}-e^{2x}=0$$ (5)

2.2 If $6^{2x}=4^{x+2}$, prove that $x=\log_9 16$ (4)

2.3. Simplify the following expression:

$$\frac{\sqrt{(a-b)^3}\times\sqrt{a-b}\times\sqrt{a+b}}{\sqrt[3]{(a-b)^5}\times(a^2-b^2)^{\frac{1}{3}}}$$ (5)

[14]

QUESTION 3

3.1. Three friends, Vusi, Luvuyo, and Sbusiso, went to gamble at a casino. Vusi won half of what Sbusiso won, and Sbusiso won R80 more than what Luvuyo won. If the three friends won R710 in total, how much did each of the friends win, respectively? (5)

3.2. Given the following compound interest formula, make i the subject of the formula:

$$P = \frac{A}{\left(1+\frac{i}{m}\right)^{n\times m}}$$

(3)

3.3. Solve for x and y simultaneously in the following two equations:

$$x + y = 3$$
$$2x^2 + 4xy - y = 15$$

(5)
[13]

QUESTION 4

Consider FIGURE A below which shows $P(2;3)$, $Q(-1;0)$ and $R(x;y)$ as the vertices of ΔPQR. $M\left(2;-\frac{3}{2}\right)$ is the midpoint of line QR and $N(a;b)$, the midpoint of PR.

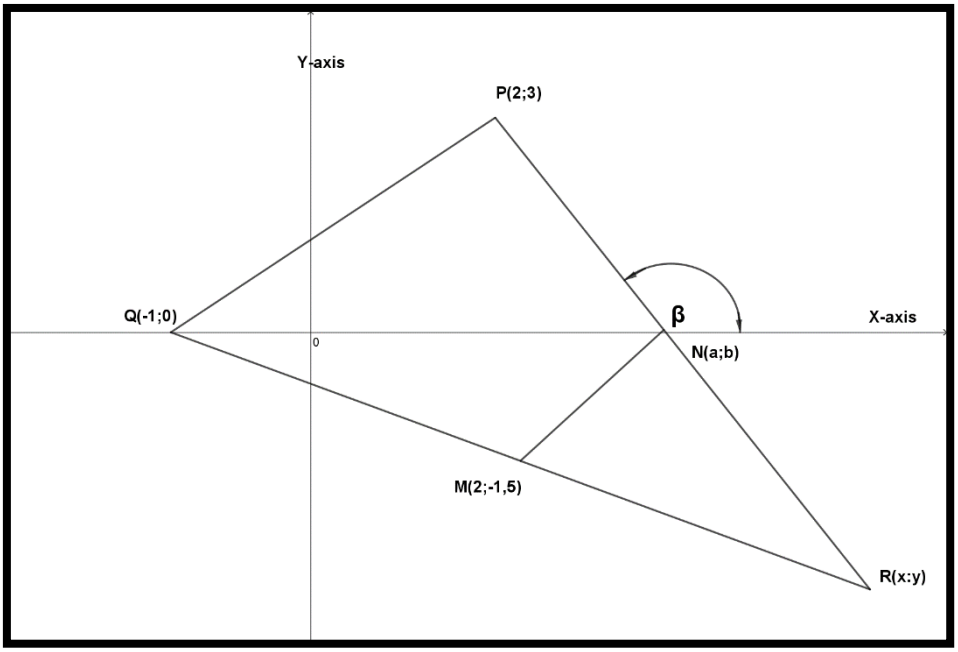

FIGURE A

Determine the following:

4.1 The co-ordinates of R (3)
4.2 The co-ordinates of N, the midpoint of PR (2)
4.3 The distance between P and Q in the simplest surd form (2)
4.4 The equation of line MN in the form of $y = mx + c$ (4)

4.5 The size of the obtuse angle β, the angle of inclination of line PR (4)

4.6 The equation of a line passing through point P in the form of $y = mx + c$, which is perpendicular to PR (3)

[18]

QUESTION 5

5.1 Draw the graph of $100x^2 + 16y^2 = 400$, showing the intercepts on the axes. (3)

5.2 FIGURE B below shows the function $y = x^2 - 3x + 2$. Determine the co-ordinates of the turning point at D.

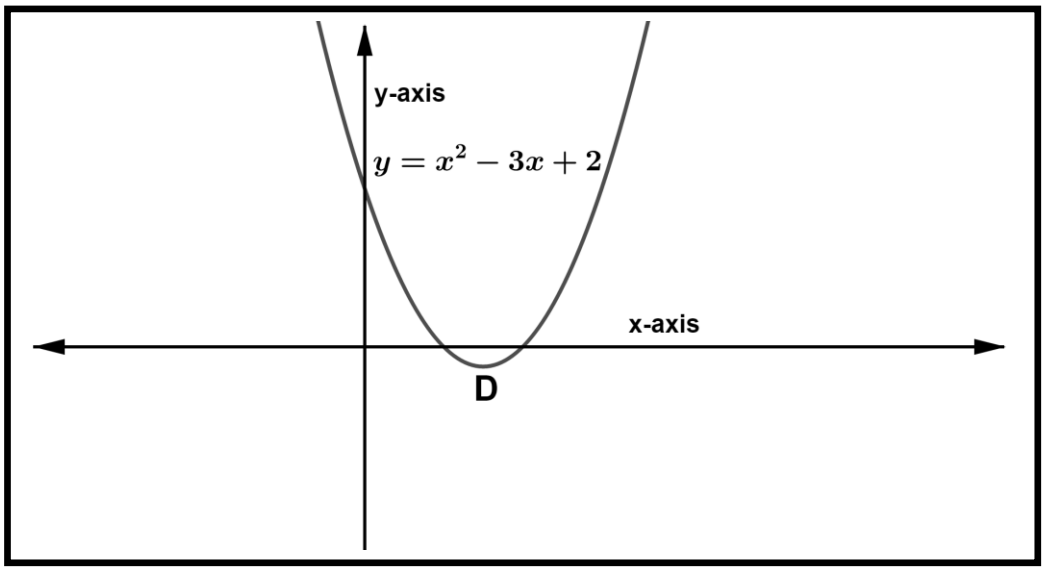

FIGURE B (2)

5.3 Determine $\frac{dy}{dx}$ of the function of $y = \frac{\sqrt{x}}{2} + \frac{1}{3x^3} - \pi x$ by using the rules of differentiation. The final answer should be with positive exponents and in surd form where applicable. (5)

5.4 FIGURE C below represents a sketch of the function defined by $f(x) = -x^3 - 2x^2$. Determine the co-ordinates of the turning point at A.

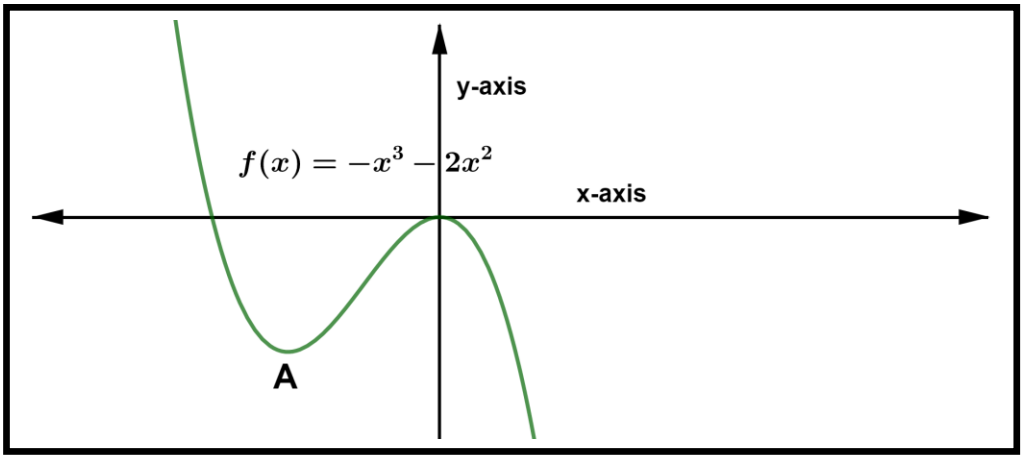

FIGURE C

(4)

[14]

QUESTION 6

6.1 Use trigonometric identities to prove that

$\text{cosec}\,B \times \sec B - \cot B = \tan B$ (4)

6.2 Calculate the value(s) of x that will satisfy the equation for $0° \leq x \leq 360°$

$4\cos^2 x + 4\sin^2 x = 5 - \tan x$ (4)

6.3 FIGURE D below represents a quadrilateral ABCD. $A\hat{B}C = 70^o$ and $A\hat{C}B = 60^o$ while AB = 4 cm, AD = 5 cm and DC = 6 cm. Determine the following:

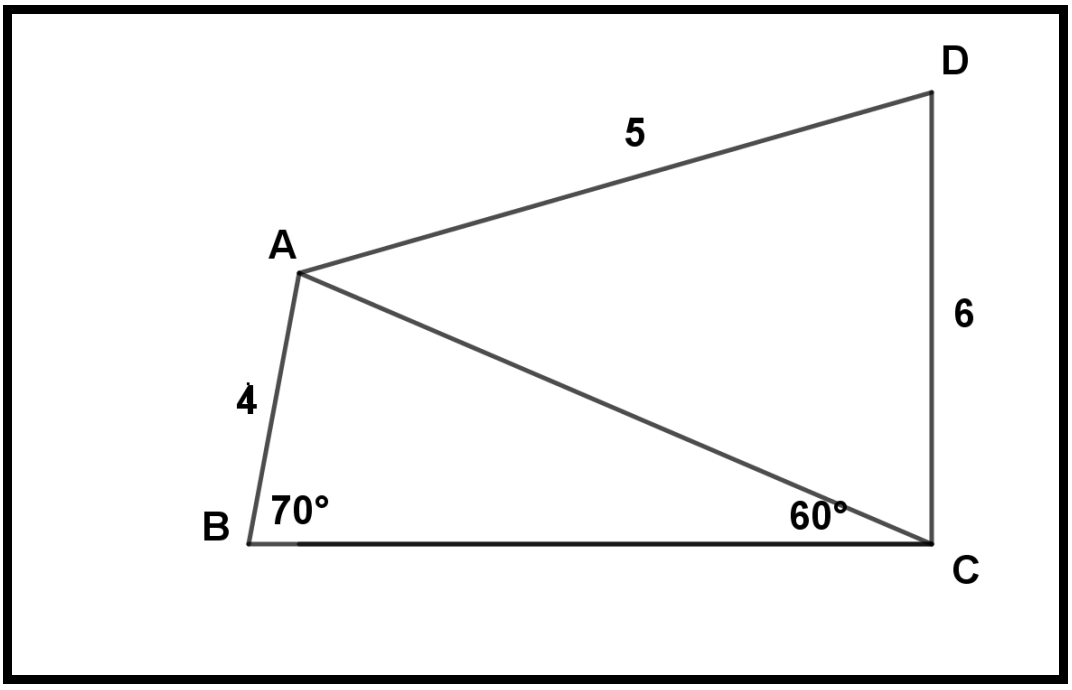

FIGURE D

Determine the following:

6.3.1 The length of AC (2)

6.3.2 $A\hat{D}C$ (3)

6.4 Draw the graphs represented by the following trigonometric equations on the same set of axes for $0° \leq x \leq 180°$.

$$f(x) = \frac{1}{2}\sin x \text{ and } g(x) = -2\cos x$$

Show the coordinates of the turning points and intercepts with the axes.

(6)

[19]

TOTAL: 100

MATHEMATICS N3

FORMULA SHEET

Any applicable formula may also be used.

1. **Factors**

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

2. **Logarithms**

$\log ab = \log a + \log b$

$\log \frac{a}{b} = \log a - \log b$

$\log_b a = \frac{\log_c a}{\log_c b}$

$\log a^m = m \log a$

$\log_b a = \frac{1}{\log_a b}$

$\log_a a = 1 \therefore 1n\ e = 1$
$a^{\log_a t} = t \therefore e^{1n\ m} = m$

3. **Quadratic formula**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4. **Parabola**

$y = ax^2 + bx + c$

$y = \frac{4ac - b^2}{4a}$

$x = \frac{-b}{2a}$

5. **Circle**

$x^2 + y^2 = r^2$

$D = \frac{x^2}{4h} + h$

$x = \sqrt{4Dh - 4h^2}$

6. **Straight line**

$y - y_1 = m(x - x_1)$
Perpendicular: $m_1 \cdot m_2 = -1$
Parallel lines: $m_1 = m_2$
Distance: $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Midpoint: $P = \left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2} \right)$
Angle of inclination: $\theta = tan^{-1} m$
Gradient: $m = \frac{y_2 - y_1}{x_2 - x_1}$

7. **Differentiation**

$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\frac{d}{dx}(x^n) = nx^{n-1}$

Max/Min

For turning points: $f'(x) = 0$

8. **Trigonometry**

$$\sin\theta = \frac{y}{r} = \frac{1}{\operatorname{cosec}\theta} \qquad \cos\theta = \frac{x}{r} = \frac{1}{\sec\theta} \qquad \tan\theta = \frac{y}{x} = \frac{1}{\cot\theta}$$

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \qquad a^2 = b^2 + c^2 - 2bc\cos A$$